Front. Phys., 2013, 8(6): 771–793
DOI 10.1007/s11467-013-0329-5

REVIEW ARTICLE

Gravitational wave astronomy

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Received February 20, 2013; accepted April 6, 2013

We are entering a new era of gravitational-wave astronomy. The ground-based interferometers have reached their initial design sensitivity in the audio band. Several upper limits have been set for anticipated astrophysical sources from the science data. The advanced detectors in the US and in Europe are expected to be operational around 2015. New advanced detectors are also planned in Japan and in India. The first direct detections of gravitational waves are expected within this decade. In the meanwhile, three pulsar timing array projects are forming an international collaboration to detect gravitational waves directly in the nanoHertz range using timing data from millisecond pulsars. The first direct detection of nanoHertz gravitational waves are also expected within this decade. In this paper, we review the status of current gravitational-wave detectors, possible types of sources, observational upper limits achieved, and future prospects for direct detection of gravitational waves.

Keywords gravitational waves, gravitational-wave detectors, gravitional astronomy

PACS numbers 04.80.Nn, 95.75.-z, 97.80.-d, 97.60.Gb

Frontiers of Physics

Contents

1 Introduction

Gravitational waves (GW) are a prediction of Einstein's theory of General Relativity [1], and in general of theories with a finite propagation speed for gravitational interaction. Although most motion of masses will generate these "ripples of space time", a conspiracy of constants of Nature make these perturbations very small – spacetime is very stiff, and detecting its perturbations needs very sophisticated measurement technologies.

For several decades now, physicists around the world have searched for gravitational waves designing and building different kinds of detectors. In the late 60's, Joseph Weber built the first gravitational wave detectors, "resonant bar detectors", and his claim about seeing coincident transients between bars in Maryland and Argonne started a flurry of groups around the world building similar detectors of increasing sensitivity [2]. Although no other experiment confirmed the claimed transients, and their amplitude made them very unlikely to be astrophysical, a new era for the direct experimental detection of gravitational waves had started.

The understanding of gravitational wave amplitudes and frequencies expected from astrophysical origin and thus possible detection rates has greatly improved from the 70s, and has presented a great challenge to the experimental field; see Section 2 on Astrophysical Sources. However, several current efforts are promising to detect gravitational waves this decade: these are advanced interferometric ground-based detectors and pulsar timing (see Sections 3 and 5). Other efforts for space based detectors have great prospects for detecting many strong and frequent signals, but the proposed instruments are very expensive and are still searching for the appropriate funding from space agencies in the USA and in Europe (see Section 6).

There are several ground based interferometric detectors around the world (see Section 3), and there are many important results obtained already from this network of detectors (see Section 4).

The technological progress achieved in the construction of the current generation of detectors, and the knowledge gained in astrophysical sources and in data analysis techniques promises that the next years will generate the exciting first detection of gravitational waves, and start a new era of gravitational wave astronomy.

2 Astrophysical sources

The possible astrophysical sources for gravitational waves have been reviewed extensively previously [3–9]. For the most up-to-date reviews, we refer interested readers to Ref. [6] for sources for ground-based interferometers, to Ref. [5] for the space mission eLISA, and to Ref. [8] for pulsar timing arrays.

2.1 Impulsive sources for ground-based interferometer

2.1.1 Compact binary coalescences

Compact binary coalescence is one of the most important sources for ground-based interferometers. It includes coalescences of binaries formed by neutron stars (NS-NS), by black holes (BH-BH), or by neutron stars and black holes (NS-BH). Among them, NS-NS binary systems are probably the most reliable ones given the handful actual observations of these systems in various EM channels [10]. Gravitational wave signals are treated differently according to three phases of these binary coalescences: the inspiral, merger and the final ring-down phase of the produced BH. For NS-NS binaries, most of the detection signal-to-noise ratio (SNR) is carried by the inspiral phase. The waveform at the inspiral phase is fairly well understood theoretically. In particular, the frequency and amplitude of the wave will increase monotonically with time, forming a characteristic "chirping" signal. For systems with higher masses such as BH-BH binaries, SNR contributions from the merger and ring-down phases are increasingly more important. The expected detectable distance for NS-NS binaries is around 150–200 Mpc and the most probable event rate is around 40 per year for advanced LIGO/Virgo detectors [11].

We recall that the dominant contribution to the GW emission by a distribution of mass $\rho(\vec{x})$, at a distance R by the observer, is given by the time variation of its quadrupole component: more precisely,

$$
\begin{aligned}
h_{ij} &= -\frac{2G}{c^4 R}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\int \rho(\vec{x})x_i x_j \mathrm{d}V \\
&= -\frac{2c}{R}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\int \rho(\vec{x})x_i x_j \mathrm{d}V \Big/ \left(\frac{c^5}{G}\right) \qquad (1)
\end{aligned}
$$

where the factor $\frac{c^5}{G} = 3.63\times10^{52}\mathrm{J} = 2.03\times10^5 M_\odot c^2/\mathrm{s} \equiv L_o$ is a luminosity of 10^5 solar masses per second! It is therefore evident that stellar size distributions of masses are required to generate sizable signals.

The coalescence of compact binaries is particularly interesting for its efficiency, and the signal's strength can be readily estimated, in fact: assuming for simplicity a pair of objects with equal mass m, in a circular orbit characterized by an angular frequency ω and a semi-axis a, the equivalence of gravitational and centripetal forces translates into Kepler's law $\omega^2 = Gm/(4a^3)$, which yields

$$
h_{ij} \propto \frac{2\,(Gm)^2}{c^4 a\, R} = \frac{(2Gm)^{5/3}}{c^4 R}\omega^{2/3} \qquad (2)
$$

hence the signal increases if the objects' separation a decreases; and this in fact happens, because energy is radiated. In linearized general relativity, an energy momentum $t_{\mu\nu}$ can be associated to the GW, and the energy flux through a sphere centered on the source is

$$
\begin{aligned}
L_{\mathrm{GW}} &\equiv -\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{G}{5c^5}\langle \dddot{Q}_{ij}\dddot{Q}^{ij}\rangle \\
Q_{ij} &\equiv \int \rho\left(x_i x_j - \frac{1}{3}x^k x^k \delta_{ij}\right)\mathrm{d}V \qquad (3)
\end{aligned}
$$

which for a binary system in circular orbit gives

$$L_{\rm GW} = \frac{128}{5}\frac{G}{c^5}(ma^2\omega^3)^2 = \frac{32G^{7/3}}{5\sqrt[3]{4}c^5}(m\omega)^{10/3} \tag{4}$$

The two masses possess $E_{kin} = m\,(a\omega)^2$ and $E_{pot} = -Gm^2/(2a)$, hence

$$E = -Gm^2/(4a) = -\frac{G^{2/3}m^{5/3}}{4^{2/3}}\omega^{2/3} \tag{5}$$

comparing with Eq. (4) we obtain

$$\frac{\dot\omega(t)}{\omega(t)^{11/3}} = \frac{48\,2^{2/3}G^{5/3}m^{5/3}}{5c^5} \tag{6}$$

or

$$\omega(t) = \omega(t_0)\left[1 - \frac{256\,G^{5/3}\mu\,M^{2/3}}{5\,c^5\omega(t_0)^{-8/3}}(t-t_0)\right]^{-3/8} \tag{7}$$

where reduced (μ) and total (M) masses have been introduced. The negative exponent tells that after a time τ,

$$\tau = \frac{5c^5}{256G^{5/3}\mu\,M^{2/3}\omega(t_0)^{8/3}} \tag{8}$$

the angular frequency goes to infinite, signaling a breakdown of the approximation, close to the collision. Integrating the expression for ω we can find the phase and instantaneous frequency of the signal as

$$\phi\,(t) = \frac{16\pi\nu_0\tau\,(\nu_0)}{11}\left[1-\left(1-\frac{t}{\tau(\nu_0)}\right)^{5/8}\right]$$
$$\nu\,(t) = \nu_0\left(1-\frac{t}{\tau\,(\nu_0)}\right)^{-3/8} \tag{9}$$

in which ν_0 is the GW frequency (twice the orbital frequency) at $t_0 = 0$ and $\tau\,(\nu_0)$ is the time to coalescence:

$$\tau(\nu_0) \equiv \frac{5}{256}\frac{c^5}{G^{5/3}}\frac{(\pi\nu_0)^{-8/3}}{\mathcal{M}^{5/3}} \tag{10}$$

The quantity $\mathcal{M} \equiv \mu^{3/5}M^{2/5}$ is called "chirp mass", to express the fact that its value determines the "chirping" evolution of the signal, whose typical shape is shown in Fig. 1: an initial "inspiral" phase, followed by a "merger", during which the objects collide, and a "ringdown" during which damped sinusoids are emitted by the resulting object, which settles in a new stable state.

All this treatment is simplified: it assumes purely Newtonian physics, which of course tells nothing about the merger and ringdown phase: a much more complex treatment is required to compute accurately enough the signal, even in the inspiral phase (see Ref. [12] for a review). However, many characteristics can be deduced on the basis of these simple formulas: in particular, it is immediate to estimate the signal's duration in a certain frequency band $[\nu_l,\, \nu_u]$ as:

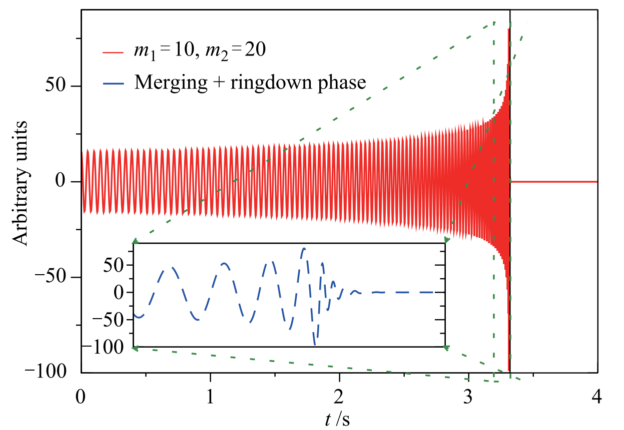

Fig. 1 The final seconds of the signal emitted by a pair of (non-spinning) black-holes (10+20 $M_\odot$) in coalescence.

$$t_u - t_l = \tau\,(\nu_0)\left[\left(\frac{\nu_l}{\nu_0}\right)^{-8/3} - \left(\frac{\nu_u}{\nu_0}\right)^{-8/3}\right] \tag{11}$$

extending the sensitivity towards lower frequencies ($\nu_l \to 0$) increases the signal's observable duration, faster than $\frac{1}{\nu_l^2}$.

Coalescences therefore appear on ground based detectors as impulsive events just because the intereferometers' sensitivity is limited by thermal and seismic noise, so that we start observing only above ν_l, a few tens of Hz. In reality, like the famous Hulse–Taylor binary pulsar, the associated sources emit gravitational waves for millions of years, of which we potentially observe only the final few tens of seconds. For more massive sources (larger $\mathcal{M}$), the observation time becomes shorter, to the point that large mass binaries are just not accessible to ground observation by first generation instruments.

