COMPARATIVE STUDY BETWEEN VARIOUS REDUCED MODEL AND MODAL ANALYSIS OF VISCOELASTIC ROTORS

A thesis submitted to National Institute of Technology, Rourkela in partial fulfillment for the degree of

Master of Technology
in
Mechanical Engineering

by

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CERTIFICATE

This is to certify that the thesis entitled, "**Comparative study between various reduced model and modal analysis of viscoelastic rotors**", which is submitted by **Mr. Yogesh Verma** in partial fulfillment of the requirement for the award of degree of M.Tech in Mechanical Engineering to **National Institute of Technology, Rourkela** is a record of candidate's own work carried out by him under my supervision. The matter embodied in this thesis is original and has not been used for the award of any other degree.

Date: 2/06/2014

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Yogesh Verma

ABSTRACT

The present work includes the study of dynamic characteristics on a flexible rotor shaft system. This arises due to the internal material damping of rotor bearing system, which produces a tangential force on a rotor and increasing with the rotor spin speed. Due to these dynamic characteristics of rotor is influence which destabilizes the rotor shaft system. Under this dynamic behaviour of rotor shaft system is studied to get the dynamic nature of rotor shaft system. This can be estimated in terms of Campbell diagram, modal damping factor, mode shape and directional frequency response function. These plots are obtained by using the matlab software by solving a eigenvalues problem.

finite element approach plays a significant role in modelling continuum system after discretizing it into some finite number of element. More number of elements or enhancing the mesh size give better accuracy in results. But discretizing the system into infinite elements inherits the swelling size of system matrices. Substantial increment of the system matrices sometime causes very high mathematical complications and takes an unwanted computational time. Model reduction is techniques for reducing the degree of freedom from the full system model to produce a reduced model but its dynamic characteristics is maintain. System equivalent reduction process, Improved reduced system, Guyan reduction are used to reduce the large system of equation of motion to fewer degree of freedom. The full system model also includes internal and external damping and gyroscopic effect. Since it is not practical to measure all degree of freedom, so the model is reduced using model reduction techniques. The reduced model is used to plot Campbell diagram, unbalance response using matlab software and comparison is done with original system to show its effectiveness.

NOMENCLATURE

$[M_T]$	Translational mass matrix
$[M_R]$	Rotary inertia matrix
[G]	Gyroscopic matrix
$[K_B]$	Bending stiffness matrix
$[K_C]$	Skew symmetric circulatory matrix
σ_x	Bending stress
ε_x	Strain in x-direction
$\dot{\varepsilon}$	Strain rate
R	Position vector of displaced centre rotation
M_{yy}	Bending moment in y-direction
M_{zz}	Bending moment in z-direction
M	Global Mass matrix
C	Global Damping matrix
K	Global Stiffness matrix
F	Force vector
Q	Nodal displacement vector
X	State vector
u and v	Displaced position of cross-section
E	Young modulus
q_n, x_n	Full degree of freedom
T	Transformation matrix
q_1, q_2	Co-ordinate of full and master degree of freedom
K_1, K_2	Stiffness matrices at state1 and state2.
M_n, K_n, C_n	Mass, stiffness and damping matrix with full degree of freedom
m ,s	Master and slave degrees of freedom
mm	Master- master degree of freedom
ms	Master-slave degree of freedom
sm	slave-master degree of freedom
ss	slave- slave degree of freedom
x_m	Co-ordinate of master degree of freedom
x_s	Co-ordinate of slave degree of freedom
T_s	Transformation matrix of Guyan reduction
T_i	Transformation matrix of IRS
T_u	Transformation matrix of Serep
X_m	Selected master degree of freedom in Eigen vector
X_s	Selected slave degree of freedom in Eigen vector

P	Modal participation vector
A	Mode of interest
f_y, f_z	Forces in y and z direction
λ	Eigen values
u ,v	Right hand and left hand Eigen vector
ψ , l	Right hand and left hand Eigen vector
δ_{ir}	Kronecker delta
η_r	Modal force vector
η_v	Coefficient of viscous damping
Ω	Rotor spin speed
M_r, K_r, C_r	Mass, stiffness and damping matrix

