

Thermo-mechanical Analysis of Carbon Nanotube based Functionally Graded Timoshenko Beam

Thesis submitted in partial fulfillment of
the requirements for the degree of
Bachelor of Technology in Mechanical Engineering

By

Swarup Sahoo (110ME0325)

Kartik Naik (110ME0314)

Saurav Sekhar Sahoo (110ME0324)

Under the guidance of

Prof. Tarapada Roy





National Institute of Technology

ROURKELA

CERTIFICATE

This is to certify that the thesis entitled, “**Thermomechanical Analysis of Carbon Nanotube Based Functionally Graded Timoshenko Beam**” submitted by Mr. Swarup Sahoo (110ME0325), Mr. Kartik Naik (110ME0314) and Mr. Saurav Sekhar Sahoo (110ME0324) in partial fulfillment of the requirements for the award of Bachelor of Technology in Mechanical Engineering at the National Institute of Technology, Rourkela is an authentic work carried out by them under my supervision and guidance. To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/ Institute for the award of any Degree or Diploma.

Date: 01-05-2014

Prof. Tarapada Roy

Department of Mechanical Engineering

National Institute of Technology

Rourkela – 769008

ACKNOWLEDGEMENT

I wish to express my sincere gratitude to **Prof Tarapada Roy** for his inspiring encouragement, guidance and efforts taken throughout the entire course of this work. His constructive criticism, timely help, and efforts made it possible to present the work contained in this Thesis.

We are grateful to **Prof. S.K. Sarangi**, Director, and **Prof K.P Maity**, Head of the Department, Mechanical Engineering, for their active interest and support.

We would also like to thank Mr. Benedict Thomas, Ph.D.scholar for his constant help in understanding of the technical aspects of the project.

Last but not the least, we wish to express our sincere thanks to all those who directly or indirectly helped us at various stages of this work.

Swarup Sahoo

110ME0325

Kartik Naik

110ME0314

Saurav Sekhar Sahoo

110ME0324

ABSTRACT

This analytical work deals with prediction of the stresses developed in a Functionally Graded Timoshenko Beam that has been reinforced with Carbon Nanotubes (CNTs), which is subjected to thermal and mechanical loads. High temperatures have been applied to the upper and lower surfaces of the beam with a certain temperature difference between the two layers for the formation of a temperature gradient. The physical properties of the constituent elements of the beam material vary with temperature and further a variation in temperature leads to development of stresses in a beam. The constituent materials are alumina as the ceramic material, as well as the matrix material, of the functionally graded beam and single walled CNTs as the reinforcement material. Further the physical properties of the beam would vary along the thickness direction according to the volume fraction of the constituents of the beam. In this analysis the volumetric fraction varies according to power law. Temperature-dependent and temperature-independent material properties were obtained layer wise by dividing the entire thickness of the beam into ten layers. Thermal stresses were obtained using temperature-dependent and temperature-independent material properties for each layer for different slenderness ratios and compared.

CONTENTS

Certificate	ii
Acknowledgement	iii
Abstract	iv
List of Figures and Tables	vi
Nomenclature	vii
Chapter 1: Introduction	
1.1 Composites	2
1.2 Carbon Nanotubes	2
1.3 Functionally Graded Materials	3
1.3.1 Applications of FGM	4
1.4 Beam Theories	6
1.4.1 Euler-Bernoulli Beam Theory	6
1.4.2 Timoshenko Beam Theory	6
Chapter 2: Literature Review	
2.1 Functionally graded materials	8
2.2 CNT based functionally graded materials	9
2.3 Thermo-mechanical analysis of FG beam	10
Chapter 3: Motivation and Objectives	15
Chapter 4: Methodology	
4.1 Material modeling	16
4.1.1 Rule of Mixture	16
4.1.2 Power Law Material Distribution	16
4.2 Temperature Profile	17
4.3 Temperature Dependence of Material Properties	18
Chapter 5: Results and Discussion	
5.1 Results	25
Chapter 6: Conclusions & Scope of future work	
6.1 Conclusions	33
6.2 Scope of future work	33
References	34

List of Figures:

- Fig.1 Roll-up vector defining the structure of carbon nanotubes. (a) Graphene lattice and (b) carbon nanotube.
- Fig.2. Schematic of FG beam with used coordinates
- Fig.3 Distribution of ceramic volume fraction through the thickness for various power law indices.
- Fig.4. Axial, bending and shearing deformation characteristics of a beam according to FSDT
- Fig. 5. Variation of temperature along the z direction
- Fig 6. Variation of young modulus in the thickness direction
- Fig7. Variation of stress in the thickness direction
- Fig8. Variation of strain in the thickness direction
- Fig9. Variation of strain with respect to temperature
- Fig10. Variation of stress with respect to temperature

List of Tables:

- Table 1. The constants in the cubic fit of Alumina
- Table 2. Material properties of Single Walled CNT
- Table 3. Constants in Cubic fit for Young's Modulus of (10x10) SWCNT (10, 10) SWCNT
- Table 4. Volume fraction and young's modulus variation for different material distribution
- Table 5. Effect of CNT volume fraction on the stress and strain ($k=0.5$)
- Table 6. Stresses and strains for variation in power law index keeping $(L/h) = 10$

NOMENCLATURE

Although all the principal symbols used in this thesis are defined in the text as they occur, a list of them is presented below for easy reference.

V_m	Volume fraction of ceramic (Al_2O_3)
z	Z axis
h	Thickness of beam
K	Thermal conductivity of the material
V_{cnt}	Volume fraction of CNT
$P(z)$	Material property of FG-beam at a layer in z coordinates.
P_{cnt}	Material Property of Carbon Nanotubes
P_m	Material property of matrix, which is the ceramic material Al_2O_3
$P_0, P1, P2, P3, P_{-1}$	Constants in cubic fit of the material property
T_m	Temperature of metal
T_{cnt}	Temperature of CNT
$\eta(z)$	Conductivity function
C	Composition dependent variable
κ_{cm}	Difference between Conductivity of CNT and ceramic material
κ_m	Conductivity of the ceramic material (Al_2O_3)
κ_{cnt}	Conductivity of the CNT
$P(T)$	Physical properties with respect to temperature
E	Young's Modulus
ρ	Density
ν	Poisson's ratio
α	Coefficient of thermal expansion

u_0	Axial displacement of reference plane
w	transverse displacement of reference plane
$\frac{\partial w}{\partial x}$	Rotations due to bending in Z- direction
ϕ_y	Rotations due to shearing
ε_{xx}	Normal Strain in X direction
$\sigma_{xx}(z)$	Axial stress on the surface with distance z from mid surface
ν	Poisson's ratio
M_{Ty}	Resultant moment in Y axis due to thermal effects
M_{My}	Mechanical moment about Y-axis
I_y	Moment of inertia about Y axis
b	Width of the beam
h	Thickness of the beam
I_0	Moment of inertia of reference plane
E_{cm}	Difference between Young's modulus of CNT and ceramic
α_{cm}	Difference between coefficient of thermal expansion of CNT and ceramic
C_1	Deformation coefficient in X direction
C_2	Deformation coefficient in X direction
C_3	Deformation coefficient in X direction

Chapter 1

Introduction

INTRODUCTION

1.1. Composites

Composites are simply solid mixtures of two or more constituent substances. Typically materials with significant differences in physical and chemical properties are chosen for a mixture. This helps in obtaining a material that is superior in qualities from the individual constituents of the mixture. However it may also lead to deterioration in certain qualities, but generally such combinations are avoided. A composite gives a combination of desired properties accumulated from various constituent elements and hence finds more usability than the individual elements. Fiber reinforced polymers, metal composites; metal-ceramic composites are some of the types of vastly used composites.

Composites may be divided into two groups on the basis of homogeneity, as simple composites and functionally graded composites. Simple composites are homogeneous in nature over the entire volume. The volumetric composition of any elementary section of the composite would be same as any other section. Hence all the physical properties are same throughout its volume. But in a functionally graded material, the composition of the constituent materials vary according to a predefined mixing rule. The mixing could be exponential or follow a power law. So the volumetric composition at point of the material may not be the same as any other point and hence the properties will also vary accordingly.

