THE STRATEGIC TIMING OF R&D AGREEMENTS

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ABSTRACT. We present a model of endogenous formation of R&D agreements among firms in which also the timing of R&D investments is made endogenous. The purpose is to bridge two usually separate streams of literature, the noncooperative formation of R&D alliances and the endogenous timing literature. This allows to consider the formation of R&D agreements over time. It is shown that, when both R&D spillovers and investment costs are sufficiently low, firms may find difficult to maintain a stable agreement due to the strong incentive to invest noncooperatively as leaders. In such a case, the stability of an R&D agreement requires that the joint investment occurs at the initial stage, thus avoiding any delay. When instead spillovers are sufficiently high, cooperation in R&D constitutes a profitable option, although firms also possess the incentive to sequence their investment over time. Finally, when spillovers are asymmetric and the knowledge mainly leaks from the leader to the follower, to invest as follower becomes extremely profitable, making R&D alliances hard to sustain unless firms strategically delay their joint investment in R&D.

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1. Introduction

Research agreements among firms competing in the same markets have since long become a fairly widespread form of industrial cooperation. Many countries have set up cooperative R&D programmes for some well defined research areas, both at a national and at an international level. The US, for example, has created in the mid-1980s the *National Cooperative Research Act* (NCRA) which aims to protect and regulate R&D alliances among firms. Also the EU encourages and subsidizes R&D cooperation between participants belonging to EU countries with initiatives aimed at enhancing inter-firm research cooperation, such as the *Eureka* and *EU Framework Programmes*. Important research agreements go back to the early eighties, such as the *Microelectronics and Computer Technology Corporation* (MCC), formed in 1982 to conduct research related to information technology, and the *Bell Communications Research* created in 1984 by seven regional US telephone companies.

The economic literature provides a strong empirical evidence on the existence of such arrangements also in more recent years, and the analysis of the effects of cooperation on innovation has emerged as an important research topic. A clear understanding of this phenomenon is indeed crucial for consideration of technology and industrial policies.

As is well known, a research agreement is an alliance between firms in order to coordinate their research and development activities in a joint project, and to share, to some established degree, the knowledge obtained from this common effort. Therefore, the creation of such research agreements allows the firms not only to coordinate their research efforts but also to improve information-sharing. Many reasons may push firms to form research cartels. First, innovation is expensive, and the possibility of cost sharing and avoidance of duplication can strongly cut the expenses to each member. Second, the risk for a firm that its own innovation programmes will not produce valuable results is reduced since a research agreement has greater possibilities of diversification and each member can share risks with the other members. Third, the members of a research alliance can acquire a greater competitive advantage than nonmembers, which implies that there can be a strong danger in being left out of such cartels (see on this topic, Baumol 1992; see also Katz and Ordover 1990, Hernan, Marin and Siotis 2003, and Alonso and Marin 2004 for interesting empirical studies). Research coalitions may also have socially beneficial effects, such as the internalization of technological spillovers, which in general produces an increase in the aggregate level of R&D, and the elimination of duplication efforts, which clearly leads to a reduction in research expenditures.

Departing from d'Aspremont and Jacquemin (1988) pioneering work, a number of papers have analyzed the effects of research alliances in models with endogenous R&D (see, among others, Katz and Ordover 1990, Kamien *et al.* 1992, Suzumura 1992, Petit and Tolwinski, 1997, 1999). However, in these models, the creation of research agreements is exogenously assumed.

More recently, the endogenous coalition formation literature has attempted to endogenize the formation of R&D cartels by applying noncooperative models of coalition formation (see Bloch 2003 and 2004, Yi and Shin 2000 and Yi 2003). Usually, in these models, at a first stage R&D coalitions are assumed to set their investment to maximize their joint profit, and at a second stage firms compete individually in the product market. A crucial aspect to assess the stability of a given structure of agreements among firms is the sign of the externalities of R&D investments which, in turn, depend on the level of spillovers. For sufficiently high spillovers, forming a research cartel reduces the underinvestment in R&D,

since the externalities due to the nature of public good of R&D investments are internalized. Thus, alliances of firms can invest more than small groups, and this, in turn, may trigger some firms to stay out and free ride on the existing cartels. Moreover, different R&D alliance formation rules may yield different outcomes in terms of stability of cooperation (see, for instance, Yi & Shin 2000). In some cases, the whole industry alliance of firms investing in R&D can be stable, especially if there are no synergies and if, after breaking the agreement, all firms end up investing as singletons (see, for instance, Yi 2003, Bloch 2003 and Marini 2008 for surveys). However, the stability of alliances is no longer guaranteed if firms are assumed to decide endogenously the timing of their investment

The endogenous-timing approach was first introduced by Hamilton and Slutsky's (1990) within a duopoly game. In their *extensive game with observable delay*, the authors describe a two stage set-up in which, at a preplay stage, two players (duopolists) decide independently whether to move early or late in the basic game (e.g., a duopoly quantity game). If both players announce the same timing, that is (*early*, *early*) or (*late*, *late*), the basic game is played simultaneously. If the players' time-announcements differ, the basic game is played sequentially, with the order of moves as announced by the players. Hamilton and Slutsky's main results are that the two leader-follower configurations (with either order of play) constitute pure subgame perfect equilibria of the extended game only if at least one player's payoff as follower weakly dominates her corresponding payoff in the simultaneous game. When, conversely, the payoff of a follower is lower than in the simultaneous case, the only pure strategy subgame Nash equilibrium prescribes that both players play simultaneously the basic game. In a symmetric duopoly with single-valued and monotone best-replies, if firms actions are *strategic complements* (with increasing best-replies) the follower's payoff dominates that of the leader, and therefore that of the simultaneous case. When instead actions are *strategic substitutes* (with decreasing best-replies) the opposite holds and a first-mover advantage arises.[1]

A few recent papers have introduced asymmetric spillovers in a model *à la* d'Aspremont & Jacquemin (1988) by assuming that firms sequence their R&D activities. While some of these papers assume a given *exogenous* timing for the investment game (Goel 1990, Crampes and Langinier 2003, Halmenschlager 2004, Atallah 2005, De Bondt 2007) some other papers endogenize the timing of investment (Amir *et al.* 2000, Tesoriere 2008) by adopting a framework *à la* Hamilton and Slutsky (1990). The degree of technological spillovers is thus shown to be crucial for these games to possess strategic substitutes vs. strategic complements attributes and give rise to simultaneous vs. sequential endogenous timing R&D equilibria (see Amir *et al.*, 2000). However, these models do not consider explicitly the possibility for firms to form research agreements.

Our purpose in this paper is to bridge these otherwise separate streams of literature, the noncooperative formation of R&D agreements and the endogenous timing literature, with the aim to study the formation of research alliances when the timing of R&D investments is endogenous. Our approach is novel in that it allows for a far more complete picture of R&D agreements, by considering the possible formation of these agreements over time. Firms may prefer to wait and enter a research coalition at a subsequent moment of time. As observed by Duso *et al* (2010), where an interesting empirical analysis is performed, firms at each period

[1]See also Dowrick (1985), Boyer and Moreaux (1987), Amir (1995), Amir and Grilo (1995), Amir, Grilo and Jin (1999), von Stengel (2004) and Currarini and Marini (2003, 2004) for various leader-follower and simultaneous payoffs comparisons.

in time weight the benefits against the costs of being a research cartel member. For example, a larger number of participants (i.e. a larger pool of learning) may increase the benefits of entering a research cartel (Bloch 1995, Veugelers 1998).[2] Moreover, the incentive to join an R&D agreement is stronger in high-tech industries, due to higher gains from cooperation and knowledge transmission (Cassiman and Veugelers 2002). These empirically relevant issues have been neglected in theoretical studies.

Our approach aims to fill this gap. It makes it possible to analyze whether the possibility to cooperate in R&D across time can change the results of the existing R&D literature, in particular, the results concerning the endogenous formation of R&D agreements.

This paper is organized as follows. Section 2 introduces the setup adopted in the paper. Section 3 and 4 apply the model to the game *à la* d'Aspremont & Jacquemin (1988) with symmetric and asymmetric R&D spillovers and present the main results. Section 5 concludes.

2. The Model

The typical approach to R&D collaboration among firms usually assumes that at a first stage a firm can form an R&D alliance with its competitors and at a second stage the formed alliance decides cooperatively its joint level of investment in R&D. At a third and final stage, every firm sets noncooperatively its strategic market variable, typically quantity or price, to compete oligopolistically with all other firms. Our aim is to introduce a variant of this setup assuming that at the first stage a firm decides not only whether to form or not an R&D agreement, but also the timing of its investment in R&D. More specifically, both the R&D agreement formation process and the timing of the investment are made endogenous. Introducing endogenous timing basically determines at which stage a single firm or an R&D cartel will play its investment in R&D.

2.1. **R&D Alliances & Timing Formation Game.**

We imagine that at a pre-play stage, denoted with t_0, every firm sends simultaneously a message to its rival announcing both its intention to form or not an R&D alliance as well as its preferred timing for the R&D investment. Every firm's message set M_i can be denoted as:

$$M_i = [(\{i,j\}, t_1), (\{i,j\}, t_2), (\{i\}, t_1), (\{i\}, t_2)] \quad i = 1,2 \text{ and } j \neq i. \tag{2.1}$$

The message space contains 16 different message profiles $\mathbf{m} \in M_1 \times M_2$, which, in turn, may induce the following set of nonempty R&D timing-partitions $P(\mathbf{m})$,

$$\mathcal{P} = [(\{1,2\}^{t_1}), (\{1,2\}^{t_2}), (\{1\}^{t_1}, \{2\}^{t_1}), (\{1\}^{t_2}, \{2\}^{t_2}), (\{1\}^{t_1}, \{2\}^{t_2}), (\{1\}^{t_2}, \{2\}^{t_1})].$$

Differently from Hamilton and Slutsky's (1990) endogenous timing game applied to a model *à la* d'Aspremont and Jacquemin's (1988) (see Amir *et al.* 2000), here it is assumed that the two firms may also form an R&D cartel at period t_1 or t_2.[3] We assume that in order to form a research alliance with a given timing of investment in R&D requires the *unanimity* of firms' decisions: when firms send messages indicating both the same R&D coalition and the same investment timing, they will sign a binding agreement to invest at the prescribed

[2] On average, four firms enter a research joint venture (RJV) yearly. The average entry decreases with the age of these RJVs (Duso et al. 2010).

[3] Note that allowing the two firms to play their cooperative investment strategy at different stages, one at period t_1 and the other at period t_2, does not alter the basic results of the analysis.

time; otherwise, they will invest as individual firms with the timing prescribed by their own messages. Formally, for $i, j = 1, 2$ and $j \neq i$

$$\begin{cases} P(\mathbf{m}) = \{1,2\}^{\tau} \text{ if } m_i = m_j = (\{i,j\}, \tau) \text{ and} \\ P(\mathbf{m}) = (\{i\}^{\tau_i}, \{j\}^{\tau_j}) \text{ if } m_i \neq m_j. \end{cases}$$

The above rule prescribes that if both firms agree to form the same alliance and to invest with the same timing, the alliance will be created and thus will invest at that given time. Conversely, if one firm disagrees, either on the alliance or on the investment timing, both firms will play as singletons the R&D investment game, with the timing depending on their message. The described R&D agreement formation rule reflects an exclusive membership rule, where the consensus of all members is required to complete the agreement.[4] In what follows, we formally introduce our model.

2.2. **The Investment Game.** Once every firm has sent a message m_i and a timing-partition, denoted $P(\mathbf{m}) \in \mathcal{P}$, has been induced on the set of firms, every firm decides its cooperative or noncooperative investment according to the timing prescribed by $P(\mathbf{m})$. At this stage, as well as at the following stages, it is assumed that a firm cannot manipulate its level of investment in order to convince its rival to renegotiate the timing-partition decided at stage t_0.

As in d'Aspremont & Jacquemin (1988) every firm is assumed to set a finite level of investment $x_i \in X_i \subset R_+$ affecting its profit function via its production cost $c_i(x_1, x_2)$ which, in turn, influences the final market competition between individual firms. Denoting with $q_i \in [0, \infty)$ the final market competition variable (here quantity), a firm profit function can be written as $\pi_i(\mathbf{q}(\mathbf{x}))$.

In a research agreement $\{1,2\}^{\tau}$ firms will therefore set cooperatively their level of investment at stage $\tau = t_1$ or t_2 i.e.

$$\mathbf{x}^{c^{\tau}} = \left(x_1^{c^{\tau}}, x_2^{c^{\tau}}\right) \tag{2.2}$$

such that, for every $i, j = 1, 2$ and $j \neq i$

$$x_i^{c^{\tau}} = \arg\max_{x_i} \sum_{i=1,2} \pi_i\left(\mathbf{q}\left(x_i, x_j\right)\right)$$

given the profile of quantities $\mathbf{q} = (q_1, q_2)$ which will be optimally chosen in the final market stage.

If the firms play simultaneously as singletons at stage $\tau = t_1$ or t_2, the appropriate equilibrium concept will be the Nash equilibrium $x^{\tau *}$ of the simultaneous investment game played at stage τ

$$\mathbf{x}^{\tau *} = \left(x_1^{\tau *}, x_2^{\tau *}\right) \tag{2.3}$$

such that, for every $i = 1, 2$ and $j \neq i$

$$x_i^{\tau *} = \arg\max_{x_i} \pi_i\left(\mathbf{q}\left(x_i, x_j\right)\right).$$

[4] For a discussion on which coalition formation rule may be more appropriate according to the specific context, see the material contained in Hart & Kurz (1983), Yi (2003) and Ray (2007).

