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Il sottoscritto Travaglini Giuseppe, codice fiscale TRVGPP64B14H501K, nato a Roma provincia di Roma il 14-02-1964, attuale residenza a Roma provincia di Roma, indirizzo Via Emilio Draconzio n° 6, c.a.p 00136, telefono 06 35403509 oppure Cell. 347 4414200, ed E-Mail giuseppe.travaglini@uniurb.it, consapevole delle sanzioni penali previste dall'art. 76 del D.P.R. n. 445/00 per le ipotesi di falsità in atti e dichiarazioni mendaci:

D I C H I A R A

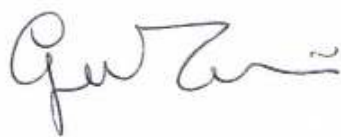
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Roma, 10 novembre 2012.

Il dichiarante, Giuseppe Travaglini



FINANCIAL CONSTRAINTS AND INVESTMENT DECISIONS

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ABSTRACT

In what follows we show that liquidity constraints can affect a firm's investment even when the constraints are not currently effective. This happens when, at any given time, the firm believes that internal finance is likely to become a constraint in the future. In these circumstances, the value of the firm becomes a non-monotonic functional form of the fundamental. Thus, in a dynamic setting, the potential barrier to internal liquidity expansion exerts a global effect on the firm's investment policy, lowering its desired investment profile (Classification JEL: E22, E51).

I INTRODUCTION

The traditional theory of investment assumes that capital markets are frictionless and that firms have access to an infinite supply of funds at the required interest rate. Under this assumption, external investors can replicate at no additional cost to themselves, the financial structure of a firm's original investment. It follows that the capital structure of the firm has no effect on its market value (Modigliani and Miller, 1958). Obviously, if capital structure does not change the value of the firm, *ex-ante* evaluation of investment plans is simplified, the current value of a project being exclusively determined by technology and by prevailing market prices. In this context, investment decisions depend solely on real variables, such as expected profits, while financial decisions assume a secondary role in the firm's activities. These assumptions inform both the intertemporal optimizing model of investment with (or without) adjustment costs (Eisner and Strotz, 1963; Lucas, 1967), and the neoclassical interpretation of Tobin's *q* (Tobin, 1969; Hayashi, 1982).

The most interesting (and realistic) case, however, is when external and internal resources are *not* perfect substitutes. If, for example, asymmetric information makes it difficult for external lenders to evaluate the profitability of the firm's investment plans, the market interest rate for external funds may be substantially higher than the opportunity cost of the corresponding internal resources. In these circumstances investment decisions will depend on the financial structure of the firm and, in particular, on the internal liquidity generated by current profits

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(Myers, 1984; Fazzari e Athey, 1987). There is a considerable empirical literature supporting this view (Fazzari *et al.* 1988; Blanchard *et al.* 1993; Chirinko, 1997).¹ These works tend, however, to play down the influence of *expected* profits on current investment decisions—emphasizing *current* profit flows. In other words, it disregards a distinctive aspect of any dynamic economic environment, namely the role of *expectations* in affecting the current behaviour of firms. In particular, it fails to attach sufficient importance to the way in which, in imperfect capital markets, expectations concerning *future* financial resources can influence *current* investment policy. According to the financing hierarchy theory,² for example, it is often argued that:

to the extent that managers control sufficient internal funds to finance all profitable investment projects, investment demand models based on a representative firm in a perfect capital market apply (Fazzari *et al.*, p. 150, 1988).

But, in this scenario, the optimal policy for the forward-looking firm may well be to exclude from its set of potential investments any project that will, sooner or later, require external funds.

In what follows we will show that liquidity constraints can affect firm's investment policy *even when these constraints are currently slack*. This happens when, at a given time, the firm expects that internal finance will become a constraint in the future. In these circumstances, the value of the firm becomes a non-linear function of the fundamental with the potential barriers to liquidity expansion exerting a global effect on the firm's investment decisions, and lowering its desired investment profile.³

In this perspective, we should take into account the contributions by Dixit (1989, 1991) on investment decisions with *price ceilings*, the paper by Whited (1992) on financing constraints and corporate investment, the paper by Zeldes (1989) on the dynamic interrelationships between consumption and liquidity constraints, and the models of investment with threat of liquidation (Milne and Robertson, 1996).

