

# **POWER SYSTEM FREQUENCY ESTIMATION USING LINEAR AND NONLINEAR TECHNIQUES**

**A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS  
FOR**

**THE DEGREE OF  
MASTER OF TECHNOLOGY  
IN  
ELECTRICAL ENGINEERING**



**By**

**NILESH SHINDE  
212EE5264**

**NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA**

**A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS  
FOR**

**THE DEGREE OF  
MASTER OF TECHNOLOGY  
IN  
ELECTRICAL ENGINEERING**



**By**

**NILESH SHINDE  
212EE5264**

**UNDER THE GUIDANCE OF  
PROF. P.K. RAY**

**DEPARTMENT OF ELECTRICAL ENGINEERING**

**NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA**



NATIONAL INSTITUTE OF TECHNOLOGY  
ROURKELA  
CERTIFICATE

*This is to certify that the thesis entitled, “**POWER SYSTEM FREQUENCY ESTIMATION USING LINEAR AND NONLINEAR TECHNIQUES**” submitted by **Nilesh Shinde** in partial fulfilment of the requirements for the award of Master of Technology Degree in Electrical Engineering with specialization in “**Industrial electronics**” at National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance. To the best of my knowledge, the matter embodied in this Project review report has not been submitted to any other university/ institute for award of any Degree or Diploma.*

**Date:**

**(Prof. P. K. Ray)**

*Dept. of Electrical Engineering  
National Institute of Technology,  
Rourkela-769008*

# ACKNOWLEDGEMENT

I express my deepest gratitude to my project guide **Prof. P.K. Ray** whose encouragement, guidance and support from the initial to the final level enabled me to develop an understanding of the subject.

Besides, we would like to thank to **Prof. A.K. Panda**, Head of the Electrical Engineering Department, National Institute of Technology, Rourkela for providing their invaluable advice and for providing me with an environment to complete our project successfully.

I am deeply indebted to all faculty members of Electrical Engineering Department, National Institute of Technology, Rourkela, for their help in making the project a successful one.

Finally, I take this opportunity to extend my deep appreciation to my **family** and **friends**, for all that they meant to me during the crucial times of the completion of my project.

Date: 26.05.2014

Place: Rourkela

**NILESH SHINDE**

ROLL NO: 212EE5264

NATIONAL INSTITUTE OF TECHNOLOGY

ROURKELA

# CONTENTS

---

ACKNOWLEDGEMENT	i
CONTENTS	ii
ABSTRACT	iv
LIST OF FIGURES	v
ABBREVIATIONS	vi
<b>CHAPTER-1 INTRODUCTION</b>	<b>1</b>
1.1 Background	1
1.2 Literature review	2
1.2.1 Review of power system frequency estimation	2
1.2.2 Soft computing approaches to power system frequency estimation	6
1.3 Motivation of project work	7
1.4 Objective of thesis	7
1.5 Thesis organization	8
<b>CHAPTER-2 MATHEMATICAL ANALYSIS-LINEAR ESTIMATION</b>	<b>9</b>
2.1 Power system frequency estimation	9
2.1.1 Introduction	9
2.2 Frequency estimation using LS algorithm	10
2.3 Simulation result of LS algorithm	10
2.4 Frequency estimation using RLS algorithm	11
2.5 simulation result of RLS algorithm	13

2.6 Frequency estimation using LMS algorithm	14
2.7 Steps for frequency estimation using LMS algorithm	16
2.8 Simulation result of LMS algorithm	17
<b>CHAPTER-3 MATHEMATICAL ANALYSIS-NONLINEAR ESTIMATION</b>	18
3.1 Frequency estimation using NLS algorithm	18
3.2 Simulation result of NLS algorithm	20
3.3 Frequency estimation using NRLS algorithm	22
3.4 Simulation result of NRLS algorithm	23
3.5 Frequency estimation using NLMS algorithm	24
3.6 Simulation result of NLMS algorithm	25
<b>CHAPTER-4 SUMMARY AND COLCLUSIONS</b>	26
4.1 Summary of the project work	26
4.2 Conclusions	27
4.3 Future scope of the work	28
<b>REFERENCES</b>	

## **ABSTRACT:**

---

In an electrical power system frequency is an important parameter. The frequency of operation is not constant but it varies depending upon the load conditions. In the operating, monitoring and controlling of electric device power system parameters are having great contribution. So it is very important to accurately measure this slowly varying frequency. Under steady state conditions the total power generated by power stations is equal to system load and losses. Frequency can deviate from its nominal value due to sudden appearance of generation-load mismatches. Frequency is a vital parameter which influences different relay functionality of power system. This study was made to estimate the frequency of measuring voltage or current signal in presence of random noise and distortion. Here we are first using linear techniques such as complex least mean square (LMS), least square (LS) and recursive least square (RLS) algorithm for measuring the frequency from the distorted voltage signal. Then comparing these results with nonlinear techniques such as nonlinear least mean square (NLMS), nonlinear least square (NLS), nonlinear recursive least square (NRLS) algorithms. The performances of these algorithms are studied through simulation.

## LISTS OF FIGURES:

---

Figure no		Page no
2.1	Frequency estimation using LS algorithm	10
2.2	Frequency estimation using RLS algorithm	13
2.3	LMS filter structure	14
2.4	Frequency estimation using LMS algorithm	17
3.1	Frequency estimation using NLS algorithm	21
3.2	Frequency estimation using NRLS algorithm	23
3.3	Frequency estimation using NLMS algorithm	25



## **ABBREVIATIONS:**

---

DFT: Discrete Fourier Transformation

DSO: Digital Storage Oscilloscope

FFT: Fast Fourier Transformation

LMS: Least Mean Square

LS: Least Square

RLS: Recursive Least Square

NLMS: Nonlinear Least Mean Square

NLS: Nonlinear Least Square

NRLS: Nonlinear Recursive Least Square

RL load: Resistive- Inductive Load

# CHAPTER-1

---

## INTRODUCTION

### 1.1 Background:

Frequency in electrical power system is an important operating parameter which is required to remain constant because it reflects the whole situation of the system. Frequency can show the active energy balance between generating power and load. Therefore frequency is considered as an index for operating power systems in practical. In power system frequency is not constant but changes according to load condition. Ideally this frequency should be constant, but due to noise, sudden appearance of load-generation mismatches and increasing use of nonlinear load, the frequency of operating system is not constant. Component reactance change results due to deviation in system frequency from its desired value which influences different relay functionality of power system such as server damage or reactive power reduction occurs in system devices. So the frequency plays an essential role in operating, monitoring and controlling of any power system device. Basically digitalized samples of current or voltage signal are used for available frequency estimation techniques. Generally, the voltage signal is used for frequency estimation because it is less contort than the line current. Considering the purely sinusoidal power system voltage signal, the frequency can be defined as time between two zero crossing. However in practice, the measured signals are available in the distorted form and many techniques are available for estimation of the frequency. Further, many power electronic equipment and arc furnaces etc. generate disturbances in the power systems. Therefore it is necessary for utilities to develop a realistic approach for measurement of frequency in presence of noises. Since frequency variation is a dynamic phenomenon, the conventional phasor estimation techniques such as Discrete Fourier Transform (DFT), Least Square (LS) and Kalman Filtering may not be appropriate for achieving accurate frequency estimation

under dynamic conditions. Zero crossing technique, discrete Fourier transform, phase lock loop, least square error; orthogonal finite impulse response filtering, Kalman filtering and iterative methods are some techniques in this area. Soft computing technique such as genetic algorithm and artificial neural network are also use for power system frequency estimation. Also for protection and control of the power system devices it is essential to have a knowledge of frequency, therefore it is necessary that how we are estimating the frequency without significant delay. It is therefore absolutely necessary to develop a realistic approach for frequency measurement in presence of disturbances.

This paper represented estimation of frequency from a distorted voltage waveform. By considering different situation of power system the distortion of the system signal is further enhanced. An algorithm used here are complex least mean square (LMS) using three phase voltage signal, LS estimator and RLS estimator. The first two estimators use batch processing and third one is online processing.