When firms play sequentially, the relevant equilibrium investment will be a Stackelberg (subgame perfect) equilibrium, i.e. the profile

$$\mathbf{x}^{\sigma *} = \left(x_i^{\sigma *}, g_j\left(x_i^{\sigma *}\right)\right) \tag{2.4}$$

such that, for the leader (henceforth firm i)

$$x_i^{\sigma *} = \arg\max_{x_i} \pi_i\left(\mathbf{q}\left(x_i, g_j\left(x_i\right)\right)\right)$$

and for the follower (firm j), $g_j : X_i \rightarrow X_j$ is the best-reply mapping,

$$g_j\left(x_i\right) = \arg\max_{x_j} \pi_j\left(\mathbf{q}\left(x_i, x_j\right)\right).$$

Note that for the investment game to be well-defined, all equilibria in (2.2), (2.3) and (2.4) must exist and be unique.

2.3. **The Market Game.** Once the two firms have either formed a research cartel or have chosen their R&D investment as singletons at t_1 or t_2, they set their market variable in the last stage of the game (denoted with t_3). We assume competition in quantities and a unique Cournot equilibrium among firms, given the equilibrium level of investment $\mathbf{x}^{c^\tau}$, or $\mathbf{x}^{\tau *}$ or $\mathbf{x}^{\sigma *}$ decided at stages 1, 2 or both. In particular, the Cournot quantity profile is simply the vector

$$\mathbf{q}^* = \left(q_1^*, q_2^*\right)$$

such that, for every firm $i = 1, 2$ and $j \neq i$

$$q_i^* = \arg\max_{q_i} \ \pi_i(q_i, q_j^*).$$

2.4. **Strategies.** Firm strategies in the described multi-stage game can formally be expressed as follows. When the investment game is played simultaneously, either at stage t_1 or t_2, every firm $i \in N$ strategy set is a triple $\Sigma_i^{sim} = (m_i, x_i, q_i)$ where, in turn, m_i is firm i-th message, such that $m_i = (S_i, \tau_i) \in (\{i\}, \{i, j\}) \times (t_1, t_2)$ with S_i being a nonempty coalition selected by the firm i, $x_i : M_i \times M_j \rightarrow X_i$ is the investment choice (a mapping from the message space to a given investment level), and $q_i : X_i \times X_j \rightarrow R_+$ the output choice, i.e. a mapping from the firm investments to a positive level of output. When the investment game is played sequentially, the strategy sets are triples $\Sigma_i^{\mathrm{seq}} = (m_i, x_i, q_i)$ and $\Sigma_j^{\mathrm{seq}} = (m_j, g_j, q_j)$, for the i-th leader and the j-th follower respectively, where the follower investment choice is a mapping $g_j : M_i \times M_j \times X_i \rightarrow X_j$.

2.5. **Stable R&D Agreements.** Given the equilibrium quantities of the market game played by firms at stage t_3, and given the level of investment decided simultaneously or sequentially at stages t_1 and/or t_2 either by the research cartel or by individual firms, all firms receive a profit that, with a slight abuse of notation, can be denoted as $\pi_i(\mathbf{q}^*\left(\mathbf{x}^*(P(\mathbf{m}))\right))$, where $\mathbf{q}^*\left(\mathbf{x}^*(P(\mathbf{m}))\right)$ indicates the equilibrium quantity profile when an investment profile, as defined by (2.2), or by (2.3) or finally by (2.4) is decided by the firms in a given partition $P(\mathbf{m})$ induced by the message profile $\mathbf{m}$ sent at stage t_0.

At this stage we need to make explicit a concept of equilibrium for the message game played at stage t_0. For this purpose, we introduce two different equilibrium concepts. The first is a standard Nash equilibrium of the R&D partition-timing game. The second introduces a coalitional stability requirement, implying that a structure $P(\mathbf{m})$ is stable if and only if the message profile $\mathbf{m}$ is a strong Nash equilibrium, i.e., cannot be improved upon by an alternative message announced by a firm or by a group of firms, here the grand coalition.

Formally, when a given timing-partition P is *Nash stable*, the profile $\theta^* = (\mathbf{m}^*, \mathbf{q}^*, \mathbf{x}^*)$ is a subgame perfect Nash equilibrium (SPNE) of the entire game. When, as additional requirement, the message profile $\mathbf{m}$ played at t_0 is also strong Nash, θ^* is again SPNE, with the property to be Pareto-efficient for the two firms.

Definition 1. *(Nash stability) A feasible R&D timing-partition $P \in \mathcal{P}$ is Nash stable if $P = P(\mathbf{m}^*)$, for some $\mathbf{m}^*$ with the following properties: there exists no $m_i^{'} \in M_i$ for every firm $i = 1, 2$ and $j \neq i$ such that*

$$\pi_i(\mathbf{q}^*(\mathbf{x}^*(P(m_i^{'}, m_j^*))) > \pi_i\left(\mathbf{q}^*\left(\mathbf{x}^*(P(\mathbf{m}^*)\right)\right).$$

Definition 2. *(Strong Nash stability) A feasible R&D timing-partition $P \in \mathcal{P}$ is strongly stable if $P = P(\widehat{\mathbf{m}})$, for some $\widehat{\mathbf{m}}$ with the following properties:*

$$\pi_i(\mathbf{q}^*(\mathbf{x}^*(P(\widehat{\mathbf{m}}))) \geq \pi_i(\mathbf{q}^*(\mathbf{x}^*(P(m_i^{'}, \widehat{m}_j)))$$

for $m_i^{'} \in M_i$ and if

$$\pi_i(\mathbf{q}^*(\mathbf{x}^*(P(m_i^{'}, \widehat{m}_j))) > \pi_i(\mathbf{q}^*(\mathbf{x}^*(P(\widehat{\mathbf{m}})))$$

thus

$$\pi_j(\mathbf{q}^*(\mathbf{x}^*(P(m_i^{'}, \widehat{m}_j))) < \pi_j(\mathbf{q}^*(\mathbf{x}^*(P(\widehat{\mathbf{m}})))$$

for $i, j = 1, 2$ and $j \neq i$.

Note that a strong Nash equilibrium message profile $\widehat{\mathbf{m}}$ is both a Nash equilibrium and a Pareto-optimum.

We are now ready to apply our framework to d'Aspremont & Jacquemin's (1988) well-known model.

3. Duopoly with Symmetric Spillovers

Following d'Aspremont & Jacquemin (1988), we assume a linear inverse market demand function

$$P(Q) = \max\{0,\ a - bQ\},$$

with $Q = \sum_{i=1}^{2} q_i$ and a linear cost function for every firm decreasing in R&D investment,

$$c_i(x_i, x_j) = (c - x_i - \beta x_j) \tag{3.1}$$

for $j \neq i$, and $c \geq x_i - \beta x_j$. In this setup, the learning which results from investment in R&D characterizes the production process, implying that marginal and unit costs decrease as the investment in R&D increases. We allow for the possibility of imperfect appropriability (i.e. technological spillovers between the firms), by introducing a spillover parameter $\beta \in [0, 1]$. Obviously the case of no spillovers ($\beta = 0$) may only arise in a situation of strong intellectual protection. More frequently, however, involuntary information leaks occur due to reverse engineering, industrial espionage or by hiring away employees of an innovative firm. The cases of partial to full spillovers can be modelled by setting $0 < \beta \leq 1$. At this stage the parameter β in (3.1) is assumed to be identical for all firms. However, in Section 4, this parameter, though exogenously given, will differ due to the cooperative versus non-cooperative nature and to the timing properties of the R&D investment game.

Moreover, we assume a simple quadratic cost function for the investment in R&D given by

$$I_i(x_i) = \gamma \frac{x_i^2}{2},$$

with $\gamma > 0$. This guarantees decreasing returns to R&D expenditure (see e.g. Cheng 1984; d'Aspremont and Jacquemin 1988). As a result, under Cournot competition in the product market, and setting for simplicity $b = 1$, the last stage profit function for each firm $i = 1, 2$ can be obtained as a function of (x_i, x_j):

$$\pi_i\left(\mathbf{q}^*\left(x_i, x_j\right)\right) = \frac{\left(a - c + (2-\beta)\, x_i + (2\beta - 1)\, x_j\right)^2}{9} - \frac{\gamma}{2} x_i^2. \tag{3.2}$$

Note that in this setup, for sufficiently high R&D spillover rates ($\beta > 1/2$), there are positive R&D cartel-externalities, as the formation of a cooperative agreement affects positively all remaining firms. Only in this case firms' cooperative choice implies social efficiency, which instead is not guaranteed for $\beta < 1/2$.

3.1. **Main Assumptions.** Some assumptions are now introduced to ensure the existence and uniqueness of all stages equilibria as well as to simplify comparative statics.

A.1 (*Quantity stage constraint*). $(a/c) > 2$.

A.2 (*Profit concavity and best-reply contraction property*). $\gamma > 4/3$.

A.3 (*Boundaries on R&D efforts*) For every firm, $X_i = [0, c]$. Moreover, for $\beta < 1/2$: $\gamma > \frac{a(2-\beta)(\beta+1)}{4.5c}$ and for $\beta > 1/2$: $\gamma > \frac{a(\beta+1)^2}{4.5c}$.

As explained in detail in the Appendix, assumption A.1 simply ensures that the last stage Cournot equilibrium quantities for both firms are unique and interior, with associated positive profits.

Assumption A.2 guarantees both the strict concavity of every firm non cooperative payoff (3.2) in its own investment x_i (guaranteed for $\gamma > \frac{8}{9}$) as well as a contraction property on every firm best-replies $g_i(x_j)$, which requires that $\gamma > \frac{4}{3}$.

Assumption A.3 prescribes a compact R&D investment set for every firm and imposes some Inada-type conditions to obtain interior investment equilibria in all noncooperative (simultaneous or sequential) and cooperative R&D games (see Amir *et al.* 2000, Amir *et al.* 2011, Tesoriere 2008 and Stepanova and Tesoriere 2011).[5]

Note that by assumption A.2 every firm payoff is strictly concave in its own investment choice and thus best-replies are single-valued and continuous. Investment spaces are compact by A.3 and therefore a Nash equilibrium exists by Brower fixed-point theorem. The contraction property implied by A.2 ensures uniqueness of the Nash equilibrium $\mathbf{x}^{\tau *}$. The existence of a Stackelberg equilibrium $\mathbf{x}^{\sigma *}$ - a subgame perfect Nash equilibrium (SPNE) of the sequential R&D game - is guaranteed by both firm continuous payoffs and continuous best-replies, thus implying that a firm as leader faces a continuous maximization problem over a closed set. Then, by the Weierstrass theorem, a SPNE equilibrium exists. Its uniqueness is generically ensured by the fact that firm best-replies are single-valued and monotone and all firm payoffs are strictly monotone in their rival investment. In fact, for every $i = 1, 2$ with $j \neq i$, by A.1 and by (3.2)

$$\frac{\partial \pi_i\left(\mathbf{q}^*\left(\mathbf{x}\right)\right)}{\partial x_j} = \frac{2}{9}\left(2\beta - 1\right)\left(a - c + 2x_i - x_j - \beta x_i + 2\beta x_j\right) \gtrless 0 \text{ for } \beta \gtrless \frac{1}{2}.$$

[5]For a detailed description of the consequences occurring to the simultaneous investment game when these boundaries are violated, see, for instance, Amir (2011).

Hence, along the best-reply $g_j(x_i)$ of its rival's, no multiple argmax are possible for a i-th firm acting as leader. The follower firm will instead act *à la* Nash and, by the property of best-replies, its investment choice will be uniquely defined. Moreover, the strict concavity of every firm profit, under the additional constraint that the two firms select the same collusive investment, implies that also the joint R&D cartel profit is strictly concave. In this case, this is maximized by a unique investment profile $\mathbf{x}$.

In the next section we characterize all stable R&D agreements with endogenous timing reached by the two firms. We then extend the symmetric set-up to the case of asymmetric spillovers. This can offer a broader view on a recent stream of literature concerning the endogenous timing under asymmetric spillovers (De Bondt and Vandekerckhove 2008, Tesoriere 2008).

3.2. **Cooperative R&D.** The R&D cartel made of all firms $N = \{1, 2\}$ (i.e. the grand coalition) investing cooperatively in R&D is assumed to maximize the sum of firms' profits, i.e.

$$\sum_{i=1}^{2}\pi_i\left(\mathbf{q}^*\left(\mathbf{x}\left(\{1,2\}^\tau\right)\right)\right) = \sum_{i=1}^{2}\left\{\frac{1}{9}\left[a-c+(2-\beta)\,x_i+(2\beta-1)\,x_j\right]^2-\gamma\frac{x_i^2}{2}\right\}. \tag{3.3}$$

where $\mathbf{x} = (x_i, x_j)$ is any arbitrary profile of R&D investment carried out simultaneously by the two firms either at $\tau = t_1$ or at $\tau = t_2$, for $i = 1, 2$ and $j \neq i$. Following much of the literature, we will assume henceforth that the level of investment that maximizes (3.3) is equal for every firm, i.e., is such that $x_i^{c^\tau} = x_j^{c^\tau}$.[6]

Maximizing the profit of the R&D cartel in (3.3), and given the constraint of symmetric behaviour, a firm cooperative investment can be easily obtained as

$$x_i^{c^\tau}\left(\{1,2\}^\tau\right) = \frac{2(a-c)(1+\beta)}{9\gamma-2(1+\beta)^2} \tag{3.4}$$

with an associated (equal split) equilibrium profit

$$\pi_i^C\left(\mathbf{q}^*\left(\mathbf{x}^{c^\tau}\left(\{1,2\}^\tau\right)\right)\right) = \frac{\gamma(a-c)^2}{9\gamma-2(1+\beta)^2}. \tag{3.5}$$

3.3. **Noncooperative Simultaneous R&D.** Differentiating (3.2) and exploiting the symmetry of firm payoffs, the noncooperative level of investment can be obtained as

$$x_i^{\tau *}\left(\{1\}^\tau, \{2\}^\tau\right) = \frac{2(a-c)(2-\beta)}{9\gamma-2(2-\beta)(1+\beta)} \tag{3.6}$$

for $\tau = 1, 2$, with the noncooperative equilibrium profit of every firm i given by:

$$\pi_i^N\left(\mathbf{q}^*\left(\mathbf{x}^{\tau *}\left(\{1\}^\tau, \{2\}^\tau\right)\right)\right) = \frac{\gamma(a-c)^2(9\gamma-2(\beta-2)^2)}{(9\gamma-2(2-\beta)(1+\beta))^2}.$$

By the Pareto-efficiency of $x_i^{c^\tau}\left(\{1,2\}^\tau\right)$ (for the two firms) we can establish the following Lemma.