In lower mass systems, like pairs of neutron stars for which we expect individual masses $O\,(1M_\odot)$, the signal is dominated by the inspiral phase, well approximated by post-newtonian approximants, so that standard matched filtering allows to search for events in the detectors' noise: a search complicated by the fact that signal parameters are not known a priori, so that the space of possible masses and spins needs to be sampled.

We have focused on the inspiral phase, however possibly the most interesting physics will come from the observation of the merger phase. For binaries made of two neutron stars, or a neutron star and a black hole, during the merger the objects will be disrupted and the details of their composition will become relevant, determining the actual shape of the GW signal, which could allow for instance to draw conclusions on the equation of state of the nuclear matter [13].

However, the merger phase is very complicated to model, and requires sophisticated simulations which take into account general relativity and nuclear physics in order to predict the emitted GW [14] and possibly a simultaneous γ-ray burst [15].

For binary black holes, the problem is conceptually simpler because the only relevant equations are those of general relativity: however, the deviations from the simple Newtonian formulas become relevant not just for the merger, but also for the inspiral phase, and an accurate calculation of the effects has called for a substantial theoretical effort, contributing to the development of the field of Numerical Relativity. Actually, NR was born in the '80s of the 20th century to study the core-collapse events [16], but later an enormous effort was undertaken to confront compact binary collisions. One important milestone was the simulation of an head-on BH-BH collision [17, 18], whereas later the progress in the simulation methodologies made possible to study realistic collisions in three dimensions, including eventually spin effects, which are expected to be important in binary black-holes: it is not possible to account here for the rich literature on the subject, and the reader could consider for instance the reviews [19, 20].

From the point of view of gravitational wave searches, the results of Numerical Relativity have been exploited to assess the sensitivity of the search waveforms [21], and a significant effort is still ongoing to further translate the knowledge gained by several NR groups into waveforms usable for the data analysis: in the future, this effort is likely to lead to an even closer collaboration of theorists and experimentalists.

2.1.2 Other impulsive signals, also called "bursts"

Besides coalescing binaries, we expect that other sources emit gravitational signals of short duration, like GW outburst generated from asymmetric core collapse of massive stars during supernova event. The dynamics of core collapse during a supernova is extremely complex and any predictions on its GW production are far from being conclusive. The best effort from numerical simulations show that the signal is probably burst-like with a duration within hundreds of milliseconds. It has been argued that within a few years of observation, the advanced detectors can possibly detect a few such events in our local universe up to a few Mpc [22].

2.2 Continuous sources for ground-based interferometers

2.2.1 Periodic signals

Spinning compact objects emit gravitational waves when their mass distribution displays a time-varying quadrupolar component. Examples are freely precessing neutron stars, which exhibit a momentum of inertia along one axis $\hat{z}$ different from the ones along the other two axes. The deviation from spherical shape can for instance be caused by internal magnetic stress for a fast-spinning neutron star (e.g., Refs. [39, 40]) or by accretion onto neutron stars residing within low mass X-ray binaries [41]. For known non-accreting pulsars, targeted searches could yield tens of detections within one year's observation [42]. A few known bright persisting accreting systems could also be detectable and the chance of detection will be improved if their spin and orbital parameters are known in advance [43].

In terms of the source's parameters, a periodic signal is emitted with a characteristic amplitude, as seen at distance R,

$$h_c = \frac{2G_N}{c^4}\frac{\epsilon I_{xx}}{R}\Omega^2, \quad \text{with} \quad \epsilon \equiv \frac{I_{zz} - I_{xx}}{I_{xx}} \tag{12}$$

where Ω is the angular frequency of the spinning object, and ϵ is called its "ellipticity". A similar formula holds for a tri-axial rotator, in which case one has

$$h_c = \frac{2G_N}{c^4}\frac{\epsilon I_{zz}}{R}\Omega^2 \quad \text{with} \quad \epsilon \equiv \frac{I_{xx} - I_{yy}}{I_{zz}} \tag{13}$$

In either case, the formula is more easily read as

$$h_c \simeq 10^{-26}\frac{I_{xx}}{10^{38}\text{kg } m^2}\frac{10 \text{ kpc}}{R}\left(\frac{f}{100 \text{ Hz}}\right)^2\frac{\epsilon}{10^{-6}} \tag{14}$$

exposing typical values for I, and plausible values for ϵ. This formula tells us that searches for such signals are confined to galactic sources only, whose number is estimated to be very large, though: 10^8–10^9 neutron stars in the Milky Way alone, on the basis of the supernova rate (about 1 every 30 years), the Milky Way age (about 13 billion years) and the assumption that most supernovae originate NS and not BH.

The signal emitted is simply a periodic wave, in general with pulsation Ω and 2Ω, but the detection is made difficult by the Doppler effect due to Earth rotation and revolution: even considering just the Earth's revolution motion $\vec{r}(t)$ as a circular orbit with angular frequency ω_m, the phase of a periodic signal with pulsation ω received from direction $\hat{n}$ is altered as

$$\phi(t) = \omega\left(t + \frac{\hat{n}\cdot\vec{r}(t)}{c}\right) = \omega(t + \beta\cos\omega_m t) \tag{15}$$

where $\beta = O(1\text{AU}/c)$; the signal itself can be expanded in a series of side-bands

$$\cos(\omega(t + \beta\cos\omega_m t))$$

$$= \sum_{k=-\infty}^{+\infty} (-1)^k J_{2k}(\omega\beta)\cos(\omega + 2k\omega_m)t$$
$$- \sum_{k=-\infty}^{+\infty} (-1)^k J_{2k+1}(\omega\beta)\sin(\omega + (2k+1)\omega_m)t \qquad (16)$$

The amplitude of each sideband depends on the argument $\omega\beta$ of the Bessel function: for $\omega\beta \ll \pi/2$, only the closest two matter; otherwise, energy gets dispersed on a large number of bands. To show this, consider the formula

$$\frac{\sum_k k^2 J_k(x)}{\sum_k J_k(x)} = \frac{x^2}{2} \qquad (17)$$

which relates the variance of the index k in the distribution of $J_k(x)$ to the square of the argument x: for large x, large values of k are important. Considering the revolution motion of Earth, $\beta \sim 5 \times 10^2$ and for $\omega = 2\pi \times 100$ Hz we obtain $x \sim 3 \times 10^5$, and a similar order for the standard deviation of k: hundreds of thousands of sidebands.

The Doppler effect can be corrected for, only if the source location is known with high accuracy, hence signals associated with known pulsars can be searched for with a higher sensitivity. However, despite a reduced sensitivity, searches for periodic signals emitted by unknown neutron stars are still worthwhile, since a large number of such objects is expected to exist, and there could be sources close enough to be detectable.

The matter is further complicated by the evolution of the fundamental period of the source: a rotating neutron star will lose energy as

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\frac{G}{5c^5}\langle \dddot{Q}_{ij}\dddot{Q}_{ij}\rangle = -\frac{32G}{5c^5} I_{zz}^2 \epsilon^2 \Omega^6 \qquad (18)$$

given that $E = \frac{1}{2} I_{zz}\Omega^2$, it follows

$$\dot{\Omega} = -\frac{32G}{5c^5} I_{zz}\epsilon^2\Omega^5 \qquad (19)$$

hence for the gravitational frequency

$$\dot{\nu} \simeq -5 \times 10^{-13} \frac{I_{zz}}{10^{38}\mathrm{kg}\, m^2}\left(\frac{\nu}{100\ \mathrm{Hz}}\right)^5 \left(\frac{\epsilon}{10^{-6}}\right)^2 \mathrm{Hz/s} \qquad (20)$$

or equivalently for the rotation period

$$\dot{P} \simeq 10^{-14} \frac{I_{zz}}{10^{38}\mathrm{kg}\, m^2}\left(\frac{P}{1\ \mathrm{ms}}\right)^{-3} \left(\frac{\epsilon}{10^{-6}}\right)^2 \qquad (21)$$

A measured braking $\dot{P}$ provides therefore an upper limit on ϵ:

$$\epsilon \leqslant \epsilon_{max} \equiv 3 \times 10^{-9}\left(\frac{P}{1\ \mathrm{ms}}\right)^{3/2}\left(\frac{\dot{P}}{10^{-19}}\right)^{1/2} \qquad (22)$$

As an example, for the Crab $\dot{P} = 38$ ns/day $= 4.4\times 10^{-13}$ hence

$$\epsilon_{max}(\mathrm{Crab}) \simeq 1.2 \times 10^{-4} \qquad (23)$$

Note that in deriving Eq. (22) we have assumed that the braking is due solely to GW emission. This does not account for the EM emission, which is likely to be the dominant mechanism for the loss of rotational energy, thus altering also the dependence of the braking on the period P [44]. We do not discuss here this added complication, which however is taken into account in the data analysis, when exploring the space of possible frequency evolutions.

2.2.2 Stochastic background

Besides periodic signals, we anticipate the existence of stochastic signals, either of cosmological origin, for instance emitted shortly after the Big Bang [45], or of astrophysical origin, for instance due to an incoherent superposition of impulsive signals emitted by a large population of distant sources [46]. It has been shown [47] that the advanced detectors are likely to detect the stochastic signals caused by binary coalescences and that these coalescence signals could form a “foreground” potentially masking the GW background due to cosmological sources.

In any case, the signal seen by a detector l would be

$$h_l(t) = \sum_p \int \mathrm{d}\Omega F_l^p(\hat{\Omega}) \int_{-\infty}^{+\infty} \mathrm{d}f \mathrm{e}^{\mathrm{i}2\pi f\left(t - \frac{\vec{x}_l\cdot\hat{\Omega}}{c}\right)} \tilde{h}_p(f, \hat{\Omega}) \qquad (24)$$

where $\vec{x}_l$ specifies the location of the detector in a reference frame, whereas $\hat{\Omega}$ is a unit vector representing the direction of the source, and F_l^p represents the detector's l antenna pattern, for polarization p; $\tilde{h}_p$ would represent the p component of the stochastic signal from direction $\hat{\Omega}$.

