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**D I C H I A R A**

che sebbene il lavoro in collaborazione:

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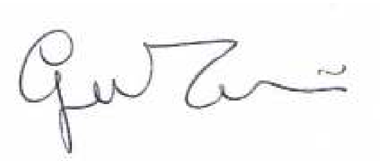
presentato ai fini della valutazione scientifica nazionale è il risultato di uno sforzo comune di ricerca, il medesimo Giuseppe Travaglini ha curato la stesura dei **paragrafi 2, appendice 1.**

Roma, 10 novembre 2012.

Il dichiarante, Giuseppe Travaglini



# Chapter 1
# Investment, Productivity and Employment in the Italian Economy

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**Abstract** This paper analyzes the effect of institutional structure, regulations, technological progress, and labour market flexibility on productivity in the Italian economy within the framework of the representative agent model of Saltari and Travaglini (2007). The core model is shown to be too restrictive to provide a good representation of the Italian economy. Broadening the view of the way in which firms take account of the costs of changing the labour force and investment achieves a more satisfactory representation of the dynamics of the productive sector of the economy while still retaining the spirit of the core model. Institutional or market structures, regulations, and other factors are incorporated in the system through modifications to the production function, the demand and supply functions for labour. A full-information, Gaussian estimator of a differential equation system is used throughout. As the constraints on the system arise from both macro-economic theory and the institutional structure of the Italian economy, this estimator provides a much more stringent test of all the hypotheses embedded in the model than many other studies. The model provides a foundation for a study of the extent to which, over time, changes in regulations or market structure might allow firms to reallocate resources to take better advantage of the skills available in the labour force within the context of a segmented labour market with varying efficiencies. The model lends itself to a policy analysis of the effects of these changes on the workings of the labour market as the ease with which firms may change their labour force determine the dynamics of the interaction between firms and labour and the path over time of labour and capital themselves.

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## 1.1 Introduction

The aim of this study is to investigate the effect of the institutional structure, regulations, technological progress, and labour market flexibility on productivity in the Italian economy within the context of a tightly defined macro-economic model. The core model is based on the representative agent model of Saltari and Travaglini (2007).

The core model (called ST below) is derived from maximising the intertemporal profit function of a firm with respect to the labour/capital ratio, with the value function determining investment, both subject to deterministic costs of adjustment. A simple function for real wages closes the model. The steady state may be derived from the first order conditions so that differentiating with respect to the parameters of the system allows both a comparative steady state analysis and an analysis of stability in the neighborhood of the steady state.

The core model assumes the value of the firm is normalized by capital stock which means it cannot be estimated as a dynamic system as it stands. Also, it did not allow differentiation between the different issues being investigated. For those reasons it was modified to allow aggregation over firms to the macro level and to incorporate costs of investment directly in the behaviour function. The wage determination equation was reformulated as a simple non-tatonnement process which helps differentiate demand and supply effects on the system. The Hamiltonian of this extended or augmented model (called STA below) provides first order conditions very similar to the core (ST) model and hence it has a similar steady state. The differential equations that form this model can be estimated directly by a full information procedure so all the constraints inherent in the theory are imposed within that procedure and hence there is full consistency between the estimated parameters and model and the theory. Moreover, the estimators use either the non-linear model directly or, for linear or linearized differential equation models, a stochastically equivalent discrete model which is satisfied by the observations generated by the continuous system irrespective of the observation interval of the sample. Thus the properties of the parameters of the differential equation system are given directly by the non-linear model or may be derived from the sampling properties of the discrete model.

The derivation of this model does not take account of the specific institutional structures in the economy nor of regulations imposed on firms or the labour market that affect the workings and flexibility of the labour market. Thus it still precludes investigation of some of the issues of concern. In order to address these issues, a more general causal model of the production sector was specified. This model (called STW below) again has a very similar steady state (if it exists) to the models above, but although it is based on optimizing the profit function of the firm subject to the usual constraints, it is not Hamiltonian and hence the question of its stability is much more complex.

The models in this study are derived from or based directly on economic theory, particularly the theory of the firm, and do not take account of the specific institutional or market structure within which the system operates. These institutional or market structures, regulations, and other factors may be incorporated in the system by appropriate modifications to the functions of the model such as, in this case, the production function, the demand and supply functions for labour, and the overall labour market function that brings together demand and supply to determine the wage rate (or it's rate of change). In a more general model, price determination could also be introduced.

The specific issues of interest are:

1. The effect of a segmented labour market on productivity where the different segments have different efficiencies. Over time, and with changes in regulations or market structure more generally, firms may be able to reallocate resources to take better advantage of the skills available in the labour force.
2. The effect of institutions on the structure of the labour market, including the way in which it operates, and the impact of changes in regulations on the workings of the market, the ease or otherwise with which firms may change their labour force and hence the associated costs. Regulations affect the function that embodies the interaction between firms and labour as well as the costs embedded in the functions that determine labour and capital themselves.
3. The effect of changes in technology on productivity and employment.
4. The effect of the differential in efficiency of skilled and unskilled labour, and the extent to which firms can utilize skills, on the productivity and profitability of the firm.

Part of this study was to estimate and test the joint hypotheses underlying the core model using macroeconomic data of the Italian economy. In investigating the issues above, it is necessary to have some base model which can incorporate additional hypotheses and allow them to be tested with enough precision that they can be distinguished. It was found that when the core model was estimated subject to all the constraints imposed by the theory underlying the model, it was rejected by the data. This meant that alternative models, as much as possible in the spirit of the underlying core model, had to be developed and tested. Modifying the model by replacing the Cobb-Douglas production function of the core model by a CES improved the estimates but was not sufficient to give a model which could be estimated precisely enough for the purposes of this research. It was necessary to broaden the view of the way in which firms take account of the costs of changing both the labour force or investment, and hence in their optimal choice of technology, in order to achieve a more satisfactory representation of the dynamics of the productive sector of the economy. These results raise the question of whether some of the models being used in this field are justifiable.

A feature of this research is that the steady state of even the more complex models are essentially the same as the core model and are functions of the parameters of the system. Thus the effect of changes in those parameters may be derived immediately. The dynamic properties of the model written in terms of (logarithmic) deviations about the steady state may then be calculated.

The core model is given in Appendix 1. Section 2 develops this model so it is suitable for econometric purposes. Some comments on the estimation procedure, and the estimates of the augmented (STA) core model, are given in Section 3. Section 4 discusses variants of this model and gives estimates of the two major variants.

## 1.2 Augmented Saltari-Travaglini model with investment in the objective function

This model is based directly on Saltari and Travaglini (2007). The value of the firm is maximized taking into account the costs of changing employment and investment and assuming the production function is Cobb-Douglas with constant returns to scale. Let $L$ be employment, $K$ the fixed capital stock, and the labour/capital ratio $n = \frac{L}{K}$. It is assumed that the derivatives of employment and capital can be changed by the firm so let $z = \dot{n}$ and $I = \dot{K}$ with costs of adjustment $c$ and $h$ respectively. Initially $I$ is considered as net investment but it could be defined as gross with a depreciation factor. In Saltari and Travaglini (2007) the size of the firm was normalized but in this study capital is made explicit; no distinction is made between firms increasing in size and an increase in the number of firms.

