

# **THE FEEDFORWARD PLUS DECOUPLING CONTROL DESIGN FOR TITO SYSTEM**

*A Thesis submitted for partial fulfillment for the degree of*

**Shaktikanta Nayak**

*Master of Technology*

*In*

Electronics and Instrumentation Engineering



**Department of Electronics and Communication Engineering  
National Institute of Technology, Rourkela**

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**By**

**SHAKTIKANTA NAYAK**

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**Prof. Tarun Kumar Dan**



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# CERTIFICATE

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This is to certify that the project entitled, “**Feedforward plus Decoupling control Design for TITO system**” submitted by **Shaktikanta Nayakis** an authentic work carried out by him under my supervision and guidance for the partial fulfilment of the requirements for the award of **Master of Technology in Electronics and Instrumentation Engineering during session 2012-14** at **National Institute of Technology, Rourkela**. A bonafide record of research work carried out by them under my supervision and guidance.

The candidates have fulfilled all the prescribed requirements.

The Thesis which is based on candidates’ own work, have not submitted elsewhere for a degree/diploma.

In my opinion, the thesis is of standard required for the award of a Master of Technology Degree in Electronics and Instrumentation Engineering.

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(Prof. T. K. Dan)

Place:

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Shaktikanta Nayak

(212EC3159)

*Dedicated to my Respected Parents Pratap Ch. Lenka  
and Sunanda Lenka, and Lovely Brother*

*Devidatta Nayak and my  
BelovedFriends.*

## **ABSTRACT**

Proportional-Integral-Derivative (PID) controllers are commonly used controllers in industry for their relative ease of use and it also provides the satisfactory performance in industrial processes. It exploits in case of tight performances by employing additional functionalities such as antiwindup, set-point weight, feedforward action. These functionalities are developed for Single-Input-Single-output (SISO) systems. A system with multi inputs and multi outputs are alloyed as MIMO system. Industrial systems like Chemical reactors, Heat exchangers and Distillation column etc. are the best examples of MIMO systems. To perform an operation in SISO system is easier than the operation in MIMO system. The cause of this complication occurs due to the coupling or interactions in between the input and output variables. Hence to minimize the interactions, in this paper we are going to discuss about Decoupling for a TITO system. And the feedforward controller is used for following the set-point.

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## **Acronyms**

SISO: Single-Input-Single-Output

TITO: Two-Input-Two-Output

MIMO: Multi-Input-multi-Output

RGA: Relative Gain Array

PI: Proportional Integral

PID: Proportional Integral Derivative

# CHAPTER: 1

## **Introduction**

## 1.1 INTRODUCTION:

Proportional-Integral-Derivative (PID) controllers are commonly used controllers in industry for their relative ease of use and it also provides the satisfactory performance in industrial processes. It exploits in case of tight performances by employing additional functionalities such as antiwindup, set-point weight, feedforward action. These functionalities are developed for Single-Input-Single-output (SISO) systems.

A system with multi inputs and multi outputs are alloyed as MIMO system. Industrial systems like Chemical reactors, Heat exchangers and Distillation column etc. are the best examples of MIMO systems. To perform an operation in SISO system is easier than the operation in MIMO system. The cause of this complication occurs due to the coupling or interactions in between the input and output variables [1].

## 1.2 OBJECTIVE:

In this thesis we have proposed the method for reducing the process interactions occurring in a TITO system. For reducing the pairings between the controlled variables and manipulated variables we are using a decoupler control scheme in this.

By using the decoupler we can minimize or reduce the interactions in the system. A feed forward controller is also used for the set point tracking. For the design of feed forward controller we have taken a reference model which transfer function is the nonminimum-phase part of process transfer function.

# CHAPTER: 2

## **Overview on TITO system**

## 2.1 SISO SYSTEM:

In case of Single-input-Single-Output (SISO) systems, it is very easy and convenient in implementation. Many concepts which are exploited for the purpose of SISO processes are such as input-output inversion, bang-bang control, set-point weight etc. let us consider standard distributed control system ( DCS ) blocks for implementation of feedforward control law in a SISO process.

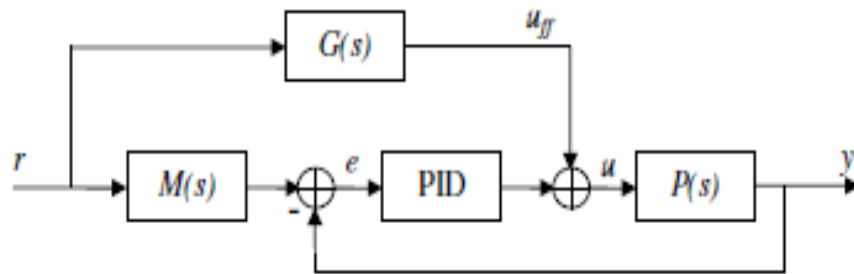


Fig 2.1: The PID plus feedforward control process for SISO system.

$M(s)$  is the reference model which gives us desired response for the set-point change. It must contain the nonminimum-phase part of transfer function of the process which is  $P(s)$ . Hence the  $G(s)$  is chosen,

$$G(s) = \frac{M(s)}{P(s)}$$

This method, the feedforward contains the process model and lead/lag network. So as far in the case of TITO processes we cannot consider this strategy[1].

## 2.2 TITO SYSTEM:

This type of system is commonly known as Two-Input-Two-Output system. It is a MIMO (Multi-Input-Multi-Output) type of system. In the case of SISO systems, we have only single manipulating variable to control the single output variable. But in cases of MIMO [2] systems we have to control a number of inputs over a no of output variables at a time. Let's take an example of MIMO process i.e. A two-input-two-output coupled tank system.

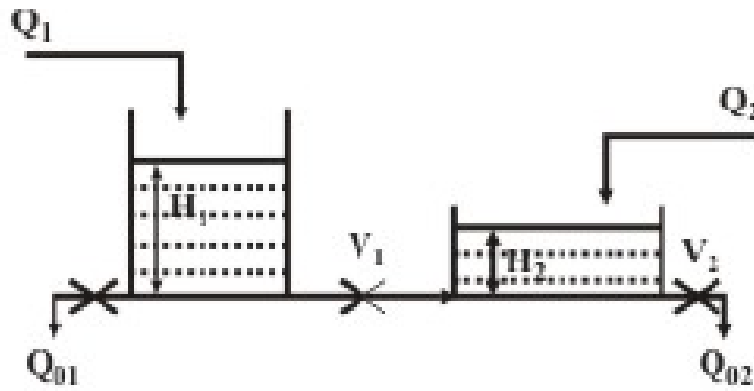


Fig: .2.2.A two-input-two-output interacting tank system.

As in the above figure  $Q_1$  and  $Q_2$  are the inputs i.e. flow rate and  $H_1$ ,  $H_2$  are the outputs i.e. change in the liquid levels of the two tanks.

Here the controlled variables are  $Q_1$  and  $Q_2$

And the manipulated variables are  $H_1$  and  $H_2$ .

Here each manipulated variable depends on the two controlled variable.

### **2.3 WHY IMPLIMENTATION OF TITO IS DIFFICULT THAN SISO?**

- Control loop interactions and Process interactions.
- Pairing of controlled variables and manipulated variables.
- Tuning of multiloop PID controller.
- Reduction of control loop interactions.
- Decoupling and multivariable control strategies.
- The multiloop control system may not provide best control even if the process interactions are significant.

## 2.4 TRANSFER FUNCTION MODEL OF TITO SYSTEM:

Let's consider general block diagram of TITO system or 2x2 systems.

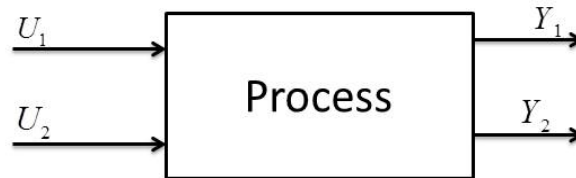


Fig 2.3: Block diagram of TITO system

Here  $U_1$  and  $U_2$  are the two controlled variables or inputs and  $Y_1$  and  $Y_2$  are the two outputs or manipulated variables [3] [4].

According to the above diagram two possible controller pairings can be obtained i.e.

$$U_1 \text{ With } Y_1, U_2 \text{ with } Y_2 \quad (1-1 / 2-2 \text{ pairing})$$

$$U_1 \text{ With } Y_2, U_2 \text{ with } Y_1 \quad (1-2 / 2-1 \text{ pairings})$$

Let's take a glance at the block diagram for TITO or 2x2 system.

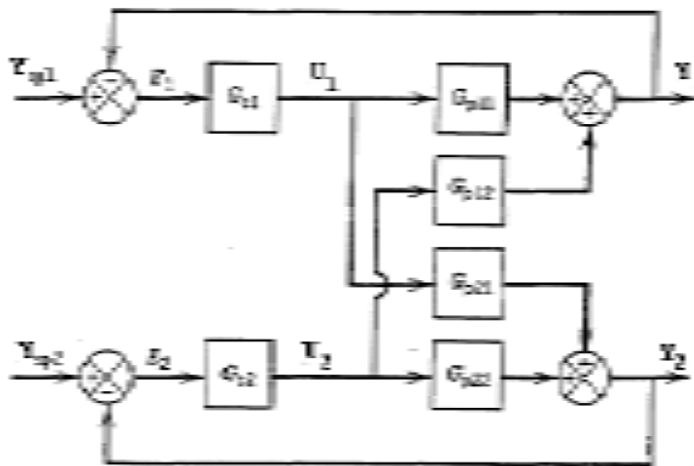


Fig 2.3.1: 1-1 / 2-2 controller pairing.



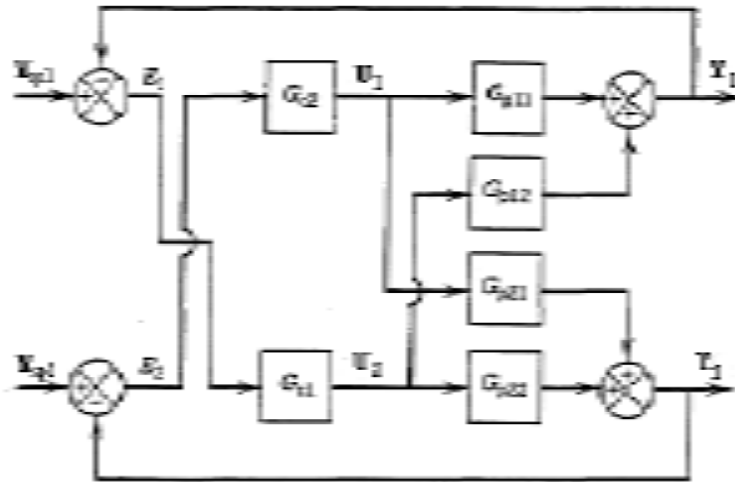


Fig 2.3.2: 1-2 / 2-1 controller pairing.

