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A characterization of absolutely summing operators by means of McShane integrable functions

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Abstract

Absolutely summing operators between Banach spaces are characterized by means of McShane integrable functions.

Keywords: Pettis integral; McShane integral; Absolutely summing operator

1. Introduction

In [15] the McShane integral of functions taking values in a locally convex space is studied. In particular (see [15, Theorem 4]) Fréchet spaces for which Henstock lemma holds true for every McShane integrable function have been characterized. The proof of the lemma is based on the property that a nuclear operator is absolutely summing.

In [4] Diestel characterized absolutely summing operators between Banach spaces by means of Pettis integrable strongly measurable functions.

It is known that the class of Pettis integrable strongly measurable functions is strictly included in the class of McShane integrable functions. Diestel at the Symposium on Real Analysis and Measure Theory in Ischia (2002) observed that with the technique of [15, Theorem 4], it could be possible to characterize absolutely summing operators by means of McShane integrable functions. In this paper we continue the investigation of such operators and we prove the characterization conjectured by Diestel (Theorem 5). Applying a recent

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result of [7] we also characterize absolutely summing operators in super-reflexive Banach spaces (Corollary 7).

2. Definitions and notations

Let $(\Omega, \mathcal{T}, \mathcal{F}, \mu)$ be a nonempty compact finite Radon measure space (see [10]), i.e.:

(i) $(\Omega, \mathcal{F}, \mu)$ is complete;
(ii) $\mathcal{T}$ is a Hausdorff topology such that $\mathcal{T} \subset \mathcal{F}$ and Ω is compact;
(iii) $\mu(E) = \sup\{\mu(F)\colon F \subseteq E,\ F \text{ is compact}\}$ for every $E \in \mathcal{F}$.

Unless specified otherwise, the terms "measure," "measurable" and "almost everywhere" (briefly "a.e.") are referred to the measure μ. For a set $E \in \mathcal{F}$, we denote by χ_E the characteristic function of E. A *generalized McShane partition* (or simply a *Mc-partition*) in Ω is a countable (eventually finite) set of pairs $P = \{(E_i, \omega_i)\colon i = 1, 2, \ldots\}$, where $(E_i)_i$ is a disjoint family of measurable sets of finite measure and $\omega_i \in \Omega$ for each $i = 1, 2, \ldots.$ If $\mu(\Omega \setminus \bigcup_i E_i) = 0$, we say that P is a *Mc-partition of* Ω.

A *gauge* on Ω is a function $\Delta : \Omega \to \mathcal{T}$ such that $\omega \in \Delta(\omega)$ for each $\omega \in \Omega$. We say that a *Mc*-partition $\{(E_i, \omega_i)\colon i = 1, 2, \ldots\}$ is *subordinate to a gauge* Δ if $E_i \subset \Delta(\omega_i)$ for $i = 1, 2, \ldots.$

From now on X and Y are real Banach spaces with dual, respectively, X^* and Y^*.

A function $f : \Omega \to X$ is said to be *Pettis integrable* on Ω, if $x^* f$ is Lebesgue integrable on Ω for each $x^* \in X^*$ and for every measurable set $E \subset \Omega$ there is a vector $\nu_f(E) \in X$ such that $x^*(\nu_f(E)) = \int_E x^* f\, d\mu$ for all $x^* \in X^*$. In this case $\nu_f(\Omega)$ is the Pettis integral of f and the map $E \to \nu_f(E)$ is the indefinite Pettis integral of f. The symbol $\mathcal{P}(X)$ will denote the set of Pettis integrable functions $f : \Omega \to X$, with the usual seminorm $\|f\|_{\mathcal{P}(X)} = \sup\{\int_\Omega |x^* f|\, d\mu\colon x^* \in B_{X^*}\}$, where B_{X^*} is the unit ball of X^*. The subset of $\mathcal{P}(X)$ consisting of maps which are strongly measurable is denoted by $\mathcal{P}_m(X)$.

