SHEAR DEMAND AND SHEAR DEFORMATION IN EXTERIOR BEAM-COLUMN JOINTS

A Thesis
Submitted by

MD Zeeshan Ali
(212ce2035)

In partial fulfillment of the requirements
for the award of the degree of

Master of Technology
In
Civil Engineering
(Structural Engineering)

Department of Civil Engineering

National Institute of Technology Rourkela

Orissa -769008, India

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Dr. PRADIP SARKAR

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National Institute of Technology Rourkela

Orissa -769008, India

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NATIONAL INSTITUTE OF TECHNOLOGY
ROURKELA, ORISSA -769008, INDIA

CERTIFICATE

This is to certify that the thesis entitle "**Shear Demand and Shear Deformation in Exterior Beam-Column Joints**" being submitted by **Md. Zeeshan Ali (Roll No. 212CE2035)** in the partial fulfilment of the requirement for the award for the degree of **MASTER OF TECHNOLOGY IN CIVIL ENGINEERING (STRUCTURE)** at the National Institute of Technology, ROURKELA is an authentic work carried out by him under my guidance and supervision. To the best of my knowledge no part of this thesis has been submitted for any other University/Institute for the award of any degree or diploma.

Place: Rourkela, Odisha

Date:

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ABSTRACT

Keywords: Beam-column joint, RCC, Crossed-rebar, Prestress, ANSYS, Shear force

Beam-column joint is the gap in the modern ductile design of building. Especially under the earthquake loading this is more susceptible to damage. Due to brittle nature of failure this type of failure cannot be afford. Since 1970’s this areas is under the light of research, but with the paper of Park and Paul, It got momentum. But still due to versatile nature of the joints core behaviour, the problem is still persisting.

The entire researchers till 1970’s believed that RCC beam-column joints behave as rigid joint. So in none of the pre 1970 building codes, they had not provided the confining reinforcement in the joints. With lot of damage and destruction of building due to shear force under earthquake force most of the code committee to introduce the confinement in the joints.

But recently due to use of high grade of concrete and better quality control in the RCC structures, confinements in the joints as per the new provision of codes leading us to the problem of the congestion. It has been observed at many construction sites that this congestion leads to poor workmanship at the joints, which actually making the joint more vulnerable than previous. Researcher has been working on this area to counter act by Increasing the size of the joints, Using the steel fiber in the joints, Using GRFP to wrap the joints, Prestressing the beam including the joint, Using of the crossed rebar at the joint cores. Due to prestressing of joint through the beam has not been so effective and economical, the present paper come up with the direct way of prestressing the joints. This paper tries to combine the benefits of the crossed rebar and prestressing in the joints together.

The present work is divided into two phase. In first phase few sample of normal low and medium high building has been chosen and designed according to the IS 456:2000(LSD) and shear force are calculated as per ACI 352-02. From this phase we come to conclusion that first two stories have higher shear force demand and these are the joints more susceptible to congestion and prestessing of joint core should be implemented to these joints only.

In the second phase two exterior beam-column joint from previous experimental programme. They were model and analyse using ANSYS v13. Improvement in the ultimate load and failure pattern has been detailed in the thesis. From this phase we come to conclusion that this new technique is more effective than the previous prestressing technique of joints.

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Chapter 1

Introduction

1.1 BACKGROUND:

Past is witness to many devastation and destruction of structure due to joint failures due to earthquakes. Beam-column joint has not been area of research for many decades because scientist believes that beam column joint behave as rigid joint with no deformation contributed by it. Beam-column joint has no problem in itself until the dead and live loads are concern. As soon as lateral loads, *i.e*. seismic force, comes into picture it will become a critical problem. This problem has not been solved completely till date. It can be seen how the time has evolved to witness the development in the understanding of the beam-column joint core behaviour, specially related to shear force and shear deformation. Still we have translucent vision about this area. In the following discussion an endeavour is just tried to remove the dust from this area so as to make it as clear as pure water.

As we know that, practically we can't construct the structure earthquake-proof, so there must be way out to earthquake problem. And we are fortunate enough that the solution come in only one term and that is ductility. Make the structure enough ductile and forget about the force which is going to come on it. So in short the solution to the problem of earthquake is ductility. So whatever going to come in the way of ductility and your structure you have to kill that, simple enough to understand? So in this process of removing our enemy through the research of 70 years in the seismic design, only beam-column joint shear failure is left behind. Before getting into the objective and scope of the project work on the beam-column joints an introduction is presented in the following sections.

What is beam to column joint?

The portion of the column where beam is use to join it is called beam-column joint. Beam-column joints are classified into three types based on the number of beams ending into the column

i) Interior Beam-Column joints

ii) Exterior Beam-Column joints

iii) Corner Beam-Column joints

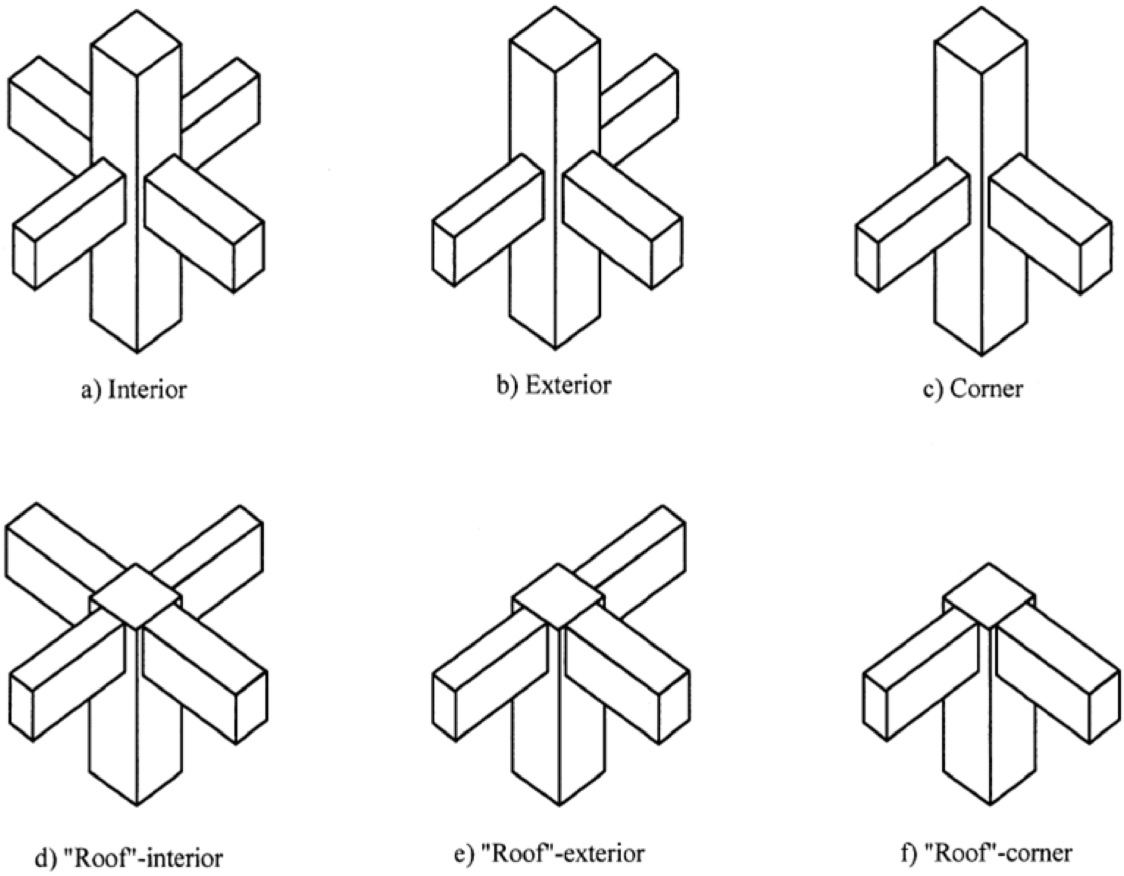


Fig: 1.1.1 Types of Beam-Column Joints (***ref: ACI 352-02***)

Background problem with the beam-column joints:

Beam-column joint is subjected to very high shear forces due to pulling of top rebar and pushing of bottom rebar’s or vice versa in the concrete structure especially during the earthquake loading. These very high shear force leads to the brittle damage, which can’t be accepted in the earthquake resistant building which has to be ductile in nature to deal with unseen forces. Building damaged by the joint failure is shown in Fig 1.1.2.

Fig: 1.1.2: Failure of Structure due to Shear Failure of the Joints (***Ref: webinar by Ben Deaton)***

These failures on the technical ground can be classified into three types as mentioned below:

i) Shear failure of the joint before plastic hinge in the beam, J.

ii) Shear failure of the joint after the plastic hinge in the beam, BJ.

iii) Bond failure of the longitudinal due to slippage of the bar due to excess tension in the bar.

From through study of the literatures on the beam-column joints it was interpreted that these individual or the combination of failure are depend on the sets of few parameter which are presented in the tabular form below.

The researchers are mainly concern about three things about the beam to column joints.

i. Deformation due to joint behaviour,
ii. Joint shear demand and
iii. Joint shear capacity.

Table 1.1 Factors affecting three different types of failure			
S. No.	J	BJ	BOND failure
1.	Longitudinal area of steel	Longitudinal area of the steel	Size of the column#
2.	Depth of the beam	Depth of beam	Diameter of bar
3.	Width of bay	Width of bay	Grade of concrete
4.	Height of story	Height of the story	Grade of the steel
5.	Height of the building	Height of the building	
6.	Lateral loading	Lateral loading	
7.	Column to beam capacity ratio*	Column to beam capacity	
8.	Presence of the slab+	Presence of the slab+	
9.	Confinement due to steel and the member##	Confinement due to steel and the member##	
10.	Types of joint**	Type of joint**	Type of joint**

***** If the column to beam moment capacity is large enough, say more than 2 then the joint failure will be shifted to the beam even the joint is under-designed. But current code recommend for the 1.2 factor. And so research is going to make this coefficient as close as to 1, without changing the concept of the strong-column-weak-beam.

As there is the chance of the slippage of the beam bar along the column, so the column bar along the beam but it has never been consider in any of the code; the probable reason for this may be the axial compressive force in the column.

******ACI 352-02 segregate the beam to column connection into 2 type; one is the type 1,which has to design for the strength only, and other is type 2 which has to be design for both strength and ductility. In this report we be discussing on the 2nd type of the joints only.

+ The slab contribution to the joint shear was fist consider in the ACI 352-02. Earlier (1960-1980's) set-up consisting of beam and column joint only has been criticized in the last 2 decades. So now a day the subassembly consist of the slab-beam-column connection are rigorously being studied. As per today slab contribution to the joint has not been fully understand, but there is monotonic voice from the scientist in the positive direction.

##Confinement reinforcement can be reduced up to 50% if the confinement for the interior joints is provided by the lateral member. So, interior joint is less venerable the other two type of the joint.

Fig. 1.1.3 Damage in the exterior joints *Kacholi earthquake*, 1999 (***Ref: webinar by Ben Deaton***)

Earlier in 1970's no codes has provided the joint confinement which leads to the major devastation like *Kacholi earthquake Turkey, 1999* and many more which had change the thought of researcher about joints, as shown in the Fig. 1.1.4.

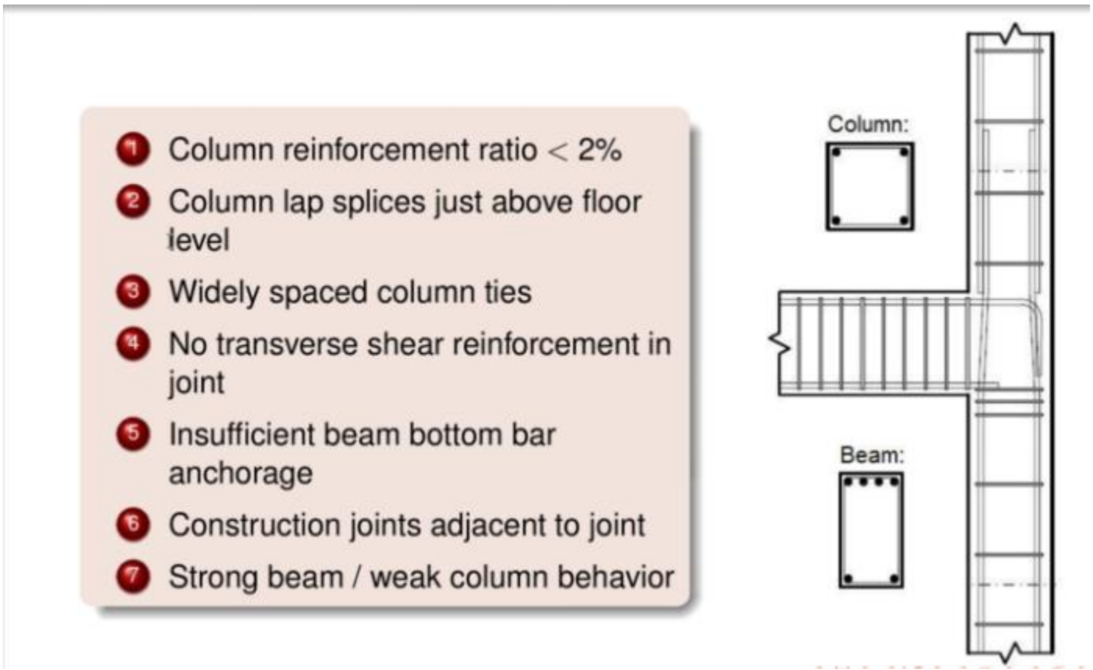


Fig. 1.1.4: Reinforcement detailing according to the Pre 1970Nonseismic Building Codes. (***Ref: Beres et.al., 1996***)

To prevent the damage due to joint shear failure they come up with idea of joint confining with the rebar. Confining the Beam-column joints isn't so easy because there are already rebar coming from three directions. With the extra provision as per the present codes confining stirrups leads to the problem of the congestion as shown in the Fig. 1.1.5.

Fig. 1.1.5: Congestion at the Beam-Column Joints (***ref.: www.concreteconstruction.net***)

The contribution of the slab in preventing the shear damage in the joints is very significant according to the ACI 352-02 but for the simplicity in the present studies it contribution has been ignored.

There are two very different ways to look any civil engineering problems. One is well known force based approach and other most important and obviously the difficult one is the deformation based approach. If you are just a civil engineer force base approach will be enough for you, but as earthquake engineer you have to be more acquainted with the deformation based approach. And the concept of ductility came from this approach. Earlier earthquake analysis was forced based so most of the paper published on the prediction of shear strength of the beam-column joint but as present earthquake analysis mostly relay on the performance of the building so inelastic deformation criteria is gaining momentum. Most of the recent papers are based on the deformation behaviour of joint under earthquake loading. ***Mitra (2008)*** in his peer report classified the first beam-column research as first generation beam-column joint and later as second generation beam-column joints.

Global behaviour of the structure depends on the individual behaviour of the many components of the structure and their relative damages. If every component behaves in the ductile manner, the global behaviour will be ductile, but if even one of them can't pass the required ductility criteria, the whole structure has to suffer. We are fortunate enough that we have sorted out the problem linked to every components of the structure except the one i.e. beam-column joints. So because of the reason, this becomes very important to go along with it to explore it and find the solutions. In seismic design, reinforced concrete structures must perform satisfactorily under severe load conditions. To withstand large lateral loads without severe damage, structures need enough deformation and better energy dissipation capacity instead of strength. It is commonly accepted that it is uneconomical to design reinforced concrete structures for the greatest possible earthquake ground motion without damage. Therefore, the need for strength and ductility has to be weighed against economic constraints. Ductility is an essential property of structures responding in-elastically during severe earthquakes. Ductility is defined as the ability of sections, members and structures to deform in-elastically without excessive degradation in strength or stiffness. The most common and desirable sources of inelastic structural deformations are rotations in potential plastic hinge regions. An energy dissipation mechanism should be chosen so that the desirable displacement ductility is achieved with smallest rotation demands in the plastic hinges. Development of plastic hinges in frame columns is usually associated with very high rotation demand and may result in total structural instability (global failure).

While for the same maximum displacement in a structural frame system, the rotation demand in the plastic hinges would be much smaller if they developed in the beams. For getting an efficient performance of beam at beam-column joints we need to give proper anchorage which will provide proper dissipation of energy and ductility to the structure. Otherwise the failure may occur due to the poor anchorage at the joint by pulling out of the beam longitudinal bars from the joints.

Current design philosophy requires that beam-column joints have sufficient capacity to sustain the maximum flexural resistance of all the attached members. The mechanism of force transfer within beam column joint of a rigid frame during seismic events is known to be complex involving bending in beams and columns, shear and bond stress transfer in the joint core. To provide proper anchorage of beam at the joint, various countries like India, USA provides special detailing on and near hinged zones. The primary aim of joint design must be to suppress a shear failure. This often necessitates a considerable amount of joint shear reinforcement, which may result in construction difficulties. Current seismic code details for reinforced concrete structures are often considered impracticable by construction and structural engineers because of its installation and the difficulties in placing and consolidating the concrete in the beam column joint regions For high seismic zones, load reversals in the joint can lead to significant bond deterioration along straight bar anchorages; therefore, American Concrete Institute (ACI) and Indian Standard(IS) requires that standard hooks be used to anchor longitudinal reinforcement terminated within an exterior joint. The use of standard hooks results in more steel congestion, making the fabrication and construction more difficult.

In the present scenario of the earthquake engineering raising above the rigid beam-column joints is very important (for the actual prediction of the behaviour of the building under lateral loading especially deformation based loading like earthquake) but also very challenging (till date nobody has been come up with the satisfactory mathematical model of how the overall performance of building because of shear deformation of individual beam-column joints). Till date many scientists tried to incorporate the beam-column joint shear stress-strain behaviour to the classical beam plastic hinge to model the actual behaviour of the structures. But they are also diverse in their opinion.

While for the same maximum displacement in a structural frame system, the rotation demand in the plastic hinges would be much smaller if they developed in the beams. For getting an efficient performance of structure at beam-column joints we need to give proper anchorage, which will provide proper dissipation of energy and ductility to the structure. Otherwise the failure may occur due to the poor anchorage at the joint by pulling out of the beam longitudinal bars from the joint. If we compare the vulnerability of the all three types of joints then by the past experience we will find that exterior joint is more prone to damage. Because the exterior joint is less confined and subjected to high shear demand. So according to my study preventing the damage in the exterior joint is more critical than other joints. So my study is especially directed to these types of joint only.

1.2 MECHANICS OF BEAM-COLUMN JOINT CORE: SHEAR FORCE

Shear force is very critical in the earthquake resistance design of the structure because of it induce brittle failures. But if the structure is subjected to lateral force due to wind or earthquakes most of the shear force is being concentrated in the joint cores, which leads to the brittle failure of the many structure in the past earthquakes. Even though the mechanic of the calculation of the shear force in the joint core is very simple it had been ignore for many decades with the wrong assumption of the rigid joint behaviour. The detail mathematical formulae to calculate the shear force demand and shear force capacity has been well presented in Chapter 2.

1.3 MECHANICS OF BEAM-COLUMN JOINT: SHEAR DEFORMATION

Deformation of the joints contributes significant lateral drift of the story and the global story displacement. But due incapability to calculate the shear deformation most of the code till present assume the rigid joint behaviour of the joint. Which may sometime leads to significant error in the calculation of the max story displacement. Estimation or calculation of lateral story drift due to shear deformation of the joint is very challenging. From the past many scientist has tried to solve this riddle. They proposed many different type of models starting with the rigid joint assumption, matrix method based on the central line analysis, implementation of the panel zone concept to add the shear deformation, adding rotational hinge and the use of full scale finite element analysis etc. with every advancement they are moving forward to the accurate estimate of the shear deformation. Detailed version will be discussed in the literature review section. Here we will over view the status of estimation and contribution of shear deformation in the global deformation of the building. Following are the deformation model propose in the timeline orders

1. Conventional rigid joint model
2. ASCE/SEI 41-06 joint model
3. Modelling inelastic joint action within the beam-column element
4. Rotational hinge models
5. Continuum models and FEM

1.4 FINITE ELEMENT ANALYSIS:

FEA is a powerful computational technique for approximate solutions to a variety of complex "real-world" engineering problems having complex domains subjected to general boundary conditions. FEA has become an essential step in the design or modelling of a physical phenomenon in various engineering disciplines including civil engineering, aeronautical engineering and many more. The second phase of this project is completed with the finite element software ANSYSv13. An introduction about the finite element method has been presented in following sections.

Background:

According to Wikipedia, exact date for the origination of the finite element method is very hard to say, but this method serves as the greatest tool to solve the complex and impossible structural analysis problems. Its origination is believed to the work ***by A. Hrennikoff*** and ***R.***

Courant. In China, in the later 1950s and early 1960s, based on the computations of dam constructions, K. Feng suggested a systematic numerical method for solving partial differential equations. The method was called the finite difference method based on variation principle, which was another independent invention of finite element method. Although the approaches used by these pioneers are different, they share one essential characteristic, mesh discretization of a continuous domain into a set of discrete sub-domains, usually called elements.

Finite element method:

This is a procedure for the numerical solution of the equations that govern the problems found in nature. In mathematical term, FEM is a numerical technique for solving partial differential equations. Any natural problem which can be model into partial differential equations can be solved through this methods i.e. structural problems, computational fluid dynamic problems, electromagnetic problems etc. But it should always be remember that, this is an approximate method and result must be validated before use. When referred to the analysis of structures the FEM is a powerful method for computing the displacements, stresses and strains in a structure under set of loads.

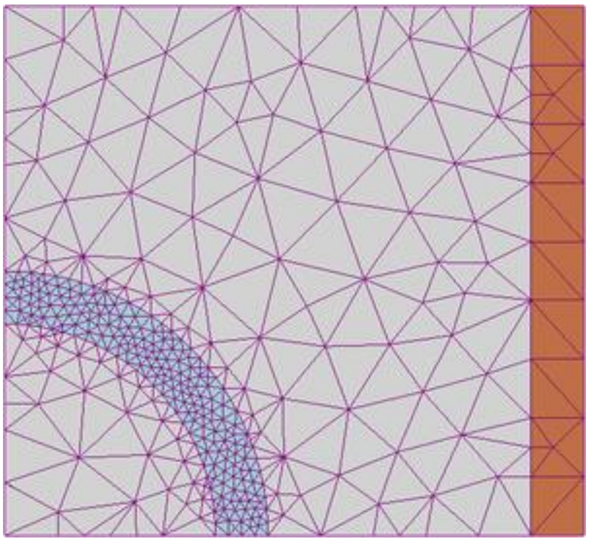


Fig. 1.4.1: Meshing in the FEM (***ref: www.wikipedia.com***)

Finite element:

A finite element can be visualized as a small portion of a continuum. In FEM the structural elements/ model i.e. beam, slab, wall etc. are meshed into small units called finite elements. As you can see in Fig. 1.4.1

FEA is one of the economical ways to perform the virtual experiment without the costly lab setup. Also this is very much reliable with the advancement in the speed and accuracy of the computing. But as the FEM is the approximate method the validation of it results are very challenging. A very deep and expert knowledge of structural behaviour and FE is required to reach at the authenticated and most reliable results. One must proceed with very systematic manner. The first step in the solution of a problem is the identification of the problem itself i.e. which is more relevant physical phenomena influencing the structure? Is the problem is static or dynamic nature? Are the kinematics or the material properties is linear or non-linear? Geometric non-linearity should be incorporated or not? What is the level of accuracy? What are the results sought?

According to ***Onate (2009)*** FEM is based on three types of models: conceptual models, structural models and numerical method/model. Computational methods such as the FEM are applied to conceptual models of the real problem, and not to the actual problem itself. Even experimental methods in structural laboratories make use of the scale reproductions of the conceptual model chosen (also called physical models) unless the actual structure is tested in real size, which rarely occurs. A conceptual model can be developed once the physical nature of a problem is clearly understood. In this one can exclude superfluous details depending upon the accuracy and results required.

After selecting a conceptual model of a structure, the next step for the numerical study is the definition of a structural model (sometime called mathematical model). The structural model must include three fundamental aspects such as the geometrical description, mathematical expression of the physical laws and property of material and loads and constraints.

Next is the numerical method such as FEM. The application of the FEM invariably requires its implementation in the computer code. And the outcome of the process is computation model.