Lemma 1. *Under high (low) spillover rate $\beta > \frac{1}{2}$ ($\beta < \frac{1}{2}$) the cooperative investment level is higher (lower) than the simultaneous Nash investment level, i.e. $x_i^{c^\tau} > x_i^{\tau *}$ ($x_i^{c^\tau} < x_i^{\tau *}$).*

Proof. See the Appendix. □

[6] As shown by Salant and Shaffer (1998,1999), for certain values of the parameters, the joint profit maximization may easily imply unequal R&D investments for the two firms.

3.4. **Sequential R&D Investment Game.** Using again (3.2) we can easily obtain the best-reply of the j-th firm playing as follower the investment game:

$$g_j(x_i) = \frac{2(2-\beta)(a-c-(1-2\beta)x_i)}{9\gamma - 2(\beta-2)^2}. \tag{3.7}$$

Therefore the leader and the follower equilibrium investment levels are given by

$$x_i^{\sigma *}\left(\{i\}^{t_1}, \{j\}^{t_2}\right) = \frac{2\left(2-\beta\right)\left(a-c\right)\left(3\gamma+2\beta^2-2\right)\left(6\beta+3\gamma-2\beta^2-4\right)}{\Delta}$$

$$x_j^{\sigma *}\left(\{i\}^{t_1}, \{j\}^{t_2}\right) = \frac{2\left(2-\beta\right)\left(a-c\right)\Gamma}{\Delta}$$

where

$$\Gamma = \left(26\beta\gamma - 20\gamma - 12\beta - 4\beta^2 + 12\beta^3 + 9\gamma^2 - 4\beta^4 - 8\beta^2\gamma + 8\right)$$

and

$$\begin{aligned}\Delta = {} & 160\gamma - 216\gamma^2 + 81\gamma^3 + 32\beta^5 - 8\beta^6 - \beta^4\left(20\gamma+16\right) + \beta^3\left(64\gamma - 64\right) \\ & + \beta\left(216\gamma^2 - 224\gamma + 32\right) + \beta^2\left(24\gamma - 54\gamma^2 + 56\right) - 32\end{aligned}$$

with associated equilibrium profits given by

$$\pi_i^L\left(\mathbf{q}^*\left(\mathbf{x}^{\sigma *}\left(\{i\}^{t_1}, \{j\}^{t_2}\right)\right)\right) = \frac{\left(a-c\right)^2\gamma\left(6\beta+3\gamma-2\beta^2-4\right)^2}{\Delta}$$

$$\pi_j^F\left(\mathbf{q}^*\left(\mathbf{x}^{\sigma *}\left(\{i\}^{t_1}, \{j\}^{t_2}\right)\right)\right) = \frac{(a-c)^2\gamma(9\gamma+8\beta-2\beta^2-8)\Gamma^2}{\Delta^2}$$

Comparing R&D equilibrium investment levels under assumptions A.1-A.3, we can state the following:

Proposition 1. *(i) When firm R&D investments are strategic substitutes ($\beta < \frac{1}{2}$) there exists a $\beta^*(\gamma)$ and a $\overline{\gamma}$ such that, for $\beta < \beta^*(\gamma)$ and $\gamma < \overline{\gamma}$,*

$$x_i^{\sigma *} > x_i^{\tau *} > x_i^{c^\tau} > x_j^{\sigma *}.$$

(ii) When firm R&D investments are strategic substitutes ($\beta < \frac{1}{2}$) and $\beta \geq \beta^*(\gamma)$ or $\gamma \geq \overline{\gamma}$

$$x_i^{\sigma *} > x_i^{\tau *} > x_j^{\sigma *} \geq x_i^{c^\tau}.$$

(iii) When firm R&D investments are strategic complements ($\beta > \frac{1}{2}$),

$$x_i^{c^\tau} > x_i^{\sigma *} > x_j^{\sigma *} > x_i^{\tau *}$$

for $i = 1, 2$ and $j \neq i$.

Proof. See the Appendix. □

The above proposition provides a full ranking of firm equilibrium investment levels, as it combines the well-known results by d'Aspremont and Jacquemin (1988), who compare cooperative and simultaneous noncooperative R&D levels, with Amir *et al.* (2000) analysis, focussing on sequential vs. simultaneous noncooperative outcomes. Lemma 1 has already proven that for low (high) spillover rates $x_i^{\tau *} < x_i^{c^\tau}$ ($x_i^{c^\tau} > x_i^{\tau *}$) which, combined with Amir's *et al.* (2000) results, implies that $x_i^{\sigma *} > x_i^{\tau *} > x_i^{c^\tau}$ ($x_i^{c^\tau} > x_i^{\tau *}$ and $x_i^{\sigma *} > x_j^{\sigma *} > x_i^{\tau *}$).

Proposition 1 completes this ranking by also including the cooperative investment levels. It can be noticed (see expression (3.7)) that the level of spillover is crucial to determine the slope of the follower's best-reply in the investment game. Thus, when the spillover rate is very low (case (*i*)), the follower's best-reply is extremely steep (and negatively sloped) and this strongly contracts its equilibrium investment, which is thus even lower than that resulting under a cooperative agreement. A firm investing as leader at stage t_1 can therefore profitably expand its investment, and this occurs in particular when the cost of the investment (which depends on γ) is very low and an investor is very unlikely to be imitated (low β). Under such circumstances, being a leader may become more profitable than investing cooperatively. When instead spillover rates start to increase, the cooperative investment overcomes that of the follower, although the leader's investment remains very high. Finally, for $\beta > 1/2$, cooperation implies the efficient and highest level of R&D investment, regardless of the level of investment costs.

In what follows we perform some comparisons of the firm payoffs obtained in the different investment games by combining the results of Lemma 1 and Proposition 1 above, with Amir's *et al.* (2000) analysis. We recall that in Amir's *et al.* (2000) paper, the following ranking is established for simultaneous and sequential payoffs in the symmetric case:

$$\pi_i^L\left(\mathbf{x}^{\sigma *}\right) > \pi_i^N\left(\mathbf{x}^{\tau *}\right) > \pi_j^F\left(\mathbf{x}^{\sigma *}\right) \text{ for } \beta < \frac{1}{2} \tag{3.8}$$

$$\pi_j^F\left(\mathbf{x}^{\sigma *}\right) > \pi_i^L\left(\mathbf{x}^{\sigma *}\right) > \pi_i^N\left(\mathbf{x}^{\tau *}\right) \text{ for } \beta > \frac{1}{2} \tag{3.9}$$

where L, N and F denote the leader/Nash simultaneous/follower roles, respectively, in the different R&D investment games.[7] By the efficiency of the profile $\mathbf{x}^{c^{\tau}}$, we also know that $\pi_i^C\left(\mathbf{x}^{c^{\tau}}\right) > \pi_i^N\left(\mathbf{x}^{\tau *}\right)$. Moreover, the following lemma proves that for $\beta < \frac{1}{2}$ ($\beta > \frac{1}{2}$) a follower (leader) payoff can never be greater than that of a firm in a cooperative agreement.

Lemma 2. *Under high (low) spillovers $\beta > \frac{1}{2}$ ($\beta < \frac{1}{2}$) the profit of a firm in an R&D agreement is always higher than the profit of a leader (follower), namely, $\pi_i^C\left(\mathbf{x}^{c^{\tau}}\right) > \pi_i^L\left(\mathbf{x}^{\sigma *}\right)$ ($\pi_i^C\left(\mathbf{x}^{c^{\tau}}\right) > \pi_j^F\left(\mathbf{x}^{\sigma *}\right)$).*

Proof. See the Appendix. □

The following two propositions complete the full ranking of firm payoffs in all different cases and for all levels of spillover rates.

Proposition 2. *When firm R&D investments are strategic substitutes ($\beta < \frac{1}{2}$): (i) there exists a $\beta^*(\gamma)$ and a $\overline{\gamma}$ such that, for $\beta < \beta^*(\gamma)$ and $\gamma < \overline{\gamma}$, the profit obtained by a firm playing as leader in a sequential investment game is higher than that obtained in a cooperative R&D agreement, and the following ranking arises*

$$\pi_i^L\left(\mathbf{x}^{\sigma *}\right) > \pi_i^C\left(\mathbf{x}^{c^{\tau}}\right) > \pi_i^N\left(\mathbf{x}^{\tau *}\right) > \pi_j^F\left(\mathbf{x}^{\sigma *}\right).$$

(ii) When, instead $\beta \geq \beta^*(\gamma)$ or $\gamma \geq \overline{\gamma}$ or both, the following ranking arises:

$$\pi_i^C\left(\mathbf{x}^{c^{\tau}}\right) \geq \pi_i^L\left(\mathbf{x}^{\sigma *}\right) > \pi_i^N\left(\mathbf{x}^{\tau *}\right) > \pi_j^F\left(\mathbf{x}^{\sigma *}\right).$$

Proof. See the Appendix. □

[7] In what follows we maintain the convention that firm i indicates the leader while firm j indicates the follower in the sequential investment game.

Figure 1 and 2 illustrate the effect of β on the investment levels and on $\left(\pi_i^C\left(\mathbf{x}^{c^\tau}\right) - \pi_i^L\left(\mathbf{x}^{\sigma *}\right)\right)$, the difference between the profit obtained by a firm in a cooperative agreement and that obtained by the leader in a sequential game. When firm investments are strategic substitutes ($\beta < 1/2$) there exists a narrow range of the spillover rate (between 0 and β^*) for which being leader, and thus expanding the investment, turns out to be extremely profitable. This occurs only when the cost to invest in R&D is extremely low ($\gamma < \overline{\gamma}$).

[FIGURE 1 AND 2 APPROXIMATELY HERE]

The proposition that follows completes our findings for the model with symmetric spillovers by comparing the firms' profitability under different arrangements when R&D investments are strategic complements.

Proposition 3. *When firm investments are strategic complements ($\beta > \frac{1}{2}$) the profit obtained by a firm in a cooperative R&D agreement is always higher than the profit obtained by a firm investing as follower in the sequential investment game, and the following ranking arises*

$$\pi_i^C\left(\mathbf{x}^{c^\tau}\right) > \pi_j^F\left(\mathbf{x}^{\sigma *}\right) > \pi_i^L\left(\mathbf{x}^{\sigma *}\right) > \pi_i^N\left(\mathbf{x}^{\tau *}\right).$$

Proof. See the Appendix. □

As it can be observed in figure 3, for $\beta > 1/2$, the highest level of investment is selected by the research cartel. Under the sequential game the follower free-rides on the leader investment and gains a higher profit. However, as shown in figure 4, the difference between the cooperative payoff and the follower payoff is positive and monotonically increasing within the interval for β under analysis.

[FIGURE 3 AND 4 APPROXIMATELY HERE]

Finally, the next two propositions characterize all Nash and strongly stable timing-partitions according to Definitions 1 and 2.

Proposition 4. *(Nash stability) (i) When the spillover rate $\beta < \beta^*(\gamma)$, and $\gamma < \overline{\gamma}$, the Nash stable timing-partitions are given by*

$$\mathcal{P}\left(\mathbf{m}^*\right) = [\left(\{1,2\}^{t_1}\right), \left(\{1\}^{t_1}, \{2\}^{t_1}\right)].$$

(ii) When $1/2 > \beta \geq \beta^*(\gamma)$ or $\gamma \geq \overline{\gamma}$ or both, the Nash stable timing-partitions are, instead, given by

$$\mathcal{P}\left(\mathbf{m}^*\right) = [\left(\{1,2\}^{t_1}\right), \left(\{1,2\}^{t_2}\right), \left(\{1\}^{t_1}, \{2\}^{t_1}\right)].$$

(iii) Finally, for $\beta \in (1/2, 1]$, the Nash stable timing-partitions are given by

$$\mathcal{P}\left(\mathbf{m}^*\right) = [\left(\{1,2\}^{t_1}\right), \left(\{1,2\}^{t_2}\right), \left(\{1\}^{t_1}, \{2\}^{t_2}\right), \left(\{1\}^{t_2}, \{2\}^{t_1}\right)].$$

Proof. See the Appendix. □

It is obvious that, if we require the *strong stability of* timing-partitions, by symmetry all noncooperative partitions in which firms invest simultaneously *à la* Nash are Pareto-dominated by the cooperative partitions. Forming a cooperative research agreement to coordinate costly investments in R&D is clearly more profitable than playing the symmetric investment game *à la* Nash. If, however, $\beta < \beta^*(\gamma)$, we have proven that being leader in the investment game yields a higher profit than playing cooperatively, and therefore the only timing-partition that remains *strongly stable* is the grand coalition investing at time t_1. A cooperative agreement, to be stable, requires that firms anticipate strategically their joint investments.