Signals of cosmological origin are expected to be stationary, isotropic and gaussian, which means that all we can know about $\tilde{h}$ is embodied in the correlation

$$\langle \tilde{h}_p(f, \hat{\Omega})\tilde{h}_q^*(f', \hat{\Omega}')\rangle = \frac{1}{4\pi}\delta(f - f')\delta^2(\hat{\Omega} - \hat{\Omega}')\delta_{pq}\frac{1}{2}H(f) \qquad (25)$$

where $H(f)$ is a one-sided spectral density. Could we search for such a signal with a single detector? The resulting contribution to the noise at the output of the detector can be easily computed from the correlation

$$\langle h_l(t)h_l(t')\rangle = \sum_p \frac{1}{4\pi}\int \mathrm{d}\Omega\,[F_l^p(\hat{\Omega})]^2 \cdot \int_{-\infty}^{+\infty} \mathrm{d}f\,\mathrm{e}^{\mathrm{i}2\pi f(t-t')}\frac{1}{2}H^2(f) \tag{26}$$

which shows that, apart factors of the order of unity, $H^2(f)$ directly contributes to the detector's spectral noise density. Starting from the definition of the energy density of GW

$$\rho_{gw} = \frac{c^3}{32\pi G}\langle \dot{h}_{ij}(t)\dot{h}^{ij}(t)\rangle \tag{27}$$

it is immediate to show that

$$\frac{\mathrm{d}\rho_{gw}}{\mathrm{d}\log f} = \frac{\pi c^3}{2G} f^3 H^2(f) \tag{28}$$

It is customary to introduce the quantity

$$\Omega_{\mathrm{GW}}(f) \equiv \frac{1}{\rho_c}\frac{\mathrm{d}\rho_{gw}}{\mathrm{d}\log f} \tag{29}$$

in terms of the critical energy density ρ_c, in turn defined as

$$\rho_c = \frac{3H_0^2 c^3}{8\pi G} \tag{30}$$

where the Hubble constant $H_0 = h_0 \times 100 \times \mathrm{km/(s{\cdot}Mpc)}$. One obtains

$$h_0^2 \Omega_{\mathrm{GW}}(f) = \frac{4\pi^2 h_0^2}{3H_0^2} f^3 H^2(f) \tag{31}$$

as a dimensionless measure of the spectral strength of the GW background, independent on the experimental uncertainty on h_0 (recall the most recent estimates $h_0 \simeq 0.714 \pm 0.016$ [48]). Taking into account existing bounds, one should aim at $h_0^2\Omega_{\mathrm{GW}}(f) \leqslant 10^{-6}$, that is

$$H(f) \leqslant \sqrt{\frac{3}{4}\frac{H_0}{\pi h_0}} f^{-3/2} \times 10^{-6} \simeq 10^{-21}\frac{\mathrm{Hz}}{f^{3/2}} \tag{32}$$

a quantity to be compared with the detector's sensitivity: at 100 Hz one could detect $H(f) \simeq 3 \times 10^{-24}/\sqrt{\mathrm{Hz}}$, about a factor 10 below the best sensitivity attained so far. In fact, correlating the output of several instruments, it is possible to do much better.

2.3 GW sources for pulsar timing arrays

Stochastic background: This is by far the most popular GW source for the Pulsar Timing Arrays (PTAs). The background sources include GWs from cosmic strings, the inflationary era (e.g., Ref. [45]), and from coalescing binaries of supermassive black holes of $M \geqslant 10^8 M_\odot$ at redshifts $z \lesssim 2$ (e.g., Refs. [49, 50]). Recent phenomenological studies [51, 52], using a merger-only scenario for galaxy evolution at $z \leqslant 1$, argued for the possibility of "imminent" detection of the background using existing PTAs. A more conservative estimation [53] also implies a good chance for the background to be detected within the next few years. In any case, the PTA observations will be able to test the proposed scenarios within the next few years.

Resolvable binaries of supermassive black holes: It is anticipated that at frequencies higher than ~ 1/yr, individual resolvable GW sources from binaries of supermassive black holes of 10^8–10^{10} $M_\odot$ at low redshift of $z \leqslant 1.5$ can be detected out of the stochastic background [54, 55]. The vast majority of these sources reside at redshift $0.1 \leqslant z \leqslant 1$ [54]. With a timing resolution of ~ 10 ns as promised by the proposed Square Kilometer Array (SKA) [54, 56], at least one of these sources should be detected over five years of observation. A higher detection rate is possible for the scenario suggested in Refs. [51, 52].

Burst sources: these are loosely defined as emitting signals with duration much shorter than the observation period. It includes GWs from the final inspiral phase of binaries of supermassive black holes up to the merger phase, or the close passage of one supermassive black hole in a highly elliptical or unbound orbit about another supermassive black hole [57] or cusps on cosmic strings. These events could last for months to years. The event rate for these sources has not been discussed in literature, and is highly uncertain.

GW memory: a permanent distortion to the space and time metric after the binary black hole merger [58]. The observable for PTAs would be a discontinuous jump of the pulse frequency. It has been shown that such sources are in principle detectable for BH binaries of $10^8 M_\odot$ out to the redshift of $z \sim 1$ [59–61]. The event rate is highly uncertain, though: it has been argued [61] that with optimistic models, there is probably a marginal to modest chance for an individual frequency jump to be detectable by existing PTAs for 10 years' observation and the rate can be an order of magnitude better with SKA.

2.4 GW sources for space interferometer eLISA

Ultra-compact stellar-mass binaries: These would be the most numerous sources in the eLISA band. Several thousands of stellar-mass close binaries in our Galaxy are expected to be detected individually [62] while the combined signals of the millions of compact binaries will form a foreground signal [62, 63]. Among them, the double white dwarf binaries are the most numerous ones. The individually detectable ones include a few tens of those whose GWs can be detected with a few weeks to months observation and are already well observed in the electromagnetic bands [64].

Coalescing binaries of massive black holes: eLISA is sensitive to black hole binaries of total rest-frame mass $M \sim 10^4$–10^7 $M_\odot$ up to a redshift $z \leqslant 10$ for SNR$\geqslant 10$ and possibly up to $z \sim 20$ if such sources exist. The predicted detection rate ranges from a few up to few hundred events per year (see e.g., Ref. [65]).

Extreme Mass Ratio Inspirals (EMRIs): This is the inspiral of a stellar mass compact object (black hole, neutron star, or white dwarf) into a massive black hole of 10^4–$10^7 M_\odot$. It has been shown [66] that eLISA could observe a few tens of EMRIs over its two year mission lifetime at redshifts $z < 0.5$.

3 Ground-based interferometers

Ground-based interferometric detectors are the most promising experiments for detecting gravitational waves in the next several years, especially the LIGO [67] and Virgo [68] "advanced" detectors currently being installed. There is also a detector being installed in Japan, KAGRA [69, 70] and a design for a future European detector, the "Einstein Telescope" [71–73]. The GEO detector [74] described below has been in operation since 2001. An Australian consortium, ACIGA, is looking for funds for a detector in the Gingin facility in Western Australia.

The world-wide community of scientists working with the experiments and data analysis of the ground-based detectors undertakes a very collaborative effort. The LIGO Scientific Collaboration (LSC) [75] includes the LIGO Laboratory, the GEO Collaboration and ACIGA as some of its more than 900 members in 17 different countries, developing technologies for future detectors, operating and diagnosing the data of the current detectors, and analyzing the data taken in science runs. The Virgo Collaboration has more than 200 members from several European countries, and since 2007 has a data sharing agreement with the LSC [76]. LSC, Virgo and the KAGRA Collaboration also have an agreement to plan sharing data when sensitive detectors are in operation [78].

At the time of writing this article, only one interferometric detector is operating, GEO600 in Ruthe, Germany [77]. The interferometer facility has 600 m long arms, and the current interferometer is a dual-recycled interferometer (using power and signal recycling), as well as a homodyne detection system and quantum squeezing [79]. Unlike the LIGO and Virgo detectors, it does not have resonant Fabry-Perot cavities in its arms, but it has folded arms making a Michelson interferometer with 1200 m long arms. Its best sensitivity is near 1 kHz, with an amplitude spectral density better than 3×10^{-21} in a 2 kHz bandwidth. The GEO detector has pioneered many techniques like signal recycling, monolithic fused silica suspensions, electrostatic actuation for suspended masses, and the use of squeezing over long periods of time.

There are two LIGO Observatories in the US operated by groups at Caltech and MIT, in Hanford, Washington, and in Livingston, Louisiana [80]. Both facilities have perpendicular, 4 km long interferometer arms, with light beams within high vacuum enclosures. Until 2010, the Hanford Observatory hosted two interferometers in its facility, one 4 km long and another 2 km long. The three LIGO detectors took data at their designed sensitivity with record worldwide sensitivity between November of 2005 and October of 2007, in the LIGO fifth "science run" S5. Although no gravitational waves were detected, many significant upper limits were set (see Section 4).

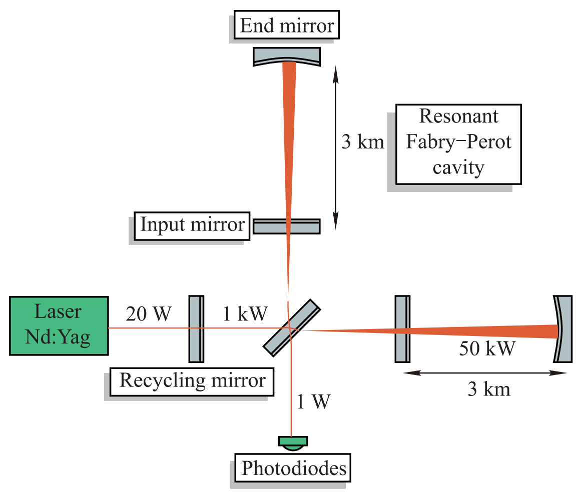

Fig. 2 Initial Virgo Fabry–Perot power-recycled interferometer, from `http://www.ego-gw.it/virgodescription/`. The initial LIGO detectors used the same configuration, although its arms were 4 km long and the power in the different cavities slightly different than the ones shown in the figure.

The "initial" LIGO detectors [81] were power-recycled Fabry–Perot Michelson interferometers, and used heterodyne detection with phase modulation of the infrared laser light. The test masses in the Fabry–Perot arm cavities were 10 kg-fused silica masses suspended on metal wires, and the suspension frame was attached to a passive seismic isolation system. The 4 km detectors at Hanford and Livingston achieved a sensitivity of about $2\times10^{-23}/\sqrt{\text{Hz}}$ near 150 Hz, and a sensitivity better than

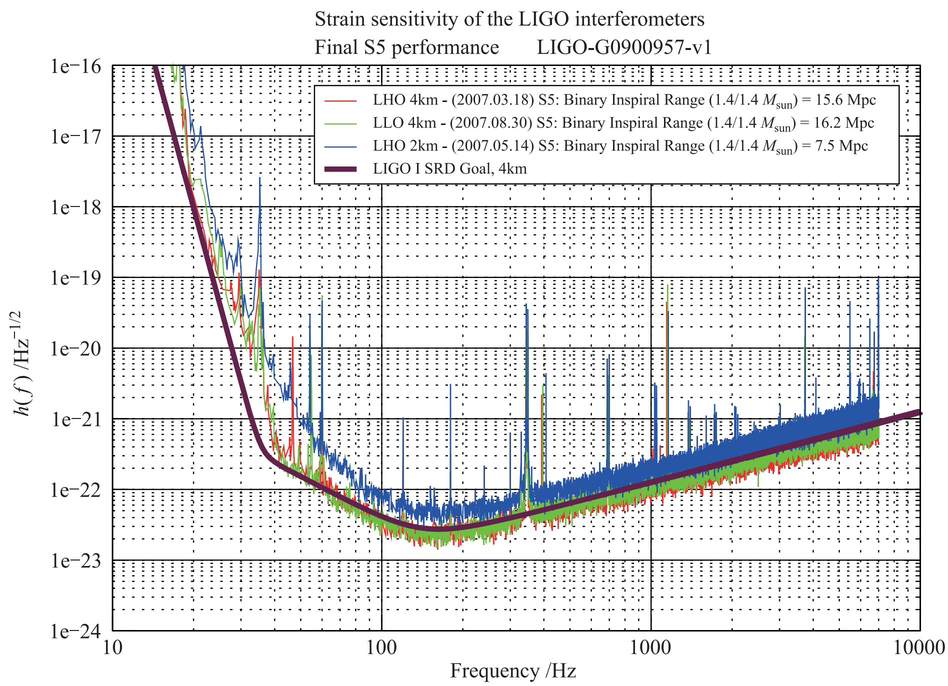

Fig. 3 Sensitivity of LIGO detectors in 2007 (LIGO-G0900957 in dcc.ligo.org).