The paper is organized as follows. In Section II we consider the effects of shifts in the product demand curve on prices and internal finance. Section III examines the optimal behaviour of firms with certainty and an upper price barrier. Section IV generalizes these results for the case of uncertainty with upper and lower price barriers. Section V outlines the conclusions of the analysis.

II INTERNAL LIQUIDITY AND PRICE BARRIERS

To analyze the relationship between financial structure and investment decisions, let us consider the behaviour of a firm in a perfectly competitive, homogeneous,

¹ An empirical investigation of data for Italian firms can be found in Calcagnini (1993).

² These ideas are pioneered by Myers and Majluf (1984), Greenwald, Stiglitz and Weiss (1984) and Myers (1984).

³ In a famous survey of the credit market Jaffee and Stiglitz (1990) argue that 'the anticipation of future credit rationing may have current effects, even when there is no credit rationing at the present. Thus the impact of credit rationing cannot be assessed just by looking at those periods in which there is direct evidence for its presence' (p. 874).

product market, with zero entry and exit costs and with reversible inputs. Assume, further, that access to the credit market is rationed in such a way that the firm has to rely on internal resources to finance its projects and that optimal investment decisions depend, among other elements, on market demand.

At market level, shifts in the product demand curve have two effects: a direct effect on the equilibrium price for the product, and an indirect effect on profit and internal liquidity. Taking a long-run view, in which profits are, nonetheless, yet positive, the zero cost of market entry will ensure that the number of firms in the market rises, and that incumbents can enlarge their capital stock. The larger the number of firms in an industry, the greater will be the supply of the product and the lower its equilibrium price. This competitive mechanism affects firms' current profit and internal finance, changing their incentives to implement new investment plans. More precisely, if the industry demand shock is favourable, incumbents expand their production and new firms enter the market. This competitive mechanism shifts the supply curve to the right, preventing the price from increasing beyond the upper trigger value \bar{P} . The threshold \bar{P} thus represents an upper barrier on the price. If, on the other hand, the shock is unfavourable the demand curve shifts to the left and the equilibrium price decreases. As a result, the number of firms falls and the supply curve shifts to the left, dampening the initial drop in price. It follows that the current price is never lower than a minimum level \underline{P} , which represents a lower bound. Hence, the demand shock translates into internal liquidity which rises as the price tends to the maximum \bar{P} , and decreases when the current price converges towards the lower barrier \underline{P} . This dynamic affects investment. As long as the price lies between \bar{P} and \underline{P} the firm is able to invest; it knows however, that as the price tends towards the upper barrier, internal liquidity will approach its maximum possible level, increasing the probability of liquidity constraints in the future.

This kind of consideration can be used to study how investment policy changes when a rational firm expects that liquidity will be constrained in the future. In examining this question, it is useful to compare the behaviour of a firm in two alternative settings. In the first case, we will consider decision-making in the presence of *certainty* and an upper barrier on price. In the second case, we will examine firms operating in the presence of *uncertainty* with both an upper and a lower price barrier. We will show that the presence of uncertainty does not change the basic mechanism: in both cases a firm that expects to run into financial constraints in the future will modify its conception of optimal current investment, so as to reduce its desired capital stock for any given period. In these circumstances the value of the firm becomes a non-linear function of the fundamental.

CERTAINTY

Before going further it is useful to summarize the main properties of the *standard* investment model with certainty, constant prices, and perfect capital markets. We use this model as a benchmark.⁴

⁴See for an example Abel (1990).

The standard model

The decision problem for the firm at time zero is to choose the investment time path that maximizes the present discounted value of profits:

$$\max_{I_s, L_s} \int_t^\infty \left[pL_s^a K_s^{1-a} - wL_s - k \left(I_s + \frac{1}{2\omega} I_s^2 \right) \right] e^{-r(s-t)} ds \tag{1}$$

subject to the accumulation constraint $dK = (I_t - \delta K_t)dt$, where δ is the depreciation rate. The production function, $L_t^a K_t^{1-a}$, has constant returns to scale, with L_t as labour input, and K_t as capital input. The product price p , the capital price k , wages w and the interest rate r are all constant. The global cost of the investment is given by the function $c(I_t) = I_t + 1/2\omega I_t^2$, and it is equal to the sum of the direct cost I_t , and the adjustment cost $1/2\omega I_t^2$, increasing and strictly convex in I_t .