1.2. Carbon Nanotubes (CNTs)

Carbon Nanotubes are allotropic forms of carbon with a cylindrical nanostructure. There are various processes for synthesis of Carbon nanotubes which include arc discharge, high-pressure carbon monoxide disproportionation, laser ablation, and chemical vapor deposition (CVD). Nanotubes have been constructed with a high length to diameter ratio of up to 132,000,000:1, which is significantly larger than any other material. CNTs are considered as the ultimate materials for advanced energy, composites, biomaterials, electronics, and optical applications due to their extraordinary physico-chemical properties. Thus, there have been tremendous efforts to develop economical production methods for CNTs, the main issues being structural quality and purity enhancement, scalability, selectivity to single-walled carbon nanotubes (SWCNTs), and diameter or chirality control. The topological arrangement of the atom in the CNT and its

overall tubular structure are the factors behind its superior properties. Carbon nanotubes have a lattice-like tubular structure comprising of periodic hexagonal network of bonded carbon atoms. The generation of the tubular structure can be visualized by rolling up a single graphite sheet while the ends of the tube are closed with fullerene like end caps. As a result, the carbon atoms in a nanotube are arranged in a hexagonal network to form a toroidal configuration, as shown in Fig. 1. [21]

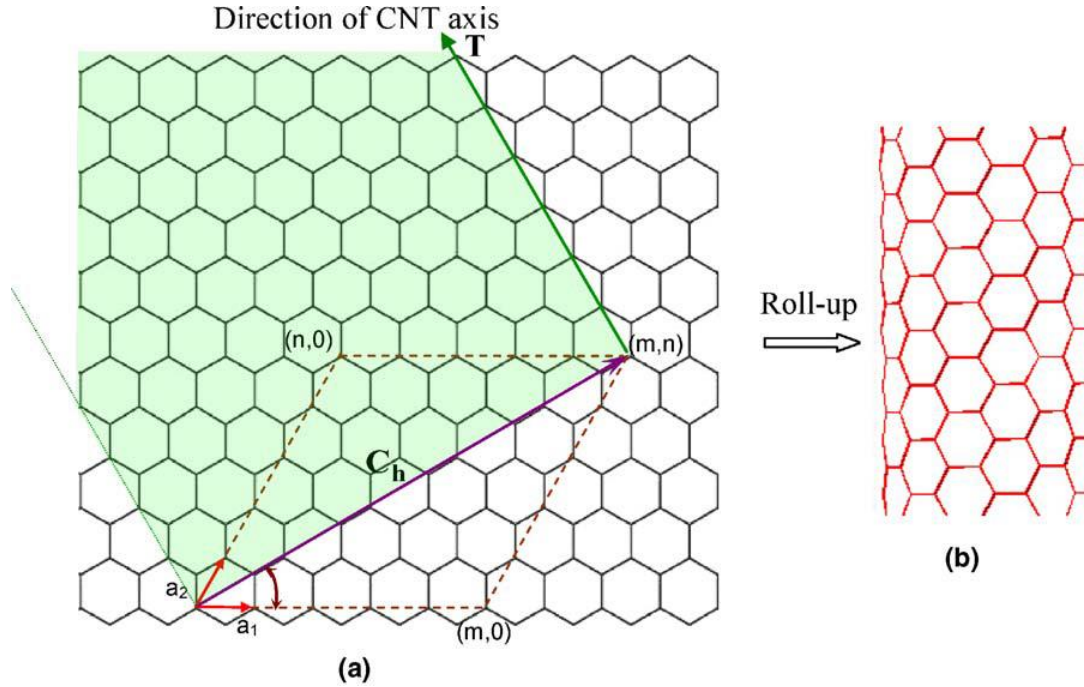


Fig. 1 Roll-up vector defining the structure of carbon nanotubes. (a) Graphene lattice and (b) carbon nanotube.

1.3. Functionally Graded Materials (FGMs)

FGMs were first used by the Japan Aerospace eXploration Agency (JAXA) in the 1980s. Since then the benefits of graded composites have motivated a lot of research in that field all around the world. Functionally Graded Materials are basically composites of two or more materials with gradually varying composition of the constituents. This variation can be mathematically controlled using various functions such as exponential function or power law. The volumetric variation gives rise to difference in material properties in the material and hence the same material can be used for multiple purposes. FGMs have better thermal and mechanical stress bearing qualities. The materials can be designed for specific functions and applications. For

example an FGM of a ceramic and a metal would have better thermal stress capacity because of the ceramic and better mechanical load bearing capacity because of the presence of metal, also the use of functionally graded (FG) coating on structural elements in high-temperature environments can effectively help to reduce possible failures induced by thermo or combined thermo-mechanical loadings. Typical examples of those elements are movable arms, tall buildings and towers and beams used in high-performance surface and air vehicles. [5]

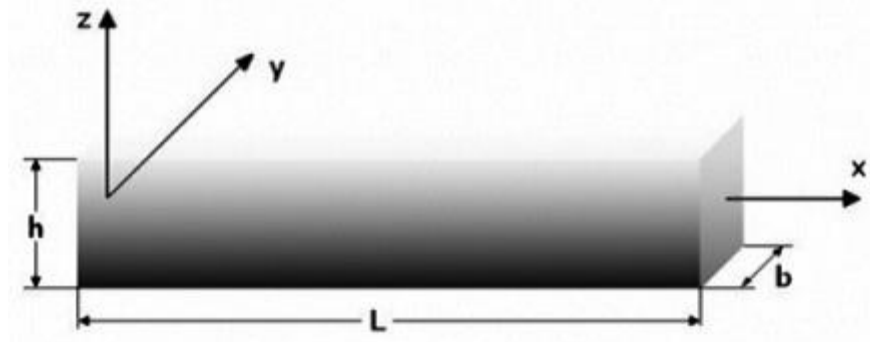


Fig.2. Schematic of a functionally graded beam [5]

1.3.1. Applications of FGM

Due to progressing of technology it is need for advanced capability of materials to become a priority in engineering field for higher performance systems. FGMs are relatively new materials and are being studied for the use in high temperature applications. FGM is an extensive variety of applications in engineering practice which requires materials performance to vary with locations within the component. The following applications exist for FGM structures.

1. Aerospace

- Aerospace skins
- Rocket engine components
- Vibration control
- Adaptive structures

2. Engineering

- Cutting tools
- Wall linings of engines
- Shafts

- Engine components
- Turbine blades
- 3. Nuclear energy
 - Nuclear reactor components
 - First wall of fusion reactor
 - Fuel pellet
- 4. Optics
 - Optical fiber
 - Lens
- 5. Electronics
 - Graded band semiconductor
 - Substrate
 - Sensor
 - Actuator
 - Integrated chips
- 6. Chemical plants
 - Heat exchanger
 - Heat pipe
 - Reaction vessel
 - Substrate
- 7. Energy conversion
 - Thermoelectric generator
 - Thermo-ionic converter
 - Fuel cells, solar cells
- 8. Biomaterials
 - Implants
 - Artificial skin
 - Prosthetics
- 9. Commodities
 - Building material

- Sports goods
- Car body
- Casing of various materials
- Air Conditioning temperature control

1.4. Beam Theories

1.4.1. Euler-Bernoulli Beam Theory:

It is also known as Classical Beam Theory, which simply formulates the load carrying and deflection characteristics of thin beams. It takes the assumption that the plane in the cross section of a thin beam is infinitely rigid and no deformation takes place in this plane. It does not take into consideration, the shear deformation of the beam. Further when a beam is deformed the plane of any cross section is assumed to be plane and normal to the deformed axis. All these assumptions are experimentally proven to be true for long slender beams i.e. length to thickness ratio is greater than 20.

1.4.2. Timoshenko Beam Theory:

Developed by Ukrainian born scientist and engineer Stephen Timoshenko, this theory takes into account the shear effect in a beam and also considers the rotational inertia effect in beams. This theory is particularly used in thick beams with length to thickness ratio as lower than 20. Here the shear effects cannot be neglected. If the shear modulus approaches infinity and rotational effects are neglected then this theory converges towards an ordinary beam theory. It is highly useful for analyses of high frequency of beams when the wavelength nearly reaches the length of the beam. In reality the plane of the cross section curves when the beam is bent. However Timoshenko beam theory still accepts this assumption but it does not allow the assumption that the plane of the cross section remains normal to the bent axis of the beam.

1.5. Present Work

The present work deals with the thermo-mechanical analyses of carbon nanotube based functionally graded Timoshenko beams. The FG beam is graded in thickness direction using simple power law to obtain material properties. The temperature distribution is assumed according to Fourier's law of heat conduction and obtained using the power law. Using this temperature distribution, Temperature dependent material properties are also obtained. Stresses and strains are obtained using the above material properties. FGMs with temperature-dependent material properties being taken into account.