Let the value of the firm be

$$\max_{z,I} \int_t^{\infty} e^{-\rho s}\{(An^{1-\alpha} - wn - \frac{c}{2}(\frac{z}{n})^2)K - (1 + \frac{h}{2}I)I\}ds \qquad (2.1)$$

subject to the definitional equations above for the control variables. Function (2.1) may be written

$$\max_{z,I} \int_t^{\infty} e^{-\rho s}\{(An^{1-\alpha} - wn - \frac{c}{2}(\frac{z}{n})^2) - (1 + \frac{h}{2}I)k\}Kds \qquad (2.1a)$$

where $k = \frac{I}{K}$. This allows (2.1) to be interpreted both as the objective function of an individual firm at the micro level or the aggregate at the macro level on the assumption of the firm being a representative agent. For theoretical studies of a single firm, $K$ is often assumed to be normalized to 1 for simplicity but that is unnecessary. The term inside $\{...\}$ in (2.1a) is the value function of the single firm per unit capital; if the initial capital stock is normalized, $I$ and $k$ are the same and the final $K$ in the expression disappears but otherwise $I$ refers to the *level* of net investment by the single firm. Hence under normalization $K$ disappears from the value function.

At the macro level, the value function is aggregated across firms to give a total capital stock $K$ but in this case investment $I$, and costs of investment, must be interpreted as the aggregate level. Moving from micro to the macro level is not just a matter of multiplying the (normalized) value of the firm by the number of firms $K$ but of noting that, because the model is no longer normalized and the interpretation of $I$, $K$ becomes explicit in the value function itself via $k$. The first order conditions below apply to both interpretations.

It is useful (as a minor simplification) to transform the control variable by defining $\ell = \frac{\dot{n}}{n} = D\ln n$. This does not change the profit function but the constraint on the state variable $n$ becomes $\dot{n} = \ell n$ and the inter-temporal objective function is optimized with respect to $\ell$ rather than $z$.

The Hamiltonian becomes

$$H = e^{-\rho t}\{(An^{1-\alpha} - wn - \frac{c}{2}\ell^2)K - (1+\frac{h}{2}I)I\} + \nu_1 \ell n + \nu_2 I \tag{2.2}$$

Where required, it will be assumed $\nu_i = \mu_i e^{-\rho t}$ so $\dot{\nu}_i = \dot{\mu}_i e^{-\rho t} - \rho\mu_i e^{-\rho t}$.

The first order conditions are:

$$\frac{\partial H}{\partial \nu_1} = \dot{n} = \ell n, \tag{2.2.1}$$

$$\frac{\partial H}{\partial \nu_2} = \dot{K} = I, \tag{2.2.2}$$

$$\frac{\partial H}{\partial n} = e^{-\rho t}(-\dot{\mu}_1 + \mu_1\rho) = e^{-\rho t}\{A(1-\alpha)n^{-\alpha} - w\}K + e^{-\rho t}\mu_1\ell, \tag{2.2.3}$$

$$\frac{\partial H}{\partial K} = e^{-\rho t}(-\dot{\mu}_2 + \mu_2\rho) = e^{-\rho t}\{An^{1-\alpha} - wn - \frac{c}{2}\ell^2\}, \tag{2.2.4}$$

$$\frac{\partial H}{\partial \ell} = -e^{-\rho t}(c\ell K - \mu_1 n) = 0, \tag{2.2.5}$$

$$\frac{\partial H}{\partial I} = -e^{-\rho t}(1 + hI - \mu_2) = 0. \tag{2.2.6}$$

Thus

$$\mu_1 = \tfrac{c}{n}\ell K,\ \dot{\mu}_1 = \tfrac{c}{n}(\dot{\ell}K + \ell\dot{K} - \ell^2\dot{K}), \tag{2.2.5a}$$

$$\mu_2 = 1 + hI,\ \dot{\mu}_2 = h\dot{I}. \tag{2.2.5b}$$

From (2.2.3) and (2.2.4)

$$\dot{\mu}_1 = \mu_1(\rho - \ell) - \{A(1-\alpha)n^{-\alpha} - w\}K, \tag{2.2.3a}$$

and

$$\dot{\mu}_2 = \mu_2\rho - \{An^{1-\alpha} - wn - \frac{c}{2}\ell^2\}. \tag{2.2.4a}$$

If required, this reduces to a second order system in $n$ and $K$. $\mu_1$ is essentially the same as $q$ in Saltari and Travaglini (2007). If $q^* = \frac{\mu_1}{K}$, (2.2.3a) becomes

$$\dot{q}^* = \rho q^* - A(1-\alpha)n^{-\alpha} - w - q^* D\ln K \text{ and } \ell = \tfrac{n}{c}q^*\ . \tag{2.2.3a}$$

Alternatively, for estimation purposes, (2.2.1), (2.2.3) and (2.2.5a) give

$$\dot{\ell} = \ell(\rho - k) - \frac{n}{c}\{A(1-\alpha)n^{-\alpha} - w\}, \tag{2.3.1}$$

and, similarly, (2.2.2), (2.2.4) and (2.2.6a) give

$$\dot{k} = k(\rho - k) - \frac{1}{hK}\{An^{1-\alpha} - wn - \frac{c}{2}\ell^2 - \rho\} \tag{2.3.2}$$

Assuming that wages are determined by marginal product of labour but are sticky, the model may be closed with a wage determination equation such as,

$$D\ln w = \gamma \ln\left(\frac{A(1-\alpha)n^{-\alpha}}{w}\right) + \lambda_w \tag{2.3.3}$$

where the numerator is the marginal product of capital and $\lambda_w$is the long run rate of growth of wages. The latter term is necessary for consistency in a model with growth; alternatively, a corresponding term could be introduced within the logarithm giving

$$D\ln w = \gamma \ln\left(\frac{A(1-\alpha)n^{-\alpha}}{we^{-\lambda_w/\gamma}}\right). \tag{2.3.3a}$$

It was found during estimation that a second order function, which gives a "humped" adjustment functions so that the peak adjustment to wages does not occur immediately, was preferable. Thus

$$D^2 \ln w = \gamma_1 \ln\left(\frac{A(1-\alpha)n^{-\alpha}}{w}\right) - \gamma_2(D\ln w - \lambda_w). \tag{2.3.4}$$

If investment is gross and capital depreciates at a fixed rate $\delta$ the capital equation (2.2.2) becomes

$$\dot{K} = I - \delta K \tag{2.2.2b}$$

and so (2.2.4) has an extra term $-\delta\mu_2 e^{-\rho t}$; hence (2.2.4a) becomes

$$\dot{\mu}_2 = \mu_2(\rho - \delta) - (An^{1-\alpha} - wn - \frac{c}{2}z^{\frac{2}{n}}). \tag{2.2.4b}$$

In order for the model to be a plausible representation of a developed economy, it is necessary to introduce growth in some form; for simplicity, technical progress was introduced into the production function by replacing $A$ by $A_0e^{\lambda_1 t}$ where $\lambda_1$ is the rate of technical progress.

The model has a steady state if there exists a solution of the form $x(t) = x^*e^{\mu_x t}$ for all variables. Let the rate of growth of the labour force be $\lambda_2$. The rate of growth of the capital stock is $k^*$, and as all terms in $\{...\}$ in (2.3.2) must be independent of $t$, the first term in that expression gives $k^* = \lambda_1/(1-\alpha) + \lambda_2$; as the left hand side of equation (2.3.2) is zero, multiplying through by $hK$ shows that for a steady state to exist $\rho$ must equal to $k^*$. From (2.3.3) the steady state rate of growth of wages is $\lambda_1/(1-\alpha)$ so that in efficiency units, wages are constant. Thus for consistency $\lambda_w = \lambda_1/(1-\alpha)$. The term $\{A(1-\alpha)n^{-\alpha} - w\}$ in (2.3.1) is zero and hence the term $\{\ldots\}$ in (2.3.2) becomes $\{A\alpha n^{1-\alpha} - \frac{c}{2}\ell^2 - \rho\}$ which again is independent of $t$.