As we all know we have two manipulated variable and two controlled variables here, hence 4 transfer functions are required.

$$\frac{Y_1(s)}{U_1(s)} = G_{p11}(s), \frac{Y_1(s)}{U_2(s)} = G_{p12}(s) \quad (1)$$

$$\frac{Y_2(s)}{U_1(s)} = G_{p21}(s), \frac{Y_2(s)}{U_2(s)} = G_{p22}(s) \quad (2)$$

Hence the input-output relations can be written as

$$Y_1(s) = G_{p11}(s)U_1(s) + G_{p12}(s)U_2(s) \quad (3)$$

$$Y_2(s) = G_{p21}(s)U_1(s) + G_{p22}(s)U_2(s) \quad (4)$$

This system can also be defined in state variable form i.e.

$$\begin{aligned} \dot{x} &= f(x, u) \\ y &= f(x, u) \end{aligned}$$

Here,  $x$  is state vector,  $y$  and  $u$  are the output and input vectors respectively.

We can linearize the system and it can be written in state-space form is,

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \quad (5)$$

Let's assume there are  $m$  no of inputs and  $p$  no of outputs in a system, then the system's transfer functionmatrix is  $p \times m$ . When output is less than the input ( $m > p$ ) then the system is known as *overactuated* system. When output is more than the inputs, this type of system is known as *underactuated* system. When the no of inputs are equal to the no of outputs then the system is square.

In (5) the input vector  $u$  is of dimension  $m$ , and the output vector  $y$  is of dimension  $p$  and  $x$  is  $n$ -dimensional vector [4].

By taking the Laplace transfer of in zero initial condition,

$$sIx(s) = Ax(s) + Bu(s)$$

Or 
$$x(s) = (sI - A)^{-1} Bu(s) \quad (6)$$

And 
$$y(s) = Cx(s) + Du(s) \quad (7)$$

Implying (6) into (7) we can conclude that,

$$y(s) = (C(sI - A)^{-1} B + D)u(s) \quad (8)$$

Hence  $G(s)$  can be obtained from the above equations i.e.

$$G(s) = C(sI - A)^{-1} B + D \quad (9)$$

# CHAPTER: 3

# DECOUPLING

As we all discussed earlier that, in a TITO system we have to control two controlled and two manipulated variables. Due to this reason process interactions are shown in this case. Process interactions cause undesirable interactions between two control loops. Hence we face the problems like [5],

- Controller tuning seems to be more difficult.
- Closed –loop systems are going to be destabilized.

### **3.1 ALTERNATIVE STRATEGIES:**

To avoid the interactions between the two control loops we can do,

- ✓ “Detune” the feedback controllers.
- ✓ We have to consider in case of the selection in manipulated or controlled variables.
- ✓ We can use different types of multivariable control schemes.
- ✓ We can take consideration of a decoupling control system scheme.

### **3.2 WHY DECOUPLING?**

- ❖ These controllers are designed by using the simple process model such as steady state model or transfer function model.
- ❖ Basically it allows changing in setpoint that result in affecting on the desired controlled variables.
- ❖ By using decoupler we can compensate the process interactions and thus it reduces the interactions in control loops.
- ❖ We can eliminate the complicated loop interactions which results in the system that the change in the process variables have no influence on another process variable.

As we discussed above in a TITO system the major problem we face is the interactions between the loops. We can minimize the problem of this kind of interactions by designing of a perfect decoupler. Study of a decoupling system can be done by a most reliable or by a most effective method i.e. Relative Gain Array (RGA) method.

### 3.3 RGA:

This method is commonly known as Relative Gain Array method, and is proposed or introduced by Bristol in 1966 [4]. This method is commonly used for

- The measure for process interactions.
- It can also show the pairing of control loops.
- We can predict the behavior of control pair ness.
- By the help of this method, selection of controlled manipulated variables pairing can possible
- It shows the method in which the input can be coupled with the output with minimal interactions.

Let's consider a process is having N no of manipulated variables then there are N! no of ways of having controlled loops. RGA can be defined as [4] [6],

$$\text{RGA} = \begin{pmatrix} \lambda_{11} & \dots & \lambda_{1n} \\ \vdots & \ddots & \vdots \\ \lambda_{n1} & \dots & \lambda_{nn} \end{pmatrix}$$

Here,  $\lambda_{ij} = (\text{open loop gain between } y_i \text{ and } u_j) / (\text{closed loop gain between } y_i \text{ and } u_j)$

$$\lambda_{ij} = \frac{\left. \frac{\Delta y_i}{\Delta u_j} \right|_U}{\left. \frac{\Delta y_i}{\Delta u_j} \right|_Y} \quad (10)$$

The entries in this matrix can satisfy 2 properties,

1.  $\sum_{i=1}^N \lambda_{ij} = 1$  , for all  $j = 1, 2, 3 \dots n$  (summation of columns)
2.  $\sum_{j=1}^N \lambda_{ij} = 1$  , for all  $i = 1, 2, 3 \dots n$  ( summation of rows )

For linearized steady-state model,

$$\begin{aligned} y_1 &= G_{11}u_1 + G_{12}u_2 \\ y_2 &= G_{21}u_1 + G_{22}u_2 \end{aligned} \quad (11)$$

Open loop gain:

$$\left[ \frac{\Delta y_1}{\Delta u_1} \right]_{u_2} = G_{11}$$

And in case of closed loop gain,  $y_2 = 0$

$$\text{Hence, } y_1 = G_{11}u_1 - \frac{G_{12}G_{21}}{G_{22}}u_1$$

Closed loop gain:

$$\left[ \frac{\Delta y_1}{\Delta u_1} \right]_{y_2} = G_{11} - \frac{G_{12}G_{21}}{G_{22}}$$

Hence from the above we can conclude that,

$$\lambda_{11} = \frac{1}{1 - \frac{G_{12}G_{21}}{G_{11}G_{22}}}, \quad (12)$$

In a TITO or 2x2 systems, only  $\lambda_{11}$  is calculated. The other elements can be determined by the method that “The sum of the elements in row or column adds up to unity”.

In the year of 1987, Skogestad proposed that we can evaluate RGA in matrix form, for systems larger than the 2x2. The method proposed by him is,

$$RGA = G [G^T]^{-1}$$

Where G= Process matrix,

$[G^T]^{-1}$  = Inverse of transpose of G matrix.

### **3.3.1 Calculation of RGA In MATLAB:**

% enter the process matrix.

```
>> g = [1 1; a b c d]
```

```
>>rga = g(*inv g ' )
```

$g'$  = transpose matrix of g

In this project, we are using feedforward and feedback controller for the perfect decoupling where feedforward is used for the set point following purposes and feedback controller is a decentralized PID controller.

### **3.4 FEEDFORWARD CONTROLLER:**

When disturbances occur in the plant, it affects the measured process output. So a necessary and corrective action should be taken in such a manner that the controlled variables can be controlled, in a way that it must has no effect on the manipulated variable. This can be done by implementing a sensor, which can measure or detect the disturbance before the output changes. Hence for this kind of problems a feedforward controller is introduced as a remedy [7].

For better understanding let's take an example of the furnace system. In this system the main disturbance occurs due to the process fluid flow rate. When there is increase in fluid flow rate then the heat duty is also more required. By implementing the feedforward controller we can control the fuel gas flow rate immediately when there is change in the process flow rate. The feedforward controller can be developed based on disturbances models and process transfer functions. In the case when there is no models present then the feedforward controller can do the work of a tunable gain plus lead- lag controller.

### **3.4.1 Advantages of feedforward controller:**

- ❖ The deviations or disturbances can be corrected before they affect the system or plant.
- ❖ It plays the role of a perfect controller in ideal cases.
- ❖ Disturbances are not required to propagate through the system.

### **3.4.2 Disadvantages of feedforward controller:**

- ❖ Infinitely accurate models are required.
- ❖ Infinitely accurate measurements are also needed in this case
- ❖ Disturbances which occurred within the system cannot be compensated.

### **3.4.3 Factors needed before implementing of a Feedforward Controller:**

For designing a feedforward controller, we should keep an eye on some of the factors that are going to be discussed below [8].

- ✓ The controller should be stable and physically it must be realizable.
- ✓ For perfect feedforward compensation, the time delays of the disturbance or deviations should be greater than the time delays of the process. If it cannot be done then the perfect compensation cannot be achieved.
- ✓ In case of presence of a RHP zero in the process, it should be factored out before the implementation of the controller.
- ✓ In case of higher order process than the disturbance, time constants should be neglected before designing the feedforward controller.
- ✓ When dynamics of the deviation and process are in a same time scale, then static feedforward controller are commonly used for their better performances.
- ✓ Feedforward controller can be designed either with PID type (classical feedback) or with IMC structure.
- ✓ Feedforward controller has no effect on the stability of a closed loop system, assuming that the feedforward controller is stable.
- ✓ Disturbance feedforward controller has to be causal and system inversion.

Let's take some of the examples on feedforward controller.



### **3.4.4 Ex:1 First-order process with disturbance.**

Let the first-order plant is [4],

$$s_p(s) = \frac{k_p}{t_p s + 1}$$

And the transfer of deviation is,

$$s_d(s) = \frac{k_d}{t_d s + 1}$$

Hence the feedforward controller is,

$$s_{cf}(s) = \frac{s_d(s)}{s_p(s)} = - \left[ \frac{k_d}{t_d s + 1} / \frac{k_p}{t_p s + 1} \right]$$

$$\text{Or } s_{cf}(s) = - \left( \frac{k_d}{k_p} \right) \frac{t_p s + 1}{t_d s + 1}$$

From this example we can conclude that, in this case feedforward controller is acting as an lead-lag controller.