Chapter 2

Literature Review

LITERATURE REVIEW

Functionally Graded materials are useful in aerospace industry for its thermal properties, multi functionalities. The addition of carbon nanotubes in to FG beam enhances its mechanical and thermal properties. Many researchers reported functionally graded beams based on different theories and developed new methods of solutions which are listed below:

2.1 Functionally graded materials:

A Chakraborty et al. [1] have studied the thermo-elastic behavior of functionally graded beam structures based on the first-order shear deformation theory and these properties are varying along its thickness. The governing differential equations are used to construct interpolating polynomials for the element formulation. To determine various stresses, both exponential and power-law variations of material property distribution are used. Thermal behaviors of functionally graded beam (FGB) by taking the distribution of material properties in exponential function were analyzed by GH Rahimi and AR Davoodinik [2]. The steady state of heat conduction with exponentially and hyperbolic variations through the thickness were consider for the use of thermal loading. They found that thermal behavior of both isotropic beam and functionally graded beam depend up on the temperature distribution.

B.V. Sankar [3] has obtained an elasticity solution for a functionally graded beam subjected to transverse loads. The exponential variations of the elastic stiffness coefficient were used to find solution for the elasticity equations. On the basis of the assumption that plane sections remain plane and normal to the beam axis, a simple Euler–Bernoulli type beam theory was developed. T. Prakash et al. [4] have investigated the post buckling behavior of functionally graded material (FGM) skew plates under thermal load based on the shear deformable finite element method. The volume fractions and the properties of the constituent materials have estimated by using Mori–Tanaka homogenization method. The temperature was assumed to vary exponentially through the thickness and Poisson ratio was held constant. Mohammad Azadi [5] has studied a finite element method (FEM) free and forced lateral vibration of beams made of functionally graded material. For functionally graded (FG) beams with various boundary conditions, the natural frequencies were obtained. His results were compared with the analytical solution and the results for ANSYS and NASTRAN software. Numerical results were obtained

to show the influences of the temperature dependency of the materials properties, the volume fraction distribution, the geometrical parameters, the boundary conditions.

Arani et al. [6] were together reported thermal Stress Analysis of a Composite Cylinder Reinforced with FG SWCNTs. Based on Mori-Tanaka method, Thin -walled cylinder was subjected to a thermal field. They presented nano-composite characteristics to be transversely isotropic which can only be applied where SWCNTs are uniformly distributed. The higher order governing equation was solved in order to obtain the distribution of displacement and thermal stresses in radial, circumferential and axial directions. Their results indicate that FG distributions of SWCNTs have significant effect on thermal stresses and displacements in axial, radial and circumferential directions. K. Nirmala et al. [7] have derived analytical expressions for the thermo-elastic stresses in a three layered composite beam system having a middle layer of functionally graded material. Numerical scheme of discretizing the continuous FGM layer and treating the beam as a discretely graded structure has also been discussed.

2.2 CNT based functionally graded materials:

Heshmati and Yas [8] have studied the vibrational properties of functionally graded nano-composite beams reinforced by randomly oriented straight single-walled carbon nanotubes (SWCNTs) under the actions of moving load. The material properties of the beam were investigated using Eshelby–Mori–Tanaka approach. The system of equations of motion was derived by using Hamilton’s principle under the assumptions of the Timoshenko beam theory. A Shooshtari et al. [9] have studied thermo-mechanical dynamic characteristics of SWCNT-Reinforced Composite Plates. Based on the multi-scale approach, numerical values were carried out for CNTRC plates and uniformly distributed CNTRC plates under different values of the nanotube volume fractions. The results also show that the natural frequencies are reduced but the nonlinear to linear frequency ratios are increased by increasing the temperature rise. The natural frequencies were obtained for nonlinear problem. Wang et al. [10] have investigated the effect of environmental temperature on elastic properties of armchair and zigzag single walled carbon nanotubes based on a method of molecular structural mechanics. The force constant values of the bonds stretching, bonds angle bending and torsional resistance, the corresponding basic parameters of a truss of the single-walled carbon nanotubes were obtained in different environmental temperatures. It is noted that the Young’s modulus of single-walled carbon

nanotubes is more sensitive to environmental temperature than the shear modulus. Hui-Shen Shen and Chen-Li Zhang [22] presented the thermal buckling and postbuckling behavior for functionally graded nano-composite plates reinforced by single-walled carbon nanotubes (SWCNTs) subjected to in-plane temperature variation.

K.M. Liew et al. [11] have proposed a temperature-related higher-order gradient continuum theory for predicting the mechanical properties of single-walled carbon nanotubes at various temperatures. Their results indicated that the temperature effect influences the axial Young moduli of zigzag SWCNTs less than those of the other types. Y.C. Zhang et al. [12] have reported the results the effect of environmental temperature on mechanical properties of multi walled carbon nanotubes (CNTs), by means of a molecular structural mechanics model in which the covalent bonds are treaded as dimensional Euler–Bernoulli beam. Their results obtained by means of nano-scale finite element simulation reveal that the Young’s modulus and Poisson’s ratios of multi-walled carbon nanotubes decreases significantly with the increase of environmental temperatures.

2.3 Thermo-mechanical analysis of FG beam:

J N Reddy et al. [13] have investigated the dynamic thermo-elastic response of functionally graded cylinders and plates. A finite element model of the formulation was developed for the formulation of thermo-mechanical coupling. They solved are the heat conduction and the thermo-elastic equations for a functionally graded axisymmetric cylinder subjected to thermal loading and thermo-elastic boundary value problem using the first-order shear deformation plate theory. Both problems are studied by varying the volume fraction of a ceramic and a metal using a power law distribution.

G.R. Liu et al. [14] have presented a mesh free model for the active shape control as well as the dynamic response repression of the functionally graded material plate containing distributed piezoelectric sensors and actuators. The element-free Galerkin method was used to derive the shape functions using the moving least squares method. The static deflection and vibration control through a closed control loop were realized by introduced a constant displacement and velocity feedback control algorithm. J. Woo et al. [15] have analyzed solution for the post buckling behavior of plates and shallow cylindrical shells made of functionally graded materials under edge compressive loads and a temperature field. The fundamental

equations for thin rectangular shallow shells of FGM were obtained using the von Karman theory for large transverse deflection, and the solution was obtained in terms of mixed Fourier series. It was found that thermo-mechanical coupling effects and the boundary conditions play a major role in dictating the response of the functionally graded plates and shells under the action of edge compressive loads.

Y. Kiani and M.R. Eslami [16] have analyzed Buckling of beams made of functionally graded materials (FGM) under thermo-mechanical loading. They assumed that the mechanical and thermal nonhomogeneous properties of beam vary smoothly by distribution of the power law index across the thickness of the beam. The equilibrium and stability equations for an FGM beam were derived and the existence of bifurcation buckling was examined. In each case of boundary conditions and loading, the critical buckling temperature of the beam was presented.

A. Fallah et al. [17] have investigated thermo-mechanical buckling and nonlinear free vibration analysis of functionally graded beams on nonlinear elastic foundation. Nonlinear governing partial differential equations of motion were derived based on Von Karman strain–displacement relation together with Euler–Bernoulli assumptions. The effects of vibration amplitude, boundary conditions, nonlinear elastic foundation, material inhomogeneity, geometric parameter and thermal loading were analyzed by the thermo-mechanical buckling and nonlinear free vibration analysis of the FG beams.

J.D. Fidelus et al. [18] have examined thermo-mechanical properties of epoxy-based nano-composites based on low weight fractions of randomly oriented single- and multi-walled carbon nanotubes. There is no significant change in the glass transition temperature of SWCNT/epoxy nano-composites was observed, compared to that of the epoxy matrix. The mechanical properties examined were the tensile Young's modulus by Dynamic Mechanical Thermal Analysis and the toughness under tensile impact using notched specimens. Yas and Heshmati [19] worked on dynamics of functionally graded nano composite beams with randomly oriented single walled carbon nano tubes. They found that under the action of moving load, CNT-FGM beam with symmetrical distribution gives superior properties than that of unsymmetrical distribution. Simon Caraballo [20] has studied various design approaches for analyzing functionally graded material structures. Based on spatial temperature variation governing equations were derived to solve a two-node beam element for functionally graded

materials based on first order shear deformable (FOSD) theory. The thermal stresses distribution was influenced by the temperature dependency thermo-elastic material properties. The best layer thicknesses for the beam models were came by taking some specific factor of safety, which combines both the strength and mass of the beam.