Without costs of adjustment, the steady state solution of the model is given by wages $w$ and the return on capital $\rho$ being equal to the corresponding marginal products. With costs, the steady state levels are

$$n^* = \psi^{\frac{1}{1-\alpha}} \text{ and } w^* = A_0(1-\alpha)\psi^{-\frac{\alpha}{1-\alpha}}$$

where

$$\psi = \frac{1}{A_0\alpha}\left[\rho + \frac{c}{2}\left(\frac{\lambda_1}{1-\alpha}\right)^2\right]$$

The assumption of a Cobb-Douglas production function with constant returns to scale means that the steady state level of the capital stock is indeterminate and is a function of initial values. For a given steady state value of employment $L^*$ there is a corresponding steady state level of capital stock $K^* = L^*/n^*$.

For analytical purposes, such as questions of stability either in a classical or non-classical sense, it is useful to write the model in terms of deviations about the steady state, if it exists. The underlying model above has the non-autonomous form

$$Dy(t) = f\{y(t), t; \theta\} \tag{2.4}$$

where $\theta$ is the vector of parameters; under appropriate conditions, there is a transformation of variables that allows it to be written as the autonomous or non-autonomous system

$$Dx(t) = \phi\{x(t), t; \theta\}\ . \tag{2.5}$$

Let $x_\ell = \ell - \ell^*$, $x_k = k - k^*$, $x_\omega = D\ln w - \frac{\lambda_1}{1-\alpha}$, $x_n = \ln(n/n^*) + \frac{\lambda_1}{1-\alpha}t$, $x_w = \ln(w/w^*) - \lambda_w t$ and $x_K = \ln(K/K^*) - (\frac{\lambda}{1-\alpha} + \lambda_2)t$ be the (logarithmic) deviations from the steady state $\omega = D\ln w$. Thus

$$\dot{x}_\ell = (x_\ell + \ell^*)(\rho - x_k - k^*) - A_0\frac{1-\alpha}{c}\psi\left(e^{(1-\alpha)x_n} - e^{x_n + x_w}\right)\ , \tag{2.6.1}$$

$$\dot{x}_k = (x_k + k^*)(\rho - x_k - k^*) \tag{2.6.2}$$

$$-\frac{1}{hK^*}e^{-x_K - k^* t}\{A_0\psi e^{(1-\alpha)x_n} - A_0(1-\alpha)\psi e^{x_w + x_n} - \frac{c}{2}\left(x_\ell + \frac{\lambda_1}{1-\alpha}\right)^2 - \rho\},$$

$$\dot{x}_\omega = -\gamma_1\alpha x_n - \gamma_1 x_w - \gamma_2 x_\omega\ , \tag{2.6.3}$$

with three definitional equations

$$\dot{x}_n = x_\ell\ , \tag{2.6.4}$$

$$\dot{x}_K = x_k\ ,$$

$$\dot{x}_w = x_\omega\ . \tag{2.6.6}$$

The first terms in (2.6.1) and (2.6.2) simplify if the steady state condition $\rho = k^*$ is imposed.

Linearizing in terms of deviations about the steady state, with $x_j = 0$ for all $j$, gives

$$\dot{x}_\ell = x_\ell(\rho - k^*) - x_k \ell^* + A_0 \frac{1-\alpha}{c} \psi(\alpha x_n + x_w) \;, \tag{2.7.1}$$

$$\dot{x}_k = x_k(\rho - 2k^*) + \frac{1}{hK^*}\{A_0 \alpha \psi - \frac{c}{2}\left(\frac{\lambda_1}{1-\alpha}\right)^2 - \rho\} e^{-k^* t} x_K \tag{2.7.2}$$

$$+ \frac{1}{hK^*} A_0 (1-\alpha) \psi e^{-k^* t} x_w + \frac{c}{hK^*} \frac{\lambda_1}{1-\alpha} e^{-k^* t} x_\ell \;,$$

$$\dot{x}_\omega = -\gamma_1 \alpha x_n - \gamma_1 x_w - \gamma_2 x_\omega \;, \tag{2.7.3}$$

$$\dot{x}_n = x_\ell, \tag{2.7.4}$$

$$\dot{x}_K = x_k, \tag{2.7.5}$$

$$\dot{x}_w = x_\omega. \tag{2.7.6}$$

As $t$ becomes large, the exponential in $t$ goes to zero.

## 1.3 Estimation

It is assumed throughout that at the macro-economic level the Italian economy can be represented by a continuous system as in (2.2) or (2.2.1) - (2.2.4) and (2.3.3) above, and the data used are discrete observations of the continuous trajectory at equidistant (quarterly) periods. The estimators used are all full-information maximum-likelihood and estimate the parameters of the system defined above using either the continuous model directly or a discrete models stochastically equivalent to that system. Thus the parameters of the estimated models are the same as the parameters of the specified differential equation system. Owing to the derivation of the first order conditions of the profit function (2.2) these models are heavily over-identified and thus provide a powerful test of the joint hypotheses inherent in (2.2). Similar comments apply to the models below.

Full-information maximum-likelihood estimators were used throughout, an exact discrete estimator of a linear (or linearized) system and a Gaussian estimator of a non-linear system[4]. These are described in Wymer (2006) and a more general discussion of these techniques is in Wymer (1996, 1997). The properties of full-information maximum likelihood estimators of linear models are more developed than those for non-linear models but a non-linear estimator eliminates any bias arising from linearization and provides an estimate of any biases. Moreover, lineariza-

[4] The programs used here are part of the WYSEA System Estimation and Analysis package. Specifically, they were an approximate discrete estimator (Resimul), the exact discrete estimator (Discon) and a non-linear exact estimator (Escona). Eigenvalues of a linear system and Lyapunov exponents of a non-linear system may also be calculated.

tion may sometimes lead to parameters becoming unidentified, or poorly identified in that the asymptotic standard errors become very large; this is less likely with a non-linear estimator.

The data are described in the Data Appendix below.

Assuming that the data are generated by the process (2.2) or (2.3.1), (2.3.2) etc. above, the model with second order derivatives of $n$ and $K$ may be estimated directly[5].

The model used for estimation is (2.3.1), (2.3.2), (2.3.3) or (2.3.4) in terms of $\ln(n)$, $\ln(K)$, and $\ln(w)$ but written as a first order system with $D\ln(n) = \ell$ and $D\ln(K) = k$, and $D\ln(w) = \omega$ where (2.3.4) is used. Although the model may be estimated in linear or non-linear form, it was decided initially to linearize the system about sample means (that is, $\bar{\ell}$, $\bar{k}$, $\overline{\ln n}$, $\overline{\ln K}$, and $\overline{\ln w}$); this linear model may be estimated subject to all of the constraints inherent in the underlying theory as well as those arising from the linearization. Alternatively, the model could have been linearized about the steady state. In either case, the estimated parameters are those of the theoretical model. For simplification only, time $t$ is defined to have mean zero; thus $\bar{t}$ drops out of the linearization.

