

Determination of optimal distributed generation plant capacity in a micro-grid using fuzzy linear programming

A THESIS SUBMITTED IN PARTIAL FULFILLMENT

OF THE REQUIREMENT FOR THE DEGREE OF

**Master of Technology
In
Electrical Engineering**

SUBMITTED BY

PERVEZ AHMAD

212EE5394



Under the Supervision of

Prof. S. Ganguly

Department of Electrical Engineering

National Institute Of Technology Rourkela

June 2014



National Institute of Technology Rourkela

Certificate

This is to certify that the thesis entitled “**DETERMINATION OF THE OPTIMAL DISTRIBUTED GENERATION PLANT CAPACITY IN A MICRO-GRID USING FUZZY LINEAR PROGRAMMING**” by **PERVEZ AHMAD (212EE5394)**, in partial fulfilment of the requirements for the award of the degree of **Master of Technology in ELECTRICAL ENGINEERING** with specialization in **INDUSTRIAL ELECTRONICS** during session 2012-2014 in the Department of Electrical Engineering, National Institute of Technology Rourkela, is an authentic work carried out by him under our supervision and guidance.

To the best of our knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any Degree or Diploma.

Date-

Place-Rourkela

Prof. S. Ganguly

Department of Electrical Engineering

National Institute of Technology, Rourkela

Acknowledgement

With a deep sense of gratitude, I wish to express my sincere thanks to my guide **PROF S. GANGULY**, Department of Electrical Engineering, National Institute of Technology, Rourkela for giving me the opportunity to work under his guidance on this project. I truly appreciate and value his esteemed guidance and encouragement from the beginning of this project. I am extremely grateful to him. His knowledge and company at the time of crisis would be remembered lifelong.

I want to thank all my teachers for providing a solid background for my studies and research thereafter. They have been great sources of inspiration to me and I thank them from the bottom of my heart. I will be failing in my duty if I do not mention the laboratory staff and administrative staff of this department for their timely help. I also want to thank my parents. I would like to share this moment of happiness with my parents and family member. They rendered me enormous support during the whole tenure of my stay in NIT Rourkela. Finally, I would like to thank all whose direct and indirect support helped me to completing my semester project report in time.

I would like to thank our department for giving me the opportunity and platform to make my effort a successful one.

PERVEZ AHMAD

212EE5394

Contents

Certificate.....	II
Acknowledgements	III
List of Figures.....	VII
List of Tables.....	VIII
List of Abbreviations	IX
Abstract.....	X

Chapter 1

INTRODUCTION

1.1 Motivation	1
1.2 literature survey	2
1.3 Objective of the thesis.....	5
1.4 Organization of the thesis.....	6

Chapter 2

Distributed generation

2.1 Introduction.....	7
2.2 Various technologies for distributed generation.....	8
2.3 DG system with modular size	11
2.4 Comparison between main DG technologies	13

Chapter 3

Distributed generation planning using linear programming

3.1 Introduction.....	15
3.2 linear programming	16
3.3 Mathematical model of linear programming	17
3.4 General expression of linear programming model.....	17
3.5 Assumption in linear programming.....	18
3.6 Determination of the optimal capacity of DG using LP	18
3.6.1 Assumption taken in formulation of linear mathematical model.....	19
3.6.2 Associated cost with DGs plant.....	19

3.6.3 Mathematical formulation.....	20
3.7 solution of LP using simplex method.....	21
3.8 Result.....	23

Chapter 4

Determination of the optimal DG capacity using Fuzzy linear programming

4.1 Introduction	25
4.2 implementation of fuzzy logic.....	26
4.2.1 Fuzzification.....	26
4.2.1 Fuzzy inference rule.....	26
4.2.3 Defuzzification	27
4.3 Fuzzy linear programming.....	27
4.3.1 Types of fuzzy linear programming.....	28
4.3.2 Assumption taken in formulation of fuzzy mathematical model.....	28
4.3.3 General model of fuzzy linear programming.....	28
4.3.4 Variation in cost for DG.....	29
4.4 Mathematical formulation of FLP.....	29
4.4.1 FLP model with fuzzy objective and crisp constraints.....	29
4.4.2 Procedure to solve FLP.....	30
4.4.3 Membership function for the DG plant.....	30
4.5 Defuzzification techniques.....	33
4.6 Modified objective function.....	35
4.7 FLP model with fuzzy objective and fuzzy constraints.....	40
4.8 Cost membership function for DG.....	41
4.9 Modified objective function and constraints.....	46

Chapter 5

Conclusions and scope for future work

5.1 Conclusions.....	52
5.2 Scope for future work.....	53
References.....	54

List of figures

Chapter 3

Fig 3.1 flow chart of simplex algorithm.....	22
--	----

Chapter 4

Fig 4.1 fuzzy logic phases.....	27
Fig 4.2 cost membership function of wind power plant.....	31
Fig 4.3 cost membership function for solar power plant.....	32
Fig 4.4 cost membership function for diesel power plant.....	32
Fig 4.5 cost membership function of solar power unit for FLP.....	41
Fig 4.6 cost membership function of wind power unit for FLP.....	42
Fig 4.7 cost membership function of diesel power unit for FLP.....	43
Fig 4.8 membership function for no. of wind power unit for FLP.....	43
Fig 4.9 membership function for no. of solar power unit for FLP.....	45
Fig 4.10 membership function for no. of diesel power unit for FLP.....	46

List of tables

Chapter 2

Table 2.1: DG modular size.....	12
Table 2.2 merits and demerits of various dg technologies.....	13

Chapter 3

Table 3.1 optimal capacity of DG using linear programming	24
---	----

Chapter 4

Table 4.1 defuzzified value for DG.....	35
Table 4.2 optimal capacity of FLP with fuzzy objective using mean-max method.....	36
Table 4.3 optimal capacity of FLP with fuzzy objective using centroid method.....	37
Table 4.4 optimal capacity of FLP with fuzzy objective using first or last of maxima method.....	38
Table 4.5 optimal capacity of FLP with fuzzy objective using weighted average method.....	39
Table 4.6 optimal capacity of FLP with fuzzy objective using max method.....	48
Table 4.7 optimal capacity of FLP with fuzzy objective using symmetric method.....	50
Table 4.8 optimal capacity of FLP with fuzzy objective using centroid method.....	51

ABBREVIATIONS AND ACRONYMS

DG	DISTRIBUTED GENERATION
FLP	FUZZY LINEAR PROGRAMMING
LP	LINEAR PROGRAMMING
MF	MEMBERSHIP FUNCTION
IC	INTERNAL COMBUSTION
MT	MICRO-TURBINE
RES	RENEWABLE ENERGY SOURCES

ABSTRACT

Distributed generation (DG) generates electricity on small scale close to end user of power. There are many economical, technical and environmental benefits of using DG. By suitable placement of DGs at optimal location with optimal size, benefits of DG can be maximize.

In the thesis work objective is to find the optimal capacity of distributed generation plant in a micro-grid to minimize the cost function. Three different cases are considered to determine the cost function. In first case it is assumed that price of installation cost of DG unit is fixed and based on the constraints on the capacity limit of DG, linear mathematical model is developed. Here the method is applied on solar, diesel and wind power unit.

In second case uncertainty in installation cost of DG has been included. To deal with uncertainty fuzzy logic is used and membership function generated which defuzzified by different methods and converted into linear mathematical programming.

Finally uncertainty included in number of DG unit used and in installation cost also, so that both objective function and constraints become fuzzy. For this condition mathematical equation is developed and defuzzified to convert fuzzy linear programming (FLP) problem into linear programming (LP). In this case new method of defuzzification is used which is called as symmetric method. LP problem is solved by simplex algorithm. A constraint has putted on power generation limit of each DG units. Finally comparison has been made among all the de-fuzzification techniques and between linear programming and fuzzy linear programming.

Chapter I

Introduction

1.1 MOTIVATION

To get the technical, economic and environmental benefit for consumer and utility, it is important to know the optimal siting and sizing of DG. Because of continuous increase in petroleum Fuel price and environmental damage, DG plays important role in distributed generation planning. Apart from environmental advantages there are many benefits of DG, since it is nearer to consumer, transmission loss is less. Power quality improvement, increase in network capacity and improvement in voltage profile are also the important factor to use the DG in planning. Moreover DG is available in different modular units, from few KW to several MW.

Improper allocation of DGs leads to increase in power loss, decline in power quality hence it is a big challenge for distribution system network operators (DNO) On the other hand if distributed energy sources are placed in an optimal way they may improve the system's parameters, and may bring technical benefits like improvement of the power quality, decrease of energy loss as well as essential economic benefits like lower cost. Economic benefits are the most important parameter in present scenario. Since different plant has different capacity and associated cost is different for different plant, it becomes necessary to determine the optimal capacity of each plant to minimize the cost and maximize the profit.

The main objective of the proposed study is to determine the optimal capacity of DGs plant so as the cost function is minimized .It is assumed that installation cost of various plant is uncertain and after the use of fuzzy logic we can determine the capacity of each plant hence the main point is to deal the situation under uncertainty .we proceed further and include uncertainty in number of plant as well as uncertainty in generation capacity of each plant so that we can solve much complex system with the help of fuzzy logic. The associated cost in generating the power is uncertain for all the three units. Fuzzy logic is used to deal with uncertainty. Fuzzy logic is very important tool in decision making. We can deal with the situation where the system is complex and it is difficult to formulate the mathematical model, since it follows linguistic rules i.e. rules expressed by words.

1.2 literature survey

Today distributed generation system are much in demand because of several advantage like reduction of on-peak operating cost, deferral of network upgrades, lower losses and lower transmission and distribution costs. It is also environment friendly. It reduces greenhouse gas emission. There has been a wide range of modular size from 5 KW to 500 MW [1].

In recent years the integration of DG units in power distribution network has increased. To optimize electrical distribution network operation it is necessary to provide the best location and size of DG. Installation of multiple DGs, objective is to minimize the total power, design variable are location and size are some of the common characters tics of optimal distributed generation plant [2].

Distributed power unit can be directly connected to consumer. It responds very quickly to system problem that can help in reduction in investment and can become more reliable. It can also reduce transmission losses, power wheeling cost and certain environmental emission costs. Economic efficiency is one of the major concerns of distributed generation customers. With many techno-economic restrictions and scenarios we could employ a flexible and reliable planning for renewable and distributed generations [3].

It is important to select the capacity of different type of DG to ensure reliability of micro-grid system. Grid connected system are constructed and output model of various DG are built. Power of wind turbine is predicted by linear interpolation and the output is calculated using reindl, hay model [4].

To solve the linear programming various methods are developed. Initially LP problem solved by graphical analysis but this method is not suitable for large decision variables. To solve the large decision variables simplex algorithm is used. Depend on objective function and constraints different cases of simplex algorithm are developed. Big m, two phase methods are the part of simplex algorithm [5].

Different technological options are available for DG. It is important to know the cost of generation and future scope of renewable and non-renewable technologies. This paper studies the various

technology used for DG, compares the life, efficiency, capital cost, each unit and future scope of these technologies in India [6-7].

Linear programming where all coefficients are fuzzy can be solved by resolution method. In this method rank is assigned to fuzzy objective and depends on feasibility degree objective value is decided. Higher the feasibility degree worst will be the objective value. To give rank to the fuzzy objective value, fuzzy ranking method is used [8].

There are many techniques to distributed generation and each one has their own merit and demerit. Different techniques have different modular size and efficiency and based on different factor there generation cost are different from each other. Some have very low operation and maintenance cost and some have high. The penetration of DGs is very economical if their size and site are selected optimally by single or multiple objective functions under certain operating constraints. Its objective can be minimize cost or minimize power loss under different load condition. Constraints can be on no. of unit used or capacity of DGs [9].

Micro-grid integrate the distributed renewable energy. it is environment friendly and provides higher reliability, reduces transmission loss and reduced greenhouse gas emission. Optimal power operation is determined to minimize the cost function. An optimization model has been presented and three case are considered. Result is shown with no battery and 10 KW batteries [10].