As a first step, it is possible to simplify the intertemporal problem (1), maximizing the instantaneous profit with respect to L_t , with K_t constant, that is:

$$\max_{L_t} [pL_t^a K_t^{1-a} - wL_t] = hp^\theta K_t$$

where $h = (1 - a)(a/w)^{\alpha/(1-\alpha)}$, and $\theta = 1/(1 - a)$. Substituting this result in equation (1), we can set up the corresponding Hamiltonian:

$$H = \left[hp^\theta K_t - k \left(I_t + \frac{1}{2\omega} I_t^2 \right) + q_t(I_t - \delta K_t) \right] e^{-rt}$$

where q_t is the costate variable. The first order conditions with respect to capital and investment are given by:

$$K: q_t = v + \lambda \frac{dq_t}{dt} \tag{2}$$

$$I: q_t = k \left(1 + \frac{1}{\omega} I_t \right) \tag{3}$$

where $v = hp^\theta/(r + \delta)$ and $\lambda = 1/(r + \delta)$. In equation (2) the variable q_t is endogenous to the problem, whereas v is an exogenous variable depending on the value of the parameters and on the level of price p . This expression is an arbitrage condition: the firm's value q_t is given by the marginal product hp^θ , discounted at the market rate $(r + \delta)$, plus the discounted value of the capital gain dq_t/dt . Under the twin assumptions of certainty and perfect capital markets, the solution of the differential equation (2) is:

$$q_t = hp^\theta \int_t^\infty e^{-(r+\delta)(s-t)} ds = \frac{hp^\theta}{r + \delta} \tag{4}$$

that is, excluding speculative bubbles, the value of the firm is equal to the present value of the marginal product hp^θ . We call this expression the *fundamental*.

This last condition is particularly revealing because it describes the relationship between the price p and the equilibrium value of the firm q_t . Given that $a < 1$, it follows that $\theta > 1$, meaning that q_t is a monotonic function of price, and that, as long as $q_t > k(1 + (1/\omega)I_t)$ —that is until the marginal value of capital q_t is higher than its marginal cost k (see condition (3))—the investment is positive and equal to:

$$I_t = \omega \left(\frac{q_t}{k} - 1 \right) > 0.$$

Using this argument we reach a first conclusion, namely that in a perfect capital market, the sole determinants of the firm's investment decisions are price and the discount rate applied to the marginal product hp^θ . In any given period, the level of investment can change; there are, however, no financial constraints on investment decisions. Given that the firm can access an infinitely elastic supply of financial resources, its financial structure has no effect on its investment policy and none, therefore, on its value.

The standard model with upper absorbing barrier

The previous model refers to the behaviour of a representative firm, which takes the price of its product as given. But when other firms are able to enter the market, and capital markets are imperfect this is only one of the determinants of the intertemporal problem. In this latter scenario, the rational firm recognizes that a positive shock to the demand curve will initially lead to an increase in price, but that competition will prevent the price from climbing beyond the upper barrier \bar{P} . It is aware, furthermore, that internal liquidity will change in proportion to price changes. To analyze the consequences for investment decisions, it is useful to start from the very basic case of certainty with an absorbing barrier.

Let the price, P_t , grow at the deterministic rate π , such that:

$$\frac{dP_t}{P_t} = \pi dt.$$

We denote the initial price as p , so that over time $P_t = pe^{\pi t}$. The price P_t increases until it reaches the upper barrier \bar{P} , whereafter it remains constant. In this case, the trigger value \bar{P} is an *absorbing barrier*. This is a simplifying assumption which nonetheless allows us to investigate how the presence of a competitive market changes the optimal dynamic behaviour of individual firms in the presence of liquidity constraints.

At any given time, changes in price translate into changes in profit, affecting the arbitrage condition (2). This we can rewrite as:

$$q_t = ve^{\mu t} + \lambda \frac{dq_t}{dt} \quad (5)$$

where $\mu = \pi\theta$. To solve this specific problem, consider the function:

$$V_t = ve^{\mu t} \quad \text{if } t < \hat{t} = \frac{\ln m - \ln v}{\mu}, \quad \text{and}$$

$$V_t = m \quad \text{otherwise.}$$

In other words, for $t < \hat{t}$ the *fundamental value* V_t grows at rate μ , which represents the increase in profit (θ) caused by the increase in price (π). m represents the value of the upper barrier, that is the maximum value which the firm attains for $t \geq \hat{t}$.