The model linearized about sample means is:

$$D\ell = (\rho - \bar{k})\ell - \bar{\ell}k - \frac{1}{c}\{(1-\alpha)\psi - e^{\overline{\ln w}+\overline{\ln n}}\}\ln n + \frac{1}{c}e^{\overline{\ln w}+\overline{\ln n}}\ln w + \frac{1}{c}\psi\lambda_1 t$$
$$+\bar{\ell}\bar{k} - \frac{1}{c}\{\psi - (1-\alpha)\psi\overline{\ln n} - e^{\overline{\ln w}+\overline{\ln n}}(1 - \overline{\ln w} - \overline{\ln n})\} \quad (3.1.1)$$

where $\psi = A_0(1-\alpha)e^{(1-\alpha)\overline{\ln n}}$,

$$Dk = (\rho - 2\bar{k})k + \frac{\phi}{h}e^{-\overline{\ln K}}\ln K - \frac{1}{h}e^{-\overline{\ln K}}\{\psi - e^{\overline{\ln w}+\overline{\ln n}}\}\ln n \quad (3.1.2)$$
$$+\frac{1}{h}e^{\overline{\ln w}+\overline{\ln n}-\overline{\ln K}}\ln w - \frac{1}{h}A_0 e^{(1-\alpha)\overline{\ln n}-\overline{\ln K}}\lambda_1 t + \frac{c}{h}e^{-\overline{\ln K}}\bar{\ell}\ell$$
$$+\bar{k}^2 - \frac{1}{h}e^{-\overline{\ln K}}\{\phi + \phi\overline{\ln K} - (\psi - e^{\overline{\ln w}+e^{\overline{\ln n}}})\overline{\ln n} + e^{\overline{\ln w}+\overline{\ln n}}\overline{\ln w} + c\bar{\ell}^2\}$$

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[5] Several attempts were made to estimate the underlying model (2.2.1), (2.2.2) and (2.3.3) with other estimators but the extent to which the model was not consistent with the data led to these being unsatisfactory. The first order conditions give a first order non-linear differential equation model with endogenous (state) variables $n$, $K$ and $w$ and costate variables $\mu_1$and $\mu_2$. Although the costate variables are unobserved this may be estimated as a two point boundary point model with $\mu_i(t + T) = 0$ for each observation point t and T is a given horizon relative to t as in Wymer(2006).

As the system is continuous, (2.2.1), (2.2.2 ) may be replaced by the second order process in $n$ and $K$ (2.3.1), (2.3.2 ) as all observations are consistent with the latter. This non-linear model, with (2.3.3) or (2.3.4) can be estimated using a non-linear continuous estimator or linearized and estimated with a linear estimator but subject to all of the constraints inherent in the underlying model and in the linearization. Both estimators were used during this study but only the results for the linearized model are given in this Section.

**Table 1.1** Estimates of parameters

| Parameter | Estimate | Asymptotic Standard Error |
|---|---|---|
| c | 3.636 | 4.629 |
| h | 126.807 | 1.00E+05 |
| $\rho$ | 0.016 | 0.004 |
| $A_0$ | 3.840 | 24.780 |
| $\alpha$ | 0.185 | 1.663 |
| $\gamma_1$ | 0.057 | 0.098 |
| $\gamma_2$ | 0.196 | 4.866 |
| $\lambda_1$ | 0.016 | 0.012 |
| $\lambda_2$ | −0.001 | 0.122 |
| p | 1.056 | 2.657 |

where $\phi = A_0 e^{(1-\alpha)\overline{\ln n}} - e^{\overline{\ln w}+\overline{\ln n}} - \frac{c}{2}\overline{\ell}^2 - \rho$,

$$D\omega = -\gamma_1 \alpha \ln n - \gamma_1 \ln w - \gamma_1 \lambda_1 t - \gamma_2 \omega + \gamma_1 \{\ln A_0 + \ln(1-\alpha)\} + \gamma_2 \frac{\lambda_1}{1-\alpha}, \tag{3.1.3}$$

$$Dn = \ell, \tag{3.1.4}$$

$$D\ln K = k, \tag{3.1.5}$$

$$D\ln w = \omega. \tag{3.1.6}$$

Full-information maximum-likelihood estimates of this model are given in Table 1.1.

The Chi-square value of the likelihood ratio test is 990.6 with 14 degrees of freedom; the critical value at the 5 per cent level is 23.7.

These estimates give some idea of the values of the parameters[6] of the core theoretical model but the asymptotic standard errors are large and the likelihood ratio test rejects the hypothesis that the model represents the system that generated the data. Almost all parameters are not significantly different from zero but the large asymptotic standard errors show that the true values of the parameters could lie within a wide range. The parameter $p$ is merely a scaling factor in the wage equation needed to equate (approximately) the mean marginal product of labour and the mean wage rate and has no economic significance.

Given the values of variables in the model, the cost of adjustment $c$ of the labour/capital ratio seems particularly low. This may indicate a misspecification of the cost of adjustment term in the (discounted long-term profit) objective function of the firm.

It should be noted that the full-information estimation procedure used here imposes all the conditions implicit in the underlying theoretical model as defined in equations (3.1.1), (3.1.2) and (3.1.4) as well as imposing the constraints that arise in linearization. This provides consistent estimation of all parameters in the system

[6] To interpret these parameters, the mean values of the variables are approximately K=3000 (€bn), L = 20 (m), n = 0.007 (employees per unit capital), and w = 6 ( € '000 per employee per quarter). Real output,Y, used in the models below, is approximately 220 (€bn per quarter).

subject to all constraints. The tight, highly theoretical, specification means that the parameter set used to represent the core equations of the economy is very small and undoubtedly this leads to the data rejecting this specification.

The properties of a Cobb-Douglas production function raise the question of whether it is justifiable and the most suitable for a model of this nature. While the labour/capital ratio is well-defined, the steady state level of capital (or of labour) is not; given an assumption about the level of one variable, for instance $L^*$, immediately provides the other as $n^*$ is known. The use of this function is particularly restrictive and it has poor properties; in particular the elasticity of substitution is one. The CES is perhaps the simplest of production functions which have more satisfactory properties with the CES having an elasticity of substitution which is constant but not necessarily one and although the standard specification has constant returns to scale, that is not necessary. Comparing the two functions must take into account the way in which the functions enter each equation of the model; while the CES can, as a special case, exhibit constant returns to scale and in that sense be similar to a Cobb-Douglas, this is only one aspect of their relative properties and estimates of this, independent of the whole model, are likely to be biased.

This model was also estimated in non-linear form (2.3.1) - (2.3.3) using a full-information Gaussian estimator and also as a two-point boundary point system (2.2.1) - (2.2.6) as indicated above. These estimates were not satisfactory and again reject the joint hypothesis that the observed data were generated by this system.

## 1.4 A more general specification of core model: Saltari-Travaglini-Wymer model

Several suggestions can be made towards formulating a more representative model of the Italian economy while still retaining the strongly theoretical core. Although a number of suggestions can be made, for the purposes of this study only those that are broadly within the framework of the core model will be tested.

A CES production function has more plausible properties than the Cobb-Douglas from the viewpoint of the whole system. It is more general than the Cobb-Douglas but is amenable to analysis and, in models such as this, usually is consistent with a steady state (if that is considered important) and, subject to the specification of the whole system, provides a well defined steady state level of the capital stock as a function of parameters of the model. It can also be adapted more easily to investigate some of the issues discussed below.

Secondly, wage determination may be mis-specified. In the present model wages are assumed to adjust to the marginal product of labour and this imposes a strong constraints on the system and the parameters. A better representation may be that wages are determined by excess demand in the labour market. This process of prices adjusting to excess stocks has been found to provide a good explanation of price movements in other models: in macro models where the GDP deflator depends on excess demand for stocks of goods (inventories); with interest rates in monetary models; with copper prices to excess copper stocks in a commodity model, and similar results in other commodity markets.

A more general formulation within the same framework defines the value of the firm as

$$\max_{z,I} \int_t^{\infty} e^{-\rho s}\{f(L,K) - wL - \frac{c}{2}z^2 - (1+\frac{h}{2}I)I\}ds \tag{4.1}$$

subject to the definitional equations above for the control variables $z = \dot{L}$ and $I = \dot{K}$.