In a typical case of hydroelectric generation scheduling problem, fuzzy linear programming is used. In this problem forecast hourly load and natural inflows are taken uncertain. To determine the optimal schedule under uncertain environment fuzzy linear programming is developed in which the hourly natural inflows, hourly loads and cost are expressed as fuzzy quantity [11].

Sub gradient and method of feasible direction are used to solve linear programming problem whose left and right hand coefficient are fuzzy. Membership function are piecewise linear. Symmetric method is used to defuzzify the membership function [12].

In real world problem most of the time information is vague. To deal with uncertainty fuzzy logic was introduced. Linear programming is one of most usable techniques in the field of operation research. Membership function developed and various techniques can be used to defuzzify it. Membership function can be of triangular shape, trapezoidal, square, depends on input quantity. A method based on Yagar's parameterized is illustrated to solve fuzzy linear programming [13].

To solve fuzzy linear programming simplex method can be used. . Membership function need to be defuzzify and converted into crisp one. Then FLP become linear problem can be solved by simplex algorithm. Graphical method can also be used but have some limitation so simplex is the best method to solve the FLP [14].

Fuzzy set theory can be applied in many fields like operation research, power system, neural network, control theory, instrumentation. System where it is difficult to develop mathematical model because of uncertainty in information, fuzzy set theory plays an important role to develop model. Fuzzy set follows some properties like associative, distributive [15].

Simplex method is used to solve the fuzzy linear programming with bounded variable. The proposed method is very easy and can be implemented into programming language. The cost coefficient is trapezoidal and optimal value is also in trapezoidal. to solve the FLP it is important to know arithmetic operation of trapezoidal number [16].

To deal with uncertainty L.A.Zadeh introduces fuzzy sets. It is a methodology for expressing operational laws of a system in linguistic terms instead of mathematical equations. A fuzzy set is a class of objects with a continuum of grades of membership. Such set is characterized by a membership function which assigns to each object a grade of membership ranging between zero and one. Various notations like inclusion, union, intersection, complement, convexity are established in fuzzy sets [17].

The objective function and constraints can be fuzzy in nature i.e. if our objective function is to minimize the cost function and price of electricity is uncertain, we can include this uncertainty to solve the problem. Different methods have been given to deal with such problem. One of the tasks is to evaluate investment in small scale wind power under uncertainty It is assumed that the price of electricity is uncertain and that an owner of a property with wind resources has a deferrable opportunity to invest in one wind power turbine within a capacity range. Given model evaluates investment in a set of projects with different capacity with the inclusion of uncertainty. [18]

A linear model is developed whose objective is to maximize the DG energy. Load factor is also included in the objective function. Constraints are putted on capacity allocation, location and plant size. The result is tested on 38KV distribution network. Loss adjustment factor, effective load factor are also calculated [19].

Fuzzy variable linear programming is solved using simplex algorithm. Definition and notation of fuzzy set are defined to understand fuzzy set theory and its application. Trapezoidal membership function has been taken for the fuzzy set. Linear model with objective function and constraints are defined. Defuzzification of fuzzy set done by symmetric method and finally simplex algorithm is used to solve the problem [20].

1.3 Objective of the thesis

Objective of the thesis is to determine the optimal capacity of solar power unit, diesel power unit and wind power unit under normal and fuzzy environment to minimize the cost function. In this thesis work three cases are considered to determine the optimal capacity. Several assumptions have been taken to formulate the model into mathematical form. First case leads to formulate the linear mathematical problem while second and third case leads to the fuzzy linear programming problem. Assumption while formulating the model is given below for different case.

Problem statement I

- Price of electricity is fixed.
- Numbers of unit of DGs plant are single.
- The entire three units should generate at least 1MW unit of power.
- Diesel engine should generate power at least twice of combination of wind and solar plant.
- Each plant should generate some electricity i.e. power generation by the plant must be greater than zero.
- Each plant should generate some minimum define amount of power and we kept varying minimum limit of generation of power.

Problem statement II

- Price of electricity is uncertain.
- Rest other condition are same as case i

Problem statement III

- Price of electricity is uncertain
- Number of DGs unit is uncertain and considered in some ranges.

- Rest other condition are same as case I and ii

Hence aim is to –

- formulate linear mathematical model for case i
- Determine the optimal capacity to minimize the cost function
- formulate fuzzy mathematical model for case ii and case ii
- Use simplex algorithm to solve the LP and FLP
- Compare the different defuzzification method
- Compare and discuss all three cases.

1.4 Organization of the thesis

Chapter 2- it describes about distributed generation, its importance in present scenario were discussed. Various kinds of technologies for DGs are discussed. DGs system with modular size is presented. In the end comparison among all the technologies, their merits and demerits are discussed.

Chapter 3- renewable planning with linear programming discussed. Chapter starts with the introduction of linear programming followed by assumption in linear programming mathematical model were presented .method to solve the linear programming discussed. At the end result is shown.

Chapter 4- this chapter starts with introduction of fuzzy logic and phases of fuzzy logic.in this chapter we developed fuzzy linear mathematical model, in which case ii and case iii of chapter 1 is discussed. Membership functions for cost function, number of DG unit are developed and defuzzified be the defuzzification techniques. At the end result and conclusion are made.

Chapter 5- draws conclusion on various works presented and suggests the scope of future work.

Chapter 2

Distributed generation

2.1 Introduction

Distributed generation (DG) which are also called decentralized generation, implanted generation, dispersed generation, is a small scale power generation source. It is an approach that employs small-scale technologies to produce electricity near to the end users of power. DG includes both conventional and non-conventional types of energy resource to generate power. The development of new technologies allow the DG to leads various benefits like low transmission loss, power wheeling cost [9].

DG uses a wide range of technologies to generate power like combined heat and power plant, PV system, small scale wind turbine and other renewable resources. The main feature of DG is that it can be used as an isolated way or in an integrated way supplying energy to the remaining of the electric system. DG system Improves power quality and system reliability [3]. DG system reduces transmission loss. There are many environmental benefits of using DG like it reduces carbon emission.

Micro-grid integrates small scale distributed energy resources. It integrates distributed power generator and local renewable energy which include wind, hydro, solar power, fuel cell and bio energy. Micro-grid utilizes waste heat produced from electricity generation so that transmission efficiency improves [10]. It reduces greenhouse gas emission by avoiding unnecessary transmission loss. Apart from several advantages, there are some complications involved in generating power from DG sources for example there is instability and unpredictability in power generation from solar and wind power cause of its dependability on weather condition.

2.2 VARIOUS TECHNOLOGIES FOR DISTRIBUTED GENERATION [3]

The Distributed Generation (DG) technologies include both conventional and non-conventional type of energy sources for generating power. It play important role in DG system as an alternative distribution system planning option [1]. There are many technologies available to generate small – scale power, these are describes below.

- Reciprocal engine
- Combustion turbines
- Wind turbines
- Fuel cells
- Micro-turbines
- Hydro power
- Sterling engines
- Solar PV

2.2.1 RECIPROCAL ENGINES

Working principle and features of IC engine are as follows-

- Energy contained in a fuel is converted into mechanical power by the reciprocal engine and power is used to a rotate shaft.
- To convert the rotational motion into power, a generator is attached to the IC engine.
- High -speed IC operate at 1200-3600 rpm.
- Medium -speed IC operate at 275-1000 rpm.
- It have higher capital cost but have high efficiency.

2.2.2 COMBUSTION TURBINES

Features of CT are as follows-

- CT's efficiency depends on its size, larger the size greater will be the efficiency
- CT's are available in range of 500KW to 25 MW.
- Efficiency are in the range of 25to 45%

2.2.3 MICRO TURBINES (MT)

Important features of MT are as follows-

- Available in the range between 25 KW- 500kW
- It can be used for stand-by power, power quality and reliability, peak shaving, and cogeneration applications.
- It is well-suited for small commercial building establishments such as restaurants, hotels, small offices, retail stores, and many others.

2.2.4 STIRLING ENGINES

Important point on Stirling engines are describe below

- A Stirling engine comes under external combustion engines.
- Stirling engines are sealed systems with an inert working fluid, usually either helium or hydrogen.
- These are generally found in small sizes (1 - 25 kW).
- It is used in residential power generation, small aircraft and refrigeration.

2.2.5 FUEL CELLS

Working principle and range of fuel cells are given below-

- It converts the chemical energy from a fuel into electricity through a chemical reaction with oxygen or another oxidizing agent.
- In fuel cells hydrogen is the most common fuel, but hydrocarbons such as natural gas and alcohols like methanol are sometimes used.
- It is available in a wide range of size typically 200 KW -2 MW.

2.2.6 SOLAR PV

Working principle and properties of solar PV are as follows-

- Solar power is the conversion of sunlight into electricity either directly using photovoltaic (PV) or indirectly using concentrated solar power.
- Concentrated solar power systems use lenses or mirrors and tracking systems to focus a large area of sunlight into a small beam.
- Photovoltaic convert light into electric current using the photoelectric effect.

2.2.7 WIND POWER

Features and operation of wind power is given below-

- Wind power is the conversion of wind energy into a useful form of energy.
- It uses wind turbines to make electrical power, windmills for mechanical power, wind pumps for water pumping or drainage, or sails to propel ships.

- Large wind farms consist of hundreds of individual wind turbines which are connected to the electric power transmission network.
- Onshore wind is an inexpensive source of electricity but cheaper than fossil fuel plants.
- In isolated locations electricity provided by small onshore wind form.

2.2.8 DIESEL ENGINE POWER

- A diesel generator is the combination of a diesel engine with an electric generator to generate electrical energy.
- It is used as an alternating option where grid is failed.
- Sizing of diesel generators is critical to avoid low-load or a shortage of power and is complicated by modern electronics specifically non-linear loads.

2.3 DG SYSTEMS WITH MODULAR SIZE

The advances in DG technologies and increase in their sizes play significant role in power distribution systems. DG is very diverse and range from 1kW PV installation, 1 MW engine generators to 1000 MW offshore wind farms or more. Table- 2.1 shows the most commonly used DG and their typical module size [1].from the table it can be seen that it have very vast ranges from few KW to several MW. Solar PV has the lowest range and gas turbine has the highest range.