Proposition 5. *(Strong stability) (i) when the spillover rate* $\beta < \beta^*(\gamma)$ *and* $\gamma < \overline{\gamma}$*, the only strongly stable R&D timing-partition is*

$$\mathcal{P}(\widehat{\mathbf{m}}) = \left[\left(\{1,2\}^{t_1}\right)\right].$$

(ii) - (iii) When $1 \geq \beta \geq \beta^*(\gamma)$ *or* $\gamma \geq \overline{\gamma}$ *or both, the strongly stable R&D timing-partitions are*

$$\mathcal{P}(\widehat{\mathbf{m}}) = \left[\left(\{1,2\}^{t_1}\right), \left(\{1,2\}^{t_2}\right)\right].$$

Proof. See the Appendix. □

Our results depart from those obtained in the previous literature. In particular, since in our set-up firms can form a strategic alliance to invest cooperatively in R&D, differently from Amir *et al.* (2000) the alliance of firms always constitutes a SPNE of the whole game. However, our model suggests that in forming alliances firms have to consider carefully the effect of timing. If a group of firms procrastinates its cooperative investment, it may risk a defection by a partner breaking the alliance to invest as leader. To avoid this problem, firms have to anticipate strategically their joint investment in R&D. As illustrated in detail, this happens only when investing in R&D is not very costly and spillovers are very low. For higher spillovers, to discipline the stability of a research cartel is easier and time-constraints for the investment are no longer required. Our model also shows that, without requiring Pareto-optimality, the noncooperative simultaneous (or sequential) configurations are also stable under low (high) spillover rate, $\beta < 1/2$ ($\beta > 1/2$), as already established in Amir *et al.* (2000).

3.5. **An Extension to n-symmetric Firms.** The extension of our model to n-symmetric firms would allow to check the stability of more complex alliances between firms coordinating their investment in R&D. However, including more than two firms in our analysis with endogenous timing makes the model highly unmanageable. Only intuitive conclusions can be drawn with the help of our previous analysis and some well known existing results. A first observation concerns the whole industry R&D agreement (grand coalition of firms) investing at stage t_2, i.e., using the model notation, the timing-partition $P = \left(\{N\}^{t_2}\right)$ which is formed when at stage t_0 all firms $i = 1, 2, .., n$ send the message $m_i = (\{N\}, t_2)$. This partition can be strongly stable if every individual firm investing as follower at stage t_2 would be better off than any firm participating to an R&D agreement investing at stage t_1 as leader. Thus, any coalition $S \subset N$ of firms that deviates from the grand coalition $\left(\{N\}^{t_2}\right)$ by sending one of these alternative messages, $m_S^{'} = (\{S\}, t_2)$ or $m_S^{''} = (\{S\}, t_1)$, would induce either the simultaneous partition

$$P(m_S^{'}) = (\{S\}^{t_2}, \{j\}_{j \in N \setminus S}^{t_2}), \tag{3.10}$$

where all j-th firms outside S are singletons or, analogously, the sequential partition

$$P(m_S^{''}) = (\{S\}^{t_1}, \{j\}_{j \in N \setminus S}^{t_2}). \tag{3.11}$$

However, if firms in coalition S cannot improve upon partition $(\{N\}^{t_2})$ by playing as leaders in (3.11) they would not improve *a fortiori* by playing simultaneously in (3.11). Therefore, if we show that in the partition (3.11) all firms within the research cartel S (regardless of its size) do not improve upon the cooperative partition $(\{N\}^{t_2})$, the stability of the grand coalition agreement is proved as a result. When investment decisions are strategic complements, it can be proved that the payoff of a symmetric firm playing as singleton follower against the coalition S playing as leader is always higher than the payoff of every firm in S.[8] Hence, given the efficiency of the grand coalition, it would be impossible for any coalition S to improve by deviating as leader, given that followers would improve even more their payoffs. Similarly, it can be shown that when R&D investments are strategic substitutes ($\beta < 1/2$) a coalition $S \subset N$ made of followers is beaten by individual firms investing as leaders, and therefore the partition $(\{N\}^{t_1})$ - made by the grand coalition of firms investing at stage 1- is strongly stable. The strong stability of these two cooperative timing-partitions already observed in our duopoly model thus extends to an analogous endogenous timing game played by n-symmetric firms.

4. Duopoly with Asymmetric Spillovers

Introducing asymmetric spillovers equals to introducing a higher degree of realism into the model. As is well known (see e.g. Atallah 2005), asymmetries may derive from differences in protection practices, from geographical localization (e.g. Petit *et al.* 2009), from product differentiation (Amir *et al.* 2000), or from sequential moves in the R&D game, as in R&D models with endogenous timing (Tesoriere 2008). Other sources of asymmetry can arise from different technological capabilities, as in Amir and Wooders (1999, 2000), where knowledge may leak only from the more R&D-active firm to the rival, or from a better absorption capacity influencing the outcome of a technological race, as in De Bondt and Henriques (1995).

The spillover asymmetry arising in our model stems instead from the cooperative versus the non-cooperative nature of the R&D game and from the timing of the R&D investment process. The parameter β_i, ($0 \leq \beta_i \leq 1$) will represent henceforth the *incoming* spillover for firm $i = 1, 2$. Moreover, let β_i^N denote the firm spillover rate under simultaneous noncooperative R&D, β_i^C the spillover rate under R&D cooperation, and β_i^L, β_j^F the spillover rates for the leader and the follower in the sequential investment game, with $i, j = 1, 2$, $i \neq j$.

Our assumptions on spillovers asymmetry are based on the following considerations:

(i) When the two firms invest simultaneously and noncooperatively at stage one or two their spillover rate is assumed to be symmetric and lower than or equal to 0.5 (i.e., $\beta_1^N = \beta_2^N \leq 0.5$). The idea is that the competition in R&D and the simultaneity of firm decisions do not allow for a high amount of knowledge transmission.

(ii) When a noncooperative sequential investment in R&D takes place, the spillover rate can be though to be favorable to the firm playing as follower and unfavorable to the firm playing as leader (i.e. $\beta_j^F > \beta_i^L$). In particular we shall set $\beta_j^F > 0.5$ and $\beta_i^L \leq 0.5$. A sequential order of moves in the R&D investment game implies a greater amount of

[8]See for a formal proof of this fact, Currarini and Marini (2003, 2004).

knowledge leaking out from the leader to the follower than *vice versa*. The rationale is that knowledge leaks also through imitation, thus leading to a strong advantage for the firm that is able to observe the first mover innovative outcome. Therefore, benefits from spillovers should be lower for a first mover (see also Tesoriere 2008). Moreover, we assume that sector-specific features determining the intensity of knowledge diffusion affect to the same extent the incoming spillover for the leader in the sequential game (i.e. β_i^L) and the incoming spillovers for both firms in the simultaneous noncooperative game (i.e., $\beta_i^N \quad i = 1, 2$). Therefore we will set $\beta_i^L = \beta_i^N$.

(iii) When the two firms play cooperatively and form a research cartel, they generally also agree to share to some extent the knowledge obtained from their joint R&D effort. It seems realistic to assume that they might agree to fully share their knowledge, and therefore their spillover rates will be symmetric and sufficiently high (i.e. $\beta_1^C = \beta_2^C$ close or equal to one). Moreover, we assume that knowledge leaks occurring mainly through imitation and favouring the follower in a sequential game are less intense if compared with the voluntary exchange of technological knowledge typical of a research agreement. Thus, we maintain that $\beta_i^C > \beta_j^F$, for $i, j = 1, 2, i \neq j$.

Taking into account all the above inequalities, our assumptions on the relationship among spillover values can be summarized as follows:

$$1 \geq \beta_i^C > \beta_j^F > \beta_i^L = \beta_i^N \geq 0 \quad i = 1, 2, j \neq i \tag{4.1}$$

with $\beta_i^L = \beta_i^N \leq 0.5$ and $\beta_j^F > 0.5$.

4.1. **Main assumptions.** Also in this section we introduce some assumptions needed to ensure the existence and uniqueness of equilibria at all stages:

B.1 (*quantity stage constraint*). As in the case of symmetric spillovers, $a/c > 2$.

B.2 (*Profit concavity and best-reply contraction property*). Again, $\gamma > 4/3$.

B.3 (*Boundaries on R&D efforts*) For every firm $i = 1, 2$, $X_i = [0, c]$ and $\gamma > \frac{2a\left(\beta_i^C+1\right)^2}{9c}$, for $0.5 < \beta_i^C \leq 1$.

Assumption B.1 does not vary with respect to the symmetric case. Assumption B.2 deals with the strict concavity of every firm's profit (3.2) with respect to own investment x_i. Given our assumptions on spillovers in the noncooperative simultaneous game, i.e. $\beta_i^N \leq 0.5$, the SOC requires that $\gamma > \frac{8}{9}$. Likewise, the SOC for profit maximization in the case of a research cartel, given that $\beta_i^C \leq 1$, requires, in the most stringent case, that $\gamma > \frac{8}{9}$ as well. The contraction properties on both firms best-replies $g_i(x_j)$ and $g_j(x_i)$ introduced in section 3.1 are still valid and implied by $\gamma > 4/3$ (see the Appendix). Assumption B.3 follows the same logic of A.3. As previously discussed, from the strict concavity of profits it comes out that each best reply is single-valued and continuous. The existence of a Nash equilibrium is therefore guaranteed. The contraction property in B.2 ensures uniqueness of the Nash equilibrium. In order to have interior equilibria it has to hold true that:

$$\frac{\partial \pi_i \left(\mathbf{x}\left(\mathbf{q}^*\right)\right)}{\partial x_i} = 2(2 - \beta_j)[a - 2c(1 - 2\beta_i)] > 0$$

for $i, j = 1, 2, i \neq j$. It suffices assumption B.1 for the above expression to be strictly positive.

4.2. **Noncooperative Sequential R&D with Asymmetric Spillovers.** Since only in the case of a game with sequential moves at the investment stage our calculations differ from the symmetric case analyzed in the previous sections, we shall deal henceforth extensively with this scenario. Our main aim is to investigate whether the asymmetry in the transmission of knowledge between firms is relevant for the endogenous formation of research alliances.

Using an asymmetric-spillover specification, every firm's objective function at the market game stage is given by

$$\pi_i = (a - (q_i + q_j))q_i - (c - x_i - \beta_j x_j)\, q_i - \gamma \frac{x_i^2}{2}$$

with $i, j = 1, 2$ and $i \neq j$. Solving the game by backward induction, every firm's payoff at the investment stage can be obtained as:

$$\pi_i \left(\mathbf{q}^* \left(x_i, x_j\right)\right) = \frac{1}{9} \left[(a - c) + (2 - \beta_j)\, x_i + (2\beta_i - 1)\, x_j\right]^2 - \gamma \frac{x_i^2}{2}. \tag{4.2}$$

Differentiating (4.2) we get the best-reply for the follower in the investment game:

$$g_j(x_i) = \frac{2(2-\beta_i^L)[(a-c) - (1-2\beta_j^F)x_i]}{9\gamma - 2(2-\beta_i^L)^2}$$

Hence, the sequential equilibrium investment levels for the two firms are given by:

$$x_i^{\sigma *} \left(\{i\}^{t_1}, \{j\}^{t_2}\right) = \frac{2}{9} \frac{\frac{a-c+2(a-c)C}{A} \left(2 + \frac{B}{A} - \beta_j^F\right)}{\gamma - \frac{2}{9}\left(\frac{2-\beta_j^F + \left(2\beta_j^F - 1\right)B}{A}\right)^2}$$

$$x_j^{\sigma *} \left(\{i\}^{t_1}, \{j\}^{t_2}\right) = \frac{2\left(2 - \beta_i^L\right)\left(\alpha + 2\beta_j^F\right)}{9\gamma + 8\beta_i^L - 2\left(\beta_i^L\right)^2 - 8}$$

$$\begin{aligned} \text{where } A &= 9\gamma + 8\beta_i^L - 2\left(\beta_i^L\right)^2 - 8, \\ B &= 2\left(2\beta_j^F - 1\right)\left(2 - \beta_i^L\right)\left(2\beta_i^L - 1\right), \\ C &= \left(2 - \beta_i^L\right)\left(2\beta_i^L - 1\right). \end{aligned}$$

Let the assumptions on spillovers in equation (4.1) as well as assumptions B.1-B.3 hold. Comparing firm R&D equilibrium investment levels under asymmetric spillovers, we can state:

Proposition 6. *There exists a* $\tilde{\beta} \in (0, 1/2)$ *such that, if* $\beta_i^N = \beta_i^L \leq \tilde{\beta}$*, then* $x_j^{\sigma *} \geq x_i^{c^\tau} > x_i^{\tau *} > x_i^{\sigma *}$*. If instead* $\beta_i^N = \beta_i^L \geq \tilde{\beta}$*, then* $x_i^{c^\tau} \geq x_j^{\sigma *} > x_i^{\tau *} > x_i^{\sigma *}$*, for* $i, j = 1, 2$ $i \neq j$*.*

Proof. See the Appendix. □

An illustration of this result is shown in Figure 5. To give an intuition, when spillovers are low, the asymmetry between the incoming spillover of the leader (β_i^L) and that of the follower (β_j^F) is strong (since β_j^F is always greater than 0.5). Therefore, the leader has the lowest incentive to invest in R&D since the high outgoing spillover effect overcomes the first-mover advantage effect. Conversely the follower takes advantage of a high learning opportunity and of low knowledge leaks. Moreover, in this case, the R&D investment of the follower overcomes that of the cooperative firm, since a competition effect prevails. Conversely, when spillovers are high, the asymmetry between leader and follower decreases. In this case a free-riding effect may prevail for both players and the cooperative outcome may become convenient, since cooperation between firms succeeds in internalizing knowledge externalities.

Firms profits could be compared only via numerical simulations. In what follows the numerical values assigned to the parameters are as follows: $a = 38$, $c = 18$, $\gamma = 2$. In addition, we assume that in the case of cooperation firms agree to share a high amount of technological knowledge. Thus we assign a constant value $\beta_i^C = 0.8$. Moreover we set the incoming spillover of the follower such that $1 \geq \beta_i^C > \beta_j^F > 0.5$ (for instance $\beta_j^F = 0.6$ as in Figures 5 and 6).