Thus the Hamiltonian is

$$H = e^{-\rho t}\{f(L,K) - wL - \frac{c}{2}z^2 - (1+\frac{h}{2}I)I\} + \nu_1 z + \nu_2 I. \tag{4.2}$$

As above, let $\nu_i = \mu_i e^{-\rho t}$ so $\dot{\nu}_i = \mu_i e^{-\rho t} - \rho\mu_i e^{-\rho t}$

The first order conditions are:

$$\frac{\partial H}{\partial \nu_1} = \dot{L} = z, \tag{4.2.1}$$

$$\frac{\partial H}{\partial \nu_2} = \dot{K} = I, \tag{4.2.2}$$

$$\frac{\partial H}{\partial L} = e^{-\rho t}(-\dot{\mu}_1 + \mu_1\rho) = e^{-\rho t}\{\frac{\partial f}{\partial L} - w\}, \tag{4.2.3}$$

$$\frac{\partial H}{\partial K} = e^{-\rho t}(-\dot{\mu}_2 + \mu_2\rho) = e^{-\rho t}\frac{\partial f}{\partial K}, \tag{4.2.4}$$

$$\frac{\partial H}{\partial z} = -e^{-\rho t}(cz - \mu_1) = 0, \tag{4.2.5}$$

$$\frac{\partial H}{\partial I} = -e^{-\rho t}(1 + hI - \mu_2) = 0. \tag{4.2.6}$$

Thus $\mu_1 = c\ell L$ and $\mu_2 = 1 + hkK$ and the model reduces to

$$\dot{\ell} = \ell(\rho - \ell) - \frac{1}{cL}(\frac{\partial f}{\partial L} - w), \tag{4.3.1}$$

and

$$\dot{k} = k(\rho - k) - \frac{1}{hK}(\frac{\partial f}{\partial K} - \rho). \tag{4.3.2}$$

If wages are assumed to be determined by demand and supply but again, as above, are sticky, an appropriate function (in logarithmic form) would be

$$\ddot{w} = g(L^d, L^s) - \alpha\dot{w} \tag{4.3.3}$$

where $L^d$ is the demand for labour (defined as the inverse of the production function or derived from Hamiltonian optimisation) and $L^s$ is supply. The function

$g(\ldots)$ is defined to take account of the structure of the labour market and it's affect on wage determination. Thus this can be viewed as a non-tatonnement process which depends on excess demand and the structure of the labour market.

If the supply function is

$$L^s = L_0 w^{\beta_4} e^{\lambda_2 t}, \tag{4.4}$$

equation (4.3.3) could then become

$$D^2 \ln w = \gamma_1 \ln \left( \frac{L^d}{L_0 e^{\lambda_2 t} w^{\beta_4}} \right) - \gamma_2 (D \ln w - \lambda_w) \tag{4.5}$$

where the numerator is the demand for labour $L^d$ defined as the inverse of the production function and the denominator is a supply function $L^s$ where the labour force is defined to grow (or decline) at a steady rate $\lambda_2$ and vary according to the real wage rate with elasticity $\beta_4$. The wage rate $w$ is defined in units corresponding to the definition of $L$.

$L_0$ is a parameter representing the base labour force (at $t = 0$) and $\lambda_2$ the rate of growth of the labour force. If $w$ is real wages, then $\beta_4$is the elasticity of the supply of labour with respect to real wages; depending on the definition of wages in the model it may be necessary to correct for efficiency units in which case that factor becomes $(we^{-\lambda_1 t})^{\beta_4}$. Demand for labour presents more of a problem in the present model. A production function $Y = f(L,K)$ can be inverted to give $L = g(Y,K)$ which shows the amount of labour required to produce a given level of output $Y$ using a given capital stock $K$. In a more complete macro model with output endogenous (perhaps as a function of aggregate demand) the numerator in (4.5) is just $L^d = g(Y,K)$**.**

The formulation in (4.1) in which the costs of adjusting labour is defined in terms of $\ell$ (or similarly in terms of $\dot{L}$) may not be satisfactory. The real costs, from the point of view of the firm, is in deviations of actual labour from the optimal level, that is $|L - L^d|$ and these costs may not be symmetric.

If the production function $f(K,L)$ is defined as CES then

$$Y = \beta_3 [K^{-\beta_1} + (\beta_2 e^{\lambda_1 t} L)^{-\beta_1}]^{-1/\beta_1}, \tag{4.6}$$

so that

$$\frac{\partial f}{\partial L} = \beta_2 e^{\lambda_1 t} \beta_3 \left[ 1 + \left( \beta_2 e^{\lambda_1 t} \frac{L}{K} \right)^{\beta_1} \right]^{-\frac{1+\beta_1}{\beta_1}}, \tag{4.6a}$$

and

$$\frac{\partial f}{\partial K} = \beta_3 \left[ 1 + \left( \beta_2 e^{\lambda_1 t} \frac{L}{K} \right)^{-\beta_1} \right]^{-\frac{1+\beta_1}{\beta_1}}, \tag{4.6a}$$

and these are substituted into (4.3.1) and (4.3.2).

The steady state may be derived as above.

**Table 1.2** Estimates of parameters

| Parameter | Estimate | Asymptotic Standard Error |
|---|---|---|
| c | 6.908 | 65.354 |
| h | −0.134 | 198.869 |
| s | 0.211 | 0.091 |
| $\rho$ | 0.000 | 0.001 |
| $\beta_1$ | 0.955 | 3.203 |
| $\ln\beta_2$ | 0.047 | 21.696 |
| $\ln\beta_3$ | −0.752 | 21.450 |
| $\beta_4$ | 0.392 | 2.314 |
| $\gamma_1$ | 0.000 | 0.001 |
| $\gamma_2$ | 0.691 | 0.159 |
| $\lambda_1$ | 0.000 | 0.000 |
| $\lambda_2$ | 0.023 | 0.023 |
| $\ln(L_0)$ | 0.453 | 0.368 |

This model may be estimated directly in non-linear form or linearized about sample means or the steady state. In all cases the estimator imposes all the constraints on the parameters of the system both from theory and, if linearized, from the linearization.

Full-information Gaussian estimates of the non-linear model, again subject to all the constraints imposed by theory, are given in Table 1.2:

The elasticity of substitution, $1/(1+\beta_1)$, is 0.512 with asymptotic standard error 0.838.
Variants of the this model, and full-information estimates of a linearized version, give broadly similar results. Again, the likelihood ratio test shows this model is inconsistent with the Italian economy generating the data so the joint hypotheses underlying the model must be rejected.

These results are consistent with other research in the field for other economies and must raise doubts whether such models can be justified. It is suggested that the constraints of the Hamiltonian optimisation of the objective function which is the basis of these models is just too stringent to explain the dynamic behaviour of a developed economy. In particular, the hypothesis that the costs of changing either labour or capital is a function of only the derivative (proportional or otherwise) of the control variables may be too simplistic or not robust enough to provide a satisfactory explanation of the behaviour of the firm. For instance, rather than costs depending only on the derivative of the appropriate variable, for instance capital or employment, the discrepancy between current levels of employment and some medium term target may be more appropriate. As employment provides a flow of services, this deviation is the integral of any shortfall, or over-supply in those services; other factors are the discrepancy in current services and the rate of change of the control variable. This is a feature of control systems and is similar to the Phillips proposal of integral, proportional and derivative macro policies. While the objective function could be extended to incorporate these factors this rapidly becomes mathematically intractable.

Instead of introducing adjustment costs into the profit function, a two step optimization process may be a better representation of the behaviour of a firm. The firm first optimizes an objective function to give the optimal medium to long run levels of capital and labour given output, wages, cost of capital etc., and then minimizes a cost function to take account of the deviation of the firm from it's optimal position and to allow for uncertainty as in Bergstrom (1984).

Let $\tilde{K} = ax(t), \tilde{I} = \delta ax(t)$ be the optimal medium term or steady state levels of the capital stock $K(t)$ and investment $I(t)$ derived from Hamiltonian optimization as in (2.3) but without costs of adjustment; $x(t)$ is a vector of non-random functions of variables exogenous to the firm and $a$ is a vector whose elements are functions of the parameters of the underlying objective function. As the values of $x(t)$ are not known with certainty, it is assumed implicitly that the firm views $x(t)$ as the conditional expectations of $x(t+s)$ for all $s > 0$, so $x(t+s), -\infty < t < \infty$ is treated as a martingale process.