Table 2.1: DG MODULAR SIZE [1]

SI NO.	DG TECHNOLOGY	TYPICAL AVAILABLE POWER MODULE SIZE
1	COMBINED CYCLE GAS TURBINES	35-400 MW
2	INTERNAL COMBUSTION ENGINES	5KW-10 MW
3	COMBUSTION TURBINE	1-250 MW
4	MICRO-TURBINE	35KW -1 MW
5	FUEL CELLS	200KW-2 MW
6	BATTERY STORAGE	0.5-5 MW
7	HYDRO POWER	1-25 MW
8	WIND TURBINES	200W-3 MW
9	SOLAR PV	20 W -100 KW
10	SOLAR THERMAL POWER PLANT	1-10 MW
11	BIOMASS GASIFICATION BASED	100KW-20 MW
12	OCEAN ENERGY	0.1-1 MW

2.4 COMPARISON BETWEEN MAIN DG TECHNOLOGIES

Table 2.2 merits and demerits of various dg technologies [1]

SI NO.	DG TECHNOLOGY	MERITS	DEMERITS
1	FUEL CELL	<ul style="list-style-type: none"> • High efficiency • Near zero emission • Low noise • Fast load response 	<ul style="list-style-type: none"> • Pure hydrogen need • High cost • Fuel required processing • Low durability
2	MICRO-TURBINE	<ul style="list-style-type: none"> • Low emission • Light weight • Small size • Low noise 	<ul style="list-style-type: none"> • High cost • Relatively low efficiency • Limited to low temperature
3	WIND TURBINE	<ul style="list-style-type: none"> • Low energy loss • Save land use • No fuel demand • Low production cost 	<ul style="list-style-type: none"> • Affected by wind speed • Variable power output noise • High investment cost
4	SOLAR PV	<ul style="list-style-type: none"> • Low maintenance • No fuel demand • Environmental friendly 	<ul style="list-style-type: none"> • High investment cost • Affected by solar radiation
5	GAS TURBINE	<ul style="list-style-type: none"> • Low emission • High reliability 	<ul style="list-style-type: none"> • Low efficiency at low load • High pressure gas

6	RECIPROCATING-ENGINE	<ul style="list-style-type: none"> • Fast start up • Low investment 	<ul style="list-style-type: none"> • Relatively higher emission • High maintenance
7	SMALL HYDRO	<ul style="list-style-type: none"> • Free and renewable source of energy • Short installation time • No impact on river ecosystem 	<ul style="list-style-type: none"> • Power output depends upon availability of water • Can't meet require load demand • Affected by flood
8	BIOMASS PLANT	<ul style="list-style-type: none"> • Reduces dependency on fossil fuels • Reduces greenhouse gas emission • Uses renewable sources 	<ul style="list-style-type: none"> • Expensive • Limited source • Cause pollution
9	CHP	<ul style="list-style-type: none"> • High efficiency • Save energy loss • Integration various fuels • Low emission 	<ul style="list-style-type: none"> • Increased investment cost • Need reasonable plan • Complex technology need • Decrease flexibility

From above table 2.1 it can be seen that solar PV generates lowest power and combine gas turbine cycle have highest capacity of generating power. From table 2.2 we can say that fuel cell, solar PV, wind and small hydro are emission free DGs require no fuel and are eco-friendly. The most suitable DGs considering environmental concerns, fuel cost, maintenance costs and output power are renowned as biomass, small hydro, SPV etc. [1].

Chapter 3

Distributed generation planning using linear programming

3.1 Introduction

Distributed generation (DG) which include renewable energy sources (RES) and cogeneration, is an important trend in energy market all over the world. The contribution of DG in energy generation is increasing especially on the level of medium and low voltage. It is a big challenge for distribution system network operators (DNO) because DG may impact the reliability, loss of energy voltage level, and obstruct the operation of the power grid [2].

On the other hand if distributed energy sources are placed in an optimal way, they may improve the system's parameters and may bring technical benefits for e.g. improvement of the voltage level, decrease of energy loss as well essential economic benefits for e.g. lower cost, because of limitation of energy purchase from the system or suspended reinforcement of the power grid and significant income from energy sales [10].

Different DG plant have different generation capacity based on the modular size and each plant has different installation cost associated with it before connecting to the grid. On the basis of requirement of energy needed and have constraint on it , we can put these situation into linear equation and can solve with linear programming subject to condition that there is a linear relationship among variables[1].

In this research work linear programming is used to optimize the cost function which is related to distributed generation of three different units' i.e. solar power unit, wind power unit and diesel power unit. Based on the capacity of power generation of each DG, different equations are formed .single unit of solar plant, wind plant and diesel unit are considered for LP.

3.2 linear programming

George B. Dantzig in 1947 introduced linear programming. It is a method of optimizing an objective function by solving a system of linear equations where the solution is depends on constraints (Danzig 1949). The first polynomial-time algorithm for systems of linear equations was given by Khachiyan in 1979. LP problem consists of an objective function and a set of constraints. Feasible solution is the solution which satisfies all the constraints [8]. Most solution algorithms start by finding a feasible solution and then move from one feasible solution to another until the objective function has been optimized i.e. maximized or minimized.

LP models can be considered as a special kind of decision model in which constraints is used to define the decision space, the objective function is used to define the goal (utility) and the type of decision is assumed under certainty.

LP refers to the problem in which both the objective function to be optimized and all the constraints should be linear in terms of the decision variables i.e. there should be no square root, second degree polynomials. Simple models of linear programming can easily be solved graphically and this is particularly helpful, as it makes the intuition behind the process very clear but if numbers of variables are more than graphical method will become complex hence simplex method is used to solve the linear programming[20].

3.3 Mathematical model of linear programming

A mathematical model of linear programming consists of an objective function and a set of constraints expressed in the form of a system of equations or inequalities [8]. Components of linear programming model are discussed below.

3.3.1 Decision variable

It is a mathematical symbol that represents the level of action of a firm.

3.3.2 Objective function

An equation which describes relationship between objectives of the firm in terms of decision variables and is either to be maximized or minimized called objective function. Objective function may be single objective function or multiple objective functions.

3.3.3 Constraints

It is the restrictions placed on the firm by the operating environment stated in linear relationships of the decision variables.

3.3.4 Parameter

Constant and numerical coefficient of objective function and constraints are the parameters of linear programming.

3.4 General expression of linear programming model

Objective function-

$$\text{Minimize or maximize} \quad c_1x_1 + c_2x_2 + \dots + c_nx_n = z \quad \text{-----} \quad (3.1)$$

Subject to constraints-

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &= b_2 \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &= b_m \end{aligned} \quad \text{-----} \quad (3.2)$$

Here 'z' is the expression being optimized. The variables $x_1, x_2, x_3, \dots, x_n$ are called decision variables. Here all constraints are equal; it may be less than or greater than also. a_{ij} is the coefficient of the decision variable.

3.5 Assumptions in linear programming

Equation of any kind cannot be considered as linear it must follow some property. These are as follows -

1- **Proportionality**- there must be a linear relationship among variables of objective function and variables of constraints.

For example- $8x$ is double the value of $4x$.

2- **Additivity** – there must be no relationship among the variables of objective function and constraints i.e. all variables must be independent from each other.

3- **Divisibility** - In normal LP decision variables are integer but it can be fraction also. Linear integer programming is used to tackle fractional decision variables.

4- **Certainty** - also called the deterministic assumption. In problem every parameter should be defined clearly. Most of the time coefficients and parameters are in fuzzy.

3.6 Determination of optimal capacity of DG using linear programming [18-19]

In this research work three plants are considered; solar power plant, wind power plant and diesel power plant. All the three plants are connected to grid. Since different plant has different generation capacity and in order to maintain the power demand some constraints are put on the generation capacity of each plant. It is assumed that cost of generating the power from different sources are fixed.

i.e. there is no uncertainty in cost of generating the electricity. Our objective is to minimize the cost subject to some condition of power demand. Cost in Rs/KW has been taken.

3.6.1 Assumption taken in formulation of linear mathematical model

- 1- Cost of generating electricity is fixed.
- 2- Numbers of unit of DGs plant are single.
- 3- The entire three units should generate at least 1MW unit of power.
- 4- Diesel engine should generate power at least twice of combination of wind and solar plant.
- 5- Each plant should generate some electricity i.e. power generation by the plant must be greater than zero.
- 6- Each plant should generate some minimum define amount of power and we kept varying minimum limit of generating power.

3.6.2 Associated cost with DGs plant [6-7]

Rangan Banerjee and Avizit Nayak reviews the different technological options available for DG their current status and evaluates them based on the cost of generation and future potential in India [6-7]. The cost generation is dependent on the load factor and the discount rate. It includes operation and maintenances cost also. Installation cost for different plant taken in Rs / KW and as follows-

1. Cost of wind power generator (C_w) = 50,000 Rs /kW
2. Cost of solar power generator (C_s) = 144,000 Rs/kw
3. Cost of diesel power generator (C_c) = 25000 Rs /kW

3.6.3 Mathematical formulation

Based on the given condition objective function and subject to constraints are developed and formulated in mathematical form which are given below.

Objective function- our objective is to minimize the cost function .here cost coefficient of the objective function are crisp value i.e. single valued [18]. Mathematically objective function can be represented as-

$$\text{Min } \sum(C_w X_w + C_s X_s + C_c X_c) \quad \text{-----} \quad (3.3)$$

Subject to constraints-

$$1. \quad X_w + X_c + X_s \geq 1000\text{KW} \quad \text{-----} \quad (3.4)$$

$$2. \quad X_w + X_s \leq 0.5X_c \quad \text{-----} \quad (3.5)$$

$$3. \quad X_w \geq 100 \text{ (KW)} \quad \text{-----} \quad (3.6)$$

$$4. \quad X_c \geq 100 \text{ (KW)} \quad \text{-----} \quad (3.7)$$

$$5. \quad X_s \geq 100 \text{ (KW)} \quad \text{-----} \quad (3.8)$$

Where –

C_w : cost of wind power generator (Rs/kw)

C_s : cost of solar power generator (Rs/kw)

C_c : Cost of diesel engine generator (Rs/kw)

X_w : Capacity of wind power generator unit (kW)

X_s : Capacity of solar power generator unit (kW)

X_c : Capacity of diesel power generator unit (kW)

Modified objective function

$$\text{Min } \Sigma(50000X_w + 144000X_s + 25000X_c) \quad \text{-----} \quad (3.9)$$

Our goal is to find the capacity of each plant i.e. find the value of X_w , X_s and X_c such that objective function will be minimized, from the above objective equation it can be seen that optimal value will depend upon cost coefficient of power plant. The less cost coefficient more will be optimal value.

To solve above linear equation we need to convert above equation into general form. To convert above inequality into equality we need to add slack and surplus variable depend upon the type of equation [19]. After adding slack and surplus variable we can write general form of the above equation which is as follows-

After converting inequality into equality equation become-

$$X_w + X_c + X_s - S_1 + A_1 = 1000 \quad \text{-----} \quad (3.10)$$

$$X_w + X_s - 0.5X_c + S_2 = 0 \quad \text{-----} \quad (3.11)$$

$$X_w - S_3 + A_2 = 100 \quad \text{-----} \quad (3.12)$$

$$X_c - S_4 + A_3 = 100 \quad \text{-----} \quad (3.13)$$

$$X_s - S_5 + A_4 = 100 \quad \text{-----} \quad (3.14)$$

3.7 Solution of linear programming by simplex algorithm

There are many methods to solve linear programming but the most efficient method is simplex algorithm. It is simple and computationally efficient method. Other method is graphical method. Graphical method is preferable where numbers of variable are two, when number of variable increases graphical method become complex as it is difficult to locate point in three dimensional spaces, whereas simplex algorithm can take 'n' no. of decision variable to solve LP [20]. Hence simplex algorithm is the most efficient method to solve the linear programming. Flow charts of simplex algorithm are shown in next page.