After that frequency estimation is done using some nonlinear techniques such as Nonlinear Least Mean Square (NLMS), Nonlinear Least Square (NLS), and Nonlinear Recursive Least Square (NRLS). Then by comparing the results of both linear and nonlinear method, shows advantages and disadvantages of both techniques.

## **1.2 Literature review:**

### **1.2.1 Review of power system frequency estimation:**

A prony's method with digital algorithm has been proposed by T. Lobos and J. Rezmer et al [1] to estimate the frequency of power system in the year 1997. In this approach at first using Fourier technique algorithm the distorted voltage signal is filtered and by assuming constant frequency the coefficient of the filters are calculated. So filter coefficient is not exact because of deviation in power system frequency. To overcome the filter effect here Balckman and hamming

window are used where Blackman window is worked effectively in this approach. After that using prony's estimation method the output signal of filter is processed to calculate the system fundamental frequency. This algorithm was tested on online by assuming the deviation of frequency up to 2Hz in presence of higher harmonics. The response time of this new technique is equal to 3 to 4 times of the fundamental component.

P.K.Das et al [2] implemented Extended Complex Kalman Filter (ECKF) for frequency estimation from distorted power signal in 1999. In this paper discrete values of 3-phase voltage signal of power system are considered and then by well-known  $\alpha - \beta$  transformer signal converted into complex voltage vector form. From the complex voltage vector a nonlinear state space is formulated which is further computed to true state of model iteratively by the use of Extended Complex Kalman Filter (ECKF) with harmonics distortion and significant noise. The convergence speed of this method is reduced by three cycles and this can also be further improved significantly if we considered harmonics in the state space formulation. Here the error in estimation of Frequency is close to .01Hz to .02Hz in presence of noise. This approach is worked well for decay or rise and step change in the frequency. This technique gives an idea of various situation of power system.

Huang et al. [3] proposed a robust algorithm based on an Extended Complex Kalman Filter (ECKF) for the estimation of power system frequency. This algorithm is called robust because it suppressed the abnormalities such that noises and disturbances in power systems of measurement and efficiency of frequency estimation are enhanced. They have verified the proposed approach using test signals, signals recorded from an arc furnace, signals obtained from the stainless steel factory and signals generated in the laboratory. It also shows that this new robust ECKF works much better than ECKF. Design of an Extended Complex Kalman Filter (ECKF) [4], [5] is discussed for measurement frequency of power signal. During the change in signal parameters, the covariance matrix and Kalman Gain should reset to track them quickly. Authors have used decision block of hysteresis type to solve such type of problem. By the nature of noise and that of

convergence the authors determined hysteresis band. Then by using an experimental setup from the signal generator through an unshielded ribbon cable test signal is derived. A signal frequency is first measured by DSO (Digital Storage Oscilloscope). The step change in frequency is realized with introduction of delay subroutines by allowing the sudden jump in the signal generator. Kalman Filter rectifies the distortion in the transmission line, which is introduced due to instrumentation cable and CT (or PT).

Least mean squares (LMS) algorithms are the class of adaptive filter used to imitate a filter by estimating the coefficients of filter which are related to generating the least mean squares of the error signal (difference between the actual and assumed signal). As the Least Mean Square algorithm does not use the precise values of the expectations, the weights would never attain the optimal weights in the absolute sense, but a convergence is possible in mean.

Both Least Square (LS) and Recursive Least Square (RLS) approaches have been applied for estimation of frequency. In order to avoid poor tracking ability of Block wise Least Square (BLS), a sliding window block wise least square method with an adjustable window length is recommended to extend the LS approach for frequency estimation of the system. This approach performs the Least Square algorithms significantly and has an excellent tracking ability for steady state performance and abrupt parameter changes. Djuric et al. [6] proposed an algorithm, which is derived from Zero crossing technique, and Fourier is applied to sine or cosine components of original signal, which is degraded by higher harmonics. This Fourier and Zero crossing techniques show high measurement accuracy over a wide frequency change. The proposed algorithm is verified using simulation, fields and laboratory tests. For frequency estimation in power system, a method [7] based on adaptive notch filter has been proposed. A voltage or current signal with noise and d.c. component is taken. Its performance is studied in various situations such as step variations of frequency, in presence of harmonics, oscillatory variations of amplitude, oscillatory variations of frequency etc. Its performance is also compared with a phase-locked loop (PLL). By comparing it

has been found that dynamic performance of the proposed approach is quit faster than that of using Phase Lock Loop. Due to its simpler structure, it can be used for both software and hardware environments. This method [8] is based on implementation of two orthogonal digital filters. It provides almost accurate estimate up to a resolution of 0.01-0.02 Hz for near nominal, nominal and off-nominal frequencies in about 20ms. This technique is also tested with voltage signals from a power system and from a dynamic frequency source. It requires less computation and also it is suitable for microprocessor-based relays. Karimi-Ghartemani and Iravani [9] implemented a method based on phase locked loop for frequency estimation in power system. The main features of this method are robustness with respect to harmonics, immune to noise, simple structure, and having negligible steady state error.

M.S.Sachdev and M.M.Giray et al [15] describe a least square technique for determining power system frequency. A least square approach is used for solving the solution of overdetermined system. The meaning least square is that overall solution minimizes sum of squares of the errors. A least square problem classifies into two categories that are linear and nonlinear residuals are linear in all unknowns or not. In linear LS it can be evaluated in finite no. of operations and it is having closed form solution.

A nonlinear Least Square (LS) technique is employed for measuring the frequency of electrical power system by R. Chudamani, Krishna et al [10]. Nonlinear least square technique is proposed for the fundamental frequency estimation. Frequency estimate is obtained by minimizing the squared error between actual signal and desired signal. Here estimation of fundamental frequency is carried out by performing 1-Dimensional search over a range of allowable frequency. The voltage signal used here is modelled by implementing Fourier series. The above technique is very much flexible for estimation of the frequency in presence of harmonics either selectively or in total.

### **1.2.2 Soft Computing approaches to Power System Frequency Estimation:**

A technique based on neural network has been proposed in [11] for applications of frequency in real time a power system network. They have shown frequency as a weight of neural network and managed it through an appropriate learning process. They have judged the steady state accuracy and dynamic behaviour of technique. The change in power system frequency within very less time can also be track using this.

A technique based on fuzzy linear regression is given in [12] for the frequency and harmonics evaluation in a power system network, so for frequency estimation and harmonics estimation components of voltage signal digitized voltage signals is used as fuzzy numbers. They have examined the effects of sampling frequency, degree of fuzziness on the parameters estimated and data window size. They have examined the above method using simulated data.

Genetic algorithm and Neural network have been used in [13] for frequency estimation in power system. In that algorithm, by genetic algorithm the learning of weights of neural network was carried out. They have compared the performance of this technique with the conventional error back propagation and LMS algorithm. But they found that the proposed algorithm works effectively over other two. They have examined the performance using simulation data only.

An adaptive neural network is presented in [14] for power system frequency estimation. For recognizing parameters of a discrete signal model of a power system voltage, authors have used a linear adaptive neuron Adaline. They have adjusted parameter learning to have a stable difference error equation. By using Proposed algorithm frequency over a wide range of frequency changes can be track. This algorithm tracks the frequency at different situations of power system and also immune to presence of disturbances and noise in signal.

### **1.3 Motivation of Project work:**

As discussed before, due to sudden mismatch of generation-load and frequent use of nonlinear load in electrical device, electrical power system environment is contaminated by random noise, harmonics and reactive power disturbance. As a result of this there is deviation of fundamental frequency from its normal value and elevates harmonics level in the power system network which is not wanted. It is a tough task to estimate the exact frequency of voltage in presence of random noise. Although complex LMS algorithm is used for power system frequency estimation but attention has been not given to estimation of frequency in various power system condition, which motivated to estimate frequency in different conditions. As discussed above, Linear techniques such as Least mean square, Least Square and Recursive Least Square have been applied for power system frequency estimation however Nonlinear estimation by all above methods has not been done in this field. So it was motivated to carry out an estimation of power system frequency nonlinear methods.