### **3.4.5 Ex 2: First-order with dead time and deviation:**

Let the plant TF is,

$$s_p(s) = \frac{k_p e^{-\theta_p s}}{t_p s + 1}$$

Deviation TF is,

$$s_d(s) = \frac{k_d e^{-\theta_d s}}{t_d s + 1}$$

Hence the feedforward controller is,

$$s_{cf}(s) = \frac{s_d(s)}{s_p(s)} = - \left[ \frac{k_d e^{-\theta_d s}}{t_d s + 1} / \frac{k_p e^{-\theta_p s}}{t_p s + 1} \right]$$

$$\text{Or } s_{cf}(s) = - \left( \frac{k_d}{k_p} \right) \frac{t_p s + 1}{t_d s + 1} e^{-(\theta_d - \theta_p)}$$

### **3.5 FEEDBACK CONTROLLER:**

In this type of controller mainly the output of the system is controlled by its measurement through a feedback signal. First the feedback signal is collected from the output of the system, and then it is compared with the reference signal of the system. Hence an error signal is generated which is then corrected by a controller and produce the systems control input [9].

In a feedback control system three most of the main components: a plant, the object which is to be controlled, a sensor which is used for the measurement of the output, and last of all a controller which is used for generating the plant's input. A common block diagram of a feedback control system is,

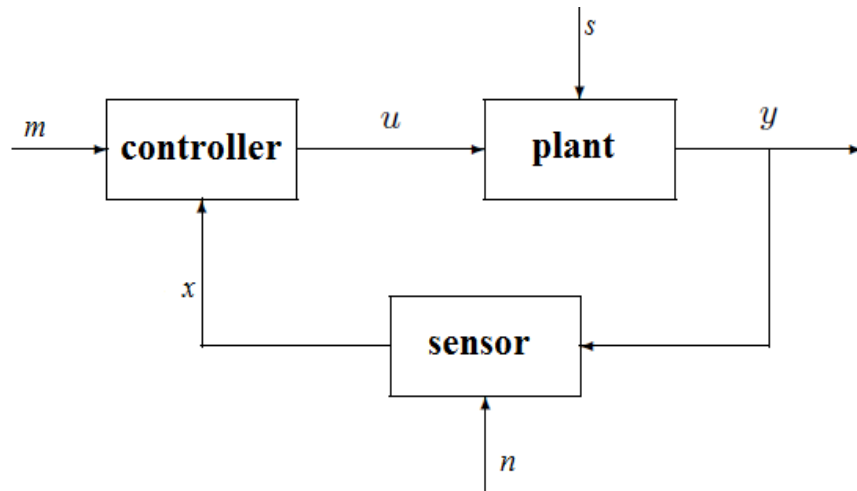


Fig 3.1: Feedback control system.

As per the above figure,

$m$  = reference input

$x$  = sensor output

$u$  = plant input

$s$  = external disturbances

$y$  = plant output or measured signal

$n$  = sensor noise

Here the three signals which are from the outside are  $m$ ,  $s$ ,  $n$ . These signals are called exogenous inputs of the system. The above plant information can be described in two-dimensional vector form like,

$$y = P \begin{pmatrix} s \\ u \end{pmatrix} \quad (13)$$

1x2 TF (transfer matrix) of P is

$$P = [P_1 \ P_2] \quad (14)$$

Hence we get,

$$y = P_1 s + P_2 u$$

Hence the controller equations can be taken from this in the form of,

$$\begin{aligned} y &= P (s + u) \\ x &= F (y + n) \\ u &= C (m - x) \end{aligned} \quad (15)$$

### **3.5.1 Why Feedback control?**

The basic advantages which we get by using this controller is, the system response is insensitive to external deviations (e.g. pressure, temperature etc.) and the variations in internal parameters (e.g. tolerance of the component) of the system can be predicted. Any kind of process models are not required in this type of controller designing. It also helps in stabilizing of an unstable plant [10].

#### ***I. Minimizing the deviations from affecting the output:***

Let's assume that we have to maintain the level of a liquid tank but an predictable disturbance is occurring due to a drain valve. Then for minimizing the disturbance we can use a level sensor to measure the level of the liquid and control the inlet flow of the tank.

#### ***II. Improve system performances by the help of model uncertainty:***

Mostly, for a model(mathematical model) of a system open-loop control is used. It specifies the desired signals for outputs and inverts the model to determine the input. For this reason uncertainty in the plant is shown. Hence disturbances are seen in between the desired output and the actual output.

#### ***III. For stabilizing the unstable system:***

Manyof the plants which having open-loop are unstable systems such as aircrafts, nuclear reactors, level measurement in a tank. For this kind of systems feedback controller is used for stabilizing.

### **4.4.1 Disadvantages of feedback control:**

- The system's instability will increase due to the feedback.
- The analysis of the system loops will be more complex and implementation will be more difficult.
- Due to the complexity the system will no more cost effective and will be expensive in nature.

In this paper, we are using a decentralized PID controller for the feedback loop. And the loop is merged with the feedforward controller for following the set-point. In case of a multivariable process the complexity of the system is more and according to this the system demands multi loop control solution. PID controllers are used for its wide availability and simplicity in nature.

#### **4.4 Proportional Controller:**

This type of control system is the linear type feedback control system. This type of controller is commonly denoted as P-type controller. The main fundamental of this kind of control system is, the controller output is directly proportional to the error signals generated by the process. The error signal is the difference of set point and variables of the process [11].

It can be denoted as,

$$Y_{out} = k_p e(t) \quad (16)$$

Where,

$Y_{out}$  = controller output

$k_p$  = proportional gain

$e(t)$  = error signal (Set point – process variable)

#### **4.4 Integral controller:**

This type of controller is commonly denoted as I-type of controller. In this controller the output of the controller is the proportional of integral of the error signal. It is required when a steady offset is seen from the reference signal. By using this type of controller we can eliminate the offset value without using the excessive controller gain [12].

It can be denoted as,

$$Y(t) = k_i \int e(t) dt \quad (17)$$

Where,

$Y(t)$  = controller output

$k_I$  = controller gain

#### **4.4 Derivative controller:**

This type of controller is commonly denoted as D-type of controller. It has the control on the rate of change on the error signal. When process variable will overshoot on set point of the system then this type of controller is used and reduces the output as much as we needed. The implementation of this type of controller is more difficult than the other two controllers. Feedback in DC motor control is an example of derivative controller [12-13].

It can be denoted as,

$$Y_t = k_D \frac{de}{dt} \quad (18)$$

Where,

$Y_t$  = controller output

$k_D$  = derivative control gain

#### **4.4 Proportional plus Integral controller:**

This type of controller is commonly alloyed as PI- type of controller. This type of controller acts like, P controller acts on proportional to the error and I controller acts as integral of the error. The system of open loop transfer function of type 1 or having the error of zero steady state is commonly used PI controller. Derivative action also helps in improvement of system responses [13].

Mathematically it can be denoted as,

$$Y_t = k_c e(t) + k_I \int_0^t e(t) dt \quad (19)$$

Or,

$$Y_t = k_c \left[ e(t) + \frac{1}{T_i} \int_0^t e(t) dt \right] \quad (20)$$

Where,

$Y_t$ =controller output

$T_i$ =integral time

$$k_I = \frac{k_c}{T_i}$$

#### 4.4 Proportional-Integral-Derivative controller:

This type of controller is known as PID-type of controller. This type of controllers is mostly used in the plants. Here the P quickly responds to the change in error. I type works slowly but help in removing the offset between the output of the plant and reference. D type again speeds up the system by the help of control action which is proportional to the rate of change of the feedback.

Mathematically it is denoted as,

$$Y_t = k_c e(t) + k_I \int_0^t e(t) dt + k_D \frac{de(t)}{dt} \quad (21)$$

Or,

$$Y_t = k_c \left[ e(t) + \frac{1}{T_I} \int_0^t e(t) dt + t_D \frac{de(t)}{dt} \right] \quad (22)$$

## **4.4 TUNING OF PID CONTROLLER:**

The tuning based on the values of  $k_c, T_i, T_D$ . Several methods are proposed for tuning methods of PID controller [13] [3] [14].

- I. Ziegler Nichols method.
- II. Cohen Coon method.
- III. Decentralized relay feedback method.
- IV. Colonial competitive algorithm.
- V. Genetic algorithm.

### **4.4.1 Ziegler Nichols method:**

This method was introduced by *Nathaniel B. Nichols*. In this method the integral and derivative gain are set to zero. Proportional controller is set to in a way that, it increases the magnitude by which the controller output is having a continuous oscillation. For the slight large value the system will act like unstable system. This type of gain of the proportional controller is known as critical gain.

The values given by Ziegler Nichols for PID type is,

$$k_I = 2k_P / T_u, \quad c = 0.60k_u, \quad k_D = k_c T_u / 8$$

Where,

$T_u$  = oscillation period

$T_u$  = controller gain

### **4.4.1 Cohen Coon method:**

This method was proposed by Cohen Coon in 1953. This method mainly based on the First order plus time delay model. By this method the system is set to have the response of  $1/4$  decay ratios. The main problem in this method is, if there is small change in the parameters then the system will be unstable.



The values given by this method is,

$$k_c = \frac{1}{k_p} \frac{\tau}{\theta} \left( \frac{4}{3} + \frac{\theta}{4\tau} \right)$$

$$T_i = \theta \frac{32 + 6(\theta / \tau)}{13 + 8(\theta / \tau)}$$

#### 4.4.1 Decentralized Relay feedback method:

This method is used in the system as it simultaneously having the relay tests in all closed loops of the system. This method also identifies the interactions in multivariable processes. The main advantage of this method is it will help out in finding the desired critical point. The critical points of the system are based on the complexity of the system [15].

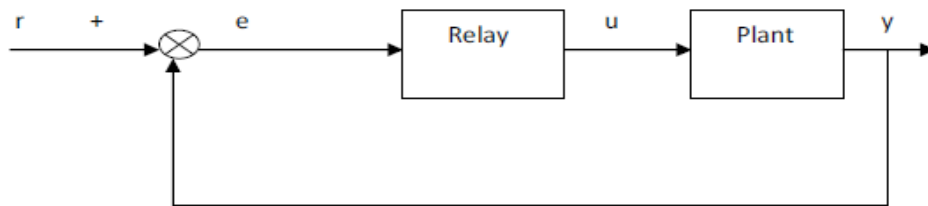


Fig 3.2: General block diagram of relay feedback

This method is a one shot experiment method. The model equation can be easily obtained by this method.