As depicted in figure 6, there exists a value $\hat{\beta} \in (0, 1/2)$, such that the following payoff ranking emerges:

$$\pi_j^F\left(\mathbf{x}^{\sigma*}\right) > \pi_i^C\left(\mathbf{x}^{c^\tau}\right) > \pi_i^N\left(\mathbf{x}^{\tau*}\right) > \pi_i^L\left(\mathbf{x}^{\sigma*}\right)$$

for $\beta_i^L = \beta_i^N \leq \hat{\beta}$. As a result, in this case the *Nash equilibrium* timing-partitions are

$$\mathcal{P}\left(\mathbf{m}^*\right) = [\left(\{1,2\}^{t_2}\right), \left(\{1\}^{t_2}, \{2\}^{t_2}\right)],$$

while the strongly stable partitions are given by

$$\mathcal{P}(\widehat{\mathbf{m}}) = \left[\left(\{1,2\}^{t_2}\right)\right].$$

When instead $\beta_1^L = \beta_i^N \geq \hat{\beta}$, the following payoff ranking comes out:

$$\pi_i^C\left(\mathbf{x}^{c^\tau}\right) > \pi_j^F\left(\mathbf{x}^{\sigma*}\right) > \pi_i^N\left(\mathbf{x}^{\tau*}\right) > \pi_i^L\left(\mathbf{x}^{\sigma*}\right)$$

and then

$$\mathcal{P}\left(\mathbf{m}^*\right) = [\left(\{1,2\}^{t_1}\right), \left(\{1,2\}^{t_2}\right), \left(\{1\}^{t_2}, \{2\}^{t_2}\right)],$$
$$\mathcal{P}(\widehat{\mathbf{m}}) = \left[\left(\{1,2\}^{t_1}\right), \left(\{1,2\}^{t_2}\right)\right].$$

[FIGURES 5 AND 6 APPROXIMATELY HERE]

These results can be explained by considering that joint cooperative agreements across time or at time t_1 are particularly at risk when there is a strong advantage to be follower in the R&D investment game. As a matter of fact, firms prefer to wait and observe the rival's move rather then trying to reach an agreement. This happens in particular when spillovers are extremely unbalanced (i.e. when $\beta_1^L = \beta_i^N \leq \hat{\beta}$) towards firms that wait before investing, thus conferring a strong "follower advantage".

Our findings complement the few existing results (Amir et al., 2000; Tesoriere, 2008) on endogenous sequencing in R&D investment with asymmetric spillovers. In particular, Tesoriere (2008) considers only the noncooperative case with extreme spillovers ($\beta_1^L = \beta_i^N = 0$ and $\beta_j^F = 1$). Under these values he proves that the only timing configuration which is a SPNE involves simultaneous noncooperative play at the R&D stage (with zero spillovers). In contrast, in our setup the noncooperative simultaneous configuration may not be the only Nash stable timing-partition and in addition is never strongly stable, as firms prefer to form an R&D cartel than playing (suboptimally) as singletons the investment game.

5. Concluding Remarks

This paper constitutes a first attempt to bridge two usually distinct streams of the economic literature, one dealing with the endogenous formation of R&D agreements, the other with the endogenous timing of R&D investments in a model with spillovers *à la* d'Aspremont and Jacquemin (1988). This is done by introducing a new set-up in which firms express both their intention to form or not an alliance as well as the timing of their effort in R&D. This allows to assess the stability of research cartels against deviations occurring across time. Every firm can express its willingness to play cooperatively or noncooperatively as leader or follower according to the circumstances. Our results show that the nature of the interaction among the firms in the investment game plays an important role. In particular, under symmetric spillovers and when the level of spillovers is extremely low (and thus R&D investments are highly substitutes) both firms want to play the investment game as leaders and, as a result, they may easily end up investing simultaneously either cooperatively or noncooperatively. In this case, any cooperative agreement, to be stable, must contain a commitment to invest at time 1. A cooperative agreement of this sort would remain stable also against deviations by coalitions of firms, if we include in the model a number of symmetric firms higher than two. When instead R&D investments are strategic complements, our model predicts that both sequential (noncooperative) and simultaneous (cooperative) R&D configurations are stable against individual deviations. However, only cooperative agreements are strongly stable and, in this case, the timing of investment seems irrelevant for the stability of cooperation. Finally, when spillovers are asymmetric and favourable to the firm investing as follower, the model shows that an R&D agreement to be stable requires that the joint investment is strategically delayed as to avoid that a firm may break the agreement to exploit the existing "second-mover advantage". This occurs, in particular, when the incoming spillover of the follower is much higher then that of the leader.

6. Appendix

Proofs of Lemmata and Propositions

Proof of Lemma 1. For every $i, j = 1, 2$ with $j \neq i$

$$\pi_i\left(x_i^{c^\tau}(\mathbf{q}^*), x_j^{c^\tau}(\mathbf{q}^*)\right) > \pi_i\left(x_i^{\tau *}(\mathbf{q}^*), x_j^{\tau *}(\mathbf{q}^*)\right) \geq \pi_i\left(x_i^{c^\tau}(\mathbf{q}^*), x_j^{\tau *}(\mathbf{q}^*)\right)$$

where the first inequality is due to the Pareto-efficiency of $\mathbf{x}^{c^\tau}$ and the second by the Nash equilibrium property. Thus, by monotone negative (positive) externalities for $\beta < \frac{1}{2}$ $(\beta > \frac{1}{2})$, it follows that $\mathbf{x}^{c^\tau} < \mathbf{x}^{\tau *}$ $(\mathbf{x}^{c^\tau} > \mathbf{x}^{\tau *})$. ■

Proof of Proposition 1. (i)-(ii) For $\beta \in [0, 1/2)$, the following equation

$$\left(x_j^{\sigma *} - x_i^{c^\tau}\right) = \frac{2(a-c)(2-\beta)\Gamma}{\Delta} - \frac{2(a-c)(1+\beta)}{9\gamma - 2(1+\beta)^2} = 0 \tag{6.1}$$

can be solved for $\beta^*(\gamma) = \frac{7}{5} - \frac{3}{10}\sqrt{2}\sqrt{5\gamma + 2}$, which is strictly positive for $\gamma \leq \overline{\gamma}$, where $\overline{\gamma} = 16/9$. Condition A.3 for $\beta < 1/2$ requires that $\gamma > \frac{a(2-\beta)(\beta+1)}{4.5c}$ and since this constraint reaches its maximum for $\beta = 1/2$, it follows that for $\gamma \in \left[\frac{a}{2c}, \frac{16}{9}\right]$, there exists a $\beta^*(\gamma) \in [0, 1/2)$ for which $\left(x_j^{\sigma *} - x_i^{c^\tau}\right) < 0$. It can be checked that the defined interval for γ is compatible with a market size-cost ratio $a/c \leq 32/9$. Moreover, by (6.1) for $1/2 > \beta > \beta^*(\gamma)$

and/or for a $\gamma > 16/9$, $\left(x_j^{\sigma*} - x_i^{c^\tau}\right) > 0$. Combining these facts with Amir's *et al.* (2000) ranking on leader-follower and Nash simultaneous investments, the results follow. (iii) For $\beta \in (1/2, 1]$, by (6.1), it turns out that $\left(x_j^{\sigma*} - x_i^{c^\tau}\right) < 0$. Moreover it can be easily checked that

$$\text{sign } \left(x_i^{\sigma*} - x_j^{\sigma*}\right) = \text{ sign } 2\gamma\left(2\beta - 1\right)^2 > 0$$

which holds for any β and, thus, also for $\beta \in (1/2, 1]$. Again, combining the above fact with Amir's *et al.* (2000) results, the ranking between R&D investments is proven. ■

Proof of Lemma 3. Suppose by contradiction that for $\beta > \frac{1}{2}$

$$\pi_i^C\left(\mathbf{x}^{c^\tau}\right) < \pi_i^L\left(\mathbf{x}^{\sigma*}\right)$$

and by (3.8)

$$\pi_i^C\left(\mathbf{x}^{c^\tau}\right) < \pi_i^L\left(\mathbf{x}^{\sigma*}\right) < \pi_j^F\left(\mathbf{x}^{\sigma*}\right).$$

It follows that

$$\sum_{i=1}^{2}\pi_i^C\left(\mathbf{x}^{c^\tau}\right) < \pi_i^L\left(\mathbf{x}^{\sigma*}\right) + \pi_j^F\left(\mathbf{x}^{\sigma*}\right)$$

contradicting the efficiency of profile $x_i^{c^\tau}\left(\mathbf{q}^*\right)$. Similarly, let for $\beta < \frac{1}{2}$

$$\pi_i^C\left(\mathbf{x}^{c^\tau}\right) < \pi_j^F\left(\mathbf{x}^{\sigma*}\right)$$

and by (3.9)

$$\pi_i^C\left(\mathbf{x}^{c^\tau}\right) < \pi_j^F\left(\mathbf{x}^{\sigma*}\right) < \pi_L\left(\mathbf{x}^{\sigma*}\right).$$

which again implies

$$\sum_{i=1}^{2}\pi_i^C\left(\mathbf{x}^{c^\tau}\right) < \pi_i^L\left(\mathbf{x}^{\sigma*}\right) + \pi_j^F\left(\mathbf{x}^{\sigma*}\right),$$

which is a contradiction. ■

Proof of Proposition 2. (i) By Lemma 2 and 3 and by (3.8)-(3.9) we know that, under low spillover rates, ($\beta < \frac{1}{2}$), either

$$\pi_i^L\left(\mathbf{x}^{\sigma*}\right) > \pi_i^C\left(\mathbf{x}^{c^\tau}\right) > \pi_i^N\left(\mathbf{x}^{\tau*}\right) > \pi_j^F\left(\mathbf{x}^{\sigma*}\right). \tag{6.2}$$

or

$$\pi_i^C\left(\mathbf{x}^{c^\tau}\right) > \pi_i^L\left(\mathbf{x}^{\sigma*}\right) > \pi_i^N\left(\mathbf{x}^{\tau*}\right) > \pi_j^F\left(\mathbf{x}^{\sigma*}\right). \tag{6.3}$$

For $\beta \in [0, 1/2)$ the following equation

$$\pi_i^C\left(\mathbf{x}^{c^\tau}\right) - \pi_i^L\left(\mathbf{x}^{\sigma*}\right) = \frac{\gamma(a-c)^2}{9\gamma - 2(1+\beta)^2} - \frac{\gamma(a-c)^2\left(6\beta + 3\gamma - 2\beta^2 - 4\right)^2}{\Delta} = 0 \tag{6.4}$$

has only one root $\beta^*(\gamma) = \frac{7}{5} - \frac{3}{10}\sqrt{2}\sqrt{5\gamma + 2}$, requiring that $\gamma < 16/9$ to be positive. Since by A.3 $\gamma > \frac{a(2-\beta)(\beta+1)}{4.5c}$ and such constraint reaches its maximum for $\beta = 1/2$, we conclude that for $\gamma \in \left[\frac{a}{2c}, \frac{16}{9}\right]$ there exists a $\beta^*(\gamma) \in [0, 1/2)$ ensuring that the inequality $\left(\pi_i^C - \pi_i^L\right) < 0$ holds true. (ii) For $\beta \in [0, 1/2)$, when either $\beta \geq \beta^*$ or $\gamma > \frac{16}{9}$ or both, it can be assessed that

$$\pi_i^C\left(\mathbf{x}^{c^\tau}\right) - \pi_i^L\left(\mathbf{x}^{\sigma*}\right) = \frac{\gamma(a-c)^2}{9\gamma - 2(1+\beta)^2} - \frac{\gamma(a-c)^2\left(6\beta + 3\gamma - 2\beta^2 - 4\right)^2}{\Delta} \geq 0. \tag{6.5}$$

The payoffs ranking can therefore be completed as stated in (6.3)-(6.2) using Lemma 2 and Amir's *et al.* (2000) results. ■

Proof of Proposition 3. By Lemma 2 and 3 and by (3.8)-(3.9) we know that under high spillover rates ($\beta > \frac{1}{2}$), either

$$\pi_j^F\left(\mathbf{x}^{\sigma *}\right) > \pi_i^C\left(\mathbf{x}^{c^\tau}\right) > \pi_i^L\left(\mathbf{x}^{\sigma *}\right) > \pi_i^N\left(\mathbf{x}^{\tau *}\right) \tag{6.6}$$

or

$$\pi_i^C\left(\mathbf{x}^{c^\tau}\right) > \pi_j^F\left(\mathbf{x}^{\sigma *}\right) > \pi_i^L\left(\mathbf{x}^{\sigma *}\right) > \pi_i^N\left(\mathbf{x}^{\tau *}\right). \tag{6.7}$$

For $\beta \in [1/2, 1]$ and $\gamma \in \left(\frac{a(\beta+1)^2}{4.5c}, \infty\right)$, the equation

$$\left(\pi_i^C - \pi_j^F\right) = \frac{\gamma(a-c)^2}{9\gamma-2(1+\beta)^2} - \frac{\gamma(a-c)^2(9\gamma+8\beta-2\beta^2-8)\left(26\beta\gamma-20\gamma-12\beta-4\beta^2+12\beta^3+9\gamma^2-4\beta^4-8\beta^2\gamma+8\right)^2}{\Delta^2} = 0$$

is solved only for $\beta = 1/2$. It can be checked that for any other spillover rate $1 \geq \beta > 1/2$, the difference $\left(\pi_i^C - \pi_j^F\right)$ is positive and increases monotonically in β. Only for $\gamma \to +\infty$, it occurs that $\left(\pi_i^C - \pi_j^F\right) \to 0$. ■