$10^{-22}/\sqrt{\text{Hz}}$ in about 1 kHz bandwidth, as shown in Fig. 3. This sensitivity was enough to detect a binary neutron star system at an average distance of 15 Mpc away (the distance to the Virgo cluster). The sensitivity was limited at low frequencies by seismic noise, and at high frequency by sensing "shot" noise.

The Virgo detector in Cascina, Italy, is operated by the European Gravitational Observatory [84]. The vacuum facility has perpendicular 3km long arms. The "initial" Virgo detector began taking data in 2007, together with LIGO detectors, with the first Virgo science run (VSR1), and a last science run (VSR4) in 2010 [82]. In 2011 it began installing an "advanced" detector in the same facility. Similar to LIGO detectors, the initial Virgo detector was a power-recycled Fabry–Perot interferometer. Differently from LIGO, however, Virgo uses a multi-stage pendulum approach that provide better seismic isolation than LIGO at lower frequencies [83].

In 2008, both LIGO and Virgo detectors installed some upgrades to test technologies for the future advanced detectors, and to improve the detectors' sensitivity for another joint data taking run. In 2008, a joint LIGO S6 (with two 4km detectors) and Virgo VSR2 science runs started, with typical sensitivities shown in Fig. 4.

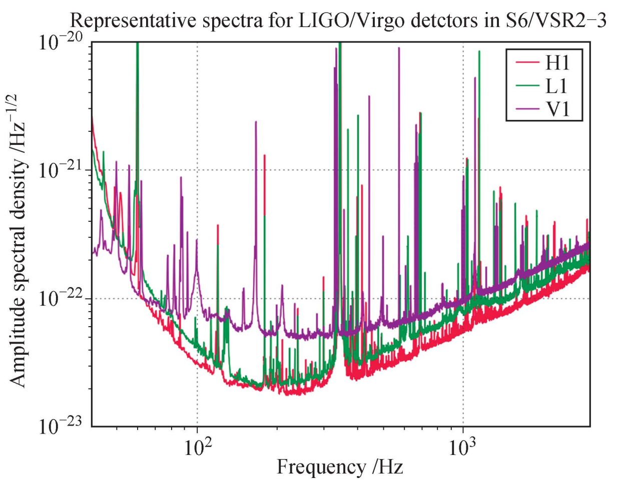

Fig. 4 Representative spectral density curves for LIGO H1 and L1 and Virgo V1 detectors during S6 and VSR2-3, from LIGO-T1100338 in dcc.ligo.org.

In 2008, the installation of "advanced" LIGO detectors was started, and is currently underway, making very good progress. Advanced LIGO detectors use a higher power laser to reduce the shot noise at high frequencies, and a sophisticated active seismic isolation system as well as quadruple pendulums to reduce the seismic noise at low frequencies. It also uses fused silica "monolithic suspensions" for the last pendulum stage of suspended test masses, to reduce the brownian motion limit at low and mid frequencies [85, 86], and signal recycling for improved sensitivity. The advanced LIGO detectors have a designed sensitivity about ten times better than initial LIGO (much more at lower frequencies), and thus will be capable of observing sources from a spatial volume a thousand times larger. These detectors will have an ultimate average sensitivity to binary neutron stars to a distance of 200 Mpc, and the expected detection rates are several ten per year [87].

Although three LIGO 4km detectors were to be installed, one of the two detectors to be installed at the Hanford Observatory is now planned to be installed in a 4 km facility similar to LIGO Observatories in India that would be constructed soon [88]. This will provide a network with significantly improved sky localization of sources [89].

An advanced Virgo detector is also currently being installed, using higher power laser, monolithic suspensions, and signal recycling [90]. Its expected ultimate average sensitivity to binary neutron stars is to a distance of 130 Mpc.

It is expected that the advanced LIGO detectors at Hanford and Livingston will begin operating in 2015, and gradually achieve the expected sensitivities with many plausible detections. The Virgo detector will soon join the network, improving the chances for detection and providing localization. The network will be further increased with the LIGO-India and KAGRA detectors. Since the detection probability increases with the number of detectors, it is very likely that gravitational wave detections will happen in this decade, starting the new era of gravitational wave astronomy.

4 Observational results from interferometers and prospects

The first generation of large scale interferometers has been operated now over several years, with LIGO instruments being the first to achieve the planned sensitivity in 2005, followed by Virgo in 2007.

In this part we wish to recapitulate the observational results obtained, which also give a sense of the steady progress made possible by the parallel improvement of the detectors and of the analysis methods, and we briefly comment on the future directions of development.

A first, broad classification of searches is possible, considering the two wide classes of sources; in Section 4.1 we discuss the impulsive ones, like compact binary coalescences or supernova events, and in Section 4.2 the continuous ones, like the periodic signals by spinning, distorted neutron stars, and the stochastic signals expected for instance as cosmological remnants.

In Section 4.3 we briefly mention the most recent developments and observational prospects for the impending advanced detectors era.

4.1 Impulsive sources

For convenience of discussion, in Section 4.1.1 we consider searches for coalescing binaries, whereas in Section 4.1.2 we discuss generally unmodeled signals, which were not triggered by external events. We reserve to Section 4.1.3 a discussion of externally triggered searches, carried out to look for coincidences with electromagnetic and neutrino events.

4.1.1 Compact binary coalescences

The first relevant observational results on systems with masses in the range 1–3 $M_\odot$ have been placed using LIGO S1 data [91]: the sensitivity could cover only the Milky Way and its satellites, the Magellanic Clouds, and allowed to establish an upper limit on the rate $R < 1.7 \times 10^2$/year/MWEG (Milky Way Equivalent Galaxy). Extra-galactic sources, up to about 1.5 Mpc, became accessible with S2 data: in Ref. [92], using less than 15 days of run, the same rate limit could be pushed down to $R < 47$/year/MWEG. Essentially the same limit ($R < 49$/year/MWEG) was obtained in a coincidence search between LIGO S2 data and TAMA300 data [93].

Data of the same S2 run allowed to search also for primordial black-holes, with masses in the range 0.2–1 $M_\odot$, which could possibly harbor in the galactic halo: a coincident search using the two LIGO 4km instruments [94] allowed to set a rate limit $R < 63$/year/MWH (Milky Way Halo). On the other side of the mass spectrum, for larger component masses in the range 3–20 $M_\odot$, with the same S2 data, no events were detected within a 1 Mpc radius [95]; in this study, a phenomenological family of waveforms was used to capture effects beyond the inspiral phase. Poor models of the source population prevented to set a firm upper limit, although in a class of BBH systems a tentative limit $R < 35$/year/MWEG could be estimated.

In larger mass systems, spin effects are anticipated to be relevant, and induce not only changes in the signal's phase, but also a precession of the orbit, resulting into a modulation of the signal's amplitude: such effects were systematically investigated using LIGO S3 data [96], exploring the asymmetric range $1M_\odot < m_1 < 3M_\odot$, $12M_\odot < m_2 < 20M_\odot$. This search was sensitive to sources in the Milky Way and to a good fraction of sources in galaxies M31 (Andromeda) and M33 (Triangulum), establishing an upper limit $R < 15.9/\text{year}/L_{10}$, where $L_{10} = 10^{10}$ times the luminosity in the blue of the Sun, as a scale of galaxy size.

A search for the inspiral phase of coalescing binary signals, neglecting spin effects, but combining both S3 and S4 data (1364 hours), resulted in improved limits on several classes of sources [97], thanks to the improved search range, boosted to tens of Mpc. No signals were

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detected, and with specific hypotheses about the population of binaries, upper limits were derived: for primordial black hole binaries with masses in the range $0.35M_\odot < m_1, m_2 < 1.0M_\odot$, a limit $R < 4.9\,\mathrm{yr}^{-1}\,L_{10}^{-1}$; for binary neutron stars in the range $1.0M_\odot < m_1, m_2 < 3.0M_\odot$, a limit $R < 1.2\,\mathrm{yr}^{-1}\,L_{10}^{-1}$; and for "stellar-mass" black holes with masses in $3.0M_\odot < m_1, m_2 < M_{max}$, with $m_1 + m_2 < M_{max}$ and $M_{max} = 40.0M_\odot, 80.0M_\odot$ in S3 and S4 respectively, a limit $R < 0.5\,\mathrm{yr}^{-1}\,L_{10}^{-1}$.

With the S5 run, thanks to LIGO achieving design sensitivity, the searches were much improved: the first year of data [98] allowed to search for low mass binaries, with $M_{tot} \in [2, 35]M_\odot$ and $1.0M_\odot < m_1$, with a range as far as 150Mpc. Again, with specific hypotheses on the distribution of sources, the search derived upper limits on NS-NS, NS-BH and BH-BH coalescences: respectively, $R_{\text{NS-NS}} < 3.9 \times 10^{-2}\,\mathrm{yr}^{-1}\,L_{10}^{-1}$, $R_{\text{NS-BH}} < 1.1 \times 10^{-2}\,\mathrm{yr}^{-1}\,L_{10}^{-1}$ and $R_{\text{BH-BH}} < 2.5 \times 10^{-3}\,\mathrm{yr}^{-1}\,L_{10}^{-1}$. These limits were further refined by the analysis of 186 days of the second S5 year [99], achieving $R_{\text{NS-NS}} < 1.4 \times 10^{-2}\,\mathrm{yr}^{-1}\,L_{10}^{-1}$, $R_{\text{NS-BH}} < 3.6 \times 10^{-3}\,\mathrm{yr}^{-1}\,L_{10}^{-1}$ and $R_{\text{BH-BH}} < 7.3 \times 10^{-4}\,\mathrm{yr}^{-1}\,L_{10}^{-1}$.

In the second half of 2007 Virgo performed its first scientific run (VSR1), and the combined analysis with LIGO S5 data allowed to further improve the limits [100], down to $R_{\text{NS-NS}} < 8.7 \times 10^{-3}\mathrm{yr}^{-1}L_{10}^{-1}$, $R_{\text{NS-BH}} < 2.2 \times 10^{-3}\mathrm{yr}^{-1}L_{10}^{-1}$ and $R_{\text{BH-BH}} < 4.4 \times 10^{-4}\mathrm{yr}^{-1}L_{10}^{-1}$.

The S5 and VSR1 data were exploited also for a search for Intermediate Mass Binary Black Holes (IMBH) with $M_{tot} \in [100, 450]M_\odot$ and $m_1/m_2 \in [1, 4]$; no signals were detected, and the best limit achieved (for $m_1 = m_2 = 88M_\odot$)was $R < 0.3 \times 10^{-6}\,\mathrm{yr}^{-1}\mathrm{Mpc}^{-3}$, for non-spinning sources.