#### 4.4.1 Colonial Competitive algorithm:

This method is commonly known as CCA. It is used in the system for tuning of PID controller. It also helps in characterization of the property of the material from a sharp notch test. The algorithm used for this method is [16],

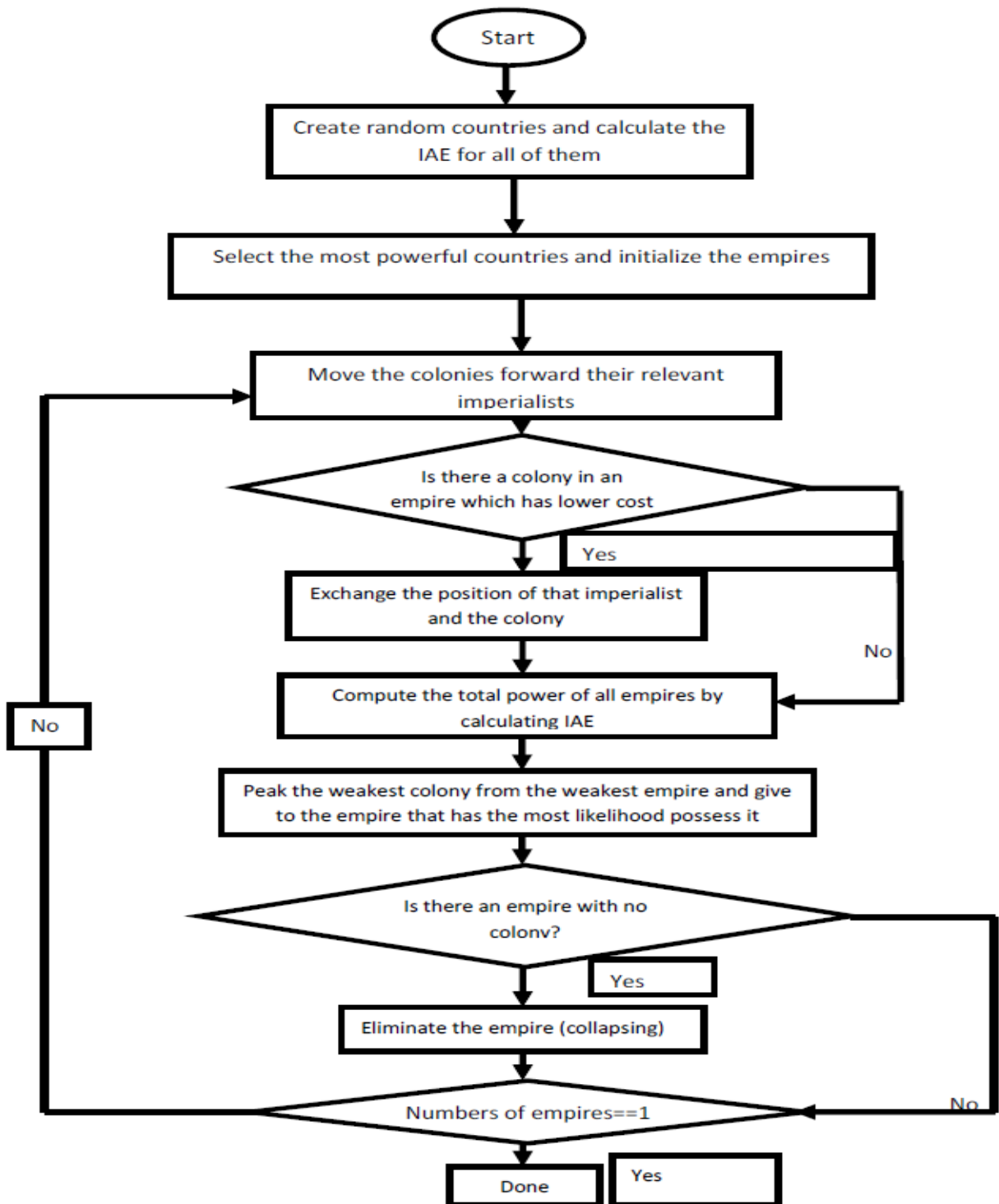


Fig 3.3: Algorithm for CCA

#### **4.4.1 Genetic Algorithm:**

This method always copies the natural evolution and it is an global search method. By using this algorithm in tuning we always get that optimum controller is evaluated every time for the system. The algorithm id defined as,

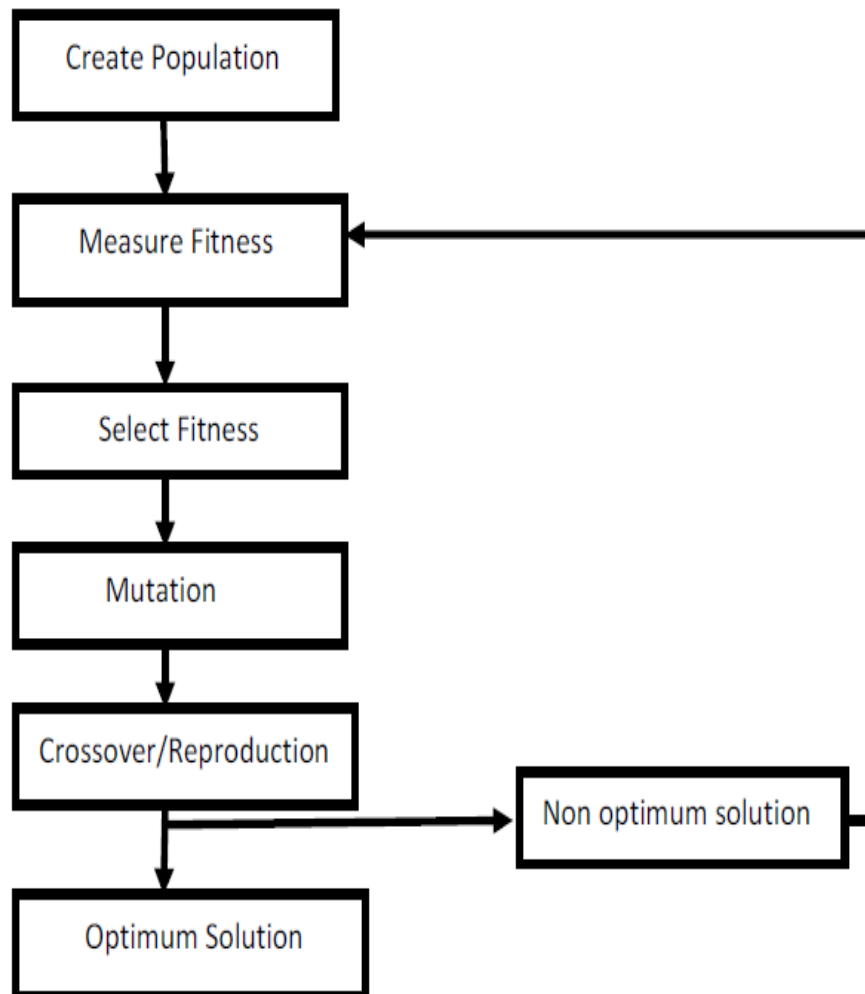


Fig 3.4: Genetic algorithm.

# CHAPTER: 4

## **Performance Analysis**

**&**

**Results**

## **4.1 Control strategy in TITO system:**

In this paper we proposed a design methodology to avoid the process interactions in the 2x2 or TITO system. According to this methodology a decoupler is used in this case. In case of a two-input-two-output system, as it is more complex than the SISO process, hence avoiding of the process interaction is very necessary for getting the perfect output. In this paper, we are going to use a PID controller with a decoupler [17]. PID controllers are used for having a control action on the coupled process. A feedforward controller is also used in this for following the set-point.

### **4.1.1 Methodology:**

Let's consider a TITO system which is linear in nature, time invariant, and continuous time system. The transfer function of this kind of TITO system can be denoted as,

$$G(s) = \begin{pmatrix} \frac{A_{11}}{T_{11}s + 1} e^{-K_{11}s} & \frac{A_{12}}{T_{12}s + 1} e^{-K_{12}s} \\ \frac{A_{21}}{T_{21}s + 1} e^{-K_{21}s} & \frac{A_{22}}{T_{22}s + 1} e^{-K_{22}s} \end{pmatrix}$$

$G(s)$  is the transfer function of the TITO system.

As it is a 2x2 system hence for reducing the interactions between the processes a perfect decoupling is needed. Hence a decoupler should be implemented. Let's assume that  $C(s)$  is denoted for decoupler. Then  $c(s)$  is written as [1],

$$C(s) = \begin{pmatrix} C_{11}(s) & C_{12}(s) \\ C_{21}(s) & C_{22}(s) \end{pmatrix}$$

Where,

$$C_{11}(s) = e^{-v_{11}(K_{22}-K_{21})s} \quad (23)$$

$$C_{12}(s) = -\frac{A_{12}T_{11}s+1}{A_{11}T_{12}s+1} e^{-v_{12}(K_{12}-K_{11})s} \quad (24)$$

$$C_{21}(s) = -\frac{A_{21}T_{22}s+1}{A_{22}T_{21}s+1} e^{-v_{21}(K_{21}-K_{22})s} \quad (25)$$

$$C_{22}(s) = e^{-v_{22}(K_{11}-K_{12})s} \quad (26)$$

And,

$$v_{11} = \begin{cases} 1, K_{22} - K_{21} \geq 0 \\ 0, K_{22} - K_{21} < 0 \end{cases} \quad (27)$$

$$v_{12} = \begin{cases} 1, K_{12} - K_{11} \geq 0 \\ 0, K_{12} - K_{11} < 0 \end{cases} \quad (28)$$

$$v_{21} = \begin{cases} 1, K_{21} - K_{22} \geq 0 \\ 0, K_{21} - K_{22} < 0 \end{cases} \quad (29)$$

$$v_{22} = \begin{cases} 1, K_{11} - K_{12} \geq 0 \\ 0, K_{11} - K_{12} < 0 \end{cases} \quad (30)$$

Let the resulting transfer function of the decoupled system is denoted as H(s),

Hence,  $H(s) = G(s).C(s)$

$$\text{Hence, } H(s) = \begin{pmatrix} H_{11}(s) & 0 \\ 0 & H_{22}(s) \end{pmatrix}$$

Where,

$$H_{11}(s) = \frac{A_{11}}{T_{11}s + 1} e^{-[K_{11} + v_{11}(K_{22} - K_{21})]s} - \frac{A_{12}A_{21}}{A_{22}} \frac{T_{22}s + 1}{(T_{12}s + 1)(T_{21}s + 1)} e^{-[K_{12} + v_{21}(K_{21} - K_{22})]s} \quad (31)$$

And,

$$H_{22}(s) = \frac{A_{22}}{T_{22}s + 1} e^{-[K_{22} + v_{12}(K_{12} - K_{11})]s} - \frac{A_{12}A_{21}}{A_{11}} \frac{T_{11}s + 1}{(T_{12}s + 1)(T_{21}s + 1)} e^{-[K_{21} + v_{22}(K_{11} - K_{12})]s} \quad (32)$$

The parallel coupled system  $H_{ii}(s)$  can be obtained by the difference between the FOPDT (first order plus dead time) systems and SOPDT (second order plus dead time) system with a negative zero.  $H_{ii}(s)$  represents the dynamics of the system. Simply  $H_{ii}(s)$  can be written as,

$$\tilde{H}_{ii}(s) = A_i \frac{-T_{zi}s + 1}{(T_{1i}s + 1)(T_{2i}s + 1)} e^{-\tau_i s}, \quad i = 1, 2 \quad (33)$$

Where,

$$\tau_1 = \min\{K_{11} + v_{11}(K_{22} - K_{21}), K_{12} + v_{21}(K_{21} - K_{22})\}$$

$$\tau_2 = \min\{K_{22} + v_{12}(K_{12} - K_{11}), K_{21} + v_{22}(K_{11} - K_{12})\}$$

Here the parameters  $A_i, T_{zi}, \tilde{T}_{1i}$ , and  $\tilde{T}_{2i}$  can be identified by the least square method to the step response of system  $H_{ii}(s)$ ,  $i = 1, 2$ .

