Process Algebraic Architectural Description Languages: Generalizing Component-Oriented Mismatch Detection in the Presence of Nonsynchronous Communications

Addendum to:
"Handling Communications in Process Algebraic Architectural Description Languages: Modeling, Verification, and Implementation"
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Abstract

In the original paper, we showed how to enhance the expressiveness of a typical process algebraic architectural description language by including the capability of representing nonsynchronous communications. In particular, we extended the language by means of additional qualifiers enabling the designer to distinguish among synchronous, semi-synchronous, and asynchronous ports. Moreover, we showed how to modify techniques for detecting coordination mismatches such as the compatibility check for star topologies and the interoperability check for cycle topologies, in such a way that those two checks are applicable also in the presence of nonsynchronous communications. In this addendum, we generalize those results by showing that it is possible to verify in a component-oriented way an arbitrary property of a certain class (not only deadlock) over an entire architectural type having an arbitrary topology (not only stars and cycles) by considering also behavioral variations, exogenous variations, endogenous variations, and multiplicity variations, so to deal with the possible presence of nonsynchronous communications. The proofs are at the basis of some results mentioned in the book "A Process Algebraic Approach to Software Architecture Design" by Alessandro Aldini, Marco Bernardo, and Flavio Corradini, published by Springer in 2010.

1 Introduction

In [3], we showed how to enhance the expressiveness of a typical process algebraic architectural description language by including the capability of representing nonsynchronous communications. We focused on PADL [4, 1] and extended it by means of additional qualifiers enabling the designer to distinguish among synchronous, semi-synchronous, and asynchronous ports.

Semi-synchronous ports are not blocking. A semi-synchronous port of a component succeeds if there is another component ready to communicate with it, otherwise it raises an exception so as not to block the component to which it belongs. For example, a semi-synchronous input port can be used to model accesses to a tuple space via input or read probes. A semi-synchronous output port can instead be used to model the interplay between a graphical user interface and an underlying application, as the former must not block whenever the latter cannot do certain tasks requested by the user.

Likewise, asynchronous ports are not blocking because the beginning and the end of the communications in which these ports are involved are completely decoupled. For instance, an asynchronous output port can be used to model output operations on a tuple space. An asynchronous input port can instead be used to model the periodical check for the presence of information received from an event notification service. The semantic treatment of asynchronous ports requires the addition of implicit repository-like components that implement the decoupling.

As far as verification is concerned, in [3] we showed how to modify techniques for detecting coordination mismatches. In particular, we addressed the compatibility check for star topologies and the interoperability check for cycle topologies, both introduced in [4], in such a way that those two checks are applicable also in the presence of nonsynchronous communications.

This is accomplished by viewing certain activities carried out through semi-synchronous and asynchronous ports as internal activities when performing the above mentioned checks. The reason is that each such activity has a specific outcome and takes place at a specific time instant when considered from the point of view of the individual component executing that activity. However, in the overall architecture, the same activity can raise an exception (if the port is semi-synchronous and the other ports are not ready to communicate with it) or can be delayed (if the port is asynchronous and the communication is buffered). Thus, if we do not regard exceptions and all the activities carried out through asynchronous ports as internal activities at verification time, the compatibility or interoperability check may fail even in the absence of a real coordination mismatch.

In Thms. 4.7 and 4.9 of [3], we showed results based on adaptations of the compatibility check for star topologies and the interoperability check for cycle topologies, which correspond to Thms. 4.5 and 5.2 of [4]. However, in Thms. 4.14, 4.15, 4.23, 4.25, and 4.26 of [1], the results of [4] were generalized by showing that it is possible to verify in a component-oriented way an arbitrary property of a certain class (not only deadlock) over an entire architectural type having an arbitrary topology (not only stars and cycles) by considering also behavioral variations, exogenous variations, endogenous variations, and multiplicity variations.

In this addendum to [3], we extend the general results of [1] to deal with the possible presence of nonsynchronous communications. Observing that Prop. 5.1 of [2] is a slight extension of Thm. 4.7 of [3] and that Props. 5.2 and 5.3 of [2] are slight extensions of Thm. 4.9 of [3], this work gives rise to Thm. 5.1 and Cors. 5.1, 5.2, 5.3, and 5.4 of [2], which respectively extend Thms. 4.14, 4.15, 4.23, 4.25, and 4.26 of [1].

2 Arbitrary Topologies and General Properties

As in [1, 2], we start with an architectural description $\mathcal{A}$ that has an arbitrary topology and a property $\mathcal{P}$ that belongs to the class Ψ of properties each of which (i) is expressed only in terms of the possibility/necessity of executing certain local interactions in a certain order through a logic that does not allow negation to be freely used and (ii) comes equipped with a weak behavioral equivalence $\approx_{\mathcal{P}}$ coarser than weak bisimilarity that preserves $\mathcal{P}$ and is a congruence with respect to static process algebraic operators. Following the notation used in [2], we denote by $\mathcal{F}_{C_1,\dots,C_n}$ the frontier of a set of AEIs $\{C_1, \dots, C_n\}$, by $\mathcal{CU}_C$ the cyclic union of an AEI C, and by $\mathcal{CU}(\kappa)$ the set of cyclic unions generated by a cycle covering algorithm κ; moreover, subscript bbm, standing for before behavioral modifications, replaces bbv, standing for before behavioral variations.

Theorem 5.1 of [2] Let $\mathcal{A}$ be an architectural description and $\mathcal{P} \in \Psi$ be a property for which the following two conditions hold:

1. For each $K \in \mathcal{A}$ belonging to an acyclic portion or to the intersection of some cycle with acyclic portions of the abstract enriched flow graph of $\mathcal{A}$, K is $\mathcal{P}$-compatible with every $C \in \mathcal{B}_K - \mathcal{CU}_K$.

2. If $\mathcal{A}$ is cyclic, then there exists a total cycle covering algorithm κ such that for each cyclic union $\{C_1, \dots, C_n\} \in \mathcal{CU}(\kappa)$:

   (a) If $\mathcal{F}_{C_1,\dots,C_n} = \emptyset$, then there exists $C_j \in \{C_1, \dots, C_n\}$ that $\mathcal{P}$-interoperates with the other AEIs in the cyclic union.

   (b) If $\mathcal{F}_{C_1,\dots,C_n} \neq \emptyset$, then every $C_j \in \mathcal{F}_{C_1,\dots,C_n}$ $\mathcal{P}$-interoperates with the other AEIs in the cyclic union.

   (c) If no $C_j \in \mathcal{F}_{C_1,\dots,C_n}$ is such that $[\![C_j]\!]^{\text{pc;wob}}_{\mathcal{A}}$ satisfies $\mathcal{P}$ and there exists $C_g \in \{C_1, \dots, C_n\} - \mathcal{F}_{C_1,\dots,C_n}$ such that $[\![C_g]\!]^{\text{pc;wob}}_{\mathcal{A}}$ satisfies $\mathcal{P}$, then at least one such C_g $\mathcal{P}$-interoperates with the other AEIs in the cyclic union.

Then $[\![\mathcal{A}]\!]^{\text{pc;}\#\mathcal{A}}_{\text{bbm}}$ satisfies $\mathcal{P}$ iff so does $[\![C]\!]^{\text{pc;wob}}_{\mathcal{A}}$ for some $C \in \mathcal{A}$.

Proof We proceed by induction on the number $m \in \mathbb{N}$ of cycles in the abstract enriched flow graph of $\mathcal{A}$:

- If $m = 0$, then the abstract enriched flow graph of $\mathcal{A}$ is acyclic. We prove the result by induction on the number $s \in \mathbb{N}_{\geq 1}$ of stars in the abstract enriched flow graph of $\mathcal{A}$:

  - If $s = 1$, then there is only one star in the abstract enriched flow graph of $\mathcal{A}$, which we assume to be composed of the AEIs $K, C_1, \dots, C_n$ and centered on K. In order to avoid trivial cases, let us assume $n > 0$. We distinguish among the following three cases:

    * Case i: $[\![K]\!]_{\mathcal{A}}^{\mathrm{pc;wob}}$ satisfies $\mathcal{P}$. By virtue of condition 1, since K is $\mathcal{P}$-compatible with all the AEIs in $\mathcal{B}_K$, from Prop. 5.1 of [2] we derive that also $[\![K, \mathcal{B}_K]\!]_{K,\mathcal{B}_K}^{\mathrm{tc};\#K,\mathcal{B}_K;K} \,/\, \bigcup_{l=1}^{n} (H_{K,C_l} \cup E_{K,C_l})$ satisfies $\mathcal{P}$. Since $[\![K, \mathcal{B}_K]\!]_{K,\mathcal{B}_K}^{\mathrm{tc};\#K,\mathcal{B}_K;K} = [\![\mathcal{A}]\!]_{\mathrm{bbm}}^{\mathrm{tc};\#\mathcal{A};K}$, it holds that $[\![\mathcal{A}]\!]_{\mathrm{bbm}}^{\mathrm{pc};\#\mathcal{A}}$ satisfies $\mathcal{P}$ too, because $\mathcal{P}$ does not contain any free use of negation.