Given that q_t is a function of t , the dynamic relationship between q_t and t can be expressed through the fundamental value V_t , that is $q_t = q(V_t)$. In this setting, as long as $t < \hat{t}$ changes in q_t over time can be written in the form:

$$\frac{dq(V_t)}{dt} = \frac{dq}{dV} \frac{dV}{dt},$$

or equivalently:

$$\frac{dq(V_t)}{dt} = \mu q_V V_t.$$

Hence, at any time between t and \hat{t} , we can express changes in q as a function of V :

$$q(V) = V + \lambda \mu q_V V. \tag{6}$$

The solution of this first order differential equation is:

$$q(V) = \frac{V}{1 - \lambda \mu} + AV^{1/\lambda \mu}. \tag{7}$$

To determine the constant A , note that as time approaches \hat{t} , the fundamental $V(\cdot)$ tends to m —the firm’s maximum value. Given, however, that q is a function of V , it must be verified that, as time approaches \hat{t} , the function $q(\cdot)$ also tends to m . If this condition is violated, dq_t/dt would have no derivative in \hat{t} , implying a violation of the no arbitrage condition (6). Thus, as $q(V)$ approaches the upper barrier m , the derivative of q with respect to V tends to zero. In technical terms, this means:

$$q_V(m) = 0,$$

that is:

$$q_V(m) = \frac{1}{1 - \lambda \mu} + \frac{1}{\lambda \mu} Am^{1/\lambda \mu - 1} = 0,$$

or more properly:

$$A = - \frac{\lambda \mu}{1 - \lambda \mu} m^{1 - 1/\lambda \mu} < 0. \tag{8}$$

The constant A is negative. Intuitively, this is because the upper barrier blocks price increases, preventing profit and internal liquidity from climbing beyond their respective upper trigger levels. In other words, the constant A measures the present value of the latent investment projects lost by the firm because of liquidity constraints. Of course, the rational firm anticipates all this. To confirm this proposition substitute condition (8) in (7), replacing V , λ and μ in (8) with explicit expressions, therefore obtaining the equilibrium condition:

$$q_t = \frac{hp^\theta e^{\pi\theta t}}{r + \delta - \pi\theta} - \left[\left(\frac{hp^\theta e^{\pi\theta t}}{r + \delta} \right)^{1/\lambda\mu} \frac{\pi\theta}{r + \delta - \pi\theta} m^{1 - (r + \delta)/\pi\theta} \right]. \tag{9}$$

Solution (9) shows that with an upper absorbing barrier the firm's *equilibrium value* is smaller than the fundamental $hp^\theta e^{\pi\theta t}/(r + \delta - \pi\theta)$. Note that, if a firm can invest without financial constraints, and if the price p is constant, solution (9) is equivalent to (4), the constant A being equal to zero so as to avoid speculative bubbles.

We can now use this interpretation of equation (9), to explain the trajectories of q_t and V_t over time. Figure 1 plots these two functions against t . Applying equation (2) or (5) we see that the value of the firm, q_t , is an intertemporal