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Chapter 1

INTRODUCTION

1.1 Background and importance

Rotor dynamics is a branch of system dynamics deals with mechanical devices in which at least one part is defined as rotor, which rotates with some angular momentum. Following the ISO definition a rotor is a body suspended through a set of cylindrical hinges or bearings that allows it to rotate freely about an axis fixed in space. It deals with behaviour of high speed rotary machines which extending from very large systems like steam power plant rotors for example, a turbo generator, to very minor systems like enigma machine, with a variety of rotors is used in centrifuge, steam turbine, motors etc. Genta [1]. The principal component of rotor-dynamic system is the shaft or rotor with bearing and disk. The shaft or rotor is the rotating component of the system rotating at higher speed in order in order to maximize the power output. Rotors with bearing support to restrain their spin axis in one or other rigid way to a fixed position in space which are usually known as fixed rotor, whereas the rotor which is not constrain is defined as free rotor. The parts of machine which do not rotate are usually defined as stator. Rotors are the main causes of vibration in most of the rotating machineries. At higher speeds of rotor, vibrations caused by the mass unbalance results in some severe problems Rao [2]. So it is necessary to decrease or to minimize these vibrations for operational protection and stability. It can be ended by suitable assessment of dynamics of system. The dynamic behaviour of a mechanical system must be examined in its design phase so that you can determine whether it will present a satisfactory

Performance or not in its condition of the planned operation. The natural frequencies, damping factors and vibration modes of these systems can be determined analytically, numerically or experimentally.

The simplest model of rotor is used to study the dynamics behaviour of rotor consists of a point mass connected to a massless shaft. It dynamics behaviour was carefully studied by Jeffcott [3], it is often referred to as Jeffcott rotor.

Unlike the viscoelastic structures (which do not spin) viscoelastic rotors are acted upon by rotating damping force generated by the internal material damping, which tends to disrupt the rotor shaft bearing system by generating a tangential force which is proportional to the rotor spin speed. Thus a reliable model is required which consider constitutive relationship of a rotor material by taking into account the internal material damping for understanding a dynamic behaviour of viscoelastic rotor. Gunter's [4, 5] work on internal material damping. It's work give the idea of destabilize of rotor due to internal material damping (viscous) as the rotor spin speed increases. Dimentberg [6] work included both viscous and hysteretic internal damping, hysteretic internal damping destabilize the rotor at all speed.

Modal analysis is based on the severe mathematical treatment to convert multidegree of freedom into single degree of freedom. Therefore understanding of modal behaviour of rotor modal analysis is necessary by which we came to know about the importance of directivity and mode shapes with the help of modal matrices like modal mass, modal stiffness and modal damping.

Due to the higher demand of improving the performance of high speed rotating machinery the influence of rotor dynamics is increased. Increased power output through the use of high speed, more flexible rotor has been increased the need, at all stage of design. The acceptable performance of a turbo machine depends on the adequate design and operation of the rotor supporting a bearing. Turbo machine also include other mechanical element which provide stiffness and damping characteristic affect the dynamics of the rotor and shaft system.

The rotor dynamics of turbo machine comprises of structural analysis of rotor (shaft and disks) and design of bearing that determine the best dynamic performance under given operating condition. Rotor dynamic instabilities have become common as the speed and power of high speed turbo machinery is increased. Sometime, these instabilities resulting in increasing vibration amplitude. So it is essential to minimize the vibration for the operational protection and stability. The purpose of modal analysis is to get an idea about the dynamic behaviour of the system. By the use of modal analysis we find out critical and stability limit speed from this we can limit this vibration or minimize it.

Model reduction is necessary for the higher order system having large degree of freedom by this techniques we reduce the original system in to a reduce model by different techniques, with a reduce model we deal with a fewer degree of freedom as compared to the original system by preserving all its dynamic characteristics in a reduced model.