    * Case ii: $[\![K]\!]_{\mathcal{A}}^{\mathrm{pc;wob}}$ does not satisfy $\mathcal{P}$, but there exists $C_j \in \mathcal{B}_K$ such that $[\![C_j]\!]_{\mathcal{A}}^{\mathrm{pc;wob}}$ satisfies $\mathcal{P}$. By virtue of condition 1, C_j is $\mathcal{P}$-compatible with K:
    $$([\![C_j]\!]_{\mathcal{A}}^{\mathrm{pc};\#K} \parallel_{\mathcal{S}(C_j,K;\mathcal{A})} [\![K]\!]_{C_j,\mathcal{B}_{C_j}}^{\mathrm{tc};\#C_j}) \,/\, (H_{C_j,K} \cup E_{C_j,K}) \ \approx_{\mathcal{P}} \ [\![C_j]\!]_{\mathcal{A}}^{\mathrm{pc;wob}}$$
    where we observe that:
    $$\begin{aligned} [\![K]\!]_{C_j,\mathcal{B}_{C_j}}^{\mathrm{tc};\#C_j} &= [\![K]\!]_{C_j,\mathcal{B}_{C_j}}^{\mathrm{pc};\#\mathrm{C_j}} \,/\, \varphi_{K,\mathrm{async}}(\mathcal{OALI}_K) \\ &\approx_{\mathcal{P}} [\![K]\!]_{\mathcal{A}}^{\mathrm{pc};\#\mathrm{C_j}} \,/\, (\mathit{Name} - \mathcal{V}_{K;C_j}) \,/\, \varphi_{K,\mathrm{async}}(\mathcal{OALI}_K) \end{aligned}$$
    By virtue of condition 1, since K is $\mathcal{P}$-compatible with all the AEIs in $\mathcal{B}_K$, from Prop. 5.1 of [2] we derive in particular that:
    $$[\![K, \mathcal{B}_K - \{C_j\}]\!]_{K,\mathcal{B}_K}^{\mathrm{tc};\#K,\mathcal{B}_K;K} \,/\, \bigcup_{l=1, l\neq j}^{n} (H_{K,C_l} \cup E_{K,C_l}) \ \approx_{\mathcal{P}} \ [\![K]\!]_{\mathcal{A}}^{\mathrm{pc;wob}}$$
    Since $[\![K]\!]_{\mathcal{A}}^{\mathrm{pc};\#\mathrm{C_j}}$ – occurring in the former of the last two equalities – is given by $[\![K]\!]_{\mathcal{A}}^{\mathrm{pc;wob}}$ – occurring in the latter of the last two equalities – in parallel with the buffers associated with the originally asynchronous local interactions of K attached to C_j, from the last equality we derive that:
    $$\begin{gathered} [\![K]\!]_{\mathcal{A}}^{\mathrm{pc};\#\mathrm{C_j}} \,/\, (\mathit{Name} - \mathcal{V}_{K;C_j}) \,/\, \varphi_{K,\mathrm{async}}(\mathcal{OALI}_K) \\ \approx_{\mathcal{P}} \\ [\![K, \mathcal{B}_K - \{C_j\}]\!]_{K,\mathcal{B}_K}^{\mathrm{tc};\#K,\mathcal{B}_K;K} \,/\, \bigcup_{l=1, l\neq j}^{n} (H_{K,C_l} \cup E_{K,C_l}) \,/\, (\mathit{Name} - \mathcal{V}_{K;C_j}) \,/\, \varphi_{K,\mathrm{async}}(\mathcal{OALI}_K) \end{gathered}$$
    Thanks to the last two hidings, all the actions but those in $\varphi_{K;C_j}(\mathcal{LI}_{K;C_j})$ are hidden, hence the first hiding in the right-hand term above is redundant and we obtain that:
    $$\begin{gathered} [\![K, \mathcal{B}_K - \{C_j\}]\!]_{K,\mathcal{B}_K}^{\mathrm{tc};\#K,\mathcal{B}_K;K} \,/\, \bigcup_{l=1, l\neq j}^{n} (H_{K,C_l} \cup E_{K,C_l}) \,/\, (\mathit{Name} - \mathcal{V}_{K;C_j}) \,/\, \varphi_{K,\mathrm{async}}(\mathcal{OALI}_K) \\ \approx_{\mathcal{P}} \\ [\![K, \mathcal{B}_K - \{C_j\}]\!]_{K,\mathcal{B}_K}^{\mathrm{tc};\#K,\mathcal{B}_K;K} \,/\, (\mathit{Name} - \mathcal{V}_{K;C_j}) \,/\, \varphi_{K,\mathrm{async}}(\mathcal{OALI}_K) \end{gathered}$$
    By definition of totally closed semantics, we then derive that:
    $$\begin{gathered} [\![K, \mathcal{B}_K - \{C_j\}]\!]_{K,\mathcal{B}_K}^{\mathrm{tc};\#K,\mathcal{B}_K;K} \,/\, (\mathit{Name} - \mathcal{V}_{K;C_j}) \,/\, \varphi_{K,\mathrm{async}}(\mathcal{OALI}_K) \\ \approx_{\mathcal{P}} \\ [\![K, \mathcal{B}_K - \{C_j\}]\!]_{K,\mathcal{B}_K}^{\mathrm{tc};\#K,\mathcal{B}_K;K} \,/\, (\mathit{Name} - \mathcal{V}_{K;C_j}) \end{gathered}$$
    Hence, summarizing, we have proved that:
    $$[\![K]\!]_{C_j,\mathcal{B}_{C_j}}^{\mathrm{tc};\#C_j} \ \approx_{\mathcal{P}} \ [\![K, \mathcal{B}_K - \{C_j\}]\!]_{K,\mathcal{B}_K}^{\mathrm{tc};\#K,\mathcal{B}_K;K} \,/\, (\mathit{Name} - \mathcal{V}_{K;C_j})$$
    From the first equality at the beginning of this case and the congruence property of $\approx_{\mathcal{P}}$ with respect to static process algebraic operators, we obtain that:
    $$\begin{gathered} ([\![C_j]\!]_{\mathcal{A}}^{\mathrm{pc};\#K} \parallel_{\mathcal{S}(C_j,K;\mathcal{A})} ([\![K, \mathcal{B}_K - \{C_j\}]\!]_{K,\mathcal{B}_K}^{\mathrm{tc};\#K,\mathcal{B}_K;K} \,/\, (\mathit{Name} - \mathcal{V}_{K;C_j}))) \,/\, (H_{C_j,K} \cup E_{C_j,K}) \\ \approx_{\mathcal{P}} \\ [\![C_j]\!]_{\mathcal{A}}^{\mathrm{pc;wob}} \end{gathered}$$
    Since $\approx_{\mathcal{P}}$ preserves $\mathcal{P}$, the left-hand term of the previous equality satisfies $\mathcal{P}$. From the fact that $\mathcal{P}$ does not contain any free use of negation, we derive that $[\![C_j, K, \mathcal{B}_K - \{C_j\}]\!]_{K,\mathcal{B}_K}^{\mathrm{tc};\#K,\mathcal{B}_K;C_j,K}$

satisfies $\mathcal{P}$. Since $[\![C_j, K, \mathcal{B}_K - \{C_j\}]\!]_{K,\mathcal{B}_K}^{\mathrm{tc};\#K,\mathcal{B}_K;C_j,K} = [\![\mathcal{A}]\!]_{\mathrm{bbm}}^{\mathrm{tc};\#\mathcal{A};C_j,K}$, it holds that $[\![\mathcal{A}]\!]_{\mathrm{bbm}}^{\mathrm{pc};\#\mathcal{A}}$ satisfies $\mathcal{P}$ too, because $\mathcal{P}$ does not contain any free use of negation.