Both detectors improved further, particularly at low frequencies, and the analysis of LIGO S6 and Virgo VSR2, 3 data provides on low-mass binaries the best results so far [101]: assuming $M_{tot} \in [2, 25]M_\odot$ the search had a reach for NS-NS as far as 40 Mpc, and better for larger mass systems, obtaining $R_{\text{NS-NS}} < 1.3 \times 10^{-4}\mathrm{yr}^{-1}\mathrm{Mpc}^{-3}$, $R_{\text{NS-BH}} < 3.1 \times 10^{-5}\mathrm{yr}^{-1}\mathrm{Mpc}^{-3}$ and $R_{\text{BH-BH}} < 6.4 \times 10^{-6}\mathrm{yr}^{-1}\mathrm{Mpc}^{-3}$. The same data allowed to improve limits on binary black hole events, with an analysis which exploited all phases (inspiral, merger, ringdown) of the signal [102]: for $M_{tot} \in [25, 100]M_\odot$, the search achieved a range of 300 Mpc for (20, 20) BBH systems, and set a limit $R < 3.3 \times 10^{-7}\mathrm{yr}^{-1}\mathrm{Mpc}^{-3}$ for systems with non-spinning components in the $19-28\ M_\odot$ range.

4.1.2 Other impulsive signals

For signals of unknown shape, it is generally convenient to define the signal strength not in terms of amplitudes, but of other measures like the "root sum square" (rss) amplitude, defined for a signal $h(t)$, as received by the detector, as

$$h_{rss} \equiv \sqrt{\int |h(t)|^2\,\mathrm{d}t} \tag{33}$$

note that $h(t) = F_+h_+(t) + F_\times h_\times(t)$ in terms of the polarizations of the waveform and the detector's antenna patterns $F_{+,\times}$: hence h_{rss} at the detector is in general different from the "power content" of the signal, possibly defined as

$$h^{sig}_{rss} \equiv \sqrt{\int (|h_+(t)|^2 + |h_\times(t)|^2)\mathrm{d}t} \tag{34}$$

The relation with the real energy flux, across an area element $\mathrm{d}A$, depends on the signal waveform, as

$$\frac{\mathrm{d}^2E_{\mathrm{GW}}}{\mathrm{d}A\,\mathrm{d}t} = \frac{c^3}{16\pi G_N}\langle(\dot{h}_+(t))^2 + (\dot{h}_\times(t))^2\rangle \tag{35}$$

or equivalently

$$\frac{\mathrm{d}^2E_{\mathrm{GW}}}{\mathrm{d}A\,\mathrm{d}f} = \frac{\pi c^3}{2G_N}f^2[|\tilde{h}_+(f)|^2 + |\tilde{h}_\times(f)|^2] \tag{36}$$

which can be related to h^{sig}_{rss}. For instance for a sine-Gaussian signal with central frequency f_0 and quality factor Q

$$h_{sg}(t) = h_c \sin(2\pi f_0 t)\exp[-(2\pi f_0 t)^2/(2Q^2)] \tag{37}$$

emitted by a source at (non-cosmological) distance r, one obtains

$$E_{\mathrm{GW}} = \frac{r^2c^3}{4\pi}(2\pi f_0)^2h^2_{rss} \tag{38}$$

Another possible measure of amplitude is a *characteristic* strain amplitude

$$h_{char} \equiv f_c|\tilde{h}(f_{char})| \tag{39}$$

where $\tilde{h}$ is the Fourier transform of the signal and f_{char} is some characteristic frequency; for instance, the frequency at which $|\tilde{h}|$ peaks, or at which the detector's sensitivity is best.

A first search with interferometers was carried out on LIGO S1 data, focusing on signals between 4ms and 100ms of duration, in the [150, 3000] Hz band [103]: with sensitivities in the range $h_{rss} \sim 10^{-19}$–$10^{-17}\mathrm{Hz}^{-1/2}$, calibrated on gaussian and sine-gaussian signals, the search reported less than 1.6 events / day.

With better LIGO S2 data, and improved search methods based on wavelet decomposition, higher sensitivities were achieved [104]: focusing again on signals much shorter than 1s, in the 100–1100 Hz range, signals as weak as $h_{rss} \sim 10^{-20}$–$10^{-19}\,\mathrm{Hz}^{-1/2}$ could be searched for in about 10 days of data, and an upper limit of 0.26 events / day could be established. At these sensitivities, supernovae events could be detected only up to about 100 pc, and black-hole mergers up to about 1 Mpc: therefore a non-detection was expected. With 8 days of S3 data [105], the search sensitivity in the same bandwidth was improved, to achieve a firm $h_{rss} \sim 10^{-20}\mathrm{Hz}^{-1/2}$.

The LIGO S2 data were also searched in coincidence with data from the TAMA detector [106], considering signals of millisecond duration: no signal was found in 473 hours of coincident data, with a sensitivity $1-3\times 10^{-19}\mathrm{Hz}^{-1/2}$ in the frequency range 700–2000 Hz. The most interesting result of this study was the development of methods for analyzing data from heterogeneous instrumements.

Another interesting methodological study was carried out using data from the S3 LIGO run in coincidence with the Auriga resonant bar detector [107]: a little more than 100 hours of data could be exploited, and the search was about as sensitive as the LIGO S2 analysis, but allowed to exploit also times at which only one LIGO detector was operating.

Better LIGO S4 data allowed a much improved search [108]: the bandwith was extended to 64–1600 Hz and the sensitivity was at least one order of magnitude better than with the S2 search, allowing for instance to potentially detect the merger of two $10M_\odot$ black holes (which could emit $E_{\mathrm{GW}} \sim 0.7M_\odot c^2$) up to about 1.4 Mpc, and up to 60 Mpc for two $50M_\odot$ black holes. With about two weeks of data, an upper limit on the rate of 0.15 events/day could be set.

The same LIGO S4 data were analyzed also in coincidence with GEO600 data [109], covering about one month of calendar time and focusing on the 768–2048 Hz frequency band. The study allowed to demonstrate how to include detectors with different sensitivities in a network, without being limited by the least sensitive instrument.

A search for signals emitted by cosmic string cusps was carried out with S4 data as well [110], in about two weeks of data: the search allowed to place limits on string tension and strings reconnection probability, albeit weaker than previous, indirect ones [111].

A single detector search was performed with the Virgo detector using C7 commissioning data [112]: the development of extensive veto procedures allowed to achieve a sensitivity $h_{rss} \sim 10^{-20}\mathrm{Hz}^{-1/2}$ even with a single instrument, corresponding to a maximum detection distance for BH-BH merger of about 2.9 Mpc; an upper limit of less than 1.1 events per day was thus set. The same Virgo C7 data, in coincidence with the Auriga, Nautilus and Explorer resonant bar detectors, was exploited to carry out a joint search [113], targeting signals emitted by the galactic center, mainly to develop methods for coincidence search with detectors of widely different bandwidth.

With the S5 run, the LIGO detectors achieved the design sensitivity and the first calendar year was exploited to search for bursts in the 1–6 kHz frequency band [114]: an analysis of 161.3 days of triple-coincident data allowed to set an upper limit of 5.4 events/year, at a sensitivity which could be as low as $h^{50\%}_{rss} \simeq 3\times 10^{-21}\mathrm{Hz}^{-1/2}$ for sine-Gaussians in the most sensitive frequency window, as measured by a 50% detection efficiency. From the astrophysical point of view, this sensitivity would have allowed to detect a core-collapse only if occurring significantly closer than 1 kpc.

The same first year S5 data were exploited to extend the burst search to lower frequencies [115], covering the 64–2000 Hz band: using double and triple coincident data, the livetime was about 286 days and the sensitivity was remarkably better than previous results, with h_{rss} in the range $6\times 10^{-22}\mathrm{Hz}^{-1/2}$ to a few times in $10^{-21}\mathrm{Hz}^{-1/2}$, setting an upper limit of less than 3.6 events/year on strong GW bursts. With respect to the S4 run, this study allowed to improve the sensitivity, in terms of energy of the source, by about a factor 5.

Virgo joined the LIGO and GEO600 detectors for the final part of the S5 run, contributing its VSR1 data: the same 64–2048 Hz bandwidth was searched for events, using 266 days of data with at least two detectors in operation [116]. Combining the analysis with the previous S5 search, the overall upper limit was pushed down to 2.0 events/year, with a similar sensitivity in the range 6×10^{-22}–$2\times 10^{-20}\mathrm{Hz}^{-1/2}$.

The joint LIGO S6 run, and Virgo VSR2, VSR3 runs allowed to further improve the limits, achieving the best results so far for an untriggered search [117]: over a total observation time of 207 days, signals shorter than 1s were targeted over a 64–5000 Hz frequency band, setting an upper limit on the rate of 1.3 events/year, with a sensitivity analogous to the one of the previous run, in the range 5×10^{-22}–$1\times 10^{-20}\mathrm{Hz}^{-1/2}$ depending on the test waveform. These sensitivities would allow to detect, with 50% efficiency, sources emitting an energy of about $2.2\times 10^{-8}M_\odot c^2$ at 10 kpc, a plausible scale for a galactic supernova.

4.1.3 Triggered searches

An issue with interferometric detectors of gravitational waves is that there is no way to measure the noise background in a single instrument, since GWs cannot be "shielded"; moreover, a single instrument has a rather poor directionality. Both issues are partially remedied by a network of detectors: introducing time-shifts between the different datasets allows to estimate the background of coincident events, at least in the regime of low trigger-rate; by requiring, instead, precise time delays and constraining the relative amplitudes, one can "aim" the analysis, favoring specific directions in the sky.

The value of these procedures becomes evident when we consider the search for events in GW data which are coincident with external triggers, for which we generally know precisely both the sky localization and the timing.

A first such analysis with interferometric data was performed to search events coincident with GRB030329: this γ-ray burst was detected on March 29, 2003 by the HETE-2 satellite [118], and its redshift [119] ($z = 0.1685$) and distance [120] ($d \sim 800$ Mpc) were both estimated thanks to a bright after-glow. The spectrum characteristics allowed to associate this long GRB with a supernova, named SN2003dh [121]. An analysis was performed using LIGO S2 data [122]: only the two Hanford instruments were operating at the event time, and the background could be estimated by analyzing data not only "on-source", but also "off-source", that is at different epochs; this is important since for coincident instruments it cannot be excluded the presence of correlated noise, that wouldn't be accounted for in a background estimation made introducing a relative time-shift in the data. The GW search was carried out in the 80–2048 Hz frequency band, looking for excess correlated power in the data of the two instruments, and achieved a sensitivity limit $h_{rss} \sim 6 \times 10^{-21}\mathrm{Hz}^{-1/2}$, detecting no coincident events: which is not surprising, since at this sensitivity this relatively distant source should have emitted an energy in GW $E_{\mathrm{GW}} \sim 2$–$3 \times 10^4 M_\odot c^2$ in order to be detectable, far more than one could reasonably expect.