A function $f : \Omega \to X$ is said to be *Birkhoff integrable* on Ω, with Birkhoff integral ω, if for every $\varepsilon > 0$ there is a partition $(E_i)_{i \in \mathbb{N}}$ of Ω into measurable sets of finite measure such that $\sum_{i \in \mathbb{N}} \mu(E_i) f(\omega_i)$ is unconditionally summable and

$$\left\| \sum_{i \in \mathbb{N}} \mu(E_i) f(\omega_i) - w \right\| < \varepsilon$$

whenever $\omega_i \in E_i$ for $i = 1, 2, \ldots.$

A function $f : \Omega \to X$ is said to be *McShane integrable* on Ω (see [11, Proposition 1E]), if there exists a vector $w \in X$ satisfying the following property: given $\varepsilon > 0$ there exists a gauge Δ on Ω such that

$$\left\| \sum_{i=1}^{p} \mu(E_i) f(\omega_i) - w \right\| < \varepsilon$$

for each finite McShane partition $\mathcal{P} = \{(E_i, \omega_i)\colon i = 1, \ldots, p\}$ of Ω subordinate to Δ. We denote by $\mathcal{M}(X)$ the family of McShane integrable functions on Ω endowed with the seminorm $\|f\|_{\mathcal{M}(X)} = \sup\{\int_\Omega |x^* f|\, dt\colon x^* \in B_{X^*}\}$ and we set $w = (Mc)\int_\Omega f$.

It is known that the family of McShane integrable function is a subset of that of Pettis integrable ones ([13, Theorem 2C] and [11, Theorem 1Q]).

A function $f:\Omega \to X$ is said to be *McShane variationally integrable* (briefly *MV-integrable*) on Ω (see [6, Definition 1]), if it is Pettis integrable and for each $\varepsilon > 0$ there exists a gauge Δ such that if $\mathcal{P} = \{(E_i, \omega_i):\ i = 1, \dots, p\}$ is a partition of Ω subordinate to Δ, we have

$$\sum_{i=1}^{p} \bigl\| \mu(E_i) f(\omega_i) - \nu_f(E_i) \bigr\| < \varepsilon.$$

We denote by $MV(X)$ the family of all MV-integrable functions on Ω. It follows by the definition that $MV(X) \subseteq \mathcal{M}(X)$. It was proved that if Ω is a compact interval of the real line (see [3,17]) or, in general, a compact finite Radon measure space (see [6]) the class of MV-integrable functions coincides with the class of Bochner integrable ones.

We denote by $\mathcal{B}(X)$ the set of Bochner integrable functions $f:\Omega \to X$, endowed with the norm $\|f\|_{\mathcal{B}(X)} = \int_\Omega \|f(t)\|_X \, d\mu$. It is known that $(\mathcal{B}(X), \|\cdot\|_{\mathcal{B}(X)})$ is a Banach space.

Let $\mathcal{L}(X, Y)$ be the space of continuous linear operators from X to Y. An operator $u \in \mathcal{L}(X, Y)$ is said to be *absolutely summing* if there is a constant $c \geqslant 0$ so that, for every choice of an integer n and vectors $\{x_i\}_{i=1}^{n}$ in X, we have

$$\sum_{i=1}^{n} \bigl\| u(x_i) \bigr\|_Y \leqslant c \sup_{x^* \in B_{X^*}} \sum_{i=1}^{n} \bigl| x^*(x_i) \bigr|. \tag{1}$$

The least c for which inequality (1) always holds is denoted by $\pi(u)$. Let $u \in \mathcal{L}(X, Y)$. Then define U from $\mathcal{M}(X)$ to $\mathcal{M}(Y)$ (or from $\mathcal{B}(X)$ to $\mathcal{B}(Y)$) by

$$(Uf)(\omega) = u\bigl(f(\omega)\bigr).$$

Then as it is seen in [11] u “lifts” to an operator $U \in \mathcal{L}(\mathcal{M}(X), \mathcal{M}(Y))$ (or, as it is seen in [8], to an operator $U \in \mathcal{L}(\mathcal{B}(X), \mathcal{B}(Y))$.