### **1.4 Objective of the thesis:**

The objectives of the thesis are as follows:

- To analyse the Least Square algorithm for frequency estimation in a range of frequency.
- To analyse the Recursive Least Square algorithm for frequency estimation.
- To estimate the frequency of power system using LMS algorithm and compare the performance of this algorithm with the previous algorithms and show how it is effective than LS and RLS algorithm.



In case of frequency estimation research has been done using several linear techniques for estimating the frequency. But estimation of the frequency by using nonlinear techniques has not been done upto that extent. So here in this paper we have also estimated the frequency using some nonlinear techniques.

- To analyse the Nonlinear Least Square algorithm for the frequency estimation.
- To analyse the Nonlinear Recursive Least Square algorithm for the frequency estimation.
- To analyse the Nonlinear Least Mean Square algorithm for the frequency estimation.

### **1.5 Thesis organization:**

**Chapter-1** It consists of an introduction of power system frequency. It also includes a brief literature review on power system frequency estimation and it focus on motivation and objective of the project.

**Chapter-2** It deals with the mathematical analysis using linear techniques such as complex LMS, Least Square and Recursive Least Square algorithm for frequency estimation in power system. This chapter also includes of simulation results of each algorithm.

**Chapter-3** It consists of mathematical analysis using nonlinear techniques such as Nonlinear LMS, Nonlinear Least Square and Nonlinear Recursive Least Square algorithm for estimating the power system frequency. And it also contained the simulation results obtained by each algorithm.

**Chapter-4** It deals with the conclusion and suggestion for the future work.

**Chapter-5** It contains the references.

# CHAPTER-2

---

## MATHEMATICAL ANALYSIS-LINEAR ESTIMATION

### **2.1 Power system frequency estimation:**

#### **2.1.1 Introduction:**

In power system with no loss performance is considered for estimation. So it is significant to have a purely sinusoidal voltage or current signal for a better power quality. But in practical, it decline due to over voltage, variation in frequency, source type, under voltage, harmonics, generation load mismatches. Therefore it is needed to estimate the frequency fast and accurately for better power quality in presence of noise and other disturbances .Digitized samples of supply voltage are used in most of the methods for frequency estimation in power system. Basically frequency is defined as the time between two zero crossing of a purely sinusoidal voltage. But in real, signals are not available in pure form but are distorted. Hence there are various methods for estimation of frequency. Zero crossing detection and calculation of the number of cycles within a particular time interval are some basic methods for finding the frequency of a purely sinusoidal voltage waveform. Discrete Fourier transformation (DFT), Kalman filtering, least square error method, and iterative approaches (2, 6-11) are some of well-known approaches in this area. In this chapter complex Least Mean Square, Least Square and Recursive Least Square has been implemented for power system frequency estimation.

## 2.2 Frequency Estimation using Least Square (LS) Algorithm:

The least square is a standard method for the approximate solution of overdetermined systems. Overdetermined system is the system which having more no. of equations than unknowns. Least square means the overall solution minimizes the sum of the squared error. Least square approach has two types linear and nonlinear depending on whether residuals are linear in all unknowns or not. Linear least square is the linear combination of the parameters i.e. it consists of the linear equations. Linear least square has a unique solution.

## 2.3 Simulation result of Least Square (LS) Algorithm:

The figure shows the plot of frequency Vs No. of iteration. Here estimated frequency comes to be 50 Hz.

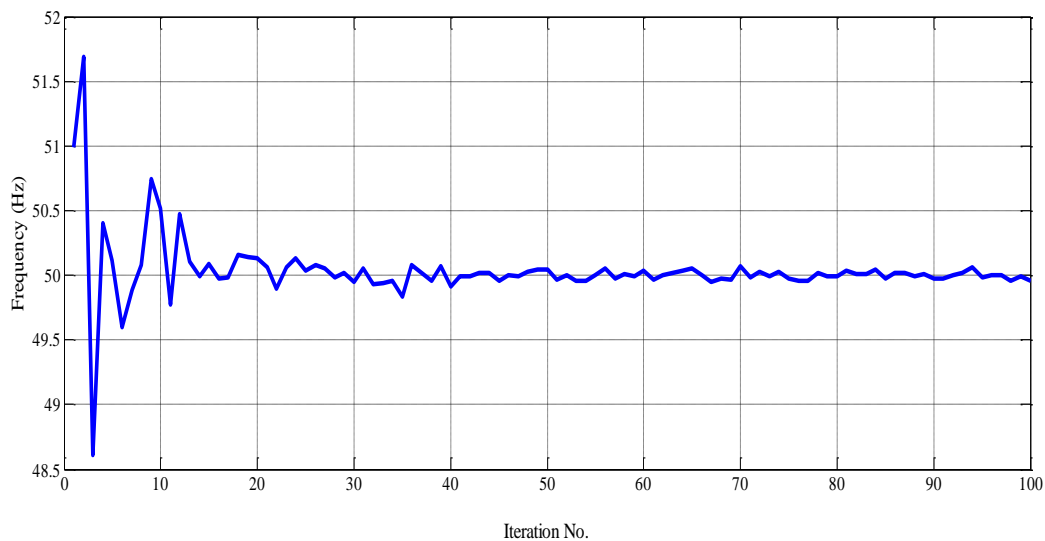


Fig.2.1 Frequency estimation using LS Algorithm

Least square method is used here for frequency estimation. By using Matlab simulation has been done. This method has good accuracy but convergence time is more. There is delay of 2 to 3 cycles in frequency estimation.

## 2.4 Frequency Estimation using Recursive Least Square (RLS) Algorithm:

A distorted power system signal with noise can be given as,

$$y(t) = A_1 \sin(\omega_0 t + \phi_1) + \varepsilon(t) \quad (2.1)$$

To estimate the power system signal  $y(t)$ , the frequency ( $f_0$ ), amplitude ( $A_1$ ), and phase ( $\phi_1$ )

equation (2.1) can also be written in discretized form as follow

$$y(k) = A_1 \sin \omega_0 kT \cos \phi_1 + A_1 \cos \omega_0 kT \sin \phi_1 + \varepsilon(k)$$
$$y(k) = [\sin \omega_0 kT \quad \cos \omega_0 kT][\alpha \quad \beta]^T + \varepsilon(k) \quad (2.2)$$

Where,

$$\alpha = \theta_{11} = A_1 \cos \phi_1$$

$$\beta = \theta_{21} = A_1 \sin \phi_1$$

By simplifying equation (2.2) given in regression form as,

$$y(k) = H(k)\theta + \varepsilon(k) \quad (2.3)$$

Where  $\varepsilon(k)$  is noise signal

By applying RLS estimation technique the parameters can be computed as follows

$$\hat{\theta}(k) = \hat{\theta}(k-1) + K(k)\varepsilon(k) \quad (2.4)$$

$\hat{\theta}(k)$  = current value of the estimate

$\hat{\theta}(k-1)$  = past value of the estimate

$K(k)$  = Kalman gain

Therefore measurement error is given by

$$\varepsilon(k) = y(k) - H(k)^T \hat{\theta}(k-1) \quad (2.5)$$

Updating of gain  $K$  is done using following expression

$$K(k) = P(k-1)H(k)[\eta I + H(k)^T P(k-1)H(k)]^{-1} \quad (2.6)$$

$P(k)$  is the error covariance matrix and  $\eta(0 < \eta < 1)$  is the forgetting factor

Covariance matrix is updated as

$$P(k) = [I - K(k)H(k)^T]P(k-1) / \eta \quad (2.7)$$

All Equations from (2.4) to (2.7) are initialized at  $k=0$ , initial covariance matrix  $P(0)$  is taken very large i.e.  $P = \delta I$  Where  $I$  is the square identity matrix and  $\delta$  is large number.