Proof of Proposition 4. (i) By proposition 2, for $\beta \in [0, \beta^*(\gamma)) < 1/2$ and $\gamma < \overline{\gamma}$, investing as leader at stage t_1 is more profitable for firms than forming a cooperative agreement. As a result, the message $\mathbf{m} = \left(\{1,2\}^{t_2}\right)$ cannot be Nash-stable, because a firm i can profitably deviates with an alternative message $m_i' = (\{i\}, t_1)$, thus inducing the timing-partition $\left(\{i\}^{t_1}, \{j\}^{t_2}\right)$. Similarly all sequential timing-partitions $\left(\{i\}^{t_1}, \{j\}^{t_2}\right)$ can profitably be objected by the j-th firm who, instead of playing as follower, would prefer to invest simultaneously. This is feasible if it sends the message $m_j' = (\{j\}, t_1)$, and thus induces the timing-partition $\left(\{i\}^{t_1}, \{j\}^{t_1}\right)$. Therefore we remain with only two partitions $\left(\{1,2\}^{t_1}\right)$ and $\left(\{1\}^{t_1}, \{2\}^{t_1}\right)$ that cannot be profitably objected by any individual firm. (ii) We know by proposition 2 that when $\beta \in [\beta^*(\gamma), 1/2)$ the payoff gained in a cooperative agreement is higher than that obtained by a leader (follower or simultaneous) firm, and therefore both cooperative timing-partitions $\left(\{1,2\}^{t_1}\right)$ and $\left(\{1,2\}^{t_2}\right)$ are Nash-stable. Also the simultaneous partition $\left(\{1\}^{t_1}, \{2\}^{t_1}\right)$ cannot be objected by individual deviations. (iii) For $\beta \in (1/2, 1]$, by proposition 3 the payoff gained in a cooperative agreement is the highest obtainable by a firm, and thus, both cooperative timing-partitions $\left(\{1,2\}^{t_1}\right)$ and $\left(\{1,2\}^{t_2}\right)$ are Nash-stable. Also the sequential partitions $\left(\{1\}^{t_1}, \{2\}^{t_2}\right)$ and $\left(\{1\}^{t_2}, \{2\}^{t_1}\right)$ cannot be profitably objected neither by the leader nor by the follower (see proposition 3), and the result follows. ■

Proof of Proposition 5. (i) This result easily follows from proposition 2 and by the fact that all other timing-partitions are Pareto-dominated by a cooperative agreement, with the exception of the sequential partition $\left(\{i\}^{t_1}, \{j\}^{t_2}\right)$. However, since by proposition 2, $\pi_i^N\left(\mathbf{x}^{\tau *}\right) > \pi_j^F\left(\mathbf{x}^{\sigma *}\right)$ for $\beta < 1/2$, the sequential partition $\left(\{i\}^{t_1}, \{j\}^{t_2}\right)$ can profitably be objected by the follower, who prefers to invest simultaneously and, that, by sending the message $m_j' = (\{j\}, t_1)$ can induce the simultaneous partition $\left(\{i\}^{t_1}, \{j\}^{t_1}\right)$. However, the latter partition can, in turn, be objected by a message $\left(\{1,2\}^{t_1}\right)$ sent by both firms, and therefore, is not strongly stable. Finally, also the partition $\left(\{1,2\}^{t_2}\right)$ can be objected by a firm sending an alternative message $m_i' = (\{i\}, t_1)$, hence inducing the relatively more profitable sequential timing-partition $\left(\{i\}^{t_1}, \{j\}^{t_2}\right)$. (ii) By proposition 2 and 3 it follows that for $\in [\beta^*(\gamma), 1]$ all sequential and simultaneous Nash timing-partitions payoffs are dominated by cooperative agreements. As a result, the two message profiles $\mathbf{m} = (\{i,j\}, t_1), (\{i,j\}, t_1))$

and $\mathbf{m} = ((\{i,j\}, t_2), (\{i,j\}, t_2))$ are both strongly undominated and the two cooperative partitions $(\{1,2\}^{t_1}), (\{1,2\}^{t_2})$ are both strongly stable. ■

Proof of Proposition 6. Consider first the equilibrium investment levels under the extreme assumptions that $\beta_i^L = 0$, $\beta_i^N = 0$, $\beta_j^F = 1$, $\beta_i^C = 1$, $i = 1,2$ with $j \neq i$. Thus we obtain:

$$x_i^{\sigma*}\big|_{\beta_i^L=0,\beta_j^F=1} = \tfrac{2(a-c)(3\gamma-4)^2}{(112\gamma-162\gamma^2+81\gamma^3-32)} \tag{6.8}$$

and

$$x_j^{\sigma*}\big|_{\beta_i^L=0,\beta_j^F=1} = \tfrac{4(a-c)(9\gamma-8)\gamma}{(112\gamma-162\gamma^2+81\gamma^3-32)}. \tag{6.9}$$

Moreover, substituting the above values for the spillover parameters into $x_i^{\tau*}$ and $x_i^{c^\tau}$, as derived in Section 3, we have that:

$$x_i^{\tau*}\big|_{\beta_i^N=0} = \tfrac{4(a-c)}{(9\gamma-4)} \tag{6.10}$$

and

$$x_i^{c^\tau}\big|_{\beta_i^C=1} = \tfrac{4(a-c)}{(9\gamma-8)} \tag{6.11}$$

for $i = 1,2$.
Then,

(a) By simply comparing (6.10) and (6.11), we obtain that $x_i^{c^\tau}\big|_{\beta_i^C=1} > x_i^{\tau*}\big|_{\beta_i^N=0}$.

(b) Considering Eqs (6.9) and (6.8), it comes out that $(x_i^{\sigma*} - x_j^{\sigma*})\big|_{\beta_i^L=0,\beta_j^F=1} = -9\gamma^2 - 8\gamma + 16 < 0$ iff $\gamma > -4/9 + 4\sqrt{10}/9$. This condition is implied by the SOC of firm i competing *a' la Stackelberg* at the R&D investment stage - evaluated at $\beta_i^L = 0$, $\beta_j^F = 1$ - which requires that $(-112\gamma + 162\gamma^2 - 81\gamma^3 + 32) < 0$.

(c) Also, $x_j^{\sigma*}\big|_{\beta_i^L=0,\beta_j^F=1} > x_i^{c^\tau}\big|_{\beta_i^C=1}$ iff $\gamma > 4/3$, and this is implied by assumption B.2.

(d) Finally, $x_i^{\tau*}\big|_{\beta_i^N=0} > x_i^{\sigma*}\big|_{\beta_i^L=0,\beta_j^F=1}$ for $\gamma > 4/9 + 4\sqrt{2}/9$, which, as shown above, is always respected.

Combining all inequalities above, we have

$$x_j^{\sigma*}\big|_{\beta_i^L=0,\beta_j^F=1} > x_i^{c^\tau}\big|_{\beta_i^C=1} > x_i^{\tau*}\big|_{\beta_i^N=0} > x_i^{\sigma*}\big|_{\beta_i^L=0,\beta_j^F=1}.$$

We now examine the ranking of R&D investments when $\beta_i^L = \beta_i^N = 0.5$, still maintaining the assumptions that $\beta_j^F = \beta_i^C = 1$. We obtain:

(i) $x_i^{c^\tau}\big|_{\beta_i^C=1} - x_i^{\tau*}\big|_{\beta_i^N=1/2}) = 3(a-c)(3\gamma+2) > 0$.

(ii) $x_i^{\tau*}\big|_{\beta_i^N=1/2} - x_i^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F=1} = (162\gamma^3 - 256.5\gamma^2 + 195.75\gamma - 63.875)$. This expression is strictly positive for any $\gamma \geq 0.35$.

(iii) $(x_i^{c^\tau}\big|_{\beta_i^C=1} - x_j^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F=1}) = (81\gamma^3 - 58.5\gamma^2 + 45\gamma - 18)$, which is strictly positive for $\gamma > 1/2$.

(iv) $(x_j^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F=1} - x_i^{\tau*}\big|_{\beta_i^{NC}=1/2}) = 9(3\gamma^2 - 3\gamma + 0.5) > 0$ for $\gamma > 1/2$.

It suffices to take into account the conditions under A.2 as to the feasible values of γ to guarantee that all inequalities sub (i)-(iv) hold. Therefore, for $\beta_i^L = \beta_i^N = 0.5$ and $\beta_j^F = \beta_i^C = 1$, the ranking among equilibrium investments is such that:

$$x_i^{c^\tau}\big|_{\beta_i^C=1} > x_j^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F=1} > x_i^{\tau*}\big|_{\beta_i^N=1/2} > x_i^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F=1}.$$

Let now introduce the more general hypotheses that $\beta_i^L = \beta_i^N < 0.5$ and $1 \geq \beta_i^C > \beta_j^F > 0.5$, $i, j = 1, 2 \;\; j \neq i$. In what follows, we show that the ranking obtained for $\beta_i^L = \beta_i^N = 0$, $\beta_j^F = \beta_i^C = 1$ and the one obtained for $\beta_i^L = \beta_i^N = 1/2$, $\beta_j^F = \beta_i^C = 1$ are general, i.e. they hold true for all spillover rates assumed.

First, we examine the ranking of R&D investments when $\beta_i^L = \beta_i^N = 0.5$ (and $0.5 < \beta_j^F < \beta_i^C = 1$). It is easy to see that:

(1) $x_i^{c^\tau}\big|_{\beta_i^C=1} - x_i^{\tau*}\big|_{\beta_i^N=1/2} = 3(a-c)(3\gamma+2) > 0$.

(2) Moreover,

$$x_i^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F} - x_j^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F} = \frac{2(1-2\beta_j^F)(a-c)(\beta_j^F-\gamma-2)}{(2\gamma-1)(8+2\beta_j^{F2}-8\beta_j^F-9\gamma)} < 0$$

due to the SOC for the profit maximization problem when firms compete simultaneously at the investment stage and the constraints hold on γ as stated above.

(3) Also,

$$x_i^{c^\tau}\big|_{\beta_i^C=1} - x_j^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F} = \frac{2(a-c)[9\gamma^2+(10\beta_j^{F2}-22\beta_j^F+10)\gamma-16-12\beta_j^{F2}+32\beta_j^F]}{(2\gamma-1)(9\gamma-8)(9\gamma-8-2\beta_j^{F2}+8\beta_j^F)} > 0$$

due to the SOC and the assumed constraints on γ (see B.2).

(4) Then, we obtain that

$$(x_j^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F} - x_i^{\tau*}\big|_{\beta_i^N=1/2}) = \frac{4(a-c)(2-\beta_j^F)(2\beta_j^F-1)}{3(2\gamma-1)(9\gamma-8-2\beta_j^{F2}+8\beta_j^F)} > 0$$

and finally

$$x_i^{\tau*}\big|_{\beta_i^N=1/2} - x_i^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F} = \frac{2(a-c)(2+3\gamma-\beta_j^F)(2\beta_j^F-1)}{3(2\gamma-1)(9\gamma-8-2\beta_j^{F2}+8\beta_j^F)} > 0.$$

As a result,

$$x_i^{c^\tau}\big|_{\beta_i^C=1} > x_j^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F} > x_i^{\tau*}\big|_{\beta_i^N=1/2} > x_i^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F}.$$

The same ranking holds also for any value of β_i^C such that $0.5 < \beta_j^F < \beta_i^C < 1$. This can be proven considering that

$$(x_i^{c^\tau} - x_j^{\sigma*})\big|_{\beta_i^L=1/2,\beta_j^F=\beta_i^C=\beta>1/2} = \frac{2(a-c)(1-2\beta)[2\beta^3+2(\gamma-1)\beta^2-(17\gamma-4)\beta+17\gamma-9\gamma^2]}{(2\gamma-1)(9\gamma-8-2\beta+8\beta)}.$$

The above expression is strictly positive since the term in square brackets at the numerator is negative (and decreasing in β), the second term at the denominator is the SOC for simultaneous competition at the investment stage (see B.2), and the third term at the denominator is negative for any $\beta > 0.5$ due to the constraints on γ (see B.2). Now, $\beta_i^C > \beta_j^F > 0.5$ implies that x_i^C increases as well. Thus, *a fortiori*, $x_i^{c^\tau}\big|_{\beta_i^C} > x_j^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F}$. Therefore:

$$x_i^{c^\tau}\big|_{\beta_i^C} > x_j^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F} > x_i^{\tau*}\big|_{\beta_i^N=1/2} > x_i^{\sigma*}\big|_{\beta_i^L=1/2,\beta_j^F}$$

for any value of β_i^C and β_j^F such that $0.5 < \beta_j^F < \beta_i^C < 1$.

Now we consider the ranking at $\beta_i^L = \beta_i^N = 0$ and we let $0.5 < \beta_j^F < \beta_i^C \leq 1$ Note that

$$x_i^{c^\tau}\big|_{\beta_i^C=1} - x_j^{\sigma*}\big|_{\beta_i^L=0,\beta_j^F} = \frac{8(a-c)(3\gamma-4)(2\beta_j^F-1)[(3\beta_j^F-6)\gamma+4]}{(9\gamma-8)(72\gamma^2\beta_j^F+160\gamma-18\beta_j^{F2}\gamma^2-216\gamma^2-32+81\gamma^3-48\beta_j^F\gamma)} = 0$$

iff $\beta_j^F = 1/2$. Now, let $\beta_i^C = 1-\epsilon$, with ϵ sufficiently small. It is easy to see that $\frac{\partial x_i^{c^\tau}}{\partial \beta_i^C} > 0$. Therefore, $x_i^{c^\tau}\big|_{\beta_i^C} < x_j^{\sigma *}\big|_{\beta_i^L=0,\beta_j^F=1/2}$. Moreover, letting β_j^F be greater than 1/2, directly implies that $x_i^{c^\tau}\big|_{\beta_i^C} < x_j^{\sigma *}\big|_{\beta_i^L=0,\beta_j^F}$, since $x_j^{\sigma *}$ is monotonically increasing in β_j^F.