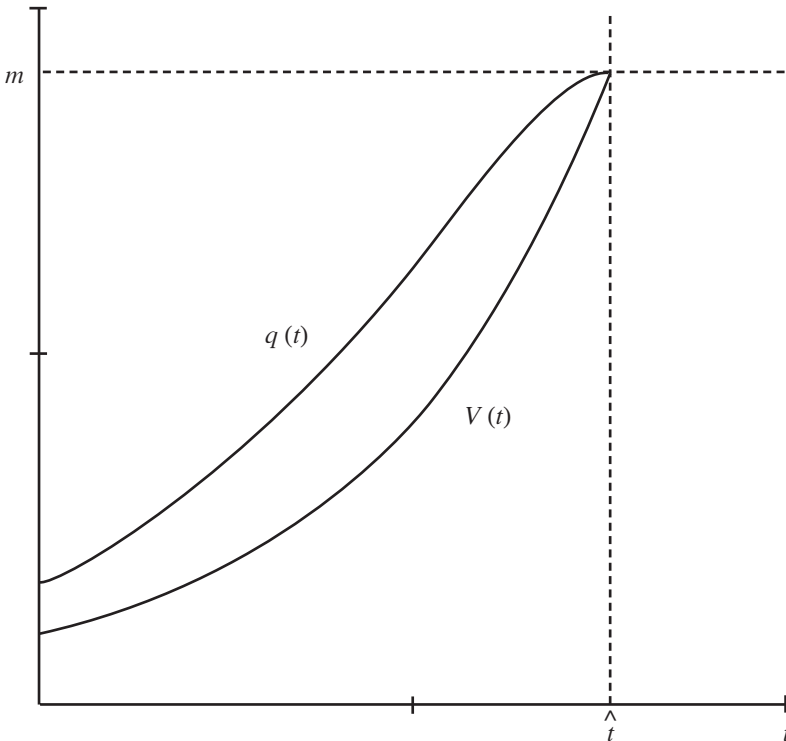


Figure 1. The value of the firm with absorbing barrier.

equilibrium relationship which depends on the fundamental and on the capital gain. Given an initial value for V_t , any future increase (or decrease) in dq_t/dt implies a corresponding increase (or decrease) in current q_t . If, at some particular time, the firm knows with certainty that the future price will be no higher than the upper barrier \bar{P} , it *anticipates* that in the long run the fundamental value will not grow beyond the maximum trigger value m . This information will affect dq_t/dt —the change in the firm’s value over time. As \hat{t} draws closer, the upper barrier exerts an ever stronger influence on current investment decisions; after a certain date, q_t becomes a concave function of time. In other words, as the price approaches the upper barrier the firm realizes that future investment plans will be constrained by the availability of internal funds. A rational firm will anticipate this trend in the fundamental; as a result future constraints will be reflected in the firm’s current value. In these circumstances, it is not surprising that as time tends to \hat{t} , q_t converges smoothly to m , becoming tangent at the trigger value m , in \hat{t} , in such a way as to satisfy the no-arbitrage condition.

IV UNCERTAINTY AND BARRIERS

Let us now extend the previous model to the case of uncertainty with an upper and a lower price barrier. More precisely, assume that the evolution of the price under a free float follows the stochastic geometric Brownian motion:

$$\frac{dP_t}{P_t} = \pi dt + \sigma dz \tag{10}$$

where, this time, π is the drift $E(dP_t/P_t) = \pi dt$, and σ is the (constant) variance parameter. The term dz is the increment of the standard Wiener process, with mean $E(dz) = 0$, and variance $E(dz)^2 = dt$.

Uncertainty transforms problem (1) into a maximization problem for *expected profits*; given that the functional form q is a function of V this changes the no-arbitrage condition, which can now be written as:

$$q(V) = V + \lambda \frac{E[dq(V)]}{dt} \tag{11}$$

The fundamental V is, however, a function of the basic stochastic process P . Hence, to solve equation (11) we must derive the evolution of $V(P)$ with respect to price. Applying Ito’s Lemma to $V(P)$, and then using the derived process dV to obtain the stochastic dynamics of $q(V)$, we obtain an explicit expression for $E[dq(V)]$ which makes it possible to solve equation (11). The general solution is:

$$q(V) = \frac{V}{1 - \lambda\mu - \lambda^2\theta(\theta - 1)\sigma^2} + A_1V^{\alpha_1} + A_2V^{\alpha_2} \tag{12}$$

where $\alpha_1 > 0$ and $\alpha_2 < 0$ are the two roots of the characteristic equation.⁵

⁵The solution of the differential equation (11) is derived in the Appendix.

A comparison between solution (12) and (7) shows that uncertainty does not change the basic properties of the previous model. As in the previous case, the general solution comprises two components: the complementary function and the fundamental solution. The expression for $q(V)$ can be interpreted as follows:

$$\frac{V}{1 - \lambda\mu - \lambda\frac{1}{2}\theta(\theta - 1)\sigma^2}$$

is the present value of the expected profit when the P process is without bounds, while $q(V)$ is the same when the process P is limited by the upper and lower bounds.

If the price dynamics (10) can change freely, then the constants A_1 and A_2 must be set to zero so as to avoid speculative bubbles. In this case the value of the firm is equal to the fundamental value:

$$\frac{V}{1 - \lambda\mu - \lambda\frac{1}{2}\theta(\theta - 1)\sigma^2}.$$

Note that if $\sigma = 0$ this solution coincides with the fundamental solution of expression (7). It should further be noted that if the price dynamics is determined exclusively by the stochastic component, that is if $\pi = 0$ and $dP/P_t = \sigma dz$, then the solution without barriers corresponds to the Abel (1983) solution.⁶

The solution with upper and lower reflecting barriers

We can now investigate how the firm's equilibrium value changes when competition imposes upper and lower reflecting bounds on price. In the presence of uncertainty the P process drifts upwards but also fluctuates randomly. When we allow P to fluctuate downwards (upwards) after hitting the upper bound \bar{P} (the lower bound \underline{P}), then the limit prices are reflecting barriers. In this case, the future evolution of P , and hence of V , is stochastic even when $P = \bar{P}$ (or alternatively $P = \underline{P}$).