There are two SISO systems in the process, which can be designed by the method is,

$$R_i(s) = \frac{\mu_i T_{zi}s + 1}{(\lambda_i s + 1)\mu_i + 1} e^{-\tau_i s} \quad (34)$$

Where,  $R_i(s)$  denotes the reference model.

As per the equations,

$$\mu_i = \begin{cases} \mathbf{1}, T_{zi} \geq \mathbf{0} \\ \mathbf{0}, T_{zi} < \mathbf{0} \end{cases}$$

Where  $i=1, 2$ .

According to this we can obtain the feedforward function  $P(s)$ . Where  $P(s)$  can be identified by the method is

$$P_i(s) = \frac{R_i(s)}{\tilde{H}_i(s)} = \frac{(\tilde{T}_{1i}s + 1)(\tilde{T}_{2i}s + 1)}{A_i(\lambda_i s + 1)^{\mu_i + 1}(-v_i T_{zi}s + 1)} \quad (35)$$

Where,

$$v_i = \begin{cases} \mathbf{0}, T_{zi} \geq \mathbf{0} \\ \mathbf{1}, T_{zi} < \mathbf{0} \end{cases}$$

As per the equation  $\lambda_1$  and  $\lambda_2$  are time constant parameters. By these parameters we can speed up the process response. We can select the values as the fraction of the time constants of the open loop i.e.  $T_1$  and  $T_2$ . We can increase the values of  $\lambda$  unless until the requirement is satisfied.

**Example:**

Let's consider a transfer function for a TITO system (i.e. distillation column)

$$G(s) = \begin{pmatrix} \frac{22.89}{4.57s + 1} e^{-0.2s} & \frac{-11.64}{1.81s + 1} e^{-0.4s} \\ \frac{4.69}{2.17s + 1} e^{-0.2s} & \frac{5.80}{1.80s + 1} e^{-0.4s} \end{pmatrix}$$



The decouple transfer function  $D(s)$  can be defined as,

$$D(s) = \begin{pmatrix} v_1(s) & d_{12}v_2(s) \\ d_{21}(s)v_1(s) & v_2(s) \end{pmatrix}$$

Where,

$$v_1(s) = \begin{cases} 1, & K_{21} \geq K_{22} \\ e^{K_{21}-K_{22}}, & K_{21} < K_{22} \end{cases}$$

$$v_2(s) = \begin{cases} 1, & K_{12} \geq K_{11} \\ e^{K_{12}-K_{11}}, & K_{12} < K_{11} \end{cases}$$

$$d_{12}(s) = \frac{-g_{12}(s)}{g_{11}(s)} e^{-(K_{12}-K_{11})s}$$

$$d_{21}(s) = \frac{-g_{21}(s)}{g_{22}(s)} e^{-(K_{21}-K_{22})s}$$

Here,

$$K_{11} = 0.2$$

$$K_{12} = 0.4$$

$$K_{21} = 0.2$$

$$K_{22} = 0.4$$

As per our transfer function,

$$v_1(s) = e^{-0.2s} \quad v_2(s) = 1$$

$$d_{12}(s) = \frac{2.330s+0.51}{1.81s+1} e^{-0.2s} \quad d_{21}(s) = \frac{-1.458s+1}{2.17s+1}$$

Hence D(s) is,

$$D(s) = \begin{pmatrix} e^{-0.2s} & \frac{2.3307s + 0.51}{1.81s + 1} e^{-0.2s} \\ \frac{-1.458s - 0.81}{2.17s + 1} & 1 \end{pmatrix}$$

The decoupled system H(s) can also be defined as per the methodology discussed above. Hence H(s) can be written as,

$$H_{11}(s) = \frac{22.89}{4.57s + 1} e^{-0.4s} + \frac{16.938s + 9.41}{(3.927s^2 + 3.98s + 1)} e^{-0.4s}$$

And

$$H_{22}(s) = \frac{5.8}{1.8s + 1} e^{-0.4s} + \frac{10.876s + 2.38}{(3.927s^2 + 3.98s + 1)} e^{-0.4s}$$

If we apply least square method according to the methodology, to step response of the system then,

$$\tilde{H}_{11}(s) = \frac{109.207s + 32.31}{(12.020s^2 + 7.29s + 1)} e^{-0.4s}$$

$$\tilde{H}_{22}(s) = \frac{6.021s + 2.23}{(1.97s + 1)^2} e^{-0.4s}$$

So far reference model M(s) can be chosen as, by selecting the  $\lambda_1$  and  $\lambda_2$  values as  $1/12^{\text{th}}$  of  $T_{11}$  and  $T_{22}$ .

$$P_1(s) = \frac{1}{0.38s + 1} e^{-0.4s}$$

And

$$P_2(s) = \frac{1}{0.15s + 1} e^{-0.4s}$$

The feedforward controller  $p(s)$  can be defined as per the method is,

$$P_1(s) = \frac{0.360612s^2 + 0.218s + 0.03}{1.2844s^2 + 3.76s + 1} \quad \text{and} \quad P_2(s) = \frac{1.746405s^2 + 1.773s + 0.45}{0.405s^2 + 2.85s + 1}$$

The general block diagram representation can be shown as below,

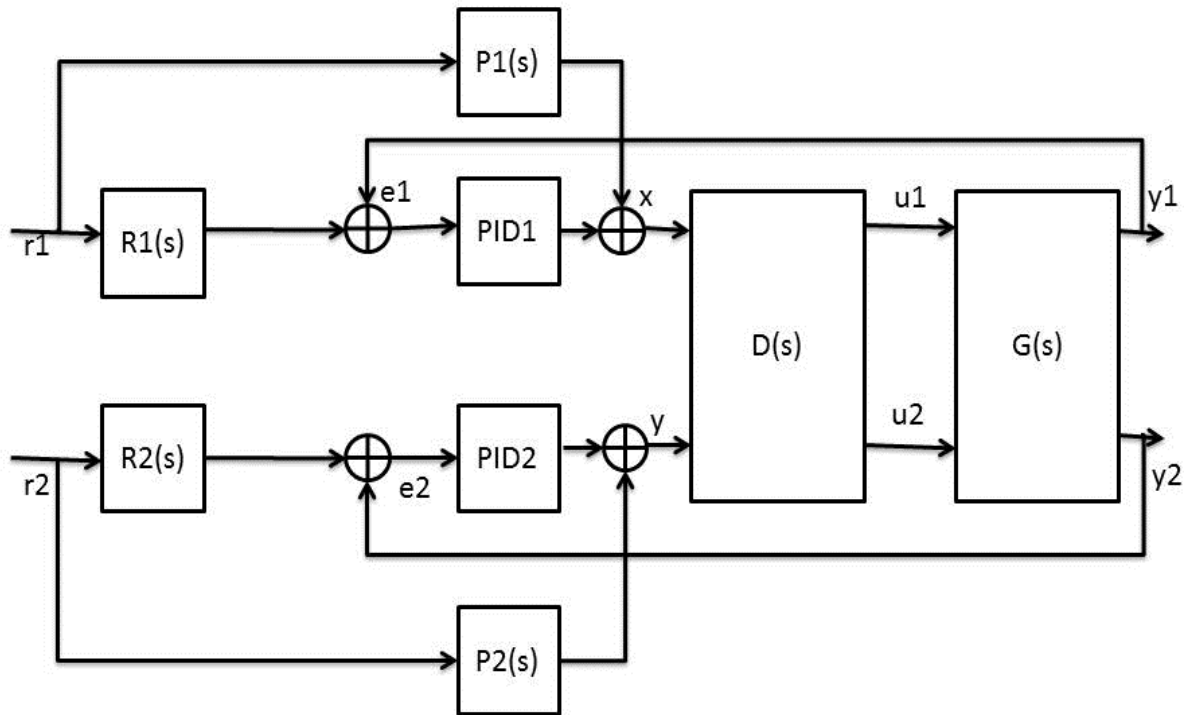
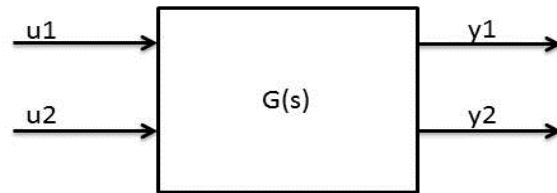


Fig 4.1: Feedforward plus decoupling control design system

As per the above diagram,



Hence  $P(s)$  can be represented as,

$$\frac{y_1}{u_1} = \left( \frac{22.89}{4.57s + 1} \right) e^{-0.2s}$$

$$\frac{y_1}{u_2} = \left( \frac{-11.64}{1.81s + 1} \right) e^{-0.4s}$$

$$\frac{y_2}{u_1} = \left( \frac{4.69}{2.17s + 1} \right) e^{-0.2s}$$

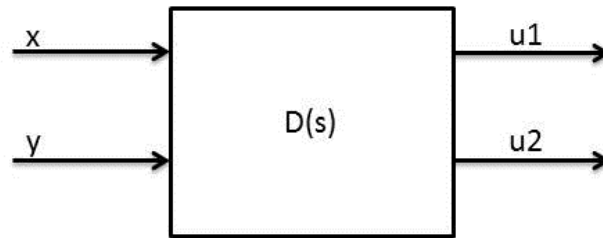
$$\frac{y_2}{u_2} = \left( \frac{5.80}{1.80s + 1} \right) e^{-0.4s}$$