Chapter 3

Motivation

and

Objectives

MOTIVATION AND OBJECTIVES

Since the discovery of FGMs in Japan in 1984 for space exploration, where a 10mm sheet could withstand a temperature 2000 K with a thermal gradient of 1000 K, the possible uses of functionally graded materials has increased manifold. While mostly used in aircrafts and aerospace industries, these materials have a wide potential for use in industries where high temperature processes take place. But at the same time the use of metals in the FGMs make them heavy and less efficient. For this cause Carbon Nanotubes (CNTs) are excellent choices for a replacement to metals in the FGM. CNTs are light in weight and at the same time behave like metals in all the other aspects. This combination of ceramic and carbon nanotubes may find a large scale use if this analysis yields desirable results, hence this analysis may have a potential industrial impact.

The objectives of the present work are:

- Mathematical modelling of a functionally graded beam by taking into account the dimensions, volumetric concentrations and temperature conditions of the beam. The properties of the beam with respect to temperature distribution and volume fractions.
- Determination of temperature dependent material properties of the beam.
- To find the stresses and strains due to temperature variation in the beam.
- Comparative study of effect of slenderness ratios and power law indices on the stresses developed.
- To plot stresses and strains with respect to the temperature and thickness.

Chapter 4

Methodology

METHODOLOGY

The aforementioned beam has been mathematically modelled using power law function of the FGM (the variation of volume fraction has been shown in Fig.1) using Timoshenko beam theory (First Order Shear Deformation Theory) and applying the physical properties of the CNTs and the ceramic material, i.e. Young's Modulus, coefficient of thermal expansion, Poisson's Ratio and Thermal conductivity. The results for the properties and temperature variation have been found using property relations with respect to temperature and volume fraction.

4.1 Material Modelling:

4.1.1 Rule of mixture

An FGM can be modelled using various functions for the distribution of the constituent materials in a beam, these functions are also known as rule of mixture. Majorly the exponential function and power laws are used. An exponential function gives exponentially increasing volume fraction of one element and similarly decreasing volume fraction of the other component. In such a distribution the volume fraction of one of the components would never be unity i.e. a hundred percent pure layer of one of the constituents would never be reached. Whereas in a power law distribution the constituents follow a power law of a particular index as required by a designer. Such distributions could be linear, quadratic, cubic or of any fractional index also. For our analysis we have used a power law. And the index has been chosen as 1.5 as that is most suited for a high temperature applications.

4.1.2 Power law material distribution

To calculate the material properties, initially the volume fraction variation should be considered. The volume fraction is assumed to vary according to simple power law along the thickness direction only, given by [17]

$$V_m = \left(\frac{1}{2} + \frac{z}{h} \right)^k \text{ and } V_{cnt} = 1 - V_m \quad (1)$$

Here k represents the power law index, h is thickness and z is co-ordinate ($-h/2 \leq z \leq h/2$), along the thickness direction. This volumetric relationship gives the composition of a layer of a beam

at a particular coordinate in the z axis. Further this constitution relationship affects the properties of the beam in a layer wise manner.

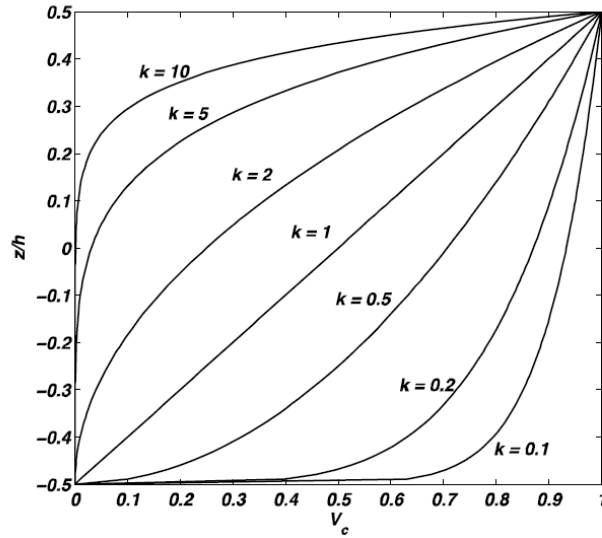


Fig.3 Distribution of ceramic volume fraction through the thickness for various power law indices. [16]

The physical properties of an FG-Beam varies according to the given relation [5]

$$P(z) = P_{cnt} + (P_m - P_{cnt}) \left(\frac{1}{2} + \frac{z}{h} \right)^k \quad (2)$$

Where,

P = Material property of FG-beam at a layer in z coordinates

P_m = Material property of matrix, which is the ceramic material Al_2O_3

P_{cnt} = Material Property of Carbon Nanotubes.

4.2. Temperature Distribution:

The temperature profile of the beam represents the gradient of temperature over the thickness of the beam. As the temperatures applied on the upper and lower surfaces of the beam are different hence due to conductivity and constituent relationship, the temperature would vary over the thickness of the beam.

The pure ceramic layer is subjected to a temperature of 700 K, whereas the pure CNT layer is subjected to 1000 K. The boundary conditions for temperature are [4] as follows

$$\begin{aligned} T = T_m &= 700 \text{ K} \quad \text{at} \quad z = h/2 \\ T = T_{cnt} &= 1000 \text{ K} \quad \text{at} \quad z = -h/2 \end{aligned} \quad (3)$$

The temperature profile for the beam over the thickness is given [4] as

$$T(z) = T_m + (T_{cnt} - T_m)\eta(z) \quad (4)$$

Where $\eta(z)$ is a conductivity and composition dependent variable and is given [4] as

$$\begin{aligned} \eta(z) &= \frac{1}{C} \left[\left(\frac{2z+h}{2h} \right) - \frac{\kappa_{cm}}{(k+1)\kappa_m} \left(\frac{2z+h}{2h} \right)^{k+1} + \frac{\kappa_{cm}^2}{(2k+1)\kappa_{cm}^2} \left(\frac{2z+h}{2h} \right)^{2k+1} \right. \\ &\quad \left. - \frac{\kappa_{cm}^3}{(3k+1)\kappa_{cm}^3} \left(\frac{2z+h}{2h} \right)^{3k+1} + \frac{\kappa_{cm}^4}{(4k+1)\kappa_{cm}^4} \left(\frac{2z+h}{2h} \right)^{4k+1} - \frac{\kappa_{cm}^5}{(5k+1)\kappa_{cm}^5} \left(\frac{2z+h}{2h} \right)^{5k+1} \right], \\ C &= 1 - \frac{\kappa_{cm}}{(k+1)\kappa_m} + \frac{\kappa_{cm}^2}{(2k+1)\kappa_{cm}^2} - \frac{\kappa_{cm}^3}{(3k+1)\kappa_{cm}^3} + \frac{\kappa_{cm}^4}{(4k+1)\kappa_{cm}^4} - \frac{\kappa_{cm}^5}{(5k+1)\kappa_{cm}^5}; \end{aligned} \quad (5)$$

and $\kappa_{cm} = \kappa_{cnt} - \kappa_m$.

T_c and T_m denote the temperature of ceramic and metal, respectively.

κ_{cnt} = Conductivity of carbon nanotubes.

κ_m = Conductivity of the ceramic material. (Al_2O_3)

4.3. Temperature dependent Material Properties:

The beam model in our problem is subjected to high temperatures. Both the ceramic and CNT are used in high temperature ranges, hence their properties might change according to temperature. Now this rise would simply be a function of temperature and a few predetermined constants of a particular material. The variation of physical properties with respect to temperature is given as [5]

$$P(T) = P_0 \left(P_{-1}T(z)^{-1} + 1 + P_1T(z) + P_2T(z)^2 + P_3T(z)^3 \right) \quad (6)$$

Where P_0, P_{-1}, P_1, P_2 and P_3 are constants in the cubic fit of the material property. These properties are obtained for each individual constituent. The temperature dependent property values are used to find the constitution dependent material property of the beam at different values of z .

