Development of Efficient Radar Pulse Compression Technique for Frequency Modulated Pulses

Thesis submitted in partial fulfillment of the requirements for the degree of

Master of Technology

In

Electronics and Communication Engineering

By

Chandan Kumar
Roll No: 212EC6187

Department of Electronics and Communication Engineering
National Institute Of Technology Rourkela
Rourkela, Odisha, 769008

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Under the guidance of

Prof. Ajit Kumar Sahoo

Department of Electronics and Communication Engineering
National Institute Of Technology Rourkela
Rourkela, Odisha, 769008

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Chandan Kumar

Chandankr304@gmail.com

DEPT. OF ELECTRONICS AND COMMUNICATION ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA

ROURKELA – 769008, ODISHA, INDIA

CERTIFICATE

This is to certify that the thesis titled "**Development of Efficient Radar Pulse Compression Technique for Frequency Modulated Pulses**" submitted by **Mr. Chandan Kumar** in partial fulfillment of the requirements for the award of Master of Technology degree in **Electronics and Communication Engineering** with specialization in **"Signal and Image Processing"** during session 2013-2014 at **National Institute Of Technology, Rourkela** is an authentic work by him under my supervision and guidance.

Place: NIT Rourkela **Prof. Ajit Kumar Sahoo**

Date: 30th May 2014 Assistant Professor

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I certify that

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b) The work has not been submitted to any other Institute for any degree or diploma.

c) I have followed the guidelines provided by the Institute in writing the thesis.

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Chandan Kumar

30th May 2014

Abstract

Radar systems use Pulse Compression techniques to enhance the long range detection capability of long duration pulse and the range resolution capability of short pulse. Frequency and phase modulation techniques are used to increase the BW of long duration pulse to achieve better range resolution with limited peak power. Towards this purpose Linear FM chrip is the very common form of waveform. This waveform has a matched filtered Response (or ACF) with side lobe level is about -13dB. It may be improve by using methods such as windowing, adaptive filtering and optimization techniques. Windowing is used in LFM pulse Compression to reduce the side lobes. But the output SNR can be reduced by 1 to 2 dB due to windowing, this leads to reduce the false alarm rates in object detection applications. Using a train of stepped frequency pulses is an efficient method that achieves large overall BW and at the same time, maintains narrow instantaneous BW. In this method a frequency step Δf is added between successive pulses. One of the benefits of this method is that it allows us to use the duration between pulses to control the mid frequency of the other narrow band components of the radar system.

Introducing frequency step between consecutive pulses is an efficient method to enhance the BW of pulse train. The large value of Δf gives large total BW and better range resolution. However, if the product of frequency step (Δf) and pulse width becomes more than one, the stepped frequency pulse-train ACF experiences unwanted peaks, referred to as “grating lobes”. A way to reduce these grating lobes is to use LFM pulses of some bandwidth B in place of the fixed frequency pulses. We can derive a relationship between Δf, and pulse duration such that nulls are placed at points where the grating lobes have been located by analyzing AF and ACF expression.

But in above method we are still getting high side lobes near the main lobe. Nevertheless, if we use NLFM pulse instead of LFM pulse, then it becomes possible to reduce both the side lobes and the grating lobes. In this thesis work, we follow a numerical approach that permits us to design the phase of such a NLFM waveform.

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List of Abbreviations

AF	Ambiguity Function
ACF	Autocorrelation Function
BW	Bandwidth
SNR	Signal to Noise Ratio
LFM	Linear Frequency Modulation
NLFM	Non Linear Frequency Modulation
AWGN	Additive White Gaussian Noise
PSL	Peak Side lobe Level

CHAPTER 1: INTRODUCTION

The abbreviation RADAR stands for RAdio Detection And Ranging. It is an electromagnetic system that detects and locates the reflecting objects like aircraft, ships, vehicles etc. It operates on the principle of radiation of energy into space and detection of the reflected echo from the target. It can work in darkness, fog, snow and rain. Its most important feature is its ability to get target range with greater accuracy and in all weather conditions. Angle and direction of the target are obtained by the angle of reception of echo at the antenna and tracking system of the radar. The target range is a function of delay in the received signal and velocity of the target is a function of signal's Doppler shift.

Waveform design is an important area of work in the radar systems development. Two important functions that are obtained by the waveform of a radar system are range resolution and detection of maximum range. The function of the radar to separate two closely spaced targets having different range but same bearing angle is known as range resolution. It is inversely proportional to bandwidth of the transmitted signal which means a signal whose BW is larger can give a better range resolution. Mathematically the range resolution is related with BW as

$$r_{rec} = \frac{c}{2B}$$

where c denotes the light speed, and B is the BW of the signal.

Whereas for an unmodulated pulse of duration *T*, the bandwidth $B = \frac{1}{T}$

A short pulse has importance in many radar applications that gives high range resolution .But use of short pulse in radar system has some limitation. Since the BW of a pulse is inversely proportional to its pulse width, the BW of a short pulse is high. High BW will make the system complex. Further, short pulses needs more peak power. A short pulse needs more peak power to accommodate enough energy for its transmission. High power transmitters and receivers are difficult to design because of the constraints on the components to withstand high power. One way to get the solution of this problem is to convert the short pulse into a long one and using some form of modulation to increase the BW of the long pulse so that the range resolution is not compromised. This technique is called Pulse Compression and is used widely in Airborne Radar applications where high peak power is undesirable [1].

1.1 Pulse Compression

The maximum range detection in radar system depends upon the strength of the received echo. To get high strength reflected echo the transmitted pulse should have high energy for long distance transmission since it gets attenuated during the course of transmission. The product of peak power and pulse duration gives an energy estimate of the signal. High peak power and short pulse duration gives the identical energy as obtained in case of low peak power pulse and high pulse width. But to get high peak power the equipment of radar becomes bigger and heavier which increases the system cost [2].

Hence a pulse with low peak power and long width is needed to transmit for detection of long range, and at receiver the pulse should have short duration and high peak power is needed to achieve fine range resolution. The range resolution R_{rec} is expressed as

$R_{rec} = \frac{c}{2B}$ where B is the bandwidth of the transmitted pulse and c=Velocity of light.

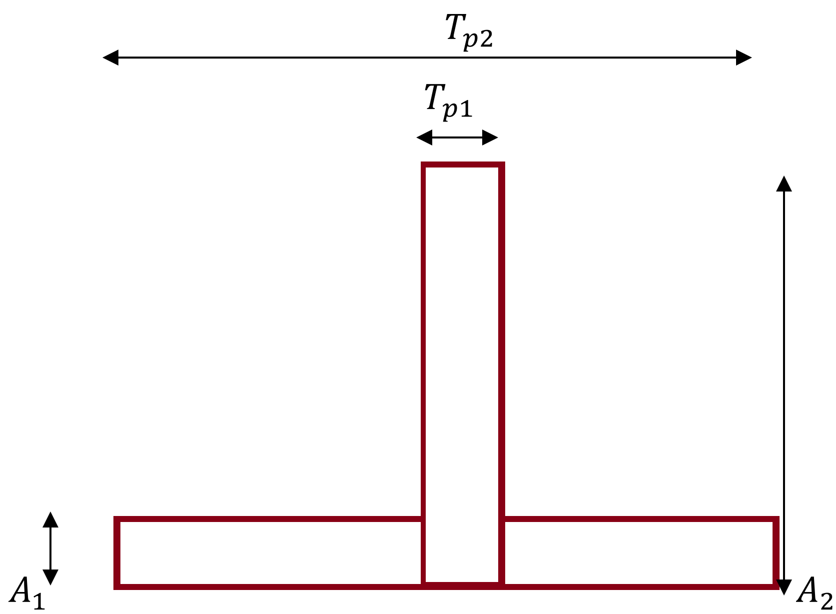


Figure 1 ***Pulses of different duration and having same energy***

And

$$P_1 T_{p1} = P_2 T_{p2}$$

Where $T_{p1} \leq T_{p2}$

So to get the advantage of large range detection ability of long pulse and better range resolution of short pulse, Pulse compression methods are used. Some modulation technique such as frequency modulation or phase modulation is used to achieve a larger BW so as to get a higher range resolution. A long duration pulse of low peak power is frequency or phase modulated before transmission and the received signal is passed through a matched filter which accumulates signal energy to a small duration of time *1/B*. The scale of compression relative to an uncompressed pulse is given by Pulse Compression Ratio (*PCR)* [3]

$$PCR = \frac{Width\ of\ the\ pulse\ before\ compression}{Width\ of\ the\ pulse\ after\ compression}$$

The compression ratio which gives a figure of merit for pulse compression is equal to the Time-BW product (TB) of the pulse [2].

1.2 Matched Filter

The reflected signal is used to get the target location in radar applications. The reflected signal is corrupted by additive white Gaussian noise (AWGN). The maximum probability of detection is depends on SNR in place of exact shape of the received signal. Hence it is necessary to maximize the SNR rather than preserving the signal shape. In receiver side to maximize the output SNR a matched filter is used. The input to a matched filter is the desired signal s(t) and additive white noise having double sided power spectral density of $\frac{N_0}{2}$.

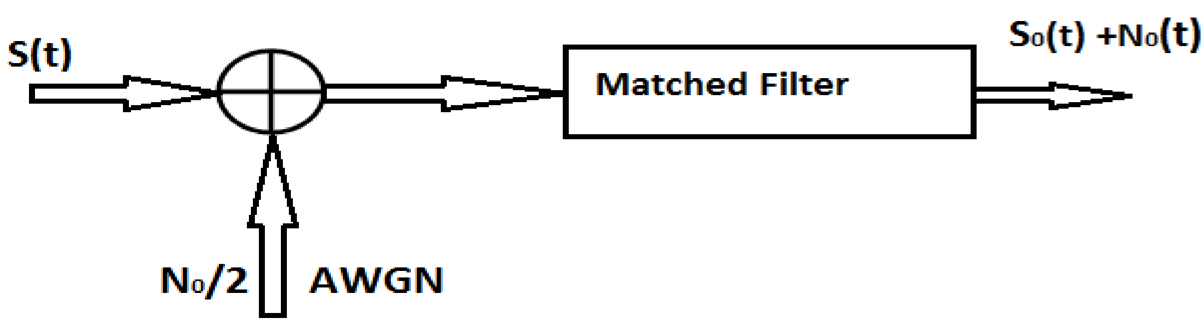


Figure 2 ***Block Diagram of Matched Filter***

Our main aim is to get the filter impulse response $h(t)$ that will maximize

$$\left(\frac{S}{N}\right)_{out} = \frac{|S_0(t_0)|^2}{n_0(t)^2} \tag{1.2.1}$$

Where Output at t_0 $S_0(t_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)S(\omega)e^{j\omega t_0}\, d\omega$ (1.2.2)

And Noise power is $n_0(t)^2 = \frac{N_0}{4\pi}\int_{-\infty}^{\infty}|H(\omega)|^2\, d\omega$ (1.2.3)

So output signal to noise ratio $\left(\frac{S}{N}\right)_{out} = \frac{\left|\int_{-\infty}^{\infty} H(\omega)S(\omega)e^{j\omega t_0} d\omega\right|^2}{\pi\, N_0 \int_{-\infty}^{\infty}|H(\omega)|^2 d\omega}$ (1.2.4)

By applying Schwarz inequality we have found that maximum output SNR will obtain when $H(\omega) = KS^*(\omega)e^{-j\omega t_0}$ (1.2.5)

This imply $h(t) = s^*(t_0 - t)$ (1.2.6)

Here we can see that the impulse response of the filter is conjugate of delayed mirror image of the signal

From equation 1.2.3 and 1.2.4 we have found that The Maximum $SNR = \frac{2E}{N_0}$

Here we can see that highest SNR at the matched filter output is a function of energy of transmitted signal not its shape.

And the matched filter output is

$S_0(t) = s(t) * h(t) = \int_{-\infty}^{\infty} s(\tau)s^*(\tau - t)\, d\tau$ Where $k = 1, t_0 = 0$

Gives the ACF of the signal $s(t)$.

1.3 Ambiguity Function

The ambiguity function (AF) gives the output of the matched filter which is matched with given finite energy signal where the received signal is delayed by time interval τ and Doppler shift ν compare to the filter expected values. The Ambiguity function is expressed as

$$|X(\tau, v)| = \iint_{-\infty}^{\infty} u(t) u^* (\mathrm{t} + \tau) \exp(\mathrm{j}2\pi v t) \mathrm{dt}$$

Where $u(t)$ denotes complex envelope of the transmitted signal, τ is time delay and υ is the Doppler shift. A positive delay τ represents that the target is far away from radar with respect to reference τ=0. A positive υ show that the target is moving towards the radar whereas a negative υ implies a receding target. The AF is widely used as a mathematical tool for visualizing radar waveform behavior. Actually, it's a chain of correlation integrals depends on detection of matched filter, where a received signal with some Doppler shifts & time delays get correlated with a reference signal having no Doppler shift or time delay [2].

1.3.1 Characteristics of Ambiguity Function -

Ambiguity Function of a signal satisfies following Characteristics [4] [5]:

1. The value of Ambiguity function is maximum at the origin

$$|X(\tau, v)| \leq |X(0,0)| = 1$$

2. It represents that the overall volume occupied by ambiguity surface is unity

$$\iint_{-\infty}^{\infty} |X(\tau, v)|^2 \, \mathrm{d}\tau \, dv = 1$$

3. Ambiguity function is symmetric with respect to origin

$$|X(-\tau, -v)| = |X(\tau, v)|$$

4. LFM effect

If a complex envelope $u\ (t)$ having AF is $|X(\tau, v)|$: namely

$$u(t) \leftrightarrow |X(\tau, v)|$$

Then adding LFM is similar to a quadratic phase addition which shows that

$u(t) * \exp(j\pi k t^2) \leftrightarrow |X(\tau, v - k\tau)|$

Fig1.4 shows the Ambiguity function plot of LFM pulse and Fig1.5 shows the Ambiguity function plot for Stepped frequency identical pulses [6].