So,

$$y_1 = \left( \frac{22.89}{4.57s + 1} \right) e^{-0.2s} \cdot u_1 + \left( \frac{-11.64}{1.81s + 1} \right) e^{-0.4s} u_2$$

$$y_2 = \left( \frac{4.69}{2.17s + 1} \right) e^{-0.2s} \cdot u_1 + \left( \frac{5.80}{1.80s + 1} \right) e^{-0.4s} u_2$$

And per the diagram D(s) can be shown like,



Hence,

$$D_{11} = \frac{u_1}{x} = e^{0.2s}$$

$$D_{12} = \frac{u_1}{y} = \frac{2.3307s + 0.51}{1.81s + 1} e^{-0.2s}$$

$$D_{21} = \frac{u_2}{x} = \frac{-1.458s - 0.81}{2.17s + 1}$$

$$D_{22} = \frac{u_2}{y} = 1$$

Or,

$$u_1 = e^{0.2s} \cdot x + \frac{2.3307s + 0.51}{1.81s + 1} e^{-0.2s} \cdot y$$

$$u_2 = \frac{-1.458s - 0.81}{2.17s + 1} \cdot x + y$$

## 4.2 Simulink diagram of TITO system without Decoupling:

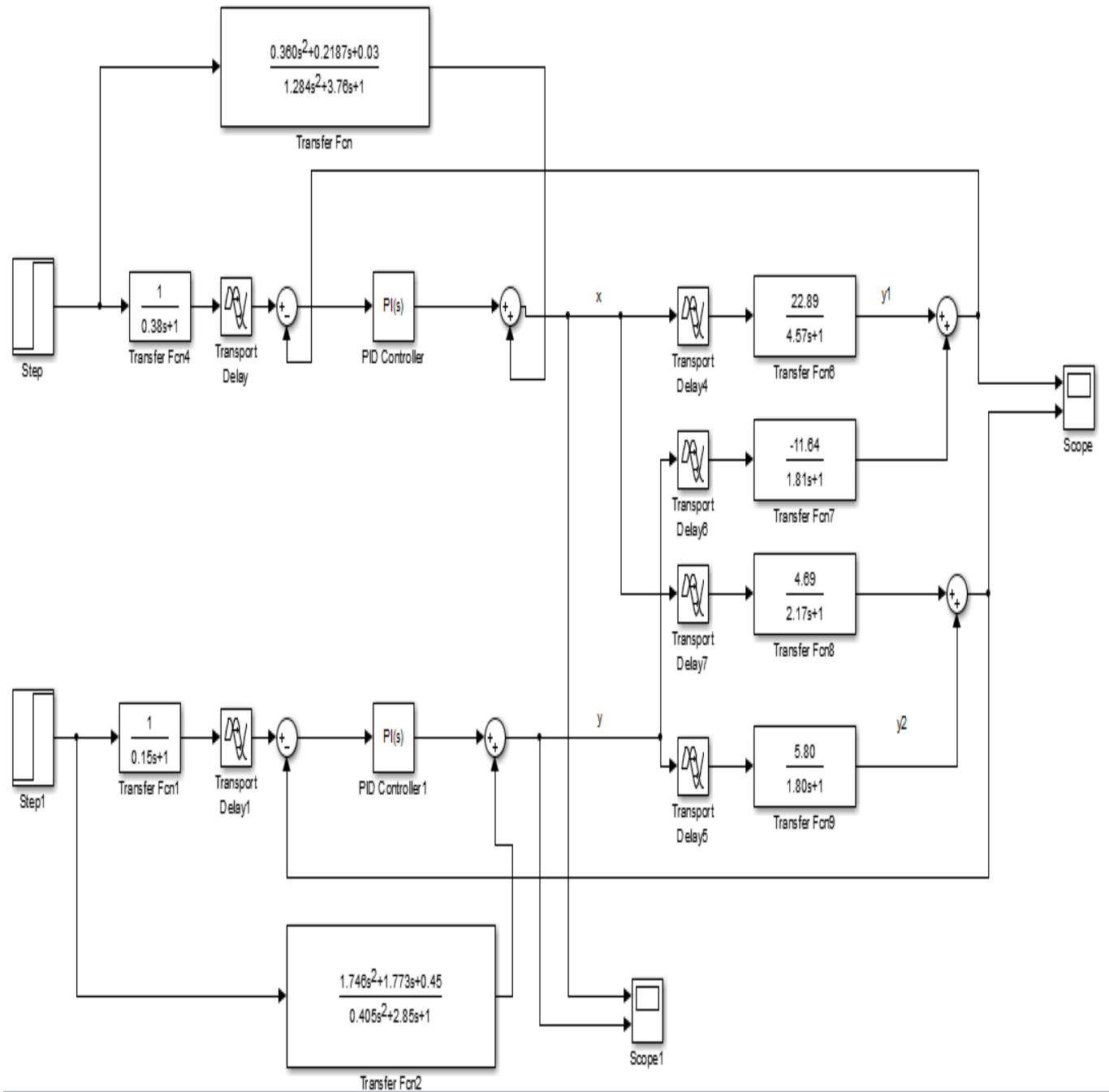


Fig 4.2: Representation of TITO system without Decoupling

**4.2.1 Simulation of TITO system without Decoupling:**

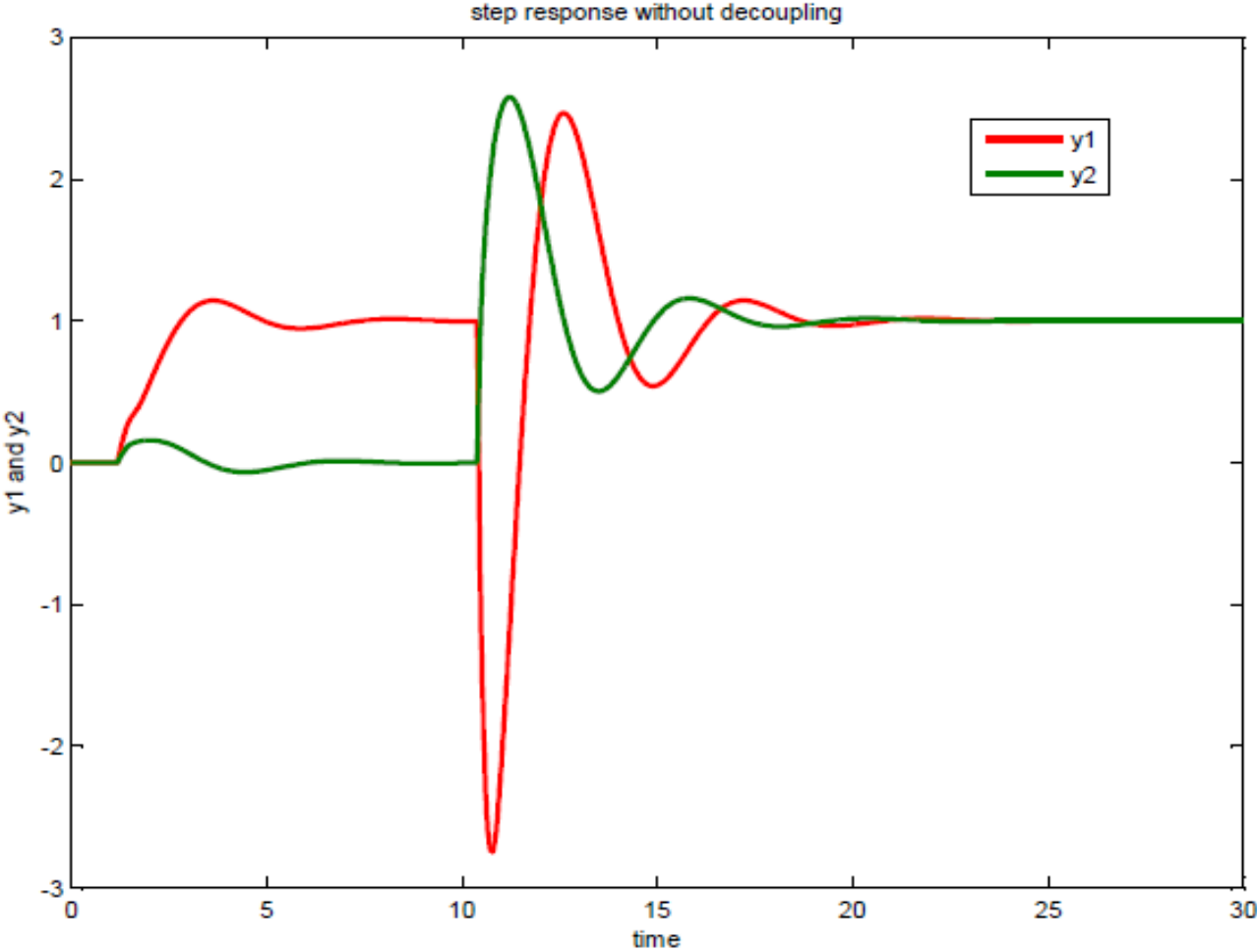


Fig 4.2.1: simulation of TITO system without Decoupling

### 4.3 Simulink diagram of TITO Decoupled system:

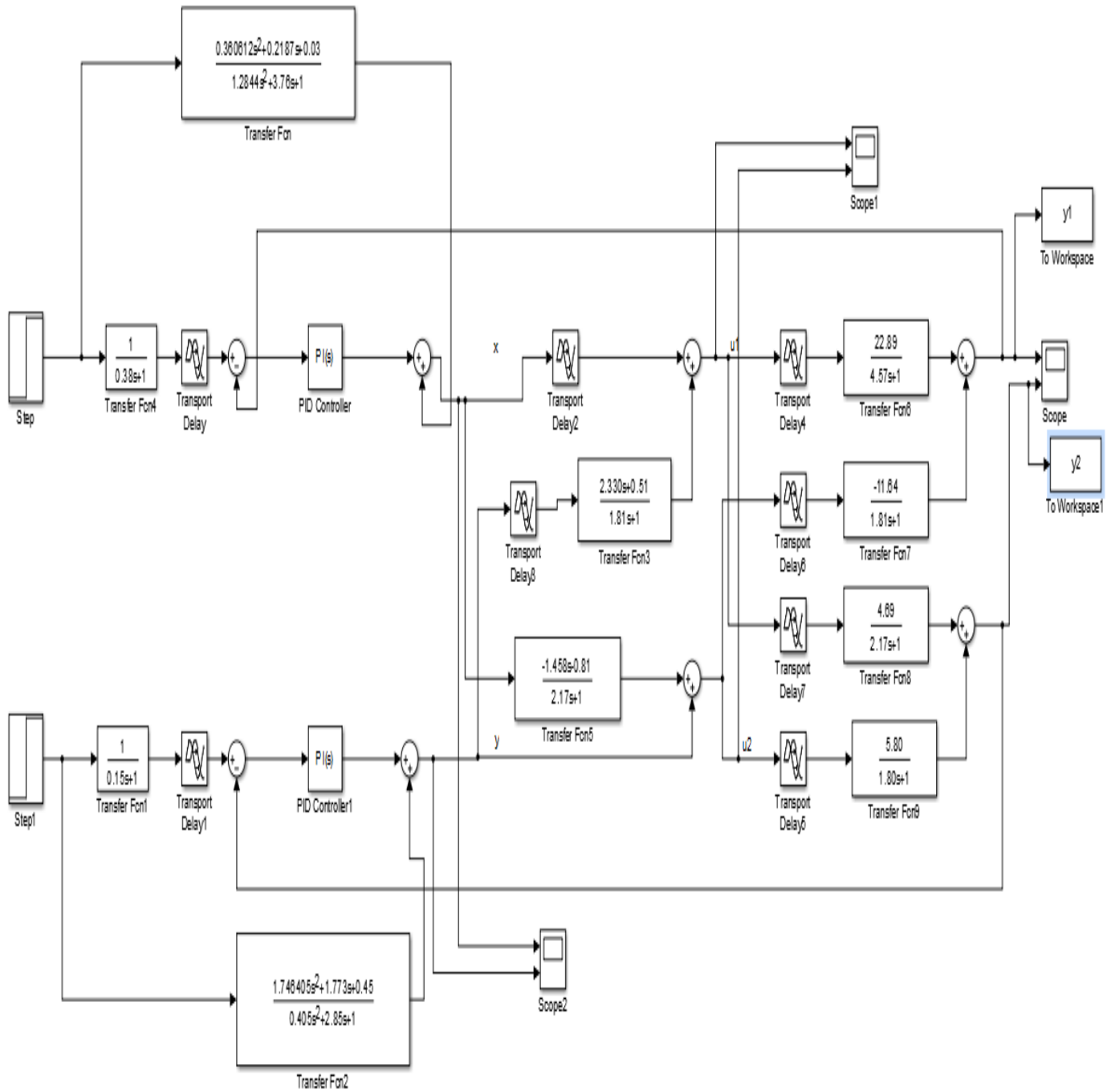