Table 1. The constants in the cubic fit of Alumina [13]

Property	P_0	P_{-1}	P_1	P_2	P_3
Modulus of Elasticity (GPa)	349.55	0	-3.853×10^{-13}	4.027×10^{-16}	-1.673×10^{-10}
Poisson's Ratio	0.2600	0	0	0	0
Coefficient of thermal expansion (K^{-1})	6.8269×10^{-6}	0	1.838×10^{-4}	0	0
Thermal Conductivity (W/mK)	-14.087	-1123.6	-6.227×10^{-3}	0	0

Table 2. Material properties of Single Walled CNT

E (GPa)	ρ (Kg/m ³)	ν	k (W/m K)	α (/ K $\times 10^{-6}$)
6151.45	2100	0.28	3500	5.1

Table 3. Constants in Cubic fit for Young's Modulus of (10, 10) SWCNT

$(L = 9.26 \text{ nm}, R = 0.68 \text{ nm}, h = 0.067 \text{ nm}, \nu_{CN}^{12} = 0.175) \text{ (GPa) (derived [22])}$

E_0	E_{-1}	E_1	E_2	E_3
6151.45	0	-4.323×10^{-4}	6.311×10^{-7}	-3.404×10^{-10}

Now in order to obtain the overall material property of an FG-Beam we have to consider both the temperature variation as well as composition variation. On one and the temperature

variation affects the individual material properties whereas the volume fraction determines the overall material properties. Therefore both the relations are combined to form the effective material property. Hence the effective material property of an FG-Beam may be given as [5]

$$P_{eff}(z, T) = P_m(T)V_m(z) + P_{cnt}(T)V_{cnt}(z) \quad (7)$$

These properties make for a realistic mathematical model of the beam with exact physical parameters to be considered during the analysis. The analysis would require these properties for finding exactness of deformations and stresses in the beam.

According to first order shear deformation theory(FSDT) , after deformation , in the FGB a point A which situated at a distance z to the middle surface will be moved to point A' (Fig.4).

Thus, the axial displacement at the point A with distance z from mid-surface ($z=0$) is given [2] as

$$u(x, z) = u_0 - z \frac{\partial w}{\partial x} + z \phi_y \quad (8)$$

Where u_0 = displacement

and w = transverse deformation from the middle surface,

The vertical line AB rotates about the Y-axis, $\frac{\partial w}{\partial x}$ and ϕ_y are the rotations due to bending and shearing and are z -direction independent.

Strain in X-direction is given [2] as

$$\varepsilon_{xx} = \frac{\partial u_0}{\partial x} - z \frac{\partial^2 w}{\partial x^2} + z \frac{\partial \phi_y}{\partial x} \quad (9)$$

For an FGB that is subjected to thermal load, the plane strain condition, stress-strain relation can be expressed [2] as

$$\sigma_{xx}(z) = \frac{E(z)}{1-\nu} \left[\frac{\varepsilon_{xx}}{1+\nu} - \alpha(z)T(z) \right] \quad (10)$$

$$\sigma_{xx}(z) = \frac{E(z)}{1-\nu^2} \left[\frac{\partial u_0}{\partial x} - z \frac{\partial^2 w}{\partial x^2} + \frac{\partial \phi_y}{\partial x} - \alpha(z)T(z)(1+\nu) \right] \quad (11)$$

Where,

$\sigma_{xx(z)}$ is the axial stress on the surface with distance z from mid surface,

ν is the Poisson's ratio and

$T_{(z)}$ is the temperature distribution along z -direction of Beam.

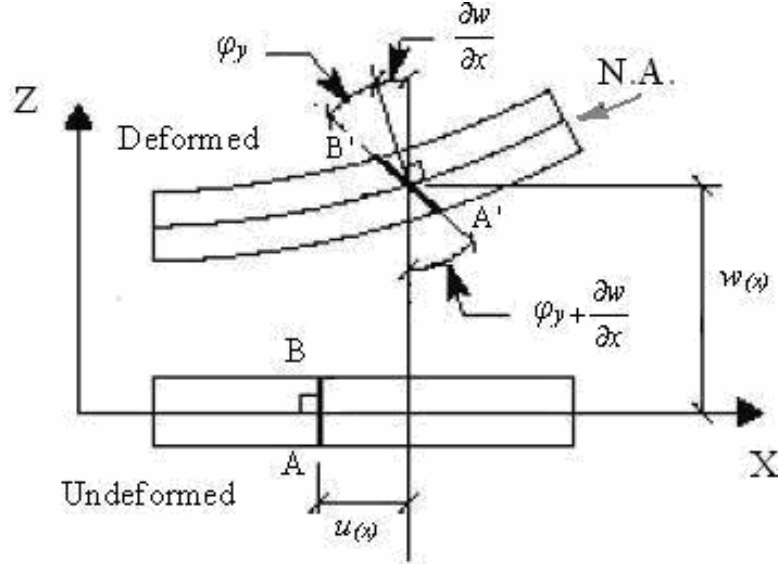


Fig.4. Axial, bending and shearing deformation characteristics of a beam according to Timoshenko Beam Theory [2]

By integrating stresses along the thickness, the resultant stress per unit length of the middle surfaces can be calculated. Assumption is that the thermal loading is only associated with distribution in z -direction. The axial resultant forces in the x -direction must be zero, when the beam is in equilibrium [2] i.e.

$$\sum F_x = 0 \Rightarrow \int_{-h/2}^{h/2} \sigma_{xx}(z) b dz = 0, \quad 0 \leq x \leq l \quad (12)$$

Where, b and l are the width and length of beam, respectively. Also in the absence of mechanical loads, the resultant moments about Y -axis appear to be effected by only thermal factor (M_{Ty}).

Thus the equilibrium equation for resultant moments can be expressed [2] as,

$$\frac{\partial^2 w}{\partial x^2} I_y \int_{-h/2}^{h/2} E(z) dz = -(M_{Ty} + M_{My}), \quad 0 \leq x \leq l \quad (13)$$

$$M_{My} = 0, \quad 0 \leq x \leq l$$

Where I_y that is the inertia moment and M_{Ty} are defined as [2] follows

$$I_y = \frac{1}{12} b h^3, \quad M_{Ty} = b \int_{-h/2}^{h/2} z E(z) \alpha(z) T(z) dz \quad (14)$$

Further, M_{My} is presented the mechanical moment about Y -axis. The Boundary Conditions of the simply supported FGM beam are

$$\begin{aligned}
w = 0, \quad M_y = 0; \quad x = 0, x = l \\
\int_{-h/2}^{h/2} \sigma_{xx}(z) \cdot z b dz = 0; \quad x = 0, x = l
\end{aligned} \tag{15}$$

Where M_y is the total moment acting on the beam about Y-axis. Conventionally the moment of inertia are given as follows,

$$\begin{aligned}
I_0 &= \int_{-h/2}^{h/2} E \left(\frac{z}{h} + 0.5 \right)^k dz, \\
I_{0T} &= \int_{-h/2}^{h/2} E \alpha \left(\frac{z}{h} + 0.5 \right)^{2k} T(z) dz, \\
I_1 &= \int_{-h/2}^{h/2} z E \left(\frac{z}{h} + 0.5 \right)^k dz, \\
I_{1T} &= \int_{-h/2}^{h/2} z E \alpha \left(\frac{z}{h} + 0.5 \right)^{2k} T(z) dz, \\
I_2 &= \int_{-h/2}^{h/2} z^2 E \left(\frac{z}{h} + 0.5 \right)^k dz, \\
I_{2T} &= \int_{-h/2}^{h/2} z \alpha \left(\frac{z}{h} + 0.5 \right)^k T(z) dz
\end{aligned} \tag{16}$$

$$C_1 = \frac{\partial u_0}{\partial x}, \quad C_2 = \frac{\partial^2 w}{\partial x^2}, \quad C_3 = \frac{\partial \varphi_y}{\partial x} \tag{17}$$

Where By substitution Eq. (14), Eq. (11) and Eq. (16) into Eq. (12), Eq. (13) and Eq. (15) we can set the equation system with the following form [2]

$$\begin{aligned}
I_0 C_1 - (C_2 - C_3) I_1 &= I_{0T} \\
I_1 C_1 - (C_2 - C_3) I_2 &= I_{1T} \\
C_2 &= (-I_{1T} \times b) / (I_y \times I_0)
\end{aligned} \tag{18}$$

Then the coefficient C_2 is directly obtained and for C_1 , C_3 we have;

$$\begin{aligned}
C_1 &= \frac{I_{1T} I_1 - I_{0T} I_2}{I_0 I_2 - I_1^2} \\
C_3 &= \frac{-I_1 I_{0T} I_y I_0 + b I_0 I_{1T} I_2 - I_0^2 I_y I_{1T} - b I_1^2 I_{1T}}{I_y I_0 (I_0 I_2 - I_1^2)}
\end{aligned} \tag{19}$$

It is evident that axial stress can be obtained from Eq. (11) by substituting for C_1 , C_2 , and C_3 from Eq. (18) and Eq. (19) as [2] follows;

$$\sigma_{xx}(z) = \frac{E(z)}{1-\nu^2} \times [C_1 - zC_2 + zC_3 - \alpha(z)T(z)(1+\nu)] \quad (20)$$

By using the above equation we can obtain stresses for the CNT based FG beam under consideration.