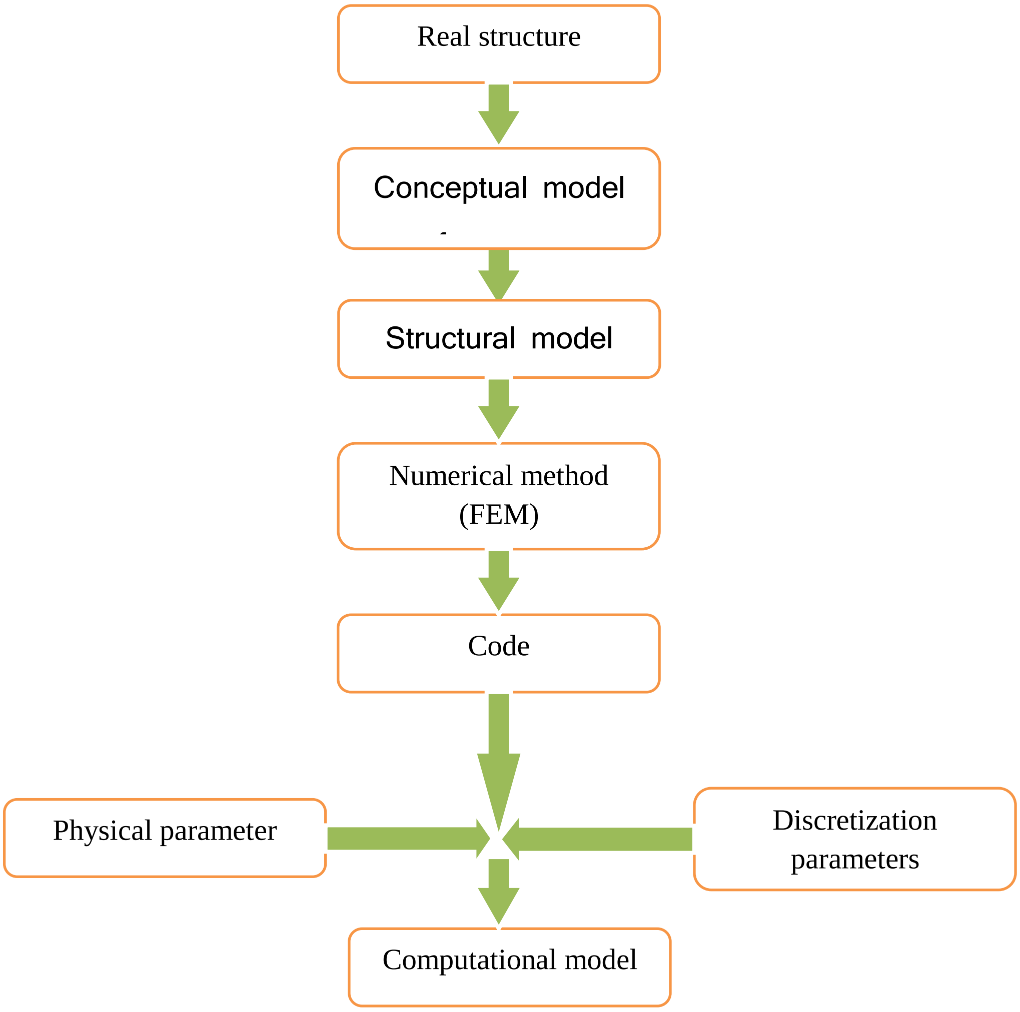


Fig.1.4.2: Flow chart showing the development of computational model in FEM ***(ref*: Onate, 2009)**

The finite element method proceeds to solve any complicated problems in following general steps:

1. The first job in the analysis of a structure by FEM is to select the element types for different parts of the structural components i.e. a choice of interpolating/ shape function according to the component in supposed to behave/deform.

2. Model the structural components with desire geometry and material properties which are going to affect the desire result.

3. Discretization of the components into a finite number of elements.

4. Then apply the desire boundaries condition and the forces field.

5. Software will develop the element stiffness matrixes for each element.

6. Then these will be combined to form a global stiffness matrix and force and displacement matrix is being generated.
7. Finally the solution of equations is done.

These above steps are made very easy with the GUI of FE software's like ANSYS, ABAQUS, DIANA etc. Each software has some features and other has another which is making one superior over another. Modelling of concrete model is very challenging task in any of the above FE software where achieving the convergence after the initial crack is very hard and need very thorough knowledge about every functions and options of the software. Every FE software has predefined elements for the given type of material and function. In ANSYS the cracking and crushing of the concrete can be model with the SOLID65, which can precisely estimate the cracks and crushing in the reinforced concrete and fairly good in predicting the failure of the section. The rebar can be model in discrete way or smeared way. Both options can fairly predict the results. In discrete modelling of the rebar LINK8 or BEAM188 can be used. In LINK you can only give the area of the rebar but in the BEAM you can directly give the shape and radius of the rebar. LINK is only defined to take either the tension or compression but the BEAM can even take the bending stress. But at that small diameter there is no significant difference in using the BEAM instead of LINK. But remember that LINK is more appropriate to use as rebar in the reinforced concrete. The modelling parts in ANSYS will be discussed in more detail in the chapter 3 under FINITE ELEMENT MODELLING.

1.5 OBJECTIVE

With introduction presented in this chapter and literature review in the next chapter the salient objective of the present study is presented below:

- To find the joint height which is more critical from the point of view of reinforcement congestion and maximum joint shear demand.
- To find the effectiveness of the direct joint prestressing to divert the failure from the joint to the beam by reducing the shear demand at the joint by combine effect of crossed rebar and prestressing.

1.6 SCOPE OF THE STUDY

Following are the scope of the present study

- As most of the congestion problem came in the high rise building but only low-rise and midrise building as it can be justified because most of the building in India fall under these range.

- Bond slip has not been considered but it is very obvious that due to confining the band capacity will also increase preventing the damage due to slip of the rebar.
- As the dynamic nature of any earthquake is most critical for the damage of the joint but for study static loads has been applied to study the effect of the prestressing of beam-column joint.

1.7 METHODOLOGY

The present work is divided in two phases. The first phase is to find the critical joints with respect to the reinforcement congestion and shear force demand. And second phase deals with the effectiveness of the direct prestressing of the beam-column joint in mitigating the brittle failure at the joint to the ductile failure in the beam. An introduction to methodology of both phase are presented here. More detailed one is presented in the chapter 3.

First Phase Methodology:

1. Few samples of the low and midrise 2D building are selected with standard dimensions and standard loading.
2. All building is being designed as per IS 456:2000(LSD).
3. Shear force has been calculated as per ACI:352-02
4. Critical joints have been shorted out on which the prestressing is being applied as going to be proposed in the phase 2.

Second Phase Methodology:

1. Two exterior beam-column joints which were going to fail at joints due to shear failure have been selected from the literature.
2. Both the joint has been modelled in ANSYS v13 as per the experiment performed in the literature to verify the result.
3. Direct prestressing is implemented in ANSYS model on both of the joints to see the improvement in shear deformation, shear strength, shear demand and failure pattern.

1.8 ORGANIZATION OF THE THESIS:

The thesis is divided into 5 chapters starting with title page, certificate, acknowledgement, table of contents, list of figures, lists of table and finally references in the last.

Chapter 1 present the overviews of the beam-column joints showing the significance of the study of the beam-column joints with the introduction of the beam column joints. The objectives and scope of the proposed research work are identified in this chapter.

Chapter 2 present the detail review of the building codes along with the literature of the beam-column joints. It is divided into four parts, 1st for general outlook of the review with code review, 2nd for the shear capacity and demand of the joints, 3rd for the shear deformation in the joint, 4th for the effect on pre-stressing the beam on joint.

Chapter 3: Methodology and present work

Chapter 4: Result and Discussion

Chapter 5: Conclusion

Chapter 2

Literature review

2.1 GENERAL:

The research in the field of the beam-column joint is gone back to 1940's. But after the 1970's the research get momentum. There are lot of papers related to this areas publish in many journals and conferences all over the world. Below we will discuss in detail, the literature review on beam-column joints. This can be classified into different broad section for easy understanding of the research motive. The present thesis has been divided the literature review into four broad sections. They are as below

1. Review of the codes.
2. Literature review on the shear capacity and demand.
3. Literature review on the shear deformation calculation.
4. Literature review on the pre-stressed joint.

The work tried to capture all the research which could be related to my present study. Some time it may look to you that unnecessary papers have be reported, but a deeper study will reveal a deep relation of the paper with the present work.

2.2 REVIEW OF THE CODES:

Before moving to the literatures, it is always good to see the stand of the various countries codes. Fortunately due to awareness and researches toward the earthquake hazards, we have many codes dealing with the beam column joints. But few are dealing in details as mention below. IS 13920:1993 (but it's going to revise soon, probably in 2015), ACI 352-2002, ACI 318.2011, NZS 3101:2006, EN 1998:2003. Except the Indian code the basic concept of all the international code is same. The general steps for the calculation of shear force and design of beam-column joints, in brief, are as follows:

a) Adopt the column width based on bond conditions for anchorage of beam bars.
b) Ensure the column to beam moment capacity ratio is adequately high (1.2 to 1.4) to achieve the desired beam yielding mechanism prior to the column rebar yielding.
c) Calculate the shear force demand in the joint from the flexural strength of the adjoining beams and the shear force in the adjoining columns.
d) Calculate the joint shear capacity from the effective joint shear area and the allowable shear stress in the joint as specified by the different codes. Verify the joint shear capacity is more than the calculated demand; if not, increase the member dimensions suitably. And Provide shear reinforcement in the joint as per the code requirements.

e) But in IS13920:1993 the just provide the special confining reinforcement depending on no. of beam merging in the joints.

But the reinforcing detailing sometime becomes so congested to hinder the workmanship of the casting concrete in the joints.

Following are the methodology followed by all the codes to calculate the shear force in the different kind of the beam-column joints.

Exterior joints:

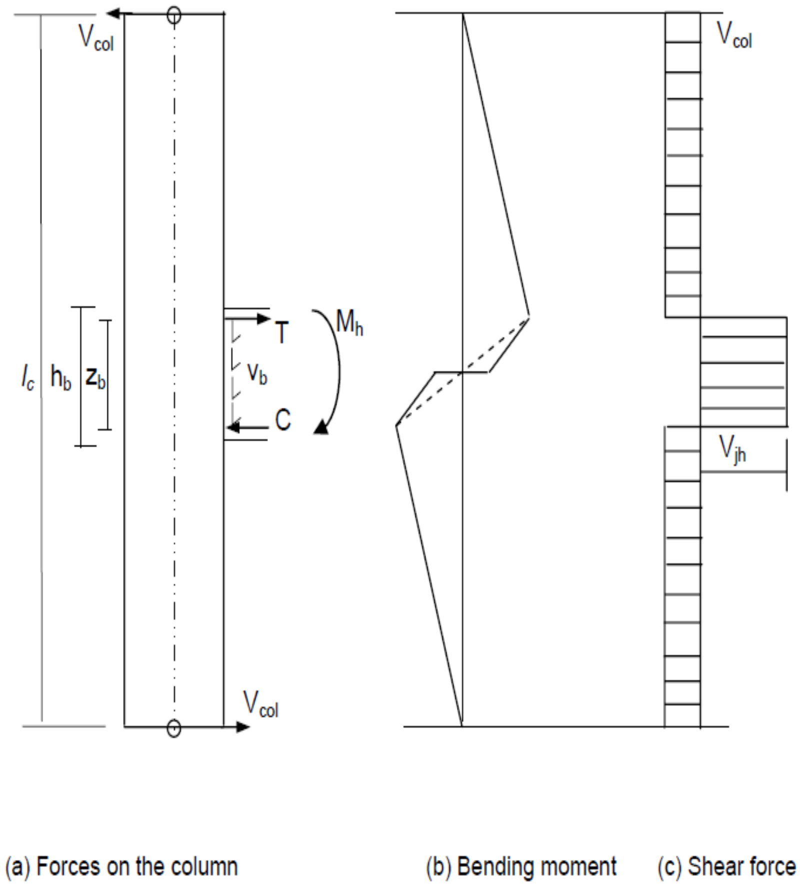


Fig: 2.2.1 Details of the forces acting at the exterior beam-column joints

Mathematically the joint shear can be calculated as following:

$$V_{col} = \frac{T_b Z_b + V_b \frac{h_c}{2}}{l_c}$$

Where, V_{col} is shear force in column.

But for the design purpose we use the simplified version of above equation as below:

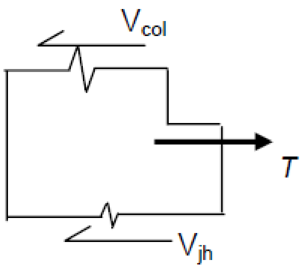
$$V_{col} = \frac{M_h}{l_c}$$

M_h is max of hogging or sagging moment in the connect beam to the column at the given exterior joint.

l_c is the centre to centre distance of the column.

The tension force in the reinforcement which is going to transfer into the joint cores can be given by following equation:

$$T_b = 1.25 f_y A_{st}$$



And finally the horizontal joint shear can be calculated by subtracting the above two

$$V_{jh} = T_b - V_{col}$$

Interior joints:

Column shear force is calculated as following in the interior joints:

$$V_{col} = \frac{2T_b Z_b + V_b h_c}{l_c}$$

But for the design purpose we use the simplified version of the equation

$$V_{col} = \frac{M_h + M_s}{l_c}$$

Tension in the longitudinal bars can be calculated as

$$T_b = 1.25 \times f_y A_{st}$$

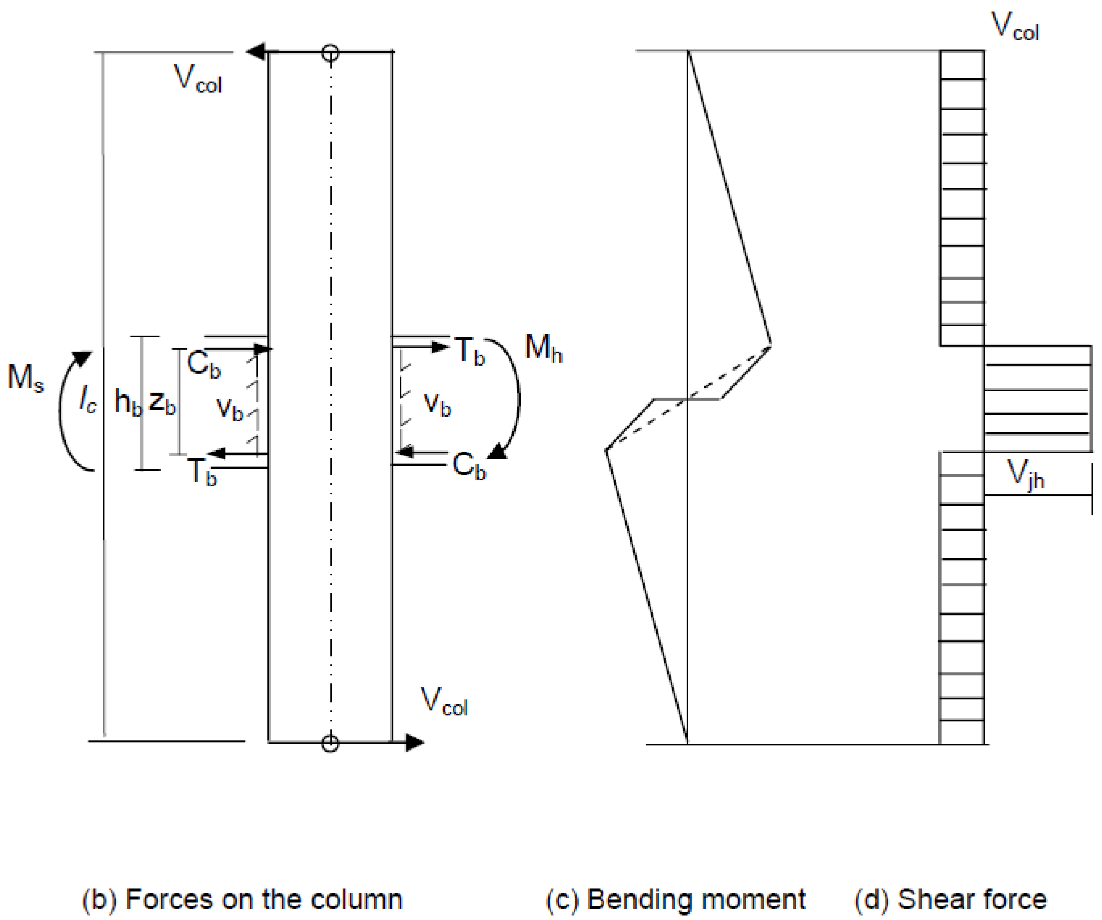
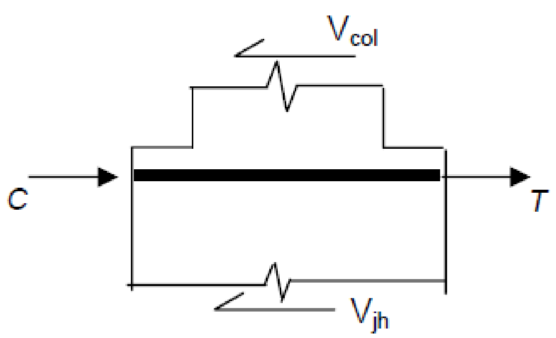


Fig: 2.2.2 Detail of forces acting at the interior beam-column joints.

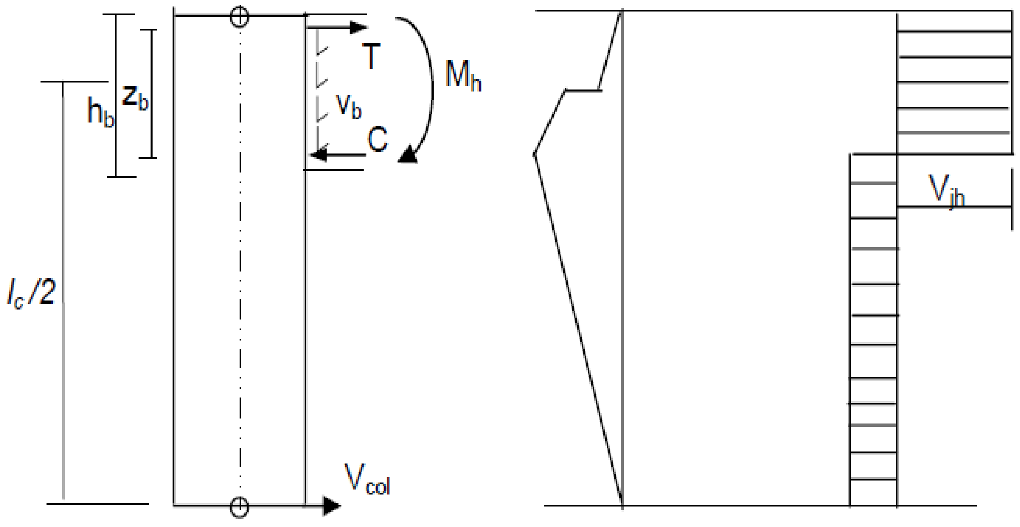
Finally the shear force in the interior joint can be calculated as below



$$V_{jh} = C + T_b - V_{col}$$

$$V_{jh} = 2 \times T_b - V_{col}$$

Corner joints:



(a) Forces in the Column (b) Bending Moment (c) Shear Force

Fig: 2.2.3 Detail of the forces acting at the corner beam-column joints

Column shear force is calculated as following in the corner joints:

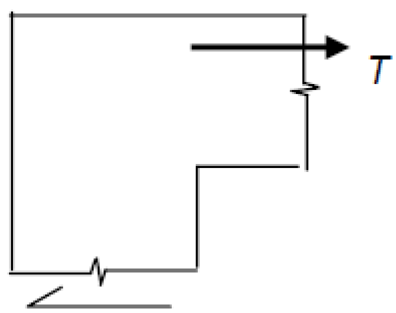
$$V_{col} = \frac{T_b Z_b + V_b h_c / 2}{l_c / 2}$$

But for the design purpose we use the simplified version of the equation

$$V_{col} = \frac{M_h}{l_c / 2}$$

Tension in the longitudinal bars can be calculated as

$$T_b = 1.25 \times f_y A_{st}$$



Finally the shear force in the interior joint can be calculated as below

$$V_{jh} = \frac{V_{col}}{2}\left(\frac{l_c}{Z_b} - 1\right) - V_b\left(\frac{h_c}{2Z_b}\right)$$

$$V_{jh} = T_b - \frac{V_{col}}{2}$$

All the codes follow the same formulae to calculate the shear force in the joints. But every country code has given their own formula to calculate the shear capacity of the joints. Which has be discussed in detail in the literature review section for the codes. Where you can find which codes has taken which parameter in their formula.

2.2.1 Indian Code IS13920:1993

Indian codes have given the following provision in regard to the beam-column joints. The clause 8 of the IS 13920 deals with the detailing of the beam-column joint irrespective of their shear demand. Following are the statement from the code.

Clause 8.1 The special confining reinforcement as required at the end of column shall be provided through the joint as well, unless the joint is confined as specified by **8.2**.

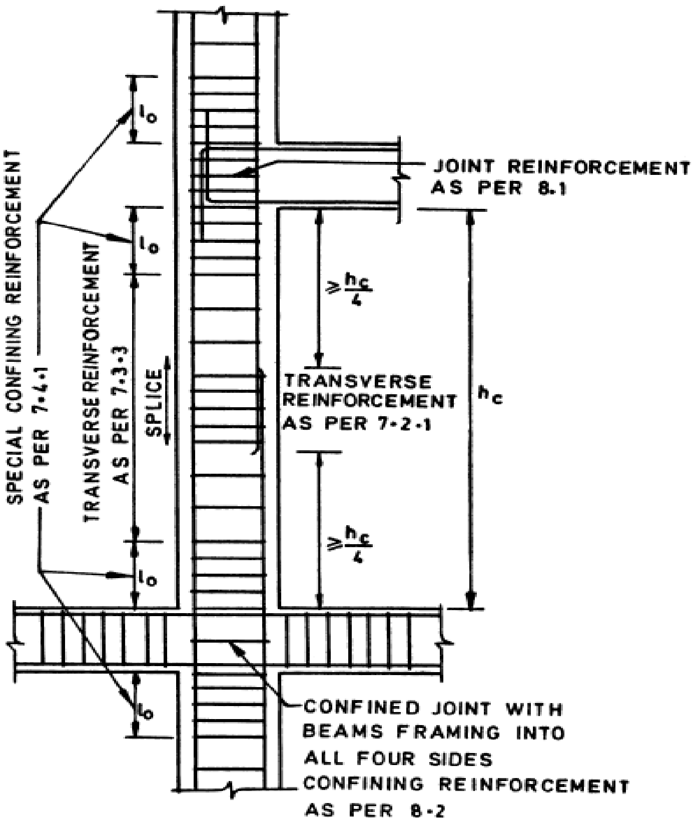


Fig: 2.2.1.1 Special reinforcement in the joints (***ref. IS 13920:1993***)

Clause 8.2 A joint which has beams framing into all vertical faces of it and where each beam width is at least 3/4 of the column width, may be provided with half the special confining reinforcement required at the end of the column. The spacing of hoops shall not exceed150 mm.

One of the drawbacks of the code is that it has not given the way to calculate the shear force and neither to calculate the shear capacity of the joint, as other international codes have mentioned. It is believed that by following the codal provisions shear capacity and the shear demand will automatically will be satisfied. Unfortunately even though the draft of IS 13920 by IIT Kanpur recommended including it the new revision has not updated this clause in the new revision which is expected to the end of 2014.

2.2.2 American Code: ACI 352-02

ACI has published a special edition of code on the beam-column joints, different from their original concrete code ACI 318:2011. This code deals with the design and detailing of the beam-column joint for both earthquake and wind loading. One of the most important things the code has given is the formula for the estimation of the shear capacity of the joints. Many researchers have reported the significant influence of the core stirrups, but this code has ignored the contribution to add for the Factor of safety. According to this code the joint shear capacity is given as below:

$$\text{Shear capacity } V_n = 0.083 \times \gamma \times \sqrt{f_c} A_j$$

Where, area of the joint $A_j = b_j \times h_c$

Width of the joint b_j should not exceed the smallest of following

$$\text{(i)} \frac{b_b + b_c}{2}$$

$$\text{(ii)} b_b + \sum \frac{mh_c}{2}$$

$$\text{(iii)} b_c$$

Where, b_b is the width of the beam heading into the joint.

b_c , is width of the column parallel to the beam

h_c , is the depth of the column perpendicular to the beam

2.2.3 Japanese code: Architectural Institute of Japan (AIJ)

The provision of the Japanese code is same as ACI 352-02, with some little modification in the formula for the calculation of the shear capacity of the joints. This can be given as following.

$$\text{Shear capacity } V_n = k \times \phi \times 0.8 (f_c)^{0.7} A_j$$

Where, k is joint shear strength factor.

ϕ , denote the effect of transverse beam(s) out of plane

f_c , cylinder compressive strength of the concrete in the joint core

A_j , area of the joints same as specified in ACI 352-02 as defines in above section

2.2.4 New Zealand Code: NZS3101

This is only code which has provision for the pre-stressed joints. This code has given provision for the calculation of the area of the steel in the joint cores, instead of calculating the shear capacity of the joint.