Simplex algorithm

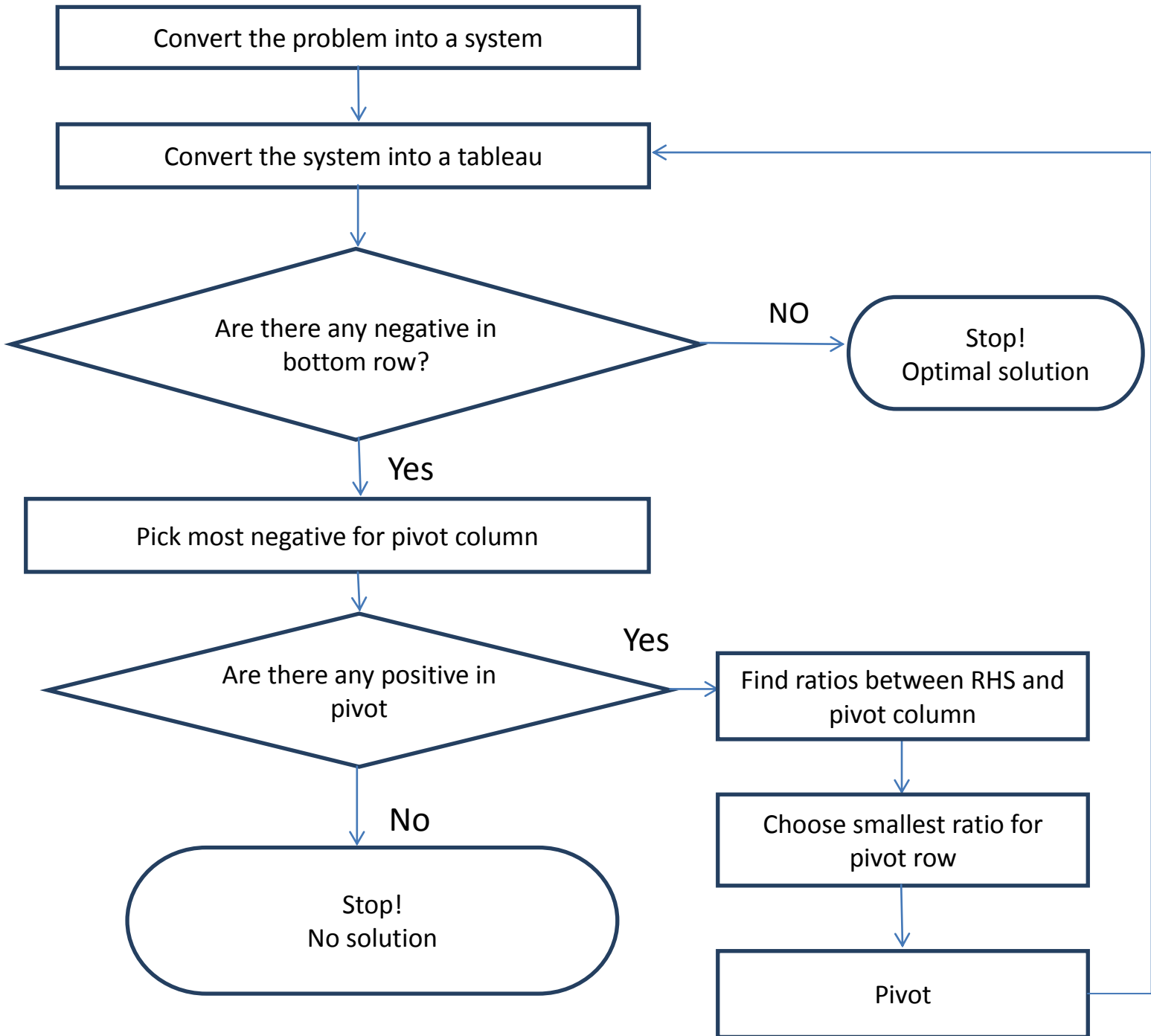


Fig 3.1 flow chart of simplex algorithm

Simplex algorithm helps to move from one vertex to another adjacent vertex which is closest to the optimal solution among all other adjacent vertices. Thus, it follows the shortest route to reach the optimal solution from the starting point. Figure 3.1 shows the step to solve the linear programming problem by simplex method [20].

At initial stage problem is converted into a system and represent it into tabular form. After representing into tabular form we checked if there is any negative in bottom row if no then iteration will be over and we will get optimal solution and if we found negative element from bottom row then pick the most negative from the bottom row to find pivot column and find whether any positive in pivot column, if there is no positive in pivot column then there will be no solution and if we got positive then we need to find the ratio between RHS and pivot column. Smallest ratio will be pivot row. Intersection between pivot row and pivot column will be pivot element [20]. Again we will repeat the whole step to until we get the optimal solution.

3.8 Result

Simplex algorithm is used to determine the capacity of DG plant. Optimal Capacity and minimized cost are shown in tabular form for under different generation capacity. From the result it can be observed that optimal capacity of diesel engine is highest among three and optimal capacity of solar and wind are minimum of its generation capacity. Optimal capacity and minimized cost with the help of simplex algorithm are shown below. From the result it can be observed that as the minimum capacity of each DG unit is increased optimal value is also increased in the same proportional for wind and solar plant, but for diesel plant in generates large power since it involves less cost. As we increase the constraint limitation of each plant, optimal value is also increased in same proportional for the case of solar and wind unit but it is increasing in large proportional in the case of diesel plant.

Optimal capacity of DG using linear programming

Table 3.1 optimal capacity of DG using linear programming

SI NO.	CAPACITY(KW)	OPTIMAL VALUE(KW)	MINIMIZED COST(Rs)
1	$X_w \geq 100, X_s \geq 100, X_c \geq 100$	$X_w = 100, X_s = 100, X_c = 800$	39400000
2	$X_w \geq 200, X_s \geq 100, X_c \geq 100$	$X_w = 200, X_s = 100, X_c = 700$	41900000
3	$X_w \geq 100, X_s \geq 200, X_c \geq 100$	$X_w = 100, X_s = 200, X_c = 700$	51300000
4	$X_w \geq 100, X_s \geq 100, X_c \geq 200$	$X_w = 100, X_s = 100, X_c = 800$	39400000
5	$X_w \geq 200, X_s \geq 200, X_c \geq 200$	$X_w = 200, X_s = 200, X_c = 800$	58800000
6	$X_w \geq 300, X_s \geq 300, X_c \geq 300$	$X_w = 300, X_s = 300, X_c = 1200$	88200000

Chapter 4

Determination of optimal DG capacity using fuzzy linear programming

4.1 Introduction

In previous chapter linear mathematical model developed in order to find the optimal capacity of DGs plant to minimize cost function. In linear model cost of generation of solar, diesel power and wind power plant are assumed as certain i.e. there is no ambiguity in cost associated in generation of power per unit KW. But in a real problem data are often fluctuate or are not known precisely. This kind of fluctuations or vagueness in the data can very well be modelled by fuzzy numbers [12].

In this chapter fuzzy linear programming model presented to find the capacity of each plant so that total generation cost will be minimize. We divided mathematical formulation of the problem in two parts .In first part uncertainty is included in cost of generation of electricity, putting other condition same as in linear mathematical model. In second part uncertainty is included in cost, number of DG unit and in generation capacity of each plant. Hence it is easy for investor in decision making [13].

As compared to classical linear programming, fuzzy linear programming contains much variation.it depends on the circumstances for which the model is to be developed. In fuzzy linear programming , it is possible to deal with real time bounded problem.

Fuzzy logic is very important tool in dealing with vagueness in data. uncertainty is included in cost function , number of DG unit and minimum generation capacity of each plant.it is necessary to know some basic terminology of fuzzy logic .In next section some basic concept of fuzzy logic are discussed , how to deal with fuzzy data and basic operation in fuzzy logic[15].

Fuzzy logic was invented and named by Lotfi Zadeh; in fuzzy logic system linguistic definition is used instead of developing mathematical model. Some system are too complex to deal and it is difficult to develop model with the help of mathematical equation , here in this case linguistic term plays very important role in modelling the system accurately. Before approaching towards fuzzy

linear programming it is important to know some basics terminology of fuzzy logic and how to implement it [16].

4.2 implementation of fuzzy logic

Fuzzy logic can be implemented in three phase, these are as follows

1. Fuzzification
2. Fuzzy Inference rule
3. Defuzzification

4.2.1 Fuzzification

It is the process of mapping of input value into fuzzy membership function where membership function is a mapping of input quantity in the ranges between zero and one. It represent the degree to which particular element belongs to defined set.

4.2.2 Fuzzy inference rule

Phase in which control rule base developed to relate the output and input based on the observation given, called as fuzzy inference rule.

Example-

If room_temp is COOL, then set fan_speed to SLOW.

If room_temp is HOT, then set fan_speed to FAST.

If room_temp is WARM, then set fan_speed to MEDIUM.

4.2.3 Defuzzification

Defuzzification is the process of conversion of fuzzy quantity into crisp quantity. To generate a useful output defuzzification is required. Systematic representation of implementation of fuzzy logic is shown below.

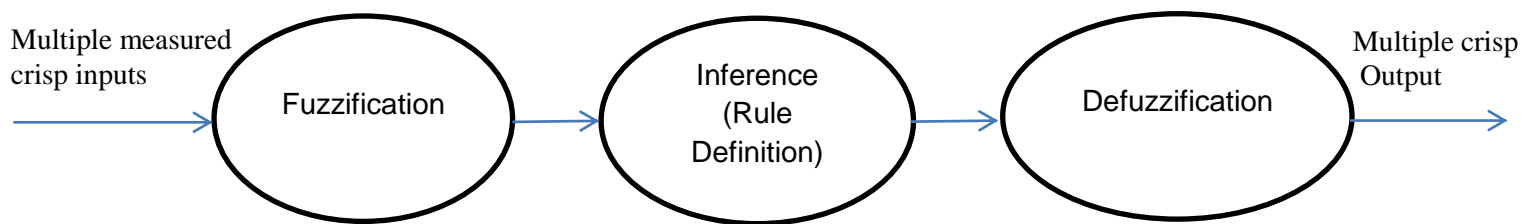


Fig 4.1 fuzzy logic phases

4.3 Fuzzy linear programming

Bellman and Zadeh (1970) proposed the concept of maximize decision making in fuzzy linear programming. Tanaka et al (1984) used concept of maximize decision making and applied in the field of mathematical programming. To solve multi-objective linear programming problems Zimmermann (1983) developed a fuzzy approach. Negoita (1970) formulated Fuzzy linear programming problem with fuzzy coefficients which are called as robust programming [16].

Problem which involve real world situation can be handled by linear programming model and based on the expert opinion many parameter can be included. It is difficult to get the accurate and precise information of the system parameter involved in real situation problem. Hence it is important to know how to develop the fuzzy model for such kind of problem [12].

Definition of fuzzy decision given by Bellman and Zadeh, fuzzy mathematical programming is one of the most active areas for researcher. Most of the works on FLP done by by Tanaka et al. and Zimmermann, last few decade observe several kinds of fuzzy linear programming problems and to

solve FLP different methods have been proposed, mostly Zimmermann et al formulated fuzzy linear programming which was extension to the linear programming problem.

4.3.1 Types of fuzzy linear programming

There are mainly four types of fuzzy linear programming-

1. Fuzzy linear programming with fuzzy constraints.
2. A general model of fuzzy linear programming.
3. Fuzzy linear programming with fuzzy co-efficient.
4. Linear programming with fuzzy objective.

In the present paper we are dealing with FLP with fuzzy coefficient as well as fuzzy objective.

4.3.2 Assumption taken in formulation of fuzzy mathematical model

All condition is same for fuzzy linear programming as we discussed in previous chapter for linear programming model.

4.3.3 General model of fuzzy linear programming

A general fuzzy linear programming can be expressed as –

$$\text{Min } z = \tilde{c}^T x$$

$$\text{S.t. } \tilde{A}x \geq \tilde{b}$$

Where \tilde{c} , \tilde{A} and \tilde{b} represent the fuzzy parameter involved in the objective function and constraints, x represent crisp decision vector. Here objective function and constraints are fuzzy in nature. it may be possible that in fuzzy linear programming only objective function are fuzzy and constraints are non-fuzzy. Sometimes only right hand side of the constraints are fuzzy and sometimes left hand side of the

constraints are fuzzy [15]. In this thesis work we are dealing with two separate cases in first case only objective function are fuzzy and in second case both objective function as well as constraints are fuzzy. We will discuss these two problems in next section.

4.3.4 Variation in cost for DG plant [6-7]

In linear programming cost coefficient was constant value. In the FLP problem we considered variation in cost coefficient of objective function. These variations are as follows-

- 1- Cost of wind power plant is in between the range of 45000Rs to 55000 Rs per KW
- 2- Cost of solar power plant is in between the range of 140000Rs to 150000 Rs per KW
- 3- Cost of diesel power plant is in between the range of 20000Rs to 30000 Rs per KW

4.4 Mathematical formulations of the FLP

In previous chapter linear programming model studied whose objective function and constraints are non-fuzzy in nature. In this section linear programming model with fuzzy objective and fuzzy constraints are discussed [17]. We deal this problem into two parts –

1. Linear programming model with fuzzy objective function only.
2. Linear programming model with fuzzy objective function as well as fuzzy constraints.

4.4.1 FLP model with fuzzy objective and crisp constraints

A general FLP model with fuzzy objective and crisp constraints is as given below. In FLP we included uncertainty in coefficient and rest other condition are same as LP.