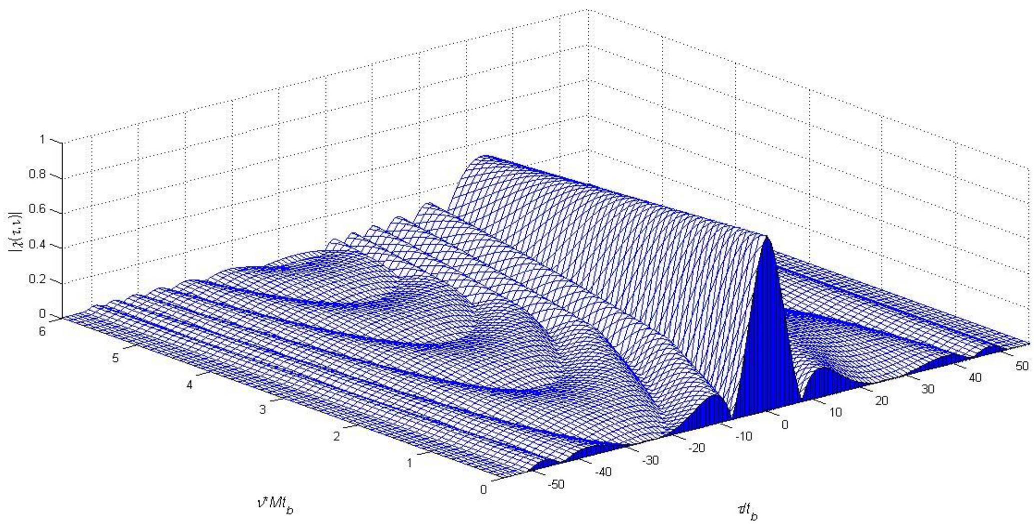


Figure 3 *AF plot of LFM pulse*

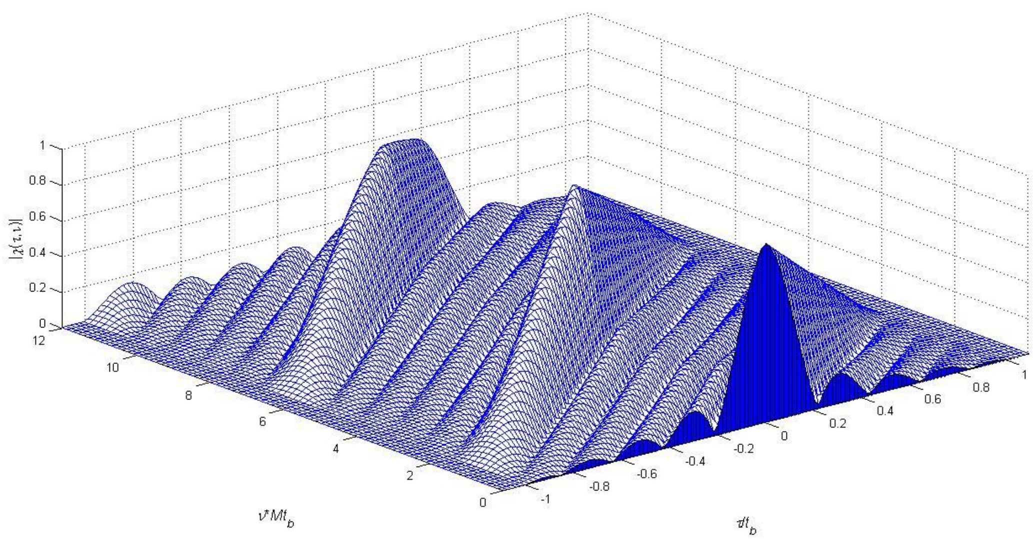


Figure 4 *AF plot for Stepped frequency identical pulse*

1.4 Radar Signals

1.4.1 Phase Modulated pulse

Unlike FM waveforms the pulse is subdivided into a number of sub pulses in the Phase modulated waveforms. Each subpulse is of equal duration and has a certain phase. The phase of each subpulse is selected in accordance with the given code sequence. The most frequently used phase-coded waveform requires two phases and it is known as binary coding. The binary code is a sequence of either $O's$ and $I's$ or $+I's$ and —$I's$. The transmitted signal phase changes between 0° and 180° with respect to the sequence of elements, Os and Is or $+Is$ and $-Is$, in the phase code, as shown in Fig. 1.5. Since the frequency of transmission is not always a multiple of the reciprocal of the subpulse interval, hence at the phase-reversal points the phase coded signal is usually discontinuous.

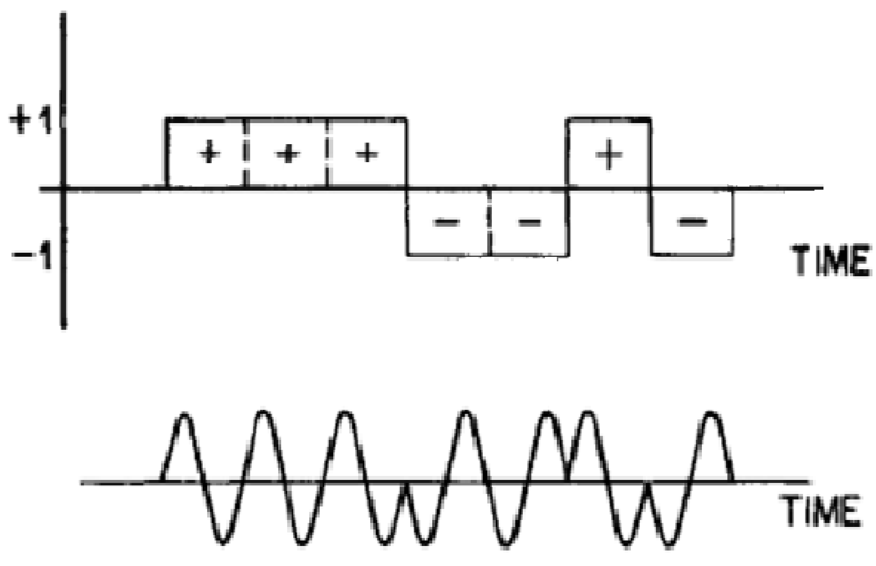


Figure 5 *phase modulated pulses*

Matched filter or correlation processor is used at the receiver side to obtain compressed pulse. The compressed pulse width at half amplitude point is usually equal to the width of the subpulse. Hence the range resolution is directly proportional to the time duration of one element of the code. The compression ratio is equivalent to the number of elements in the code, i.e the number of subpulses in the waveform [7].

1.4.2 Frequency Modulated Pulse

An ACF of unmodulated constant frequency pulse gives a triangular pulse that has poor Range Resolution because of its narrow bandwidth. Frequency modulation is an alternative way through which the spectrum of the transmitted pulse can be widened. Towards this approach LFM is a popular technique [8].

An LFM signal is a type of signal in which the transmitted signal frequency is varied over a pulse duration of T_p. This variation of the frequency from low to high or vice versa is known as “chirping”. Varying the frequency from low to high is called “up-chirp”. Similarly, varying the frequency from high to low is called “down-chirp”.

Complex envelope of LFM pulse is usually represented by

$$u(t) = \frac{1}{\sqrt{T_p}} rect(^{t}/_{T_p}) \exp(j\pi k t^2)$$

And the instantaneous phase of the LFM signal can be expressed as

$$\varphi(t) = 2\pi(f_0 t + \frac{1}{2} k t^2)$$

Where, f_0 denotes the carrier frequency and k denotes the frequency sweep rate related with the pulse duration T_p and BW B as $k = \pm \frac{B}{T_p}$

Here , Positive sign represents “up chirp” and negative sign represents “down chirp”

And the instantaneous frequency $f(t) = \frac{d\varphi(t)}{dt} = f_0 + kt$, here we have seen that instantaneous frequency of the pulse varies with time so it is known as linear frequency modulation [9] [10] [11].

The Spectrum of single LFM pulse and its matched filter response (ACF) are shown in the figure below

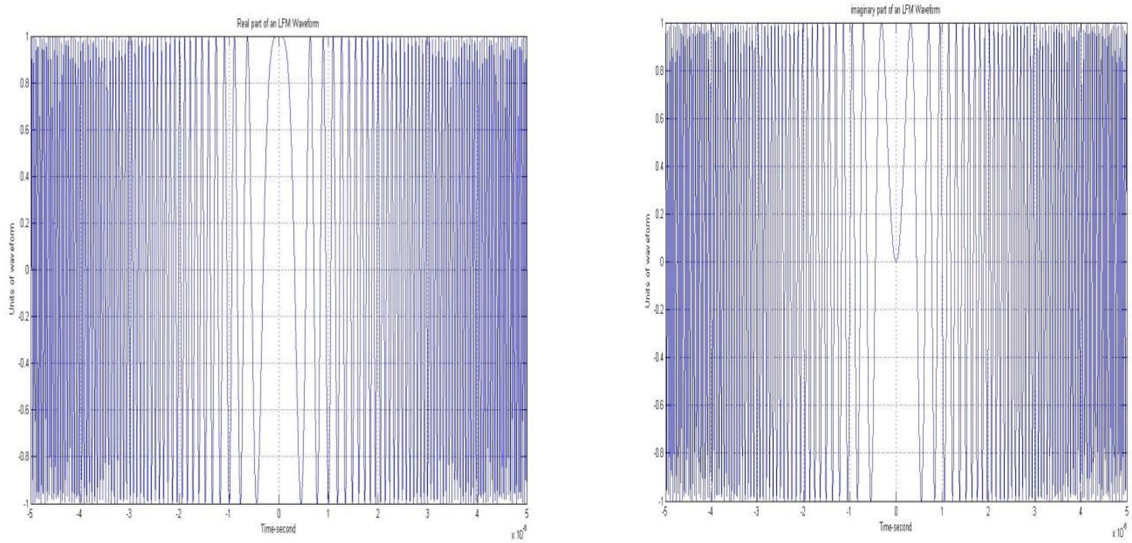


Figure 6 ***Real and Imaginary part of Single LFM pulse***

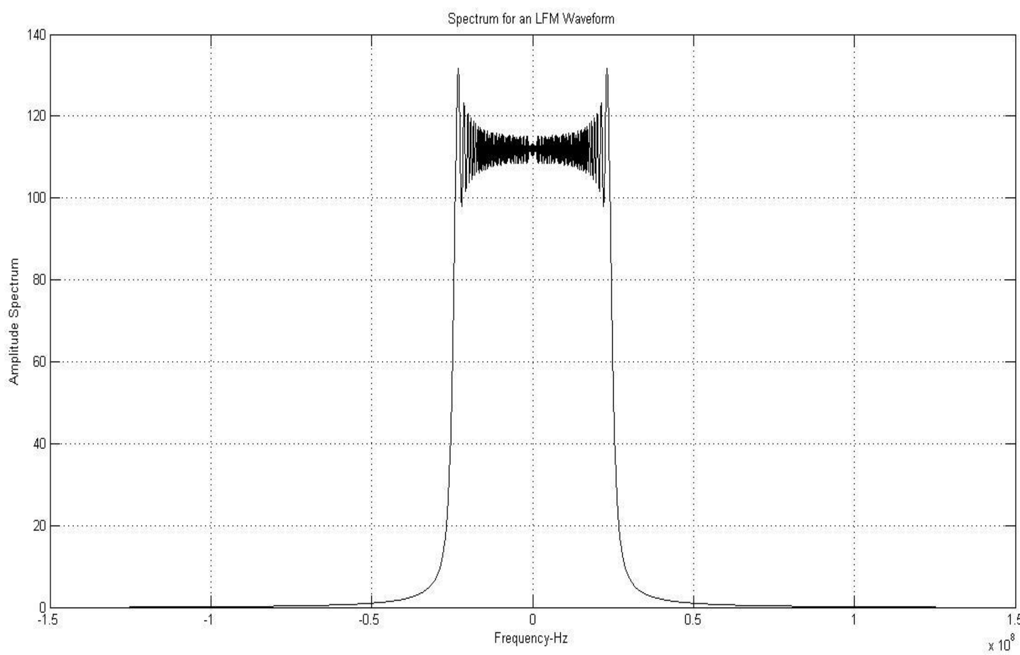


Figure 7 ***Spectrum of LFM pulse***

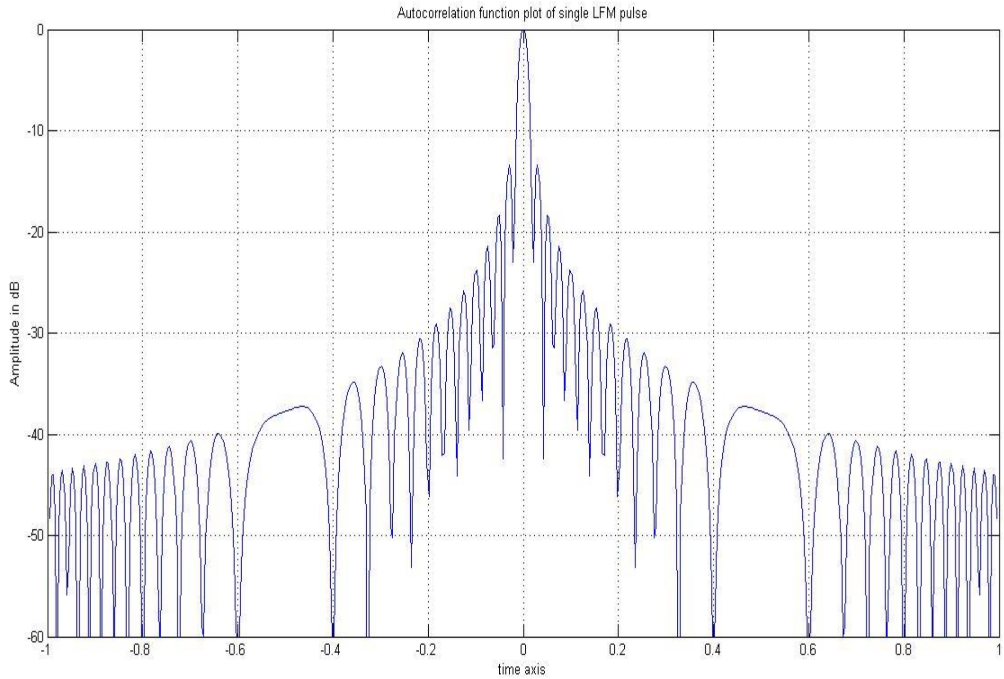


Figure 8 ***Matched Filter Response of LFM pulse in dB***

1.4.3 Drawbacks of LFM and its possible Remedies

LFM techniques increase the bandwidth of the signal thereby improving the range resolution by a factor equals to the time BW product. ACF generally having high side lobes (-13 dB below the main lobe peak) is not acceptable in certain radar applications where the number of targets are more than one that gives rise to echoes of different amplitudes. Some major techniques like time domain weighting, frequency domain weighting and NLFM are used to get lower side lobes level .The amplitude modulation of the transmitted signal is equivalent to time domain weighting that gives rise to low transmitted power thereby lowering the SNR. Frequency domain weighting broadens main lobe. NLFM overcomes the above two problems and there is no mismatch loss since the receiver is matched with the shape of the signal [4] [12] [13].