The maximum horizontal joint shear force $\mathrm{V_{jh}}^*$ shall not exceed the smaller of $0.2 f_c^{'} b_j h_c$ or $10 b_j h_c$ where h_c is overall depth of the column in the direction of the horizontal shear to be calculated and effective joint width.

$$\mathrm{V_{jh}}^* \leq \phi V_{jh}$$

The area of total effective horizontal joint shear reinforcement corresponding with, each direction of horizontal joint shear force.

$$\mathrm{A_{jh}} = \frac{V_{jh} - \phi V_{ch}}{\phi f_{yh}}$$

Where, $\phi \mathrm{V_{ch}} = V_{jh}^* \times (0.5 + \frac{C_j N^*}{A_g f_c^{'}})$

Design basis for vertical joint shear reinforcement

$$\mathrm{V_v^*} \leq \phi V_{jv}$$

$$\mathrm{A_{jv}} = \frac{\frac{V_{jh}^* h_b}{h_c} - \phi V_{cv}}{\phi f_{yv}}$$

Where, $\phi V_{cv} = 0.6 \times V_{jh}^* \frac{h_b}{h_c} + C_j N^*$

Horizontal joint shear reinforcement

(i) For Interior Joints

$$A_{jh} = \frac{6 \times V^*_{ojh}}{f'_c b_j h_c} \left(\frac{\alpha_i f_y A^*_s}{f_{yh}} \right)$$

Where, $0.85 \leq \dfrac{6 \times V^*_{ojh}}{f'_c b_j h_c} \leq 1.2$

$\alpha_i = 1.4 \alpha_n$

Or, where the beneficial effects of axial compression loads acting above the joint are included

$$\alpha_i = \left(1.4 - 1.6 \frac{C_j \times N^*_o}{f'_c A_g}\right) \alpha_n$$

α_n Depends on the sectional curvature ductility

A^*_s, is the greatest of the area of the top or bottom beam reinforcement passing through the joint. It excludes bars in effective tension flanges.

(ii) **For Exterior Joints:**

$$A_{jh} = \frac{6 \times V^*_{ojh}}{f'_c b_j h_c} \left(\frac{\beta f_y A_s}{f_{yh}} \right) \left(0.7 - \frac{C_j N^*_o}{f'_c A_g}\right)$$

Where, $0.85 \leq \dfrac{6 \times V^*_{ojh}}{f'_c b_j h_c} \leq 1.2$

N^*_o, is taken negative for axial tension in which case $C_j = 1$ must be assumed

$\beta =$ Ratio of area of compression beam reinforcement to area of tension beam reinforcement, not to be taken larger than unity.

(iii) For Prestressed Joints:

Where beam are prestressed through the joint, the horizontal joint shear reinforcement required by above

$$\Delta A_{jh} = \frac{0.7 \times P_{cs}}{f_{yh}}$$

P_{cs}, is the force after all losses in the prestressing steel.

2.2.5 European code: EU8

This code has given a more elaborated formula for the calculation of the shear capacity of the joints. This also includes the effect of axial load which has been reported in many of the literatures. For the interior joints the shear capacity formula is as below

$$V_n = \eta \times f_c \sqrt{(1 - \frac{\upsilon_d}{\eta})} \times b_j h_c$$

Where, $\eta = 0.6 \times (1 - \frac{f_{ck}}{250})$

$\upsilon_d =$ Normalized axial force in the column above the joints

$f_{ck} =$ Cube compressive strength in MPa

The width of the joint b_j is minimum of the

(i) b_c Or $b_w + 0.5 \times h_c$ if $b_c < b_w$

(ii) b_w Or $b_c + 0.5 \times h_c$ if $b_c > b_w$

For the exterior joints the above equation can be multiplied by the factor of 0.8 and everything will remain same.

Adequate confinement of both horizontal and vertical of the joint should be provided, to limit the maximum diagonal tensile stress of concrete maximum σ_{ct} to f_{ctd} , to prevent the developing of crack which leads to premature stiffness loss. Code says that in the absence of more precise model, this requirement may be satisfied by providing horizontal hoops with a diameter of not less than 6mm within the joint, such that

$$\frac{A_{sh}}{b_j} \times \frac{f_{ywd}}{h_{jw}} \geq \frac{\left(\frac{V_{jhd}}{b_j \times h_{jc}}\right)^2}{f_{ctd} + \upsilon_d f_{cd}} - f_{ctd}$$

Where, A_{sh} is the total area of the horizontal hoops

V_{jhd} = shear force demand of the joints

h_{jw} = the distance between top of the beam reinforcement and the reinforcement at the bottom of beam.

b_j = Width of the joint core

$\upsilon_d =$ Normalized axial forces of the column

$f_{cd} = \frac{f_{ck}}{1.5}$

2.3 SHEAR FORCE DEMAND AND CAPACITY:

Bakir and Boduroglu (2002) proposed a model for the prediction of the shear strength of the beam-column joints. The paper considers the three new parameters for the first time to predict the shear strength of the joint. These parameters are beam longitudinal reinforcement ratio, beam-column joint aspect ratio and the influence of stirrups ratio. It concluded that beam longitudinal reinforcement ratio has positive effect on the joint shear strength. Because the influence of beam longitudinal reinforcement ratio is taken into account, the proposed equation predicts that the joint shear strength is proportional to $(h_b/h_c)^{0.61}$.The paper also concluded that the column axial load has no effect on the shear strength but the high column axial load and high column longitudinal reinforcement is required to prevent the column failure.

Park and Mosalman (2009) given a shear strength model of the exterior beam-column joints without shear reinforcement, which can be useful in required confinement reinforcement to prevent the shear damage.

Muhsen and Umemura (2011) proposed a model to estimate the strength of the interior beam-column joint with consideration of the confinement reinforcement and axial force. The proposed model is similar to the current ACI and AJI codes with little modification in the effective area of the joint panel and considering the confinement due to axial force in the column and confinement reinforcement in the joint core. None of the codes has considered the confinement effect in the estimation of the shear strength of the beam-column joint.

Pimanmasa and Chaimahawanb (2010) present paper to prevent the beam-column joints by enlarging the joint area. The paper concluded that the joint enlargement as shown in the Fig: 2.2.1 is a very effective method to reduce the shear stress transmission in the joint panel and hence effective in preventing the damage. There has been also change in the failure mode with the relocation of the plastic hinge from the face of the beam to the face of the enlarge section. The model is well explain with the strut and tie model.

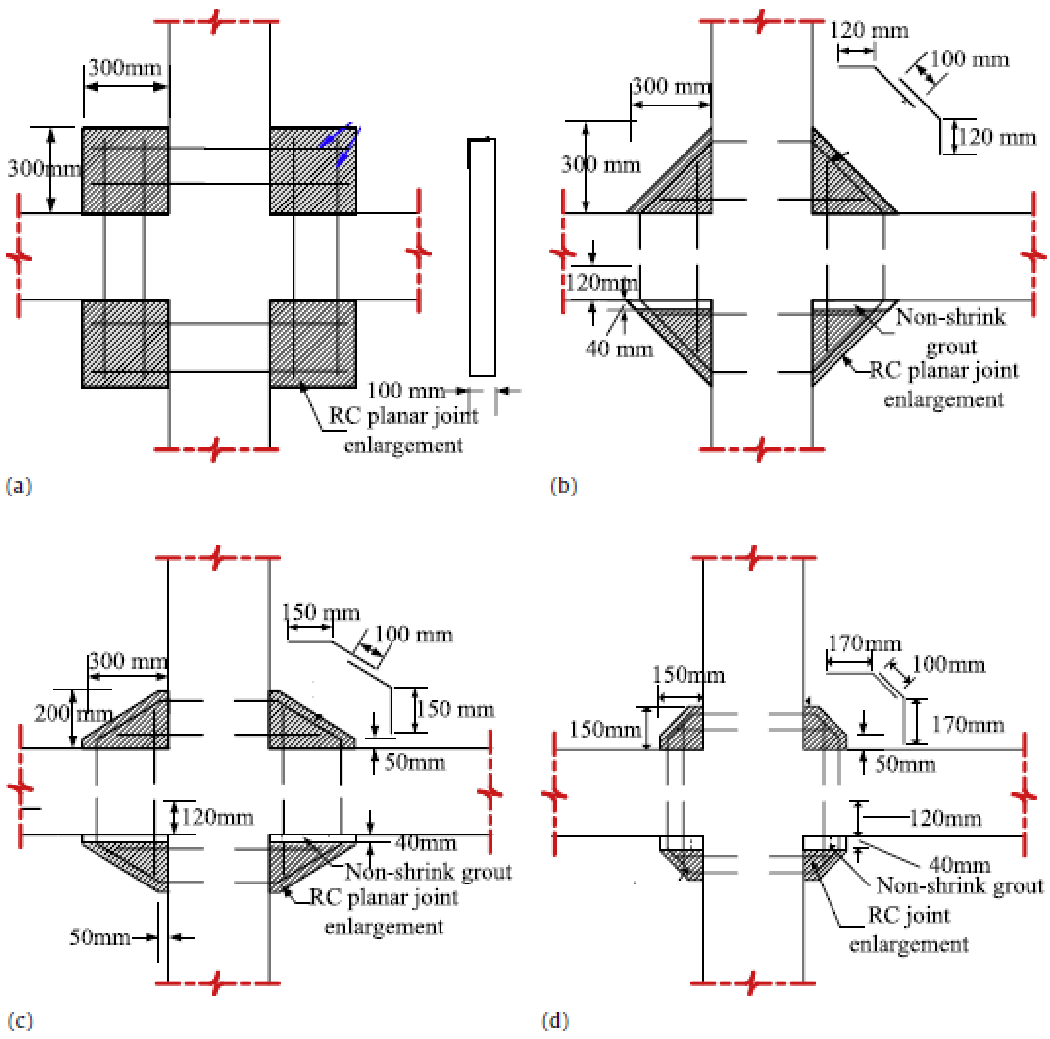


Fig 2.3.1: Dimension of different type of joint enlargement ***(ref: Pimanmasa and Chaimahawanb, 2010)***

Kang and Mitra (2012) proved that the increasing development length, head thickness and head size and decreasing joint shear demand gives better beam-column joint performance. The paper also showed that increasing rebar yield strength, joint confinement reinforcement and axial load leads to unpredictability of the performance of the beam-column joints. After going through the every parameter they found that joint shear demand and bar yield stress are two major parameters from influential point of view.

Jung et. al. (2009) has given a method to predict the deformation of the RC beam-column joints with *BJ* (joint failure after hinge formation in the beam) joint failure. Also it shows that the deformation of the joint increases with the decrease in the beam rebar. The paper has given method to calculate the ductility capacity of the beam-column joints.

2.4 BEAM-COLUMN JOINT DEFORMATION MODELS:

Modelling of the building against earthquake forces and any other types of lateral forces is based on the inelastic plastic hinge formation in the beam, slab and wall etc. But following researches proved the contrary (***Meinheit and Jirsa, 1977; Durrani and Wight, 1985; Park and Ruitong 1988; Leon, 1990; Clyde et al., 2000; Mazzoni and Moehle, 2001; Lowes and Moehle, 1999; Walker, 2001***) and showed that there are significant contributions by the beam-column joints to the overall deformation in the structure. So scientist has shown that the deformation contribution by beam-column joints can even goes up to 40% of the total deformation due to both elastic and inelastic deformation. Researcher has been trying to develop many different mathematical and FE model to accurately predict the deformation in the joint cores. As per study of different beam-column joint deformation models, the following literature review has been classified into five broad classes. This is mention below.

2.4.1. Conventional Rigid Joint Model

A common engineering practice has been to model the beam-column joints in concrete frames as rigid elements spanning the full joint dimensions. Some analysts have recognized that this model overestimates stiffness and instead have used a model in which the beam and column flexibilities extend to the joint centre-line. Studies show that the rigid joint model overestimates stiffness and underestimates drift because of ignoring join shear deformations and slip of reinforcement. The centre-line model can overestimate or underestimate stiffness. Rigid joint stiffness overestimation shortens natural period and affects the attracted seismic forces. Recent tests by ***Hassan (2011)*** showed that joint flexibility contributed significantly, up to 40%, to overall drift, especially in the nonlinear range.

2.4.2. ASCE/SEI 41-06 Nonlinear Joint Model:

ASCE/SEI 41-06 suggests modelling joints in concrete frame linear analysis using rigid links that cover partially or fully the joint dimensions. The modelling approach accounts for beam bar slip rotation using reduced flexural column and beam stiffness. For nonlinear analysis, ASCE 41 suggests a backbone curve for joint shear strain modelling, with shear strength based on the number of members framing into the joint.

However, approaches to implement this model are not described. It is clear that ASCE 41 is quite conservative in terms of estimating joint shear strength and plastic shear deformations. These backbone curves will be implemented in a cyclic model for comparison with cyclic test data in a subsequent section.

The shear strength provisions of ASCE 41 are inaccurate for unconfined exterior and corner joints because they do not account for several parameters that may affect joint strength, including joint aspect ratio, beam reinforcement ratio, axial load ratio, and bidirectional loading. The ASCE 41 nonlinear modelling parameters for unconfined joints are overly conservative, especially with high axial loads, resulting in unrealistically severe strength degradation and low drift capacity.

2.4.3. Modelling Inelastic Joint Action within the Beam-Column Element:

In this model researcher tried to model the beam or column elements such that whatever the deformation going to come in the beam-column joints can easily be predicted by the deformation in the beam or column by relating the beam or column inelastic or elastic deformation with some parameters. Many researchers has presented the papers on above philosophy like ***Townsend and Hanson (1973)***, ***Anderson and Townsend (1977)*** and ***Soleimani et al. (1979).*** As the inelastic response of the plastic-hinges are defined by the hysteretic curve. For every different beam-column joints a separate curve has to be generated. So the generalization of this model is very hard to implement.

Fillipou and Issa (1988) and ***Fillipou et al. (1988)*** proposed a model that could give due consideration to the effect of bond deterioration on the hysteretic behaviour of the joints (Fig. 2.4.3.1). The proposed model consists of a concentrated rotational spring located at each girder end. The two springs are connected by an infinitely rigid bar to form the joint sub element.

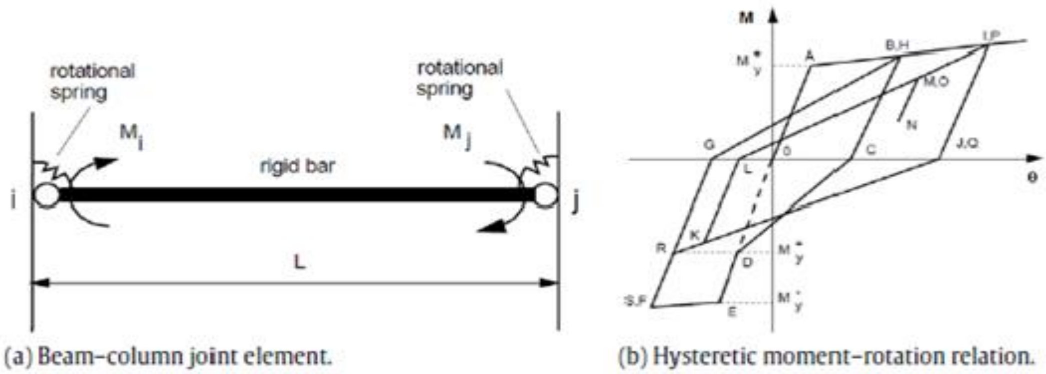


Fig. 2.4.3.1: Beam-column joint sub element (***ref:Fillipou et. al, 1988***)

2.4.4. Rotational hinge models:

Beam-column joint rotational hinge models decoupled the inelastic deformation response of the beam-column joint from beams and columns as specified in the previous models. Zero-length rotational spring elements which are being used by ***(El-Metwally and Chen 1988; Alath and Kunnath, 1995).***They connect beam elements to column elements and thereby represent the shear distortion of the beam-column joints. Many nonlinear joint models are proposed on this concept. ***Hassan (2011)*** summarizes the available macro models for joint simulation. However, some of these models may be unsuitable for older concrete building assessment, either because they were developed and calibrated for confined joints or because they are complicated to use. One of the models that may be suitable, designated the scissors model, is a relatively simple model composed of a rotational spring with rigid links that span the joint dimensions. This model is a simplification of macro model developed originally for steel panel zones. ***Alath and Kunnath (1995),*** recommend the method to calibrate the beam-column joint moment-rotation data from beam-column sub assembly test. ***El-Metwally and Chen (1998),*** given a model for predicting inelastic joints moment-rotation response under cyclic loading. Rotational-hinge model predict the deformation response of the beam-column joints moderate increase in the computational effort but unable to develop accurate calibration procedures. The model needs to develop the moment-rotation relationship to predict the deformation in the joints. The model is defined to dissipate the maximum amount of the energy through the bond-slip of the rebar.

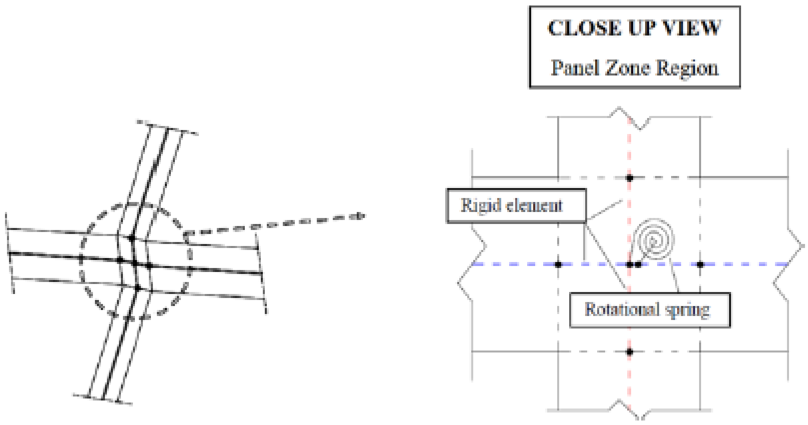


Fig. 2.4.4.1: Model for RC beam-column joints (***ref: Pampanin et al. 2003***)

Kunnath et al. (1995) modified the flexural capacities of the beams and columns of gravity load designed RC frames to model insufficient positive beam bar anchorage and inadequate joint shear capacity implicitly. The pullout moment capacity of the beam was approximated as the ratio of the embedment length to the required development length per ACI 318–89 multiplied by the yield moment of the section. ***Alath and Kunnath (1995)*** modelled the joint shear deformation with a rotational spring model with degrading hysteresis. The finite size of the joint panel was taken into account by introducing rigid links. The envelope to the shear stress–strain relationship was determined empirically. Another model has been more recently proposed by ***Pampanin et al. (2003)*** consisting of a non-linear rotational spring that permits one to model the relative rotation between beams and columns converging into the node and to describe the post cracking shear deformation of the joint panel (Fig.2.4.4.1). Beam and column elements are modelled as a one dimensional element with lumped plasticity in the end sections with an associated moment–curvature relationships defined by a section analysis. The definition of the moment–rotation relationship of the rotational spring is based on the results of experimental tests (2003). A relation between the shear deformation and the principal tensile stress in the panel region was found and transformed into a moment rotation relation to be assigned to the rotational spring. The shear deformation is assumed to be equal to the rotation of the spring and the moment is deduced as corresponding to the principal tensile stress evaluated on the basis of Mohr theory. ***Biddah and Ghobarah (1999)*** modelled the joint with separate rotational springs for joint shear and bond–slip deformations (Fig.2.4. 4.2 a). The shear stress–strain relationship of the joint was simulated using a tri-linear idealization based on a softening truss model, while the cyclic response of the joint was captured with a hysteretic relationship with no pinching effect. The model was used by ***Ghobarah and Biddah (1999)*** to perform dynamic analysis of RC frames considering joint shear deformation. ***Elmorsi et al. (2000)*** proposed an approach where beams and columns are described by elastic elements connected to the joint through the interposition of non-linear transitional elements. The effective node panel region is modelled with another element constituted by 10 joints (Fig.2.4.4.2 b). ***Youssef and Ghobarah (2001)*** proposed a joint element (Fig. 2.4.4.2 c) in which two diagonal translational springs connecting the opposite corners of the panel zone simulate the joint shear deformation; 12 translational springs located at the panel zone interface simulate all other modes of inelastic behaviour (e.g., bond–slip, concrete crushing) elastic elements were used for the joining elements. This model requires a large number of translational springs and a separate constitutive model for each spring. ***Lowes and Altoontash (2003)*** proposed a 4-node 12-degree-of -freedom (DOF) joint element (Fig.2.4.4.2 d). Eight zero-length translational springs simulate the bond–slip response of beam and column longitudinal reinforcement; a panel zone component with a zero-length rotational spring simulates the shear deformation of the joint; and four zero-length shear springs simulate the interface-shear deformations. To define the envelope for the shear stress–strain relationship of the panel zone, the modified compression field theory, MCFT (1986) was utilized. ***Lowes et al. (2005)*** later attempted to model the interface-shear based on experimental data; this effort predicted a stiff elastic response for the interface-shear. The experimental data for validation included specimens with at least a minimal amount of transverse reinforcement in the panel zone, which is consistent with the intended use of the model. Joints with no transverse reinforcement were excluded from this study. The model is not suitable for the analysis of the joints of gravity load designed frames with no transverse reinforcement. ***Altoontash (2004)*** simplified the model proposed by ***Lowes and Altoontash (2003)*** by introducing a model consisting of four zero-length rotational springs located at beam- and column joint interfaces, which simulate the member-end rotations due to bond–slip behaviour, while the panel zone component with a rotational spring remains to simulate the shear deformation of the joint (Fig.2.4.4.2 e). The development length was assumed to be adequate to prevent complete pullout. The model is still not suitable for the analysis of the joints of gravity load designed frames with no transverse reinforcement. ***Shin and LaFave (2004)*** represented the joint by rigid elements located along the edges of the panel zone and rotational springs embedded in one of the four hinges linking adjacent rigid elements (Fig.2.4.4.2 f).

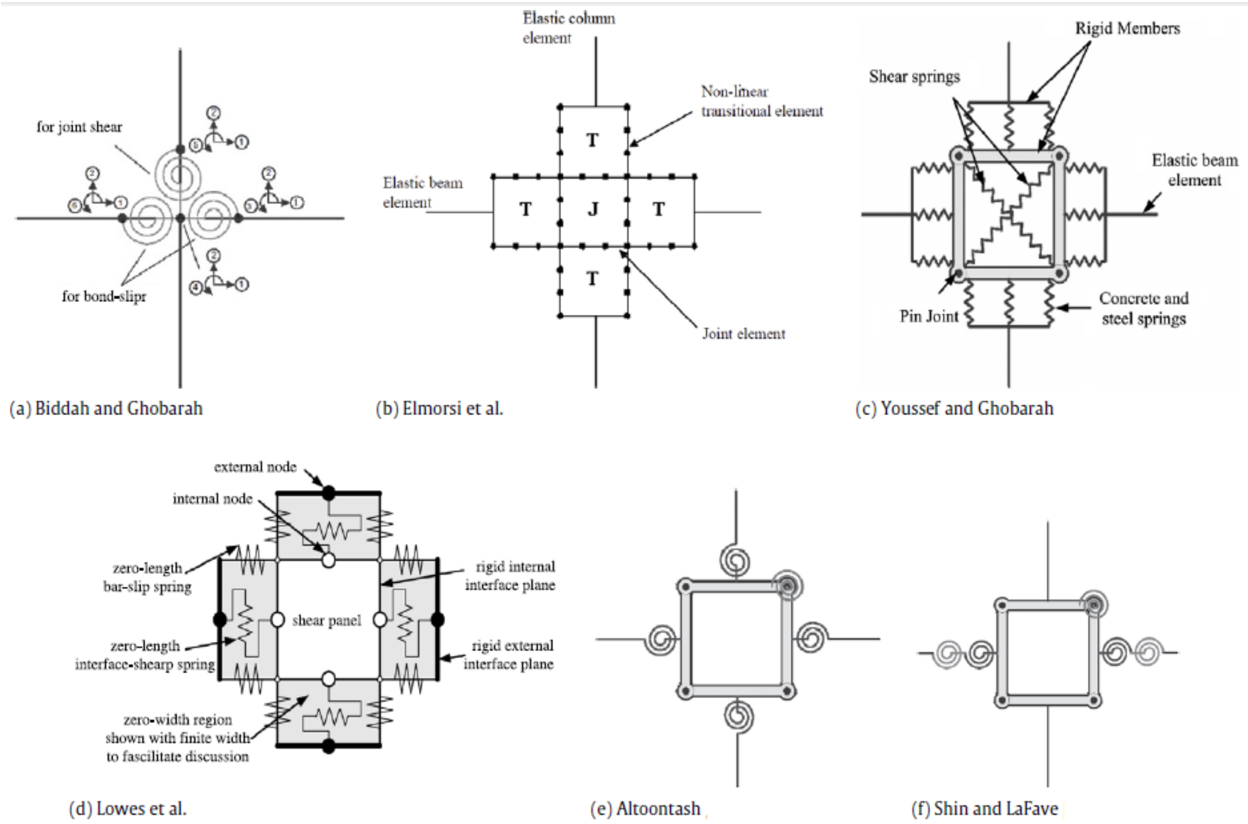


Fig. 2.4.4.2 multiple spring joints models by various researchers

The envelope to the joint shear stress–strain response was approximated by the MCFT, whereas experimental data were used to calibrate the cyclic response. Two rotational springs (in series) located at beam–joint interfaces simulate the member-end rotations due to bond–slip behaviour of the beam longitudinal reinforcement and plastic hinge rotations due to inelastic behaviour of the beam separately. The proposed joint model is intended for joints of ductile moment frames designed and detailed following modern seismic code requirements. A new model is given by for the poorly detailed reinforced concrete joints. This joint model is based on realistic deformational behaviour of the joints in structures. This makes the model more appropriate for use in analysis compared to rotational hinge models that use only a single rotational spring for modelling the joint.