Fig 4.3: Representation of the TITO decoupled system



**4.3.1 Simulation of TITO decoupled system:**

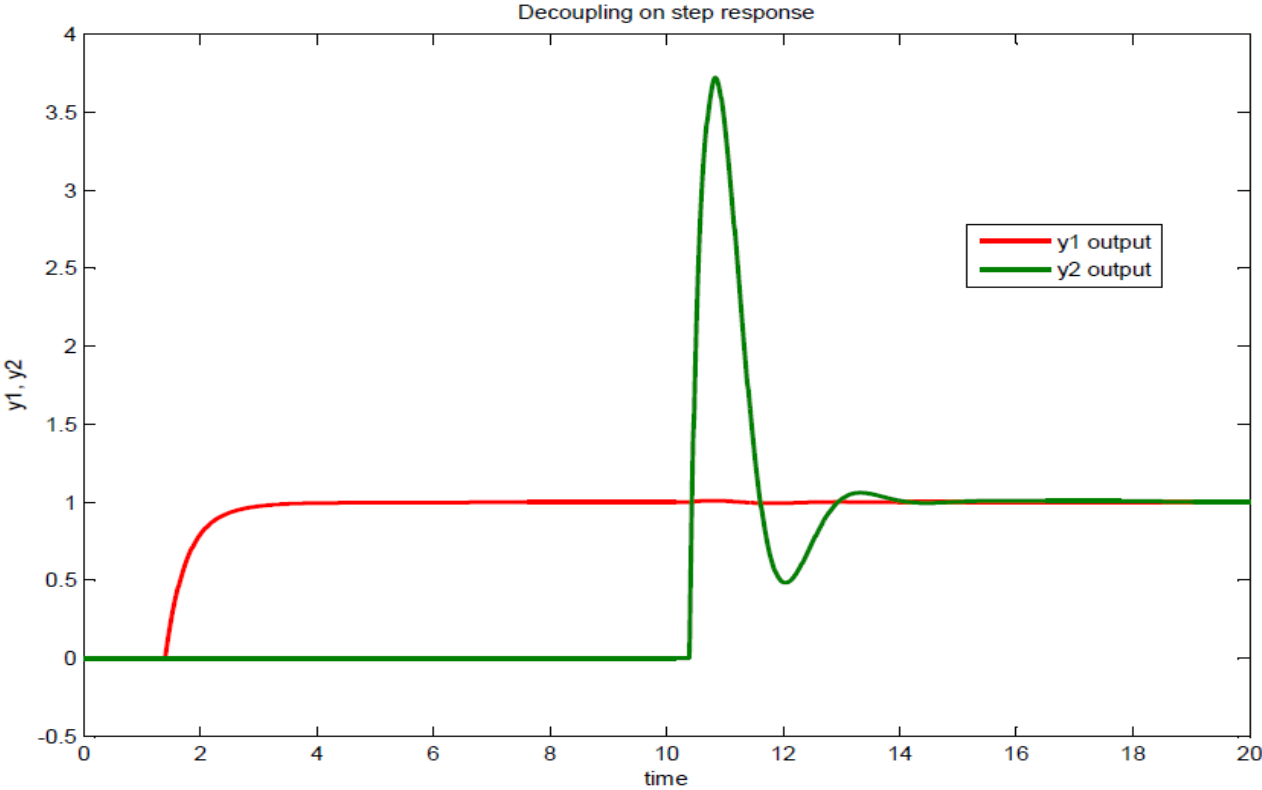


Fig 4.3.1: Simulation of TITO system

#### **4.4 Analysis by changing the values of time constant parameters ( $\lambda$ ):**

Based on the values of  $\lambda$  we can determine or choose the transfer functions of the reference model.  $\lambda$  is denoted as the desired time constant parameters. It depends upon the values of  $T_i$ , where  $i=1, 2$ .

##### **4.4.1 CASE 1:**

Let's take the values of  $\lambda = 1$ . By putting the values in the eqn. (34) we can get the reference model  $R_1(s)$  and  $R_2(s)$ .

Where,

$$R_1(s) = \frac{1}{4.57s + 1} e^{-0.4s}$$

and

$$R_2(s) = \frac{1}{1.8s + 1} e^{-0.4s}$$

##### **4.4.1.2 Simulation:**

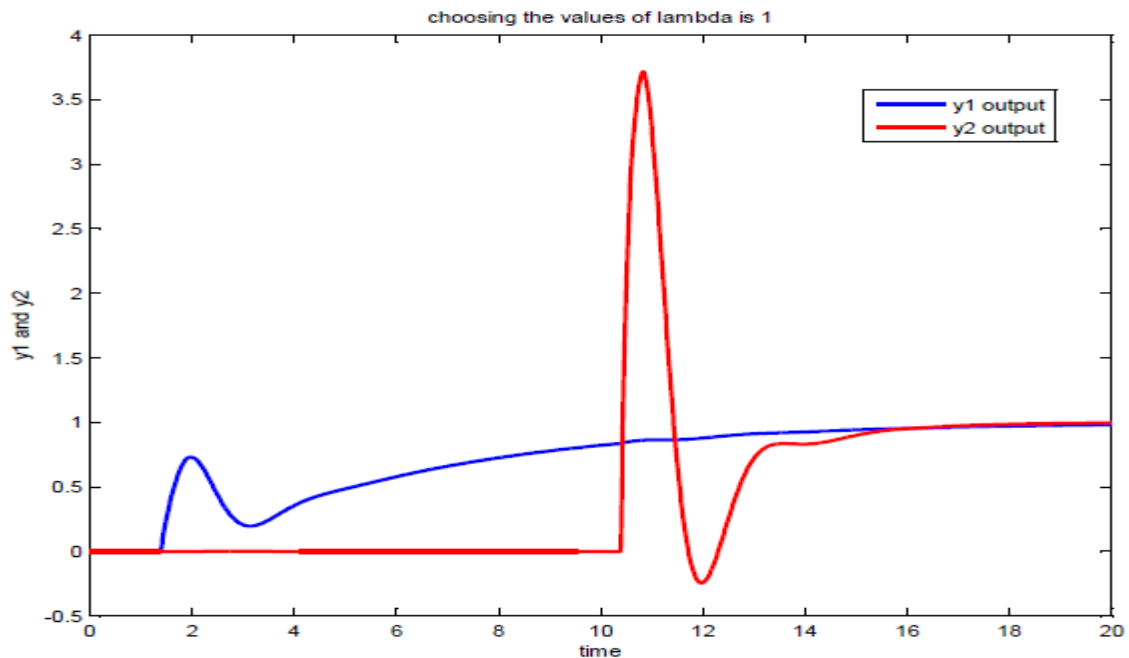


Fig: 4.4.1: choosing the values of  $\lambda = 1$

#### 4.4.2 CASE 2:

Let's take the values of  $\lambda=2$ . By putting the values in the eqn. (34) we can get the reference model  $R_1(s)$  and  $R_2(s)$  .