1.4.4 Nonlinear frequency Modulated (NLFM) pulse

Despite having several advantages the nonlinear-FM waveform has little acceptance. The waveform is designed in such a way that it provides the desired amplitude spectrum hence no time or frequency weighting is require in this NLFM waveform for range sidelobe suppression. The compatibility of Matched-filter reception and low side lobes in the design results in lowering the loss in SNR due to weighting by the general mismatching techniques. The reduction in frequency side lobes by time weighting a symmetrical FM modulation gives rise to near-ideal ambiguity function [14].

The limitations of the NLFM waveform are listed as:

(1) higher system complexity,

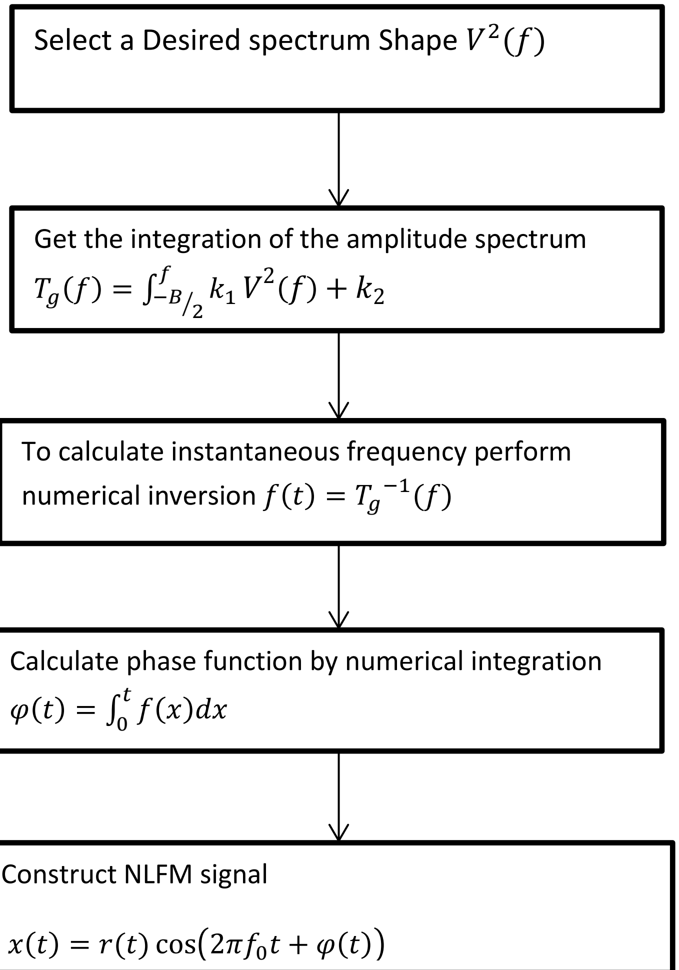
(2) limited development of NLFM generation equipments, and

(3) It requires a separate FM modulation design for each amplitude spectrum to get the desired sidelobe level.

The nonlinear waveform finds its importance in tracking system where range and Doppler are quite known because of the sharpness of the AF.

In general there are two methods through which a NLFM could be designed Stationary phase method and explicit function cluster method, stationary phase principle states that at any frequency the major improvement to the spectrum is made by that portion of the signal having instantaneous frequency [15].

- **Designing Steps of NLFM** [16]**,** [17]



The different NLFM waveforms studied with the help of stationary phase principle are

- **Raised cosine based waveform**

$$V^2(f,n) = k + (1-k)cos^n\left(\frac{\pi f}{B}\right)$$

This function is approximate to :

a. Hamming weighting function with k=0.54 &n=1
b. Hanning cosine-squared weighting function with k=0 &n=2
c. Cosine pedestal weighting function with k=0.24 & n=2

- **Cosine Spectrum Shape**

$$V^2(f,n) = cos^n\left(\frac{\pi f}{B_n}\right)$$

- **Tangent based Waveform**

  This is another spectrum shaping class based on a tangent function. Frequency modulation function for this class is

$$f(t) = \frac{B\,\tan\left(\frac{2\beta t}{T}\right)}{2tan(\beta)} \qquad -T/2 \le t \le T/2$$

  Where T is pulse duration ,B=Bandwidth and $\beta = \tan^{-1}(\alpha)$ $\alpha \ge 0$ is a time sidelobe level control factor.

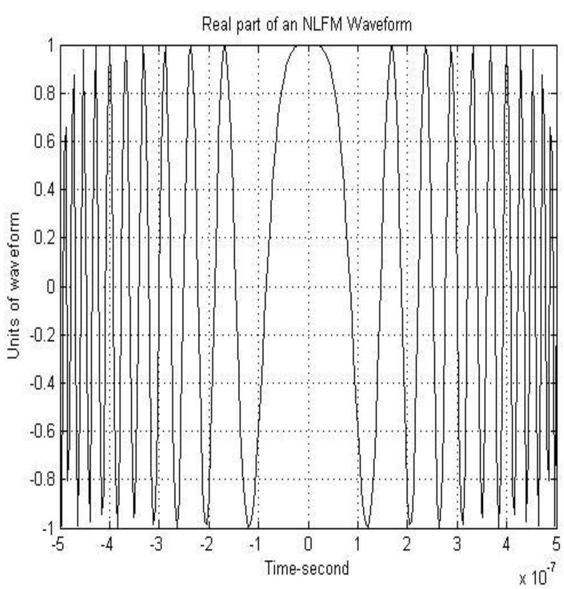
- **Hybrid Nonlinear Frequency Modulation**
- This modulation technique emerges from the research work done by M. Luszczyk, in which he suggested combining LFM and NLFM as

$$f(t) = \frac{t}{T}\left(B_l + B_c \frac{1}{\sqrt{(1--4t^2/T^2)}}\right) \quad for\ T/2 \le t \le T/2$$

  Where B_l is frequency sweep associated with LFM part and B_c is the frequency sweep associated with LFM (due to second term) when t=0 [17].

- ## ACF plot of NLFM Pulse

Figure 1.9 to 1.11 plots obtaining by applying hybrid NLFM to the single pulse



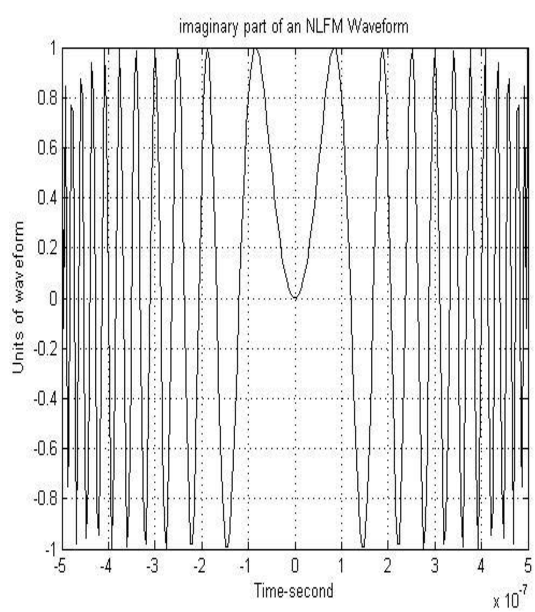


Figure 9 *Real and Imaginary part of Single NLFM pulse*

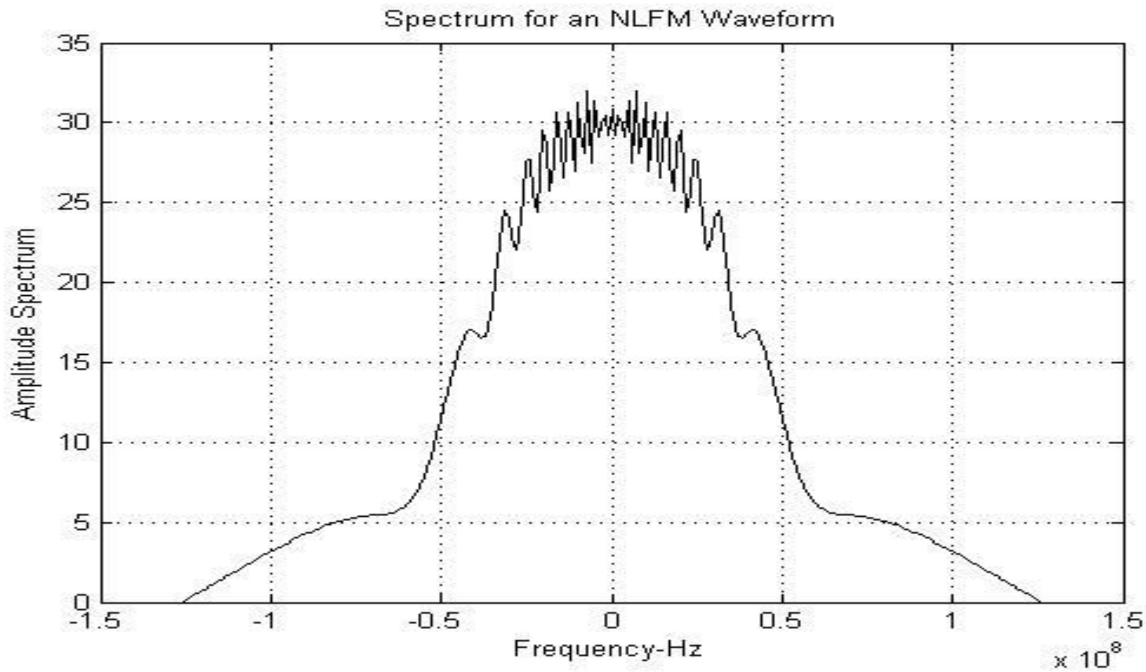


Figure 10 ***Spectrum of NLFM pulse***

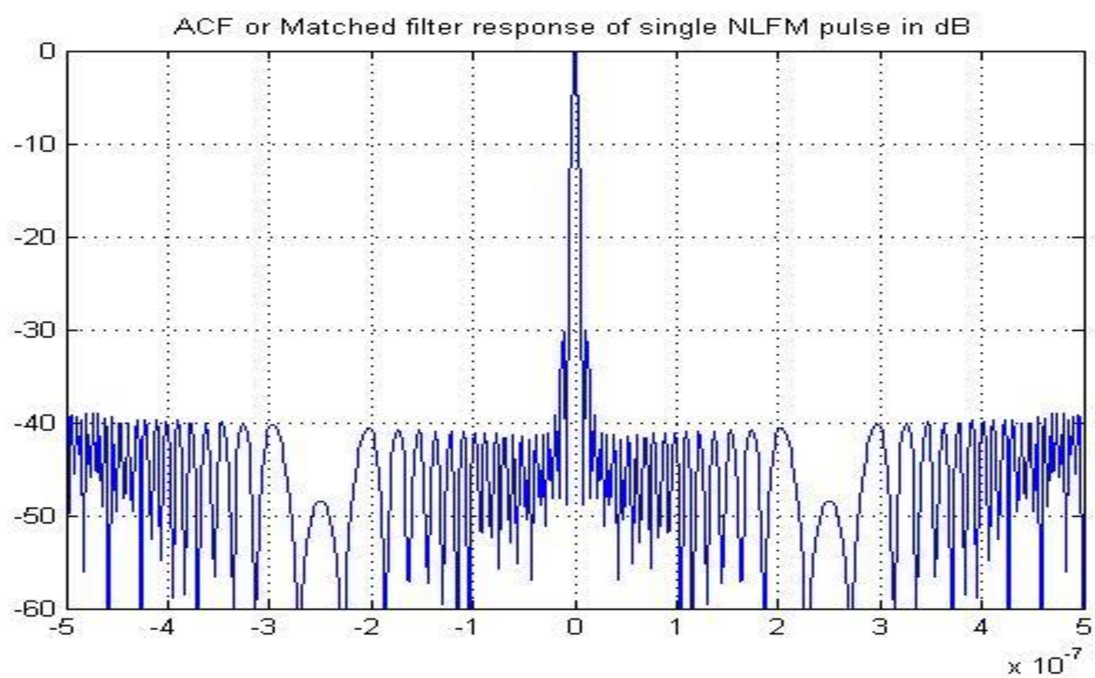


Figure 11 ***Matched Filter Response of NLFM pulse in dB***

1.5 Conclusion

Pulse compression Techniques permit us to use a low transmitter power and further achieve the better range resolution. The costs associated with pulse compression include complexity of transmitter and receiver circuits. Despite having few disadvantages the pulse compression technique is widely used of which the LFM waveform finds a major role in radar pulse compression. LFM techniques increases the bandwidth as a result range resolution of the signal is improved by a factor same as the time bandwidth product. But problem associated with LFM is that the matched filter response of such waveform suffers from high side lobe level (approx. -13 dB below main lobe). Three major techniques such as time domain weighting, frequency domain weighting and NLFM are used to overcome the above problems. The amplitude modulation of the transmitted signal is equivalent to time domain weighting that gives rise to low transmitted power thereby lowering the SNR. Frequency domain weighting broadens main lobe. NLFM overcomes the above two problems and there is no mismatch loss since the receiver is matched with the signal shape.