To focus on the consequences of this property, let d be the minimum value of the fundamental V when the price reaches the lower barrier \underline{P} , and m the maximum value of V when the price arrives at the upper barrier \bar{P} . Within these two bounds the price can change freely; once, however, it has reached one of the two extremes, its dynamics changes. The importance of the two barriers in determining the value of the firm depends on constants A_1 and A_2 in solution (12). For example, once the upper barrier m has been reached, the value of the fundamental V can only decrease, signifying a decrease in $q(V)$ —the value of the firm. In technical terms, m and d are *reflecting barriers*: when V reaches one of the two trigger values the previous trend in the firm's value is randomly reversed.

⁶ It is worthwhile to note that without constraints, uncertainty, and with constant prices, the solution (12) is equal to (4) of the standard model.

This mechanism inevitably affects the current value of the firm. As long as the current price lies within the interval $[P, \bar{P}]$, the dynamics of V are described by equation (A.1).⁷ When, on the other hand, the price reaches one of the two barriers, the evolution of dV becomes a modification of the process (A.1). In other words when $V = m$ the fundamental reaches its maximum value. This means that as V tends to m , $q(V)$ tends to its own maximum level $q(m)$; for the same reason, when V tends to d , then $q(V)$ tends to the minimum value $q(d)$. This implies that at the barriers d and m

$$q_V(d) = 0 = q_V(m). \tag{13}$$

This expression is sometimes called *smooth pasting condition*, and it is analogous to the expression derived in the second part of Section III with certainty and one absorbing barrier; now, however, we have uncertainty and two reflecting barriers.

This result shows that expression (13) is sufficient to allow the calculation of the constants A_1 and A_2 . Employing conditions (12) and (13) we obtain the expression:

$$\alpha_1 A_1 d^{\alpha_1 - 1} + \alpha_2 A_2 d^{\alpha_2 - 1} = \alpha_1 A_1 m^{\alpha_1 - 1} + \alpha_2 A_2 m^{\alpha_2 - 1}$$

and the explicit solution of (12):

$$q(V) = cV + V^{\alpha_1} \left(\frac{c}{\alpha_1} \frac{d^{\alpha_2 - 1} - m^{\alpha_2 - 1}}{d^{\alpha_1 - 1} m^{\alpha_2 - 1} - d^{\alpha_2 - 1} m^{\alpha_1 - 1}} \right) + V^{\alpha_2} \left(\frac{c}{\alpha_2} \frac{m^{\alpha_1 - 1} - d^{\alpha_1 - 1}}{d^{\alpha_1 - 1} m^{\alpha_2 - 1} - d^{\alpha_2 - 1} m^{\alpha_1 - 1}} \right)$$

where to simplify the notation:

$$c = \frac{1}{1 - \lambda\mu - \lambda^{\frac{1}{2}}\theta(\theta - 1)\sigma^2}.$$

Note that the positive root $\alpha_1 > 0$ is associated with the negative constant $A_1 < 0$ (as in the case of certainty); whereas, the negative root $\alpha_2 < 0$, is associated with the positive constant $A_2 > 0$.

The dynamics of $q(V)$ can be defined for all V in the interval $[d, m]$. Hence, the function $q(V)$ can be interpreted as representing the increased value of a firm which changes from a situation marked by scarce internal liquidity, to a situation in which liquidity has increased, but to a level where no further expansion is possible.

The dynamics of $q(V)$ are determined by the sum of three components. For small values of V the dominant component of $q(V)$ is the component with the negative root α_2 . This function is decreasing and convex. For large values of V , the component with positive root α_1 prevails. This is negative, decreasing and

⁷See the Appendix.

concave. For intermediate values of V , the component cV , contributes to the increasing portion of $q(V)$. The general form of $q(V)$ is shown in Figure 2.