In deformational behaviour of the joint, it seems most reasonable to model the contribution of joint shear deformation to overall story drift in a way that can consider the shear deformations in column and rotation in beam due to joint shear deformation. One way to model this behaviour is as shown in Fig. 2.4.4.3(a) where shear springs in the column portion and a rotational spring in the beam region are assigned. Thus, according to this model, in addition to hinges assigned at the ends of the members (beams and columns) as by most of the commercial software like STAAD.Pro and SAP2000, a joint core is modelled by dividing the frames and hinges are provided in the core region to consider the shear deformations of the joint as shown in Fig. 2.4.4.3(b). Physically, the springs have characteristics as a moment in the beam, M_b vs. shear deformation of joint, γ_j for the rotational spring and joint horizontal shear force, V_{jh} vs. shear deformation in the column portion of joint, $\Delta c = (\gamma_j hb)/2$. However, in most commercial programs that are based on matrix analysis of frame elements, it is not possible to model the reinforcement details explicitly and therefore it is not possible to calculate the horizontal joint shear force. In order to make this model suitable for implementation in such programs, the model provide the characteristics for shear springs as shear force in column, V_c vs. shear deformation in column portion of joint, $\Delta c = (\gamma_j hb)/2$. Once these characteristics are generated for the joints, the model can be implemented in the computer model of the structure so that the joint's behaviour can be taken into account. There are different ways to generate these characteristics as described under:

1. Results from experiments on beam–column joints tests.
2. Results from detailed finite element analysis of joints.
3. Analytical computation of characteristics from mechanics of the joints.

The first approach may be the most accurate one, where the tests on the joints are performed and the characteristics obtained from there are fed as the hinge characteristics in the structural model.

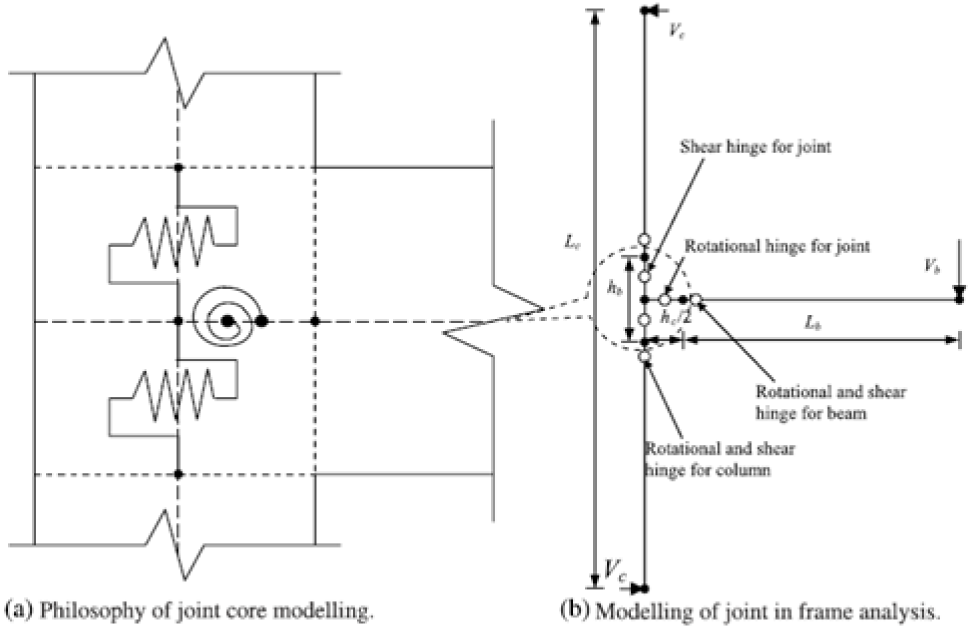


Fig.2.4.4.3: Principle behind proposed in core joint model ***(ref: Sharma et al., 2008)***.

However, this approach is cost and time prohibitive. Moreover to follow this approach for all types of the joints, in general, may not be feasible. Similarly, the second approach also can lead to good results but again is highly time consuming and computationally very demanding.

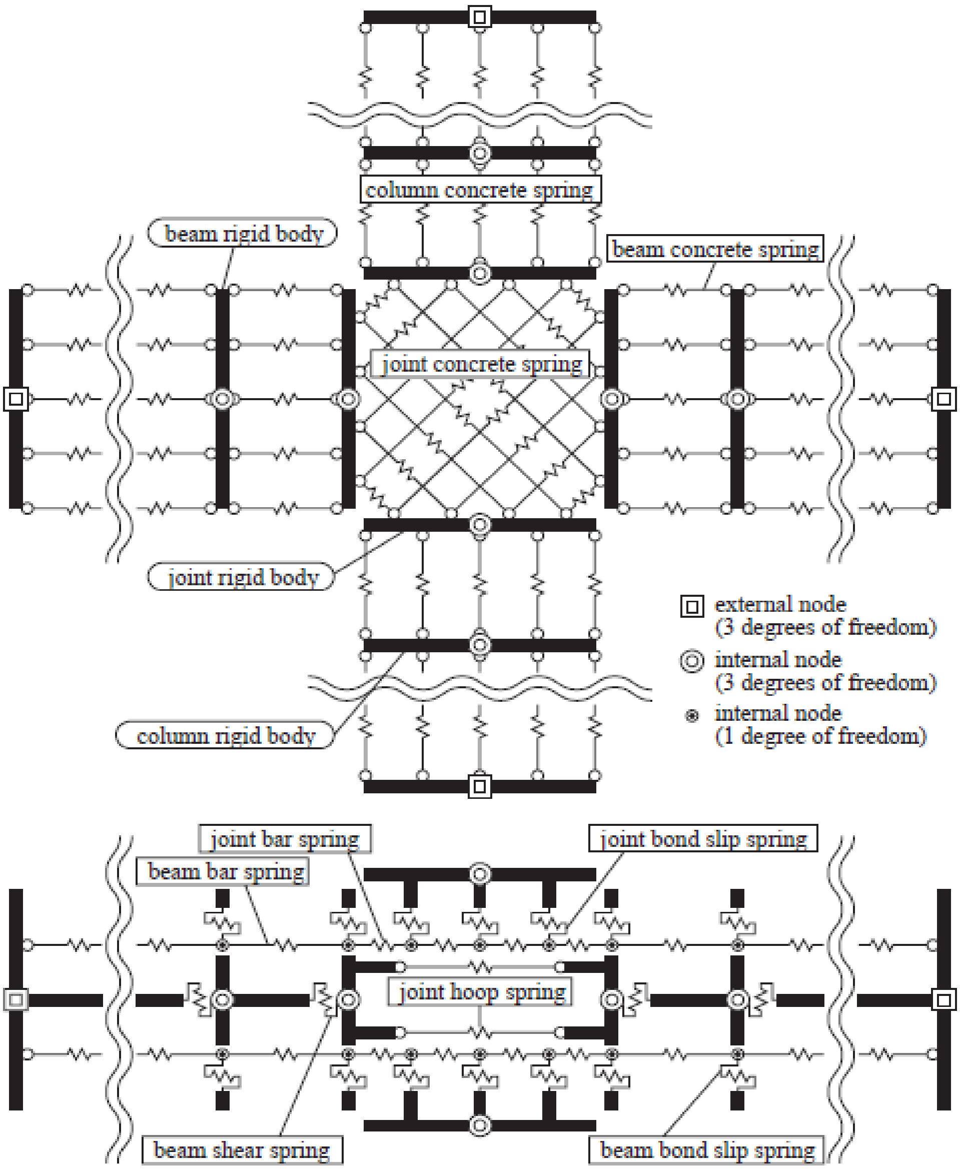


Fig. 2.4.4.4: proposed macro element of joints (***Seitora and Hitoshi, 2006)***

A new macro element for the modelling of reinforced beam-column joints in Elasto-plastic plane frame analysis is proposed by ***Seitora and Hitoshi (2006).*** The macro element defines the constitutive relationship between four nodes, each having three degrees of freedom, i.e. two translational displacements and one rotation. The parameters defining the macro element are chosen based on the dimensions, geometry and material properties of the beam-column joint. The macro element consists of axial springs and rigid bodies (Fig.2.4.4.4). Axial springs represent concrete, reinforcements, bond slip, and shear deformation, while rigid bodies represent concrete sections which remain plane after deformation. The axial springs are distributed and connected to rigid bodies or internal nodes as illustrated in Fig. 2.4.4.4. Each external and internal node in Fig. 2.4.4.4 has either 3 or 1 independent degree of freedom. The four rigid bodies (external nodes) connect to adjacent members in the frame. This results in a macro element having 12 degrees of freedom in plane frame analysis.

Developing such a model that can be used to predict the response of joints with different design details requires either a large number of data sets and a sophisticated calibration procedure or multiple models for joints with different design details. Currently, there are not sufficient data in the literature to support the development of models that are appropriate for a broad range of joint designs.

2.4.5. Continuum models:

With the advancement of the high performance computing technology researchers start using continuum-type elements to represent the inelastic deformation responses beam-column joints. These proposed elements behave as “transition element”. Which are formulated to establish compatibility between beam-column line elements that symbolize the deformation behaviour of the element outside to the joint cores and other planar continuum elements that stand for the structure inside the beam-column joints. These types of FE formulation of the joint models are very accurate in predicting the deformation contribution of the beam-column joints but at the same time need very high computational demand. But presently due to limitation in the computational advancement researcher ***(Fleury et al. 2000; Elmorsi et al. 2000)*** has taken very simple idealisation to optimize the results

There exists very few previous research which considers continuum finite elements to model and simulate behaviour of reinforced concrete beam-column connection regions, however these investigations did not account for all the local inelastic mechanisms governing beam-column connection response. Research by ***Will et al. (1972)*** was one of the first continuum finite element studies of joint regions. The investigation assumed brittle fracture for concrete and assumed linear elastic response for concrete in tension, compression and also the reinforcing steel and bond-link elements. ***Noguchi (1981)*** utilized a discrete crack approach to represent cracks in concrete. However, as had been identified later by many researchers such as ***Rots and Blauuwendraad (1989),*** one of the major drawbacks of the discrete approach for use in concrete structures is the crack propagation path needs to be well defined a-prior to the analysis. To get better accuracy of finite element modelling of concrete frame structures, a new beam-solid transition element with redefined characteristics is introduced by ***Ziyaeifar and Noguchi (2000).*** The refinement capability in this approach provides an accurate strain field approximation in the regions of high shear forces of beam and columns joint. This Finite element formulation has been extended for material nonlinearity and large deformations to account for ultimate loads and secondary effects.

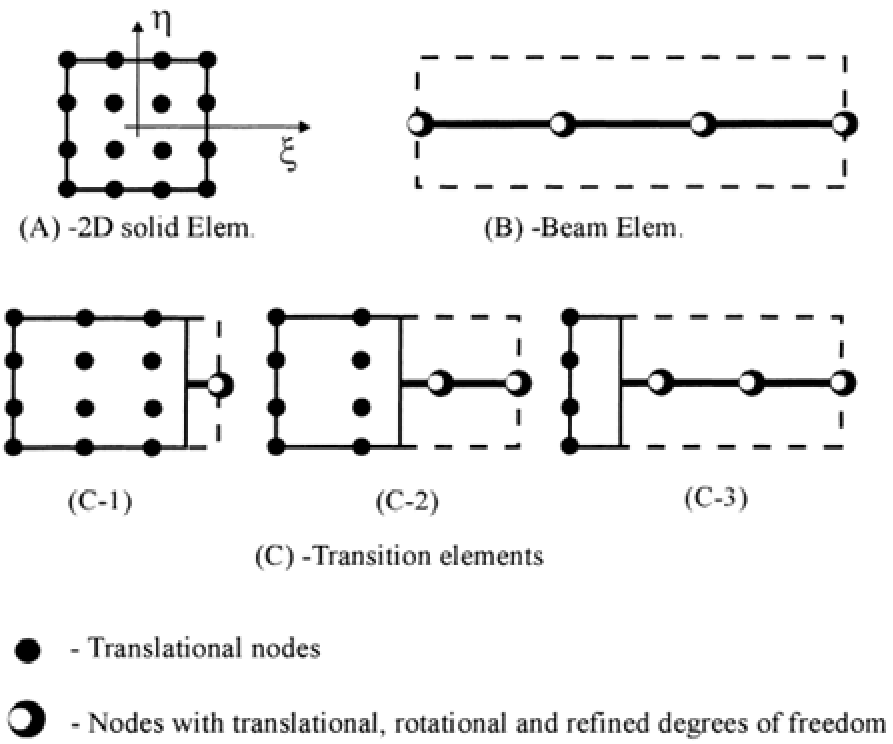


Fig. 2.4.5.1: A family of third order transition elements derived from a single formulation

Pantazopoulou and Bonacci (1994) utilized modified compressive field theory (MCFT), which primarily considers reduction in compressive strength due to tension in orthogonal direction, to represent behaviour of concrete. Even though MCFT has been used successfully in many applications, the viability of utilizing the theory as a generalized material model for reinforced beam column joints is questionable ***(LaFave and Shin, 2005; Lowes et al. 2005).*** Moreover, the direct portability of this theory for three dimensional analysis in which joints are subjected to complex 3-dimensional stress states are not easy. The model by ***Pantazopoulou and Bonacci (1994)*** however did consider frictional contact theory for simulating bond-slip between reinforcing steel and concrete, without relying on empirical bond-slip curves obtained from experimental investigations of pull out tests of reinforcement bars from concrete. ***Baglin and Scott (2000)*** and ***Hegger et al. (2004)*** utilized commercial finite element software SBETA and ATENA respectively to simulate response of beam-column connections, however these models considered perfect bond between reinforcing steel and concrete. A FE model has been used by ***Nagai et al. (1996)*** for modelling the beam-column joints which has been subject to static loading. The inelastic behaviour of interior wide column joints subjected to uniaxial loading has been investigated by ***Bing et al. (2003)*** investigated the post elastic deformation behaviour of the interior beam-column joints using 2D non-linear FE . ***Eligehausen et al. (2006) and Sharma et al. (2008)*** use the FE to develop a model for fracture in quasi static brittle material. And the model show very good result with the experimental data

A material model has been suggested in study by ***Mitra (2008)*** which is capable of considering all the local inelastic mechanisms involved in determination of a beam-column joint response. Based on the above study, it has been demonstrated that the current continuum finite element model software, such as DIANA 9.1, with the suggested material model, is capable of representing mechanistic behaviour for moderately complex problems such as three point bending, push out response of a reinforcing bar anchored in concrete, bending response of beams and so to name a few. However, it should also be noted that the current capabilities of DIANA using the suggested material model is not capable of representing extremely complex mechanisms such as the exact behaviour within the joint region demonstrating all the local inelastic mechanisms within the joint. It has also been demonstrated that if one of the local inelastic mechanisms is simplified then the analysis may converge and global response might be obtained partially. It should also be noted that a large literature exists on number of simulations of concrete structures which have been done considering empirical curves for compressive response of concrete along with degradation rules to account for tension cracks. Even though the global response can be obtained using those empirical equations but the author believes that these are not representative to identify the exact local inelastic mechanisms in complex situations such as that within a connection region. For the bond response, models from first principles of contact mechanics also need to be developed. Within the perspective of commercial finite element software, better numerical algorithms needs to be developed which can be utilized to solve situations encountered in multi-surface plasticity models. The author also suggests that these complex local inelastic responses as well as the global response may be obtained through use of explicit nonlinear finite element software such as ABAQUS and LS-DYNA.

These approaches tend to provide a good insight on the behaviour of beam–column joints but are not suitable for structural behaviour.

There are three major reasons which make this deformation model highly limited for the practical use:

1) This approach for the deformation model needs very high computational effort and making the simple analysis too time consuming. With current computational advancement it is very hard for researcher and practicing engineers to implement it with their limited facilities.

2) These types of deformation models could never meet the requirements for robustness under a wide range of joint designs and model parameters.

3) This model required many material constitutive parameters. While most of these parameters will represent fundamental material properties, but few of them cannot be easily produce leads to some kind of assumption about the material models which constitutively leads to error in the response calculation.

2.5 THE PRESTRESSED JOINTS:

Provision of prestressed beam-column joints is very limited in codes. As per the study, NZS 3101 and AIJ Code has only mentioned that location of tendon play an important role in influencing the joint shear capacity and shear deformation of the joints. Also NZS 3101 remark that prestressing of joint is not so effective. But few of the latest research have shown the positive results.

Wie Yue et. al. (2004) performed the experimental test on 7 half-scale exterior beam-column joints, to study the effect of outside and inside prestressing of joints. And come up with conclusion that outside joint prestressing is better, with increases in the failure loads somewhere from 9% to 13%. Also the shear deformation was less in the former, because total joint prestressing is coming into the pictures.

Kashiwazaki and Noguchi (2000) conducted experimental and FE analysis on four prestressed interior beam-column joints. And come up with the conclusion that prestressed has no such effect either on shear capacity and shear deformation of the joint core of interior joints.

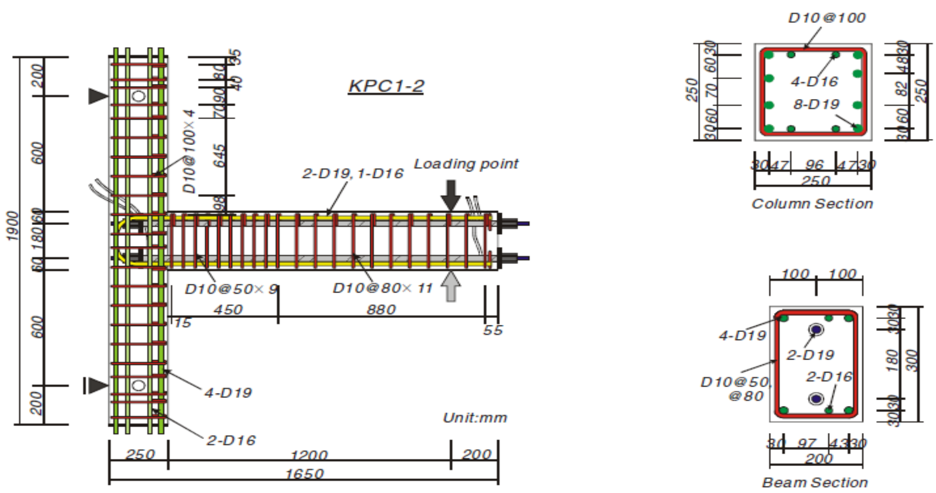


Fig.2.5.1 Reinforcement and prestressing detail of the test unit ***(Ref: Wie Yue et. al., 2004)***

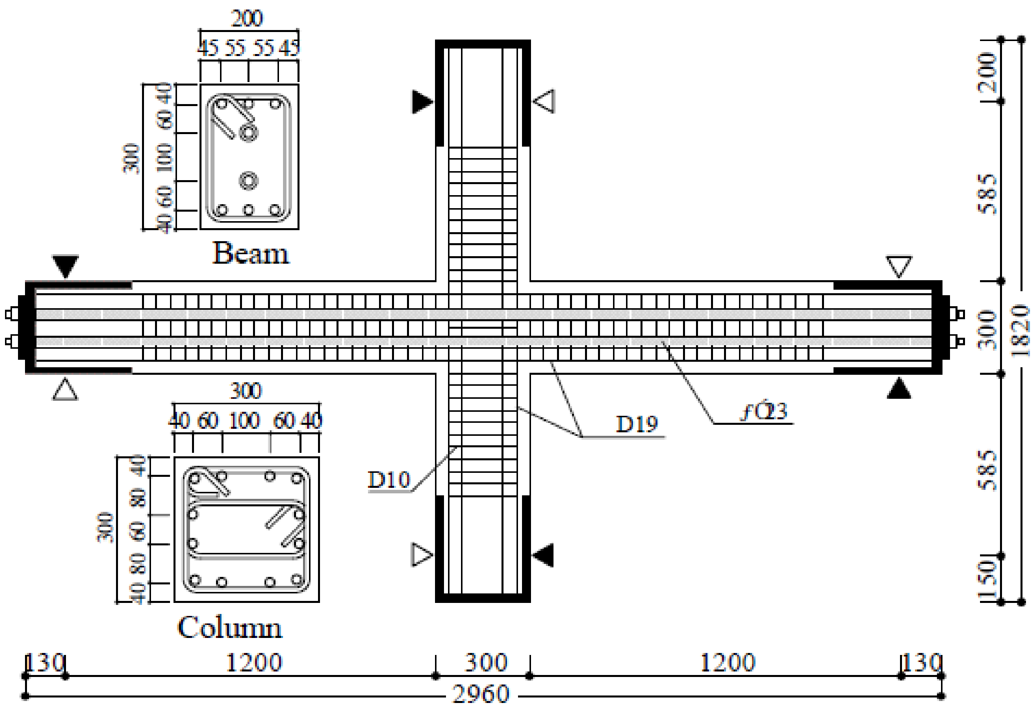


Fig.2.5.2: Reinforcement and prestressing detail of the test unit (***Reference: Kashiwazaki and Noguchi, 2000)***

Zhi-Heng et. al. (2005) performed an experimental test on the steel confined column-joints and reported the improvement in the shear capacity of the joint, delayed development of the cracks and good performance of ductility and energy dissipation.

On the basis of the experiment on eight prestessed and two non-prestressed exterior beam-column joints by***Hamaraha et. al. (2007)*** showed not any significant improvement in the deformation. There is also not any large variance in the shear force between prestressed and non-prestressed beam-column joints. Performed reversed cyclic loading tests of eight prestressed and two non-prestressed concrete exterior beam-column joint assemblies that failed inside the beam-column joints were conducted.

2.6 SIGNIFICANCE OF THE PRESENT WORK

People have been trying to use different ideas to increases the shear capacity of the joint core by many different ways. If I enlist them it could be as follow

- ⇒ Increasing the size of the joints
- ⇒ Using the steel fiber in the joints
- ⇒ Using GRFP to wrap the joints
- ⇒ Prestressing the beam including the joint
- ⇒ Using of the crossed rebar at the joint cores

It can be seen from literatures that prestressing the joint has increased the shear capacity of the joint due to increased confinement of the joint concrete. All the above literatures have prestress the whole beam passing through the joints to prestress the joint core.

But prestressing the whole beam throughout, affect the economy of the structures. Also there has not been any improvement in stopping the shear deformation, so a new direct prestressing with the plates and steel rebar's is tested, instead of prestressing all through the beam to joint.

Crossed rebar has been used to increase the performance of the joint on the concept of strut and tie models. The improvement was significant. This combine the concept of strut and tie and prestress of the joint through crossed rebar. In my model on one side the presence of crossed rebar avoid and direct crushing and direct cracking of the concrete in the joints and also due to confinement the shear strength of the concrete increase which leads to the delayed failure and preventing the undesirable shear deformation in the joints. This is very innovative because, this arrangement not just only increases the deformation but also prevent the shear deformation in the linear material zone. This way of doing the prestressing solves the both problem of shear deformation and brittle shear failure.