OBJECTIVE FUNCTION -

$$\text{Min } \Sigma[(45000, 55000)X_w + (140000, 150000)X_s + (20000, 30000)X_c] \text{ ----- (4.1)}$$

SUBJECT TO CONSTRAINTS-

$$1. X_w + X_c + X_s \geq 1000\text{KW} \text{ ----- (4.2)}$$

$$2. X_w + X_s \leq 0.5X_c \text{ ----- (4.3)}$$

$$3. X_w \geq 100 \text{ (KW)}, X_c \geq 100\text{(KW)}, X_s \geq 100\text{(KW)} \text{ ----- (4.4)}$$

Where –

X_w : Capacity of wind power generator unit (kW)

X_s : Capacity of solar power generator unit (kW)

X_c : Capacity of diesel power generator unit (kW).

4.4.2 Procedure to solve FLP

- 1- Develop membership function for the fuzzy variable coefficient.
- 2- Defuzzify the membership function
- 3- Convert FLP problem into LP.
- 4- Solve LP problem using simplex algorithm.

4.4.3 Membership function for the DG plant

Membership function characterizes the fuzziness in a fuzzy set. It is the curve of the degree of truth of a given input value .its values are in between the range of 0 to 1.it can be developed based on the intuition or numerical data. We have taken right triangular membership function for all three DG plant which are shown below [11]-

4.4.3.1 COST MEMBERSHIP FUNCTION FOR WIND POWER GENERATION UNIT

Membership function

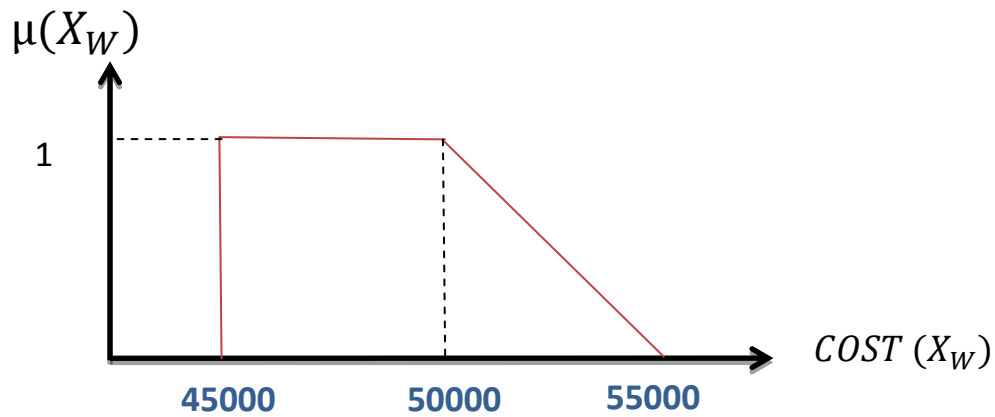


Fig 4.2 cost membership function of wind power plant

$$\begin{aligned}
 \mu(X_W) &= \begin{cases} 1 & 45000 \leq X_W \leq 50000 \\ (55000 - X_W) / 5000 & 50000 \leq X_W \leq 55000 \\ 0 & X_W \geq 55000 \end{cases} \quad \text{----- (4.5)}
 \end{aligned}$$

Membership value is one in between 45000 to 50000 and decreases to zero after 55000. Higher the membership function greater will be the chance of the cost function to lies in that range. It is important to defuzzify this membership function to convert into LP problem which can be solved by simplex method.

4.4.3.2 COST MEMBERSHIP FUNCTION FOR SOLAR POWER PLANT

Membership function

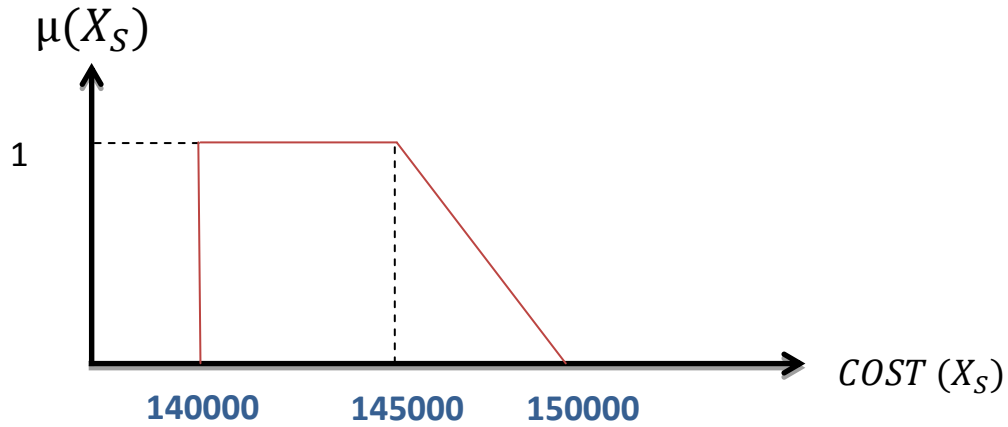


Fig 4.3 cost membership function for solar power plant

$$\mu(X_S) = \begin{cases} 1 & 140000 \leq X_S \leq 145000 \\ \frac{(150000 - X_S)}{5000} & 145000 \leq X_S \leq 150000 \\ 0 & X_S \geq 150000 \end{cases} \quad (4.6)$$

4.4.3.3 COST MEMBERSHIP FUNCTION FOR DIESEL POWER PLANT

Membership function

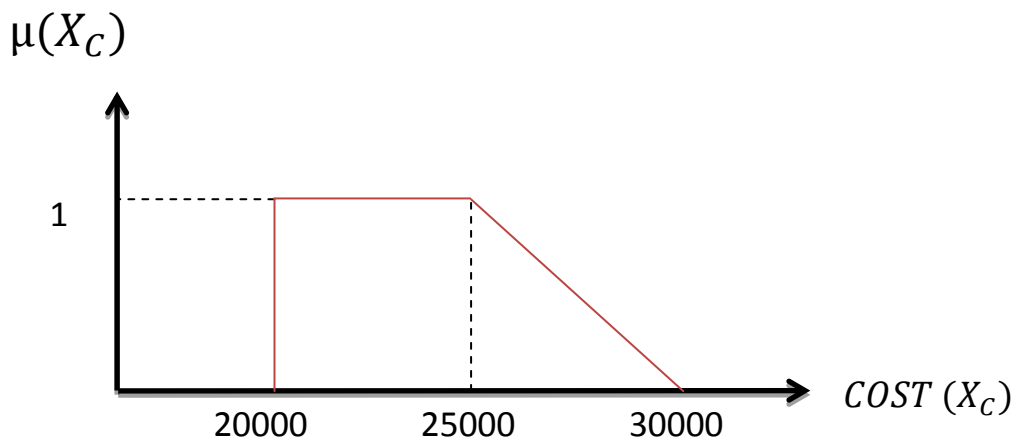


Fig 4.4 cost membership function for diesel power plant

$$\mu(X_C) = \begin{cases} 1 & 20000 \leq X_C \leq 25000 \\ (30000 - X_C)/5000 & 25000 \leq X_C \leq 30000 \\ 0 & X_C \geq 30000 \end{cases} \quad \text{----- (4.7)}$$

These interval values need to convert into crisp value i.e. single value to solve above problem. Defuzzification is the process to convert interval value into single value. After converting it into single value FLP will be converted into LP which can be solved by simplex method. There are many defuzzification techniques defuzzify the membership function. We applied mainly four techniques to defuzzify the membership function these defuzzification techniques will be discussed in next section.

4.5 Defuzzification techniques [21]

Defuzzification is the process of converting fuzzy quantity into crisp one i.e. it is the process of producing quantifiable result. There are many different methods of defuzzification some of them are given below.

- 1- centroid method
- 2- mean-max method
- 3- weighted average method
- 4- first of maxima or last of maxima method
- 5- symmetric method
- 6- max membership principle

4.5.1 Centroid method

This method is also called centre of gravity method and is the most important among all the defuzzification methods. Algebraic equation is given by-

$$z^* = \frac{\int \mu(z)z dz}{\int \mu(z) dz} \quad \text{----- (4.8)}$$

Here z^* is the defuzzified value

4.5.2 Mean –max method

This method is also called as middle of maxima and it is almost same as to max membership principle except that the locations of the maximum membership in the case of max membership principle can be non-unique. Algebraic equation is given by

$$z^* = (a+b)/2 \quad \text{-----} \quad (4.9)$$

z^* Is defuzzified value and a, b are the point at which membership function is highest. Membership function can be plateau also.

4.5.3 Weighted average method

Weighted average is the most frequently used method in fuzzy application since it is one of most computational efficient method. It is applicable to symmetric membership function only. Algebraic equation is given by-

$$z^* = \frac{\sum \mu(z)z}{\sum \mu(z)} \quad \text{-----} \quad (4.10)$$

Here z is the centroids of each symmetric membership function and z^* is the defuzzified value.

4.5.4 Max membership principle

It is also known as height method and is limited to peak output function. It algebraic expression is given by –

$$\mu(Z^*) \geq \mu(z) \quad \text{-----} \quad (4.11)$$

Here Z^* is the defuzzified value.

4.6 Modified objective function

After applying different defuzzification techniques, we get the modified objective function for different techniques which can be solved by simplex algorithm. Defuzzified values for different method are given below.

P.G.U./DEFUZZIFICATION TECHNIQUES	CENTROID	MEAN-MAX	WEIGHTED AVG.	FIRST OR LAST OF MAXIMA
SOLAR POWER	143890	142500	142500	140000
WIND POWER	48890	47500	47500	45000
DIESEL POWER	23890	22500	22500	20000

Table 4.1 defuzzified value for DG

For different defuzzification techniques different objective function are formed. We will analyze defuzzification techniques and will find a technique which gives most optimal solution. Objective function for different techniques are given below.

$$\text{Centroid method - Min } \sum(48890X_w + 143890X_s + 23890X_c) \quad \text{-----} \quad (4.12)$$

$$\text{Mean- max method - Min } \sum(47500X_w + 142500X_s + 22500X_c) \quad \text{-----} \quad (4.13)$$

$$\text{Weighted average method- Min } \sum(47500X_w + 142000X_s + 22500X_c) \quad \text{-----} \quad (4.14)$$

$$\text{First (or) last of maxima - Min } \sum(45000X_w + 140000X_s + 20000X_c) \quad \text{-----} \quad (4.15)$$

Constraints equation will be same as of LP model. In next section FLP whose objective functions as well as constraints are fuzzy are developed. Like before membership function are developed for objective function and constraints, this time uncertainty in number of DGs is also included. Therefore separate membership function will be there for number of unit of DG.

Result-

Optimize capacity using mean-max method

Table 4.2 Optimal capacity of DG with fuzzy objective using mean-max method

SI NO	CAPACITY(KW)	OPTIMAL VALUE(KW)	Z(LP)	Z(FLP)
1	$X_w \geq 100, X_s \geq 100, X_c \geq 100$	$X_w = 100, X_s = 100, X_c = 800$	39400000	37000000
2	$X_w \geq 200, X_s \geq 100, X_c \geq 100$	$X_w = 200, X_s = 100, X_c = 700$	41900000	39500000
3	$X_w \geq 100, X_s \geq 200, X_c \geq 100$	$X_w = 100, X_s = 200, X_c = 700$	51300000	49000000
4	$X_w \geq 100, X_s \geq 100, X_c \geq 200$	$X_w = 100, X_s = 100, X_c = 800$	39400000	37000000
5	$X_w \geq 200, X_s \geq 200, X_c \geq 200$	$X_w = 200, X_s = 200, X_c = 800$	58800000	56000000
6	$X_w \geq 300, X_s \geq 300, X_c \geq 300$	$X_w = 300, X_s = 300, X_c = 1200$	88200000	84000000

From the result it can be observed that FLP gives improved result and we can say that maximum power should be generated from diesel engine in order to minimize the cost function. From the centroid and mean-max method, mean – max gives better result. From the result it can be observed that as the minimum capacity of each DG unit is increased optimal value is also increased in the same proportional for wind and solar plant, but for diesel plant in generates large power since it involves less cost. As constraints capacity increased optimal value also increased in proportional way for the case of solar and wind plant but in the case of diesel plant it is increasing in large proportional.