* Case iii: no AEI in the star satisfies $\mathcal{P}$. By following the same arguments as case i, we reduce the star to the AEI K, which does not satisfy $\mathcal{P}$, from which it immediately follows that not even $[\![\mathcal{A}]\!]_{\mathrm{bbm}}^{\mathrm{pc};\#\mathcal{A}}$ satisfies $\mathcal{P}$.

– Let the result hold for a certain $s \geq 1$ and suppose that the abstract enriched flow graph of $\mathcal{A}$ is composed of $s+1$ stars. Due to the acyclicity of the abstract enriched flow graph of $\mathcal{A}$, there must be a star – say composed of the AEIs $K, C_1, \ldots, C_n$ and centered on K – that is attached only to one other star in the abstract enriched flow graph of $\mathcal{A}$ – say with C_i. Then, we distinguish among the following four cases:

* Case I: $[\![C_i]\!]_{\mathcal{A}}^{\mathrm{pc;wob}}$ does not satisfy $\mathcal{P}$, but there exists $C_j \in \mathcal{B}_K - \{C_i\}$ such that $[\![C_j]\!]_{\mathcal{A}}^{\mathrm{pc;wob}}$ satisfies $\mathcal{P}$. By considering $\mathcal{B}_K - \{C_i\}$ in place of $\mathcal{B}_K$ and following the same arguments as case ii, it is straightforward to obtain that $[\![C_j, K, \mathcal{B}_K - \{C_j, C_i\}]\!]_{K,\mathcal{B}_K}^{\mathrm{tc};\#K,\mathcal{B}_K;C_j,K}$ satisfies $\mathcal{P}$ too. Now, by virtue of condition 1, K is $\mathcal{P}$-compatible with C_i:
$$([\![K]\!]_{\mathcal{A}}^{\mathrm{pc};\#C_i} \,\|_{\mathcal{S}(K,C_i;\mathcal{A})}\, [\![C_i]\!]_{K,\mathcal{B}_K}^{\mathrm{tc};\#\mathrm{K}}) \,/\, (H_{K,C_i} \cup E_{K,C_i}) \;\approx_{\mathcal{P}}\; [\![K]\!]_{\mathcal{A}}^{\mathrm{pc;wob}}$$
Since $\approx_{\mathcal{P}}$ is a congruence with respect to static process algebraic operators, we derive that:
$$([\![C_j, K, \mathcal{B}_K - \{C_j, C_i\}]\!]_{K,\mathcal{B}_K}^{\mathrm{tc};\#K,\mathcal{B}_K;C_j,K} \,\|_{\mathcal{S}(K,C_i;\mathcal{A})} [\![C_i]\!]_{K,\mathcal{B}_K}^{\mathrm{tc};\#K}) \,/\, (H_{K,C_i} \cup E_{K,C_i})$$
$$\approx_{\mathcal{P}}$$
$$[\![C_j, K, \mathcal{B}_K - \{C_j, C_i\}]\!]_{K,\mathcal{B}_K}^{\mathrm{tc};\#K,\mathcal{B}_K;C_j,K}$$
and, as a consequence, the left-hand term of this equality satisfies $\mathcal{P}$. Note that such a term is $\approx_{\mathcal{P}}$-equivalent to $[\![K, \mathcal{B}_K]\!]_{K,\mathcal{B}_K}^{\mathrm{tc};\#K,\mathcal{B}_K;C_j,K} \,/\, (H_{K,C_i} \cup E_{K,C_i})$. Since $\mathcal{P}$ does not contain any free use of negation, we derive that also $[\![K, \mathcal{B}_K]\!]_{K,\mathcal{B}_K}^{\mathrm{tc};\#K,\mathcal{B}_K;C_j,K}$ satisfies $\mathcal{P}$ and, for the same reason, so does $[\![K, \mathcal{B}_K]\!]_{\mathcal{A}}^{\mathrm{tc};\#K,\mathcal{B}_K;C_j,K}$. Since $\mathcal{P}$ is expressed only in terms of local interactions, it holds that $[\![K, \mathcal{B}_K]\!]_{\mathcal{A}}^{\mathrm{tc};\#K,\mathcal{B}_K;C_j,K} \,/\, E_{K,\mathcal{B}_K}$ satisfies $\mathcal{P}$ too, where $E_{K,\mathcal{B}_K}$ is the set of exceptions that may be raised by semi-synchronous interactions involved in attachments between K and the AEIs in $\mathcal{B}_K$.
Now, consider the architectural description $\mathcal{A}'$ obtained by replacing the AEIs $K, C_1, \ldots, C_n$ with a new AEI K' isomorphic to $[\![K, \mathcal{B}_K]\!]_{\mathcal{A}}^{\mathrm{tc};\#K,\mathcal{B}_K;C_j,K} \,/\, E_{K,\mathcal{B}_K}$. It turns out that $\mathcal{A}'$ has an acyclic topology with one fewer star with respect to $\mathcal{A}$, so the induction hypothesis is applicable to $\mathcal{A}'$ if we show that all of its AEIs satisfy condition 1. It will then follow that $[\![\mathcal{A}']\!]_{\mathrm{bbm}}^{\mathrm{pc};\#\mathcal{A}'}$ satisfies $\mathcal{P}$ because so does $[\![K']\!]_{\mathcal{A}'}^{\mathrm{pc;wob}}$ and hence, since $\mathcal{P}$ does not contain any free use of negation, we will derive that $[\![\mathcal{A}]\!]_{\mathrm{bbm}}^{\mathrm{pc};\#\mathcal{A}}$ satisfies $\mathcal{P}$ because so does $[\![C_j]\!]_{\mathcal{A}}^{\mathrm{pc;wob}}$.
It is easy to see that K' satisfies condition 1. If C is an arbitrary AEI attached to K' because it was previously attached to C_i, by virtue of condition 1 in $\mathcal{A}$ we have that:
$$([\![C_i]\!]_{\mathcal{A}}^{\mathrm{pc};\#C} \,\|_{\mathcal{S}(C_i,C;\mathcal{A})}\, [\![C]\!]_{C_i,\mathcal{B}_{C_i}}^{\mathrm{tc};\#C_i}) \,/\, (H_{C_i,C} \cup E_{C_i,C}) \;\approx_{\mathcal{P}}\; [\![C_i]\!]_{\mathcal{A}}^{\mathrm{pc;wob}}$$
from which it follows that in $\mathcal{A}'$:
$$([\![K']\!]_{\mathcal{A}'}^{\mathrm{pc};\#C} \,\|_{\mathcal{S}(K',C;\mathcal{A}')}\, [\![C]\!]_{K',\mathcal{B}_{K'}}^{\mathrm{tc};\#K'}) \,/\, (H_{K',C} \cup E_{K',C}) \;\approx_{\mathcal{P}}\; [\![K']\!]_{\mathcal{A}'}^{\mathrm{pc;wob}}$$
because $\approx_{\mathcal{P}}$ is a congruence with respect to static process algebraic operators.
Also any such C satisfies condition 1 in $\mathcal{A}'$. Starting from the fact that by virtue of condition 1 in $\mathcal{A}$ we have that:
$$([\![C]\!]_{\mathcal{A}}^{\mathrm{pc};\#C_i} \,\|_{\mathcal{S}(C,C_i;\mathcal{A})}\, [\![C_i]\!]_{C,\mathcal{B}_C}^{\mathrm{tc};\#C}) \,/\, (H_{C,C_i} \cup E_{C,C_i}) \;\approx_{\mathcal{P}}\; [\![C]\!]_{\mathcal{A}}^{\mathrm{pc;wob}}$$
we have to prove that in $\mathcal{A}'$:
$$([\![C]\!]_{\mathcal{A}'}^{\mathrm{pc};\#K'} \,\|_{\mathcal{S}(C,K';\mathcal{A}')}\, [\![K']\!]_{C,\mathcal{B}_C}^{\mathrm{tc};\#C}) \,/\, (H_{C,K'} \cup E_{C,K'}) \;\approx_{\mathcal{P}}\; [\![C]\!]_{\mathcal{A}'}^{\mathrm{pc;wob}}$$
which can be accomplished by proving that:
$$[\![K']\!]_{C,\mathcal{B}_C}^{\mathrm{tc};\#C} \;\approx_{\mathcal{P}}\; [\![C_i]\!]_{C,\mathcal{B}_C}^{\mathrm{tc};\#C}$$
On the one hand, since K' is attached to C in $\mathcal{A}'$ because C_i is attached to C in $\mathcal{A}$, it holds that:
$$\begin{aligned}[\![K']\!]_{C,\mathcal{B}_C}^{\mathrm{tc};\#C} &\approx_{\mathcal{P}} [\![K, \mathcal{B}_K]\!]_{\mathcal{A}}^{\mathrm{tc};\#C,K,\mathcal{B}_K;C_j} \,/\, E_{K,\mathcal{B}_K} \,/\, \varphi_{C_j,\mathrm{async}}(\mathcal{OALI}_{C_j}) \,/\, (\mathit{Name} - \mathcal{V}_{C_i;C}) \\ &\approx_{\mathcal{P}} [\![K, \mathcal{B}_K]\!]_{\mathcal{A}}^{\mathrm{tc};\#C,K,\mathcal{B}_K} \,/\, E_{K,\mathcal{B}_K} \,/\, (\mathit{Name} - \mathcal{V}_{C_i;C})\end{aligned}$$