3. Main result

In the sequel we shall use of the following theorems. For the definition of perfect measure space see, for example, [18, Definition 1-3-1].

Theorem 1 [16, Theorem 9.1]. *If f is Pettis integrable and $\{\nu_f(E):\ E \in \mathcal{F}\}$ is relatively norm compact, then f is a limit of a sequence of simple functions in the norm topology of $\mathcal{P}(X)$.*

Theorem 2 [16, Theorem 9.2]. *If the measure space $(\Omega, \mathcal{F}, \mu)$ is perfect, then for each Pettis integrable function $f:\Omega \to X$, the set $\{\nu_f(E):\ E \in \mathcal{F}\}$ is relatively norm compact.*

We recall that two functions $f, g : \Omega \to X$ are *weakly-equivalent* if for all $x^* \in X^*$, $x^* f = x^* g$ a.e.

Proposition 3. *Let $u \in \mathcal{L}(X, Y)$ be an absolutely summing operator. For every $f \in \mathcal{P}(X)$, $Uf : \Omega \to Y$ is weakly equivalent to a function of $\mathcal{B}(Y)$.*

Proof. Let $s = \sum_{i=1}^{n} x_i \chi_{A_i}$ be a simple function; then

$$\|Us\|_{\mathcal{B}(Y)} = \int_\Omega \big\|(Us)(\omega)\big\|_Y \, d\mu = \int_\Omega \big\|u\big(s(\omega)\big)\big\|_Y \, d\mu$$
$$= \int_\Omega \left\| u\left(\sum_{i=1}^{n} x_i \chi_{A_i}(\omega)\right)\right\|_Y d\mu = \int_\Omega \left\|\sum_{i=1}^{n} u(x_i)\chi_{A_i}(\omega)\right\|_Y d\mu. \tag{2}$$

Applying the disjointness of $A_i's$ and the linearity of the integral we get

$$\int_\Omega \left\|\sum_{i=1}^{n} u(x_i)\chi_{A_i}(\omega)\right\|_Y d\mu = \sum_{i=1}^{n} \int_{A_i} \big\|u(x_i)\big\|_Y \, d\mu$$
$$= \sum_{i=1}^{n} \big\|u(x_i)\big\|_Y \mu(A_i) = \sum_{i=1}^{n} \big\|u\big(\mu(A_i)x_i\big)\big\|_Y$$
$$\leqslant \pi(u) \sup\left\{\sum_{i=1}^{n} \big|x^*\big(\mu(A_i)x_i\big)\big| : \; x^* \in B_{X^*}\right\} = \pi(u)\|s\|_{\mathcal{P}(X)}, \tag{3}$$

where the last inequality follows from the definition of the $\mathcal{P}(X)$-norm. Thus by (2) and (3), we get

$$\|Us\|_{\mathcal{B}(Y)} \leqslant \pi(u)\|s\|_{\mathcal{P}(X)} \tag{4}$$

for every simple function. Let $f \in \mathcal{P}(X)$. Since each finite Radon measure space is perfect (see [18, Proposition 1-3-2]), by Theorems 2 and 1 the simple functions are dense in $\mathcal{P}(X)$ with the $\mathcal{P}(X)$-norm.