Fundamental amplitude  $A_1$  and phase  $\phi_1$  are estimated after getting final estimate of  $\theta = [\alpha \ \beta]^T$

$$A_1 = \sqrt{(A_1 \cos \phi_1)^2 + (A_1 \sin \phi_1)^2}$$

$$A_1 = \sqrt{\alpha^2 + \beta^2} \quad (2.8)$$

$$\tan \phi_1 = \frac{\beta}{\alpha}$$

$$\phi_1 = \tan^{-1}\left(\frac{\beta}{\alpha}\right) \quad (2.9)$$

As we estimate this amplitude and phase then from that fundamental frequency can be estimated as

below.  $f_0$  Given by  $f_0 = \frac{\omega_0}{2\pi}$  can be estimated from  $y(k)$  by using (2.1) as,

$$(\sin \omega_0 kT + \phi_1) = \frac{y(k)}{A_1}$$

$$\omega_0 kT + \phi_1 = \sin^{-1}\left(\frac{y(k)}{A_1}\right)$$

$$f_0 = \frac{1}{2\pi kT} [\sin^{-1}\left(\frac{y(t)}{A_1}\right) - \phi_1] \quad (2.10)$$

## 2.5 Simulation result of Recursive Least Square (RLS) Algorithm:

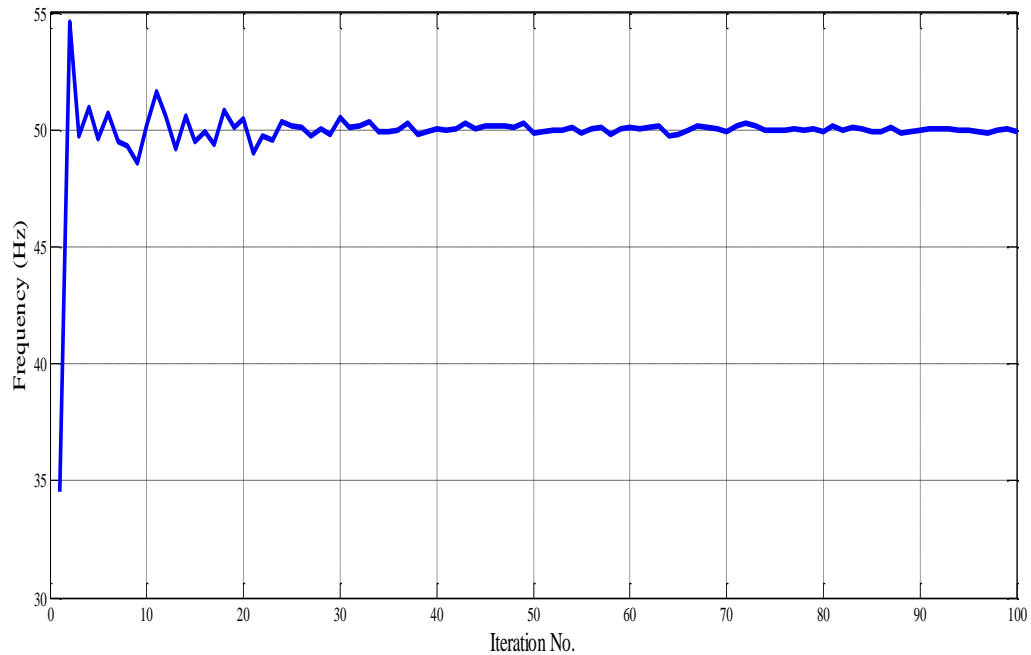


Fig.2.2 Frequency estimation using RLS Algorithm

Above figure shows the frequency estimated Vs No. of iteration. In recursive least square method frequency estimated comes to be 50 Hz. This method has a higher convergence rate than the previous one. Also time required to estimate the frequency is less with good accuracy.

## 2.6 Frequency Estimation using Least Mean Square (LMS) Algorithm:

The least-mean-square (LMS) algorithm simplification of the gradient vector and its computation is done by modifying the objective function. Due to its computational simplicity The LMS algorithm, is widely used in various applications of adaptive filtering. In order to construct a range for the convergence factor the convergence characteristics of the LMS algorithm are studied that will guarantee stability .The convergence speed of the Least Mean Square algorithm is dependent on the eigenvalue spread of the input signal correlation matrix.

The LMS algorithm is mostly used algorithm in adaptive filtering for lots of reasons. The main features are less complexity, unbiased convergence in the mean to the Wiener solution, proof of convergence in stationary environment and stable nature.

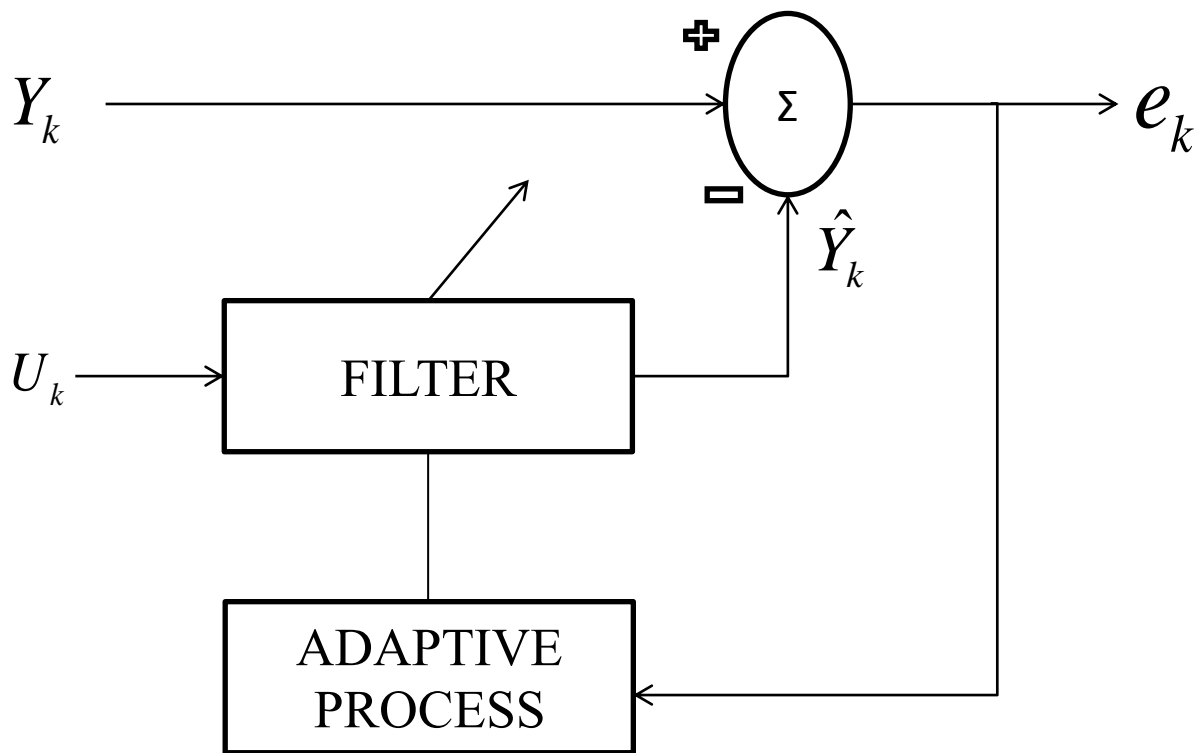


Fig 2.3 LMS filter structure

Where  $U_k$  is input data vector,

$Y_k$  is desired signal,

$\hat{Y}_k$  is an estimated signal,

$W_k$  is the weight vector,

$e_k$  is the error signal.