In the second stage of the optimisation, the firm minimizes the cost function

$$Q = \frac{1}{2}\int_t^\infty \left([\tilde{K}(s) - K(s)]^2 + c_1[\tilde{I}(s) - I(s)]^2 + c_2[\dot{I}(s)]^2\right) ds \tag{4.7}$$

subject to

$$dK(t) = I(t) - \delta K(t)dt.$$

The optimal function which minimizes $Q$ is

$$dI(t) = \gamma\alpha x(t) + \beta K(t) - I(t)dt + \zeta(dt) \tag{4.8}$$

where

$$[\gamma\beta, -\gamma] = \left[0, \frac{-1}{c_2}\right] P, \;\; \alpha = \frac{\delta - \beta}{\delta} a,$$

and $P$ is the non-negative definite second order matrix satisfying the Riccati equation

$$\begin{bmatrix} 1 & 0 \\ 0 & c_1 \end{bmatrix} + P \begin{bmatrix} -\delta & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} -\delta & 1 \\ 0 & 0 \end{bmatrix} P - P \begin{bmatrix} 0 & 0 \\ 0 & 1/c_2 \end{bmatrix} P = 0.$$

In general, it is not necessary to calculate $c_1, c_2$ but these are implicit in the parameters $\alpha$, $\beta$, $\gamma$.

The form of the cost function (4.7) may be modified to take account of deviations between actual labour being used and its optimal path. If both labour and capital are both subject to decisions of the firm, there are two optimal equations of the form (4.8) and the Riccati equations expand accordingly.

This minimization of adjustment costs provides a justification or alternative interpretation of the adjustment processes.

The model that results from these suggestions is:

$$\dot{\ell} = \alpha_1\alpha_2 \ln(\frac{\partial f}{\partial L}/w) - \alpha_1(\ell - \lambda_2), \tag{4.9.1}$$

**Table 1.3** Estimates of parameters

| Parameter | Estimate | Asymptotic Standard Error |
|---|---|---|
| $\rho$ | 0.0031 | 0.0100 |
| $\beta_1$ | 0.8068 | 0.1672 |
| $\ln\beta_2$ | 4.0189 | 0.3537 |
| $\ln\beta_3$ | −1.5648 | 0.1034 |
| $\beta_4$ | 0.3380 | 3.9525 |
| $\alpha_1$ | 1.0870 | 0.0690 |
| $\alpha_2$ | 0.0109 | 0.0036 |
| $\alpha_3$ | 0.1102 | 0.0033 |
| $\alpha_4$ | 0.0081 | 0.0029 |
| $\gamma_1$ | 0.0024 | 0.0007 |
| $\gamma_2$ | 0.8450 | 0.0367 |
| $\lambda_1$ | 0.0004 | 0.0012 |
| $\lambda_2$ | 0.0029 | 0.0003 |
| $\ln(L_0)$ | 3.8671 | 7.8336 |

$$\dot{k} = \alpha_3 \left[ \alpha_4 \left( \frac{\partial f}{\partial K} - \rho \right) + \beta_5 - k \right], \tag{4.9.2}$$

and, as in (4.3.3),

$$D^2 \ln w = \gamma_1 \ln \left( \frac{L^d}{L_0 e^{\lambda_2 t} w^{\beta_4}} \right) - \gamma_2 (D \ln w - \lambda_1). \tag{4.9.3}$$

If there were perfect competition and no risk, $\beta_5$ would be the rate of growth of fixed capital formation and hence would be the rate at which firms expect output to grow. In this specification, the real interest rate or return on capital is constant and it cannot be distinguished from $\beta_5$.

In this formulation, the question arises of the point at which the partial derivatives should be evaluated; in equilibrium this is irrelevant but out of equilibrium it is not. In the model estimated here, the partial derivative of labour is evaluated at $(L,K)$ to reflect the short term effect of the labour/capital ratio on changes in employment of the firm, but the partial derivative of capital in the investment equation is evaluated at $(Y,K)$; this is relevant to the longer term development of the firm. For the CES production function as defined above, $\frac{\partial f}{\partial K} = \beta_3 \left( \frac{Y}{\beta_3 K} \right)^{1+\beta_1}$. Full-information Gaussian estimates of the non-linear version of this model, subject to all the constraints in the specification of (4.1) to (4.3) are given in Table 1.3.

The usual Chi-square value of the likelihood ratio test cannot be calculated directly for a non-linear model but based on a linearized version of this model it is likely to be around 100 with 13 degrees of freedom; the critical value at the 5 per cent level is 22.4. It should be noted that the likelihood ratio test is biased towards rejection in small samples.

The elasticity of substitution, $1/(1+\beta_1)$, is 0.553 with asymptotic standard error 0.051. Note that the scale of $\beta_2$ depends on the relative magnitudes of capital and labour while the scale of $L_0$ depends on employment, wages and output.

All parameters have the expected sign but many are not significantly different from zero so the specification is still not satisfactory, but it should be noted that this is a much stricter test than is usually imposed in research with this class of models.

These models exclude the real interest rate, and feedbacks from price determination and output. The real interest rate, or the time discount factor, is assumed to be constant. In this model this is, in effect, represented by $\rho$ but the investment function (4.2) includes an expected growth rate and risk premium; the combined factor is $-\alpha_3\rho+\beta_5$ but $\rho$ and $\beta_5$ cannot be identified individually. Under these assumptions, the estimated value of $\rho$ above is really the joint value.

The steady state of this model can be calculated as in Section 2, and the dynamic properties derived from writing the model in terms of deviations about the steady state. Let the steady state paths be $X(t)=X^*e^{v_x t}$ so if $x=\ln X$, so (by definition) in the steady state $\dot{x}=v_x$ and $\ddot{x}=0$. Substituting this and (4.6) into (4.9.1), (4.9.2) and (4.9.3) and equating powers of $t$ gives

$$Y^*=\beta_3[K^{*-\beta_1}+(\beta_2L^*)^{-\beta_1}]^{-1/\beta_1} \quad \text{or} \quad \left(\frac{Y^*}{\beta_3K^*}\right)^{-\beta_1}=1+\left(\beta_2\frac{L^*}{K^*}\right)^{-\beta_1}. \tag{4.10a}$$

The rate of growth of $Y$ and $K$ must be the same and equal to that of the employment term $\lambda_1+\lambda$ Hence $k^*=\lambda_1+\lambda_2$ and $\ell^*=\lambda_2$ but a steady state will exist only if the elasticity of wages in the labour supply function is zero. Under that assumption, from (4.91.1) the steady state growth rate of wages is $\lambda_1$. In addition,

$$\beta_2\beta_3\left[1+\left(\beta_2\frac{L^*}{K^*}\right)^{\beta_1}\right]^{-\frac{1+\beta_1}{\beta}{}_1}=w^*, \tag{4.10b}$$

$$\alpha_4\beta_3\left(\frac{Y^*}{\beta_3K^*}\right)^{1+\beta_1}=\alpha_4\rho-\beta_5+k, \tag{4.10c}$$

$$\frac{1}{\beta_2\beta_3}\left[Y^{*-\beta_1}-(\beta_3K^*)^{-\beta_1}\right]^{-\frac{1}{\beta}{}_1}=L_0w^{*\beta_4}. \tag{4.10d}$$