Chapter 5

Results

and

Discussions

RESULTS AND DISCUSSIONS

Based on above theory a complete MATLAB code has been developed and it is applied to the beam with the simply supported end conditions and rectangular cross section with width (b) = 0.4 m and length (L) = 0.5 m. The thickness of the beam is varied so as to obtain (L/h) ratios of 5, 10, 15 and 20. The required material properties are given in Tables 1 and 2. The beam is divided into eleven layers along the thickness. The material properties are calculated for each layer using temperature-dependent and temperature-independent material properties and by applying simple power law and rule of mixture. Using above properties graph is plotted to observe the variation of Young's modulus along the thickness direction for temperature-dependent and temperature-independent material properties. It is shown in Fig. 6. It is observed that, for increasing values of CNT volume fraction, Young's modulus increases and is higher for temperature independent criterion. The temperature boundary conditions considered here are,

$$T_0 = 300^\circ K, \quad T_m = 700^\circ K, \quad T_{cnt} = 1000^\circ K$$

These boundary conditions are applied for each case. Graph is plotted to observe the temperature distribution along the thickness according to power law. It can be observed in Fig. 5. Then the required values of constants such as $I_0, I_{0T}, I_1, I_{1T}, I_2, I_{2T}, C_1, C_2$ and C_3 are evaluated for particular cases and finally the stresses and strains have been obtained.

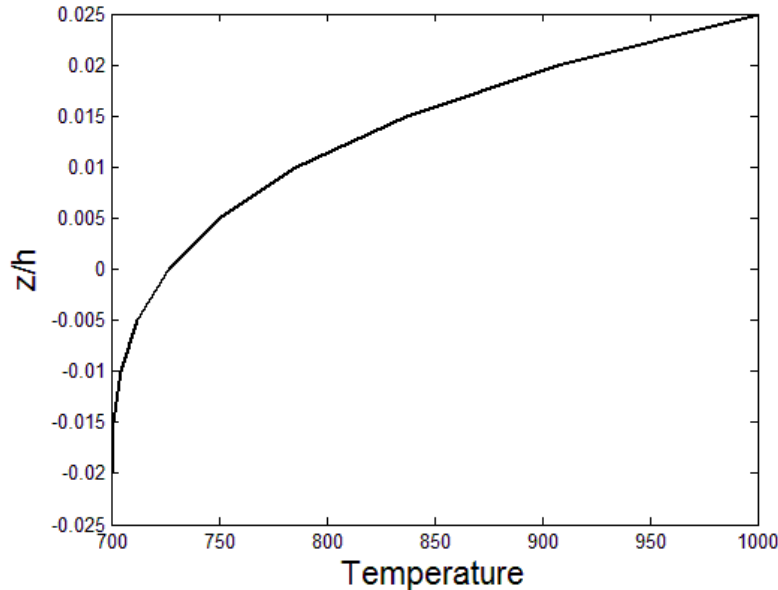


Fig 5. Variation of temperature along the thickness

Table 4: Volume fraction and young's modulus variation for different material distribution

T (K)	z/h	V_{cnt}	Young's Modulus (GPa)	
			Temperature-independent	Temperature-dependent
700	-0.5	0	349.55	304.18
700.09	-0.4	0.32	2184.27	1939.15
701.06	-0.3	0.44	2944.24	2616.24
704.40	-0.2	0.54	3527.38	3135.27
712.07	-0.1	0.63	4018.99	3571.69
726.40	0	0.70	4452.11	3954.03
750.04	0.1	0.77	4843.68	4295.93
785.92	0.2	0.83	5203.76	4603.71
837.22	0.3	0.89	5538.92	4877.88
907.36	0.4	0.94	5853.72	5111.07
1000	0.5	1	6151.45	5281.51

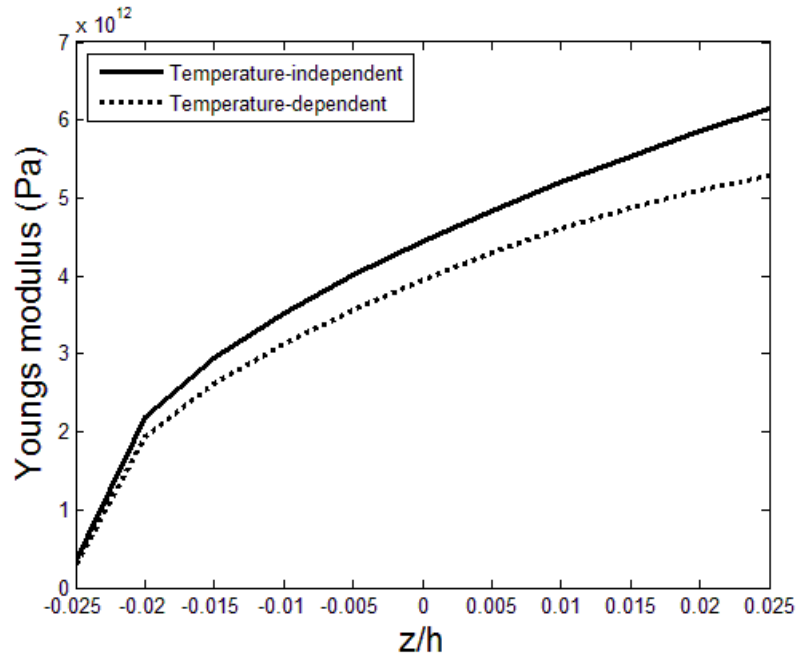


Fig 6. Variation of young modulus in the thickness direction

Table 5: Effect of CNT volume fraction on the stress and strain ($k=0.5$)

L/h	Temperature-independent		Temperature-dependent	
	Max stress (GPa)	Max Strain ($\times 10^{-3}$)	Max Stress (GPa)	Max Strain ($\times 10^{-3}$)
5	121.81	1.98	102.36	1.66
10	206.76	3.36	174.56	2.84
15	294.29	4.78	248.94	4.05

Table 6: Stresses and strains for variation in power law index keeping $(L/h) = 10$

k	Attributes		Layers									
			1	2	3	4	5	6	7	8	9	10
0.5		V_{cnt}	0.66	0.72	0.74	0.77	0.80	0.84	0.87	0.91	0.95	1.00
	TID	Stress (GPa)	30.17	26.44	14.15	4.222	27.62	55.48	87.46	123.4	163.2	206.8
		Strain	0.014	0.009	0.004	0.001	0.006	0.011	0.017	0.022	0.028	0.034
	TD	Stress (GPa)	30.79	28.27	17.77	1.666	19.03	43.77	72.16	103.9	138.4	174.6
		Strain	0.014	0.01	0.005	0.004	0.004	0.009	0.014	0.019	0.024	0.028
1		V_{cnt}	0.74	0.69	0.75	0.77	0.80	0.84	0.87	0.91	0.95	1.00
	TID	Stress (GPa)	10.55	10.25	4.672	6.217	22.48	44.26	71.84	105.7	146.7	195.7
		Strain	0.011	0.007	0.002	0.002	0.007	0.012	0.016	0.021	0.026	0.032
	TD	Stress (GPa)	11.62	12.7	8.919	0.197	13.56	32.5	56.88	87.05	123.3	164.7
		Strain	0.012	0.008	0.004	0.004	0.004	0.008	0.013	0.017	0.022	0.027
1.5		V_{cnt}	0.81	0.66	0.75	0.76	0.81	0.83	0.87	0.91	0.95	1
	TID	Stress (GPa)	5.26	4.728	1.411	5.866	18.01	35.81	60.13	92.09	133.5	187.5
		Strain	0.01	0.005	0.001	0.003	0.008	0.012	0.016	0.02	0.025	0.03
	TD	Stress (GPa)	5.981	6.484	4.741	0.43	9.977	24.76	45.67	73.79	110.6	156.4
		Strain	0.011	0.007	0.004	0.002	0.004	0.008	0.012	0.016	0.021	0.025
2		V_{cnt}	0.86	0.64	0.76	0.76	0.80	0.84	0.87	0.91	0.95	1.00
	TID	Stress (GPa)	3.602	2.637	0.264	4.926	14.29	29.16	50.89	81.17	122.7	181
		Strain	0.009	0.005	0.003	0.004	0.008	0.012	0.016	0.02	0.024	0.029
	TD	Stress (GPa)	30.17	26.44	14.15	4.222	27.62	55.48	87.46	123.4	163.2	206.8
		strain	0.01	0.007	0.003	0.005	0.004	0.008	0.012	0.015	0.02	0.024
2.5		V_{cnt}	0.9	0.63	0.76	0.76	0.80	0.83	0.87	0.91	0.95	1
	TID	Stress (GPa)	2.946	1.738	0.177	4.041	11.36	23.84	43.33	72.03	113.5	175.6
		Strain	0.008	0.004	0.003	0.004	0.008	0.012	0.016	0.02	0.024	0.029
	TD	Stress (GPa)	3.21	2.52	1.5	0.828	5.734	14.75	29.7	52.83	87.52	138.8
		strain	0.009	0.006	0.002	0.009	0.004	0.007	0.011	0.014	0.018	0.023
3		V_{cnt}	0.93	0.62	0.76	0.76	0.80	0.83	0.87	0.91	0.95	1
	TID	Stress (GPa)	2.583	1.282	0.38	3.367	9.139	19.59	37.02	64.17	105.2	170.6
		Strain	0.007	0.003	0.007	0.005	0.009	0.012	0.016	0.019	0.023	0.028
	TD	Stress (GPa)	2.385	1.593	0.67	1.017	4.497	11.21	23.06	42.55	73.56	123.5
		strain	0.007	0.004	0.001	0.001	0.004	0.007	0.01	0.013	0.016	0.02