A similar method was applied to data of LIGO S2, S3 and S4 runs to investigate 39 γ-ray bursts [123]: these events were a mixture of long bursts, generally associated with core-collapse supernovae [124], and short bursts, believed to be associated with coalescing binaries [125], but in this search no distinction was made among different signals. The search placed upper limits on h_{rss} for individual GRB events, and achieved a maximum sensitivity, for events coincident with S4 data, of the order of $1M_\odot$ at distances of tens of Mpc.

A different approach was used by the Virgo Collaboration to analyze data in coincidence with GRB050915a, a long GRB occurred during the C7 commissioning run: the data of the single instrument required to rely solely on the use of "on-source" and "off-source" observation in order to reject the background [126]. However, the search sensitivity was similar to the one for GRB030329, with a comparable limit $h_{rss} < O(10^{-20}\mathrm{Hz}^{-1/2})$, demonstrating that single instrument triggered searches are possible.

With the improvement of GW sensitivity, considerable interest was raised by GRB070201, which was a short hard γ-ray burst localized electromagnetically in the direction of the spiral arm of M31 (Andromeda), just ~ 770 kpc away! The S5 run was ongoing, but only the LIGO Hanford detectors were taking data, which were analyzed under the assumption of a coalescing binary progenitor, for which a search was carried out assuming $1 < m_1/M_\odot < 3$, $1 < m_2/M_\odot < 40$. The search could exclude at 99% level the occurrence of a coalescence in M31 (a galaxy comfortably within LIGO range), thus implying that, if associated with a coalescence, this GRB should have been emitted further away [127]. If not a coalescence in M31, the event could have been due to a soft γ-ray repeater (SGR), a source emitting periodically short (~ 0.1 s) flares of soft γ rays, with peak luminosity up to 10^{42}erg/s. These sources could be magnetars in which crust rearrangements occur, thus exciting non-radial modes which decay emitting also GW [128]. In the same paper, also a search for generic bursts of GWs was carried out, which set an upper limit of less than $4.4 \times 10^{-4} M_\odot c^2 = 7.9 \times 10^{50}$erg on the emitted energy, thus implying that the non-observation of a GW counterpart is compatible with the SGR hypothesis.

A similar case occurred with GRB 051103, an event occurring shortly before the start of S5 run: since LIGO data had been acquired, and considering the short hard nature of the signal, and the possible association with M81, about 3.6 Mpc away, two GW searches were recently carried out [129]: in the first one, it was made the assumption of a NS-NS or NS-BH progenitor system, with $m_{\mathrm{NS}} \in [1, 3]\, M_\odot$ and for the companion $m_{COMP} \in [1, 25]\, M_\odot$, searched for using matched filtering. The second search targeted generic burst events. Neither search found evidence of coincidences, and the matched filter search allowed to confidently exclude, at confidence better than 98%, an NS-NS or NS-BH coalescence in M81, under the weak assumption of a beamed electromagnetic emission, hence assuming a system favourably oriented for emitting also GW towards Earth.

A more extensive search for GW coincident with SGR events was carried out using data of the first year of S5 run [130]: the search covered the giant flare of 27 Dec. 2004 from SGR 1806–20 and 190 lesser events from SGR 1806–20 and SGR 1900+14, finding no evidence of GW counterparts. For different model waveforms it is possible to deduce limits on h_{rss}, and assuming a distance $O(10$ kpc) it was possible to set limits on the energy as low as 9×10^{45}erg, and limits on the ratio $E_{\rm GW}/E_{EM}$ as low as 30, which are compatible with some theoretical models.

The repeating nature of soft γ-ray burst was exploited for a GW search in coincidence with the 2006, March 29 SGR 1900+14 storm [131]: LIGO data acquired at the times of the bursts were "stacked" to attempt detecting a cumulative effect. Under assumptions about the emission model, this method allowed to set improved upper limits, ranging from 2×10^{45}erg to 6×10^{50}erg, on individual bursts, again assuming nominal 10 kpc distance.

The LIGO S5 and Virgo VSR1 data together allowed to carry out two separate GRB-triggered searches, appropriate for generic bursts or for inspiral signals. In Ref. [132] an analysis was carried out at times of 137 GRBs, long and short, mostly detected by the Swift satellite, looking for generic signals shorter than ~ 1 s, with cross-correlation methods. No weak GW signals were detected, neither for individual GRBs, nor for the overall sample, and upper limits could be placed on h_{rss} in the range of a few units to a few tens of $10^{-22}\text{Hz}^{-1/2}$: better limits were obtained for signals close to 150 Hz, the "sweet-spot" of detectors' sensitivities. Assuming an isotropic GW emission energy $E_{\rm GW} \simeq 0.01 M_\odot c^2$, the non-detection translates in a lower limit to the source distance which could be as large as $D = 26$ Mpc for GRB 070429B; since the smallest distance in the sample was $D \simeq 578$ Mpc for GRB 060614, this limit was still too small to constrain GRB physics. In Ref. [132] a sample of 22 short GRBs was investigated looking for coincidence with inspiral signals originating by binaries with total mass in the range $[2, 40)M_\odot$; as for the previous search, the non-detection translated into a lower limit for the source distance, whose median value was 3.3 Mpc for NS-NS systems, and 6.7 Mpc for BH-NS systems, far smaller than the expected values for the sampled population of GRB, whose median redshift peaks at $z \simeq 0.25$, which translates into distances $O\,(1$ Gpc).

A different kind of "triggered" search was carried out in correspondence of a timing glitch of the Vela pulsar, that is, a sudden change in its rotational frequency, of a few parts per million, observed in the radio spectrum on August 12th, 2006, during the LIGO S5 run: under the assumption that the glitch was due to a "star-quake", which could have excited NS's normal modes, a search for oscillatory, damped waveforms was carried out [133]. Thanks to an accurate determination of the glitch epoch, the analysis could assume a narrow "on-source" window, of 120 s, and use the rest of a long stretch of contiguous data to estimate the noise background: under the assumption of signals in the 1–3 kHz range and damping time in the [50, 500] ms range, an upper limit on the signal amplitude ranging from $6.3\times10^{-21}\text{Hz}^{-1/2}$ to $1.4\times10^{-20}\text{Hz}^{-1/2}$ could be placed, depending on the excited mode, corresponding to energy limits from 5×10^{44} to 1.3×10^{45}erg.

Data from GEO600, LIGO and Virgo detectors acquired between 2006 and 2009 were searched for GW bursts in coincidence with soft γ-ray bursts from six galactic magnetars [134]: in a sample of 1279 γ events, no evidence of coincident GW burst was found, and upper limits could be set which were particularly interesting for SGR 0501+4516, a source likely to be just 1 kpc away: the deduced energy upper limits were about 3×10^{44}erg assuming band, time-limited white-noise bursts, and about 1.4×10^{47}erg assuming f-mode ringdowns, thus improving about a factor of 10 wrt previous results.

Most recent LIGO and Virgo data, from runs S6, VSR2 and VSR3 were searched for GW events in coincidence with a sample of 154 GRBs, as usual searching both for coalescences of compact object pairs, or generic bursts [135]: under the same assumptions as in the previous S5, VSR1 search, namely $E_{\rm GW} \simeq 10^{-2} M_\odot c^2$, an overall median lower limit on the distance of $D \sim 17$ Mpc could be deduced, whereas for NS-NS and NS-BH system the median limit was 16 and 28 Mpc respectively, significantly better than a search not aided by electromagnetic observations.

4.2 Continuous sources

4.2.1 Periodic signals

The first astrophysical result, based on S1 data by LIGO and GEO600 detectors, was a limit on the emission by a specific pulsar, PSR1939+2134 [136], with known location, a study which allowed to develop and test the methodology, and provided also a robust limit on the characteristic amplitude $h_c < 2.7\times10^{-22}$.

With the progress of the detectors, it was later possible, using S2 LIGO data, to carry out a first all-sky search [137] based on a sub-optimal method, the Hough transform: the method allowed to impose upper limits, depending on the frequency, as low as $h_c < 4.43\times10^{-23}$.

Data of the same S2 run could be exploited also for carrying out a targeted search, aiming at signals potentially emitted by 28 (isolated) radio pulsars: by incorporating phase information deduced by radio observations, the search was able to set limits on h_c as low as a few 10^{-24}, limits which could be translated into ellipticities $\epsilon < 10^{-5}$ for the four closest pulsars, thus starting to probe plausible values.

Although computationally very expensive, optimal searches were also attempted on S2 data by limiting the analysis to the most sensitive portions of data [138]: a first search used 10 hours of data, covering the whole sky in the 160–728.8 Hz frequency band, and assuming $\dot{\nu} < 4 \times 10^{-10}$Hz/s; a second search used 6 hours, targeting the accreting neutron star in the low-mass x-ray binary Scorpius X-1, in the frequency bands 464–484 Hz and 604–624 Hz. It is worth noticing that a search for periodic signals is essentially carried out on single detector data: however, in this work, the background was lowered by requesting coincidence between candidate signals in LIGO Hanford and LIGO Livingston data. The search obtained limits on h_c ranging from 6.6×10^{-23} to 10^{-21}, depending on the frequency, whereas for Scorpius X-1 the limits range from 1.7×10^{-22} to 1.3×10^{-21} across the two 20 Hz frequency bands.

With the progress of the detectors, limits improved considerably: using LIGO and GEO600 data from S3 and S4 runs, combined in a coherent way, it was possible to target 78 radio pulsars, including for the first time pulsars in binary systems, thus accounting for the additional orbital signal modulation [139]. For isolated pulsars, the known upper limits were tightened by about a factor 10, obtaining $h_c < 2.6 \times 10^{-25}$ for PSR J1603–7202, and $\epsilon < 10^{-6}$ for PSR J2124–3358; noticeably, the strain upper limit for the Crab pulsar obtained was just 2.2 times larger than ϵ_{max} of Eq. (23).

The S4 data were also searched for periodic signals emitted by unknown sources, over a large frequency band [50–1000] Hz, using sub-optimal methods and limiting $\dot{\nu} \in [-10^{-8}\text{Hz/s}, 0]$; again a null result was obtained [140], which interpreted in terms of isolated neutron stars, over the whole sky and assuming isotropic distribution of spin axis, yields a best limit $h_c < 4.28 \times 10^{-24}$, near 140 Hz.

A portion (510 hours) of the same S4 data was also searched in a larger band [50–1500] Hz using an innovative technology, Einstein@Home, which distributed the search computational loads on a large number of computers (approximately 10^5) provided voluntarily by the general public [141]. The large computing power allowed to deconvolve the Doppler effect over relatively long (30 hours) data stretches, obtaining a null result with confidence that 90% of the sources in the 100–200 Hz band having $h_c > 10^{-23}$ would have been detected.

A major achievement was obtained on the Crab pulsar using the first 9 months of the LIGO S5 run: the ϵ_{max} limit was beaten [142], and the limit obtained, $\epsilon < 1.8 \times 10^{-4}$, was about 4 times smaller than ϵ_{max}, and therefore very significant, although probably still far from plausible values.