Thus let (t_n) be a sequence of simple functions converging to f in the $\mathcal{P}(X)$-norm. Then it is Cauchy in the $\mathcal{P}(X)$-norm. Moreover there is a subsequence (s_n) of (t_n) such that for each $x^* \in X^*$ (see [16, p. 235]),

$$\lim_{n\to\infty} x^*\big(s_n(\omega)\big) = x^*\big(f(\omega)\big)$$

for all $\omega \notin N_{x^*}$, $\mu(N_{x^*}) = 0$. By (4) and by the linearity of U we get

$$\|Us_n - Us_m\|_{\mathcal{B}(Y)} \leqslant \pi(u)\|s_n - s_m\|_{\mathcal{P}(X)}.$$

Therefore the sequence (Us_n) is Cauchy in $\mathcal{B}(Y)$. Since $\mathcal{B}(Y)$ is complete there is a function $h \in \mathcal{B}(Y)$ such that (Us_n) converges to h in $\mathcal{B}(Y)$. Without loss of generality we can assume that the convergence is also a.e. (see, for example, [9, p. 11]). Thus there exists a set N, $\mu(N) = 0$, such that for each $y^* \in Y^*$ and $\omega \notin N$,

$$\lim_{n\to\infty} y^*\big((Us_n)(\omega)\big) = y^*\big(h(\omega)\big). \tag{5}$$

Let $u^* : Y^* \to X^*$ be the adjoint operator of u. Since it is $weak^*$-continuous we obtain

$$\lim_{n\to\infty}\big\langle (Us_n)(\omega), y^*\big\rangle = \lim_{n\to\infty}\big\langle u\big(s_n(\omega)\big), y^*\big\rangle = \lim_{n\to\infty}\big\langle s_n(\omega), u^*y^*\big\rangle = \big\langle f(\omega), u^*y^*\big\rangle$$
$$= \big\langle u\big(f(\omega)\big), y^*\big\rangle = \big\langle (Uf)(\omega), y^*\big\rangle, \tag{6}$$

a.e. on Ω. From (5) and (6) we get that $y^*(h(\omega)) = y^*(Uf(\omega))$ a.e. on Ω. Therefore the function Uf is weakly equivalent to the Bochner integrable function h. □

The previous proposition was proved in [1, p. 230] under the hypothesis that the function f is bounded.

Our main result is next theorem which extends a characterization of absolutely summing operators due to Diestel [4]. For its proof we need the following lemma.

Lemma 4 [5, Proposition 1]. *Let $f:\Omega\to X$ be a McShane integrable function. Then for each $\varepsilon>0$ there exists a gauge Δ satisfying the condition: if $E_1,\dots,E_p$ are measurable disjoint subsets of Ω, $t_1,\dots,t_p\in\Omega$ and $E_i\subset\Delta(t_i)$, $i=1,\dots,p$, it follows that*

$$\sup_{x^*\in B_{X^*}}\sum_{i=1}^{p}\left| x^*\left[f(t_i)|E_i| - (Mc)\int_{E_i} f\right]\right| < \varepsilon.$$

Theorem 5. *Let $u\in\mathcal{L}(X,Y)$. Then the operator U is in $\mathcal{L}(\mathcal{M}(X),\mathcal{B}(Y))$ if and only if u is absolutely summing. Moreover $\|U\| = \pi(u)$.*

Proof. First we prove the sufficient part. We suppose that $u\in\mathcal{L}(X,Y)$ is absolutely summing.

Clearly the operator U is additive. We show that for every $f\in\mathcal{M}(X)$, Uf belongs to $\mathcal{B}(Y)$. Since u is absolutely summing there is $c\geqslant 0$ such that (1) holds.

Let $f\in\mathcal{M}(X)$ arbitrary and fix $\varepsilon>0$. According to Lemma 4 there is a gauge Δ such that if $\mathcal{Q}=\{(A_i,\omega_i),\ i=1,\dots,p\}$ is a partition subordinate to Δ, we have

$$\sup_{x^*\in B_{X^*}}\sum_{i=1}^{n}\left| x^*\left[f(t_i)\mu(A_i) - (Mc)\int_{A_i} f\right]\right| < \varepsilon/c. \tag{7}$$

From (1) and (7) we obtain

$$\sum_{i=1}^{n}\left\| (Uf)(t_i)\mu(A_i) - (Mc)\int_{A_i} Uf\right\|_Y$$
$$\leqslant c\sup_{x^*\in B_{X^*}}\sum_{i=1}^{n}\left| x^*\left[f(t_i)\mu(A_i) - (Mc)\int_{A_i} f\right]\right| < \varepsilon$$

for every partition $\mathcal{Q}=\{(A_i,\omega_i),\ i=1,\dots p\}$ in Ω subordinate to Δ. Thus the function Uf is *MV*-integrable and therefore it is Bochner integrable [6, Theorem 2].