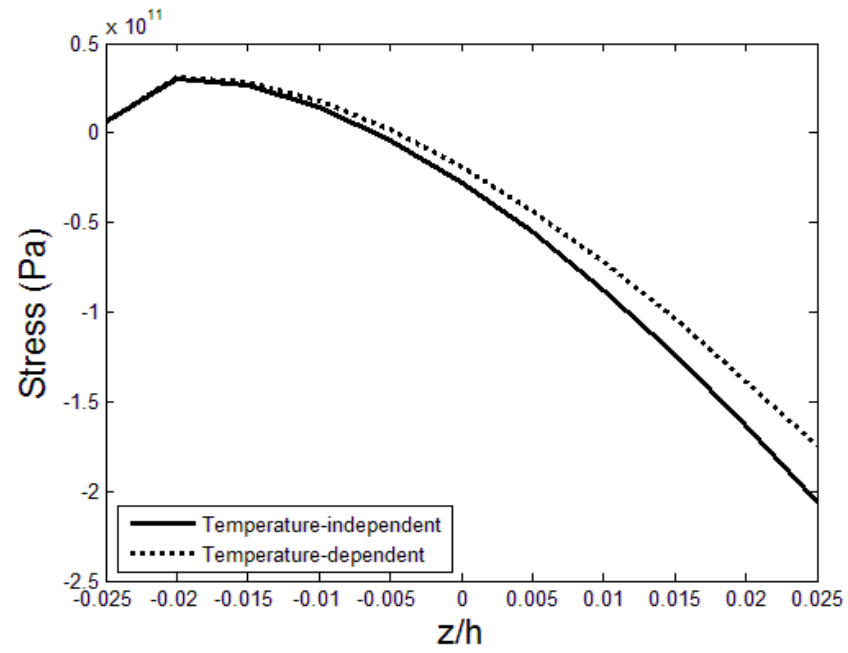


Fig 7. Variation of stress along the thickness

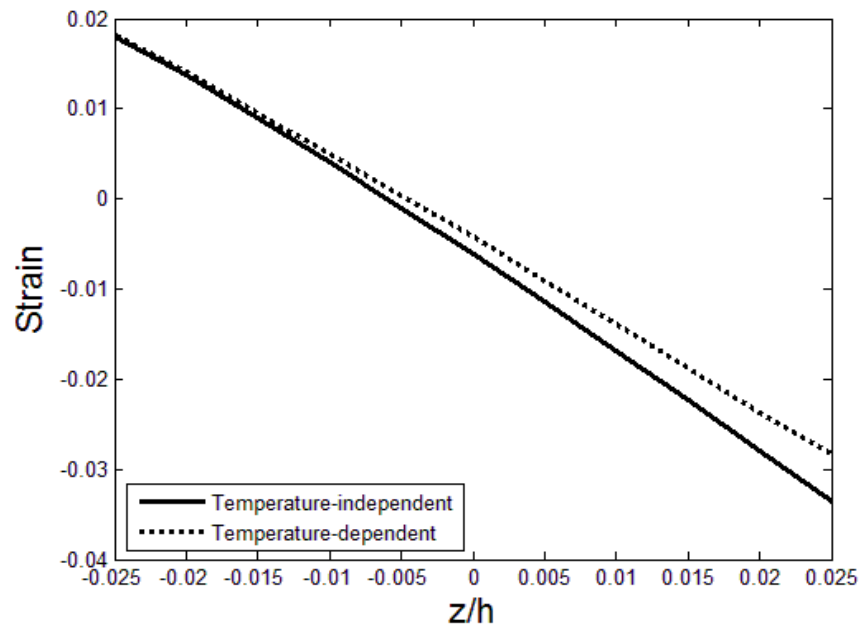


Fig 8. Variation of strain along the thickness

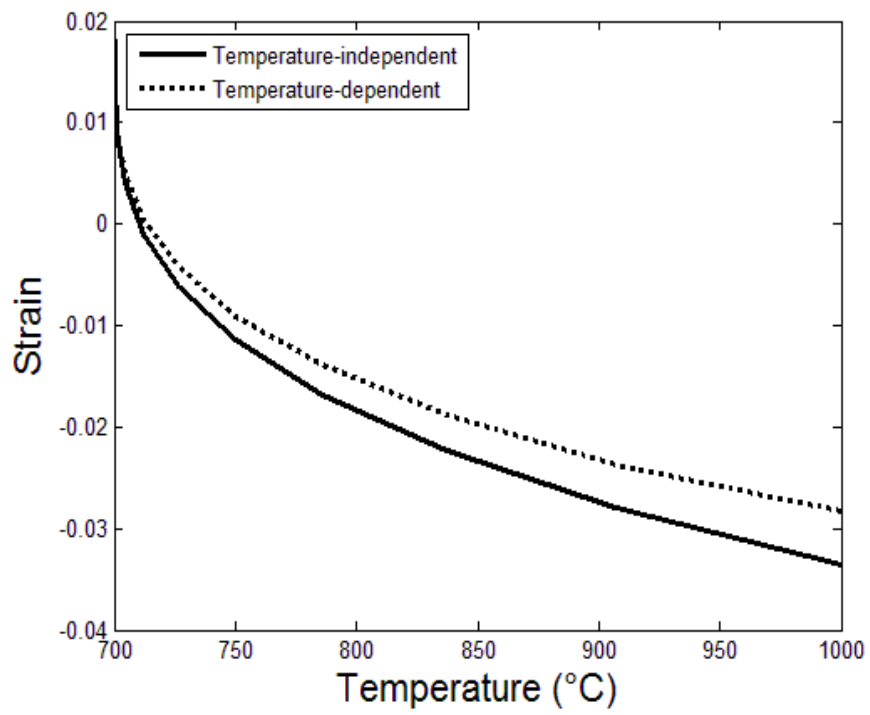


Fig 9. Variation of strain with respect to temperature

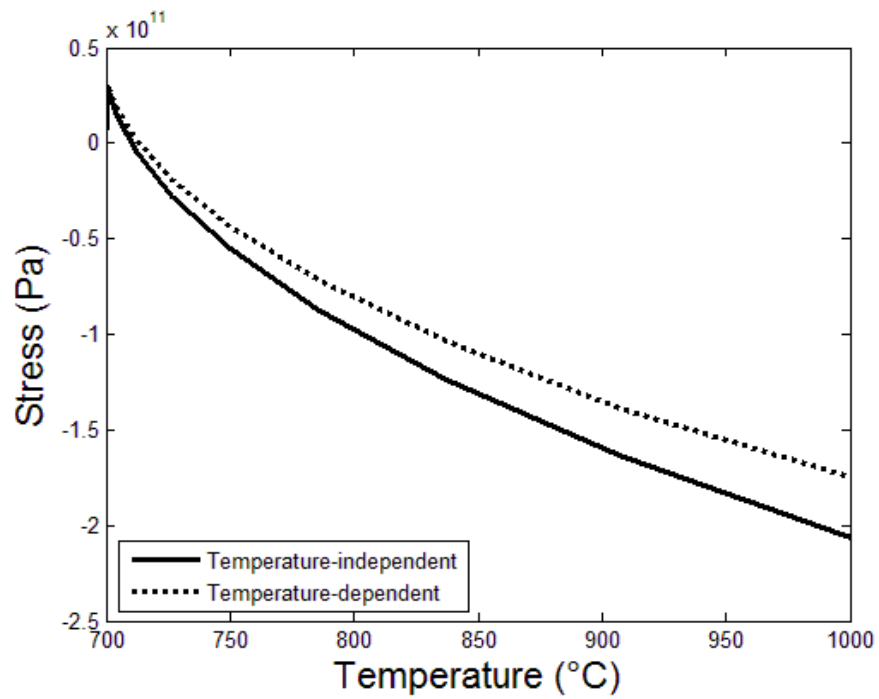


Fig 10. Variation of stress with respect to temperature

Fig. 5 shows the variation of temperature along the thickness of the beam. Here we can observe that the temperature doesn't show significant change in the ceramic dense region, which is expected as the conductivity of ceramic is low. Fig. 6 shows the variation of Young's Modulus over the thickness of the beam. The Young's modulus is in the increasing trend layer wise for both temperature-dependent and temperature-independent material properties.