On the other hand, it holds that:

$$\begin{aligned}[\![C_i]\!]^{\mathrm{tc};\#C}_{C,\mathcal{B}_C} &\approx_{\mathcal{P}} [\![C_i]\!]^{\mathrm{pc};\#C}_{C,\mathcal{B}_C} \,/\, \varphi_{C_i,\mathrm{async}}(\mathcal{OALI}_{C_i}) \\ &\approx_{\mathcal{P}} [\![C_i]\!]^{\mathrm{pc};\#C}_{\mathcal{A}} \,/\, \varphi_{C_i,\mathrm{async}}(\mathcal{OALI}_{C_i}) \,/\, (\mathit{Name} - \mathcal{V}_{C_i;C})\end{aligned}$$

and by virtue of condition 1:

$$\begin{aligned}[\![C_i]\!]^{\mathrm{pc;wob}}_{\mathcal{A}} &\approx_{\mathcal{P}} ([\![C_i]\!]^{\mathrm{pc};\#\mathrm{K}}_{\mathcal{A}} \parallel_{\mathcal{S}(C_i,K;\mathcal{A})} [\![K]\!]^{\mathrm{tc};\#C_i}_{C_i,\mathcal{B}_{C_i}}) \,/\, (H_{C_i,K} \cup E_{C_i,K}) \\ &\approx_{\mathcal{P}} [\![C_i, K]\!]^{\mathrm{tc};\#C_i,K;C_i}_{C_i,\mathcal{B}_{C_i}} \,/\, (H_{C_i,K} \cup E_{C_i,K})\end{aligned}$$

Since $[\![C_i]\!]^{\mathrm{pc};\#C}_{\mathcal{A}}$ is given by $[\![C_i]\!]^{\mathrm{pc;wob}}_{\mathcal{A}}$ in parallel with the buffers associated with the originally asynchronous local interactions of C_i attached to C, we derive that:

$$[\![C_i]\!]^{\mathrm{pc};\#C}_{\mathcal{A}} \,/\, \varphi_{C_i,\mathrm{async}}(\mathcal{OALI}_{C_i}) \,/\, (\mathit{Name} - \mathcal{V}_{C_i;C})$$
$$\approx_{\mathcal{P}}$$
$$[\![C_i, K]\!]^{\mathrm{tc};\#C,C_i,K;C_i}_{C_i,\mathcal{B}_{C_i}} \,/\, (H_{C_i,K} \cup E_{C_i,K}) \,/\, \varphi_{C_i,\mathrm{async}}(\mathcal{OALI}_{C_i}) \,/\, (\mathit{Name} - \mathcal{V}_{C_i;C})$$
$$\approx_{\mathcal{P}}$$
$$[\![C_i, K]\!]^{\mathrm{tc};\#C,C_i,K;C_i}_{C_i,\mathcal{B}_{C_i}} \,/\, E_{C_i,K} \,/\, \varphi_{C_i,\mathrm{async}}(\mathcal{OALI}_{C_i}) \,/\, (\mathit{Name} - \mathcal{V}_{C_i;C})$$

because $H_{C_i,K} \subseteq (\mathit{Name} - \mathcal{V}_{C_i;C})$. Note that the term above includes $[\![K]\!]^{\mathrm{pc;wob}}_{\mathcal{A}}$, which, by virtue of condition 1 and Prop. 5.1 of [2], satisfies:

$$[\![K]\!]^{\mathrm{pc;wob}}_{\mathcal{A}} \approx_{\mathcal{P}} [\![K, \mathcal{B}_K - \{C_i\}]\!]^{\mathrm{tc};\#K,\mathcal{B}_K;K}_{K,\mathcal{B}_K} \,/ \bigcup_{l=1,l\neq i}^{n} (H_{K,C_l} \cup E_{K,C_l})$$

Since $\approx_{\mathcal{P}}$ is a congruence with respect to static process algebraic operators, from the equalities above we derive that:

$$[\![C_i, K]\!]^{\mathrm{tc};\#C,C_i,K;C_i}_{C_i,\mathcal{B}_{C_i}} \,/\, E_{C_i,K} \,/\, \varphi_{C_i,\mathrm{async}}(\mathcal{OALI}_{C_i}) \,/\, (\mathit{Name} - \mathcal{V}_{C_i;C})$$
$$\approx_{\mathcal{P}}$$
$$[\![C_i, K, \mathcal{B}_K - \{C_i\}]\!]^{\mathrm{tc};\#C,K,\mathcal{B}_K;C_i}_{C_i,\mathcal{B}_{C_i}} \,/ \bigcup_{l=1,l\neq i}^{n} (H_{K,C_l} \cup E_{K,C_l}) \,/\, E_{C_i,K} \,/\, \varphi_{C_i,\mathrm{async}}(\mathcal{OALI}_{C_i}) \,/\, (\mathit{Name} - \mathcal{V}_{C_i;C})$$

Since $\bigcup_{l=1,l\neq i}^{n} H_{K,C_l} \subseteq (\mathit{Name} - \mathcal{V}_{C_i;C})$, by definition of totally closed semantics the right-hand term of the last equality is $\approx_{\mathcal{P}}$-equivalent to:

$$[\![K, \mathcal{B}_K]\!]^{\mathrm{tc};\#C,K,\mathcal{B}_K}_{C_i,\mathcal{B}_{C_i}} \,/ \bigcup_{l=1}^{n} E_{K,C_l} \,/\, (\mathit{Name} - \mathcal{V}_{C_i;C})$$
$$\approx_{\mathcal{P}}$$
$$[\![K, \mathcal{B}_K]\!]^{\mathrm{tc};\#C,K,\mathcal{B}_K}_{C_i,\mathcal{B}_{C_i}} \,/\, E_{K,\mathcal{B}_K} \,/\, (\mathit{Name} - \mathcal{V}_{C_i;C})$$

Since the hiding operation hides all the actions but the interactions from C_i attached to C, this term is $\approx_{\mathcal{P}}$-equivalent to $[\![K, \mathcal{B}_K]\!]^{\mathrm{tc};\#C,K,\mathcal{B}_K}_{\mathcal{A}} \,/\, E_{K,\mathcal{B}_K} \,/\, (\mathit{Name} - \mathcal{V}_{C_i;C})$. Therefore, we have shown that $[\![K']\!]^{\mathrm{tc};\#C}_{C,\mathcal{B}_C} \approx_{\mathcal{P}} [\![C_i]\!]^{\mathrm{tc};\#C}_{C,\mathcal{B}_C}$.