For LMS frequency estimation distorted power system signal with noise is represented by,

$$y(t) = A_1 \sin(\omega_0 t + \phi_1) + \varepsilon(t) \quad (2.11)$$

By discretization this can be written as,

$$y_k = \text{imag}(A_1 e^{j(\omega_k T + \phi_1)}) \quad (2.12)$$

If  $\hat{y}_k$  is estimated value of voltage at  $k^{\text{th}}$  instant so above equation becomes

$$\hat{y}_k = \text{imag}(W_k \hat{y}_{k-1}) \quad (2.13)$$

$$W_k = e^{j\hat{\omega}_k T} \quad (2.14)$$

Where  $W_k$  denotes the weight of the voltage signal,  $\hat{\omega}$  is the estimated angular frequency, error signal here is

$$e_k = y_k - \hat{y}_k \quad (2.15)$$

By alternating the complex vector  $W_k$ , this algorithm minimizes square of error at each sampling instant using equation (2.14) given as below

$$W_k = W_{k-1} + \mu_k e_k \hat{y}_k^* \quad (2.16)$$

Where \* represents the complex conjugate of a variable. For faster convergence of LMS algorithm in presence of noise step size  $\mu_k$  is varied.



For complex states the equations are modified as

$$\mu_{k+1} = \lambda\mu_k + \gamma R_k R_k^* \quad (2.17)$$

Where  $R_k$  is the autocorrelation of  $e_k$  and  $e_{k-1}$ , and  $R_k^*$  denotes the complex conjugate of  $R_k$  and it is computed as,

$$R_k = \rho R_{k-1} + (1-\rho)e_k e_{k-1} \quad (2.18)$$

Where  $\rho$  is exponential weighting parameter ( $0 < \rho < 1$ ), and  $\lambda$  ( $0 < \lambda < 1$ ) and  $\gamma > 0$  control the convergence time.  $\mu_{n-1}$  is set to  $\mu_{\max}$  or  $\mu_{\min}$  when it goes below or above of the lower and upper boundaries respectively. These values are chosen based on signal statistics. At each sampling interval, the frequency is calculated from (2.14) as

$$W_k = e^{j\hat{\omega}_k T} = \cos \hat{\omega}_k T + j \sin \hat{\omega}_k T$$

$$\sin \hat{\omega}_k T = \text{Im}(W_k)$$

$$\hat{\omega}_k T = \sin^{-1}[\text{Im}(W_k)]$$

$$\hat{f}_k = \frac{1}{2\pi T} \sin^{-1}[\text{Im}(W_k)] \quad (2.19)$$

Where  $\omega_k = 2\pi f_k$

So as described above frequency can be estimated using LMS algorithm.

Here  $\text{Im}()$  stands for imaginary part of the quantity.

## 2.7 Steps for Frequency Estimation using Least Mean Square (LMS) Algorithm:

1. Initialize  $A, \phi, \rho, \gamma, \mu, R, W, f, \lambda$
2. Generate power system signal
3. Using initial values of the weight vector estimate the discretised signal
4. Error=Actual signal – Estimated signal

5. Update the step size and auto correlation matrix using (2.17) and (2.18)
6. Update weight vector using (2.16)
7. Go to step 4, until final iteration is reached
8. Estimate the fundamental frequency using equation (2.19)

## 2.8 Simulation result of Least Mean Square (LMS) Algorithm:

LMS algorithm is applied to estimate the fundamental frequency from the sampled value of the phase voltage signal. Parameters of algorithm are  $0.01 < \mu < 0.018$ ,  $P_{initial} = 0$ ,  $r=0.99$ ,  $\gamma=0.01$  And  $\lambda$  value changes according to sampling instant.

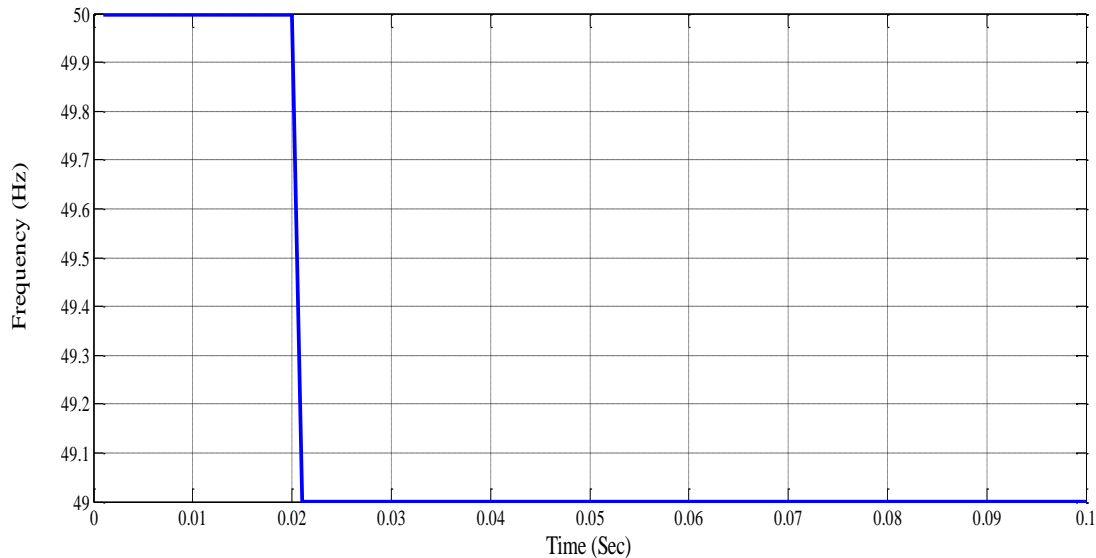


Fig.2.4 Frequency estimation using LMS Algorithm

Figure shows the simulation result obtained by implementing the above described LMS algorithm in Matlab. Here in graph step change in frequency is given and performance is observed. From the figure it is seen that during step change in frequency from 50 Hz to 49 Hz there is delay of 0.001 second. Accuracy and speed of estimation is satisfactory even in presence of noise and harmonics. Advantages of this algorithm are simplicity in formulation and computational efficiency.

# CHAPTER-3

---

## MATHEMATICAL ANALYSIS-NONLINEAR ESTIMATION

### 3.1 Power system frequency estimation using Nonlinear Least Square (NLS)

#### Algorithm:

Nonlinear Least Square estimation is the approach used for the estimation of Fundamental frequency. Here frequency is estimated by minimizing the squared error between actual signal and assumed signal. For the model  $y(k)$  is the actual signal,  $\phi(k)$  is the system structure matrix and

$\theta(k)$  is vector of parameters to be estimated.

Estimation of unknown parameter can be done by

$$\hat{\theta}_{LS} = [\phi(k)\phi(k)^T]^{-1} \phi(k)y(k) \quad (3.1)$$

Any periodic voltage signal in Fourier series can be expressed as

$$y(t) = A_0 + \sum_{n=1}^{\infty} (A_n \cos n\omega_0 t + B_n \sin n\omega_0 t) \quad (3.2)$$

Where  $\sqrt{(A_n^2 + B_n^2)}$  = Magnitude of  $n^{\text{th}}$  harmonic,

$\omega_0$  = Fundamental frequency in rad/sec

As  $y(t)$  not having any dc component  $a_0 = 0$

Generally number of harmonics should be finite suppose  $n$ .

In power system triplet harmonics are absent because generally waveforms are having half-wave symmetry and therefore they don't contain even harmonics, so  $\{1,5,7,11,\dots,n_n\}$

Total number of harmonics suppose be  $N_a$ .

For solving  $A_n$  and  $B_n$  assume that  $\omega_0$  is known, so we are assumed that  $y(t)$  is M uniformly sampled points.