CHAPTER 2: Some weighting Techniques to Optimize Sidelobe level in Radar pulse Compression

2.1 Introduction of Different Types of window function

A window function is a mathematical function that is zero-valued exterior to some selected interval, such as a rectangular window which is constant within an interval and zero outside the interval. A window function multiplied with any other function gives zero value outside the defined interval. Although weighing can be used both at the transmitter and receiver side but it is generally preferred at the receiver because weighing on transmitter side gives rise to a power loss.

Applications of window functions find many applications including filter design, spectral analysis, and beam forming. Although rectangle, triangle, and other functions can be used, non-negative smooth "bell-shaped" window functions are used in many typical applications

Some important windows functions which are mostly used in Signal processing are:

- **Rectangular window**

  The Rectangular window sequence is given by

$$w_R(n) = \begin{cases} 1, & for\ -(N-1)/2 \le n \le (N-1)/2 \\ 0\ , & othewise \end{cases}$$

- **Rectangular Window or Bartlett window**

$$\boldsymbol{w_T(n)} = \begin{cases} 1 - \dfrac{2|n|}{N-1}, & for\ -(N-1)/2 \le n \le (N-1)/2 \\ 0 \quad , & othewise \end{cases}$$

- **Hamming window**

  The mathematical equation for Hamming window is

$$w(n) = \begin{cases} 0.54 + 0.46\cos(2\pi n/(N-1))\ , & -(N-1)/2 \le n \le (N-1)/2 \\ 0 & , otherwise \end{cases}$$

- **Hanning window**

$$w_H(n) = \begin{cases} 0.5 + 0.5\cos(2\pi n/(N-1))\ , & -(N-1)/2 \le n \le (N-1)/2 \\ 0 & , otherwise \end{cases}$$

- **Blackman window**

  The Blackman window sequence is given by

$$w_B(n) = \begin{cases} 0.42 + 0.5\cos\left(\frac{2\pi n}{(N-1)}\right) + 0.08\cos\left(\frac{2\pi n}{(N-1)}\right), & -(N-1)/2 \le n \le (N-1)/2 \\ 0 & , otherwise \end{cases}$$

Figure shows the response of different windows

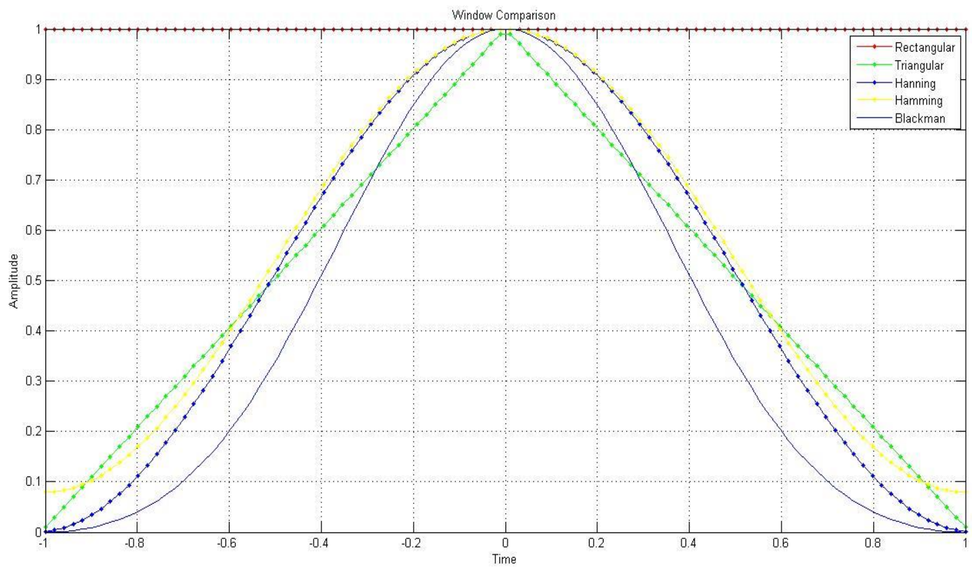


Figure 12 *Time domain Response of different window function*

Table 1.1 Gives some window parameter (peak amplitude of sidelobe (dB),Main lobe width, minimum stop band attenuation(dB)) for above windows

Table 1 *Summary of window parameter*

Window	**Peak Amplitude of side lobe(dB)**	**Main lobe width**	**Minimum stopband attenuation(dB)**
Rectangular	-13	$4\pi/N$	-21
Triangular	-25	$8\pi/N$	-25
Hanning	-31	$8\pi/N$	-44
Hamming	-41	$8\pi/N$	-53
Blackman	-57	$12\pi/N$	-74

2.2 Time and Frequency Domain weighting

Different types of windows can be employed as the weighing function to reduce side lobes in ACF of LFM pulse at the receiver. Frequency weighting is define as the process of shaping the compressed-pulse waveform by adjusting the amplitude of the frequency spectrum similarly **time weighting** is define as the process of shaping the Doppler response by controlling the shape of the envelope of the waveform. The primary aim of weighting in either domain is to reduce side lobes in the other domain. Side lobes can severely limit resolution when the relative magnitudes of received signals are large [18].

- **Frequency Domain weighting of LFM Waveforms**

The approach is to form a modified filter frequency response

$$H'(f) = w(f)X^*(f)$$

where $w(f)$ represents a window function (Hanning, Hamming, Kaiser, *etc.*), instead of using the matched filter choice $H(f) = X^*(f)$

Matched filtering can be done in the frequency domain, by combining the matched filter operation with the windowing so that required computation is reduced. Fig 2.2 shows the block diagram for frequency domain windowing for LFM sidelobe suppression. In this diagram it is shown how the window is applied to the filter frequency response before the true filtering step.

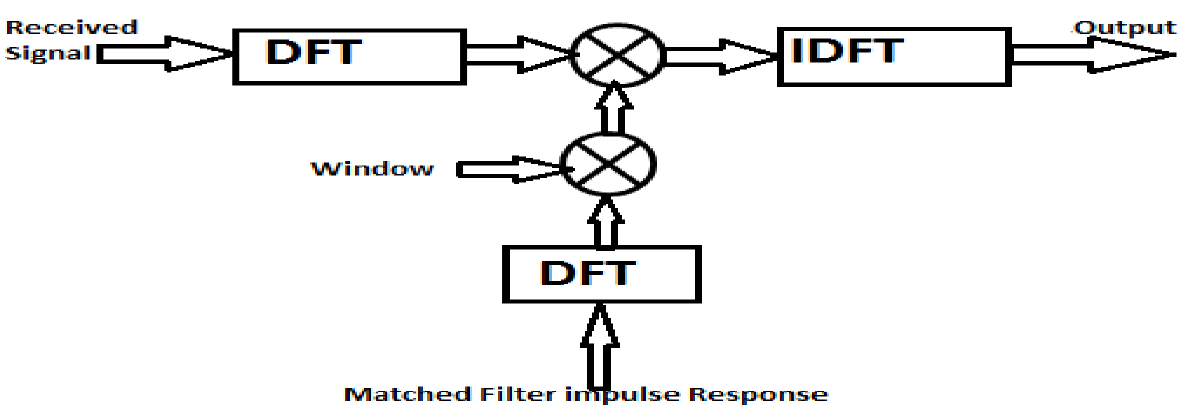


Figure 13 *Flow diagram for LFM sidelobe suppression by frequency*

- **Time Domain weighting of LFM Waveforms**

In Time domain windowing, firstly we find the conjugate of LFM pulse which itself is of matched filter impulse response (Since the matched filter impulse response is a delayed mirror image of the conjugate of the signal). But the delayed mirror image of the signal is same signal in case of even symmetric signal (like LFM signal). Now the impulse response of matched filter is multiplied with window function (Hamming, Hanning, Kaiser etc), again the output after multiplication is correlated with the LFM signal.

When we plot the above output in dB we can found that there is drastic reduction in sidelobe level (around 35dB).

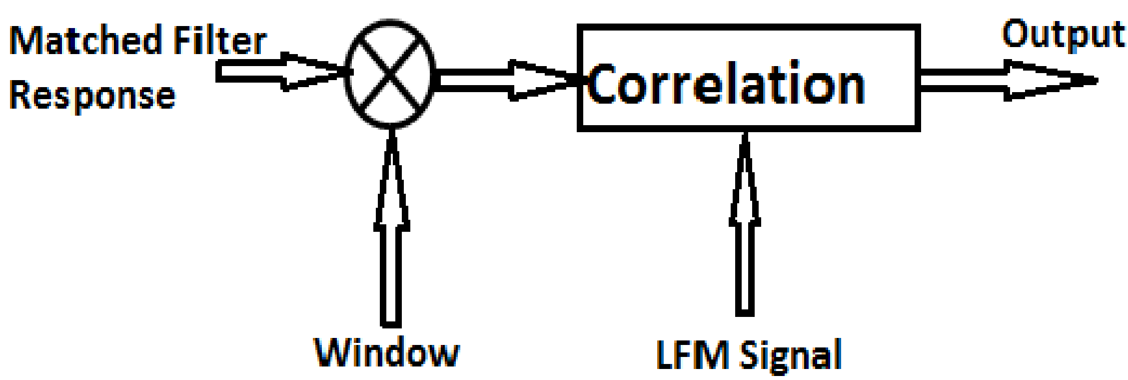


Figure 14 *Flow diagram for LFM side lobe suppression by Time domain windowing*

2.3 Drawback of Windowing

Time domain weighting is equivalent to the amplitude modulating the transmitted signal. Nevertheless such realization leads to reduction in transmitted power, and thus an SNR loss occurs. Well knows weighting windows like Hanning and Hamming are used. However the use of weighting windows may cause main lobe broadening, as a result of which directivity decreases [12].

2.4 Simulation Result

Fig2.4 shows the ACF of LFM pulse and output due to application of different type of window like hamming, hanning. This shows there is much reduction in side lobe level. Fig2.5 shows output by the application of convolutional window.

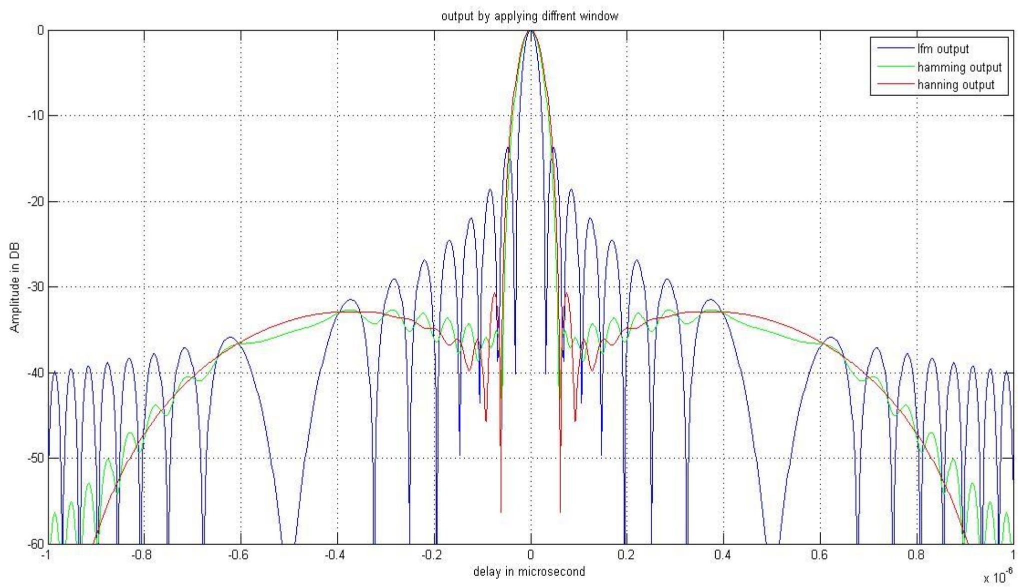


Figure 15 *LFM output (blue), LFM output after application of Hamming window (Green), LFM output after application of Hanning window (Red)*

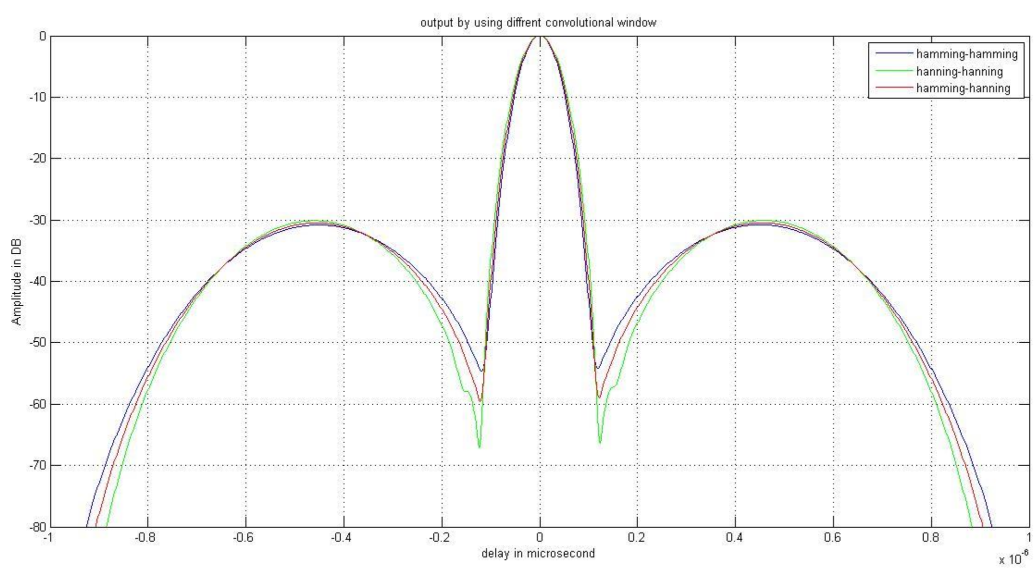


Figure 16 *Matched filter output after application of some convolutional window*

2.5 Conclusion

To reduce side lobes levels in LFM pulse different types of windows can be used as the weighting function for the matched filter receiver. The main objective of weighting in either domain is to reduce side lobes in the other domain. Frequency weighting is used for shaping the compressed-pulse waveform by adjustment of the amplitude of the frequency spectrum whereas time weighting is used for shaping the Doppler response by control of the waveform envelope shape.