Optimize capacity using centroid method

Table 4.3 Optimal capacity of DG with fuzzy objective using centroid method

SI NO.	CAPACITY(KW)	OPTIMAL VALUE(KW)	Z(LP)	Z(FLP)
1	$X_w \geq 100, X_s \geq 100, X_c \geq 100$	$X_w = 100, X_s = 100, X_c = 800$	39400000	38390000
2	$X_w \geq 200, X_s \geq 100, X_c \geq 100$	$X_w = 200, X_s = 100, X_c = 700$	41900000	40890000
3	$X_w \geq 100, X_s \geq 200, X_c \geq 100$	$X_w = 100, X_s = 200, X_c = 700$	51300000	50390000
4	$X_w \geq 100, X_s \geq 100, X_c \geq 200$	$X_w = 100, X_s = 100, X_c = 800$	39400000	38390000
5	$X_w \geq 200, X_s \geq 200, X_c \geq 200$	$X_w = 200, X_s = 200, X_c = 800$	58800000	57680000
6	$X_w \geq 300, X_s \geq 300, X_c \geq 300$	$X_w = 300, X_s = 300, X_c = 1200$	88200000	86500000

From the result it can be observed that FLP gives improved result and we can say that maximum power should be generated from diesel engine in order to minimize the cost function. From the centroid and mean-max method, mean – max gives better result. . As constraints capacity increased optimal value also increased in proportional way for the case of solar and wind plant but in the case of diesel plant it is increasing in large proportional. From the result it can be observed that as the minimum capacity of each DG unit is increased optimal value is also increased in the same proportional for wind and solar plant, but for diesel plant in generates large power since it involves less cost.

Optimize capacity using first or last of maxima method

Table 4.4 Optimal capacity of DG with fuzzy objective using first or last of maxima method

SI NO.	CAPACITY(KW)	OPTIMAL VALUE(KW)	Z(LP)	Z(FLP)
1	$X_w \geq 100, X_s \geq 100, X_c \geq 100$	$X_w = 100, X_s = 100, X_c = 800$	39400000	34500000
2	$X_w \geq 200, X_s \geq 100, X_c \geq 100$	$X_w = 200, X_s = 100, X_c = 700$	41900000	37000000
3	$X_w \geq 100, X_s \geq 200, X_c \geq 100$	$X_w = 100, X_s = 200, X_c = 700$	51300000	46500000
4	$X_w \geq 100, X_s \geq 100, X_c \geq 200$	$X_w = 100, X_s = 100, X_c = 800$	39400000	34500000
5	$X_w \geq 200, X_s \geq 200, X_c \geq 200$	$X_w = 200, X_s = 200, X_c = 800$	58800000	53000000
6	$X_w \geq 300, X_s \geq 300, X_c \geq 300$	$X_w = 300, X_s = 300, X_c = 1200$	88200000	79500000

From the result it can be observed that FLP gives improved result and we can say that maximum power should be generated from diesel engine in order to minimize the cost function. From the centroid and mean-max method, mean – max gives better result. From the result it can be observed that as the minimum capacity of each DG unit is increased optimal value is also increased in the same proportional for wind and solar plant, but for diesel plant in generates large power since it involves less cost. . As constraints capacity increased optimal value also increased in proportional way for the case of solar and wind plant but in the case of diesel plant it is increasing in large proportional.

Optimize value using weighted average method

Table 4.5 Optimal capacity of DG with fuzzy objective using weighted average method

SI NO	CAPACITY(KW)	OPTIMAL VALUE(KW)	Z(LP)	Z(FLP)
1	$X_w \geq 100, X_s \geq 100, X_c \geq 100$	$X_w = 100, X_s = 100, X_c = 800$	39400000	37000000
2	$X_w \geq 200, X_s \geq 100, X_c \geq 100$	$X_w = 200, X_s = 100, X_c = 700$	41900000	39500000
3	$X_w \geq 100, X_s \geq 200, X_c \geq 100$	$X_w = 100, X_s = 200, X_c = 700$	51300000	49000000
4	$X_w \geq 100, X_s \geq 100, X_c \geq 200$	$X_w = 100, X_s = 100, X_c = 800$	39400000	37000000
5	$X_w \geq 200, X_s \geq 200, X_c \geq 200$	$X_w = 200, X_s = 200, X_c = 800$	58800000	56000000
6	$X_w \geq 300, X_s \geq 300, X_c \geq 300$	$X_w = 300, X_s = 300, X_c = 1200$	88200000	84000000

From the results it can be seen that FLP gives improved cost value than LP. It can also be observed that different defuzzification techniques may have same optimal value like in the case of weighted average and mean-max method, depends upon the shape of membership function. . As constraints capacity increased optimal value also increased in proportional way for the case of solar and wind plant but in the case of diesel plant it is increasing in large proportional. In this case also diesel engine generated highest value of optimal capacity which satisfies the equation. From the result it can be observed that as the minimum capacity of each DG unit is increased optimal value is also increased in the same proportional for wind and solar plant, but for diesel plant in generates large power since it involves less cost.

4.7 FLP model with fuzzy objective and fuzzy constraints

In previous section we discussed FLP with fuzzy objective only. This time we are including uncertainty in no. of DG unit and in generation capacity of each plant so there will be separate membership function for each plant. Cost function and no. of DG unit under fuzzy parameter are as follows.

- cost of wind power plant (C_w)= {(45,000) (55,000)} Rs/kw
- cost of solar power plant (C_s) = {(140,000) (150,000)} Rs/kw
- cost of diesel engine plant (C_c)={ (20,000) (30,000)}Rs/kw
- No. of wind unit power plant = {(3) (12)}
- No. of solar unit power plant = {(2) (10)}
- No. of diesel engine power plant = {(2) (12)}

Equation of objective function and constraints of FLP are given as-

OBJECTIVE FUNCTION -

Min

$$\Sigma\{(45000) (55000)\} \{(3) (12)\}X_w + \{(140000) (150000)\}\{(2) (10)\}X_s + \{(20000) (30000)\}\{(2) (12)\}X_c]$$

$$\text{Min } \Sigma\{(135000) (660000)\} X_w + \{(280000) (1500000)\}X_s + \{(20000) (30000)\} X_c] \text{----- (4.16)}$$

Subject to constraints-

$$1. \quad \{(3) (12)\}X_w + \{(2) (10)\}X_c + \{(2) (12)\}X_s \geq 1\text{MW} \text{----- (4.17)}$$

$$2. \quad \{(3) (12)\} X_w + \{(2) (10)\}X_s \leq 0.5\{(2) (12)\}X_c \text{----- (4.18)}$$

$$3. \quad X_w \geq 100 \text{ KW} \text{----- (4.19)}$$

$$4. \quad X_c \geq 100 \text{ KW} \text{----- (4.20)}$$

$$5. \quad X_s \geq 100 \text{ KW} \text{----- (4.21)}$$

4.8 Cost membership function for DG

Since cost coefficients are fuzzy in nature we developed membership function for each unit. Different defuzzification method will be used to convert into crisp value and after that it can be solved by simplex algorithm. Membership functions are given below.

4.8.1 COST MEMBERSHIP FUNCTION FOR SOLAR POWER GENERATION UNIT

Membership function

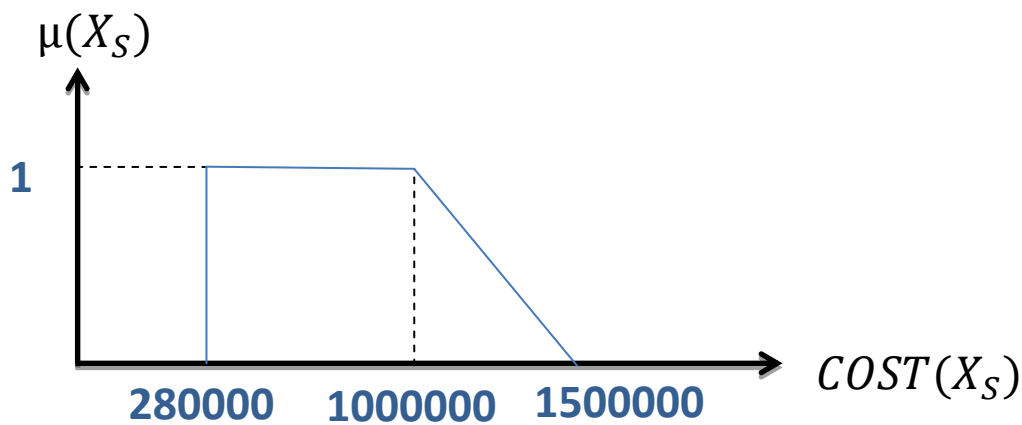


Fig 4.5 cost membership function of solar power unit for FLP

$$\mu(X_S) = \begin{cases} 1 & 280000 \leq X_S \leq 1000000 \\ \frac{(1500000 - X_S)}{500000} & 1000000 \leq X_S \leq 1500000 \\ 0 & X_S \geq 1500000 \end{cases} \quad (4.22)$$

4.8.2 COST MEMBERSHIP FUNCTION FOR WIND POWER GENERATION UNIT

Membership function

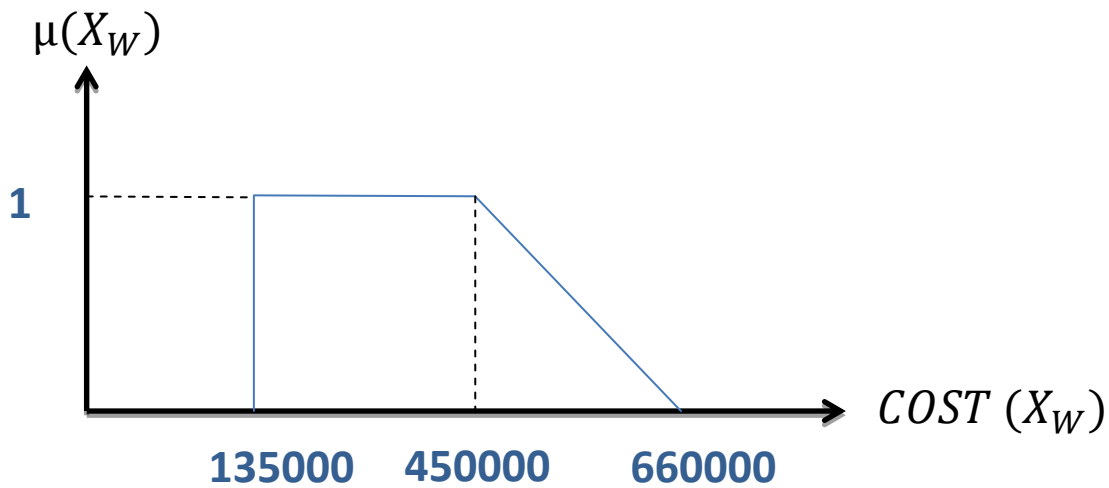


Fig 4.6 cost membership function of wind power unit for FLP

$$\mu(X_W) = \begin{cases} 1 & 135000 \leq X_W \leq 450000 \\ \frac{660000 - X_W}{210000} & 450000 \leq X_W \leq 660000 \\ 0 & X_W \geq 660000 \end{cases} \quad \text{----- (4.23)}$$

Membership function lies between zero and one. Its value is 1 in the range of 135000 to 450000 after that its value is continuously decreasing and become zero after 660000.