Let now $s=\sum_{i=1}^{n} x_i\chi_{A_i}$ be a simple function. Then with the same computation of the previous proposition we get that

$$\|Us\|_{\mathcal{B}(Y)} \leqslant \pi(u)\|s\|_{\mathcal{M}(X)}, \tag{8}$$

and the operator U is continuous on simple functions. If $f \in \mathcal{M}(X)$, then its indefinite Pettis integral is relatively compact (see [11, Corollary 3E]). Thus applying Theorem 1 simple functions are dense in $\mathcal{M}(X)$ with the $\mathcal{M}(X)$-norm. Let (t_n) be a sequence of simple functions converging to f in the $\mathcal{M}(X)$-norm. Then it is Cauchy in the $\mathcal{M}(X)$-norm, moreover there is a subsequence (s_n) of (t_n) such that for each $x^* \in X^*$,

$$\lim_{n\to\infty} x^*\big(s_n(\omega)\big) = x^*\big(f(\omega)\big)$$

for all $\omega \notin N_{x^*}$, $\mu(N_{x^*}) = 0$. By (8) and the linearity of U we get

$$\|Us_n - Us_m\|_{\mathcal{B}(Y)} \leqslant \pi(u)\|s_n - s_m\|_{\mathcal{M}(X)}.$$

Therefore the sequence (Us_n) is Cauchy in $\mathcal{B}(Y)$. Since $\mathcal{B}(Y)$ is complete there is a function $h \in \mathcal{B}(Y)$ such that (Us_n) converges to h in $\mathcal{B}(Y)$. Without loss of generality we can assume that the convergence is also a.e. So there is a set N, with $\mu(N) = 0$, so that for each $y^* \in Y^*$ and $\omega \notin N$,

$$\lim_{n\to\infty} y^*\big((Us_n)(\omega)\big) = y^*\big(h(\omega)\big). \tag{9}$$

Let $u^* : Y^* \to X^*$ be the adjoint operator of u. Since it is $weak^*$-continuous we obtain

$$\begin{aligned}\lim_{n\to\infty}\big\langle (Us_n)(\omega), y^*\big\rangle &= \lim_{n\to\infty}\big\langle u(s_n)(\omega), y^*\big\rangle = \lim_{n\to\infty}\big\langle s_n(\omega), u^*y^*\big\rangle = \big\langle f(\omega), u^*y^*\big\rangle \\ &= \big\langle u\big(f(\omega)\big), y^*\big\rangle = \big\langle (Uf)(\omega), y^*\big\rangle \end{aligned}\tag{10}$$

a.e. on Ω. From (9) and (10) we get that $y^*(h(\omega)) = y^*((Uf)(\omega))$ a.e. on Ω. Since the functions Uf and h are strongly measurable, then $h(\omega) = (Uf)(\omega)$ a.e. So the sequence (Us_n) converges to (Uf) a.e. and in $\mathcal{B}(Y)$. Furthermore

$$\|Uf\|_{\mathcal{B}(Y)} = \lim_{n\to\infty}\|Us_n\|_{\mathcal{B}(Y)} \leqslant \pi(u)\lim_{n\to\infty}\|s_n\|_{\mathcal{M}(X)} = \pi(u)\|f\|_{\mathcal{M}(X)}.$$