Time domain weighting is equivalent to the amplitude modulating the transmitted signal. Nevertheless such realization can lead to reduction in transmitted power, and thus an SNR loss can occur and the use of the weighting windows may cause the main lobe broadening, as a result of which directivity decreases.

CHAPTER 3: Stepped Frequency Train of LFM and NLFM pulses

To enhance range resolution of the transmitted radar pulse, it is necessary to widening its spectrum. One step ahead towards this is to use a coherent train of stepped frequency pulses. It is an efficient method to achieve overall BW while maintaining small instantaneous BW. This approach use a train of pulses with pulse repetition timeT_r, with a frequency step Δf between successive pulses. A interval between the pulses can be used by the radar elements to prepare for the narrow frequency step of next pulse. A large Δf between pulses gives rise to large total BW [19].

But one difficulty with this approach is when the product of frequency step Δf and pulse duration T becomes greater than one ($T\Delta f > 1$) the ACF of the stepped frequency train of pulses suffers from undesired peaks which are called as "Grating lobes". It has been noticed that using LFM pulses instead of fixed frequency pulses in the train of pulses, with their mid frequencies stepped by Δf has an effective way to reduce those grating lobes. ACF of a single LFM pulse has side lobes and nulls, while a train of pulses causes grating lobes due to the frequency steps.

From AF expression of the pulse train, a relationship between $T, B, and\ \Delta f$ can be found in such away that the nulls of LFM –ACF exactly coincide with the position of grating lobes, hence nullifying the grating lobes [20].

3.1 Ambiguity Function for Stepped frequency LFM Pulse Train

The complex envelope of a single LFM pulse having unit energy is

$$u_1(t) = \frac{1}{\sqrt{T_p}} rect\left(\frac{t}{T_p}\right) \exp(j\pi k t^2) \tag{3.1.1}$$

Where, frequency slope$(k) = \pm\frac{B_1}{T_p}$, Here '+' sign gives positive frequency slope and '-'sign stand for negative frequency slope. Let us take a frequency slope$(k > 0)$ but the result is also apply for negative slope

The AF of above single LFM pulse is

$$|X_1(\tau, v)| = \left|\left(1 - \frac{|\tau|}{T_p}\right) sinc\left[T_p(v + k\tau)\left(1 - \frac{|\tau|}{T_p}\right)\right]\right| \quad \text{where,} \quad |\tau| \le T_p \tag{3.1.2}$$

The uniform train of such LFM pulses separated by $T_r > 2T_p$ is described as

$$u_N(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} u_1(t - n\,T_r) \qquad (3.1.3)$$

where T_r is pulse repetition interval ,

for delay τ shorter than the pulse interval T_p , the AF of a pulse train is related with the AF of single pulse as

$$|X_N(\tau, v)| = |X_1(\tau, v)| \left|\frac{\sin(N\pi v T_r)}{N\sin(\pi v T_r)}\right| \qquad (3.1.4)$$

Now add LFM to the pulse train $u_N(t)$ by using an another slope k_s as

$$u_S(t) = u_N(t)\exp(j\pi k_s t^2) = \frac{1}{\sqrt{N}} \exp(j\pi k_s t^2) \sum_{n=0}^{N-1} u_1(t - n\,T_r) \qquad (3.1.5)$$

Where $k_s = \pm\frac{\Delta f}{T_r}$, $\Delta f > 0, k_s > 0$ (3.1.6)

Here '+' sign gives positive frequency step and '-'sign stand for negative frequency step. Let us take a frequency slope$(k > 0)$ but the result is also applied for negative slope as well.

From 4th property of AF addition of LFM to the signal modifies its AF as

$$|X_S(\tau, v)| = |X_N(\tau, v + k_s\tau)| \qquad (3.1.7)$$

From (3.1.4) and (3.1.6) we get

$$|X_S(\tau, v)| = |X_1(\tau, v + k_s\tau)| \left|\frac{\sin(N\pi(v+k_s\tau)T_r)}{N\sin(\pi(v+k_s\tau)T_r)}\right| \qquad (3.1.8)$$

From (3.1.2) and (3.1.7) the AF of combined signal is

$$|X_S(\tau, v)| = \left|\left(1 - \frac{|\tau|}{T_p}\right) sinc\left[T_p(v + (k + k_s)\tau)\left(1 - \frac{|\tau|}{T_p}\right)\right]\right| \times \left|\frac{\sin(N\pi(v+k_s\tau)T_r)}{N\sin(\pi(v+k_s\tau)T_r)}\right| \qquad (3.1.9)$$

In above equation the overall BW (due to LFM slope k_s which is add to original slope (k)

is$B = |k + k_s|T_p$ (3.1.10)

So from (3.1.6),(3.1.9) and (3.1.10) the AF expression can be written as

$$|X_S(\tau,v)| = \left|\left(1-\frac{|\tau|}{T_p}\right) sinc\left[T_p\left(v+B\frac{\tau}{T_p}\right)\left(1-\frac{|\tau|}{T_p}\right)\right]\right| \times \left|\frac{\sin\left(N\pi\left(v+\Delta f\frac{\tau}{T_r}\right)T_r\right)}{N\sin\left(\pi\left(v+\Delta f\frac{\tau}{T_r}\right)T_r\right)}\right| \quad (3.1.11)$$

Where $|\tau| \le T_p$

3.2 Condition to nullify grating lobes at zero Doppler cut

Zero Doppler cut of Ambiguity function is itself is the magnitude of the ACF $R(\tau)$ [21, 20]

Set v=0 in (3.1.11) gives

$$|R(\tau)| = \left|\left(1-\frac{|\tau|}{T_p}\right) sinc\left[B\tau\left(1-\frac{|\tau|}{T_p}\right)\right]\right| \times \left|\frac{\sin(N\pi\tau\Delta f)}{N\sin(\pi\tau\Delta f)}\right| \quad \text{Where} \quad |\tau| \le T_p \quad (3.2.1)$$

In (3.2.1) first term $|R_1(\tau)|$ denotes AF of a single LFM pulse

$$|R_1(\tau)| = \left|\left(1-\frac{|\tau|}{T_p}\right) sinc\left[B\tau\left(1-\frac{|\tau|}{T_p}\right)\right]\right| \quad (3.2.2)$$

And 2nd term $|R_2(\tau)|$ shows grating lobes

$$|R_2(\tau)| = \left|\frac{\sin(N\pi\tau\Delta f)}{N\sin(\pi\tau\Delta f)}\right| = \left|\frac{\sin(N\pi\tau\Delta f)}{N\pi\tau\Delta f\frac{\sin(\pi\tau\Delta f)}{\pi\tau\Delta f}}\right| = \left|\frac{sinc(N\tau\Delta f)}{sinc(\tau\Delta f)}\right| \quad (3.2.3)$$

In (3.2.3) we have seen that $|R_2(\tau)|$ is the ratio of two $sinc$ function, so $|R_2(\tau)|$ exhibits peaks or spikes only when $sinc(\tau\Delta f)$ exhibits null and $sinc(\tau\Delta f)$ has value zero

When $\pi\tau\Delta f = n\pi$

i.e $n = \tau\Delta f$

Where $n = 0, \pm1, \pm2, \pm3, \dots\dots\dots\dots \lfloor T_p\Delta f \rfloor$

Nullification of the grating lobes requires to place the grating lobes of $|R_2(\tau)|$ at the position nulls of $|R_1(\tau)|$. The approach for this involves making the coincidence of first two grating lobes with the nulls of $|R_1(\tau)|$. In some cases getting this requirement nullifies all grating lobes. The ACF obtained with fixed frequency pulses (i.e B=0) is seen in Fig.3.1,Where the grating lobes are play a major role [22].

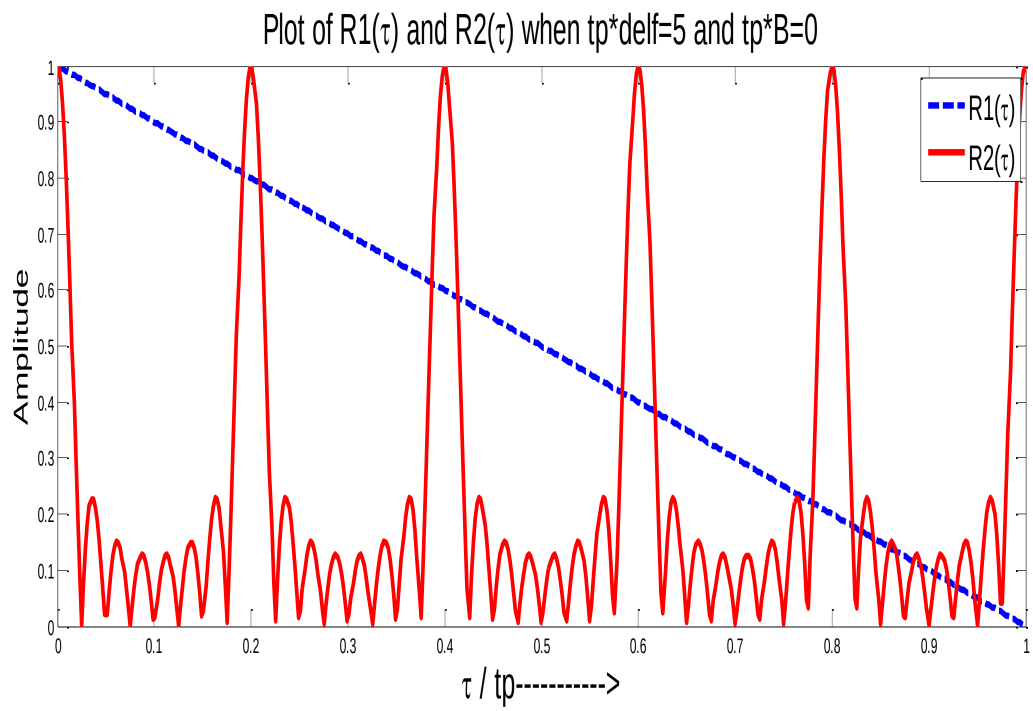


Figure 17 ***ACF plot where $R_1(\tau)$(Green) and $R_2(\tau)$ (Blue)***

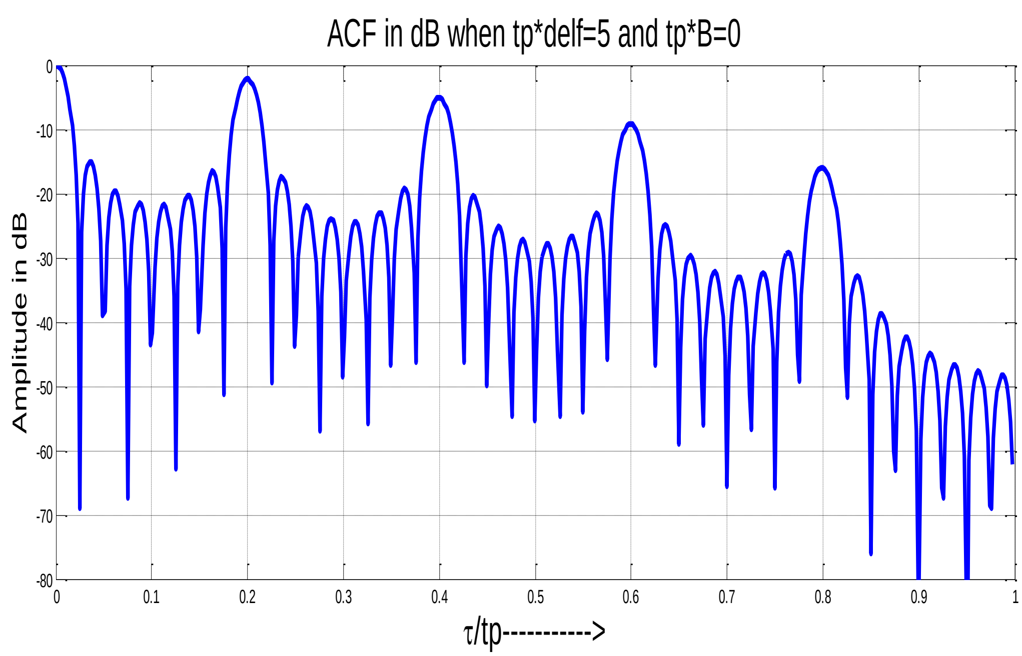


Figure 18 *ACF plot in dB*

3.3 Relation between $T\Delta f - TB$ for grating lobe Nullification

The general procedure to find the Relation between $T\Delta f$ *and* TB is divided in two phases.

- **When $T\Delta f \leq 3$**

  That is the case when there are not greater than three grating lobes. These cases are altered by the symmetry about $\tau = \frac{T_p}{2}$ of the sin argument in (3.2.2) i.e $\pi B\tau\left(1 - \frac{|\tau|}{T_p}\right)$

  The location of 1st grating lobes$\left(\tau_1 = \frac{1}{\Delta f}\right)$ in this expression can be calculated by comparing the sine argument with π.