* Case II: $[\![C_i]\!]^{\mathrm{pc;wob}}_{\mathcal{A}}$ satisfies $\mathcal{P}$. The proof straightforwardly derives from case I; in particular, K' turns out to be isomorphic to $[\![K, \mathcal{B}_K]\!]^{\mathrm{tc};\#K,\mathcal{B}_K;C_i}_{\mathcal{A}}$.
* Case III: $[\![K]\!]^{\mathrm{pc;wob}}_{\mathcal{A}}$ satisfies $\mathcal{P}$. The proof straightforwardly derives from case i and case I; in particular, K' turns out to be isomorphic to $[\![K, \mathcal{B}_K]\!]^{\mathrm{tc};\#K,\mathcal{B}_K;K}_{\mathcal{A}}$.
* Case IV: no AEI in the star satisfies $\mathcal{P}$. It is sufficient to apply the same arguments as the previous case and then observe that K' does not satisfy $\mathcal{P}$.

- Let the result hold for a certain $m \geq 0$ and suppose that the abstract enriched flow graph of $\mathcal{A}$ has $m+1$ cycles. Since the cycle covering algorithm κ of condition 2 is total, let $\mathcal{Y} = \{C_1, \ldots, C_n\}$ be a cyclic union in $\mathcal{CU}(\kappa)$ that directly interacts with at most one cyclic union in $\mathcal{CU}(\kappa)$. In the following, we let $I = \{C_g\} \cup \mathcal{F}_{C_1,\ldots,C_n}$ if there exists C_g satisfying condition 2.c, and $I = \mathcal{F}_{C_1,\ldots,C_n}$ otherwise.

  Now, we replace the AEIs $C_1, \ldots, C_n$ with a new AEI C whose behavior is isomorphic to:

  $$[\![\mathcal{Y}]\!]^{\mathrm{tc};\#\mathcal{Y};I}_{\mathcal{A}} \,/\, (\mathit{Name} - \bigcup_{C' \in I} \mathcal{V}_{C';\mathcal{A}}) \,/ \bigcup_{C' \in I} (H_{C',\mathcal{Y}} \cup E_{C',\mathcal{Y}})$$

  thus obtaining an architectural description $\mathcal{A}'$ such that:

  1. $[\![C]\!]^{\mathrm{pc;wob}}_{\mathcal{A}'}$ satisfies $\mathcal{P}$ iff so does at least one AEI in $\mathcal{Y}$. Indeed, one such AEI exists in $\mathcal{Y}$ iff, by virtue of conditions 2.b and 2.c, I includes an AEI C' that $\mathcal{P}$-interoperates with $\mathcal{Y}$ such that

$[\![C']\!]_{\mathcal{A}}^{\mathrm{pc;wob}}$ satisfies $\mathcal{P}$, which means that $[\![\mathcal{Y}]\!]_{\mathcal{A}}^{\mathrm{tc};\#\mathcal{Y};C'} \,/\, (Name - \mathcal{V}_{C';\mathcal{A}}) \,/\, (H_{C',\mathcal{Y}} \cup E_{C',\mathcal{Y}})$ satisfies $\mathcal{P}$ and hence so does $[\![C]\!]_{\mathcal{A}'}^{\mathrm{pc;wob}}$ because $\mathcal{P}$ does not contain any free use of negation.

2. C preserves condition 1. In fact, let K be an arbitrary AEI attached to C because it was previously attached to an AEI C_j of $\mathcal{F}_{C_1,\dots,C_n}$. It holds that C is $\mathcal{P}$-compatible with K and vice versa. On C side, we have that in $\mathcal{A}$:
$$([\![C_j]\!]_{\mathcal{A}}^{\mathrm{pc};\#K} \parallel_{\mathcal{S}(C_j,K;\mathcal{A})} [\![K]\!]_{C_j,\mathcal{B}_{C_j}}^{\mathrm{tc};\#C_j}) \,/\, (H_{C_j,K} \cup E_{C_j,K}) \;\approx_{\mathcal{P}}\; [\![C_j]\!]_{\mathcal{A}}^{\mathrm{pc;wob}}$$
from which it follows that in $\mathcal{A}'$:
$$([\![C]\!]_{\mathcal{A}'}^{\mathrm{pc};\#K} \parallel_{\mathcal{S}(C,K;\mathcal{A}')} [\![K]\!]_{C,\mathcal{B}_{C}}^{\mathrm{tc};\#C}) \,/\, (H_{C,K} \cup E_{C,K}) \;\approx_{\mathcal{P}}\; [\![C]\!]_{\mathcal{A}'}^{\mathrm{pc;wob}}$$
because $\approx_{\mathcal{P}}$ is a congruence with respect to static process algebraic operators.
On K side, it can be similarly shown that from:
$$([\![K]\!]_{\mathcal{A}}^{\mathrm{pc};\#C_j} \parallel_{\mathcal{S}(K,C_j;\mathcal{A})} [\![C_j]\!]_{K,\mathcal{B}_{K}}^{\mathrm{tc};\#K}) \,/\, (H_{K,C_j} \cup E_{K,C_j}) \;\approx_{\mathcal{P}}\; [\![K]\!]_{\mathcal{A}}^{\mathrm{pc;wob}}$$
we derive that:
$$([\![K]\!]_{\mathcal{A}'}^{\mathrm{pc};\#C} \parallel_{\mathcal{S}(K,C;\mathcal{A}')} [\![C]\!]_{K,\mathcal{B}_{K}}^{\mathrm{tc};\#K}) \,/\, (H_{K,C} \cup E_{K,C}) \;\approx_{\mathcal{P}}\; [\![K]\!]_{\mathcal{A}'}^{\mathrm{pc;wob}}$$
because C_j $\mathcal{P}$-interoperates with the other AEIs in $\mathcal{Y}$ due to condition 2.b.

3. If $\mathcal{A}'$ is cyclic, then condition 2 is preserved. In fact, let $\mathcal{CU}'(\kappa)$ be the set of cyclic unions for $\mathcal{A}'$ obtained from $\mathcal{CU}(\kappa)$ by replacing in each original cyclic union every occurrence of $C_1, \dots, C_n$ with C. Every cyclic union in $\mathcal{CU}'(\kappa)$ that does not include C has a corresponding topologically equivalent cyclic union in $\mathcal{CU}(\kappa)$.
Now, consider a cyclic union $\mathcal{X}' \in \mathcal{CU}'(\kappa)$ formed by the AEIs $H_1, \dots, H_m, C$. Then, $\mathcal{CU}(\kappa)$ includes a cyclic union $\mathcal{X}$ formed by the AEIs $H_1, \dots, H_m, C_j$, where $C_j \in \mathcal{F}_{C_1,\dots,C_n}$. By virtue of condition 2.b:
$$[\![\mathcal{X}]\!]_{\mathcal{A}}^{\mathrm{tc};\#\mathcal{X};C_j} \,/\, (Name - \mathcal{V}_{C_j;\mathcal{A}}) \,/\, (H_{C_j,\mathcal{X}} \cup E_{C_j,\mathcal{X}}) \;\approx_{\mathcal{P}}\; [\![C_j]\!]_{\mathcal{A}}^{\mathrm{pc;wob}}$$
Since $\approx_{\mathcal{P}}$ is a congruence with respect to static process algebraic operators:
$$[\![\mathcal{X}']\!]_{\mathcal{A}'}^{\mathrm{tc};\#\mathcal{X}';C} \,/\, (Name - \mathcal{V}_{C;\mathcal{A}'}) \,/\, (H_{C,\mathcal{X}'} \cup E_{C,\mathcal{X}'}) \;\approx_{\mathcal{P}}\; [\![C]\!]_{\mathcal{A}'}^{\mathrm{pc;wob}}$$
Therefore, if $\mathcal{F}_{H_1,\dots,H_m,C} = \emptyset$, then condition 2.a is preserved; otherwise, if $C \in \mathcal{F}_{H_1,\dots,H_m,C}$, then C preserves condition 2.b and so does each $H_l \in \mathcal{F}_{H_1,\dots,H_m,C} - \{C\}$ as from:
$$[\![\mathcal{X}]\!]_{\mathcal{A}}^{\mathrm{tc};\#\mathcal{X};H_l} \,/\, (Name - \mathcal{V}_{H_l;\mathcal{A}}) \,/\, (H_{H_l,\mathcal{X}} \cup E_{H_l,\mathcal{X}}) \;\approx_{\mathcal{P}}\; [\![H_l]\!]_{\mathcal{A}}^{\mathrm{pc;wob}}$$
we derive that:
$$[\![\mathcal{X}']\!]_{\mathcal{A}'}^{\mathrm{tc};\#\mathcal{X}';H_l} \,/\, (Name - \mathcal{V}_{H_l;\mathcal{A}'}) \,/\, (H_{H_l,\mathcal{X}'} \cup E_{H_l,\mathcal{X}'}) \;\approx_{\mathcal{P}}\; [\![H_l]\!]_{\mathcal{A}'}^{\mathrm{pc;wob}}$$
because C_j $\mathcal{P}$-interoperates with its cyclic union.
Now, let us consider condition 2.c and assume that no AEI in the frontier of $\mathcal{X}$ satisfies $\mathcal{P}$. If, by virtue of condition 2.c, there is $H_g \in \mathcal{X}$ such that $[\![H_g]\!]_{\mathcal{A}}^{\mathrm{pc;wob}}$ satisfies $\mathcal{P}$ and H_g $\mathcal{P}$-interoperates with $\mathcal{X}$, then, by virtue of the same arguments used for H_l, we immediately derive that H_g $\mathcal{P}$-interoperates with $\mathcal{X}'$, thus preserving condition 2.c.
On the other hand, if C_j is such that $[\![C_j]\!]_{\mathcal{A}}^{\mathrm{pc;wob}}$ satisfies $\mathcal{P}$, then we have shown that $[\![C]\!]_{\mathcal{A}'}^{\mathrm{pc;wob}}$ satisfies $\mathcal{P}$ and $\mathcal{P}$-interoperates with $\mathcal{X}'$. Hence, C preserves condition 2.c in the case it does not belong to the frontier of $\mathcal{X}'$.