Finally, $x_i^{\sigma *}\big|_{\beta_i^L=0,\beta_j^F}$ increases for β_j^F such that $0.5 < \beta_j^F < 1$. We proceed now by contradiction, wondering if the ranking $x_i^{\sigma *}\big|_{\beta_i^L=0,\beta_j^F} > x_i^{\tau *}\big|_{\beta_i^N=0}$ could ever be feasible. It is easily found that the inequality $x_i^{\sigma *} > x_i^{\tau *}$ contradicts the above finding, i.e. that $x_i^{\sigma *}\big|_{\beta_i^L=1/2,\beta_j^F} < x_i^{\tau *}\big|_{\beta_i^N=1/2}$ at $\beta_i^L = \beta_i^N = 0.5$, and $\beta_j^F > 0.5$, combined with the fact that $x_i^{\sigma *}$ is monotonically increasing in β_i^L. As a result,

$$x_j^{\sigma *}\big|_{\beta_i^L=0,\beta_j^F} > x_i^{c^\tau}\big|_{\beta_i^C} > x_i^{\tau *}\big|_{\beta_i^N=0} > x_i^{\sigma *}\big|_{\beta_i^L=0,\beta_j^F}.$$

The fact that both $x_j^{\sigma *}$ and $x_i^{\tau *}$ are monotonically decreasing in $\beta_i^L = \beta_i^N$, and that, conversely, $x_i^{\sigma *}$ is monotonically increasing in β_i^L completes the proof. Figure 5 illustrates this proposition by means of a numerical example. ■

Assumptions under Symmetric Spillovers

A.1 Straightforward manipulations of firms' payoffs at the quantity-stage yield

$$q_i^* = q_j^* = \frac{1}{3}\left[(a-c) + (2-\beta)\,x_i + (2\beta-1)\,x_j\right] \tag{6.12}$$

and then

$$\pi_i\left(\mathbf{q}^*\left(\mathbf{x}\right)\right) = \frac{1}{9}\left[(a-c) + (2-\beta)\,x_i + (2\beta-1)\,x_j\right]^2 - \frac{\gamma}{2}x_i^2. \tag{6.13}$$

Since $\beta \in [0,1]$ and $x_i \in [0,c]$, and given that for a firm the worst investment scenario occurs when $x_i^* = 0$, $\beta = 0$ and $x_j^* = c$, by (6.12) this yields

$$q_i^*\left(x_i = 0, x_j = c, \beta = 0\right) = \frac{1}{3}\left[(a-2c)\right]. \tag{6.14}$$

This condition implies that for

$$a > 2c$$

a unique interior (positive) Cournot profile of quantities, with associated positive equilibrium profits, always exists.

A.2 It is easily shown that the investment-stage SOCs are respected for every $i = 1,2$ for

$$\frac{\partial^2 \pi_i\left(\mathbf{x}\left(\mathbf{q}^*\right)\right)}{\partial x_i^2} = \frac{1}{9}\left(8 + 2\beta^2 - 8\beta - 9\gamma\right) < 0$$

which requires that

$$\gamma > \frac{2}{9}\left(2-\beta\right)^2$$

and then strict-concavity of $\pi_i\left(\mathbf{x}\left(\mathbf{q}^*\right)\right)$ in x_i is guaranteed for

$$\gamma > \frac{8}{9}$$

for any $\beta \in [0,1]$. Firms' best-replies are obtained instead by setting the derivative of (6.13) with respect to x_i to zero and then solving for x_i we get:

$$x_i = g_i(x_j) = \frac{2(2-\beta)(a-c+(2\beta-1)x_j)}{(9\gamma+8\beta-2\beta^2-8)}.$$

Moreover, since for every firm

$$g_i'(x_j) = -\frac{\partial^2 \pi_i(x_i(\mathbf{q}^*), x_j(\mathbf{q}^*))/\partial x_i \partial x_j}{\partial^2 \pi_i(x(\mathbf{q}^*))/\partial x_i^2} = -\frac{2(2\beta-1)(2-\beta)}{(8+2\beta^2-8\beta-9\gamma)}$$

increasing differences of $\pi_i(x_i, x_j)$ in (x_i, x_j) (and then non decreasing best-replies) are implied by $\beta > \frac{1}{2}$ and decreasing differences (and non increasing best-replies) are implied by $\beta < \frac{1}{2}$, given that

$$\frac{\partial^2 \pi_i(\mathbf{x}(\mathbf{q}^*))}{\partial x_i \partial x_j} = \frac{2}{9}(2\beta-1)(2-\beta).$$

To guarantee that uniqueness of Nash equilibrium $\mathbf{x}^{\tau *}(\mathbf{q}^*)$, a contraction condition would serve the scope. This condition is respected for $g_i'(x_j) < 1$ when the function is increasing and for $g_i'(x_j) > -1$, when the function is decreasing, thus requiring

$$g_i'(x_j) = -\frac{2(2\beta-1)(2-\beta)}{(8+2\beta^2-8\beta-9\gamma)} < 1 \tag{6.15}$$

for $\beta > \frac{1}{2}$ and

$$g_i'(x_j) = -\frac{2(2\beta-1)(2-\beta)}{(8+2\beta^2-8\beta-9\gamma)} > -1 \tag{6.16}$$

for $\beta < \frac{1}{2}$.Condition (6.15) implies

$$\left(8+2\beta^2-8\beta-9\gamma\right) > -2(2\beta-1)(2-\beta)$$

which is satisfied for

$$\gamma > \frac{2}{9}(\beta+1)(2-\beta). \tag{6.17}$$

Since the RHS in (6.17) is monotonically increasing in β, (6.17) becomes

$$\gamma > \frac{1}{2}.$$

Condition (6.16) equals to

$$-2(2\beta-1)(2-\beta) > -\left(8+2\beta^2-8\beta-9\gamma\right), \tag{6.18}$$

and thus

$$\gamma > \frac{2}{3}(\beta-1)(\beta-2). \tag{6.19}$$

Since the expression on the RHS of (6.19) is monotonically increasing in β, we get

$$\gamma > \frac{4}{3}.$$

Therefore, for any $\beta \in [0,1]$ the two firms' investment best-replies $g_i(x_j)$ are contractions for $\gamma > \frac{4}{3}$.

A.3 In order to obtain interior values for the equilibrium investment level $\mathbf{x}^*$under symmetric spillovers and in all simultaneous, cooperative and sequential games, some assumptions are in order.

(i) Using the FOC of every firm $i = 1, 2$ when playing simultaneously the investment game, we obtain that

$$\frac{\partial \pi_i \left(\mathbf{x}\left(\mathbf{q}^*\right)\right)}{\partial x_i} = \frac{2\left(2-\beta\right)}{9}\left[a - c + \left(2-\beta\right)x_i + \left(2\beta - 1\right)x_j\right] - \gamma x_i,$$

which, for $x_i = 0$ becomes

$$\frac{\partial \pi_i \left(0, x_j\right)}{\partial x_i} = \frac{2\left(2-\beta\right)}{9}\left[a - c + \left(2\beta - 1\right)x_j\right] > 0 \tag{6.20}$$

for every $x_j \in [0, c]$. As a result, for a firm to play $x_i = 0$ is never a best-reply .

(ii) Secondly, when a firm $i = 1, 2$ participates to a cooperative R&D agreement, its FOC is

$$\frac{\partial \pi_i \left(0, x_j\right)}{\partial x_i} + \frac{\partial \pi_j \left(0, x_j\right)}{\partial x_i} = \frac{2\left(2-\beta\right)}{9}\left[a - c + \left(2-\beta\right)x_i + \left(2\beta - 1\right)x_j\right] - \gamma x_i + \\ + \frac{2}{9}\left(2\beta - 1\right)\left(a - c + \left(2-\beta\right)x_j + \left(2\beta - 1\right)x_i\right)$$

which, evaluated at $\mathbf{x} = (0, x_j)$, becomes

$$\frac{\partial \pi_i \left(0, x_j\right)}{\partial x_i} + \frac{\partial \pi_j \left(0, x_j\right)}{\partial x_i} = \frac{2}{9}\left(\left(a - c\right)\left(1 + \beta\right) + 10\beta x_j - 4x_j - 4\beta^2 x_j\right) > 0$$

for every $x_j \in [0, c]$. It is thus never rational for a firm in a cooperative agreement to play $x_i = 0$, no matter what the other firm does.

(iii) For a firm $i = 1, 2$ investing as a leader, the FOC is

$$\frac{\partial \pi_i \left(x_i, g_j(x_i)\right)}{\partial x_i} = \frac{\partial \pi_i \left(x_i, g_j(x_i)\right)}{\partial x_i} + \frac{\partial \pi_i \left(x_i, g_j(x_i)\right)}{\partial x_j} g_j'(x_i) = 0. \tag{6.21}$$

Notice that for $\beta > \frac{1}{2}$ both $\partial \pi_i(x_i, x_j)/\partial x_j > 0$ and $g_j'(x_i) > 0$ while for $\beta < \frac{1}{2}$, the opposite holds, given that

$$\frac{\partial \pi_i \left(0, g_j(0)\right)}{\partial x_i} = \frac{\left(a-c\right)\left(2\beta^2 + 4 - 6\beta - 3\gamma\right)\left(2 - 2\beta^2 - 3\gamma\right)\left(2-\beta\right)}{2\left(8\beta + 9\gamma - 2\beta^2 - 8\right)^2} > 0$$

for $\gamma > \frac{4}{3}$, and expression (6.20) guarantees that a firm as a leader will always invest positively at a sequential equilibrium. Moreover, since the FOC for a follower is the same as the simultaneous Nash, at the sequential equilibrium both firms will never play a profile. $\mathbf{x}^{\sigma *}= (0, 0)$. To conclude on the role of assumption A.3, we want to be sure that both firms will never play their full cost reduction investment (corner solution), and that instead either their best-replies or their cooperative decisions always lie below their maximum rational level

$$\overline{x}_i = \overline{x}_j = \frac{c}{\beta + 1}. \tag{6.22}$$

(i) and (iii) Under noncooperative behaviour and using (6.22), this is guaranteed whenever

$$\frac{\partial \pi_i \left(\overline{\mathbf{x}}\left(\mathbf{q}^*\right)\right)}{\partial x_i} = \frac{2\left(2-\beta\right)}{9}\left(\left(a - c\right) + \left(2-\beta\right)\frac{c}{\beta+1} + \left(2\beta - 1\right)\frac{c}{\beta+1}\right) - \gamma\frac{c}{\beta+1} < 0$$

which is implied by

$$\gamma > \frac{a\,(2-\beta)\,(\beta+1)}{4.5c}. \tag{6.23}$$

As a result, for

$$\gamma > \frac{a\,(2-\beta)\,(\beta+1)}{4.5c} \Leftrightarrow x_i^{\tau*} = g_i(x_j^{\tau*}) \text{ and } \mathbf{x}^{\sigma*} = (x_i^{\sigma*}, g_j(x_i^{\sigma*})) \tag{6.24}$$

both simultaneous and sequential investment equilibria are interior and lie below the boundary points, which occurs instead for

$$\gamma \leq \frac{a\,(2-\beta)\,(\beta+1)}{4.5c} \Leftrightarrow x_i^{\tau*} = \frac{c}{\beta+1} \text{ and } \mathbf{x}^{\sigma*} = \left(\frac{c}{\beta+1}, \frac{c}{\beta+1}\right). \tag{6.25}$$

(ii) For a firm participating to a cooperative R&D agreement, its FOC evaluated at $\overline{\mathbf{x}} = \left(\frac{c}{\beta+1}, \frac{c}{\beta+1}\right)$ is

$$\frac{\partial \pi_i\left(\overline{\mathbf{x}}\left(\mathbf{q}^*\right)\right)}{\partial x_i} + \frac{\partial \pi_j\left(\overline{\mathbf{x}}\left(\mathbf{q}^*\right)\right)}{\partial x_i} = \tfrac{2(2-\beta)}{9}\left[a - c + (2-\beta)\tfrac{c}{\beta+1} + (2\beta-1)\tfrac{c}{\beta+1}\right] - \gamma x_i +$$
$$+\frac{2}{9}(2\beta-1)\left[a - c + (2-\beta)\tfrac{c}{\beta+1} + (2\beta-1)\tfrac{c}{\beta+1}\right] < 0$$

which is implied by

$$\gamma > \frac{a\,(\beta+1)^2}{4.5c}. \tag{6.26}$$

Notice that for $\beta < \frac{1}{2}$ ($\beta > \frac{1}{2}$) the cooperative constraints on γ is less (more) demanding than the noncooperative constraints. Therefore, the constraint used to avoid full cost reductions for $\beta > \frac{1}{2}$ is (6.26) while for $\beta < \frac{1}{2}$ in A.3 we can use the contraint (6.23).

Assumptions under Asymmetric Spillovers

B.1 It is easily found that equilibrium quantities as function of R&D investments are given by:

$$q_i^* = \frac{1}{3}\left[(a-c) + (2-\beta_j)\,x_i + (2\beta_i - 1)\,x_j\right] \tag{6.27}$$

for $i, j = 1, 2\ \ , i \neq j$. Therefore

$$\pi_i\left(\mathbf{q}^*\left(\mathbf{x}\right)\right) = \frac{1}{9}\left[(a-c) + (2-\beta_j)\,x_i + (2\beta_i - 1)\,x_j\right]^2 - \frac{\gamma}{2}x_i^2. \tag{6.28}$$

Substituting in Eq. (6.27) or in Eq. (6.28), $x_i^* = 0$, $x_j^* = c$, $\beta_i = 0$ and $0.5 < \beta_j < 1$, we obtain that under asymmetric spillovers the condition

$$a > 2c$$

is again needed to guarantee an interior Cournot profile of equilibrium quantities, and hence the strict positivity of equilibrium profits.