  Therefor $\quad \pi B\tau\left(1 - \frac{|\tau|}{T_p}\right) = \pi$

  This implies $\frac{B}{\Delta f}\left(1 - \frac{1}{T_p\Delta f}\right) = 1$

  $$\text{Finally } T_pB = \frac{(T_p\Delta f)^2}{T_p\Delta f - 1} \tag{3.3.1}$$

  Now When $1 < T_p\Delta f \leq 2$ the position of 1st grating lobes is in the 2nd half of the pulse duration. i.e when $1 < T_p\Delta f \leq 2$

  $1 \geq \frac{1}{T_p\Delta f} \geq \frac{1}{2} \quad or \quad T_p \geq \frac{1}{\Delta f} \geq \frac{T_p}{2}$

Finally $T_p \geq \tau_1 \geq \frac{T_p}{2}$

This shows that placing a null on it will place a symmetric null in the first half when grating lobes are absent.it is impossible to match the 1st grating lobes with the first null.

For specific value of $T_p\Delta f = 2$

$T_pB = 4$ the first and 2nd null coincide with each other at $\frac{T_p}{2}$ hence the first grating lobes matches both the 1st and 2nd null and 2nd grating lobes matches with the 3rd null

When $2 < T_p\Delta f \leq 3$ i.e $\frac{T_p}{3} \leq \tau_1 \leq \frac{T_p}{2}$

So in this case grating lobe is located in the first half of the pulse duration, so placing a null on it using (3.3.1) will place a symmetric null on 2nd half .

The 2nd grating lobes matches with 2nd null only when $T_pB = \frac{(T_p\Delta f)^2}{2(T_p\Delta f - 2)}$ (3.3.2)

From (3.3.1) and (3.3.2) we have found that

$$T_p\Delta f = 3 \; and \; T_pB = 4.5$$

This is the only condition in which first two grating lobes matched with the first two nulls.

- **General search procedure for $\boldsymbol{T\Delta f} > 3$**

Let us consider $|R_1(\tau)|$ have its mth and nth nulls (where $n > m$) exactly at the qth and rth $(r > q)$ grating lobes namely $\tau = \frac{q}{\Delta \boldsymbol{f}}$ and $\tau = \frac{r}{\Delta \boldsymbol{f}}$

Using the specific alternatives of m, n, p and q $|R_1(\tau)|$ and $|R_2(\tau)|$ gives following relationship

$$\pi B \frac{q}{\Delta f}\left(1 - \frac{q}{T_p\Delta f}\right) = m\pi \qquad (3.3.3)$$

And $\pi B \frac{r}{\Delta f}\left(1 - \frac{r}{T_p\Delta f}\right) = n\pi$ (3.3.4)

From (3.3.3) and (3.3.4)

$$\frac{q}{m}\left(1 - \frac{q}{T_p\Delta f}\right) = \frac{r}{n}\left(1 - \frac{r}{T_p\Delta f}\right) \qquad (3.3.5)$$

Which implies $T_p\Delta f = \frac{mr^2 - nq^2}{mr - mq}$ (3.3.6)

To nullify first two grating lobes put $q = 1 \; and \; r = 2$ in (3.3.6) we get

$$T_p\Delta f = \frac{4m - n}{2m - n} \qquad (3.3.7)$$

Since $m \leq n$, $q < r$ and $\lfloor T_p\Delta f \rfloor$ shows number of grating lobes. To get valid result it must be positive and greater than r.

Now from (3.3.4) for a valid $T_p\Delta f$, the product of pulse width and BW is

$$T_pB = \frac{n}{r(T_p\Delta f - r)}\left(T_p\Delta f\right)^2 = \frac{\left(mr^2 - nq^2\right)^2}{qr(r-q)(mr-nq)} \qquad (3.3.8)$$

For $q = 1, r = 2 \qquad T_pB = \frac{(4m-n)^2}{2(2m-n)}$

The ratio $\frac{B}{\Delta f}$ is known as overlap ratio which is equal to

$$\frac{B}{\Delta f} = \frac{nT_p\Delta f}{r(T_p\Delta f - r)} = \frac{(mr^2 - nq^2)}{qr(r-q)} = \frac{4m-n}{2}\,|q = 1, r = 2 \qquad (3.3.9)$$

In order to get the increase BW number of pulses $(N) > \frac{B}{\Delta f}$

For selected values of m, n, q, r (resulting with given $T_p\Delta f$ and T_pB) The ACF can be written as

$$\left|R\left(\frac{\tau}{T_p}\right)\right| = \left|\left(1 - \left|\frac{\tau}{T_p}\right|\right) sinc\left[T_pB\frac{\tau}{T_p}\left(1 - \left|\frac{\tau}{T_p}\right|\right)\right]\right| \times \left|\frac{\sin\left(N\pi T_p\Delta f\frac{\tau}{T_p}\right)}{N\sin\left(\pi T_p\Delta f\frac{\tau}{T_p}\right)}\right| \text{ where } \left|\frac{\tau}{T_p}\right| \leq 1 \qquad (3.3.10)$$

From above equation we can see that the dimensionless parameters $T_p\Delta f$ and T_pB are function of only m, n, q and r

From above relation we can get a table (Table 3.1) contains $T_p\Delta f$, $T_p\Delta B$ and $\frac{B}{\Delta f}$ for small fraction of the integer value of m & n over which the search was performed [20] [23].

Table 2 ***Some valid cases to nullify grating lobes***

m	n	q	r	$T_p\Delta f$	T_pB	B/Δf
1	1	1	2	3	4.5	1.5
2	2	1	2	3	9	3
2	3	1	2	5	12.5	2.5
3	3	1	2	3	13.5	4.5
3	4	1	2	4	16	4

When we compare ACF of stepped frequency train pulses with ACF of identical LFM pulse train, in this case the entire BW $B + (N-1)\Delta f$ is put in each place. Figure 3.3 is the plot of ACF of such signal.

3.4 Result and Discussion

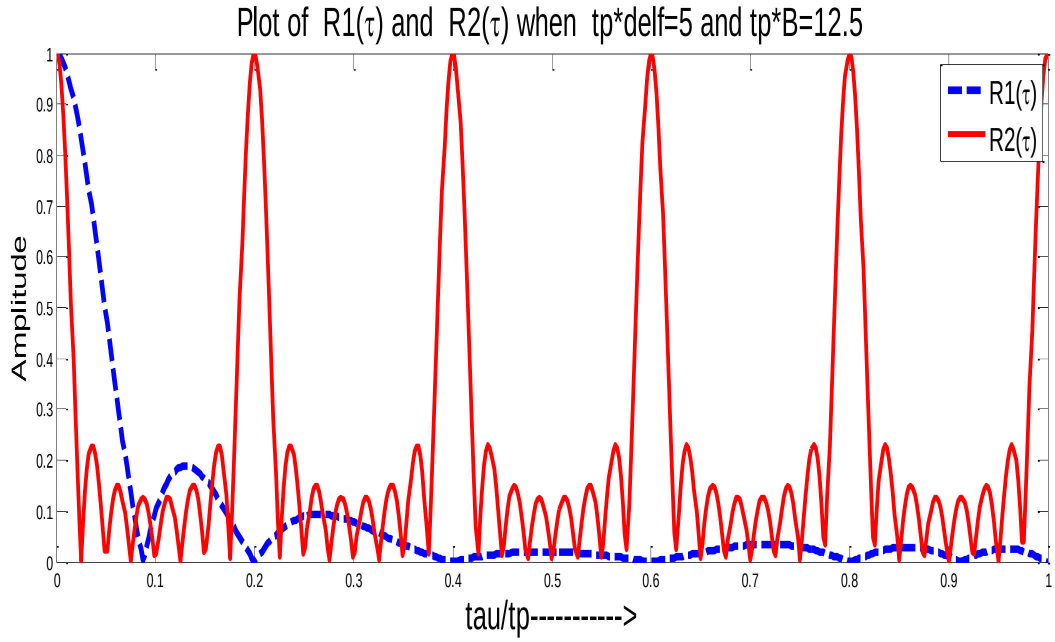


Figure 19 $|R_1(\tau)|$ *(Solid) and* $|R_2(\tau)|$ *(Dotted) when* $T_pB = 12.5$, $T_p\Delta f = 5$ *and* $N = 8$

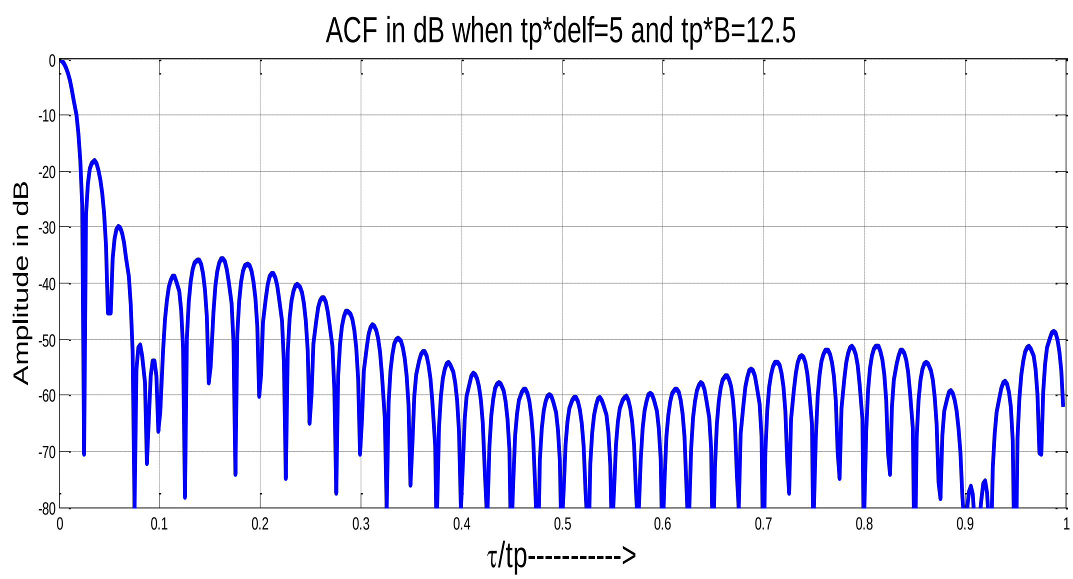


Figure 20 *Partial ACF in dB when* $T_pB = 12.5$, $T_p\Delta f = 5$ *and* $N = 8$

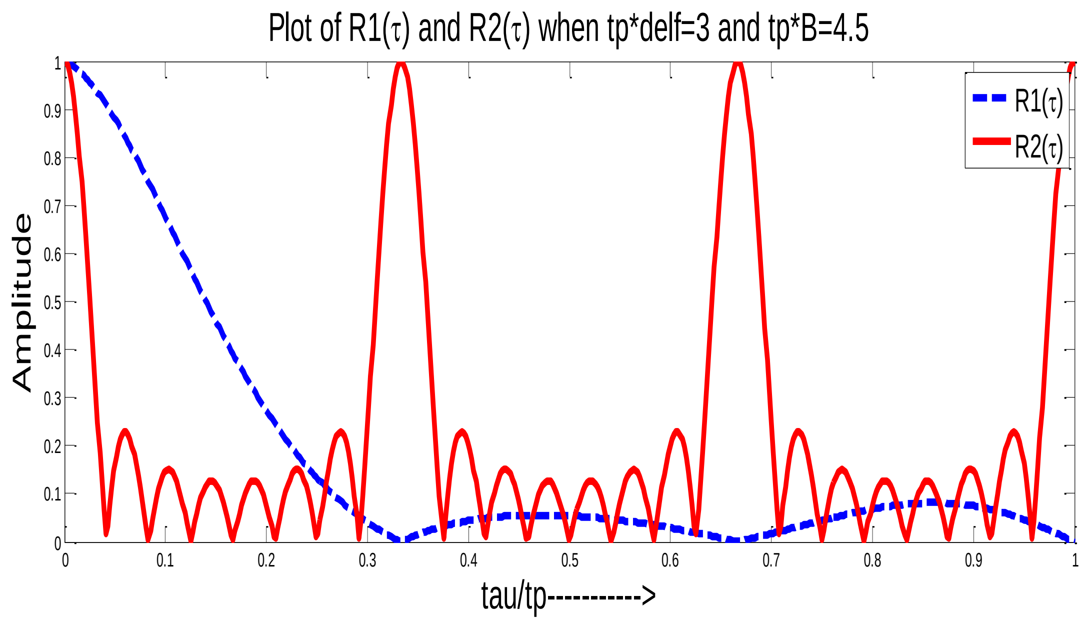


Figure 21 $|R_1(\tau)|$ *(Solid) and* $|R_2(\tau)|$ *(Dotted) when* $T_pB = 4.5$ $, T_p\Delta f = 3 and\ N = 8$