This gives M sets of equations

$$y(t_k) \approx a_0 + \sum_1^N (a_n \cos n\omega_0 t_k + b_n \sin n\omega_0 t_k) \quad (3.3)$$

Above equation in matrix form can be expressed as

$$PX = y \quad (3.4)$$

Where

$$P = [P_a \quad P_b]$$

$$P_a = \begin{bmatrix} \cos\omega_0 t_1 & \cos 5\omega_0 t_1 & \dots & \cos n_h \omega_0 t_1 \\ \cos\omega_0 t_2 & \cos 5\omega_0 t_2 & \dots & \cos n_h \omega_0 t_2 \\ \vdots & \vdots & \vdots & \vdots \\ \cos\omega_0 t_M & \cos 5\omega_0 t_M & \dots & \cos n_h \omega_0 t_M \end{bmatrix}_{M \times N_a}$$

$$P_b = \begin{bmatrix} \sin\omega_0 t_1 & \sin 5\omega_0 t_1 & \dots & \sin n_h \omega_0 t_1 \\ \sin\omega_0 t_2 & \sin 5\omega_0 t_2 & \dots & \sin n_h \omega_0 t_2 \\ \vdots & \vdots & \vdots & \vdots \\ \sin\omega_0 t_M & \sin 5\omega_0 t_M & \dots & \sin n_h \omega_0 t_M \end{bmatrix}_{M \times N_a}$$

$$X = [a_1 \quad a_5 \dots \quad a_{n_h} \quad b_1 \quad b_5 \quad b_7 \quad b_{n_h}]_{1 \times 2N_a} \quad (3.5)$$

And

$$y = [y(t_1) \quad y(t_2) \dots \quad y(t_M)]_{1 \times N_a}^T \quad (3.6)$$

No. of equations =  $2N_a$  and we need to solve M no. of samples along with  $2N_a$  equations. Noise and

error present so overdetermined system gives a better solution. ( $M > 2N_a$ )

The Least Square solution of X is given as,

$$X \approx (P^T P)^{-1} P^T y \quad (3.7)$$

If  $\omega_0$  is unknown then  $P$  is unknown least square solution becomes nonlinear. Although  $2N_a + 1$  unknowns are there, linearly entering  $2N_a$  amplitude variables can be eliminated, which results in 1-D Nonlinear least square (NLS) problem. Elimination is done by substituting (3.7) into (3.4)

$$P(P^T P)^{-1} P^T y \approx y \quad (3.8)$$

The error vector e is given as

$$e = [I - P(P^T P)^{-1} P^T] y \quad (3.9)$$

Here error e is the function of  $\omega_0$  only. So the value of  $\omega_0$  that minimizes  $\|e\|_2^2$  is taken as estimated frequency. As we have only one parameter to be estimated 1-D search is enough to locate the minimum. Estimated frequency is the frequency where the minimum occurred.

### **3.2 Simulation result of Nonlinear Least Square (NLS) Algorithm:**

Accuracy of the algorithm decides the robustness of the 1-D search. The typical behaviour of the norm of error as a function of frequency is shown in the figure below. Here the search is carried out by varying the fundamental frequency  $f_0$  in the steps of 0.1Hz. True frequency is the frequency at which the minimum of the curve occurred. For the frequency estimation we considered a voltage waveform having 10% total harmonics and additive random noise which was given as:

$$v(t) = \sin(\omega_0 t) + 0.0856 \sin(5\omega_0 t) + 0.0428 \sin(7\omega_0 t) + 0.0306 \sin(11\omega_0 t) + 0.0183 \sin(13\omega_0 t)$$

Here 3.2 KHz is the sampling frequency used for computer simulation and the fundamental frequency is taken as 50Hz. In power system half-wave symmetry waveforms are very common so it does not contain even numbered harmonics and triplen harmonics. Triplen harmonics are the 3<sup>rd</sup> and multiple of fundamental harmonics. Using above sampling frequency, maximum 32 harmonics can be taken into consideration without any aliasing, which is suitable for the any application of power system environment. Using above voltage signal we estimated the frequency by nonlinear least square technique by varying the frequency of voltage signal from 48.5 to 51.5Hz.

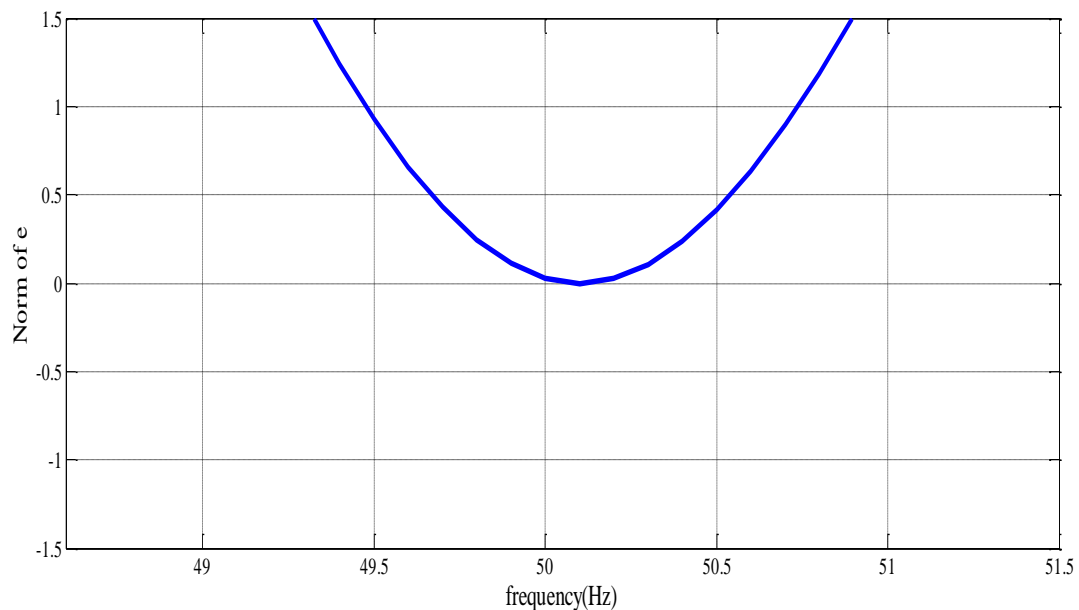


Fig.3.1 Frequency estimation using NLS Algorithm

As seen by above simulation result we found that minimum error occurred at 50.1Hz which is the estimated frequency.

### 3.3 Power system frequency estimation using Nonlinear Recursive Least Square

#### (NRLS) Algorithm:

To enhance the performance of the least square algorithm, many modification and extensions are done in literature. The performance of the algorithm improved due to modification. So algorithm has improved characteristic such as consistency of estimation, recursive calculation, less deviations and faster convergence. Nonlinear recursive least square algorithm consists of extra three steps as compared to nonlinear least square algorithm

$$P^{-1}(k+1) = P(k) + \phi^T(k+1)\phi(k+1)$$

$$K(k+1) = P(k+1)\phi^T(k+1)$$

$$\theta(k+1) = \theta(k) + K(k+1)[y(k+1) - \phi(k+1)\theta(k)] \quad (3.10)$$

All above equations are initialized by taking the initial values of  $k$ ,  $\theta(k)$  and  $P$ .

For  $P = \alpha I$ , here  $\alpha$  is very large number and  $I$  is the identity matrix of the size  $n \times n$ , where  $n$  is the number parameters to be estimated. For frequency estimation using recursive least square algorithm it consists of extra three steps of equation (3.10) added after equation (3.3).

Algorithm is initiated by taking all values zero and then unknown parameter vector is updated at each instant using the steps given in recursive least square algorithm.

### 3.4 Simulation result of Nonlinear Recursive Least Square (NRLS) Algorithm:

The simulation results of least square and Nonlinear Least square are almost same only the dissimilarity between the two is that Least Square is a batch process and Recursive Least Square is an iterative process. From the below graph it can be seen that minimum of the frequency occurred that. Practically nonlinear recursive least square have a much more attractive characteristic while using it online.