1.2 Various terminology related to this study

1.2.1 Material damping

Existence of damping in the linear system makes it viscoelastic. Viscoelasticity is a property of material that combines both elasticity and viscosity. These materials store the energy as well as dissipate it under dynamic deformation. Thus, the stress in such materials is not in phase with the strain. Due to these properties, it is extensively used in various high speed machinery applications for controlling the amplitude of resonance vibration in the system and adjusting wave attenuation and increasing mechanical life through reduction in mechanical fatigue Dutt and Nakra [19].

Some characteristics of viscoelastic materials are:

[1] Creep.

[2] Relaxation.

[3] The actual stiffness depends on the amount of application of loading.

[4] If repeated load is applied, hysteresis occurs.

There are two type of internal damping hysteretic and viscous form of internal damping, only viscous internal damping is considered and used to drive the equation of motion. This type of damping produces a tangential force on rotor and destabilizes it as the rotor spin speed increases. Under this condition dynamic performance of rotor system is studied to get the dynamics characteristics.

1.2.2 Internal Damping:

Damping is due to the rotating and non-rotating parts of the structure. Damping associated with non-rotating parts, called as external damping, has stabilizing effect on the system. And damping associated with rotating part result in instability in supercritical range. Due to rotation of rotors rotary damping arise, which increase as the spin speed is increased and act tangential to the rotor orbit. Due to this instability occur in the system. Therefore, a reliable model is required to represent the rotor internal material damping for exact estimate of stability limit of spin speed (SLS) of a rotor shaft system.

1.2.3 Modal Damping Factor:

By the modal analysis we find out the modal damping factor. It is the plot between Modal damping factor and rotor spin speed. It has the incremental value for backward whirl with respect to spin speed and has decrementing nature for forward whirl and after certain spin speed becomes negative. Positive modal damping describe stability of the system as the vibrational energy is dissipates and negative modal damping indicates the instability as rotating energy support rotor spin on addition of energy.

1.2.4 Resonant frequency:

Resonance is simply the natural frequency of a component. All the structure has a resonance frequency. Resonance problem occur in two primary ways. Critical speed occurs when a component rotates at its own natural frequency. Structural resonance occurs when some forcing frequency comes close to the resonant frequency of a structure. Structural vibration problem, it’s necessary to identify the resonant frequencies of a structure. Nowadays, modal analysis has become a common means of finding out modes of vibration of machines and structures.

1.2.5 Frequency response function (FRF):

It is a region in frequency domain where negative region is same as the positive region so the positive region is consider for the physical significance when we plot FRF.

1.2.6 Directional frequency response function (dFRF):

The traditional modal analysis for stationary structure is also applied for rotating machines. But this analysis requires a theoretical concept. Due to the rotation, gyroscopic effect appears in the system which result in non- symmetric matrices in equation of motion and, as a affect the frequency response function does not obey the Maxwell’s reciprocity theorem. In any FRF plot, the negative frequency region is identical of the positive frequency region. Therefore, it is necessary to deal with only one region of FRF merely a positive one which has some physical meaning. Thus the directivity of modes, backward and forward mode is not distinguishable in frequency domain. Therefore, a new complex modal testing is suggested by Lee [26], for modal parameter identification of rotary machines because by use of traditional method forward and backward mode is not identified. The new method uses the complex variable as an input and output source. But by use of complex modal testing separates forward and backward mode in frequency domain by plotting a directional frequency response function (dFRF) and no other testing is required.

1.3 Modal Analysis:

Modal analysis is the study of the dynamic characteristics of structure under vibration excitation. Modal analysis of rotor-shaft systems has become very popular know a days because this shows both spatial and temporal behaviours of the system in dynamic condition. Modal analysis include rotating forces in the model to achieve a more accurate modal behaviour of rotor shaft system and also used to see the influence of damping forces on the mode shape and frequency response function. Modal analysis involve both theoretical and experimental approaches He and Fu [20].

1.3.1 Modal Analysis approaches:

The modes of a system can be obtained from two very different approaches.