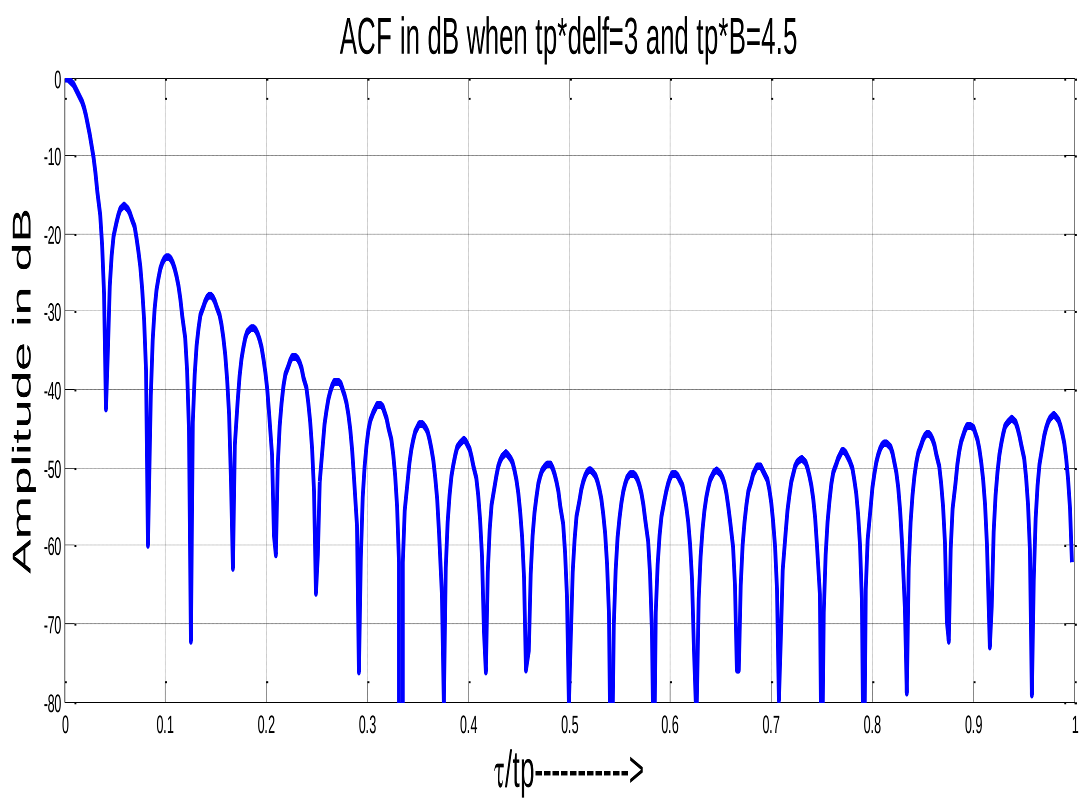


Figure 22 *Partial ACF in dB when* $T_pB = 4.5$ $, T_p\Delta f = 3 and\ N = 8$

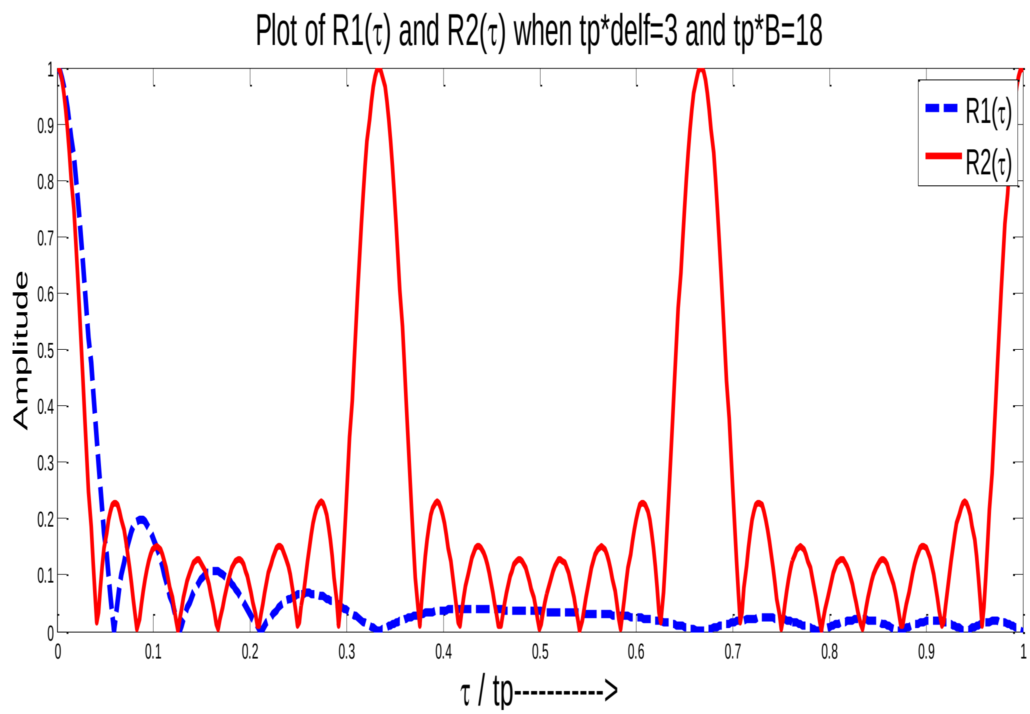


Figure 23 $|R_1(\tau)|$ *(Solid) and* $|R_2(\tau)|$ *(Dotted) when* $T_pB = 18$, $T_p\Delta f = 3$ *and* $N = 8$

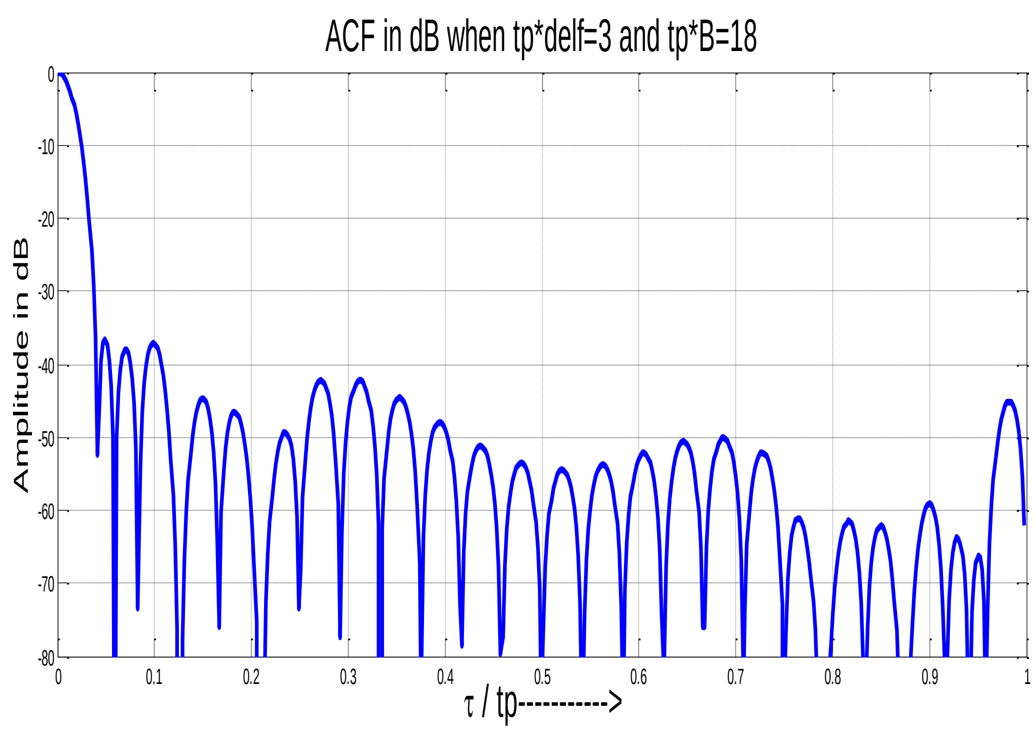


Figure 24 ***Partial ACF in dB*** $T_pB = 18$, $T_p\Delta f = 3$ ***and*** $N = 8$

3.5 Stepped Frequency train of NLFM pulses

Pulse Compression using NLFM is promising way to achieve fine range resolution, better SNR, low cost and good interference. The modulation function of NLFM has spectrum weighting function which gives the benefit that a pure matched filter gives very low sidelobes. Hence the SNR loss due to weighting or with the use of other mismatching techniques is eliminated.

NLFM has not been widely used in radar system because of difficulties associated with implementation of such waveform. But with the recent advancement of high speed DAC and large scale FPGA having high speed we can generate high precision and high performance NLFM signal [15].

In the previous chapter we have seen that the ACF of stepped frequency LFM pulse train has high side lobes near the main lobe .Within the selected construction of the stepped frequency pulse train with constant BW B, constant step Δf, constant time interval T_p , and small ratio $B/\Delta f$ suppression of few side lobe is still not possible. Nevertheless, if in the stepped frequency pulse train LFM pulse is replaced by NLFM pulse with fine characteristics than it is possible to overcome the side lobe near the vicinity of main lobe as well as grating lobe. In this section I describe a numerical approach that permit a design of phase $\varphi(t)$ of such NLFM waveform.

3.6 Autocorrelation function for Stepped frequency Train of NLFM Pulse

This technique of modulation is a result of research work done by M. Luszczyk, in which he suggested combining LFM and NLFM according to [17]

$$f(t) = \frac{t}{T}\left(B_l + B_c \frac{1}{\sqrt{(1--4t^2/T^2)}}\right) \quad for\ T/2 \le t \le T/2 \tag{4.1.1}$$

Where B_l is total frequency sweep of LFM part and B_c is the total frequency sweep of LFM (due to second term) when t=0.The technique is known as hybrid nonlinear frequency modulation.

When linear FM pulse is replaced by a non-linear FM pulse in stepped frequency train then we have found a drastic reduction in sidelobes as well as grating lobes. To begin with the analysis , the instantaneous frequency of the NLFM signal can be written as

$$f(t) = \frac{1}{2\pi}\frac{d\varphi(t)}{dt} \tag{4.1.2}$$

The above equation can be written as $\int d\varphi(t) = \int 2\pi\, f(t)dt$

From (4.1.1) and (4.1.2), finally instantaneous phase of such NLFM signal is obtained as

$$\varphi(t) = \pi k t^2 - \frac{B_c \pi T_p}{2} \times \sqrt{1 - \frac{4t^2}{{T_p}^2}} \tag{4.1.3}$$

Now, the complex envelope of a single NLFM pulse having unit energy is represented by

$$v_1(t) = \frac{1}{\sqrt{T_p}} rect\left(\frac{t}{T_p}\right) \exp\big(j\varphi(t)\big) \tag{4.1.4}$$

Again create a uniform pulse train of N such NLFM pulses separated by $T_r > 2T_p$

is represented as

$$v_N(t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N-1} v_1(t - nT_r) \tag{4.1.5}$$

The multiplication factor $1/\sqrt{N}$ is used to maintain unit energy.

We now add LFM to the entire train of pulses using an additional slope k_s and the resultant signal is

$$v_s(t) = v_N(t) exp(j\pi k_s t^2) \tag{4.1.6}$$

Where $k_s = \frac{\pm\Delta f}{T_r}$ $\quad \Delta f > 0\, , k_s > 0$

From Analysis of zero Doppler cut of AF function or matched filter response of stepped frequency train of LFM pulses (from (3.2.1),(3.2.2) and (3.2.3))The ACF of signal $v_s(t)$ or matched filter response of the stepped NLFM pulse train is represented as

$$|X(\tau)| = |X_1(\tau)| \times |X_2(\tau)| \tag{4.1.7}$$

In equation (4.1.7) the expression $|X(\tau)|$ contains product of two term first term $|X_1(\tau)|$ describe the Autocorrelation of single NLFM pulse given by

$$|X_1(\tau)| = |v(t) \otimes v^*(t)| \tag{4.1.8}$$

here $\otimes$ represents autocorrelation

and the 2nd term $|X_2(\tau)|$ describe the grating lobes $|X_2(\tau)|=\left|\frac{\sin(N\pi\tau\Delta f)}{N\sin(\pi\tau\Delta f)}\right|$

Unlike ACF of single LFM pulse ACF of single NLFM pulse has much lower side lobes. So the output of stepped frequency train of NLFM pulses have much reduction in there side lobes as well as grating lobes.

Here how much reduction in side lobes will take place that depends on value of bandwidth of LFM part, bandwidth of NLFM part, frequency steps and pulse duration [24].

3.7 Relation between parameters for Grating lobes and Side lobes Reduction

The NLFM pulse train that describe in section 4.1 is observed for the simulation study. The various values of the parameter are given in Table 4.1. Table 4.1 also describe the different peak side lobe level (PSL) for different values of T_pB_l , T_pB_c $T_p\Delta f$ and N.