4. The abstract enriched flow graph of $\mathcal{A}'$ has at most m cycles.

Then, by the induction hypothesis, the theorem holds for $[\![\mathcal{A}']\!]_{\mathrm{bbm}}^{\mathrm{pc};\#\mathcal{A}'}$. Since $\mathcal{P}$ does not contain any free use of negation, we immediately derive that the theorem holds also for $[\![\mathcal{A}]\!]_{\mathrm{bbm}}^{\mathrm{pc};\#\mathcal{A}}$. ■

3 Behavioral Variations

We continue by extending the result to behavioral variations, i.e., to instances of an AT whose observable behaviors conform to each other according to weak bisimilarity $\approx_{\mathrm{B}}$ as defined in [1, 2].

Corollary 5.1 of [2] Let $\mathcal{A}$ be an architectural description and $\mathcal{P} \in \Psi$ be a property for which the two conditions of Thm. 5.1 of [2] hold. Whenever $\approx_{\mathrm{B}} \subseteq \approx_{\mathcal{P}}$, then for each AT instance $\mathcal{A}'$ that strictly behaviorally conforms to $\mathcal{A}$ it turns out that $[\![\mathcal{A}']\!]_{\mathrm{bbm}}^{\mathrm{pc};\#\mathcal{A}'}$ satisfies $\mathcal{P}$ iff so does $[\![C]\!]_{\mathcal{A}}^{\mathrm{pc;wob}}$ for some $C \in \mathcal{A}$.

Proof Due to behavioral conformity, $[\![\mathcal{A}']\!]_{\text{bbm}}^{\text{pc};\#\mathcal{A}'} \approx_{\text{B}} [\![\mathcal{A}]\!]_{\text{bbm}}^{\text{pc};\#\mathcal{A}}$ up to an injective relabeling function that matches local interactions occurring in $\mathcal{A}'$, $\mathcal{A}$, and $\mathcal{P}$. Therefore, $[\![\mathcal{A}']\!]_{\text{bbm}}^{\text{pc};\#\mathcal{A}'} \approx_{\mathcal{P}} [\![\mathcal{A}]\!]_{\text{bbm}}^{\text{pc};\#\mathcal{A}}$ up to the same relabeling function, because $\approx_{\text{B}} \subseteq \approx_{\mathcal{P}}$, and hence $[\![\mathcal{A}']\!]_{\text{bbm}}^{\text{pc};\#\mathcal{A}'}$ satisfies $\mathcal{P}$ iff so does $[\![\mathcal{A}]\!]_{\text{bbm}}^{\text{pc};\#\mathcal{A}}$. The result then follows from Thm. 5.1 of [2]. ■

4 Exogenous Variations

We now extend the result to topological variations of exogenous nature, which take place at the topological frontier formed by architectural interactions as explained in [1, 2]. Following the notation used in [2], we denote by $\mathcal{SF}_{C_1,\dots,C_n}$ the semi-frontier of a set of AEIs $\{C_1,\dots,C_n\}$; moreover, we consider partially/totally semi-closed interacting semantics, in which architectural interactions are left visible, and the related $\mathcal{P}$-semi-compatibility and $\mathcal{P}$-semi-interoperability checks, together with the notion of exo-coverability.

Corollary 5.2 of [2] Let $\mathcal{A}$ be an architectural description and $\mathcal{P} \in \Psi$ be a property for which the two conditions of Thm. 5.1 of [2] hold. Let $\mathcal{A}'$ be an AT instance resulting from a strictly topologically conformant exogenous variation of $\mathcal{A}$ for which the following additional conditions hold:

3. For each $K \in \mathcal{A}$ belonging to an acyclic portion or to the intersection of some cycle with acyclic portions of the abstract enriched flow graph of $\mathcal{A}$, if K is of the same type as an AEI having architectural interactions at which the exogenous variation takes place, then K is $\mathcal{P}$-semi-compatible with every $C \in \mathcal{B}_K - \mathcal{CU}_K^{\mathcal{A}}$.
4. If $\mathcal{A}'$ is cyclic, then $\mathcal{A}'$ is exo-coverable by κ and, for each $C_j \in \mathcal{SF}_{C_1,\dots,C_n}$ with $\{C_1,\dots,C_n\} \in \mathcal{CU}^{\mathcal{A}}(\kappa)$, if C_j is of the same type as an AEI having architectural interactions at which the exogenous variation takes place, then C_j $\mathcal{P}$-semi-interoperates with the other AEIs in $\{C_1,\dots,C_n\}$.

Then $[\![\mathcal{A}']\!]_{\text{bbm}}^{\text{pc};\#\mathcal{A}'}$ satisfies $\mathcal{P}$ iff so does $[\![C]\!]_{\mathcal{A}}^{\text{pc;wob}}$ for some $C \in \mathcal{A}$.

Proof We show that $\mathcal{A}'$ satisfies the two conditions of Thm. 5.1 of [2], from which the result will immediately follow:

- $\mathcal{A}'$ satisfies condition 1. Consider an AEI $K \in \mathcal{A}'$ belonging to an acyclic portion or to the intersection of some cycle with acyclic portions of the abstract enriched flow graph of $\mathcal{A}'$, and an AEI $C \in \mathcal{B}_K - \mathcal{CU}_K^{\mathcal{A}'}$. We distinguish among the following four cases:
  - Both AEIs are in $\mathcal{A}$. On the one hand, if K is not an AEI having architectural interactions at which the exogenous extension takes place, then K is $\mathcal{P}$-compatible with C by virtue of condition 1 applied to $\mathcal{A}$. On the other hand, if K is an AEI having architectural interactions at which the exogenous extension takes place, then by virtue of condition 3 it holds that K is $\mathcal{P}$-semi-compatible with C in $\mathcal{A}$, from which we derive that K is $\mathcal{P}$-compatible with C in $\mathcal{A}'$.
  - $K \in \mathcal{A}$ and C is an additional AEI. By hypothesis, in $\mathcal{A}$ there is an attachment between an AEI K' and $corr(C)$, such that K' is of the same type as K and $corr(C) \in \mathcal{B}_{K'} - \mathcal{CU}_{K'}^{\mathcal{A}}$. Then, by virtue of condition 3, K' is $\mathcal{P}$-semi-compatible with $corr(C)$, from which it follows that K is $\mathcal{P}$-compatible with C.
  - K is an additional AEI and $C \in \mathcal{A}$. By hypothesis, in $\mathcal{A}$ there is an attachment between an AEI C' and $corr(K)$, such that C' is of the same type as C and $C' \in \mathcal{B}_{corr(K)} - \mathcal{CU}_{corr(K)}^{\mathcal{A}}$. Then, by virtue of condition 1, $corr(K)$ is $\mathcal{P}$-compatible with C', from which it follows that K is $\mathcal{P}$-compatible with C.
  - Both K and C are additional AEIs. By hypothesis, in $\mathcal{A}$ there are two attached AEIs $corr(K)$ and $corr(C)$ such that, by virtue of condition 1 applied to $\mathcal{A}$, $corr(K)$ is $\mathcal{P}$-compatible with $corr(C)$. As a consequence, K is $\mathcal{P}$-compatible with C.