(4.10c) can be solved to give the capital/output ratio. With $\beta_4$ non-zero, (4.10d) would give $w^*=(L^*/L_0)^{\frac{1}{\beta_4}}$ and (4.10b)..$L^*$ as a function of $Y^*$. With $\beta_4=0$, however, $L^*=L_0$ and (4.10b) gives $w^*$. Hence,

$$K^*=qY^* \quad \text{where} \quad q=\beta_3^{-\frac{\beta_1}{1+\beta_1}}\mu^{\frac{1}{1+\beta_1}} \quad \text{and} \quad \mu=\rho-(\beta_5-\lambda_1-\lambda_2)/\alpha_4, \tag{4.11a}$$

$$Y^*=\beta_2\beta_3L_0[1-(\beta_3q)^{-\beta_1}]^{\frac{1}{\beta}{}_1} \tag{4.11b}$$

$$w^*=\beta_2\beta_3[1-(\beta q)^{-\beta_1}]^{\frac{1+\beta_1}{\beta}{}_1} \tag{4.11c}$$

The model may now be rewritten in terms of (logarithmic) deviations about the steady state. If

**Table 1.4** Estimates of steady state

| Steady State | Estimate | Asymptotic standard error | Mean value |
|---|---|---|---|
| $q$ | 1.23 | 0.26 | |
| $\ln Y^*$ | 5.95 | 7.89 | 5.40 |
| $\ln K^*$ | 6.16 | 7.84 | 8.06 |
| $\ln w^*$ | 1.79 | 0.39 | 1.81 |
| $\ln L_0$ | 3.86 | 7.83 | 3.04 |

$$x_L = \ln \frac{L}{L^* e^{\lambda_2 t}},\ x_K = \ln \frac{K}{K^* e^{(\lambda_1+\lambda_2)t}},$$
$$x_w = \ln \frac{w}{w^* e^{\lambda_1 t}} \text{ and } x_Y = \ln \frac{Y}{Y^* e^{(\lambda_1+\lambda_2)t}},$$

$$\ddot{x}_L = \alpha_1\alpha_2\left(-\frac{1+\beta_1}{\beta_1}\ln\left[1-(\beta_3 q)^{-\beta_1}+(\beta_3 q)^{-\beta_1}e^{\beta_1(x_L-x_K)}\right]-x_w\right)-\alpha_1\dot{x}_L, \tag{4.12a}$$

$$\ddot{x}_K = \alpha_3\left(\alpha_4\left[\beta_3^{-\beta_1}q^{-(1+\beta_1)}e^{(1+\beta_1)(x_Y-x_K)}-\rho\right]+\beta_5-\dot{x}_K-(\lambda_1+\lambda_2)\right), \tag{4.12b}$$

$$\ddot{x}_w = \frac{\gamma_1}{\beta_1}\ln\left(\frac{1-(\beta_3 q)^{-\beta_1}}{e^{-\beta_1 x_Y}-(\beta_3 q)^{-\beta_1}e^{-\beta_1 x_K}}\right)-\gamma_1\beta_4(x_w+\ln w^*)-\gamma_2\dot{x}_w, \tag{4.12c}$$

$$x_Y = -\frac{1}{\beta_1}\ln\left((\beta_3 q)^{-\beta_1}e^{\beta_1 x_K}+\left[1-(\beta_3 q)^{-\beta_1}\right]^{-\frac{1}{\beta_1}}e^{-\beta_1 x_L}\right). \tag{4.12d}$$

Table 1.4 gives the steady state values calculated for the estimates given in Table 3 and assuming $t = 0$ at the mid-point of the sample, 1993 Q3.

The steady state levels are close to the mean values of the corresponding variables, and the actual values at the mid-point of the sample, apart from $K^*$ which is low. This suggests that the estimated value of $q$, derived from the estimates of the underlying parameters in the model, is too low. The asymptotic standard errors are large but this is due to the large standard error of $\ln L_0$. If the steady state is calculated with a given value of $L_0$ the standard errors of $\ln Y^*$ and $\ln K^*$ are 0.36 and 0.28 respectively.

The core model was derived from the optimisation of the discounted present value of the firm with respect to investment and employment under the assumption of that prices are given and output is independent of demand. In a developed economy, however, a demand driven model may be more appropriate. If the theory is modified to allow monopolistic competition with firms having some control over prices, the value of the firm would be optimized subject to the production function by choosing the level of investment in the longer term, with output (or expected output) given, and (the change of) prices and employment in the shorter term. Thus the labour/capital ratio would be a short term control variable as in the core model, but this would be dependent on output and changes in fixed capital.

The introduction of prices into the system may lead to indeterminacy but, as a first approximation to the optimal solution, prices can be determined as a markup on marginal cost but this is not unconstrained. From a macro-economic point of view, relative domestic and foreign prices determine the mix between domestic output (including output for exports) and imports; excessive markups will lead to an increase in imports and decrease in exports.

This approach paves the way formulating a more representative model of the Italian economy while still retaining the strongly theoretical core.

A demand driven model still allows for innovation. While new products will create demand, at the macro level this may be just a matter of substitution or a fulfilment of a demand waiting for a solution. For instance, the creation of new drugs may fulfil a demand for improved health care, new telephone systems fulfil a demand for more efficient communications, and containerisation of shipping was a major step in decreasing transport costs.

It is in this model that institutional or market structures, and other factors such as regulations, may be incorporated in the system by appropriate modifications to the central functions of the model, in this case, the production function, the demand and supply functions for labour $L^d$ and $L^s$, and the overall labour market function $g(.)$ as in (4.3).

In the present model the scaling factor $A_0$ or the parameters of the CES production function and the rate of technical progress $\lambda_1$ are considered fixed parameters in that they do not vary over time. This may be considered a first approximation as these parameters may not be constant but dependent on factors such as the distribution and degree of skills and education in the economy. Thus parameters that in the present model are considered fixed would become functions of a wider set of parameters and variables with the estimated values of the present parameters being some approximation to (say) the mean of these functions. For instance, if skills $S$ were thought to affect the value of $A_0$ that parameter could be replaced by the time-variant expression

$$A_0 = h(\acute{A}_0, S..; \theta)$$

where $\theta$ is a set of parameters. The function $h(.)$ must, of course, be specified explicitly; it is suggested that this be approached by setting out the properties required of $h(.)$ and finding more or less the simplest function which has these properties. The basic properties may be quite simple: how is the sign of $h$ to vary with $S$; are there any limiting factors; what are the properties of the first, or second, order derivative of $h$ with respect to $S$, and so on. Similar considerations apply to variations in $\lambda_1$ or other parameters.

Another aspect of direct relevance to this study is the question of rigidities in the labour market and the effect of regulation on the market. In the present model the parameters $\gamma_1$, $\gamma_2$ in the wage equation can be taken as a non-specific representation of such effects. If increased regulation does distort the labour market by increasing costs of adjustment, then the $\gamma$ in the model will increase with regulation and the market adjust more slowly. The Employment Protection Legislation series produced by the OECD could be used (as an exogenous variable) for this purpose.

More generally, and with more difficulty, a CES or other production function could be extended to incorporate human capital measured by some proxy such as education. One approach here is to have a two tier production function with labour $L$ and capital $K$ forming the CES but with labour then defined as a Cobb-Douglas or geometric average of two (or more) parts such as

$$L_U, L_S, L_H$$

unskilled, skilled, and highly skilled.