Theoretical modal Analysis

Theoretical Modal analysis is also known as mathematical models means “discretize” a structure by breaking it up into different parts. This process can be done by using the finite element (FE) approach. The analytical program then solves for an eigenvalue problem to get the frequency, mode shape of each mode. The solutions of the eigenvalue problems provide the modal data for the system.

Experimental modal Analysis:

This type of modal analysis, extracts the modes of vibration directly from FRF’s without having to make any assumption about the mass and stiffness distribution and without solving any eigenvalue problem. The stability and the response level of machines like aero-planes and steam turbine, cars are predicted by analytical model, must be validated experimentally.

This type of modal analysis comprises three steps: Test planning, Frequency response measurement and Modal parameter identification.

1. The first step involves the selection of structural support, type of excitation force and location of excitation force, data acquisition system to measure force. Structure is break up into different part. Accelerometer is connected at the selected nodes.

2. The impact force is applied at that location where we want to obtain FRF matrix using the exciter and corresponding response are noted using the data acquisition system. After that FRF matrix is analyze to identify modal parameters of the tested structure.

1.3.2 Literature survey on Modal Analysis:

Rotor shaft system is subjected to circulatory force and rotating force originates from several sources. Damping force of the shaft material, riveted joint in built up rotor, force generated by shrink fitted rotor assemblies and fluid film forces are crucial to be considering while dealing with rotor shaft bearing system. Tondl [21], as all these forces are tend to destabilizing the rotor shaft system above the spin speed limit. Therefore, the damping has been analysed by examine two types of model viz. viscous damping model and hysteretic damping model. Genta [1], has been reported modelling techniques for hysteretic form of material damping. Dutt [18] has reported equation of motion for a rotor-shaft system by considering linear viscoelastic model to represent the shaft material damping. After that effect of both form of damping model and the hysteretic damping model has been studied by many researchers on modal frequencies and modal damping Zorzi and Nelson [22], and Ku [23] considered the combined effect of internal viscous damping, hysteretic damping and shear deformation in the analysis. And result of forward and backward whirl speed are presented and compared it with previous papers. Better convergence of the result and high accuracy of the finite element model is presented with numerical example.

Modal analysis is based upon severe mathematical treatment, the significance of the mode shapes and directivity of the modes. There are two methods for modal testing classical and complex modal testing. The classical method is widely used for modal parameter identification of structure of all kinds, except rotary machines, limited attempt [24, 25] have been prepared to develop the modal testing method for rotary machines. The complex method is proposed for modal parameter identification of rotating machines. This method uses the complex notation which is a dominant mathematical tool used in this analysis which not only allow perfect physical understanding of forward and backward modes, but also help to separate these modes in the frequency domain. The complex method is first developed by the Lee [26].

Kim and Kessler [27], proposed complex variables to describe planar motions, which is directly relates physical motions to mathematical expression, is fully utilized in the proposed procedure for complex variable based rotor analysis. In which the equation of motion is formulated for the free vibration solution of equation of motion is defined the directional natural modes because it not only describe the frequency and the shapes but also direction of free vibrations response. For the forced vibration solution directional frequency response function is obtained which clearly allows the understanding of unique characteristics of rotor vibration.

Mesquita [28], in any frequency response function plot the negative frequency region of FRF is same as the positive frequency region so it is necessary to treat only one region merely a positive one because it have some physical meaning. Thus the directivity of modes, backward or forward cannot be distinguished with the use of traditional modal analysis. The complex modal analysis is used to distinguish the directivity of modes in frequency domain.

Chouksey [29], has reported the equation of motion with material damping in shaft is consider because both stationary and rotary damping forces in shaft system play an important roles in deciding the dynamic behaviour. Therefore, rotating damping forces originating due to material damping in shaft is consider and aims is to obtain a more complete model and a more accurate modal analysis is obtained for rotor shaft system. The effect of damping forces on the directional frequency response has been obtained.

1.4 Model Reduction:

Model reduction is a technique to reduce a large finite element system to one with fewer degrees of freedom while maintaining its vibrant features of the system. Methods such as Guyan reduction, Improved Reduced System approach and the System Equivalent Reduction Expansion process may be used for undamped and non-rotating structure.