Where,

$$R_1(s) = \frac{1}{9.14s + 1} e^{-0.4s}$$

and

$$R_2(s) = \frac{1}{3.6s + 1} e^{-0.4s}$$

##### 4.4.2.1 Simulation:

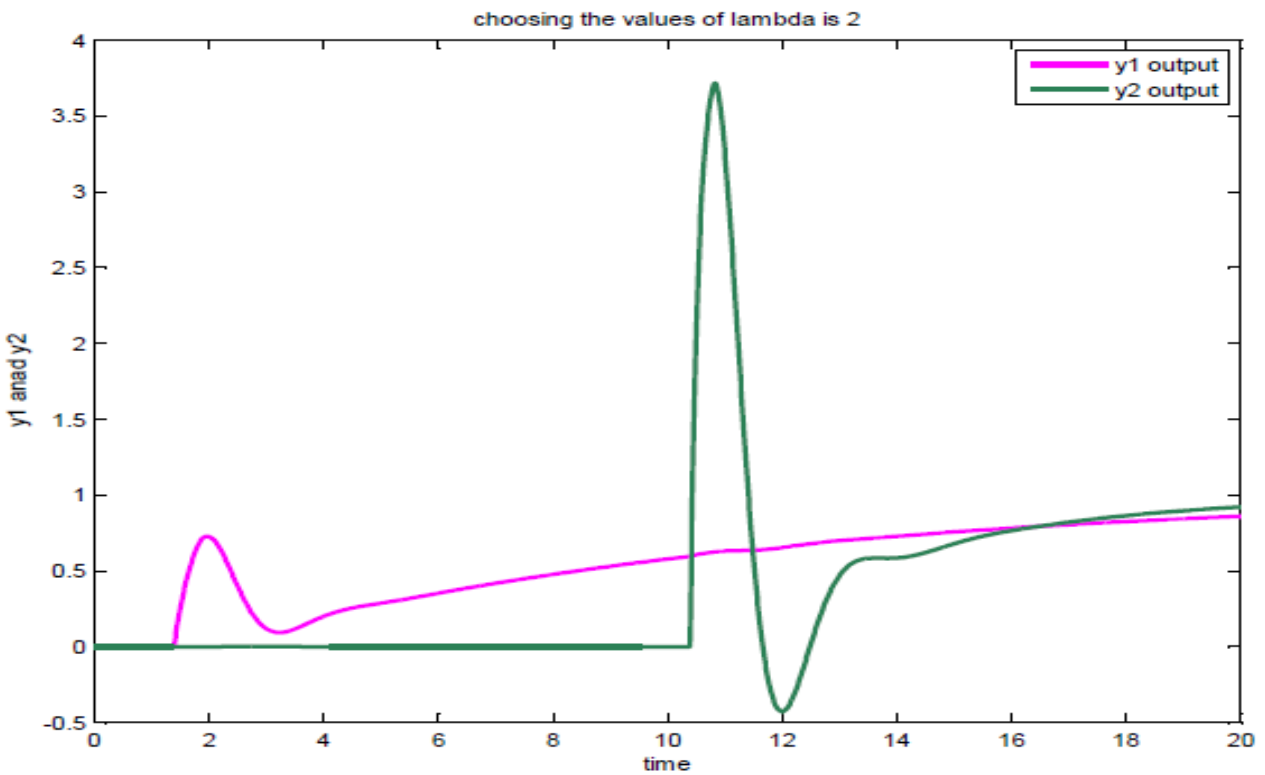


Fig: 4.4.2 choosing the values of  $\lambda = 2$

### **4.4.3 CASE 3:**

Let's take the values of  $\lambda=1/22$ . By putting the values in the eqn. (34) we can get the reference model  $R_1(s)$  and  $R_2(s)$  .

Where,

$$R_1(s) = \frac{1}{0.2077s + 1} e^{-0.4s}$$

and

$$R_2(s) = \frac{1}{0.8181s + 1} e^{-0.4s}$$

#### **4.4.3.1 Simulation:**

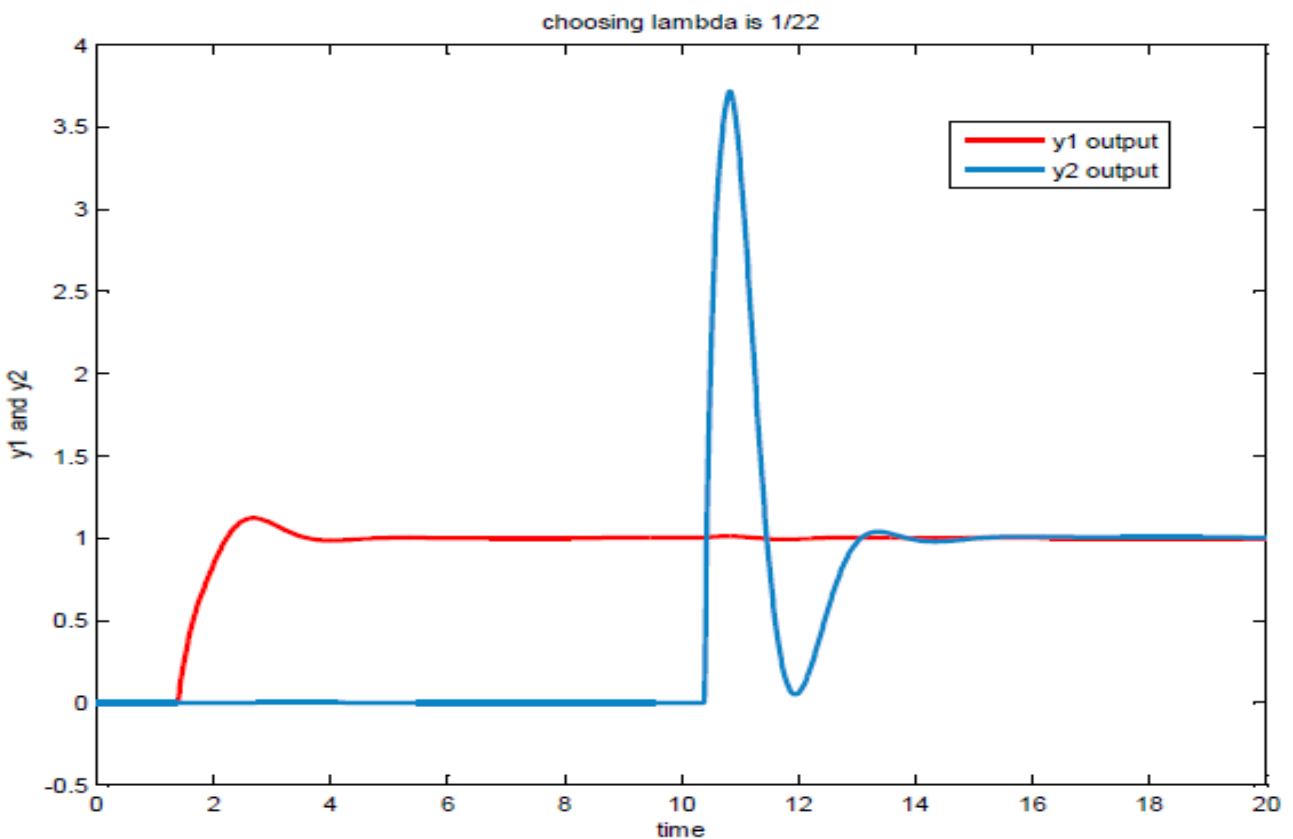


Fig: 4.4.3 choosing the values of  $\lambda = 1/22$

#### **4.4.4 CASE 4:**

Let's take the values of  $\lambda=1/30$ . By putting the values in the eqn. (34) we can get the reference model  $R_1(s)$  and  $R_2(s)$  .

Where,

$$R_1(s) = \frac{1}{0.1523s + 1} e^{-0.4s}$$

and

$$R_2(s) = \frac{1}{0.06s + 1} e^{-0.4s}$$

##### **4.4.4.1 Simulation:**

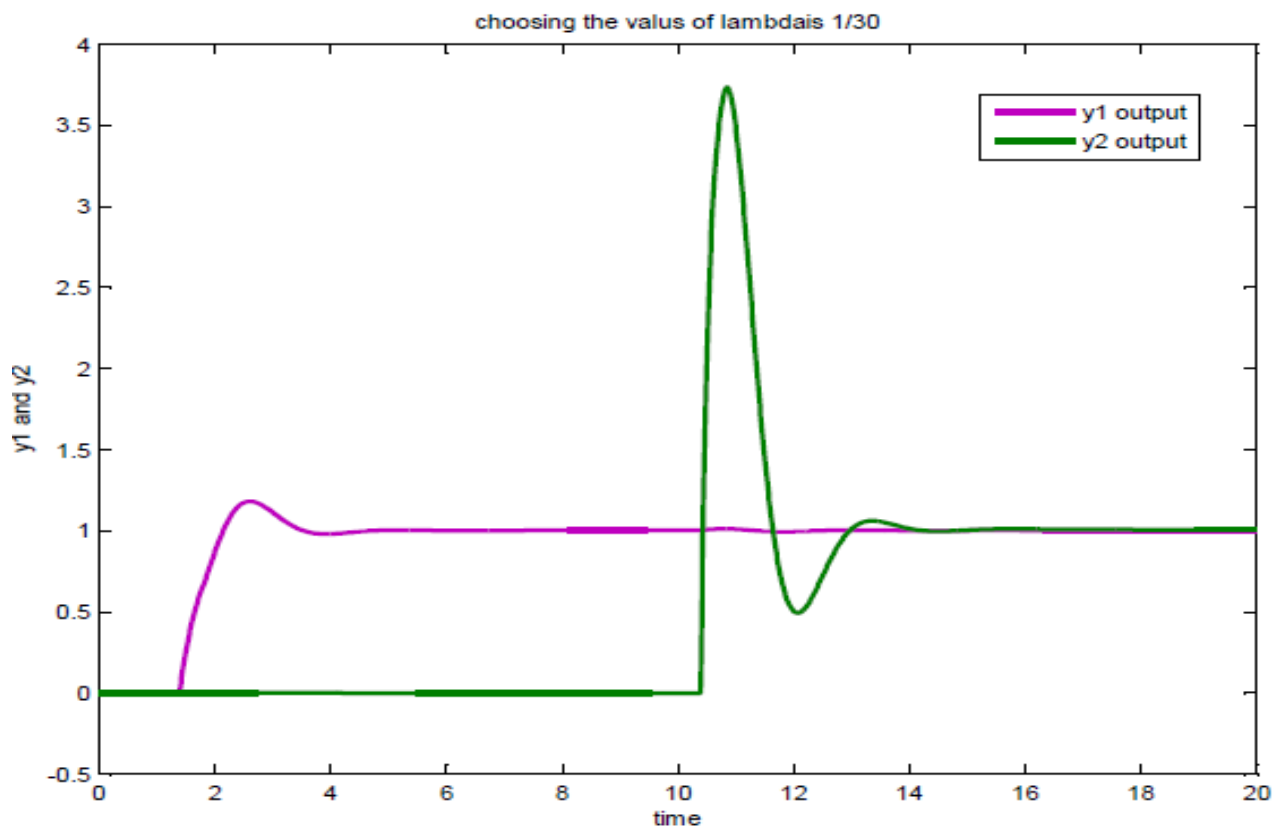


Fig: 4.4.4 choosing the values of  $\lambda=1/30$

## **5. Conclusion & future work:**

We have proposed the new method for design of feed forward control action to be applied to a TITO process. The design technique consists in decoupling in process and then approximating it into a simple transfer functions. Simulations as well as experimental results have shown the design parameters and effectiveness of the methodology. By decoupling design process we can also minimizing the process interactions here. Hence no interactions between the controlled variables and manipulated variables. As we have seen by changing the values of time constant parameter  $\lambda$  we can get the better responses. This methodology can be extended in MIMO process for better result.

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