Table 3 ***PSL (dB) with variation of parameters***

	T_pB_l	T_pB_c	$T_p\Delta f$	N	PSL (dB)
1	50	20	5	8	-70.5
2	40	20	5	8	-80.5
3	30	20	5	8	-90
4	40	30	5	8	-81.6
5	40	30	10	8	-85

3.8 Simulation Result

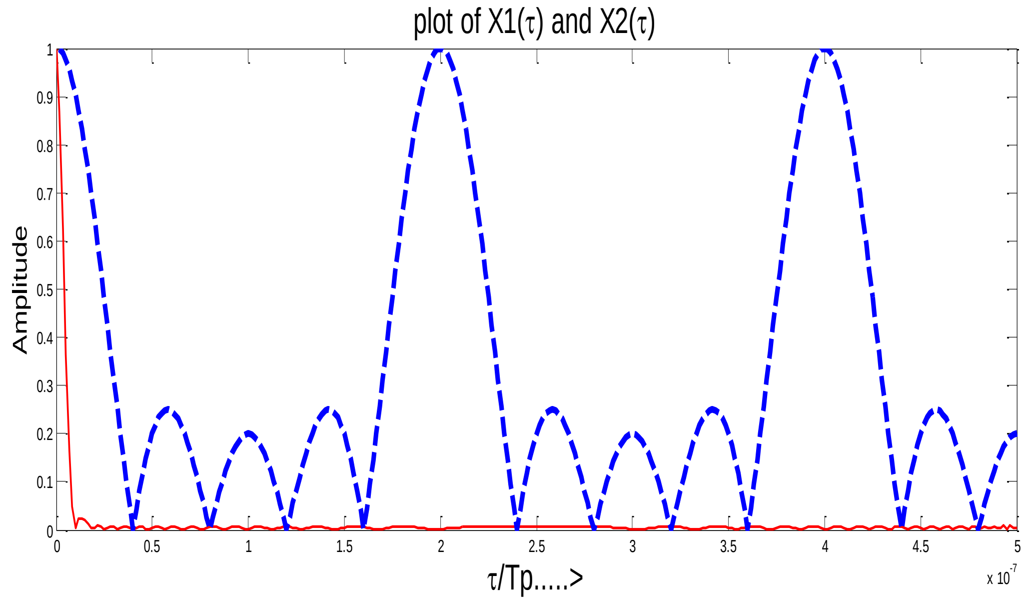


Figure 25 $|X_1(\tau)|$ *(Solid) and* $|X_2(\tau)|$ *(Dotted) when* $T_pB_l = 40, T_pB_c = 20, T_p\Delta f = 5$ *and* $N = 8$

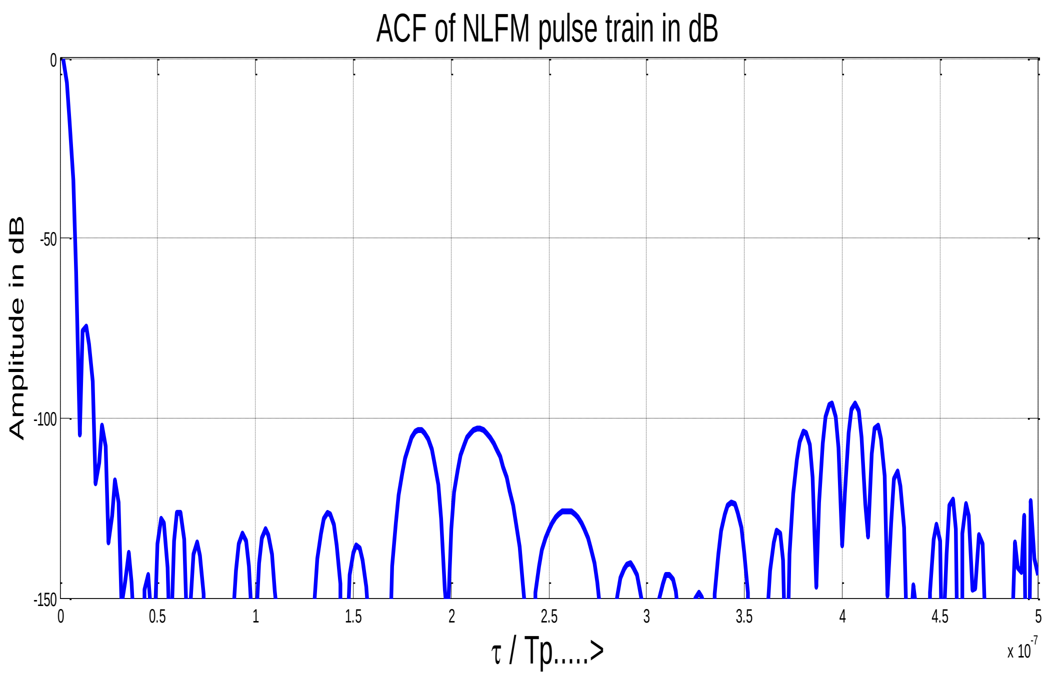


Figure 26 *Partial ACF in dB when* $T_pB_l = 40, T_pB_c = 20, T_p\Delta f = 5$ *and* $N = 8$

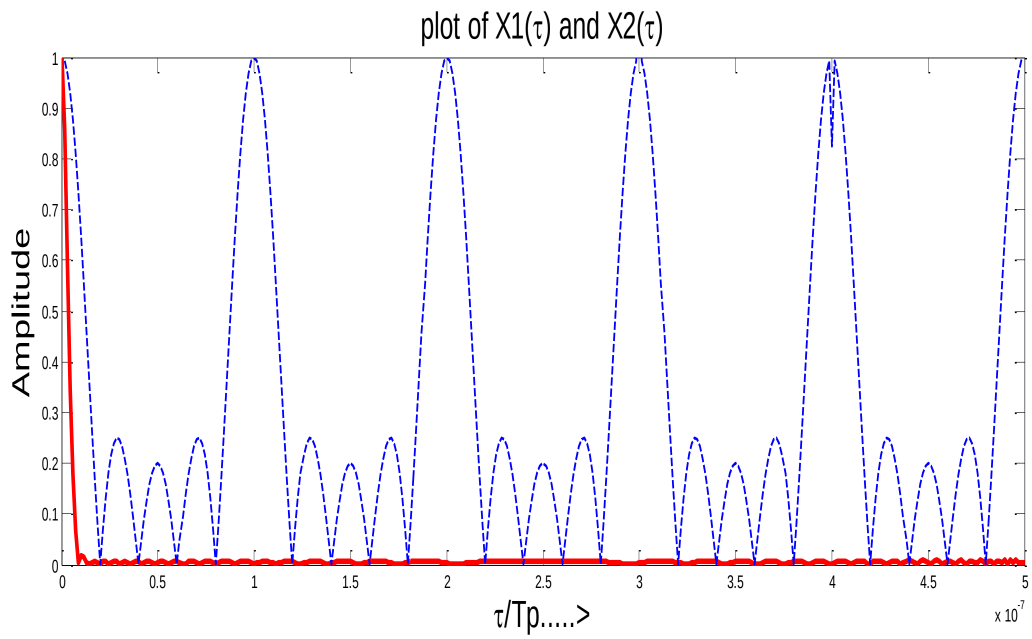


Figure 27: $|X_1(\tau)|$ *(Solid) and* $|X_2(\tau)|$ *(Dotted) when* $T_pB_l = 40$, $T_pB_c = 30$, $T_p\Delta f = 10$ *and* $N = 8$

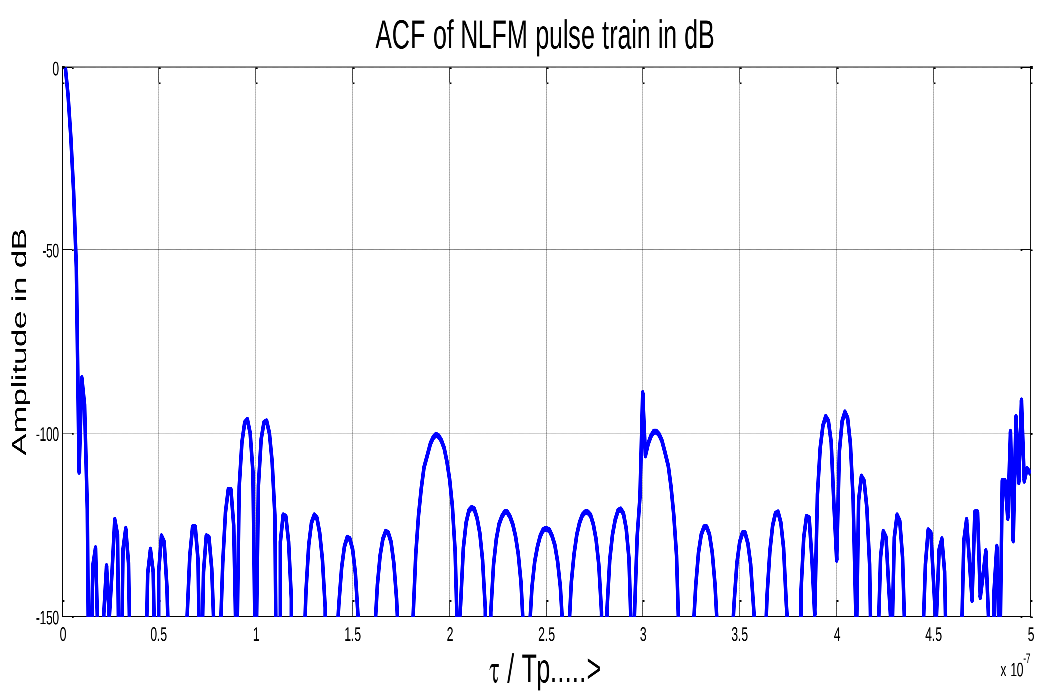


Figure 28 ***Partial ACF in dB when*** $T_pB_l = 40$, $T_pB_c = 30$, $T_p\Delta f = 10$ ***and*** $N = 8$

CHAPTER 4: Conclusion and Future Work

4.1 Conclusion

One of the important techniques used in modern radar application is frequency Stepping. It uses sequence of narrowband pulses to achieve overall bandwidth in order to get better range resolution. The main disadvantage of this technique is presence of high ambiguous peak known as "grating lobes". In this thesis i have also discussed the problem associated with high side lobes near the main lobe and have seen that if we use NLFM pulses in place of LFM pulses than it is possible to suppress these sidelobe and grating lobes. Here a numerical approach was developed to overcome the problem which arises due to the use of LFM in stepped frequency pulse train. And we have shown that when we use NLFM pulse in the stepped frequency pulse train than we have found a drastic reduction in sidelobes near the main lobe.

4.2 Future Work

In future I want to work on stretch processing in Radar, although the LFM & NLFM waveforms are very popular in radar pulse compression application because its high time-BW product gives better range Resolution. However high BW requirement of such waveform makes digital matched filtering difficult because it requires expensive, high-quality Analog to Digital converters (ADC).Stretch processing, also known as deramping, or dechirping, is an alternative to matched filtering. Stretch processing provides pulse compression by looking for the return within a predefined range interval of interest. Unlike matched filtering, stretch processing reduces the bandwidth requirement of subsequent processing [25].

Apart from this I am also interested to search for other NLFM techniques which provide better range resolution, high sidelobe reduction, less complexity and better performance.

Bibliography

[1] M. I. Skolnik, "Introduction to radar," *Radar Handbook,* p. 1990, 1962.

[2] N. Levanon and E. Mozeson, Radar signals, John Wiley \& Sons, 2004.

[3] A. K. Sahoo, G. Panda and P. M. Pradhan, "Efficient design of pulse compression codes using multiobjective genetic algorithm," in *Nature \& Biologically Inspired Computing, 2009. NaBIC 2009. World Congress on*, 2009.

[4] B. R. Mahafza, Radar systems analysis and design using MATLAB, CRC Press, 2000.

[5] C. Cook, Radar signals: An introduction to theory and application, Elsevier, 2012.

[6] E. Mozeson and N. Levanon, "MATLAB code for plotting ambiguity functions," *Aerospace and Electronic Systems, IEEE Transactions on,* vol. 38, no. 3, pp. 1064-1068, 2002.

[7] R. M. Davis, R. L. Fante and R. P. Perry, "Phase-coded waveforms for radar," *IEEE Transactions on Aerospace and Electronic Systems,* vol. 43, no. 1, p. 401, 2007.

[8] L. R. Varshney and D. Thomas, "Sidelobe reduction for matched filter range processing," in *Radar Conference, 2003. Proceedings of the 2003 IEEE*, 2003.

[9] D. K. Barton and others, Pulse compression, Artech House, 1975.

[10] M. N. Cohen, "An overview of high range resolution radar techniques," in *Telesystems Conference, 1991. Proceedings. Vol. 1., NTC'91., National*, 1991.

[11] U. K. Majumder, "Linear Frequency Modulation Pulse Compression Technique on Generic Signal Model".

[12] S. Salemian, M. Jamshihi and A. Rafiee, "Radar pulse compression techniques," in *Proceedings of the 4th WSEAS international conference on Applications of electrical engineering*, 2005.

[13] Y. K. Chan, M. Y. Chua and V. C. Koo, "Sidelobes reduction using simple two and tri-stages non linear frequency modulation (NLFM)," *Progress In Electromagnetics Research,* vol. 98, pp. 33-52, 2009.

[14] J. Johnston and A. Fairhead, "Waveform design and Doppler sensitivity analysis for nonlinear FM chirp pulses," in *IEE Proceedings F (Communications, Radar and Signal Processing)*, 1986.

[15] S. Boukeffa, Y. Jiang and T. Jiang, "Sidelobe reduction with nonlinear frequency modulated waveforms," in *Signal Processing and its Applications (CSPA), 2011 IEEE 7th International*

Colloquium on, 2011.

[16] A. W. Doerry, Generating nonlinear FM chirp waveforms for radar, United States. Department of Energy, 2006.

[17] I. C. Vizitiu, "SIDELOBES REDUCTION USING SYNTHESIS OF SOME NLFM LAWS.," *Progress In Electromagnetics Research B,* vol. 49, 2013.

[18] M. A. Richards, "Time and Frequency Domain Windowing OF LFM Pulses," 2006.

[19] I. Gladkova, "Analysis of stepped-frequency pulse train design," *Aerospace and Electronic Systems, IEEE Transactions on,* vol. 45, no. 4, pp. 1251-1261, 2009.

[20] N. Levanon and E. Mozeson, "Nullifying ACF grating lobes in stepped-frequency train of LFM pulses," *Aerospace and Electronic Systems, IEEE Transactions on,* vol. 39, no. 2, pp. 694-703, 2003.

[21] I. Gladkova and D. Chebanov, "Grating lobes suppression in stepped-frequency pulse train," *Aerospace and Electronic Systems, IEEE Transactions on,* vol. 44, no. 4, pp. 1265-1275, 2008.

[22] A. K. Sahoo and G. Panda, "A multiobjective optimization approach to determine the parameters of stepped frequency pulse train," *Aerospace science and technology,* vol. 24, no. 1, pp. 101-110, 2013.

[23] D. Chebanov, I. Gladkova and J. Weber, "Family of stepped-frequency LFM trains with low autocorrelation sidelobes," in *Proceedings of the Second IASTED International Conference on Antennas, Radar and Wave Propagation*, 2005.

[24] D. Thomas, C. Kamalanathan and S. Valarmathy, "SIDELOBE DEDUCTION BY PIECEWISE NON LINEAR FREQUENCY MODULATION WAVEFORM".

[25] M. Schikorr, "High range resolution with digital stretch processing," in *Radar Conference, 2008. RADAR'08. IEEE*, 2008.

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