Figure 2 shows the S -shaped locus representing the non-linear functional form $q(V)$, which is a tangent with respect to the values $q(d)$ and $q(m)$. It is qualitatively similar to the deterministic solution discussed in the previous section: it meets the boundaries smoothly to first order, but the solutions $q(m)$ and $q(d)$ lie, respectively, below m and above d , even when $V = m$ and $V = d$.⁸ This happens because the fundamental V can never exceed the interval $[d, m]$, and with $\sigma > 0$, and reflecting barriers, the q value of the firm will surely falls randomly below m (above d) after reaching it.⁹

More precisely, when V is higher than the lower barrier d , the value of the firm $q(V)$ increases: a larger fundamental generates greater internal liquidity and higher investment. This relationship affects the value of the firm, increasing its capital gain dq_t/dt . Once however, V passes the inflection point, the curve becomes concave: the firm *anticipates* the effect of the upper barrier on liquidity and investment at the current time. In other words, the firm perceives closeness to the upper bound as an exacerbation of the liquidity constraint. This contributes to slowing down capital gain dq_t/dt which for $V = m$ is equal to zero. Hence, the firm that appear to be more financially constrained, takes into account the uncertainty over future internal funds and it is more reluctant to invest. In turn, the firm less financially constrained exhibits a greater positive sensitivity to invest. Only when both boundaries are infinitely distant is the $q(V)$ value a linear function of the fundamental V .

This interpretation makes it possible to reach a first conclusion: namely that a *latent* liquidity constraint can affect a firm's investment decisions even when the constraint is currently slack. This is the consequence of forward-looking behaviour by the firm that anticipates the possibility of future constraints at the current time. What is remarkable here, is that the firm changes its behaviour not only when the constraint is actually effective, but even when current liquidity constraints are weak. This mechanism modifies the firm's overall investment policy.

⁸ To prove this point, consider the differential equation:

$$\frac{1}{2}\lambda s^2 q_{VV} V^2 + \lambda(\mu + \frac{1}{2}\theta(\theta - 1)\sigma^2)q_V V - q + V = 0.$$

Evaluating this for $V = m$ and $V = d$, and using the smooth pasting conditions that must hold at m and d , we find:

$$q(m) = m + \frac{1}{2}\lambda s^2 m^2 q_{VV},$$

$$q(d) = d + \frac{1}{2}\lambda s^2 d^2 q_{VV}.$$

But close to m the derivative $q_{VV} < 0$, signifying that $q(m) < m$; while close to the boundary d the derivative $q_{VV} > 0$, implying that $q(d) > d$.

⁹ Note that before q actually hits a boundary (say m) its expected change is $E(dq(V)) = [(\mu + \frac{1}{2}\theta(\theta - 1)\sigma^2)q_V V + \frac{1}{2}s^2 q_{VV} V^2] dt$. But once V reaches m , the q value is at the top of its interval, and competition will only allow V to go down, not up. If $q(V)$ were not tangent to m , at the maximum $E(dV(P))$ must change discontinuously from $(\mu + \frac{1}{2}\theta(\theta - 1)\sigma^2)V dt$ to a negative value. This discontinuity in $E(dV(P))$ would imply a discontinuous jump in $E(dq(V))$, and therefore in $q(V)$ itself. But in this situation the no-arbitrage condition would be violated, because no equilibrium path can approach such a boundary since investors would anticipate arbitrage profits.

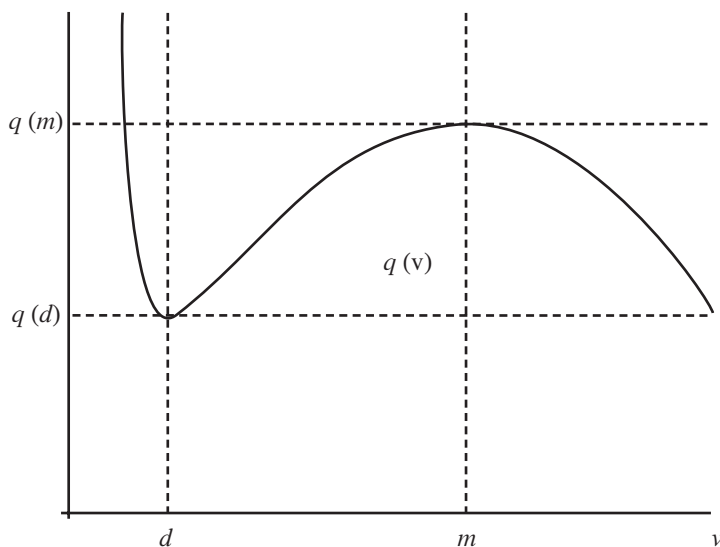


Figure 2. Barriers and firm value.