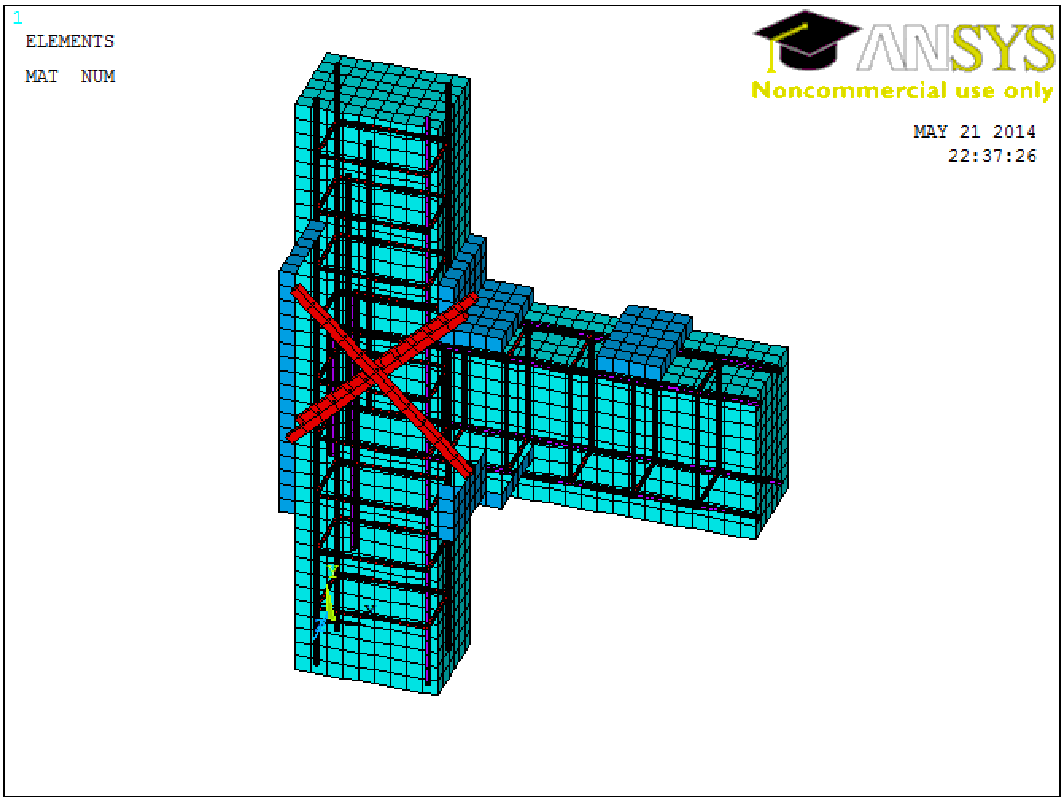


Fig.2.6.1 Arrangement of the prestressing of the joint core

Chapter 3

Methodology and Present Work

3.1 GENERAL:

My present work is divided into two phases. In the first one I have design the low to mid-story building to find the location of maximum shear force in the beam to column joints. Once we got the joint with maximum shear force we can implement the prestressing in the beam to column joints to prevent the damage and avoiding the congestion at the same time. Both the phase of the work has been discussed in detail in the section 3.2 and section 3.3 respectively.

3.2 PHASE I: JOINT WITH MAXIMUM SHEAR FORCES

As I have already discussed in the introduction section that as per the new building codes detailing of few of the beam to column joint where the maximum shear force is being inducted faced the practical problem of the congestion. This research is basically to solve that problem. So the first phase of the work is dedicated to find out the beam to column joint which may goes under maximum shear force demand under all the possible parameter variation. So I have arbitrary chosen a building of 3 story and 3 bays with 3m as the height of the story and 3m as the width of the bay. For easy reference, this building is named as *"reference building".* Many parameters have been selected from lot of literature review which are supposed to affect the shear demand of the beam to column joints. Taking these parameters studies has been done to find the influence of these parameters. All the different buildings with different parameters have been design with STAAD.Pro according to IS 456:2000 "Limit State Method" and shear force is calculated according to the ACI 352-02. Joints with the maximum shear force are shorted out where probable congestion is being expected. Final motive of this whole parametric study is to find the most critical combination of the parameters which give the most critical shear force demand at beam to column joints i.e. finding the location of most critical joint and value of shear force into that joints. Following are the range of parameters which has been taken for the parametric studies.

a. <u>Story heights</u>: it varied from 3m 3.5m and 4m in the reference buildings.
b. <u>Number of story</u> or height of the building: It is varied from 2nd story to 10th story with each as 3m of height.
c. <u>Width of the bays</u>: Bays width has chosen as 3m 4m and 5m
d. <u>Number of the bays</u>: number of bays has also be chosen as 3 4 and 5
e. <u>Grade of the concrete</u>: Grade of the concrete is taken as 30MPa, 35MPa, 40MPa, 45MPa, 50MPa, 55MPa and 60MPa.

f. Size of the beams: Size of the beam are varied from 350, 400, 450 and 500mm
g. Size of the columns: The sizes of the columns have been change from 400mm, 450mm, 500mm, 550mm and 600mm.

A step by step method for calculating the maximum shear forces in the joints is explained below.

1. A reference building of 3 story and 3 bay of 3m each has been selected
2. Following data has been used for the design of the building
    a. Reinforced concrete plain frame.
    b. Material: M25 and Fe415
    c. Type: Residential building
    d. Load:
        i. Dead load 20kN/m (excluding self-weight)
        ii. Live load 10kN/m
        iii. Earthquake load
            1. Zone= V
            2. Soil type= II
            3. Response reduction= 5
            4. Importance factor= 1
3. Design and analyzed using STAAD.Pro V8i according to IS 456:2000
4. Seven key factors are consider to study the influence on the joint shear demand for both fixed and hinge support:
    a. Story height
    b. Number of story or height of the building
    c. Width of the bays
    d. Number of the bay
    e. Grade of the concrete
    f. Size of the beam
    g. Size of the column
5. Then shear demand of the exterior joints are calculated by the simple formula mechanics as given below.

    Column shear in the joint, V_c

$$V_c = 1.4 \times \left(\frac{M_h + M_s}{h}\right)$$

Where,

$M_h \Rightarrow$ Hogging moment of the beam connecting to the joint.

$M_s \Rightarrow$ Sagging moment of the beam connecting to the joint.

$h \Rightarrow$ Height of the story.

From the equilibrium of the force in the joint, joint shear demand, V_j

$$V_j = T_1 + C_S + C_C - V_c$$

$$C_S + C_C = T_2$$

$$V_j = T_1 + T_2 - V_c$$

Where, $T_1 \Rightarrow$ Tensile force in the bar $= 1.25 \times f_y \times A_{st1}$

$T_2 \Rightarrow$ Tensile force in the bar $= 1.25 \times f_y \times A_{st2}$

$C_s \Rightarrow$ Compressive force in the steel.

$C_c \Rightarrow$ Compressive force in the concrete.

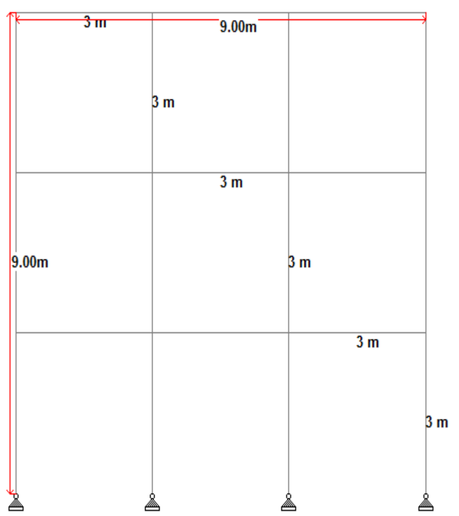


Fig 3.2.1: Dimension of the reference building

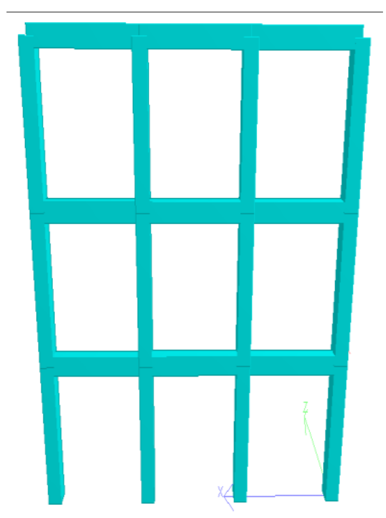


Fig. 3.2.2: STAAD.Pro 3D rendered view

3.3 PHASE II: MODELING IN ANSYS:

3.3.1. Introduction:

ANSYS is general FE software which could model the concrete and reinforced concrete with high level of accuracy. For the present study ANSYS v13.0 is being used. It is very accurate in predicting the cracks and crushing behaviour of the reinforced concrete.

Modelling in ANSYS is providing appropriate elements, defining geometry and assigning the suitable material models. Modelling is the most time consuming part of the FEM analysis. So it should be done with very care and patience. Few of the basic theory must be followed before going for the modelling in ANSYS specially of the concrete modelling. One major problem which has been encountered by the engineer/scientists working in the FEM of concrete in the convergence problem associated with it. Due to cracks, concrete is generally not able to converge so some of the convergence criteria has to be dropped to get the accurate results, ***Wolanski (2004).***

In present work an exterior beam to column joints taken from the experimental studies of ***Dar (2011)***. ***Dar (2011)*** conducted the experimental study to find the effect of different wrapping techniques on retrofitting of RCC exterior Beam to Column Joints using Ferro cement on the weak beam to column joint. First of all the exterior joint is being modelled in ANSYS as the experimental program to act as the control specimen as shown in the Fig 3.3.1. And the second ANSYS model is created with prestressing force through rebar is being applied at the joint with the help of the steel plates acting as the bearing as shown in the Fig 3.3.2. For the easy reference each exterior joint has specified B1 and D1 respectively.

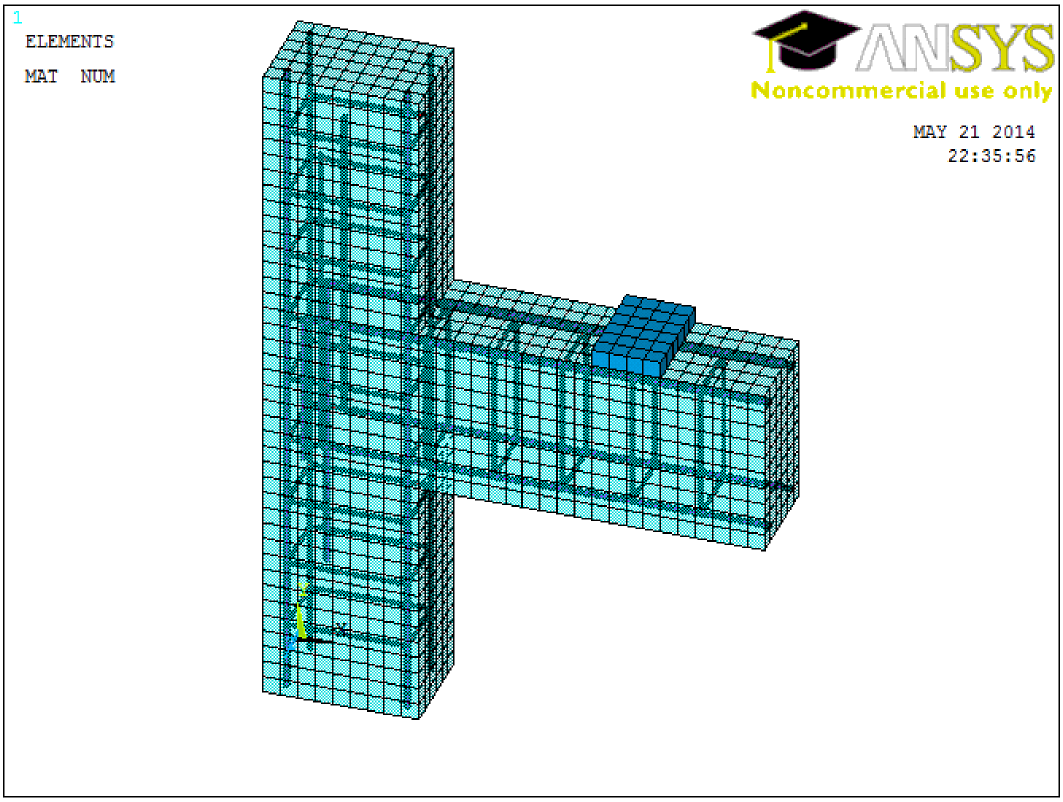


Fig. 3.3.1 ANSYS model of Exterior joint model as per the experimental setup of ***Dar (2011)*** and specify as B1

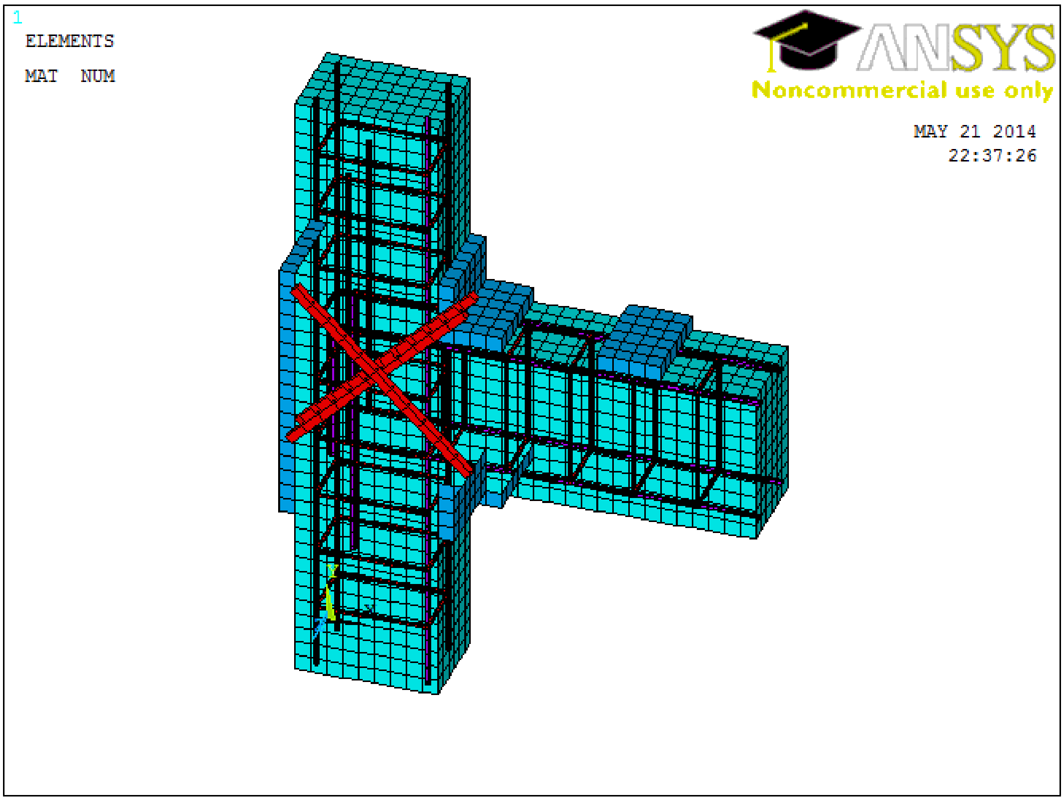


Fig.3.3.2 ANSYS model of Exterior Beam-Column Jointas proposed by present work and specify as D1 (Perspective view)

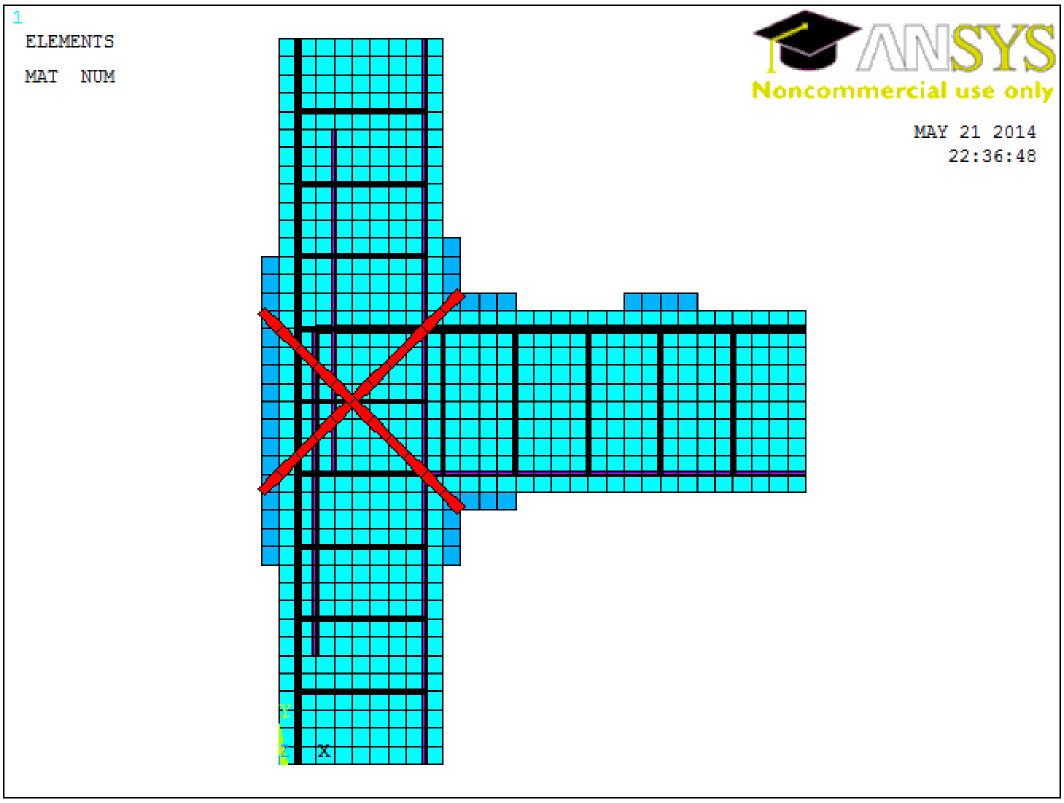


Fig 3.3.3: Side view of the proposed Exterior Beam-Column Joint by present work.

3.3.2 Assumption:

To model the real world problem into any of the FE software we have to make few assumptions to simplify the problem. Below is the assumption which has been taken during modelling of the present work.

- Concrete is assumed to be behaving as isotropic and homogeneous.
- Steel rebar and steel plate are also assumed as isotropic and homogeneous.
- Steel rebar is model as bilinear material model. With kinematic hardening model.
- No slip of rebar is assumed. Where ever the concrete element nodes and rebar nodes is coinciding it is taken as same. Leading to the perfect bonding between the concrete and rebar. And also between plate and concrete.

3.3.3 Element types

When you are working in ANSYS concrete can be better model through the element named as SOLID65. According to the ANSYS literature, this element has eight nodes with three degrees of freedom at each node – translations in the nodal x, y, and z directions. This element is capable of plastic deformation, cracking and crushing in three orthogonal directions. A schematic of the element is shown in Fig. 3.3.3.1

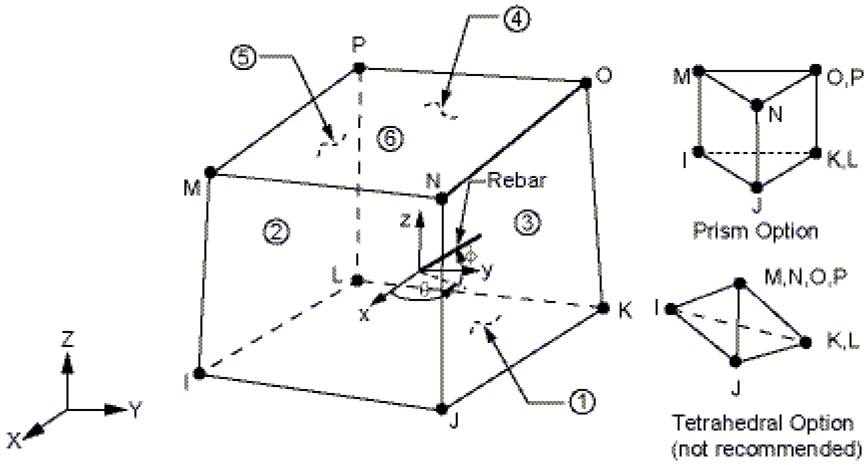


Fig. 3.3.3.1: Solid65 Element (***ANSYS,v13.0***)

A Link8 or BEAM188 element is used to model steel reinforcement. LINK180 element is a 3D spar element and it has two nodes with three degrees of freedom at each node – translations in the nodal x, y and z directions. This element is also capable of plastic deformation. This element can take either tension or compression only or both. This element can only take the square cross-section with only user can give the area of the element. But on other hand in BEAM188 you can give the desire shape from the dropdown table and can also add desire meshing to it. According to ANSYS v13, this element is based on Timoshenko beam theory. Shear deformation effects are included. BEAM188 is a linear (2-node) beam element in 3-D with six degrees of freedom at each node. The degrees of freedom at each node include translations in x, y, and z directions, and rotations about the x, y, and z directions. Warping of cross sections is assumed to be unrestrained. As this element is design for the beam behaviour but can also be used as rebar with better accuracy as compared to LINK8. The beam elements are well-suited for linear, large rotation, and/or large strain nonlinear applications.

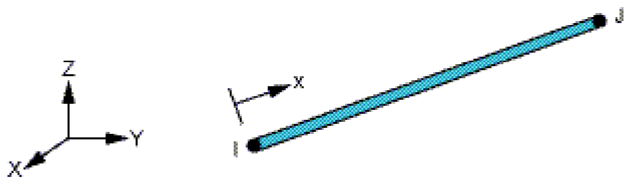


Fig. 3.3.3.2: Link8 Element (***ANSYS v 13.0***)

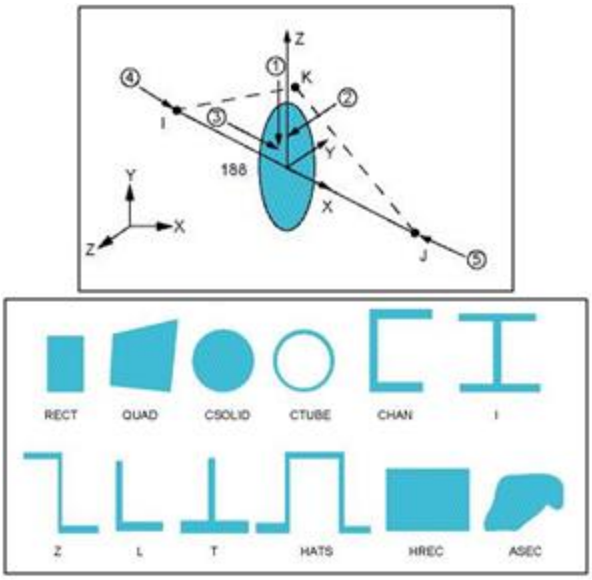


Fig. 3.3.3.3: BEAM188 Element (***ANSYS,v13.0***)

Solid steel plate has been used to apply the loads and pre-stressing in the joint. Steel plate is modelled using element called Solid185. This element is defined by eight nodes having three degrees of freedom at each node translations in the nodal x, y, and z directions. The element is capable of plasticity, hyper elasticity, stress stiffening, creep, large deflection, and large strain capabilities.

SOLID185 is available in two forms:

1. Homogeneous Structural Solid (default); and
2. Layered Structural Solid.

Homogeneous Structural Solid with simplified enhanced strain formulation is used to model steel plate for application of load. This element is shown in Fig. 3.3.3.4.

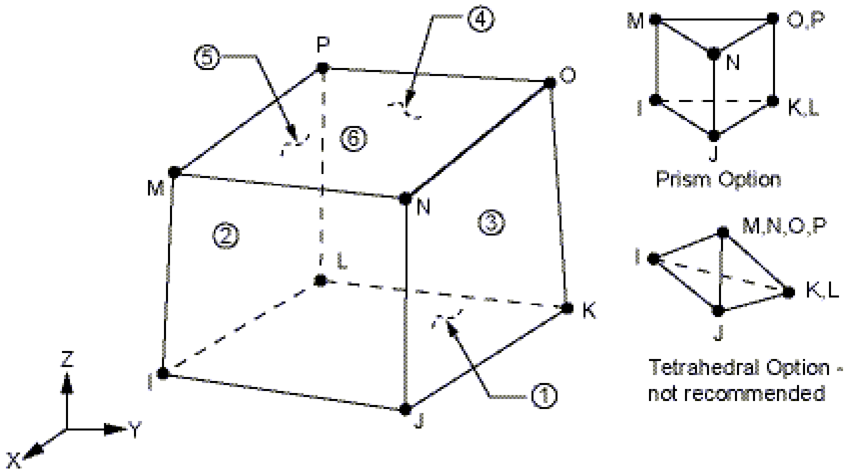


Fig. 3.3.3.3.4: Solid185 Element (Homogeneous Structural Solid) (***ANSYS v 13.0***)

The element types for the control specimen are tabulated in Table 3.3.3.1.