Therefore U is continuous and

$$\|U\| \leqslant \pi(u). \tag{11}$$

Conversely, assume that $u \in \mathcal{L}(X, Y)$ "lifts" to $U \in \mathcal{L}(\mathcal{M}(X), \mathcal{B}(Y))$. Choose $x_1, \ldots, x_n \in X$ and $A_1, \ldots, A_n$ disjoint measurable subsets of Ω, where $0 < \mu(A_i) = k$. We have

$$\begin{aligned} k\sum_{i=1}^{n}\big\|u(x_i)\big\|_Y &= \sum_{i=1}^{n}\big\|ku(x_i)\big\|_Y = \sum_{i=1}^{n}\big\|\mu(A_i)u(x_i)\big\|_Y \\ &= \sum_{i=1}^{n}\int_{A_i}\big\|u(x_i)\big\|_Y\,d\mu = \int_{\Omega}\left\|u\left(\sum_{i=1}^{n}x_i\chi_{A_i}(\omega)\right)\right\|_Y d\mu \\ &= \int_{\Omega}\left\|U\left(\sum_{i=1}^{n}x_i\chi_{A_i}(\omega)\right)\right\|_Y d\mu = \left\|U\left(\sum_{i=1}^{n}x_i\chi_{A_i}\right)\right\|_{\mathcal{B}(Y)} \\ &\leqslant \|U\|\left\|\left(\sum_{i=1}^{n}x_i\chi_{A_i}\right)\right\|_{\mathcal{M}(X)} \end{aligned}$$

$$= \|U\| \sup\left\{ \int_{\Omega} \left| x^*\left(\sum_{i=1}^{n} x_i \chi_{A_i} \right) \right| d\mu\colon\ x^* \in B_{X^*} \right\}$$

$$= \|U\| \sup\left\{ \sum_{i=1}^{n} \int_{A_i} \left| x^*(x_i) \right| d\mu\colon\ x^* \in B_{X^*} \right\}$$

$$= \|U\| k \sup\left\{ \sum_{i=1}^{n} \left| x^*(x_i) \right|\colon\ x^* \in B_{X^*} \right\}. \tag{12}$$

It follows that

$$\sum_{i=1}^{n} \left\| u(x_i) \right\|_Y \leqslant \|U\| \sup\left\{ \sum_{i=1}^{n} \left| x^*(x_i) \right|\colon\ x^* \in B_{X^*} \right\}.$$

Then u is absolutely summing and $\pi(u) \leqslant \|U\|$. Considering last inequality together with inequality (11) in the sufficient part, we obtain $\pi(u) = \|U\|$. □

If the operator U induced by $u \in \mathcal{L}(X, Y)$ is in $\mathcal{L}(\mathcal{P}(X), \mathcal{B}(Y))$ then u is absolutely summing. Indeed since the class of McShane integrable functions is included in the class of Pettis integrable ones the assert follows from the previous theorem.

As the class of strongly measurable Pettis integrable function is included in the class of McShane integrable ones [14, Theorem 17] we obtain as a corollary the characterization due to Diestel (see [4]).

Corollary 6. *Let $u \in \mathcal{L}(X, Y)$. Then the operator U is in $\mathcal{L}(\mathcal{P}_m(X), \mathcal{B}(Y))$ if and only if u is absolutely summing. Moreover $\|U\| = \pi(u)$.*

Corollary 7. *Let X be a super-reflexive Banach space and $u \in \mathcal{L}(X, Y)$. The operator U is in $\mathcal{L}(\mathcal{P}(X), \mathcal{B}(Y))$ if and only if u is absolutely summing. Moreover $\|U\| = \pi(u)$.*

Proof. By [7, Theorem 1] if X is a super-reflexive Banach space the classes of X-valued Pettis integrable functions and McShane integrable functions coincide. Therefore the assert follows from Theorem 5. □

Since any Birkhoff integrable function is McShane integrable (see [12, Proposition 4]) we get also the following

Corollary 8. *Let $u \in \mathcal{L}(X, Y)$. The operator U induced by u maps the family of Birkhoff integrable functions in the class of the Bochner ones if and only if u is absolutely summing. Moreover $\|U\| = \pi(u)$.*

The following example is an application of the previous Theorem 5.