- If $\mathcal{A}'$ is cyclic, then $\mathcal{A}'$ satisfies condition 2. We first observe that $\mathcal{A}'$ satisfies condition 2.a because, by virtue of condition 4, the exogenous variation of κ applied to $\mathcal{A}'$ cannot generate a single cyclic union with empty frontier.

  Now, suppose that $\mathcal{CU}_K^{\mathcal{A}'}$ is a cyclic union generated by the exogenous variation of κ. By virtue of condition 4, we distinguish between the following two cases:

  - If K is in $\mathcal{A}$, then $\mathcal{CU}_K^{\mathcal{A}'} = \mathcal{CU}_K^{\mathcal{A}}$ and each $C_i \in \mathcal{F}_{\mathcal{CU}_K^{\mathcal{A}'}}$ belongs to $\mathcal{SF}_{\mathcal{CU}_K^{\mathcal{A}}}$. Then, by virtue of condition 4 or by virtue of condition 2.b applied to $\mathcal{A}$, C_i $\mathcal{P}$-interoperates with the other AEIs of $\mathcal{CU}_K^{\mathcal{A}'}$. Hence, $\mathcal{CU}_K^{\mathcal{A}'}$ satisfies condition 2.b. For the same reason, if $\mathcal{CU}_K^{\mathcal{A}}$ satisfies condition 2.c, then so does $\mathcal{CU}_K^{\mathcal{A}'}$.
  - If K is an additional AEI, then $\mathcal{CU}_K^{\mathcal{A}'}$ is strictly topologically equivalent to $\mathcal{CU}_{corr(K)}^{\mathcal{A}} \in \mathcal{CU}^{\mathcal{A}}(\kappa)$. By means of an argument similar to the one applied above, it follows that $\mathcal{CU}_K^{\mathcal{A}'}$ satisfies condition 2.b because so does $\mathcal{CU}_{corr(K)}^{\mathcal{A}}$, and that if $\mathcal{CU}_{corr(K)}^{\mathcal{A}}$ satisfies condition 2.c, then so does $\mathcal{CU}_K^{\mathcal{A}'}$. ■

5 Endogenous Variations

We further extend the result to endogenous variations, which take place inside the topological frontier as explained in [1, 2]. Following [2], we consider the notion of endo-coverability.

Corollary 5.3 of [2] Let $\mathcal{A}$ be an architectural description and $\mathcal{P} \in \Psi$ be a property for which the two conditions of Thm. 5.1 of [2] hold. Let $\mathcal{A}'$ be an AT instance resulting from an endogenous variation of $\mathcal{A}$ for which the following additional conditions hold:

$\overline{3}$. For each attachment in $\mathcal{A}'$ from interaction o of an AEI C_1', which belongs to an acyclic portion or to the intersection of some cycle with acyclic portions of the abstract enriched flow graph of $\mathcal{A}'$, to interaction i of an AEI $C_2' \in \mathcal{B}_{C_1'} - \mathcal{CU}_{C_1'}^{\mathcal{A}'}$, there exists an attachment in $\mathcal{A}$ from interaction o of an AEI C_1 of the same type as C_1', with C_1 belonging to an acyclic portion or to the intersection of some cycle with acyclic portions of the abstract enriched flow graph of $\mathcal{A}$, to interaction i of an AEI $C_2 \in \mathcal{B}_{C_1} - \mathcal{CU}_{C_1}^{\mathcal{A}}$ of the same type as C_2'.

$\overline{4}$. No local interaction occurring in $\mathcal{P}$ is involved in attachments canceled by the endogenous variation.

$\overline{5}$. If $\mathcal{A}$ or $\mathcal{A}'$ is cyclic, then $\mathcal{A}'$ is endo-coverable by κ and for each cyclic union $\mathcal{CU}_C^{\mathcal{A}'}$ generated by the endogenous variation of κ:

  (a) No local interaction of the AEIs of $\mathcal{CU}_C^{\mathcal{A}}$ that $\mathcal{P}$-interoperate with the other AEIs in $\mathcal{CU}_C^{\mathcal{A}}$ by virtue of condition 2 of Thm. 5.1 of [2] is involved in attachments canceled by the endogenous variation.

  (b) No possibly added AEI in $\mathcal{CU}_C^{\mathcal{A}'}$ belongs to the frontier of $\mathcal{CU}_C^{\mathcal{A}'}$.

  (c) If $C \in \mathcal{A}$, then $[\![\mathcal{CU}_C^{\mathcal{A}'}]\!]_{\mathcal{CU}_C^{\mathcal{A}'}}^{\mathrm{pc};\#\mathcal{CU}_C^{\mathcal{A}'}} / H \approx_{\mathcal{P}} [\![\mathcal{CU}_C^{\mathcal{A}}]\!]_{\mathcal{CU}_C^{\mathcal{A}}}^{\mathrm{pc};\#\mathcal{CU}_C^{\mathcal{A}}} / H$ where H contains all local interactions of the added/removed AEIs as well as those attached to them.

Then $[\![\mathcal{A}']\!]_{\mathrm{bbm}}^{\mathrm{pc};\#\mathcal{A}'}$ satisfies $\mathcal{P}$ iff so does $[\![C]\!]_{\mathcal{A}}^{\mathrm{pc;wob}}$ for some $C \in \mathcal{A}$.

Proof We show that $\mathcal{A}'$ satisfies the two conditions of Thm. 5.1 of [2], from which the result will immediately follow thanks to condition $\overline{4}$:

- $\mathcal{A}'$ satisfies condition 1. Consider an AEI $K \in \mathcal{A}'$ belonging to an acyclic portion or to the intersection of some cycle with acyclic portions of the abstract enriched flow graph of $\mathcal{A}'$, and an AEI $C \in \mathcal{B}_K - \mathcal{CU}_K^{\mathcal{A}'}$. We distinguish between the following two cases:

– Both AEIs are in $\mathcal{A}$. The only interesting case occurs whenever K and C are not attached in $\mathcal{A}$. In this case, by virtue of condition $\overline{3}$, there exists an attachment in $\mathcal{A}$ of the same kind between an AEI of the same type as K and an AEI of the same type as C, from which the result follows by virtue of condition 1 applied to $\mathcal{A}$.

– $K \in \mathcal{A}$ and C is an additional AEI, or K is an additional AEI and $C \in \mathcal{A}$, or both K and C are additional AEIs. It is sufficient to apply the same argument illustrated above.