The behaviour of a firm subject to liquidity constraints is therefore fundamentally *different* from the behaviour of an identical firm operating on a perfect capital market. For this latter firm the value of q depends exclusively on the fundamental; for the constrained firm, on the other hand, q is determined by the *expectations* of future constraints.

It should be noted, furthermore, that the non-linear dynamics of the $q(V)$ have another implication: the effect of increasing uncertainty, σ , on a firm's value, depends on the initial value of the fundamental. In the convex part of the $q(V)$ curve increases in σ increase the value of the firm and the value of new projects. For large values of V , on the other hand, $q(V)$ is concave and the investment-uncertainty correlation is reversed. This latter property distinguishes the present model with respect both to the standard approach, where the investment-uncertainty correlation is positive,¹⁰ and to investment models with irreversible inputs, where this correlation can be negative.¹¹

V CONCLUSIONS

In this paper we have studied investment decisions by firms in the presence of liquidity constraints, showing that latent constraints can affect investment policy even when current constraints are slack. In an intertemporal context, the

¹⁰This positive effect is caused by the convexity of the profit function with respect to price (Hartman, 1972; Abel, 1983).

¹¹In this case, irreversibility stimulates firms to postpone investment (Bertola, 1988; McDonald-Siegel, 1986).

firm's forward-looking behaviour anticipates the future liquidity barrier, modifying investment policy for any particular period. The S -shaped q curve implies that competitive mechanism exerts a stabilizing effect, reducing the investment's sensitivity to a given change in fundamental, and, hence, in internal liquidity.

This result casts doubts on the proposition that if current demand for investment is unaffected by liquidity constraints, the firm will decide its investment policy as if there will be no constraints in the future. In our model, the constrained firm evaluates the probability of future constraints, and chooses, in any given period, an investment policy which is optimal in the presence of the expected constraints. It follows that the value of the firm, and of its investment intertemporal profile, are determined not only by the current but also by the future value of the fundamental.

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APPENDIX

Applying Ito's lemma to $V(P)$ we obtain the corresponding dynamics dV

$$dV = V_P(dP) + \frac{1}{2}V_{PP}(dP)^2$$

Substituting for $dP = \pi P dt + \sigma P dz$ and remembering that $V(P) = hP_t^\theta / (r + \delta)$, we obtain:

$$dV = (\mu + \frac{1}{2}\theta(\theta - 1)\sigma^2)V dt + sV dz \quad (\text{A.1})$$

where $\mu = \theta\pi$, and $s = \theta\sigma$ are proxies for the volatility of V . Given the stochastic process dV , applying Ito's Lemma to $q(V)$, and taking expectations, we obtain the explicit expression for the expected capital gain $E[dq(V)]$:

$$E(dq(V)) = [(\mu + \frac{1}{2}\theta(\theta - 1)\sigma^2)q_V V + \frac{1}{2}s^2 q_{VV} V^2] dt. \quad (\text{A.2})$$

Finally, substituting this expression in (11), we find the second order differential equation:

$$\frac{1}{2}\lambda s^2 q_{VV} V^2 + \lambda(\mu + \frac{1}{2}\theta(\theta - 1)\sigma^2)q_V V - q + V = 0 \quad (\text{A.3})$$

whose characteristic equation is:

$$\frac{1}{2}\lambda s^2 \alpha(\alpha - 1) + \alpha\lambda(\mu + \frac{1}{2}\theta(\theta - 1)\sigma^2) - 1 = 0. \quad (\text{A.4})$$

Writing $\lambda(\mu + \frac{1}{2}\theta(\theta - 1)\sigma^2) = x$ so as to simplify the notation, the two roots of (A.4) are respectively:

$$\alpha_1 = \frac{1}{2\lambda s^2} (\lambda s^2 - 2x + \sqrt{(\lambda s^2 - 2x)^2 + 8\lambda s^2}) > 0$$

and

$$\alpha_2 = \frac{1}{2\lambda s^2} (\lambda s^2 - 2x - \sqrt{(\lambda s^2 - 2x)^2 + 8\lambda s^2}) < 0.$$

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