Example. Let $X = \ell_1$ and let $x_n = (x_1^n, x_2^n, \ldots)$ such that $\sum_n x_n$ is a series in ℓ_1 converging unconditionally but not absolutely. For each $n \in \mathbb{N}$, let $I_n = (2^{-n}, 2^{-n+1})$ and define $f : [0,1] \to \ell_1$ by

$$f(t)(k) = \begin{cases} 2^n x_k^n & \text{if } t \in I_n,\ n = 1, 2, \ldots, \\ \phi & \text{otherwise,} \end{cases}$$

where ϕ denotes the null vector of ℓ_1. As f is a countably valued function, it is strongly measurable. Since $\sum_n 2^n x_n |I_n| = \sum_n x_n$ is unconditionally but not absolutely convergent, f is McShane integrable (see [14, Theorem 15]), but it is not Bochner integrable (see [2, Theorem 2]). If $i : \ell_1 \hookrightarrow \ell_2$ is the canonical immersion then i is an absolutely summing operator. Therefore applying Theorem 5 we get that the function

$$i \circ f : [0,1] \to \ell_2$$

is variationally McShane integrable and also Bochner integrable.

References

[1] A. Belanger, P.N. Dowling, Two remarks on absolutely summing operators, Math. Nachr. 136 (1988) 229–232.

[2] J.K. Brooks, Representation of weak and strong integrals in Banach spaces, Proc. Nat. Acad. Sci. USA 63 (1969) 266–279.

[3] W. Congxin, Y. Xiaobo, A Riemann type definition of the Bochner integral, J. Math. Study 27 (1994) 32–36.

[4] J. Diestel, An elementary characterization of absolutely summing operators, Math. Ann. 196 (1972) 101–105.

[5] L. Di Piazza, V. Marraffa, The McShane, PU and Henstock integrals of Banach valued functions, Czech. Math. J. 52 (2002) 609–633.

[6] L. Di Piazza, K. Musial, A characterization of variationally McShane integrable Banach-space valued functions, Illinois J. Math. 45 (2001) 279–289.

[7] L. Di Piazza, D. Preiss, When do McShane and Pettis integrals coincide?, Illinois J. Math. 47 (2003) 1177–1187.

[8] N. Dunford, J.T. Schwartz, Linear Operators, part I, Interscience, New York, 1958.

[9] L. Egghe, Stopping Time Techniques for Analysts and Probabilist, in: London Math. Soc. Lecture Notes Ser., vol. 100, Cambridge Univ. Press, Cambridge, 1984.

[10] D.H. Fremlin, Topological Riesz Spaces and Measure Theory, Cambridge Univ. Press, Cambridge, 1984.

[11] D.H. Fremlin, The generalized McShane integral, Illinois J. Math. 39 (1995) 39–67.

[12] D.H. Fremlin, The McShane and Birkhoff integrals of vector-valued functions, University of Essex, Mathematics Department, Report 92-10, 1999.

[13] D.H. Fremlin, J. Mendoza, On the integration of vector-valued functions, Illinois J. Math. 38 (1994) 127–147.

[14] R.A. Gordon, The McShane integral of Banach-valued functions, Illinois J. Math. 34 (1990) 557–567.

[15] V. Marraffa, Riemann type integrals for functions taking values in a locally convex space, preprint.

[16] K. Musial, Topics in the theory of Pettis integration, Rend. Istit. Mat. Univ. Trieste 23 (1993) 177–262.

[17] S. Schwabik, Y. Guoju, On the strong McShane integral of functions with values in a Banach space, Czech. Math. J. 51 (2001) 819–828.

[18] M. Talagrand, Pettis Integral and Measure Theory, in: Mem. Amer. Math. Soc., vol. 307, 1984.