4.8.3 COST MEMBERSHIP FUNCTION FOR DIESEL POWER GENERATION UNIT

Membership function

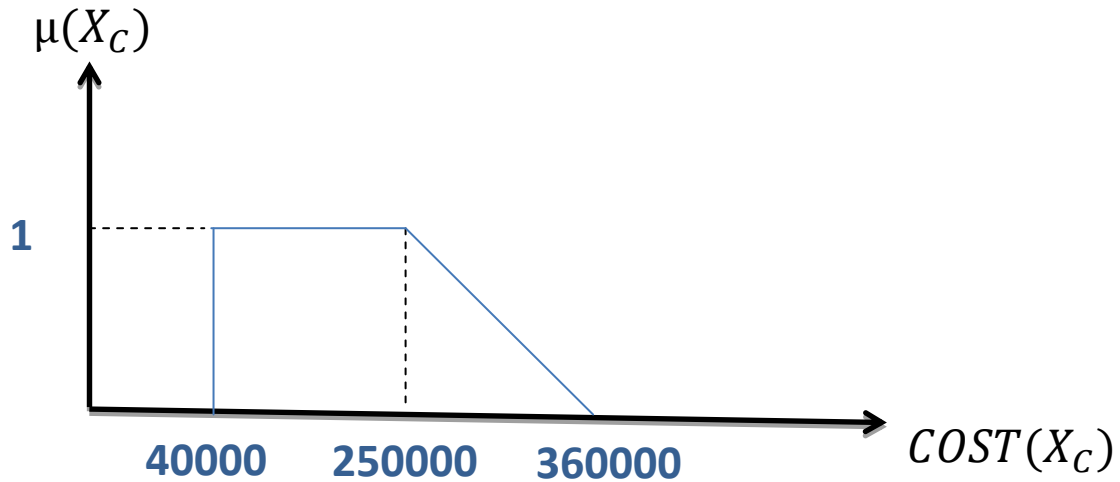


Fig 4.7 cost membership function of diesel power unit for FLP

$$\mu(X_c) = \begin{cases} 1 & 40000 \leq X_c \leq 250000 \\ (360000 - X_c) / 110000 & 250000 \leq X_c \leq 360000 \\ 0 & X_c \geq 360000 \end{cases} \quad (4.24)$$

Right triangular membership function is taken for DG unit. Its values lies between zero and one. Its value is 1 in the range of 40000 to 250000, it means there is higher possibility that cost of diesel power unit should lies in this range and after that its value is continuously decreasing and become zero after 360000.

4.8.4 MEMBERSHIP FUNCTION FOR NO. OF WIND POWER GENERATION UNIT

Membership function

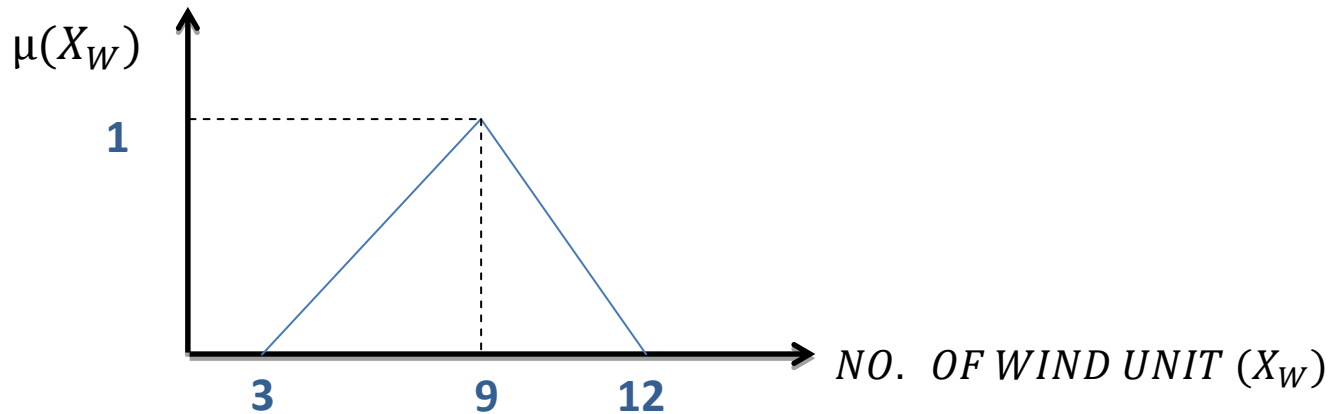


Fig 4.8 membership function for no. of wind power unit for FLP

$$\begin{aligned}
 \mu(X_W) &= \begin{cases} (X_W - 3)/6 & 3 \leq X_W \leq 9 \\ (12 - X_W)/3 & 9 \leq X_W \leq 12 \\ 0 & X_W \geq 12 \end{cases} \quad \text{-----} \quad (4.25)
 \end{aligned}$$

Membership functions are triangular in nature. Its values are continuously increasing between 3 to 9 and decreasing between 9 to 12, after that its value become zero. Membership function values are lies in between 0 to 1. maximum value of membership function is at 9. In the range of 3 to 9 membership function is continuously increasing it indicates that there is a highest possibility that number of DG unit can be used in this range while possibility decreases in range of 9 to 12.

4.8.5 MEMBERSHIP FUNCTION FOR NO. OF SOLAR POWER GENERATION UNIT

Membership function

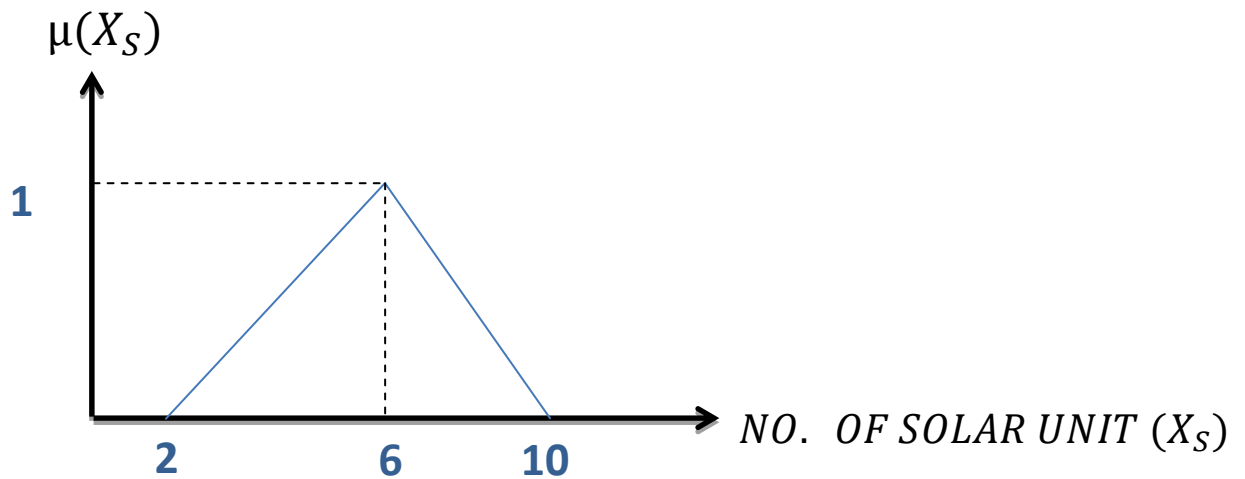


Fig 4.9 membership function for no. of solar power unit for FLP

$$\begin{aligned}
 \mu(X_s) &= \begin{cases} (X_s - 2)/4 & 2 \leq X_s \leq 6 \\ (10 - X_s)/4 & 6 \leq X_s \leq 10 \\ 0 & X_s \geq 10 \end{cases} \quad \text{-----} \quad (4.26)
 \end{aligned}$$

Membership functions are triangular in nature. Its values are continuously increasing between 2 to 6 and decreasing between 6 to 10, after that its value become zero. MF values are lies in between 0 to 1. In the range of 2 to 6 membership function is continuously increasing it indicates that there is a highest possibility that number of DG unit can be used in this range while possibility decreases in range of 6 to 10.

4.8.6 MEMBERSHIP FUNCTION FOR NO. OF DIESEL POWER GENERATION UNIT

Membership function

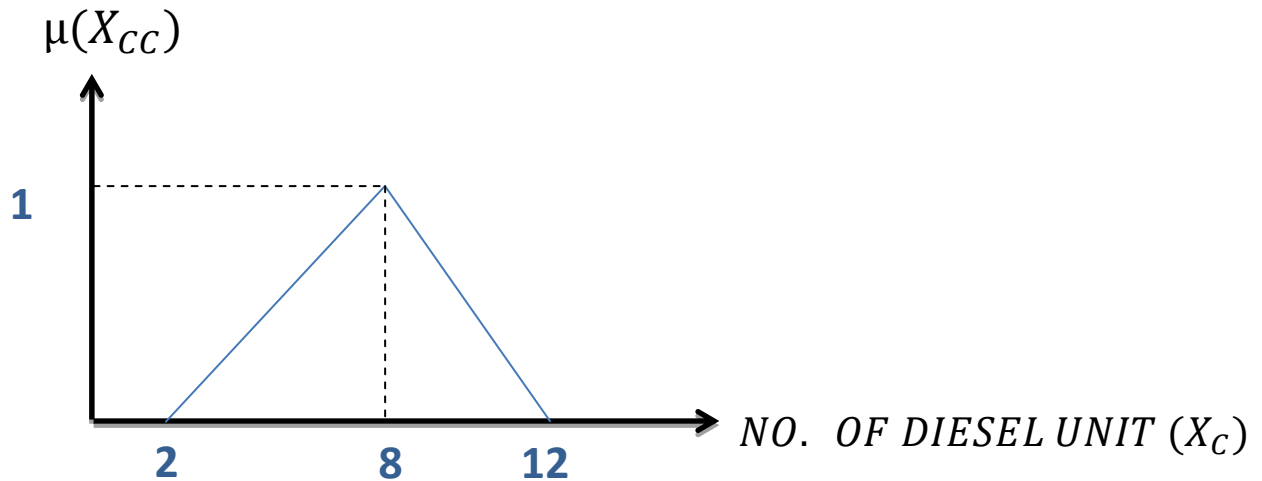


Fig 4.10 membership function for no. of diesel power unit for FLP

$$\begin{aligned}
 \mu(X_C) &= \begin{cases} (X_C - 2)/6 & 2 \leq X_C \leq 8 \\ (12 - X_C)/4 & 8 \leq X_C \leq 12 \\ 0 & X_C \geq 12 \end{cases} \quad \text{-----} \quad (4.27)
 \end{aligned}$$

Membership functions are triangular in nature. Its values are continuously increasing between 2 to 8 and decreasing between 8 to 12, after that its value become zero. MF values are lies in between 0 to 1. In the range of 2 to 8 membership function is continuously increasing it indicates that there is a highest possibility that number of DG unit can be used in this range while possibility decreases in range of 8 to 12.

4.9 MODIFIED OBJECTIVE FUNCTION AND CONSTRAINTS

Three different objective function and constraint equation are obtained after applying the different defuzzification techniques. After defuzzification of membership function FLP

converted into LP which can be solved by simplex algorithm. For different method objective function and constraints given below.