Table 3.3.3.1: Element types for the Control Specimen	
Material Type	ANSYS Element
Concrete	Solid65
Steel Reinforcement	Link8 and Beam188
Steel Plate	Solid185 (Homogeneous Structural)

3.3.4 Real Constants and Sections

Each model has their own real constant which is attached to it, which adds extra attribute to given elements. In my modelling real constant has been added to the elements solid65 and link8. In the present modelling, discrete rebar is being used to model the reinforcement, so real constants which are supposed to be for smeared type of modelling like Material Number, Volume Ratio, and Orientation Angles are set to zero as shown in Table 3.3.4.1 below. ***Fanning (2001)*** suggested for modelling of complicated reinforcement detailing the discrete reinforcement modelling will give more accurate result as compare to the smeared type of the modelling. Therefore, a value of zero was entered for all real constants which turned the smeared reinforcement capability of the Solid65 element off as suggested by past researchers like ***Ibrahim and Mahmood, (2009); Wolanski, (2004); Kachlakev et al., (2001)***. The second real constant is used with element LINK8 which is being used as pre-stressing rebar. The attributed added to this real constant are area of the cross-section and strain in the bar. In both the exterior joints same cross-section and strain is being used. Cross-section of the link8 is 200mm^2 and strain is kept as .0005mm/mm.

Table 3.3.4.1: Real constants and section of specimen B1

Real constant set/section ID	element	attributes	value
1	Solid65	Material number 1,2,3	0
		Volume ratio 1,2,3	0
		Orientation angle 1,2,3	0
		Orientation angle 1,2,3	0
2	Beam188	Section name	Rebar6
		radius	3mm
		Section subtype	Circular solid
3	Beam188	Section name	Rebar8
		radius	4mm
		Section subtype	Circular solid
4	Beam188	Section name	Rebar10
		radius	4mm
		Section subtype	Circular solid
	Solid185	Nil	nil

Table 3.3.4.2: Real constants and section of specimen D1			
Real constant set/section ID	element	attributes	value
1	Solid65	Material number 1,2,3	0
		Volume ratio 1,2,3	0
		Orientation angle 1,2,3	0
		Orientation angle 1,2,3	0
2	Beam188	Section name	Rebar6
		radius	3mm
		Section subtype	Circular solid
3	Beam188	Section name	Rebar8
		radius	4mm
		Section subtype	Circular solid
4	Beam188	Section name	Rebar10
		radius	4mm
		Section subtype	Circular solid
	Solid185	Nil	nil
5	Link8	Cross-section area	200mm^2
		strain	.0005mm/mm

3.3.5 Material models and failure criteria:

<table>
<tr><th colspan="5">Table 3.3.5.1 Material Model used in the Present Work in ANSYS</th></tr>
<tr><th>Material Model Number</th><th>Element Type</th><th colspan="3">Material Properties</th></tr>
<tr><td rowspan="17">1</td><td rowspan="17">Solid 65 (concrete)</td><td colspan="3">EX = 22361MPa</td></tr>
<tr><td colspan="3">PRXY = 0.2</td></tr>
<tr><td colspan="3">Stress-strain curve</td></tr>
<tr><td>point</td><td>stain</td><td>Stress(MPa)</td></tr>
<tr><td>1</td><td>0.0002683</td><td>6</td></tr>
<tr><td>2</td><td>0.0006485</td><td>13</td></tr>
<tr><td>3</td><td>0.0010286</td><td>17</td></tr>
<tr><td>4</td><td>0.0014087</td><td>19</td></tr>
<tr><td>5</td><td>0.0017889</td><td>20</td></tr>
<tr><td colspan="2">Open shear transfer coefficient</td><td>0.3</td></tr>
<tr><td colspan="2">Closed shear transfer coefficient</td><td>1</td></tr>
<tr><td colspan="2">Uniaxial cracking stress</td><td>3.13</td></tr>
<tr><td colspan="2">Uniaxial compressive stress</td><td>-1</td></tr>
<tr><td colspan="2">Biaxial crushing stress</td><td>default</td></tr>
<tr><td colspan="2">Hydrostatic pressure</td><td>default</td></tr>
<tr><td colspan="2">Hydro Biax crush stress</td><td>default</td></tr>
<tr><td colspan="2">Hydro Uniax crush stress</td><td>default</td></tr>
<tr><td rowspan="5">2</td><td rowspan="5">BEAM 180</td><td colspan="3">Bilinear isotropic</td></tr>
<tr><td colspan="2">EX</td><td>2E+5 MPa</td></tr>
<tr><td colspan="2">PRXY</td><td>0.3</td></tr>
<tr><td colspan="2">Yield stress</td><td>500 MPa</td></tr>
<tr><td colspan="2">Tangent modulus</td><td>0</td></tr>
<tr><td rowspan="3">3</td><td rowspan="3">BEAM 180</td><td colspan="3">Bilinear isotropic</td></tr>
<tr><td colspan="2">EX</td><td>2E+5 MPa</td></tr>
<tr><td colspan="2">PRXY</td><td>0.3</td></tr>
<tr><td rowspan="2">3</td><td rowspan="2">BEAM 180</td><td colspan="2">Yield stress</td><td>250 MPa</td></tr>
<tr><td colspan="2">Tangent modulus</td><td>0</td></tr>
<tr><td rowspan="3">4</td><td rowspan="3">SOLID165</td><td colspan="3">Linear isotropic</td></tr>
<tr><td colspan="2">EX</td><td>2E+5 MPa</td></tr>
<tr><td colspan="2">PRXY</td><td>0.3</td></tr>
</table>

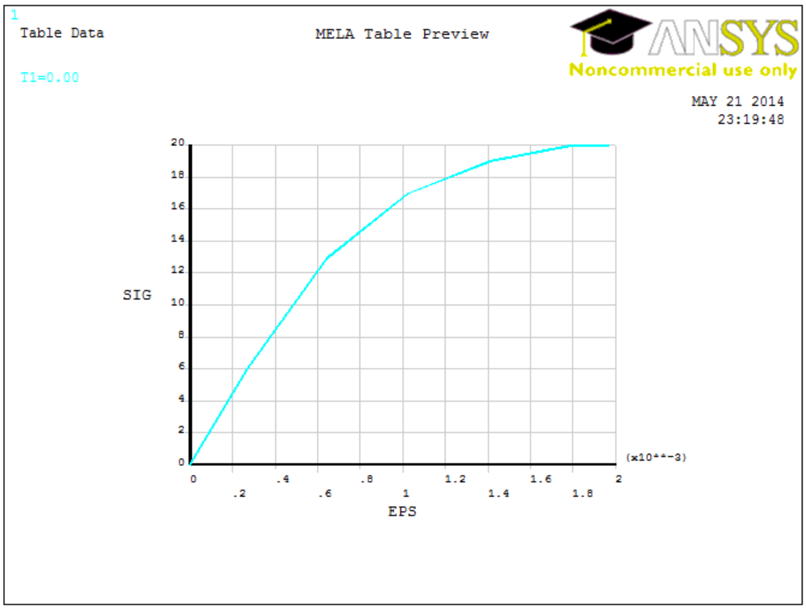


Fig 3.3.5.1 Stress-strain curve of the concrete used in the ANSYS model

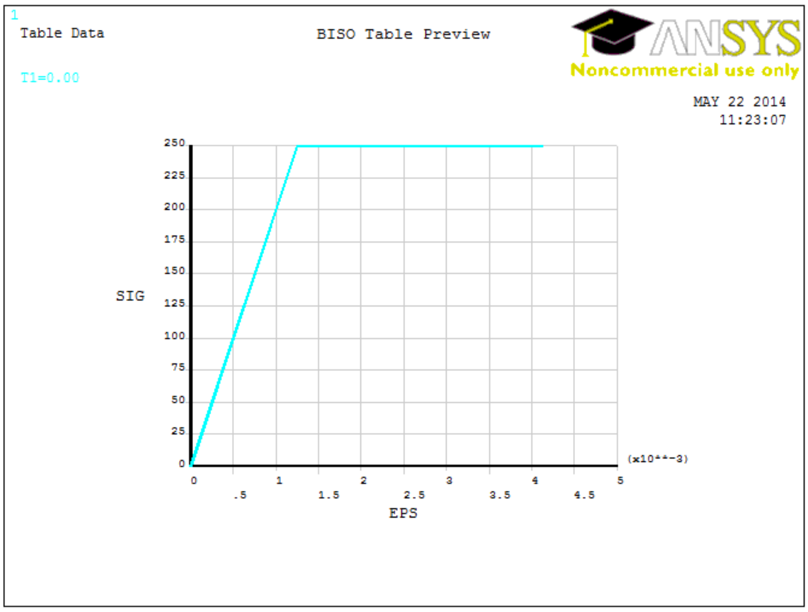


Fig. 3.3.5.2: Stress-strain curve of the isotropic bilinear model of rebar Fe250 used in the ANSYS model

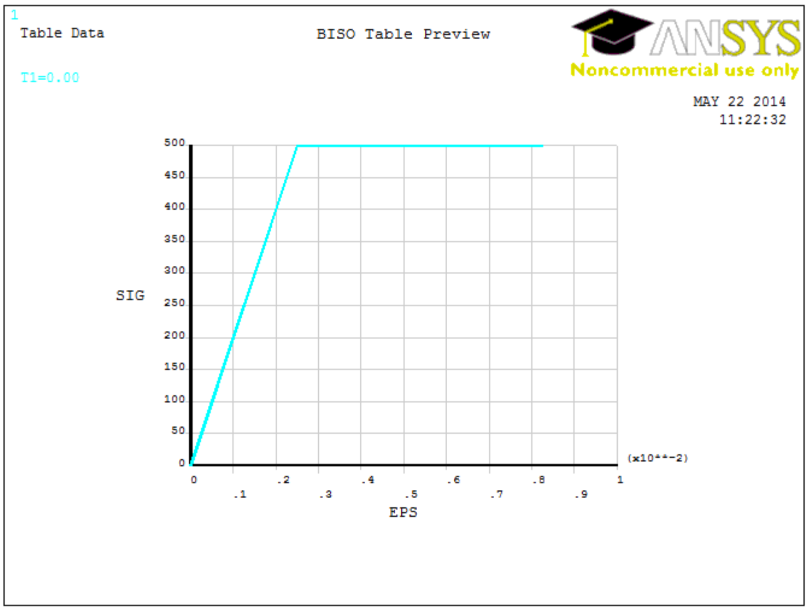


Fig. 3.3.5.3: Stress-strain curve of the isotropic bilinear model of rebar Fe500 used in the ANSYS model.

Material model number 1 is being assigned to the concrete element (Solid65 element). To fulfil the failure criteria according to VON MISES failure criteria concrete required both isotropic material properties and multi-linear isotropic material properties. In actual multi-linear isotropic material uses VON MISES failure criteria for the failure of the concrete. Similarly material of the rebar is defined using bilinear isotropic which also uses the same failure criteria for the failure of the rebar according to VON MISES failure principle.

3.3.6 Modelling:

Modelling of the Exterior Beam-column Joints B1 and D1 in ANSYS is done as per the experimental programme of ***Dar (2011)*** and the present proposed work.

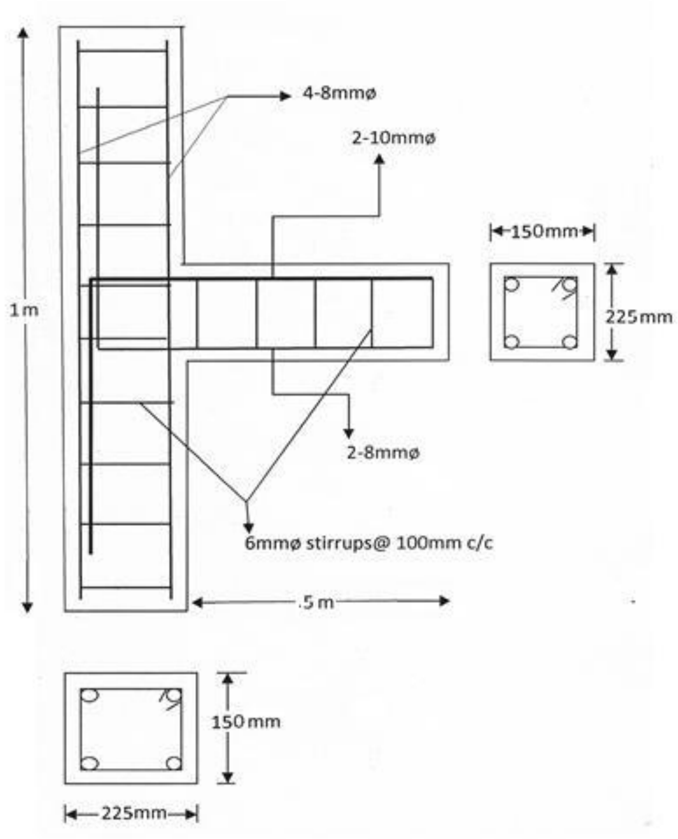


Fig. 3.3.6.1 Reinforcement Detailing and Dimension of the Exterior Beam-Column Joints, B1

(ref: Dar, 2011)

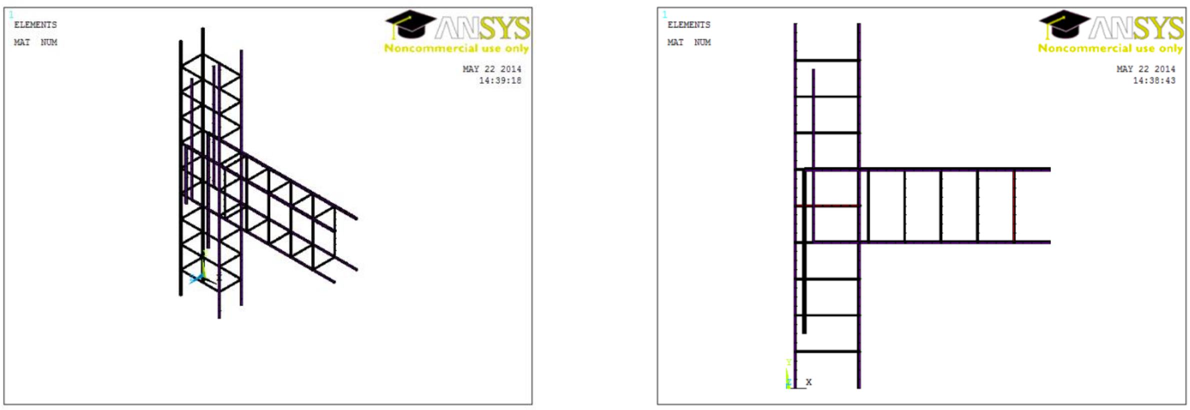


Fig. 3.3.6.2: Reinforcement Modelled for the Exterior Beam-Column Joint B1

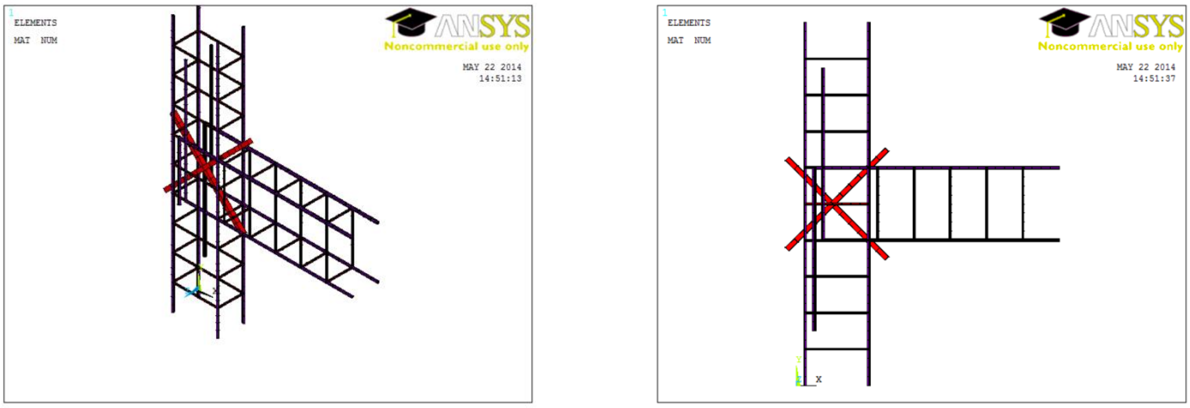


Fig. 3.3.6.3: Reinforcement Modelled for the Exterior Beam-Column Joint D1

3.3.7 Meshing:

For the better results of Solid65 element, it is always meshed as rectangular brick mesh as recommended by ***Wolanski (2004).*** So, all the concrete Solid65 elements are meshed as rectangular brick element with 25mm size. As there is no requirement of the meshing of the rebar element, it is joined as element between the spacing of the nodes created by the meshing of the concrete.

3.3.8 Load and Boundary Condition:

Both the top and the bottom of the column are fixed as per the experimental programme by ***Dar (2011)***. Beam is kept as cantilever and point loads up to failure are applied at 300mm from the face of the column with the help of steel plate to avoid crushing at the point of loading as shown in Fig 3.3.8.1. These loading and boundary conditions are kept same for both type of Exterior Beam-Column Joint i.e. B1 and D1.

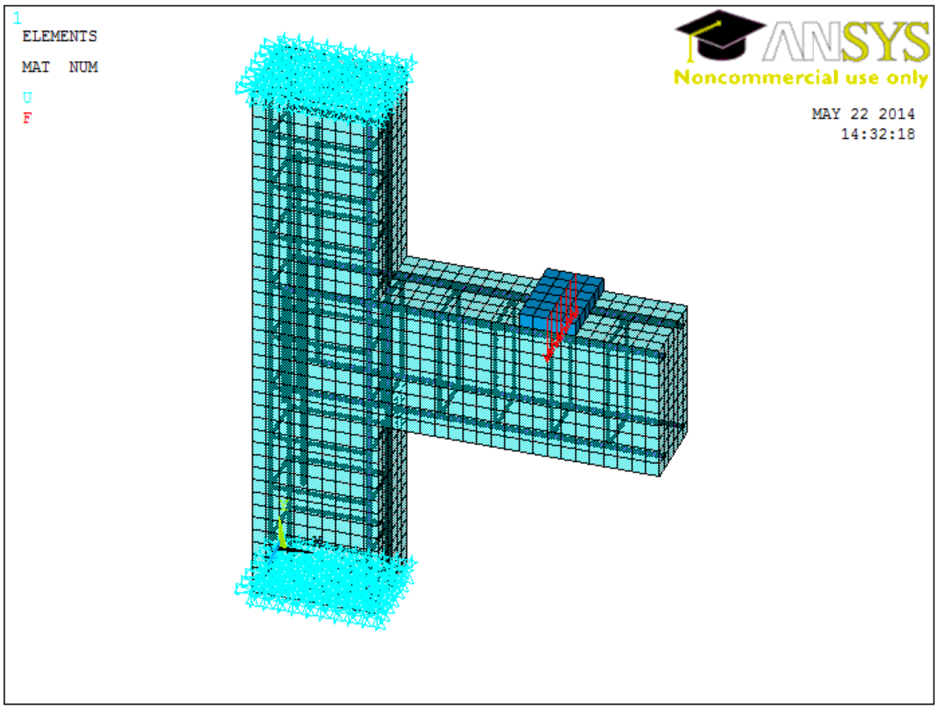


Fig. 3.3.8.1: ANSYS model showing the boundary condition and loading in the B1

3.3.9: Analysis Type and Solution Control:

Exterior Beam-Column Joint as per ***Dar (2011)*** and the proposed model of The Exterior Beam-Column Joint is analysing as the static analysis. The restart command has been used to restart the analysis with the dropped force convergence criteria after first crack to achieve the accurate result and to avoid the convergence problem due to loss of stiffness after the first crack. Following is the solution control and convergence criteria have been used.

Table 3.3.9.1: Solution control for the non-linear analysis by ANSYS	
Analysis option	Small displacement (geometry nonlinearity ignored)
Automatic time stepping	On
Write items to results file	All solution items
frequency	Write every sub steps
Equation solvers	Sparse Direct(for concrete)
Number of restart files	1
Line search	Off
Maximum number of iteration	100

All those values which are not specified here are taken as default to ANSYS (v13).

The nonlinear convergence criteria use in the analysis is being presented in the Table below. Force and deformation criteria are being used in the present nonlinear analysis.

Table. 3.3.9.2: Nonlinear convergence criteria		
Type	F	U
Ref. Value	Calculated	Calculated
Tolerance limit	0.005	0.05
Min. reference	Not applicable (-1)	Not applicable (-1)

Two different convergence criteria are being used in the whole non-linear analysis of the exterior beam-column joints B1 and D1. In the first phase of analysis before the first crack in the concrete there is being no problem of the convergence so both force and displacement criteria as mentioned in the Table 3.3.9.2. But after the first crack in the concrete, convergence was impossible with the above mention value. So after the convergence failure after the first crack, forced convergence criteria was dropped. And at the same time load steps are increased to consider the loss of stiffness due to increase in the crack of concrete.

Chapter 4

Result and discussion

4.1: PHASE I: STAAD.Pro Results

A parametric study has been done on the benchmark building to study the distribution of joint shear demand of the joints for the building designed as per IS456:2000 and detailed according to IS 13920:1993 if provision applied.

The benchmark building is selected as the 3 story and 3 bay structures. The following parameter are varied to the verified influence of these on the shear demands of the joint under the given most critical loading, which is found to be the 1.5DL+1.5EQ.

Followings are the parameter which has been checked to understand their influence on the joint shear demand. And following that the graph has been shown to discuss how they are affecting the shear force demand of the joints.

a. Support conditions
b. Story height
c. Number of story or height of the building
d. Width of the bays
e. Number of the bay
f. Grade of the concrete
g. Size of the beam
h. Size of the column

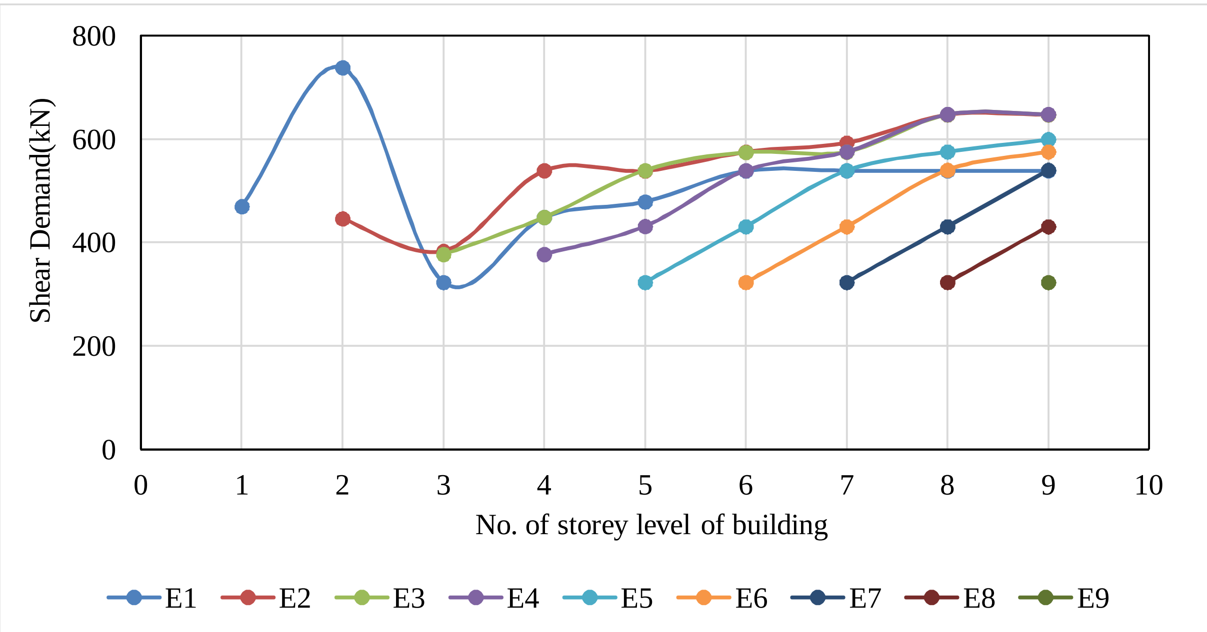


Fig 4.1: Effect of No of Storey on the Joint Shear Demand (Fixed Support)

As you can see from the figure that joint name E1 shear demand is more for only up to two-story building(fixed support) and thereafter E2 shear demand is leading. From this figure it is clear that joint shear demand of the 2nd story level is critical but the gap of difference goes on decreasing as the number of story goes on increasing.