The simulation result of Nonlinear Recursive Least Square algorithm is as shown above having frequency variation from 48.5Hz to 51.5Hz with the scale of 0.1Hz. As it is an online process so we can estimate the vector of unknown parameter in each step. Then by estimating that vector we can get error in each sample. The simulation figure is shown in fig.3.2

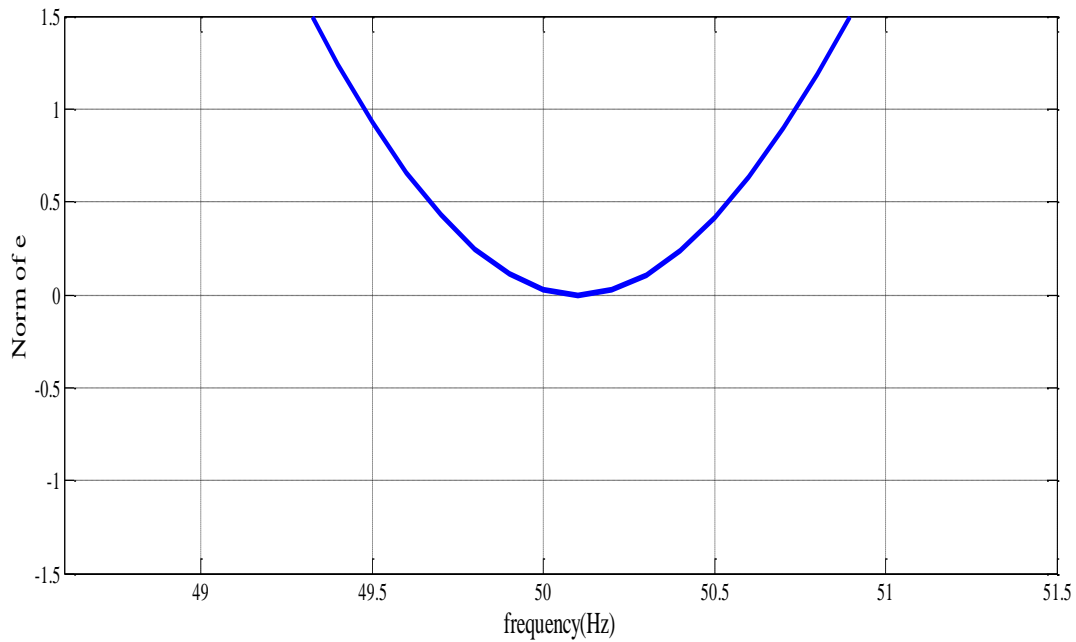


Fig.3.2 Frequency estimation using NRLS Algorithm



### 3.5 Power system frequency estimation using Nonlinear Least Mean Square

#### (NLMS) Algorithm:

Nonlinear Least Mean Square algorithm is the combination of both nonlinear least square and least mean square.

Steps for estimating the frequency are as follows,

Here all equations are initialized by taking all values zero. Sampling frequency used here is 3.2 KHz and window length is taken as 0.02.

Also  $\mu = 0.8$ ,  $\rho = 0.1$  and  $\gamma = 0.9$

System matrix  $H$  is given as,

$$H_b = \begin{bmatrix} \sin\omega_0 t_1 & \sin 5\omega_0 t_1 & \dots & \sin n_h \omega_0 t_1 \\ \sin\omega_0 t_2 & \sin 5\omega_0 t_2 & \dots & \sin n_h \omega_0 t_2 \\ \vdots & \vdots & \vdots & \vdots \\ \sin\omega_0 t_M & \sin 5\omega_0 t_M & \dots & \sin n_h \omega_0 t_M \end{bmatrix}_{M \times N_a}$$

$$H = [H_b]$$

Adaptation parameter  $\mu$  is defined as,  $\mu_{k+1} = \lambda\mu_k + \gamma R_k R_k^*$

Where,

$R_k$  is the auto correlation matrix and is given as,  $R_k = \rho R_{k-1} + (1-\rho)e_k e_{k-1}$

After that error can be find out by using,

$$e = y - (H * \theta)$$

Where  $\theta = \theta + (\mu * e)^T * (H * \theta)$

Then the frequency is estimated by using the plot of norm of error with frequency. Here the estimated frequency is the frequency at which minimum of error occurred.

Nonlinear least mean square method estimates the frequency of the given voltage signal iteratively.

### 3.6 Simulation result of Nonlinear Least Mean Square (NLMS) Algorithm:

Estimated frequency using nonlinear least mean square method is as shown in figure given below. Here the estimated frequency deviates around 50 Hz. NLMS has fast convergence rate and accuracy is good. Algorithm is simple in structure and computationally easy. It estimates the frequency of the distorted signal accurately and within time and error is also less for estimated frequency.

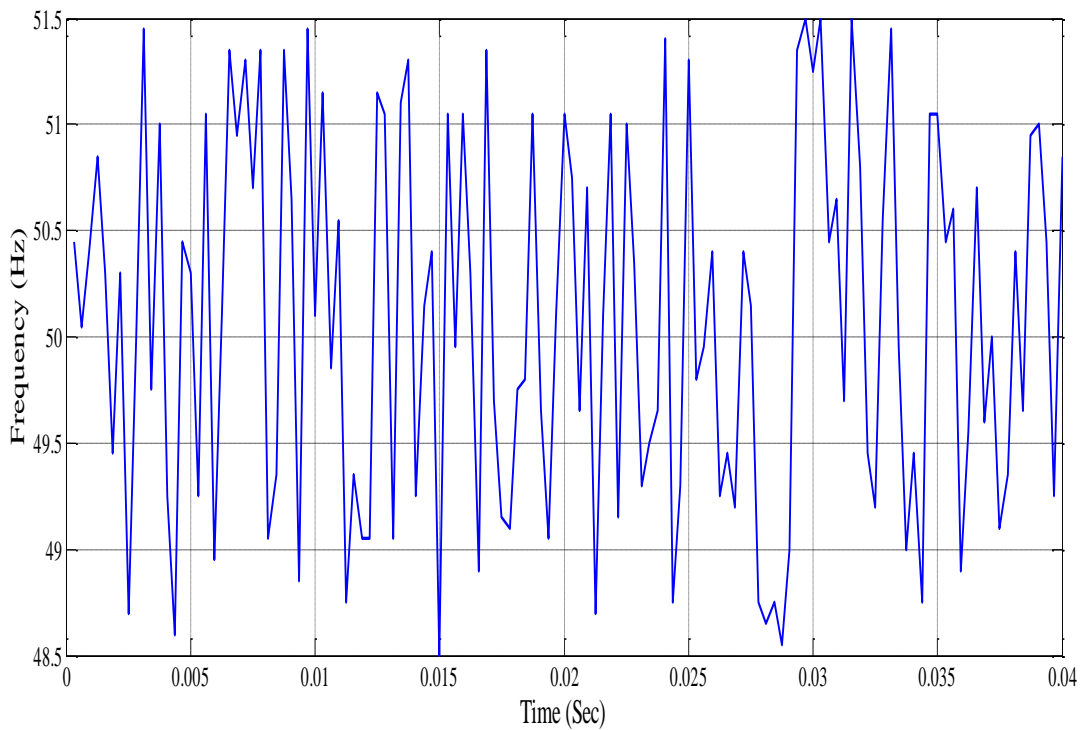


Fig.3.3 Frequency estimation using NLMS Algorithm

Frequency estimation using NLMS algorithm gives a better result as compare to NLS and NRLS. It has a good convergence characteristic and delay is very less. Therefore NLMS outperform among all three algorithms.

# CHAPTER-4

---

## SUMMARY AND CONCLUSION

### **4.1 Summary of the project work:**

This thesis mainly discovered on the frequency estimation of the power system signal. The novelty of the present work is introduction of NLMS algorithm to the estimation of frequency.

- Problems and brief review of earlier approaches for the estimation of power system frequency are discussed.
- Frequency estimation of distorted signals, firstly by using various linear techniques such as LS, RLS and LMS are presented. The performance of all three algorithms was studied through computer simulation results. By comparing the simulation results of these algorithms it was observed that LMS algorithm outperforms over other three algorithms.
- Also frequency estimation of the noisy signal using nonlinear techniques such as NLS, NRLS, NLMS are presented. We have studied all three methods and compare the results of all getting through computer simulations.