4.9.1 Objective function and constraints after defuzzification by centroid method

OBJECTIVE FUNCTION -

$$\text{Min } \sum(349375X_w + 775739X_s + 174402X_c) \quad \text{-----} \quad (4.28)$$

SUBJECT TO CONSTRAINTS-

$$8X_w + 6X_s + 7X_c \geq 1000 \quad \text{-----} \quad (4.29)$$

$$8X_w + 6X_s - 4X_c \leq 0 \quad \text{-----} \quad (4.30)$$

$$X_w \geq 100 \quad \text{-----} \quad (4.31)$$

$$X_c \geq 100 \quad \text{-----} \quad (4.32)$$

$$X_s \geq 100 \quad \text{-----} \quad (4.33)$$

4.9.2 Objective function and constraints after defuzzification by maximum method

OBJECTIVE FUNCTION -

$$\text{Min } \sum(349375X_w + 775739X_s + 174402X_c) \quad \text{-----} \quad (4.34)$$

SUBJECT TO CONSTRAINTS-

$$9X_w + 6X_s + 8X_c \geq 1000 \quad \text{-----} \quad (4.35)$$

$$9X_w + 6X_s - 4X_c \leq 0 \quad \text{-----} \quad (4.36)$$

$$X_w \geq 100 \quad \text{-----} \quad (4.37)$$

$$X_c \geq 100 \quad \text{-----} \quad (4.38)$$

$$X_s \geq 100 \quad \text{-----} \quad (4.39)$$

4.9.3 Objective function and constraints after defuzzification by symmetric method

OBJECTIVE FUNCTION -

$$\text{Min } \sum(349375X_w + 775739X_s + 174402X_c) \quad \text{-----} \quad (4.40)$$

SUBJECT TO CONSTRAINTS-

$$9X_w + 6X_s + 8X_c \geq 1000 \quad \text{-----} \quad (4.41)$$

$$3X_w + 2X_s + 2X_c \geq 1000 \quad \text{-----} \quad (4.42)$$

$$12X_w + 10X_s + 12X_c \geq 1000 \quad \text{-----} \quad (4.43)$$

$$9X_w + 6X_s - 4X_c \leq 0 \quad \text{-----} \quad (4.44)$$

$$3X_w + 2X_s - X_c \leq 0 \quad \text{-----} \quad (4.45)$$

$$12X_w + 10X_s - 6X_c \leq 0 \quad \text{-----} \quad (4.46)$$

Simplex algorithm is best method to solve the LP and FLP.in this chapter we developed membership function for cost function and no. of DG unit. Different defuzzification method is used to convert the FLP problem into LP. Three different objective function and constraints are obtained and solved by simplex algorithm. In next chapter result of LP and FLP are shown, advantage of FLP over LP are given and conclusion and discussion are studied.

Result –

Table 4.6 Optimal capacity of FLP with fuzzy objective using max method

SI NO.	CAPACITY(KW)	OPTIMAL VALUE(KW)	MINIMIZE D COST(Rs)
1	$X_w \geq 100, X_s \geq 100, X_c \geq 100$	$X_w = 100, X_s = 100, X_c = 375$	177912000
2	$X_w \geq 200, X_s \geq 100, X_c \geq 100$	$X_w = 200, X_s = 100, X_c = 600$	252090000
3	$X_w \geq 100, X_s \geq 200, X_c \geq 100$	$X_w = 100, X_s = 200, X_c = 525$	281646000
4	$X_w \geq 100, X_s \geq 100, X_c \geq 200$	$X_w = 100, X_s = 100, X_c = 375$	177912000
5	$X_w \geq 200, X_s \geq 200, X_c \geq 200$	$X_w = 200, X_s = 200, X_c = 750$	355824000
6	$X_w \geq 300, X_s \geq 300, X_c \geq 300$	$X_w = 300, X_s = 300, X_c = 1125$	533736000

Optimal capacity of FLP using symmetric method

Table 4.7 Optimal capacity of FLP with fuzzy objective using symmetric method

SI NO.	CAPACITY(KW)	OPTIMAL VALUE(KW)	MINIMIZE D COST(Rs)
1	$X_w \geq 100, X_s \geq 100, X_c \geq 100$	$X_w = 100, X_s = 100, X_c = 500$	199712000
2	$X_w \geq 200, X_s \geq 100, X_c \geq 100$	$X_w = 200, X_s = 100, X_c = 800$	286970000
3	$X_w \geq 100, X_s \geq 200, X_c \geq 100$	$X_w = 100, X_s = 200, X_c = 700$	312167000
4	$X_w \geq 100, X_s \geq 100, X_c \geq 200$	$X_w = 100, X_s = 100, X_c = 500$	199712000
5	$X_w \geq 200, X_s \geq 200, X_c \geq 200$	$X_w = 200, X_s = 200, X_c = 1000$	399425000
6	$X_w \geq 300, X_s \geq 300, X_c \geq 300$	$X_w = 300, X_s = 300, X_c = 1500$	599137000

From the results it can be observed that centroid method gives most improved value of minimization function and diesel engine contributed more since its cost coefficient function are least among three DG unit. Since we included uncertainty in cost function as well as in number of DG unit it will benefited to the investor in decision making process. From the result it can be observed that as the minimum capacity of each DG unit is increased optimal value is also increased in the same proportional for wind and solar plant, but for diesel plant in generates large power since it involves less cost.

Optimal capacity of FLP using centroid method

Table 4.8 Optimal capacity of FLP with fuzzy objective using centroid method

SI NO.	CAPACITY(KW)	OPTIMAL VALUE(KW)	MINIMIZE D COST(Rs)
1	$X_w \geq 100, X_s \geq 100, X_c \geq 100$	$X_w = 100, X_s = 100, X_c = 350$	173552000
2	$X_w \geq 200, X_s \geq 100, X_c \geq 100$	$X_w = 200, X_s = 100, X_c = 550$	243370000
3	$X_w \geq 100, X_s \geq 200, X_c \geq 100$	$X_w = 100, X_s = 200, X_c = 500$	277286000
4	$X_w \geq 100, X_s \geq 100, X_c \geq 200$	$X_w = 100, X_s = 100, X_c = 350$	173552000
5	$X_w \geq 200, X_s \geq 200, X_c \geq 200$	$X_w = 200, X_s = 200, X_c = 700$	347104000
6	$X_w \geq 300, X_s \geq 300, X_c \geq 300$	$X_w = 300, X_s = 300, X_c = 1050$	520656000

From the results it can be observed that centroid method gives most improved value of minimization function and diesel engine contributed more since its cost coefficient function are least among three DG unit. Since we included uncertainty in cost function as well as in no. of DG unit it will benefited to the investor in decision making process. From the result it can be observed that as the minimum capacity of each DG unit is increased optimal value is also increased in the same proportional for wind and solar plant, but for diesel plant in generates large power since it involves less cost.

Chapter 5

Conclusions and scope for future work

5.1 conclusions

Three different approaches have been applied to determine the capacity of DGs so as the cost function is minimized. Initially LP was used to solve determine the capacity, from the result of LP problem, it was concluded that diesel engine must generate most of the power in order to minimize the cost function whereas since cost associated with solar and wind are high hence it should generate less power that is shown in the result.

FLP problem with fuzzy objective function is studied in which price of electricity was uncertain. From the result we can say that different defuzzification techniques may give same optimal value, it depends on membership function. Weighted average and min –max method gives improved optimal value among other methods and also FLP method gives improved optimal value over LP method.

FLP problem with fuzzy objective and fuzzy constraints are solved by symmetric, centroid and max. Method. Among three method centroid method gives most suitable result. From the results it can be seen that diesel engine generated most of the power. This method will be suitable under uncertainty condition since we included uncertainty in both cost coefficients as well as in no. of DG unit, hence can be helpful in decision making under uncertainty.

5.2 Scope for future work

In this thesis work we dealt with single objective function with cost minimization and constraints was power generation limit. It can be multiple objective functions and different constraints with uncertainty included in objective function as well as in constraints. Multiple objective functions may include minimization of loss as well as maximization of profit.

Multiple objective functions with constraints in optimal distributed generation plant may include following parameter in future with the help of fuzzy linear programming.

Objective function-

1. Minimization of total power loss of the system
2. Minimization of the energy losses
3. Minimization of the voltage deviation
4. Maximization of DG capacity
5. Minimization of cost
6. Maximization of voltage limit liability

Constraints-

1. Power flow equality constraints
2. Bus voltage or voltage drop limit
3. Short circuit level limit
4. Power generation limit
5. Discrete size of DG units
6. Limited buses for DG installation

References

- 1- S. Nail, D. K. Khatod, M. P. Sharma, "Planning and Operation of Distributed Generation in Distribution Networks," *International journal of emerging technology and advance engineering* (vol.2, issue 9) pp. 34-41, September 2012.
- 2- Pavlos S. Georgilakis, Nikos D.Hatziargyrgio, "Optimal distributed generation placement in power distribution network," *IEEE Trans.on Power system*, vol. 28, NOV 23, pp. 3240-3248, Aug 2013.
- 3- S. M. Sadeghzadeh, and M. Ansarian, "Distributed Generation and Renewable Planning with a Linear Programming Model," *First international power and energy conference PEC* (48-53) on NOVEMBER 28-29, MALAYSIA, 2006.
- 4- Shujun Liu , Zaijun Wu, Xiaobo Dou1, "optimal configuration of hybrid ,solar, wind distributed generation capacity in a grid connected micro-grid,"*IEEE* 2013.
- 5- Suresh Chandra, Jayadeva, Aparna Mehta, "Numerical optimization with applications," narosa publication, 2009.
- 6- Rangan Banerjee, "Comparison of options for distributed generation in India," department of engineering and public policy, Carnegie Mellon university *Elsevier*, 2012.
- 7- Avizit Nayak, "Cost Economics of Solar kWh," *National productive council*, Kolkata, 2012.
- 8- Mariano Jimé'nez, Mar Arenas, Amelia Bilbao, M.Victoria Rodrí'guez, "Linear programming with fuzzy parameters: An interactive method resolution," *European Journal of Operational Research* (2005).
- 9- Rajendra Prasad Payasi, Asheesh K. Singh, Devender Singh, "Review of distributed generation planning: objectives, constraints, and algorithms," *International Journal of Engineering, Science and Technology* ,Vol. 3, No. 3, pp. 133-153, 2011.
- 10- Yen-Haw Chen, Yen-Hong Chen, Ming-Che Hu, "Optimal Energy Management of Micro grid Systems in Taiwan," *IEEE*, pp.1-9, 2011.
- 11- R.-H. Liang Y.-Y. HSU, "Fuzzy linear programming: an application to hydroelectric generation scheduling," *IEE Proc.-Gener. Transm. Distrib.* Vol. 141, No. 6, November 1994.

- 12- S. Effati, H. Abbasiyan, "Solving Fuzzy Linear Programming Problems with Piecewise Linear Membership Function," *Applications and Applied Mathematics: An International Journal (AAM)* Vol. 05, Issue 2, pp. 504 – 533, December 2011.
- 13- Heinrich rommelfanger, "fuzzy linear programming and application," *European Journal of Operational Research* 92, 512-527, 1996.
- 14- Amit Kumar, Pushpinder Singh, Jagdeep Kaur, "Generalized Simplex Algorithm to Solve Fuzzy Linear Programming Problems with Ranking of Generalized Fuzzy Numbers," *TJFS: Turkish Journal of Fuzzy Systems* (eISSN: 1309–1190), Vol.1, No.2, pp. 80-103, 2010.
- 15- S. H. Nasser, E. Ardil, A. Yazdani, and R. Zaefarian, "Simplex Method for Solving Linear Programming Problems with Fuzzy Numbers," *World Academy of Science, Engineering and Technology* 10, 2005.
- 16- Masoud Sanei, "The Simplex Method for Solving Fuzzy Number Linear Programming Problem with Bounded Variables," *Journal of Basic and Applied Scientific Research, J. Basic. Appl. Sci. Res.*, 3(3), pp.618-625, 2013.
- 17- L. A. Zadeh, "Fuzzy Sets," *Information and Control* 8, pp.338--353 (1965).
- 18- Fleten, Stein-Erik, Maribu, Karl Magnus, "Investment Timing and Capacity Choice for Small-Scale Wind Power under Uncertainty," *e –scholarship, university of California, Lawrence Berkeley National Laboratory*, 2004.
- 19- A. Keane, M. O'Malley, "Optimal Distributed Generation Plant Mix with Novel Loss Adjustment Factors," *IEEE*, 2006.
- 20- S.H. Nasser, and E. Ardil, "Simplex Method for Fuzzy Variable Linear Programming Problems," *World Academy of Science, Engineering and Technology* 8, 2007.
- 21- Timothy J Ross, "Fuzzy Logic with Engineering Application," *Wiley Publication*, 2010.