B.2 Given our assumptions on spillovers at the simultaneous noncooperative R&D investment game, that is, $\beta_i^N = \beta_j^N \leq 0.5$, the SOC of the investment gane do not vary and requires that, for every $i = 1, 2$

$$\gamma > \frac{2}{9}\left(2-\beta_i^N\right)^2. \tag{6.29}$$

Being the RHS of (6.29) decreasing in β_i^N, we obtain that the most stringent condition on γ requires that:

$$\gamma > \frac{8}{9}.$$

Note that this condition also guarantees that the SOC of an R&D alliance at the investment game is respected. In this case the SOC is given by

$$\gamma > \frac{2}{9}\left(1+\beta_i^C\right)^2,$$

and, being increasing in β_i^C - and given our assumptions on β_i^C- the above condition is respected for

$$\gamma > \frac{8}{9}.$$

When both firms play simultaneously the investment stage, and given that $\beta_i^N = \beta_j^N \leq 0.5$, best replies are contractions for

$$\gamma > \frac{4}{3}.$$

In addition, to guarantee the uniqueness of the sequential equilibrium, the contraction approach applied to the follower best reply requires that

$$g_j'\left(x_i\right) = \frac{2\left(2\beta_j-1\right)\left(2-\beta_i\right)}{\left(9\gamma+8\beta_i-2\beta_i^2-8\right)} < 1 \tag{6.30}$$

and given that $0.5 < \beta_j^F < 1$ and the follower's best reply is increasing, (6.30) requires

$$\gamma > \frac{2}{9}\left(2-\beta_i\right)\left(1+2\beta_j-\beta_i\right). \tag{6.31}$$

As a result, since (6.31) is increasing in β_j and decreasing in β_i, the most stringent constraint on γ becomes

$$\gamma > \frac{4}{3}.$$

B.3 First define as $(\overline{\mathbf{x}}_i, \overline{\mathbf{x}}_j)$ the point at which the boundary lines given by

$$x_i = c - \beta_i x_j$$
$$x_j = c - \beta_j x_i$$

intersect. It is easily found that:

$$\overline{x}_i = \frac{c\left(1-\beta_i\right)}{1-\beta_i\beta_j}$$
$$\overline{x}_j = \frac{c\left(1-\beta_j\right)}{1-\beta_i\beta_j}$$

Using the profit maximization problem for a firm under asymmetric spillovers, we have

$$\frac{\partial \pi_i\left(\mathbf{x}\left(\mathbf{q}^*\right)\right)}{\partial x_i}=\frac{2\left(2-\beta\right)}{9}\left[\left(a-c\right)+\left(2-\beta\right)x_i+\left(2\beta-1\right)x_j\right]-\gamma x_i=0$$

from which the following best-reply

$$g_i\left(x_j\right)=\frac{2\left(2-\beta_j\right)\left(2\left(a-c\right)+\left(2\beta_i-1\right)x_j\right)}{\left(9\gamma+8\beta_j-2\beta_j^2-8\right)}$$

is obtained. In order to show that this best-reply lies underneath the point $(\overline{x}_i, \overline{x}_j)$, it suffices to impose that, when the incoming spillover β_i is greater that $1/2$ for at least one firm, $\frac{\partial \pi_i(\overline{\mathbf{x}}_i, \overline{\mathbf{x}}_j)}{\partial x_i} < 0$. If this condition holds true, then the equilibrium R&D investment profile will lie at the interior of the full cost reduction boundary $(\overline{x}_i, \overline{x}_j)$. More specifically,

$$\begin{aligned}\frac{\partial \pi_i\left(\overline{\mathbf{x}}\left(\mathbf{q}^*\right)\right)}{\partial x_i} &= \frac{2\left(2-\beta_j\right)}{9}\left[\left(a-c\right)+\left(2-\beta_j\right)\overline{x}_i+\left(2\beta_i-1\right)\overline{x}_j\right]-\gamma= \\ &= \frac{2\left(2-\beta_j\right)}{9}\left[\left(a-c\right)+\left(2-\beta_j\right)\frac{c\left(1-\beta_i\right)}{1-\beta_i\beta_j}+\left(2\beta_i-1\right)\frac{c\left(1-\beta_j\right)}{1-\beta_i\beta_j}\right]-\gamma\frac{c\left(1-\beta_i\right)}{1-\beta_i\beta_j}<0\end{aligned}$$

and, since

$$\left[\left(a-c\right)+\left(2-\beta_j\right)\frac{c\left(1-\beta_i\right)}{1-\beta_i\beta_j}+\left(2\beta_i-1\right)\frac{c\left(1-\beta_j\right)}{1-\beta_i\beta_j}\right]=a$$

the inequality becomes

$$\frac{2a\left(2-\beta_j\right)}{9}-\gamma\frac{c\left(1-\beta_i\right)}{1-\beta_i\beta_j}<0$$

requiring that:

$$\gamma>\frac{2}{9}\frac{a\left(1-\beta_i\beta_j\right)\left(2-\beta_j\right)}{c\left(1-\beta_i\right)}. \tag{6.32}$$

Since under the sequential equilibrium, $1 > \beta_j^F > 0.5$ and $0 \leq \beta_i^L \leq 0.5$, the constraint in (6.32), which is increasing in β_i and decreasing in β_j, boils down into the following condition on γ:

$$\gamma>\frac{a}{2c} \tag{6.33}$$

which becomes the most stringent one for firm i. The boundary points required for an interior equilibrium under noncooperative behavior and simultaneous moves were derived in the previous section. By simply substituting for β_i^N, for $i = 1, 2$, we have

$$\gamma>\frac{2a\left(2-\beta_i^N\right)\left(\beta_i^N+1\right)}{9c}. \tag{6.34}$$

Moreover, since in our assumptions $\beta_i^N \leq 0.5$, and given that (6.34) is increasing in β_i^N, the (most stringent) condition on γ becomes:

$$\gamma>\frac{a}{2c}. \tag{6.35}$$

For a firm entering an R&D alliance, the constraint on γ does not vary with respect to the case with symmetric spillovers, except under the assumption $0.5 < \beta_i^C \leq 1$. As a result, the condition

$$\gamma > \frac{2a\left(\beta_i^C + 1\right)^2}{9c}. \tag{6.36}$$

boils down into:

$$\gamma > \frac{8a}{9c}. \tag{6.37}$$

Also in this case the constraint on γ required under cooperation (eq. 6.37) is the most stringent and therefore will be the one which has to be imposed. Finally, combining both constraints in B.1 and in B.3 for the sequential investment game, the most demanding condition on γ is:

$$\gamma > 1.$$

Finallly, in the noncooperative simultaneous investment stage, the same constraint on γ has to be satisfied, whilst, under the cooperative case, it is required that:

$$\gamma > 16/9$$

This is the most stringent condition used in the numerical simulations with asymmetric spillovers.

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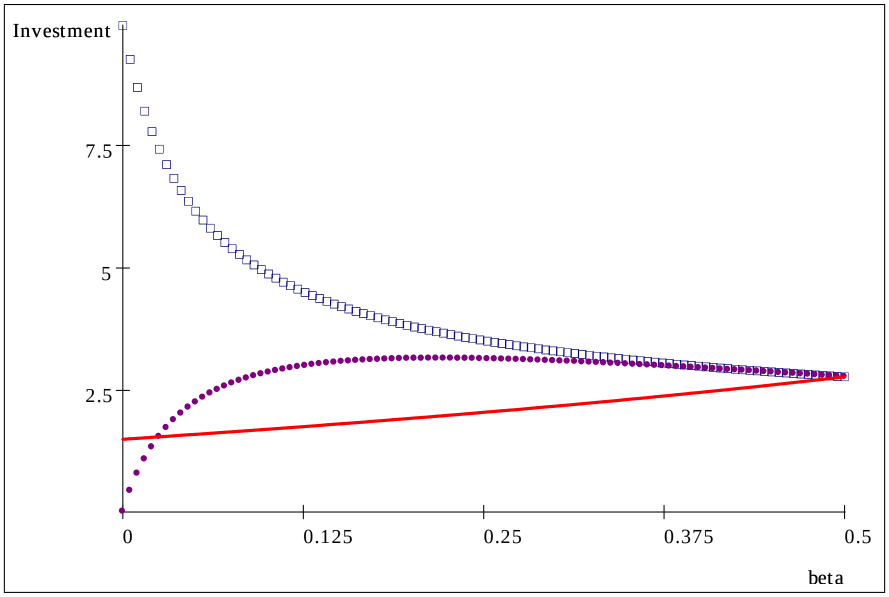

Fig 1 - R&D investment for the leader (boxed line), follower (dotted line) and cooperative firm (continuous line) for $a = 38$, $c = 18$, $\gamma = 1.7$ and $0 \leq \beta \leq \frac{1}{2}$.

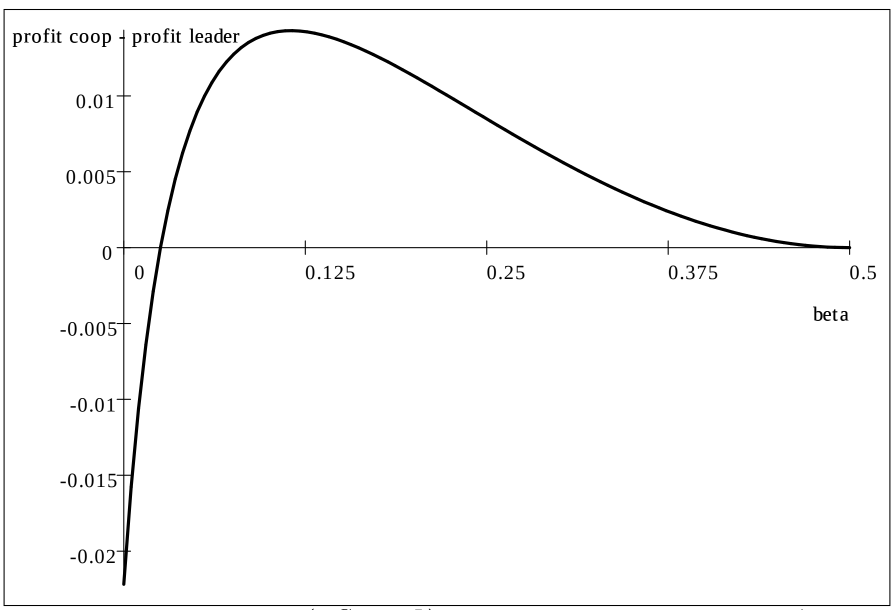

Fig 2 - Levels of $\left(\pi_i^C - \pi_i^L\right)$ for $\gamma = 1.7$ and $0 \leq \beta \leq \frac{1}{2}$.

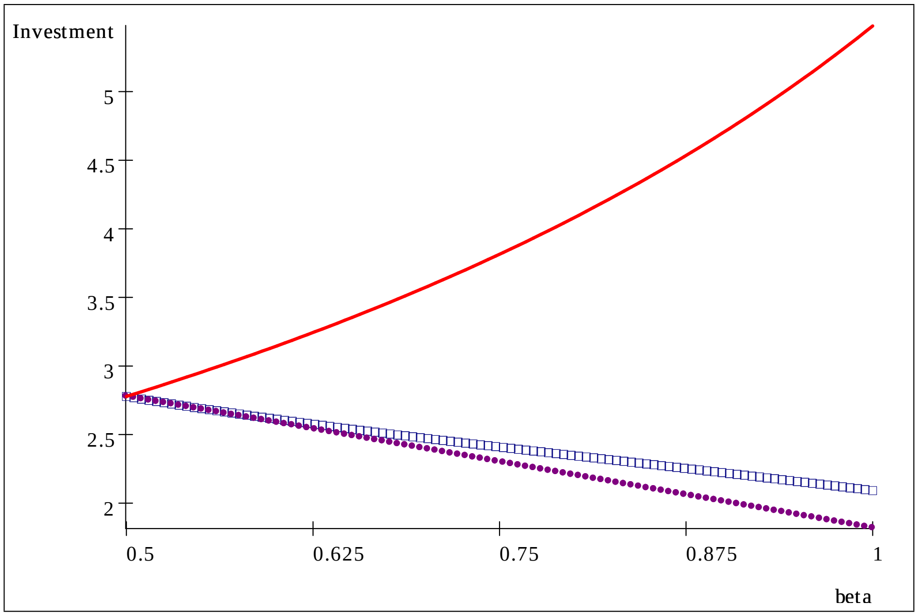

Fig 3 - R&D investments for the leader (boxed line), follower (dotted line) and cooperative firm (continuous line) for $a = 38$, $c = 18$, $\gamma = 1.7$ and $\frac{1}{2} \leq \beta \leq 1$.

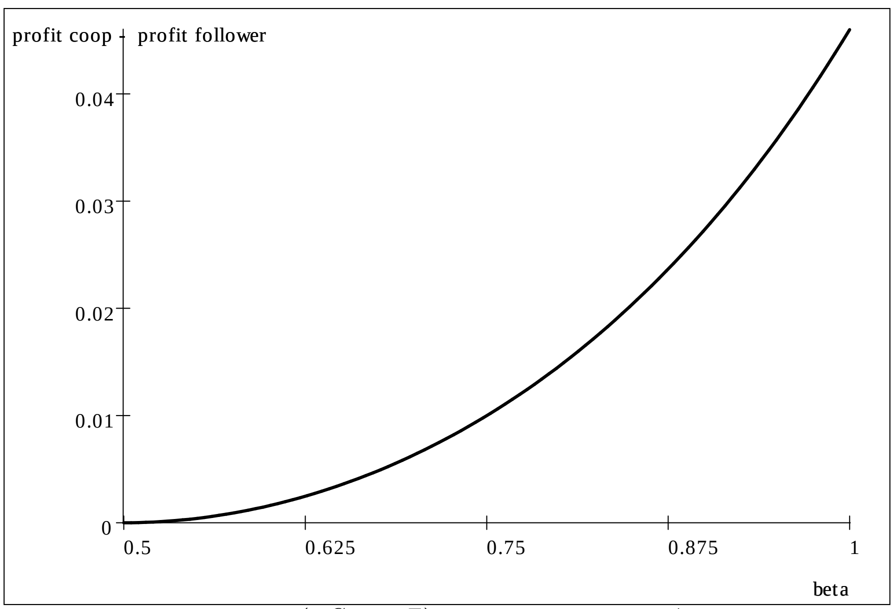

Fig 4 - Levels of $\left(\pi_i^C - \pi_j^F\right)$ for $\gamma = 1.7$ and $\frac{1}{2} \leq \beta \leq 1$.