The same early S5 data were exploited also to carry out an all-sky search in the frequency range 50–1100 Hz and with $\dot{\nu} \in [-5 \times 10^{-9}\text{Hz/s}, 0]$; the improved sensitivity allowed to set limits below 10^{-24} over a 200 Hz band, and to increase the spatial range of the search: for instance, for neutron stars with a nominal ellipticity $\epsilon = 10^{-6}$, the search was sensitive to distances as large as 500 pc.

A subset of these early S5 data, namely 840 hours from 66 days, were also searched with the Einstein@Home system in the 50–1500 Hz frequency range [143], improving over the previous search of this kind by a factor of 3: in the 125–225 Hz band, more than 90% of sources with $h_c > 3 \times 10^{-24}$ would have been detected.

The youngest known neutron star or black hole, an X-ray source which is a remnant of Cassiopeia A supernova, was also object of a targeted search using 12 days of the S5 run: the frequency range 100-300 Hz was explored, over a wide range of models for the frequency evolution. Under the assumption that the X-ray source is a neutron star, it was possible to place upper limits on $h_c < (0.7\text{–}1.2) \times 10^{-24}$ and on $\epsilon < (0.4\text{–}4) \times 10^{-4}$, beating indirect limits based on energy conservation. Interestingly, this paper also placed for the first time limits on the r-mode oscillation, potentially excited in the hypothetical young NS.

The targeted search was further extended to 116 known millisecond and young pulsars using again S5 data [144]: for the first time, for all targets radio and X-ray observations were exploited to deduce ephemerides for the phase of the signal. This search allowed to set a new limit on the Crab's GW emission, a factor seven below the spin-down limit, which means less than ~2% of the available spin-down power; for several of the other young pulsars, the limits are only slightly above the spin-down limits. The best results were obtained for J1603–720, with a limit $h_c < 2.3 \times 10^{-26}$, and for J2124–3358, with a limit $\varepsilon < 7 \times 10^{-8}$.

The all-sky search for periodic signals was further improved using the whole LIGO S5 data [145], including the portion of the run in coincidence with Virgo VSR1 data. The frequency band 50–800 Hz was searched, assuming $\dot{\nu} \in [-6 \times 10^{-9}\text{Hz/s}, 0]$. Near the "sweet spot" of the sensitivity, at 150 Hz, an upper limit of $h_c < 10^{-24}$ was obtained for linearly polarized signals, whereas at the upper end of the frequency band a limit $h_c < 3.8 \times 10^{-24}$ was obtained for all polarizations and sky locations.

Using Virgo VSR2 data, thanks to the good low-frequency sensitivity, an upper limit on the emission by the Vela pulsar could be placed [146]: on the assumption that the phase of the GW signal can be modeled on the radio emission, and that spin axis and wave polarization angles are known, it was possible to set limits in the interval $h_c < (1.9\text{–}2.2)\times10^{-24}$, thus beating the spin down limit $h_{c,max} = 3.3\times10^{-24}$ derived from the pulsar's period evolution. These limits correspond, though, to an ellipticity $\epsilon < 10^{-3}$, still fairly high. It is worth noticing that the spin down is beaten also if the assumptions on the pulsar's spin axis and wave polarization angles are released.

4.2.2 Stochastic background

The stochastic background appears in a single ground based interferometer as an additional noise, and with Eq. (32) we have seen that this would be below the instruments's noise floor, and therefore not detectable.

However, the same noise affects different instruments, and can be exposed by a cross-correlation of their outputs: this correlation, for a broad-band stochastic background, can be written as

$$\langle h_l(t)h_m(t')\rangle = \int \mathrm{e}^{\mathrm{i}2\pi f(t-t')}\gamma(r_{lm},\, f)H(f)\,\mathrm{d}f \qquad (40)$$

where γ is the so-called "overlap reduction function" depending on the product $f\times r_{lm}$, with r_{lm} the distance between the two detectors l, m. This function is 1 for co-located, parallel instruments, like the two LIGO-I detectors at the Hanford site; for distant detectors, it is similar to a sinc function of the argument $f\times r$, displaying an oscillating pattern of decorrelation.

It has not been possible, as of today, using co-located instruments to measure $H(f)$, because of the difficulty to rule out local, correlated sources of gaussian noise. The use of distant detectors, instead, made possible to set limits on $H(f)$, or equivalently on $h_0^2\Omega_{\mathrm{GW}}(f)$, using first the two LIGO sites of Livingston and Hanford, then LIGO and resonant bar detectors, and finally LIGO and Virgo detectors.

With data of summer 2002, the LSC could obtain a first rough limit [147]

$$h_0^2\Omega_{\mathrm{GW}} \lessapprox 23\pm4.6 \qquad \text{in 40–314 Hz} \qquad (41)$$

which was quickly pushed down by a factor 10^5 with the S3 run, which allowed to set [148]

$$h_0^2\Omega_{\mathrm{GW}} < 8.4\times10^{-4} \qquad \text{in 69–156 Hz} \qquad (42)$$

A correlation between the LIGO Livingston interferometer and the Allegro resonant bar allowed to set a limit also at high frequencies [149]:

$$h_0^2\Omega_{\mathrm{GW}} < 0.53 \qquad @\,915\text{ Hz} \qquad (43)$$

Further improvement was obtained during the run S4, which yielded [150]

$$h_0^2\Omega_{\mathrm{GW}} < 6.5\times10^{-5} \qquad \text{in 51–150 Hz} \qquad (44)$$

and finally during the S5-VSR1 runs, which resulted in the best limit so far [151]:

$$h_0^2\Omega_{\mathrm{GW}} < 6.9\times10^{-6} \qquad @\,100\text{ Hz} \qquad (45)$$

a result which improved over direct limits from Big Bang nucleosynthesis and from CMB. During the same run, a limit at high frequencies could be set using the LIGO and Virgo detectors [152]:

$$h_0^2\Omega_{\mathrm{GW}} = h_0^2\Omega_3\left(\frac{f}{900\text{ Hz}}\right)^3 < 0.16\times\left(\frac{f}{900\text{ Hz}}\right)^3 \quad \text{in 600–1000 Hz} \qquad (46)$$

Relaxing the assumption of complete isotropy of the stochastic background, it was possible to investigate potential anisotropies: in particular data of the S4 run were used to search for point-like sources [153], obtaining a limit, for broad band spectra, of

$$H^2(f) < 8.5-61\times10^{-49}\mathrm{Hz}^{-1} \qquad (47)$$

Data of the S5 run were searched also for the occurrence of extended sources [154], obtaining:

$$H^2(f) < 2-20\times10^{-50}\mathrm{Hz}^{-1}(\text{point sources})$$
$$H^2(f) < 5-35\times10^{-49}\mathrm{Hz}^{-1}(\text{extended sources}) \qquad (48)$$

and limits as low as 7×10^{-25} could be set on the RMS GW strain emitted from interesting sources, like the galactic center or Sco X-1, at the best sensitivity of 160 Hz.

4.3 Recent developments and prospects

We have seen in Section 4.1.3 that GW experiments have carried out several searches of signals triggered by external events. The converse starts to be true: GW candidate events are becoming targets for detectors of electromagnetic or neutrino signals.

The essential idea is to follow-up interesting GW candidates with optical, X-ray or γ observations, to look for transients that could be associated with the same source of the GW signal. The methodology of this approach has been discussed and tested in Ref. [155], where follow-ups by terrestrial telescopes have been considered, and in Ref. [156], where a satellite observatory (Swift) has been used.

Specific challenges had to be confronted: first, a network of three sites like the one formed by LIGO and Virgo detectors is capable of reconstructing the candidate's source position with rather limited accuracy, as bad as tens of degrees, depending on the candidate signal's strength. The lack of accurate localization calls for observing wide areas of the sky, using instruments generally designed with smaller fields of view, thus requiring the development of specific methods for the analysis of images.

Second, a pre-requisite of any follow-up program is a low-latency GW search, capable of delivering candidate events a few minutes after data acquisition: this requires that a number of data processing steps are performed on-time, including calibration, data quality and vetoes, trigger generation and event reconstruction.

In Ref. [155] this program has been implemented and tested, focusing on signals from coalescing pairs of neutron-stars, whereas in Ref. [156] both neutron-star binaries and un-modeled signals were targeted. In both cases, the studies demonstrated that a follow-up is possible, within latencies fully compatible with the science we are interested to harvest with multi-messenger observations.

In the advanced detector era we anticipate that this approach will be further expanded, by covering more extensively the spectrum of GW signals, and by involving several partner observatories, on Earth and in space.

Also the coincidence with neutrino observatories carries a high potential of science, and LIGO and Virgo have an active program of cooperation with instruments like Antares and ICEcube: with Antares, an attempt at identifying coincidences between high-energy neutrinos and GW events has been carried out in Ref. [157], starting the development of a methodology.

What are the future prospects? The advanced detectors, aLIGO and AdV, promise a sensitivity ten times better, in amplitude, than 1st generation instruments: when achieved, this improvement will allow to search for coalescing binary events in a space volume 1000 times larger, with a corresponding increase in the rate of events. As discussed in Ref. [158], under realistic scenario the advanced detectors should not just perform the first detection, but collect a sizable sample of events, and really start gravitational astronomy. Furthermore, for the CBC source, it has been argued that early detections of GWs with reasonable localization would be possible even before the binary merger [34–38].

Analogously, values of $h_{rss} \sim 10^{-23}\mathrm{Hz}^{-1/2}$ will be attained for impulsive sources, making possible to detect supernova events with energies released in GWs as small as a few $10^{-9} M_{\odot} c^2$, a sensitivity which should grant the detection of a galactic supernova.

The same improvement in sensitivity will allow to detect periodic signals ten times fainter, and to probe neutron-star ellipticities as small as $\epsilon \sim 10^{-9}$; for searches of stochastic background, the ten-fold improvement in sensitivity will improve the sensitivity to Ω_{GW} by a factor 100.

It is important to mention that in order to harvest all the wealth of astrophysics that advanced detectors can offer it is necessary to apply sophisticated methods of parameter estimation, whose development has started during the analysis of 1st generation detectors data, and is still ongoing: in general terms, these methods aim at constraining the various physical parameters on the basis of the characteristics of the candidate event. At large enough signal to noise ratio, such methods can rely on standard Fisher information matrix techniques [159], whereas at moderate to low signal to noise ratios, more complex methods are necessary, which estimate the probability distribution of the parameters using a Bayesian inference, and which require heavy Monte Carlo simulations. It is not possible to provide here even a short account of these developments, and the interested reader could for instance consult [160–162], for examples of papers on the subject.