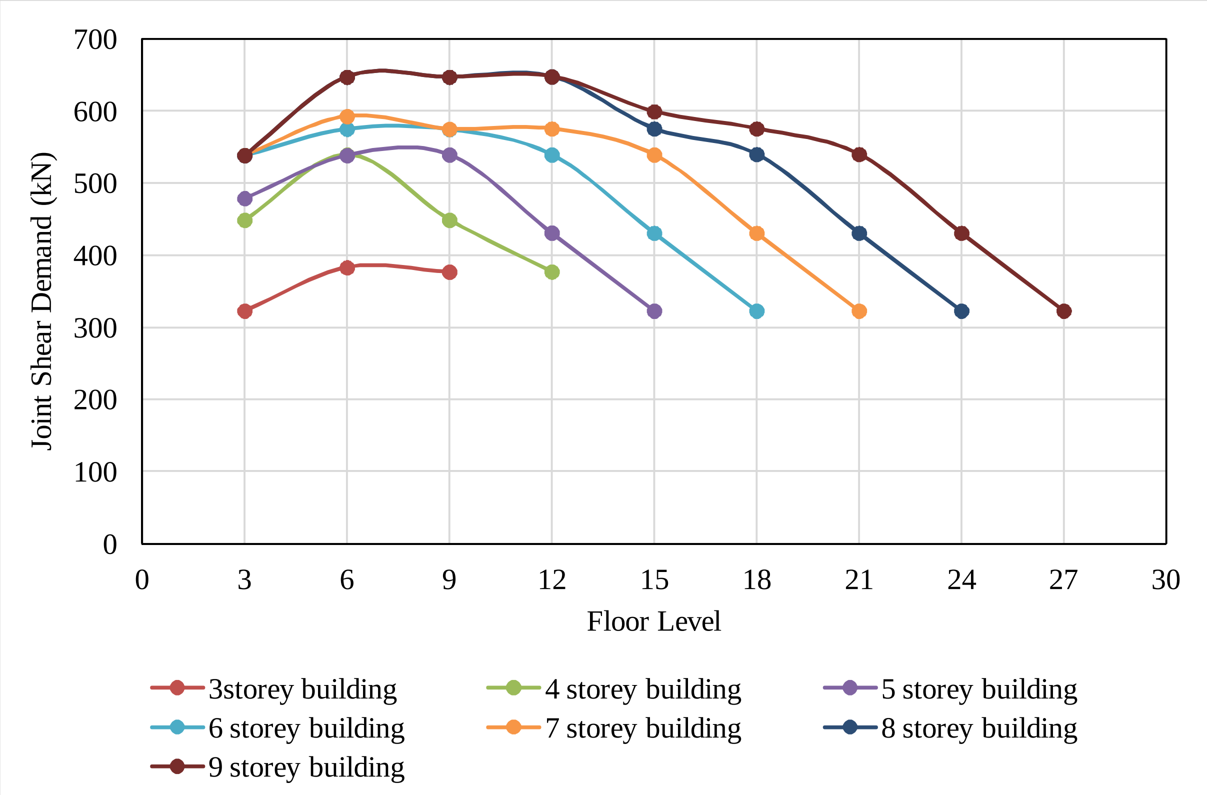


Fig. 4.2: Effect of No. Of Story on the Shear Demand of the Joint (Fixed Support)

This figure is also plotted on the same data but with respect to floor level (fixed support). As you can see that first story joint shear demand is less as compare to the above few joint but again the shear demand decrease very fast. This trend is same for all type of story.

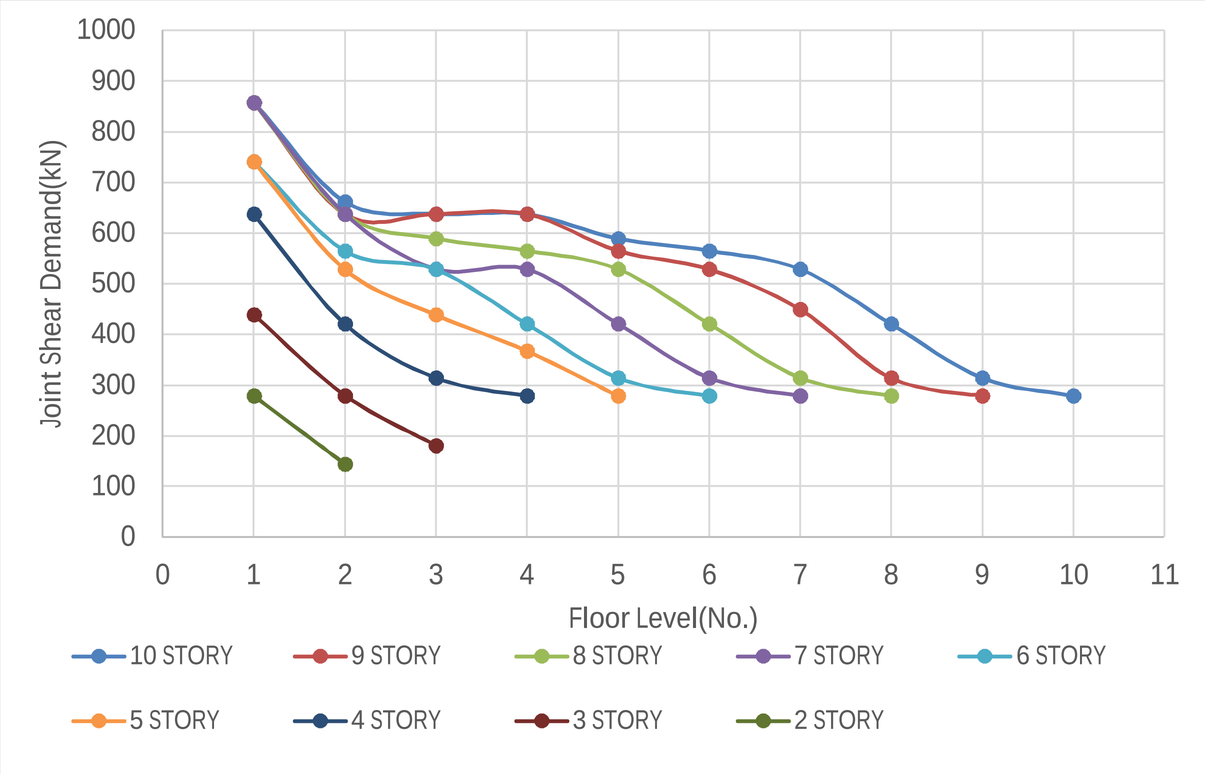


Fig 4.3: Effect of Number of Story on the Shear Force Demand (Hinge Support)

This figure shows the shear demand of the joint at the various levels with increasing number of story for the hinge support. As you can see that due to hinge support there is drastic increase in the first level of joints.

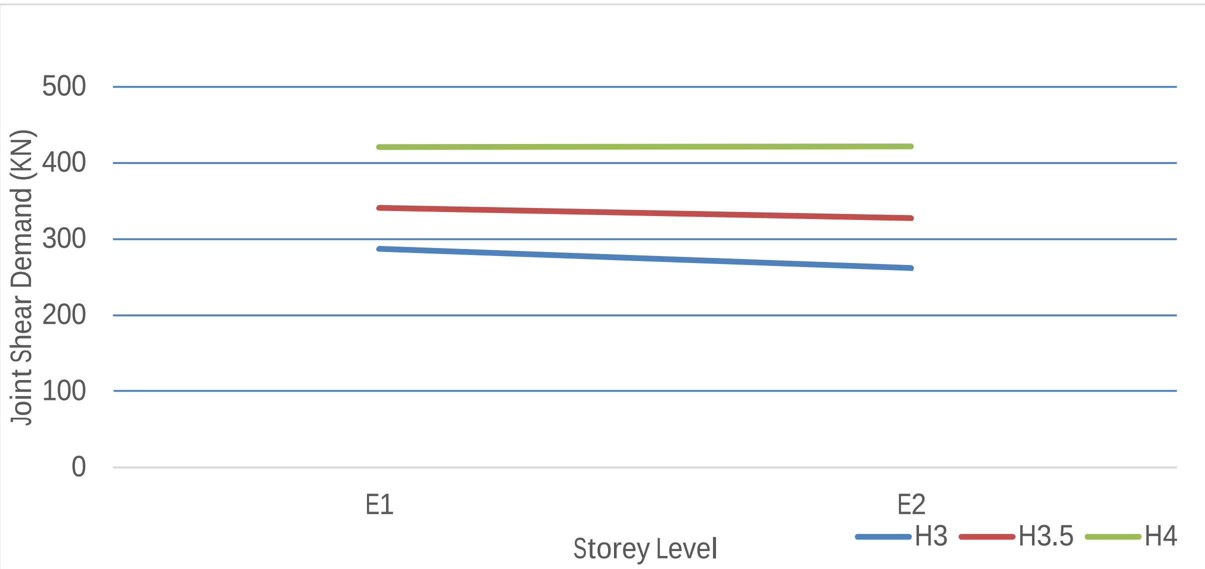


Fig 4.4: Joint Shear Demand Vs Storey Height (Fixed Support)

This figure is showing the variation of shear demand due to increase in the story height of the building with the fixed support. Form the figure we can simply interpret that increasing the height of story increase the shear demand of the building.

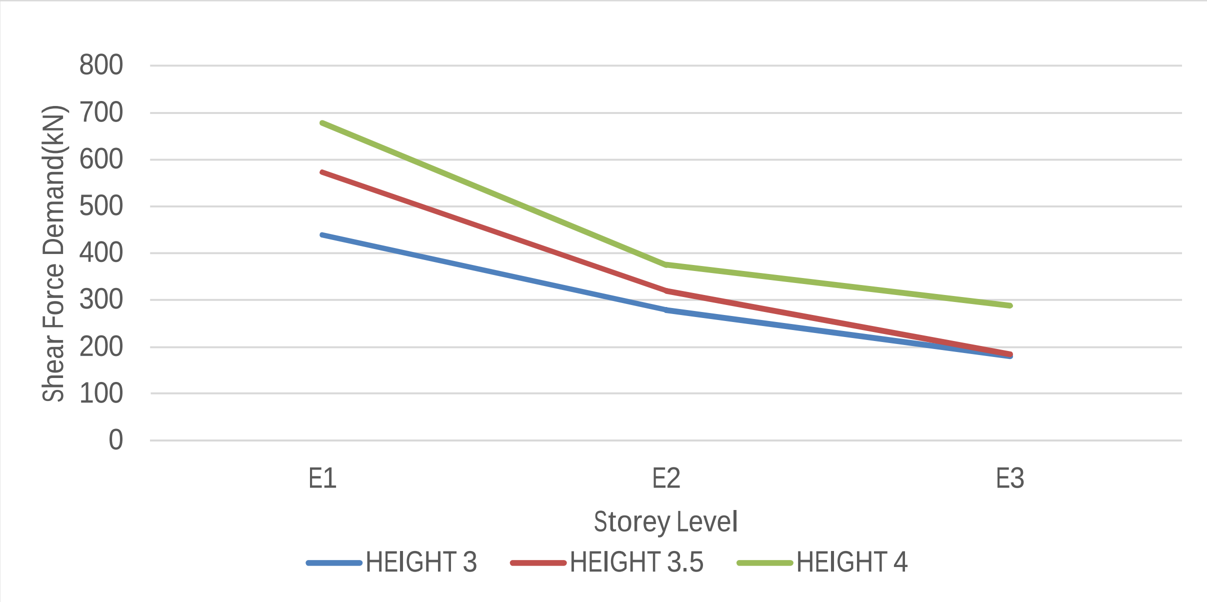


Fig. 4.5: Effect of the Height of Storey on the Shear Demand of the Joint (hinge Support)

This is same as Fig. 4.4 is plotted but this is for the hinge support. And you can directly see that hinge support increase the shear demand of the first story. We can simply say that increasing the height of the story increase the shear demand of the joints.

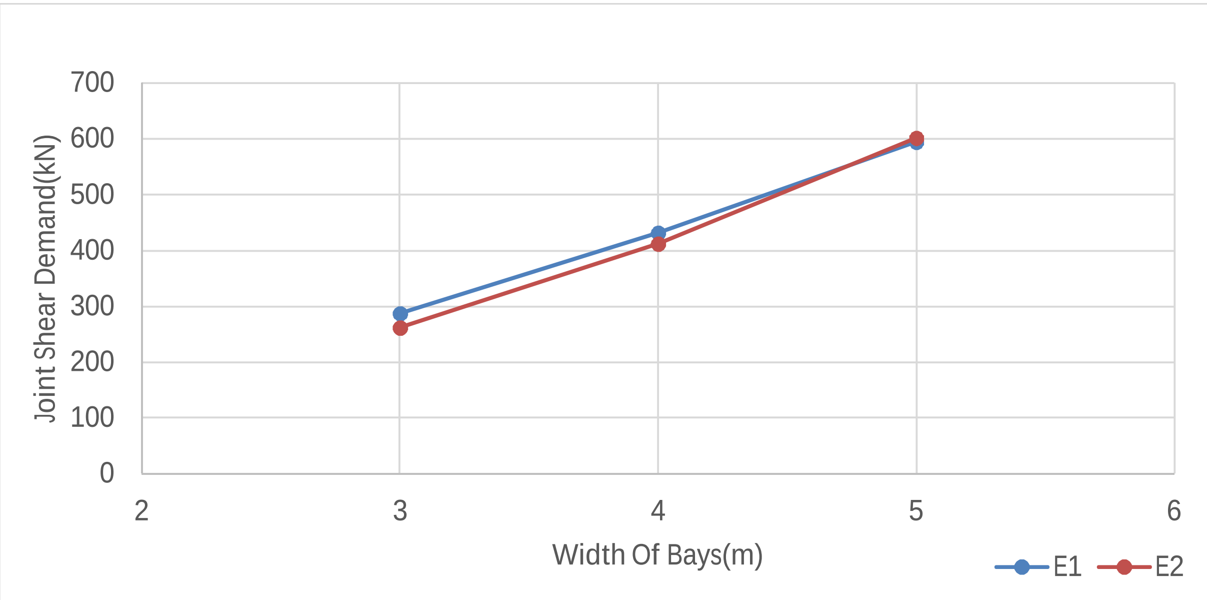


Fig. 4.6: Effect of Width of Bay on the Joint Shear Demand (Fixed Support)

This figure shows the effect of width of bay on the joint shear demand for the fixed support. This figure is clearly showing the positive effect of the width of the bay on the shear demand. As you can see the increase in the bay width from 3m to 4m the shear demand got double for both E1 and E2 joint.

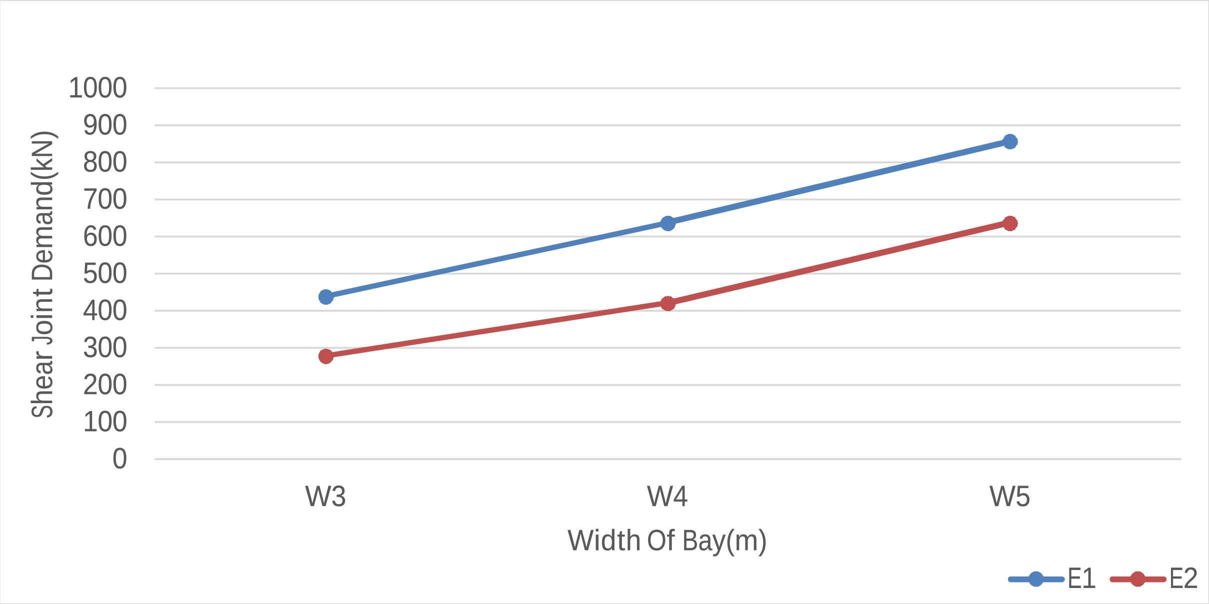


Fig. 4.7: Effect of Width of Bay on the Shear Demand (Hinge Support)

This figure is showing the same effect of bay width on the shear demand of joint but for the hinge support. And you can see the jump in the shear demand from 500kN to 900kN. This conclude that the making the support hinge increase the demand of the joint.

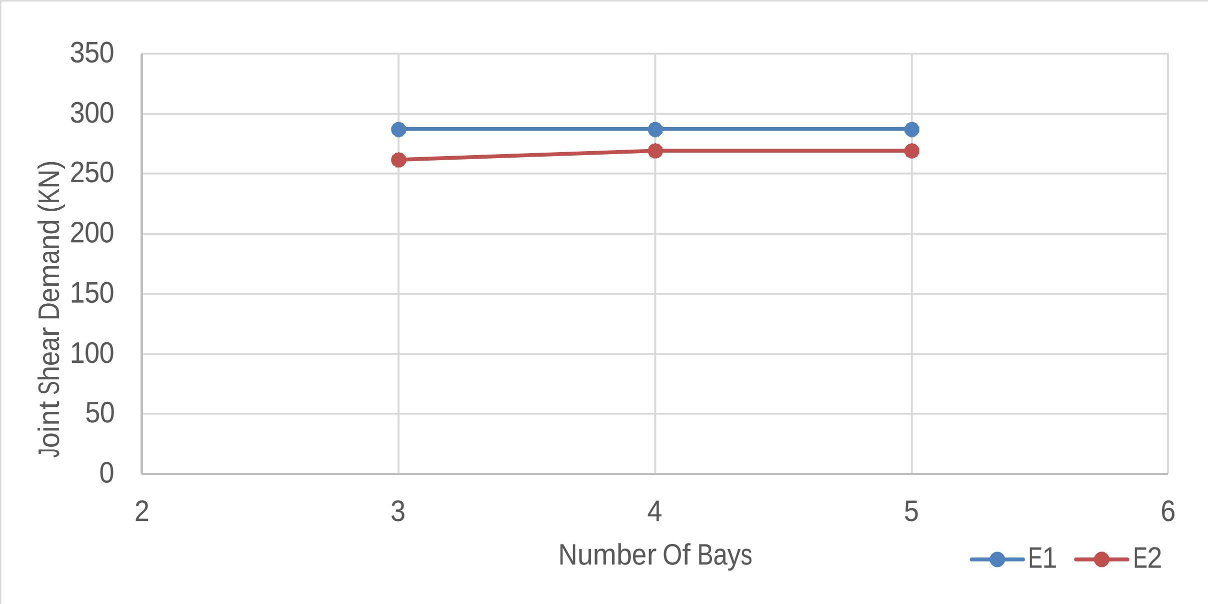


Fig. 4.8: Joint Shear Demand vs. Number of Bays (Fixed Support)

This figure is showing the effect of number of bay on the shear demand of the joints. From the figure it is clear that the increasing the number of bay has not significant effect on the joint shear demand of the joint.

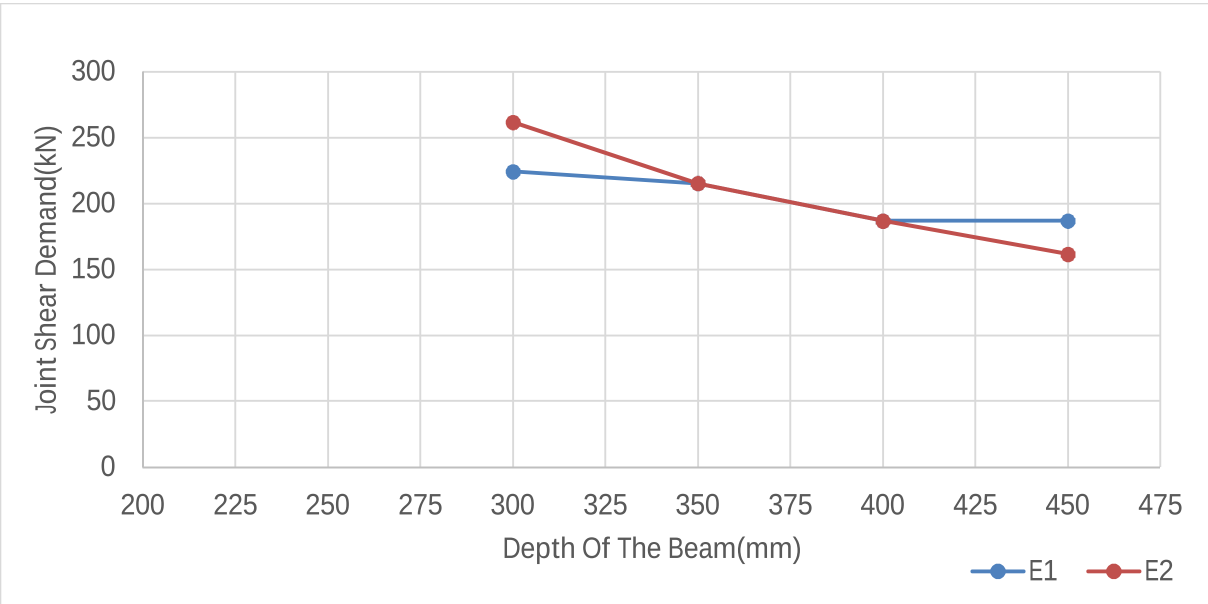


Fig 4.9: Effect of Depth of Beam on Joint Shear Demand (Fixed Support)

This figure is showing the effect of depth of beam on the shear demand of the joints. Clearly from the graph it can be proved that the increasing the depth of beam decrease the shear demand of the joint. So, if we want less shear demand at the joint we can increase the depth of beam.

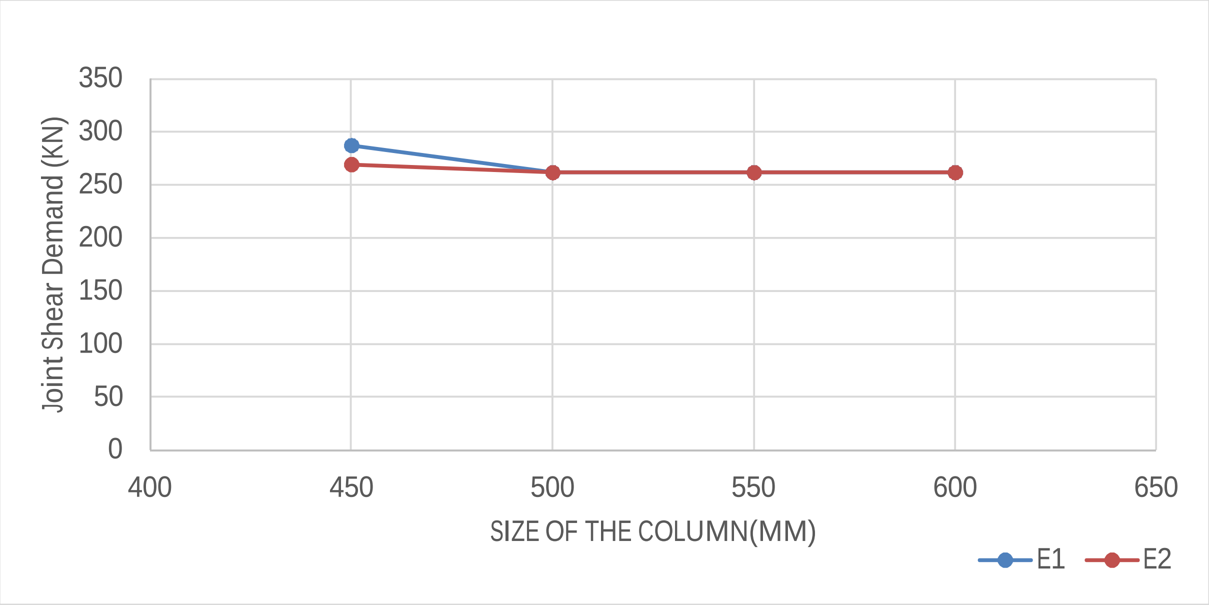


Fig. 4.10: Effect of Column Size on the Joint Shear Demand (Fixed Support)

This figure shows the effect of grade of concrete on the shear demand of the joint. As you can see that there is no significant effect on the shear demand on the joint due to change in the grade of concrete.