For instance, let $p$ be the proportion of skilled labour employed in the economy and assume that a Cobb-Douglas function representing aggregate labour or, equivalently, a geometric average of skilled and unskilled labour, is embedded in the CES production function. The labour term in the production function $(\beta_2 L e^{\lambda_1 t})$ may be replaced by the differentiated term

$$\left(\beta_{2s} p L e^{\lambda_{1s} t}\right)^p \left(\beta_{2u}(1-p) L e^{\lambda_{1u} t}\right)^{1-p}$$

or

$$(\beta_{2s} p)^p \, (\beta_{2u}(1-p))^{1-p} \, L e^{[p\lambda_{1s}+(1-p)\lambda_{1u}]t}.$$

## 1.5 Conclusions

The purpose of this research was to develop and estimate the model of the productive sector of Saltari and Travaglini (2007), derived from the optimising the value of the firm subject to a Cobb-Douglas production function and taking into account costs of changing employment and fixed capital. The resulting model was rejected by the data as a representation of the Italian economy. A modified model, replacing the Cobb-Douglas production function by a CES and generalizing the cost functions for changes in employment and investment, but remaining well within the spirit of the core model, provided more satisfactory estimates but was still rejected when estimated with the same data. It must be noted that the models were estimated using full-information, maximum-likelihood procedures subject to all the constraints inherent in the theory. These estimation procedures, and the likelihood ratio test used in this paper, provide a particularly stringent test of the joint hypotheses that the model represents the system generating the data. It is considered, on the basis of experience with the estimation of macroeconomic models of other countries, that the Saltari-Travaglini-Wymer model above provides a sufficiently good basis to continue with the investigation of the issues that are to be addressed.

The immediate task is to derive the dynamical properties of the Saltari-Travaglini-Wymer model; these may well be aperiodic. The model will then be used to further the aims of this research project in investigating the effect of institutional structure, regulations, and labour market flexibility on the productive sector of the Italian economy.

## Appendix 1. Saltari-Travaglini model. Formal derivation via Hamiltonian optimisation of a profit function

Let $n = \frac{L}{K}$, $z = \dot{n}$, and $I = \dot{K}$. Assume the costs of adjustment of $n$ and $I$ are $c$ and $h$ respectively. Initially $I$ is considered as net investment but is later defined as gross.

The profit function is:

$$\psi(L,K;Y) = Y - wL - z_2 - \frac{c}{2}(z_1)^2 - \frac{h}{2}(z_2)^2 \tag{A1.1}$$

where $z_1 = \dot{L}$, $z_2 = \dot{K}$, $k = D\ln(K)$, $\ell = D\ln(L)$.

It is assumed $Y$, $K$ and $w$ as well as the costs $c$ and $h$ to be defined as real.

$\lambda_1 =$ Harrod neutral technical progress (This could be defined as a stochastic trend if required).

$\lambda_2 =$ rate of growth of the labour force (or again defined as a stochastic trend).

Let investment be given by profit maximisation subject to a production function. In the short term, (a) labour could also be given by the same profit maximisation and the rate of change of the real wage rate a function of the excess demand for labour (that is demand minus supply) or, vice versa (b) if output is to be taken as demand determined, (very) short term labour requirements ($L$) could be determined by the inverse production function and the real wage rate a function of the marginal product of labour. The rate of time preference $\rho$ is not assumed to be equal to the real interest rate in the formal model.

Hence,

$$\max_{k,\ell} \int_t^\infty e^{-\rho s}\psi(L,K;Y)ds \tag{A1.2}$$

s.t. $Y = f(L,K)$, $z_1 = \dot{L}$, $z_2 = \dot{K}$,

so the Hamiltonian becomes

$$H = e^{-\rho t}\{f(L,K) - wL - z_2 - \frac{c}{2}(z_1)^2 - \frac{h}{2}(z_2)^2\} + \nu_1 z_1 + \nu_2 z_2. \tag{A1.3}$$

Where required, it will be assumed $\nu_i = \mu_i e^{-\rho t}$ so $\dot{\nu}_i = \dot{\mu}_i e^{-\rho t} - \rho\mu_i e^{-\rho t}$.

The first order conditions are:

$$\frac{\partial H}{\partial v_1} = \dot{L} = z_1 \qquad \text{(A1.3.1)}$$

$$\frac{\partial H}{\partial v_2} = \dot{K} = z_2 \qquad \text{(A1.3.2)}$$

$$\frac{\partial H}{\partial L} = e^{-\rho t}(-\dot{\mu}_1 + \mu_1 \rho) = e^{-\rho t}\left(\frac{\partial f}{\partial L} - w\right) \qquad \text{(A1.3.3)}$$

$$\frac{\partial H}{\partial K} = e^{-\rho t}(-\dot{\mu}_2 + \mu_2 \rho) = e^{-\rho t}\frac{\partial f}{\partial K} \qquad \text{(A1.3.4)}$$

$$\frac{\partial H}{\partial z_1} = -e^{-\rho t}(cz_1 - \mu_1) = 0 \qquad \text{(A1.3.5)}$$

$$\frac{\partial H}{\partial k} = -e^{-\rho t}(1 + hz_2 - \mu_2) = 0 \qquad \text{(A1.3.6)}$$

From (A1.3.5) and (A1.3.6)

$$\mu_1 = cz_1, \;\; \dot{\mu}_1 = c\dot{z}_1, \qquad \text{(A1.3.5a)}$$

$$\mu_2 = 1 + hz_2 \;, \dot{\mu}_2 = h\dot{z}_2. \qquad \text{(A1.3.6a)}$$

Hence, solving from (A1.3.3) and (A1.3.4),

$$\dot{z}_1 = -\frac{1}{c}\left(\frac{\partial f}{\partial L} - w\right) + \rho z_1, \qquad \text{(A1.3.3a)}$$

$$\dot{z}_2 = -\frac{1}{h}\left(\frac{\partial f}{\partial K} - h\rho z_2 - 1\right). \qquad \text{(A1.3.4a)}$$

These may be written as functions of $\ell = z_1/L, \;\; k = z_2/K$.

If wages are a (second order) distributed lag function of excess demand for labour, wage determination (in logarithmic form) would be something like

$$\ddot{w} = g(L^d, L^s) - \alpha\dot{w}. \qquad \text{(A1.4)}$$

Assume wage rates are determined by a non-tatonnement process depending on excess demand and the structure of the labour market. The wage rate w is defined in units corresponding to the definition of $L$. The demand of labour that is relevant in the wage equation could be defined as the inverse of the production in the short term as in (A1.2) or as derived from Hamiltonian optimisation.

The supply function could be

$$L^s = \gamma_4 w^{\beta_6} e^{\lambda_2 t}. \qquad \text{(A1.5)}$$

The function $g(..)$ is defined to take account of the structure of the labour market and it's affect on wage determination.

The formulation in (A1.3) in which the costs of adjusting labour is defined in terms of $\ell$ (or similarly in terms of $\dot{L}$) may not be satisfactory. The real costs, from the point of view of the firm, is in deviations of actual labour from the optimal level, that is .$|L - L^d|$ and these costs may not be symmetric.

## Data Appendix

The data used in this study are of the Italian economy, quarterly from 1980, Q2, to 2006, Q1. GDP and GNP, fixed capital, and total remuneration (wages) are defined as € bn ($10^9$ ), employment in millions of employees, any parameters of variables such as interest rates, rate of time preference, rates of growth, etc as rates per quarter in natural numbers (for instance, ten per cent per annum is represented throughout this study as 0.025). All real variables are defined with base year 2000 (so that the GDP deflator used in preparation of the data has mean value 1.0 in 2000).

All logarithms are to base e.

The stock of fixed capital is calculated from net capital formation (gross capital formation less fixed capital consumption or depreciation) divided by the GDP deflator.

The time trend has been defined with value 0.0 at the mid-point of the sample (so the mean of t is zero) to simplify linearization without affecting the properties of the model. If required, it is trivial to rebase the time trend by an appropriate adjustment of intercept terms in the model.

All series have been transformed to eliminate (to an approximation) the moving average process inherent in discrete data generated by a continuous system as discussed in Wymer (1972).

The data sources are:

- Real National Income account data: ISTAT, OECD;
- Total employment: AMECO, European Commission;
- Civilian Employment: AMECO, European Commission;
- Short_term interest_rate: OECD;
- EPL: OECD index;
- Skilled and unskilled labor force: OECD index.

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