5 Pulsar timing arrays for nanohertz GW

The scientific potential of using pulsar timing to detect GWs directly has long been recognized (e.g, Refs. [163–166]). A passing GW will affect the local space-time metric on the travel path of a radio pulse and can lead to observable fluctuations in its arrival time at Earth. For example, if the passing GW is a sinusoidal wave, the resulting timing fluctuations of the pulse arrival time will exhibit oscillatory behaviour at the same frequency (e.g., Fig. 1 in Ref. [167]). Such fluctuations can be detected by measuring pulse arrival times of well-behaved radio pulsars with sufficient timing resolution. A pulsar timing array therefore consists of a collection of well-behaved and well-observed millisecond radio pulsars. The sensitivity of a PTA lies in the frequency range of a few $\times 10^{-9}$–10^{-6} Hz. The frequency bound is determined by the duration of the pulsars' observation, that is of the order of 10 years, and by the observation frequency, roughly once every 2–3 weeks. The targeted GW wavelength is therefore much shorter than the typical distance of $\sim$ kpc for monitored millisecond pulsars. As a result, the expected timing residual $\delta\tau$ in terms of the dimensionless characteristic amplitude of the GW strain h and the GW angular frequency ω, scales roughly as $\delta\tau \sim h/\omega$.

Three individual projects using pulsar timing arrays to detect gravitational waves are collaborating to form an International Pulsar Timing Array project (IPTA) [168]. Each of the PTAs times around 20 millisecond pulsars each month. The Parkes Pulsar Timing Array

(PPTA) [169, 170] observes 20 millisecond pulsars at 2–3 weeks interval with regular monitoring commenced early 2005 [170]. PPTA has offered the most regularly monitored millisecond pulsars among all PTAs. Ten of the 20 monitored millisecond pulsars have the timing resolution better than 1 μs. The European Pulsar Timing Array (EPTA) [171–173] performs pulsar timing observations using five 100 meter class radio telescopes in Europe. These are the Jodrell Bank Observatory in UK, the Westerbork Synthesis Radio Telescope in Netherlands, the Effelsberg Telescope in Germany, the Nançay Radio Telescopes in France and the Sardinia Radio Telescope to be completed in Italy. EPTA will combine these five telescopes to make a phased array telescope for high precision pulsar timing. The North American Nanohertz Observatory for Gravitational Waves (NANOGW) also carries out regular timing of a dozen or more millisecond pulsars [174–176] on a monthly basis. Their pulsar timing data are collected with the Arecibo Observatory in Puerto Rico and Green Bank Telescope in West Virginia.

5.1 The promising future

The sensitivity of a PTA to detect GWs is going to be improved greatly with the advent of larger and more sensitive radio telescopes over the horizon. The largest-to-be single-dish 500-meter FAST telescope is under construction in China and is scheduled to see the first light in 2016. FAST promises a detection of around 300 millisecond pulsars and the timing resolution can reach around ~ 30 ns [177, 178]. The SKA, the world's largest and most sensitive radio telescope in the 21st century, promises a detection of ~ 6000 millisecond pulsars and an unprecedented timing accuracy possibly down to ~ 10 ns [179]. In the meanwhile, SKA's pathfinders, ASKAP [180, 181] in Australia and Meerkat [182] in South Africa are expected to be operational within the next 10 years. The success of these telescopes will help increase the number of known millisecond pulsars especially in the Southern Hemisphere.

It seems that even with the existing PTAs only, detections of GWs within this decade is possible. At the time of this writing, the pulsar timing data have already been used to set useful limits to possible sources. In particular, Jenet *et al.* [167] ruled out a postulated detection of a supermassive black hole binary [183] using existing pulsar timing data. The limit on stochastic GW background has been gradually improved with a new data set and with improved techniques [184–188]. If the newly proposed theory is correct, the current PTA data set can possibly already detect stochastic background [51, 52]. A more cautious investigation is also in favor of detection within the next few years [53].

5.2 Towards optimal detection and parameter estimation of individual sources

The prospects of detecting GWs from individual supermassive black hole binaries are receiving intensive interest recently. Various detection strategies have been proposed, including those for the "continuous" sinusoidal signals [189–195], the GW memory after the binary merger [61, 196, 197] and GW burst sources [57, 198]. It has become an exciting and not so distant possibility that the PTA observation cannot only be used as a tool to directly detect gravitational waves, the background or the individual sources, but also for parameter study and for cosmology.

One of the main difficulties in using PTA data to detect GWs lies in the fact that the discovery of GWs has to be accomplished with the "detections" of all other astrophysical processes that affect the arrival time of the pulses within the same data set. These include the behavior of the pulsars themselves and the interstellar scattering (see Ref. [199] for an excellent summary and fitting tools). For individual GW sources around nanoHertz frequency range, we have the observation durations that range from less than a cycle of the GW signal to at most a few tens of cycles, making data analysis extremely challenging.

The design of an optimal detection algorithm that uses the full information of the signal is complicated by the fact that the distances to pulsars involved are poorly constrained. In the timing domain, the pulsar timing residual caused by a passing GW can be written in two terms, an Earth term that is related to the GW strain at the Earth at the time of observation, and a Pulsar term related to the GW strain at an earlier time determined by the pulsar's distance and position angle relative to the GW direction. Specifically, the pulse timing residual as a function of time can be written as

$$\tau(t) = H(t) - H(t - \tau_0) \tag{49}$$

where H is some function form, $\tau_0 = (1 - \cos\theta)D/c$ with D the distance to the pulsar, c the speed of light, θ the angle between the GW source and the pulsar as viewed from the Earth. It is clear that the pulsar term modulates both the amplitude and phase of the signal. The pulsar terms of different pulsars will in general encode information of GW wavefront at different times. Even with SKA, the best distance determination would be around 0.1% for a pulsar at the distance of 1 kpc [179]. That is, the distance is uncertain by many GW wavelengths in the domain where detections of individual GW sources are expected. This means the phase modulation caused by the pulsar term is unconstrained except for a very small angle θ. Discarding the pulsar term in the detection algorithm would possibly introduce severe signal-based noise especially for persistent GW signals. On the other hand, a detection of the GW signal will help put constraints on the distances of the pulsars involved [190].

Nevertheless, the feasibility of using the pulsar terms to probe the evolution history of a GW source has been investigated [200]. Moreover, the phase modulation from the pulsar term will be extremely sensitive to the sky direction of the GW. Any information of the pulsar term can therefore help improving the angular resolution of PTAs for a GW source the same way that works for the ground-based detector network [31]. Investigations [193, 201, 202] have shown that without including the information of the pulsar terms, the one-dimension angular resolution of a PTA for an individual GW source scales roughly inversely with the signal to noise ratio, yielding tens of square degrees error ellipses for the GW sky direction at SNR$\sim$ 10. Including the knowledge of the pulsar terms (even not precisely) could possibly make orders-of-magnitude improvement.

6 Space detector for millihertz GW

6.1 eLISA

Space detectors have long been proposed to detect gravitational waves in the mHz range to evade the severe seismic and environmental noise on ground. The proposed missions include the US Big Bang Observatory [203], the Chinese Astrodynamical Space Test of Relativity using Optical Devices (ASTROD) [204, 205] and the Japanese Decihertz Interferometer Gravitational Wave Observatory (DECIGO) [206] (see Ref. [207] for the description of its pathfinder). The most established mission proposed was the Laser Interferometer Space Antenna (LISA), a joint mission of ESA and NASA until 2011 [208, 209]. LISA has now "evolved" into a new version called eLISA [210] which is also called the New Gravitational-wave Observatory (NGO). eLISA/NGO is now a joint effort of seven European countries (France, Germany, Italy, Netherlands, Spain, Switzerland, UK) and ESA, and is supported by the former US LISA project team. The current design of eLISA/NGO consists of one mother and two daughter spacecrafts forming a V-formation with the arm-length of about 1 million km. The three spacecrafts will be placed in solar orbit at the same distance from the Sun as the Earth, trailing the Earth by about 20 degrees. The aim is to launch it after 2020.

Similar to its predecessor LISA, eLISA will detect GWs over a broad band of frequencies, from about 0.1 mHz to 100 mHz. An excellent review about eLISA can be found in Ref. [8] for its mission design, orbits and the science case. The main scientific goals for eLISA include (i) tracing the formation, growth and merger history of the massive black hole population, (ii) precision measurements for massive black holes and probes of the strong gravity regime, and (iii) new physics and cosmology. eLISA will be able to probe bulk motions at times about 3×10^{-18}–3×10^{-10} s after the Big Bang, a period not directly accessible with any other technique. This allows detection of cosmological backgrounds caused by new physics. Since eLISA is going to detect a large number of astrophysical sources up to high redshifts, these sources can serve as standard sirens to establish a cosmic ladder if redshifts to these sources can be determined by other means [211, 212].

ESA's LISA Pathfinder (LPF) mission [213] is scheduled to be launched in 2015. The mission will test the key technology needed for the space detector eLISA, carrying two instruments: the LISA Technology Package (LTP) and the Disturbance Reduction System (DRS). LTP contains two identical proof masses in the form of 46 mm cubes made of gold-platinum, each suspended in its own vacuum enclosure. They shall simulate the observational arrangement for the LISA mission, with the difference that the distance between the proof masses is reduced from 5 million kilometers to 35 centimeters. The DRS includes a set of micro-rockets that aim to control the spacecraft's position to within a millionth of a millimeter.

The PathFinder Mission will lay the foundation not just for eLISA, but for many future space-borne tests of General Relativity.

7 Conclusions

In this paper we have tried to review, in a way which is necessarily incomplete and possibly partial, the status and prospects of gravitational wave astronomy. This exercise leaves a profound impression of the steady progress which has been made possible by the efforts of generations of physicists, and by the support of the funding institutions which have had the courage to invest in this long term endeavor.

Although gravitational waves are still to be directly detected, we feel that solid bases have been laid out, on which this new field of physics will be built and will certainly obtain important results. First of all, the 1st generation ground based detectors has been a tremendous success: the interferometers have achieved or surpassed their design sensitivities, a result by no means obvious when the enterprise started in the '90s. The detectors are now being upgraded towards the 2nd generation, aiming at a factor ten improvement in sensitivity; this is certainly a great challenge, but much less so if put in perspective of the progress in sensitivity steadily obtained during the recent years. Second, the community has learned how to translate theoretical predictions about the sources into concrete analysis methods, and to confront all the challenges due to imperfect instruments and uncertain predictions: this knowledge makes us confident that we will do with 2nd generation detectors the best possible science. We should also remember that the many scientific papers already published provide interesting results *per se*, that already have an impact on our knowledge of the Universe. Third, we have seen that gravitational wave observations are starting to be integrated with other observations, especially electromagnetic but also neutrino ones, a process which promises to offer a deeper and more complete understanding of the sources. Let us add that ground based interferometric detectors are not alone in this adventure: the pulsar timing arrays are already targeting a lower frequency range, where more massive sources could be emitting sufficient GW radiation to be detected in the next few years. And an enormous deal of research has been already carried out for space-based detectors, and for third generation ground detectors, that could one day change the game by providing high signal-to-noise observations of many sources.

All these considerations makes us confident that gravitational wave astronomy is truly emerging as a mature research field, of which this paper is meant to be a modest, *in itinere* review.

Acknowledgements The authors thank Dick Manchester, George Hobbs, David Blair, Jonathan Gair, Thomas Bauer, Marie-Anne Bizouard and David Shoemaker for useful comments. LW acknowledges gratefully funding support from the Australian Research Council. GG acknowledges support from the National Science Foundation, NSF-PHY 0905184 and 1205882. This paper has been assigned LIGO document number LIGO-P1300010.

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