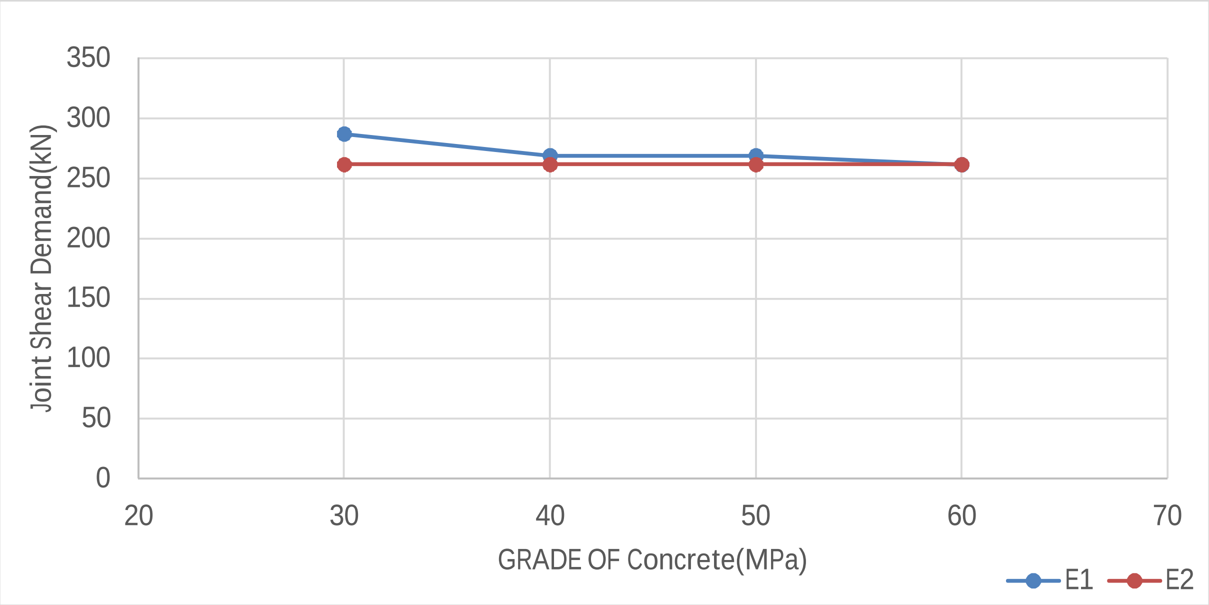


Fig. 4.11: Effect of Grade of Concrete on Joint Shear Demand (Fixed Support)

This figure is showing the effect of the column size on the shear demand of the joint. There is no significant effect of column size on the shear demand.

4.2 PHASE II: Nonlinear ANSYS Results:

Comparison of results between "The Traditional Beam-Column Joints" and "The Prestressed Beam-Column Joints":

In the following section ANSYS results are being used to demonstrate that how the prestressing the joint core as shown in Fig 3.3.3 with the normal stirrups confined joints as shown in Fig 3.3.1 as specified earlier.

B1: Exterior Beam-Column Joint with core stirrups as experimentally tested by ***Dar (2011)***

D1: Exterior Beam-Column Joint with prestressed core as proposed by the present work. There extra three rebar are crossed running through the joint with the stain of 0.005. Plates are used just as the bearing to avoid the crushing of the concrete at the corner.

1. Comparison between crack of the both joints:

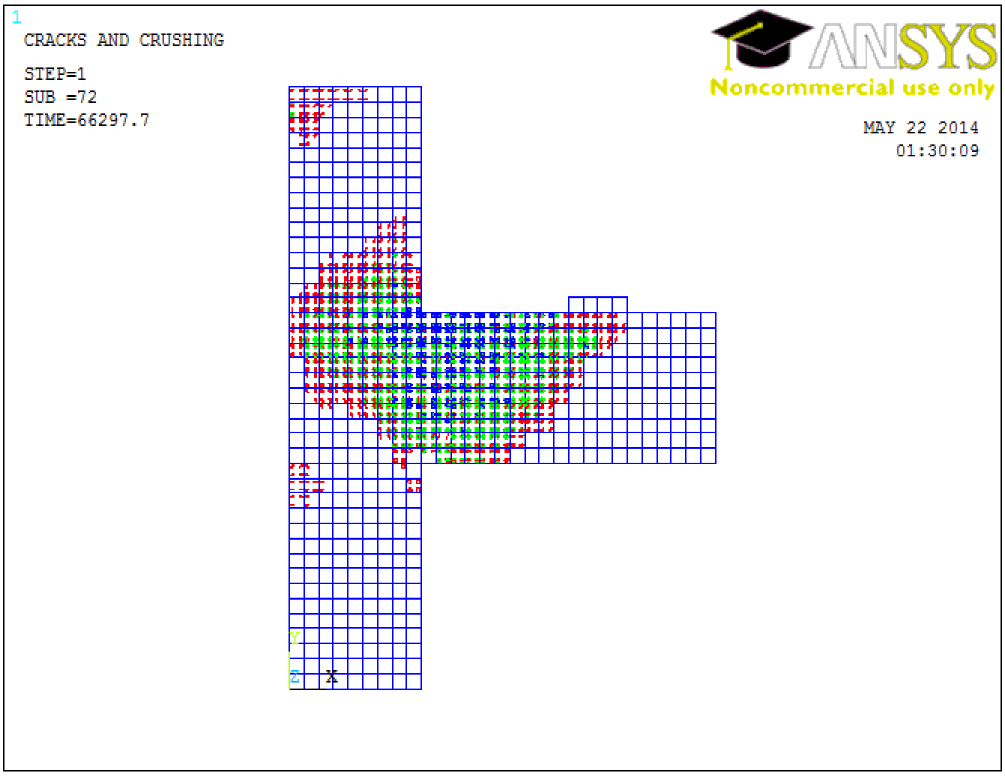


Fig. 4.12: Cracks pattern of B1 at the ultimate loads of 66.3kN

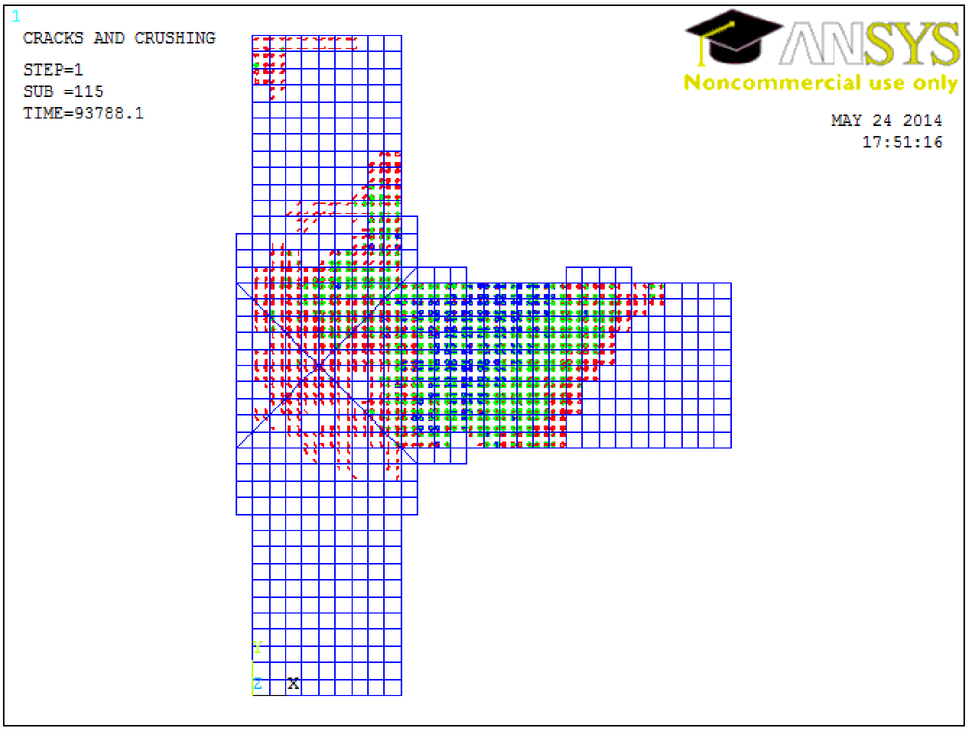


Fig. 4.13: Cracks pattern of the D1 at the ultimate load of the 93.7kN

2. Comparison of the shear stress distribution in the joints of both type:

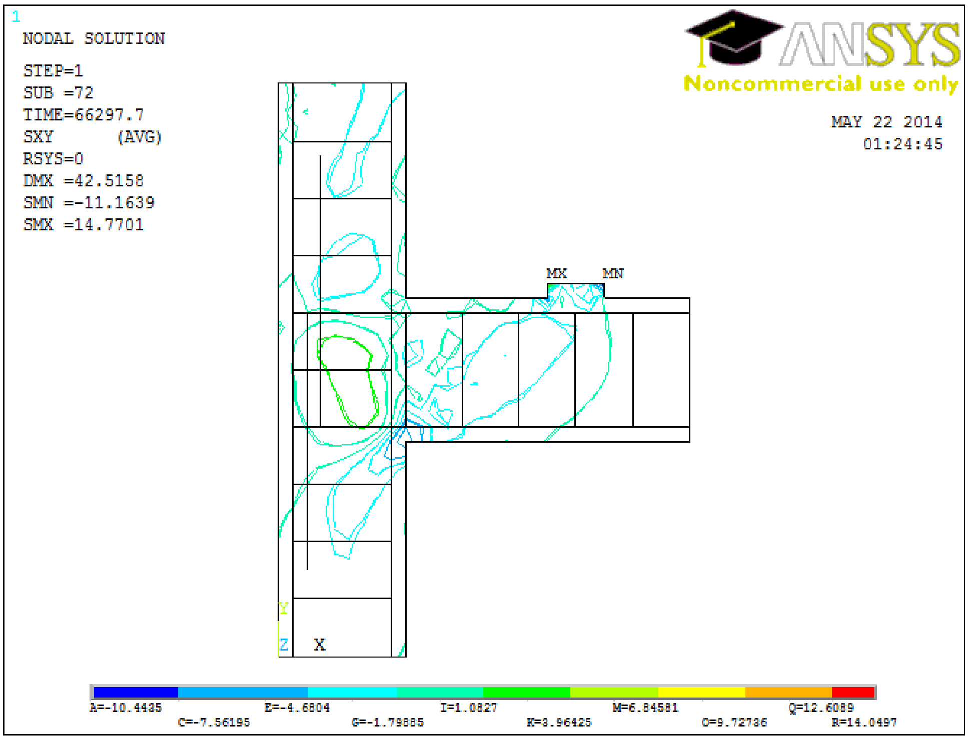


Fig. 4.14: Shear stress distribution of the B1 at the ultimate load 66.3kN

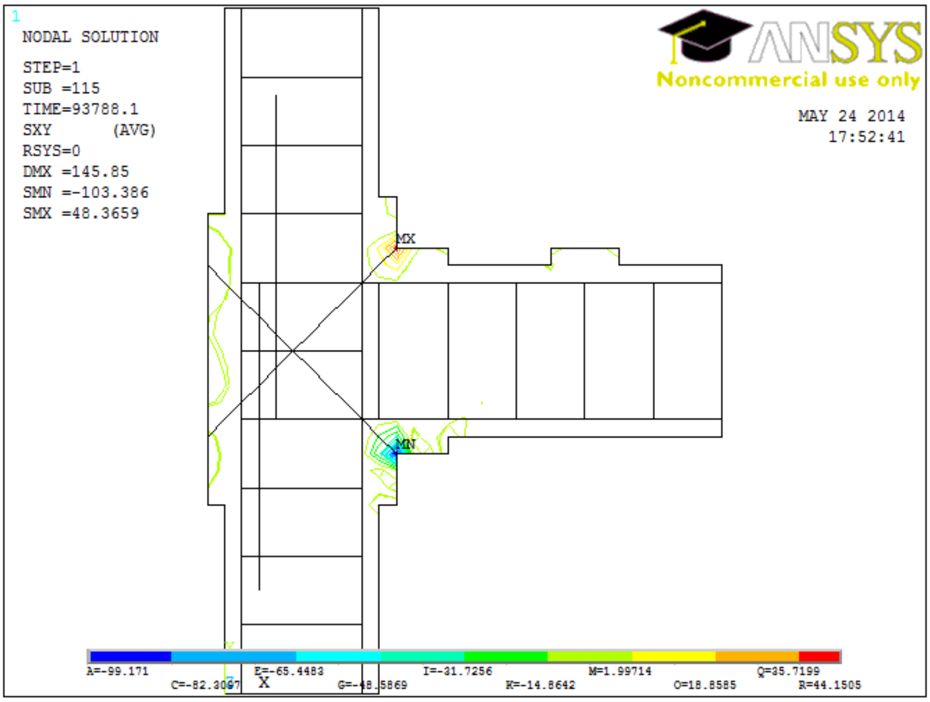


Fig. 4.15: Shear stress of the D1 at the ultimate loads of 93.7kN

By comparing the fig 4.14 and fig 4.15 it can be clearly stated that in B1 the shear force is more concentrated in the joints. This proves the experimental test data of shear failure of joint. The fig 4.15 in which prestressing are being used clearly helped in putting the shear stress out of the joint core and ultimately avoiding the shear failure of the joint.

3. Deflection comparison of the both type of the joints:

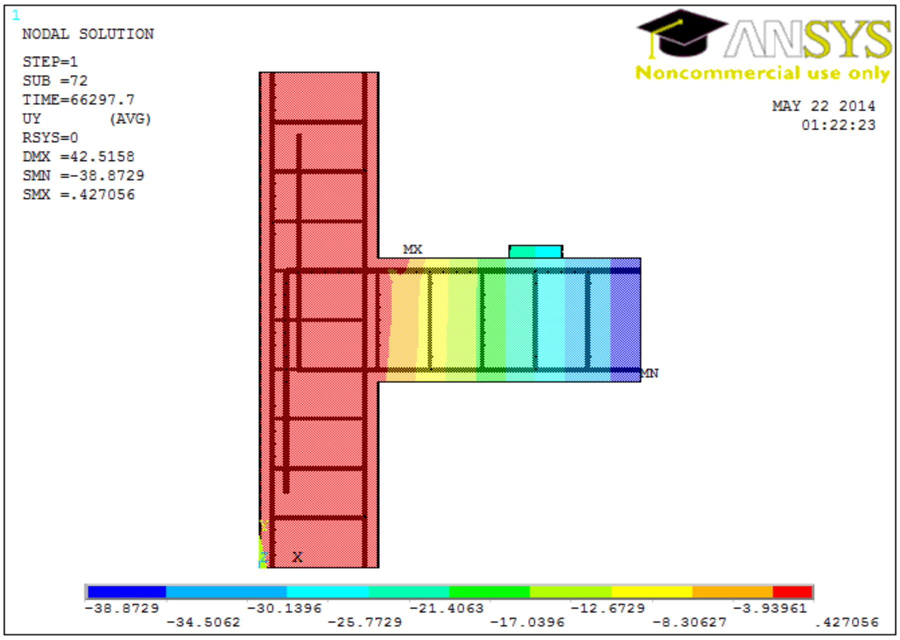


Fig. 4.16: Deflection profile of B1 at the ultimate load of 66.3kN

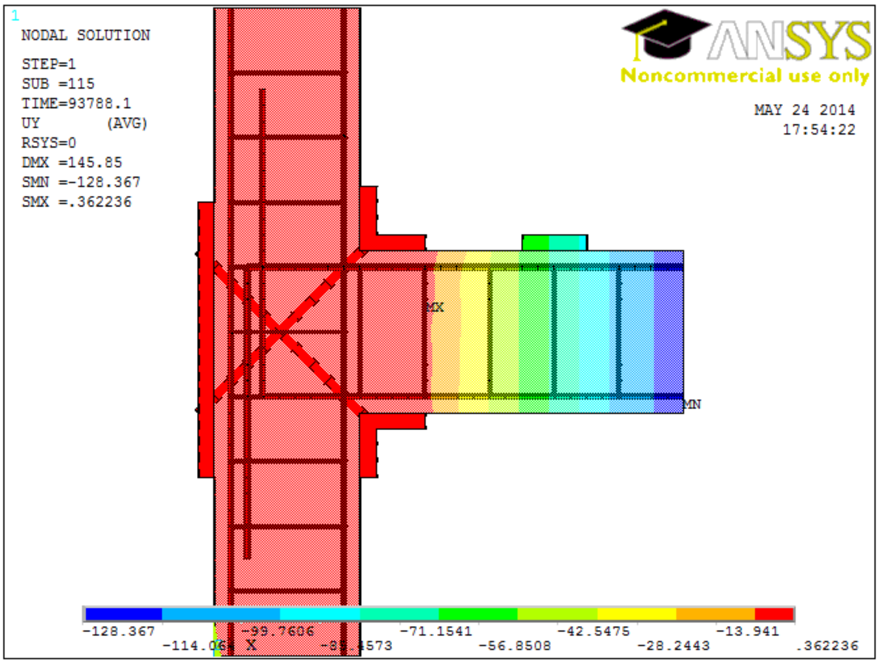


Fig. 4.17: Deflection profile of D1 at the ultimate load of 93.7kN

Comparison of the fig 4.16 and fig 4.17 shows that the prestressing of the exterior beam-column joint as proposed behave as more rigid than Dar (2011). The free end deflection of the B1 at 66.3kN is 38.3mm while in the D1 it is just 14.9 at 94.34kN.

4. Comparison of the total mechanical shear strain:

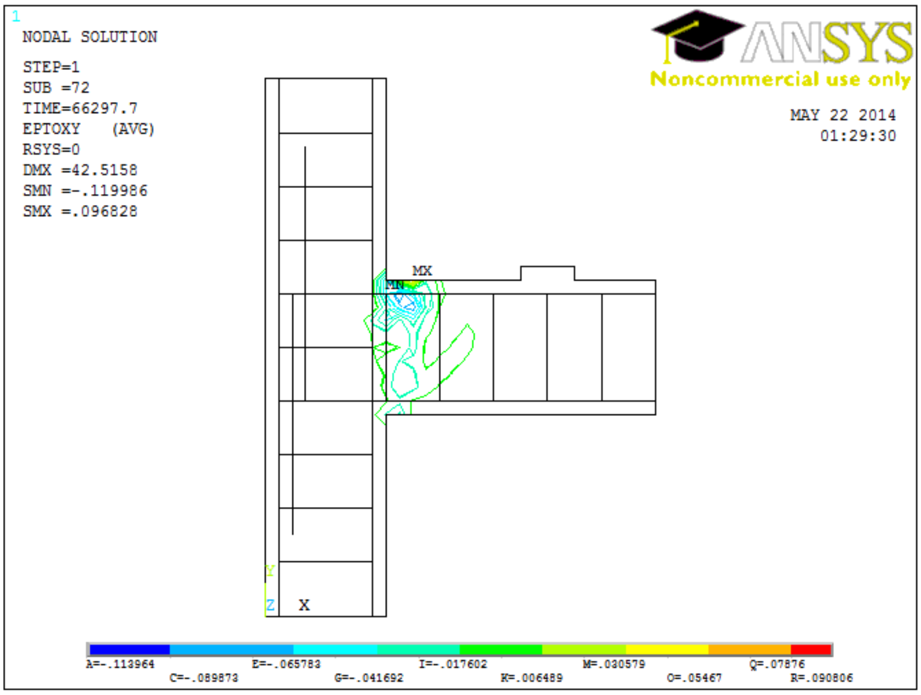


Fig. 4.18: Shear strain of the B1 at the ultimate loads of the 66.3kN

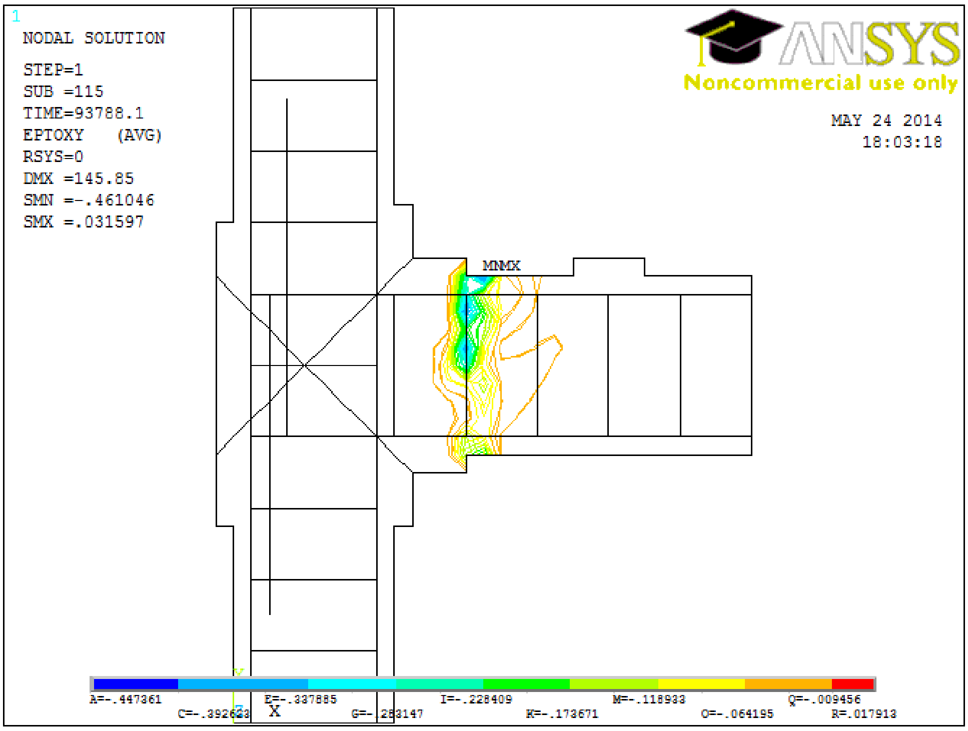


Fig. 4.19: Shear stain of D1 at the ultimate loads of the 93.7kN

5. Summary of comparisons:

Comparison Summary of the Both Beam-Column Joints			
Sl. No.		Non-prestressed joints	Pre-stressed joints
1.	Crack location	In the Joints	Shifted to the Beam
2.	Ultimate collapse load	66.3kN	93.7kN
3.	Ultimate deflection	38.8 mm	145.8mm

Chapter 5

Summary and Conclusions

5.1 SUMMARY

The objective of the present study was defined as. In order to achieve first objective a family of multi-storeyed plane frame with varying building-height, storey-height, base-width, number of bays, column and beam dimensions and grade of concrete were selected. The selected building models were analysed and design according to IS 456:2000 using commercial software STAAD.Pro. Results were analysed to find out the effect of all the above parameters on the shear force demand of critical beam-to-column joints. Also an effort has been made to detect the location of the critical joint in the multi-storeyed framed building.

To achieve the second objective an innovative joint reinforcement scheme is developed and modelled in finite element software ANSYS v13.0. Beam-column joints with conventional joint reinforcement were also modelled to compare the results of the proposed model. These models were analysed for nonlinear static behaviour. Result were presented how the new approach is effective in reducing the shear demand of the joints and hence can be used to solve the problem of congestion in the beam-column joints.

5.2. CONCLUSIONS

The following are point-wise conclusions which are being drawn from the proposed Exterior Beam-Column Joints with prestressed joint core:

- ❖ Maximum joint shear demand are located at lower portion of building, starting from second story joint for both interior and exterior joints for the fixed support.
- ❖ Maximum joint shear demand is located at first story joints for the hinge support condition for the both interior and exterior joints.
- ❖ The ratio of height of maximum shear to building height is coming out as 0.4 for the fixed support.
- ❖ Shear forces demand increases with the increase of the Number of Story, Height of Story, Width of Bays and Decreases with the Increase of Depth of Beams.
- ❖ Grade of Concrete, Number of Bays and Size of Columns has no effect on the demand of the shear forces in the beam-column joints.
- ❖ Due to prestressing the Exterior Beam-Column Joints there has been increase in the shear strength of the concrete in the joint core. But model for the calculation of the

shear strength of concrete in the prestressed beam-column joints has not been presented in the present work.

- ❖ Due to crossed prestressing with the rebar, strut and tie model has been invoked in the joints enhancing the performance of the joints. With prestressed rebar acting as tie enhances the crack resistance in the joint and consequently enhance the strut concrete performance which will act as better than without stressed post crack condition.
- ❖ Due to presence of the steel plate at the face of the Beam-Column joint, plastic hinge shifted at the edge of the plate. This shifting of the hinge toward the centre of the beam leads to the less lateral displacement at same given rotation at plastic hinge.

5.3 FUTURE SCOPE:

- ❖ Due to cross prestressing there is increase in the shear strength of the concrete in the joint core. A model can be formulated to calculate the increase in shear strength of the joint core.
- ❖ The above result clearly shows the increase in the performance of the joint due to cross-prestressing which may leads to the decrease in the joint confinement reinforcement. Further a formulation can be generated to calculate that how much reinforcement can be reduced due to this cross-prestressing.

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