- If $\mathcal{A}'$ is cyclic, then $\mathcal{A}'$ satisfies condition 2. Suppose that the endogenous variation of κ generates a single cyclic union $\mathcal{CU}_C^{\mathcal{A}'}$ with empty frontier. Then, by virtue of condition $\overline{5}.c$, $[\![\mathcal{CU}_C^{\mathcal{A}'}]\!]_{\mathcal{CU}_C^{\mathcal{A}'}}^{\mathrm{pc};\#\mathcal{CU}_C^{\mathcal{A}'}} / H \approx_{\mathcal{P}} [\![\mathcal{CU}_C^{\mathcal{A}}]\!]_{\mathcal{CU}_C^{\mathcal{A}}}^{\mathrm{pc};\#\mathcal{CU}_C^{\mathcal{A}}} / H$ and by virtue of conditions $\overline{5}.a$ and $\overline{5}.b$ it turns out that conditions 2.a and 2.c are preserved. In particular, we now show that if there exists $C_i \in \mathcal{CU}_C^{\mathcal{A}}$ that, by virtue of condition 2.a or 2.c applied to $\mathcal{A}$, $\mathcal{P}$-interoperates with the other AEIs in $\mathcal{CU}_C^{\mathcal{A}}$, then C_i $\mathcal{P}$-interoperates with the other AEIs in $\mathcal{CU}_C^{\mathcal{A}'}$. By hypothesis:
$$[\![\mathcal{CU}_C^{\mathcal{A}}]\!]_{\mathcal{A}}^{\mathrm{tc};\#\mathcal{CU}_C^{\mathcal{A}};C_i} \,/\, (Name - \mathcal{V}_{C_i;\mathcal{A}}) \,/\, (H_{C_i,\mathcal{CU}_C^{\mathcal{A}}} \cup E_{C_i,\mathcal{CU}_C^{\mathcal{A}}}) \;\approx_{\mathcal{P}}\; [\![C_i]\!]_{\mathcal{A}}^{\mathrm{pc;wob}}$$
By condition $\overline{5}.a$, $H \subseteq (Name - \mathcal{V}_{C_i;\mathcal{A}})$. Hence, the left-hand term of this equality is $\mathcal{P}$-equivalent to:
$$[\![\mathcal{CU}_C^{\mathcal{A}}]\!]_{\mathcal{CU}_C^{\mathcal{A}}}^{\mathrm{pc};\#\mathcal{CU}_C^{\mathcal{A}}} / H \,/\, (Name - \mathcal{V}_{C_i;\mathcal{A}}) \,/\, (H_{C_i,\mathcal{CU}_C^{\mathcal{A}}} \cup E_{C_i,\mathcal{CU}_C^{\mathcal{A}}})$$
and to:
$$[\![\mathcal{CU}_C^{\mathcal{A}'}]\!]_{\mathcal{CU}_C^{\mathcal{A}'}}^{\mathrm{pc};\#\mathcal{CU}_C^{\mathcal{A}'}} / H \,/\, (Name - \mathcal{V}_{C_i;\mathcal{A}'}) \,/\, (H_{C_i,\mathcal{CU}_C^{\mathcal{A}'}} \cup E_{C_i,\mathcal{CU}_C^{\mathcal{A}'}})$$
which, for the same motivations, is $\mathcal{P}$-equivalent to:
$$[\![\mathcal{CU}_C^{\mathcal{A}'}]\!]_{\mathcal{A}'}^{\mathrm{tc};\#\mathcal{CU}_C^{\mathcal{A}'};C_i} \,/\, (Name - \mathcal{V}_{C_i;\mathcal{A}'}) \,/\, (H_{C_i,\mathcal{CU}_C^{\mathcal{A}'}} \cup E_{C_i,\mathcal{CU}_C^{\mathcal{A}'}})$$
from which the result follows.

  Now, suppose that $\mathcal{CU}_C^{\mathcal{A}'}$ is a cyclic union with nonempty frontier generated by the endogenous variation of κ. We distinguish between the following two cases:

  – If $\mathcal{CU}_C^{\mathcal{A}'}$ is equal to $\mathcal{CU}_C^{\mathcal{A}}$ (resp. strictly topologically equivalent to a cyclic union $\mathcal{Y} \in \mathcal{CU}^{\mathcal{A}}(\kappa)$), then $\mathcal{CU}_C^{\mathcal{A}'}$ satisfies conditions 2.b and 2.c because so does $\mathcal{CU}_C^{\mathcal{A}}$ (resp. $\mathcal{Y}$).

  – If $\mathcal{CU}_C^{\mathcal{A}'}$ includes some of the added AEIs or $\mathcal{CU}_C^{\mathcal{A}}$ includes some of the removed AEIs, then it is sufficient to apply condition $\overline{5}$ as shown above to derive that conditions 2.b and 2.c are preserved.

■

6 Multiplicity Variations

We finally extend the result to multiplicity variations, which take place at and-/or-interactions as explained in [1, 2].

Corollary 5.4 of [2] Let $\mathcal{A}$ be an architectural description and $\mathcal{P} \in \Psi$ be a property for which the two conditions of Thm. 5.1 of [2] hold. Let $\mathcal{A}'$ be an AT instance resulting from a multiplicity variation of $\mathcal{A}$ for which the following additional conditions hold:

$\widetilde{3}$. No local interaction occurring in $\mathcal{P}$ is involved in attachments canceled by the multiplicity variation.

$\widetilde{4}$. No local or-interaction involved in the multiplicity variation is attached to a semi-synchronous uni-interaction or to an input asynchronous uni-interaction.

$\widetilde{5}$. Each local or-interaction involved in the multiplicity variation is enabled infinitely often.

$\widetilde{6}$. If $\mathcal{A}$ or $\mathcal{A}'$ is cyclic, then $\mathcal{CU}^{\mathcal{A}}(\kappa) = \mathcal{CU}^{\mathcal{A}'}(\kappa)$.

Then $[\![\mathcal{A}']\!]_{\mathrm{bbm}}^{\mathrm{pc};\#\mathcal{A}'}$ satisfies $\mathcal{P}$ iff so does $[\![C]\!]_{\mathcal{A}}^{\mathrm{pc;wob}}$ for some $C \in \mathcal{A}$.

Proof We show that $\mathcal{A}'$ satisfies the two conditions of Thm. 5.1 of [2], from which the result will immediately follow thanks to condition $\widetilde{3}$:

- $\mathcal{A}'$ satisfies condition 1. Consider an AEI $K \in \mathcal{A}'$ belonging to an acyclic portion or to the intersection of some cycle with acyclic portions of the abstract enriched flow graph of $\mathcal{A}'$, and an AEI $C \in \mathcal{B}_K - \mathcal{CU}_K^{\mathcal{A}'}$. We distinguish among the following five cases:
  - Both AEIs are in $\mathcal{A}$ and are attached through interactions that are not subject to the multiplicity variation. Then, K is $\mathcal{P}$-compatible with C by virtue of condition 1 applied to $\mathcal{A}$.
  - K is in $\mathcal{A}$ and has an and-interaction (subject to the multiplicity variation) to which the AEI C is attached. If C is in $\mathcal{A}$, then K is $\mathcal{P}$-compatible with C by virtue of condition 1 applied to $\mathcal{A}$. If C is an additional AEI, then C is of the same type as an AEI C' of $\mathcal{A}$ that is attached to K through the same and-interaction. By virtue of condition 1, K is $\mathcal{P}$-compatible with C', from which we derive that K is $\mathcal{P}$-compatible with C.
  - K is in $\mathcal{A}$ and has an or-interaction (subject to the multiplicity variation) to which the AEI C is attached. First, assume that C is in $\mathcal{A}$. By virtue of condition $\widetilde{4}$, both in $\mathcal{A}$ and in $\mathcal{A}'$ the AEI C does not raise any exception because of the attachments between K and any other AEI attached to the or-interaction. Hence, K is $\mathcal{P}$-compatible with C by virtue of condition 1 applied to $\mathcal{A}$. Second, assume that C is an additional AEI. In this case, we can apply the same argument, observing that C is of the same type as an AEI C' of $\mathcal{A}$ that is attached to K through the same or-interaction.
  - C is in $\mathcal{A}$ and has an and-interaction (subject to the multiplicity variation) to which K is attached. If K is in $\mathcal{A}$, then K is $\mathcal{P}$-compatible with C by virtue of condition 1 applied to $\mathcal{A}$. If K is an additional AEI, then it is of the same type as an AEI K' of $\mathcal{A}$ that is attached to C through the same and-interaction. By virtue of condition 1, K' is $\mathcal{P}$-compatible with C, from which we derive that K is $\mathcal{P}$-compatible with C.
  - C is in $\mathcal{A}$ and has an or-interaction (subject to the multiplicity variation) to which the AEI K is attached. First, assume that K is in $\mathcal{A}$. By virtue of condition $\widetilde{4}$, both in $\mathcal{A}$ and in $\mathcal{A}'$ the AEI K does not raise any exception because of the attachments between C and any other AEI attached to the or-interaction. Moreover, by virtue of condition $\widetilde{5}$, K eventually communicates with C through the or-interaction of C. Hence, K is $\mathcal{P}$-compatible with C by virtue of condition 1 applied to $\mathcal{A}$. Second, assume that K is an additional AEI. In this case, we can apply the same argument, observing that K is of the same type as an AEI K' of $\mathcal{A}$ that is attached to C through the same or-interaction.
- $\mathcal{A}'$ satisfies condition 2 because, by virtue of condition $\widetilde{6}$, the set of cyclic unions generated by κ for $\mathcal{A}'$ is the same as the one generated by κ for $\mathcal{A}$. ■

References

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