Figs. (7-10) show a comparative variation of stresses and strains developed in CNT based FGM Timoshenko beam for temperature dependent and independent material properties and Fig.5 shows the temperature variation along the thickness of the beam. It is observed that the stresses developed in the beam for temperature dependent material properties are marginally higher than that for the temperature independent material properties, considering that the stresses are compressive. The variation of stress along the thickness direction is shown in Fig. 7 and according to temperature profile/distribution is shown in Fig. 10. Also, strain variation along the thickness direction and temperature distribution are shown in Figs. 8 and 9 respectively. It is observed that the stresses and strains keep on increasing as the volume fraction of CNTs increase. Similarly the variation of stress and strain with respect to temperature also shows increasing trend as temperature increases but the shapes of the plots are different from what was obtained for thickness. The strain values vary almost linearly with volume fraction. The stresses and strains obtained for various power law indices show that stresses go on increasing layer wise for increasing volume fraction whereas there is no remarkable change in strain values.

Chapter 6

Conclusions

and

Scope for Future

Work

CONCLUSIONS AND SCOPE FOR FUTURE WORK

6.1. Conclusions

The analysis for thermo-mechanical stresses developed in a CNT-FG beam has been presented in this paper. Firstly the numerical analysis was conducted only using temperature independent properties of the materials for power law indices of 0.5 and 1.5. This analysis was repeated for temperature dependent material properties and both the analyses were compared to find the actual effects of temperature when it comes to application of an FGM in a high temperature scenario.

- Material properties get affected significantly when subjected to high temperatures.
- Temperature variation in ceramics is quite insignificant.
- Stress and strain developed in the beam is compressive in nature and keeps on increasing over the thickness of the beam.
- As CNT volume fraction increases the stresses developed in the beam also increase.

Thus, higher volume content of CNT in the FGM would yield better thermo-mechanical behavior, as higher content of ceramic would lead to more static temperature zones and high residual stresses. The temperature distribution and stress flow are smoother for CNT dense region than that for ceramic dense region. Hence a power law index of 0.5 is more desirable than that of 1.5.

6.2. Scope for Future Work

- Analyses could be carried out by implementing finite element method and applying mechanical loads to the beam.
- Stresses due to dynamic loading on a FG-CNT Beam could be analyzed.
- Analyses could be done to determine the optimal value of the power law index that gives the perfect material grading that would develop low residual stresses and will give smoother temperature gradation.
- Various types of ceramics could be tested with CNTs to obtain a desirable combination of materials.
- Studies could be carried out considering CNT-epoxy as the matrix material and its impact on the overall properties and variations in stresses.

REFERENCES

- [1] A. Chakraborty, S. Gopalakrishnan, J.N. Reddy, "A new beam finite element for the analysis of functionally graded materials", *International Journal of Mechanical Sciences* 45 (2003) 519–539.
- [2] GH. Rahimi & AR. Davoodinik, "Thermal behavior analysis of functionally graded Timoshenko's Beam", *IUST International Journal of Engineering Science*, Vol. 19, No.5-1, 2008, Page 105-113.
- [3] B.V. Sankar, "An elasticity solution for functionally graded beams", *Composites Science and Technology* 61 (2001) 689–696.
- [4] T. Prakash, M.K. Singha, M. Ganapathi, "Thermal post buckling analysis of FGM skew plates", *Engineering Structures* 30 (2008) 22–32.
- [5] Mohammad Azadi, "A finite element method (FEM) free and forced lateral vibration analysis of beams made of functionally graded materials (FGMs)", *Journal of Mechanical Science and Technology* 25 (1) (2011) 69~80.
- [6] A. Ghorbanpour Arani, S. Amir and V. Sadooghi, M.Mohammadimehr, "Thermal Stress Analysis of a Composite Cylinder Reinforced with FG SWCNTs", *Journal of Solid Mechanics* Vol. 3, No. 2 (2011) pp. 132-141.
- [7] K. Nirmala, P. C. Upadhyay, "Thermo-elastic Stresses in Composite Beam with Functionally Graded Layer", *Journal of REINFORCED PLASTICS AND COMPOSITES*, Vol. 24, No. 18/2005.
- [8] M.H. Yas, M. Heshmati, "Dynamic analysis of functionally graded nano-composite beams reinforced by randomly oriented carbon nanotube under the action of moving load", *Applied Mathematical Modelling* 36 (2012) 1371–1394.
- [9] A. Shooshtari, M. Rafiee, "Vibration characteristics of nano-composite plates under thermal conditions including nonlinear effects", *International Journal of Applied Research in Mechanical Engineering*, Volume-1, Issue-1, 2011.
- [10] S. Q. Zhu, X. Wang, "Effect of environmental temperatures on elastic properties of single-walled carbon nanotube", *Journal of Thermal Stresses*, 30: 1195–1210, 2007.

- [11] K.M. Liew, J.W. Yan, Y.Z. Sun, L.H. He, "Investigation of temperature effect on the mechanical properties of single-walled carbon nanotubes", *Composite Structures* 93 (2011) 2208–2212
- [12] Y.C. Zhang, X. Chen, X. Wang, "Effects of temperature on mechanical properties of multi-walled carbon nanotubes", *Composites Science and Technology* 68 (2008) 572–581
- [13] J. N. Reddy & C. D. Chin, "Thermo-mechanical analysis of functionally graded cylinders and plates", *Journal of Thermal Stresses*, 21:6, 593-626
- [14] K.Y. Dai, G.R. Liu, X. Han, K.M. Lim, "Thermo-mechanical analysis of functionally graded material (FGM) plates using element-free Galerkin method", *Computers and Structures* 83 (2005) 1487–1502
- [15] J Woo, S A Meguid, Toronto, Ontario and K. M. Liew, "Thermo mechanical post buckling analysis of functionally graded plates and shallow cylindrical shells", *Acta Mechanica* 165, 99–115 (2003) DOI 10.1007/s00707-003-0035-4
- [16] Y. Kiani and M.R. Eslami, "Thermo-mechanical Buckling of Temperature dependent FGM Beams", *Latin American Journal of Solids and Structures* 10(2013) 223 – 246
- [17] A. Fallah, M.M. Aghdam, "Thermo-mechanical buckling and nonlinear free vibration analysis of functionally graded beams on nonlinear elastic foundation", *Composites: Part B* 43 (2012) 1523–1530
- [18] J.D. Fidelus, E. Wiesel, F.H. Gojn , K. Schulte, H.D. Wagner, "Thermo-mechanical properties of randomly oriented carbon/epoxy nano-composites", *Composites: Part A* 36 (2005) 1555–1561
- [19] M. H. Yas and M. Heshmati, "Dynamic analysis of functionally graded nano composite beams reinforced by randomly oriented carbon nanotube under the action of moving load", *Applied Mathematical Modelling*, vol. 36, 2012, pp. 1371–1394.
- [20] Simon Caraballo, "Thermo-Mechanical Beam Element for Analyzing Stresses in Functionally Graded Materials", *Graduate School Theses and Dissertations*, <http://scholarcommons.usf.edu/etd/3024>.
- [21] A.L. Kalamkarov a, A.V. Georgiades, S.K. Rokkam, V.P. Veedu, M.N. Ghasemi-Nejhad, "Analytical and numerical techniques to predict carbon nanotubes properties", *International Journal of Solids and Structures* 43 (2006) 6832–6854.

- [22] Hui-Shen Shen, Chen-Li Zhang, “Thermal buckling and postbuckling behavior of functionally graded carbon nanotube-reinforced composite plates”, *Materials and Design* 31 (2010) 3403–3411.