## 4.2 Conclusions:

Based on the conducted studies in this project, the following conclusion may be drawn about the well-known above algorithm for power system frequency estimation.

- Least square approach is the simple method for estimation of power system frequency. Accuracy and speed of convergence of this algorithm is good. Least square algorithm estimates the frequency of the signal by minimizing the squared error between actual signal and assumed signal model.
- Recursive least square algorithm used for power system frequency estimation. Here single phase distorted voltage signal is used for power system frequency estimation. This algorithm has a good accuracy and a better convergence rate than least square.
- The LMS-based approach using sampled values voltage signals is simple in its formulation. Single phase distorted signal is used here for frequency estimation. According to simulation results the accuracy and speed of estimation is satisfactory even in the presence of noise and disturbances. By adapting the different situations this method works well.

Among all of the above three linear methods of frequency estimation LMS performs better in all aspect, that is it has better convergence rate, simplicity in formulation and computational efficiency.

Although we are estimating the frequency with above linear algorithms, frequency estimation using nonlinear methods has also been done in this project.

- In nonlinear estimation of frequency nonlinear least square method is first implemented for frequency estimation. The actual frequency is the frequency at which minimum of error occurred. Flexibility of nonlinear 1-D search grid of least square can be decided based on the accuracy desired. Algorithm is computationally simple.

The main advantage of this technique is that it can track the changes in the frequency within one analysis window length with a good accuracy.

- NRLS algorithm has same simulation results as that of the NLS algorithm but in NRLS frequency is updated at each sample. Therefore nonlinear RLS is an online method for power system frequency estimation. It has a better convergence rate and accuracy than NLS algorithm.
- NLMS algorithm is implemented here for power system frequency estimation. The algorithm has good accuracy and computational time is less than other techniques.

NLMS outperforms among all above algorithms. Frequency estimation using NLMS is the new technique introduced in this paper. It has good characteristics. It is simple and computationally efficient. Also this method has a higher convergence rate.

### **4.3 Future scope of the work:**

- To propose a Hardware on FPGA for real time estimation of power system Frequency.
- To collect industrial data signal and Estimate the frequency of that signal using above algorithms.

# References

- [1] T.Lobos and J. Rezmer, "Real time determination of power system frequency" IEEE transaction on instrumentation and measurement, vol. 406, No. 4, August 1997.
- [2] P. K. Dash, A. K. Pradhan, and G. Panda, "Frequency estimation of distorted power system signals using extended complex Kalman filter," IEEE Trans. Power Del., vol. 14, no. 3, pp. 761–766, Jul. 1999.
- [3] Chien-Hung Huang, Chien-Hsing Lee, Kuang-Jung Shih and Yaw-Juen Wang "Frequency Estimation of Distorted Power System Signals Using a Robust Algorithm" IEEE Transactions on Power Delivery, vol.23, no.1, pp.41-51, 2008
- [4] A. Routray, A. K. Pradhan and K. P. Rao "A Novel Kalman Filter for Frequency Estimation of Distorted Signals in Power Systems" IEEE Transactions on Instrumentation and Measurement, Vol.51, No.3, pp.469-479, 2002.
- [5] P.K.Dash, R.K.Jena, G.Panda and A. Routray "An Extended Complex Kalman Filter for Frequency Measurement of Distorted Signals" IEEE Transactions on Instrumentations and Measurement, Vol.49, No.4, pp.746-753, 2000.
- [6] V. Terzija and M. Djuric, "A numerical algorithm for direct real-time estimation of voltage phasor, frequency and its rate of change," Electr.Mach.Power Syst., vol. 24, pp. 417–428, 1996.
- [7] Mohsen Mojiri, Masoud Karimi-Ghartemani and Alireza Bakhshai "Estimation of Power System Frequency Using an Adaptive Notch Filter" IEEE Transactions on Instrumentation and Measurement, vol.56, no.6, pp.2470-2477, 2007.

- [8] T. S. Sidhu and M. S. Sachdev, "An iterative technique for fast and accurate measurement of power system frequency," IEEE Trans. Power Del., vol. 13, no. 1, pp. 109–115, Jan. 1998.
- [9] M. Karimi-Ghartemani, M. R. Iravani "Wide-range, fast and robust estimation of power system frequency" Electric Power System Research, vol.65, pp.109-117, 2003.
- [10] R. Chudamani, Krishna Vasudevan and C.S. Ramalingam, "Real time estimation of power system frequency using Nonlinear Least square," IEEE Trans. Power Del., vol. 24, no. 3, July. 2008.
- [11] T. S. Sidhu and M. S. Sachdev, "An iterative technique for fast and accurate measurement of power system frequency," IEEE Trans. Power Del., vol. 13, no. 1, pp. 109–115, Jan. 1998.
- [12] S.A. Soliman , R.A. Alammari , M.E. El-Hawary 'Frequency and harmonics evaluation in power network using fuzzy regression technique' Electric Power System Research, vol. 66, pp. 171-177., 2003.
- [13] M.Gupta, S.Srivastava and J.R.P.Gupta "Power system frequency estimation using neural network and genetic algorithm" Proceedings of Joint International Conference on Power System Technology and IEEE Power India Conference, POWERCON 2008, pp. 1-5, 12-15 Oct. 2008
- [14] P.K.Dash, D.P.Swain, A.Routray and A.C.Liew "An adaptive neural network for estimation of power system frequency" Electric Power System Research, vol.41, issue 3, pp. 203-210, 1997.
- [15] M. S. Sachdev and M. M. Giray, "A least square technique for determining power system frequency," IEEE Trans. Power App. Syst., vol. PAS-104, no. 2, pp. 437–443, 1985.

- [16] A. G. Phadke, J. Thorp, and M. Adamiak, "A new measurement technique for tracking voltage phasors, local system frequency and rate of change of frequency," *IEEE Trans. Power App. Syst.*, vol. PAS-102, no. 5, pp. 1025–1038, 1983.
- [17] A. Girgis and T. L. D.Hwang, "Optimal estimation of voltage phasors and frequency deviation using linear and nonlinear Kalman filtering," *IEEE Trans. Power App. Syst.*, vol. PAS-103, no. 10, pp. 2943–2949, 1984.
- [18] T. Lobos, "New recursive method for real time determination of basic wave form of voltage and current," *IEEE Proc. C* 136(6), (1989), 347-351.
- [19] K.F. Eichhorn and T. Lobos, "Recursive real-time calculation of basic waveforms of Signals," *IEEE Proc. C* 138(6) (1991), 469-470.
- [20] J. Xi and J.F. Chicharo, "A new algorithm for improving the accuracy of periodic signal analysis," *IEEE Trans. Instrum. Meas.* 45 (4) (1996) 827-831.
- [21] Milenko B. Djuric, Zeljko R. Djuric "Frequency measurement of distorted signals using Fourier and Zero crossing Techniques" *Electric Power System Research*, vol. 78, pp. 1407-1415, 2008
- [22]. M. Akke, "Frequency estimation by demodulation of two complex signals," *IEEE Trans. Power Del.*, vol.12, no. 1, pp. 157–163, Jan. 1997.
- [23] R. Chudamani, K. Vasudevan, and C. S. Ramalingam, "Nonlinear least squares current estimator for three phase loads," in *Proc. IEEE Conf. Industrial Technology*, Mumbai, India, Dec. 15–17, 2006, pp.2581–2586.
- [24] A. Feuer and E. Weinstein, "Convergence analysis of LMS filters with uncorrelated Gaussian data," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 33, no. 1, pp. 222–229, 1985.



- [25] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*.  
Englewood Cliffs, NJ: Prentice-Hall, 1993.
- [26] C. L. Lawson and R. J. Hanson, *Solving Least Squares Problems*. Philadelphia, PA:  
SIAM, 1995.
- [27] D. C. Lay, *Linear Algebra and Its Applications*, 3rd ed. Upper Saddle River, NJ:  
Pearson Education, 2003