O'Callahan et. al [10] suggested a improved method which is known as the Improved Reduced System (IRS) method. In this method an additional term is added to the Guyan reduction transformation to take some effect of the inertia terms. But it depends on the Guyan reduced model.

O'Callahan et. al [15] suggested other model reduction techniques known as system equivalent reduction expansion process for undamped system depend on the arbitrary selection of mode of interest.

Friswell et. al[12,13], used these reduction techniques for damped and rotating structures, compare these techniques for damped and undamped structures and discusses the errors introducing by using approaches based on the undamped model.

Das and Dutt [18], uses an improved System Equivalent Reduction Expansion Process (SEREP) are used to reduce huge linear system of equation of motion. In this equation of motion gyroscopic effect, internal material damping is included. This techniques is applied on the state space model. And nice plot of Campbell diagram, and unbalance response is plotted for reduced and original system show effectiveness of the reduced model.

1.5 Objective of the thesis:

Based on that work, the aim and scope proposed in this work are as follow:

1. The equation of motion and finite element formulation of the viscoelastic rotor are derived. Euler Bernouli beam theory is used for Finite element formulation to discritize the rotor continuum. Two element voigt model is used to incorporate the internal damping of the rotor shaft.

2. Effect of modal damping factor on rotor spin speed is analyzed. The mode shape are found using eigenvector. Further dFRF plot is obtained to explore the direction of whirl.

3. By model reduction techniques the full system model is reduced to fewer degrees of freedom. A MATLAB code is generated for the model reduction techniques and after that Campbell diagram and unbalance response is plotted and compared with the original full system model.

4. Comparison between Guyan Reduction, Improved Reduced System, and System Equivalent Reduction Expansion process is done.

1.6 Outline of the present thesis:

In present thesis chapter 2, equation of motion for damped rotor is written after discretizing it into beam finite element. Model reduction techniques are explained in detail like Guyan reduction, Improved reduced system and System equivalent reduction expansion system. A comparative study of that technique is done from Unbalance response and Campbell diagram with the help of Matlab software.

In chapter 3, Modal analysis in rotor is done to get the idea of dynamic characteristics of rotor. An example is taken to identify various modal parameters like Modal damping factor, 3-D mode shape, frequency response function and directional frequency response function.

In chapter 4, some salient conclusions and related future scopes for model reduction and modal analysis are given.

Chapter 2

A reduced model for rotor shaft system

In this chapter the mathematical modelling of viscoelastic rotor shaft is presented, where external damping from viscoelastic support is considered. The system matrix like mass matrix, stiffness matrix, gyroscopic matrix are obtain through finite element formulation. Euler Bernouli beam theory is used in this purpose. The finite element model is further used to develop reduced model. The transformation matrix for various reduced model is derived. Finally an example is taken for comparison of numerical results for different reduced model.

2.1 Finite element formulation

Finite element approach is used to model the rotor shaft system. First equation of motion is derived from the constitutive relation. The dynamic longitudinal stress and strain induced in the infinite area are σ_x and ε_x respectively. The formulation of σ_x and ε_x at an instant of time are given by Zorzi and Nelson [22].

$$\sigma_x = E\left(\varepsilon + \eta_v \dot{\varepsilon}\right); \quad \varepsilon_x = -r\cos\left[\left(\Omega - \omega\right)t\right]\frac{\partial^2 R(x,t)}{\partial x^2} \tag{2.1}$$

Figure 2.1 display the moved position of the shaft cross section (u and v)describe the displacement of the shaft centre along Y and Z direction and an element is consider of differential radial thickness of dr at a distance r (where r varies from 0 to r_0) and subtend an angle of $d(\Omega t)$ where Ω is the spin speed in rad/sec and Ωt lies from 0 to 2π at any instant of time $'t'$. Due to transverse vibration of the shaft is under two types of rotation at the same time, i.e., spin and whirl. ω is the whirl speed. O is the shaft centre when it is at rest.

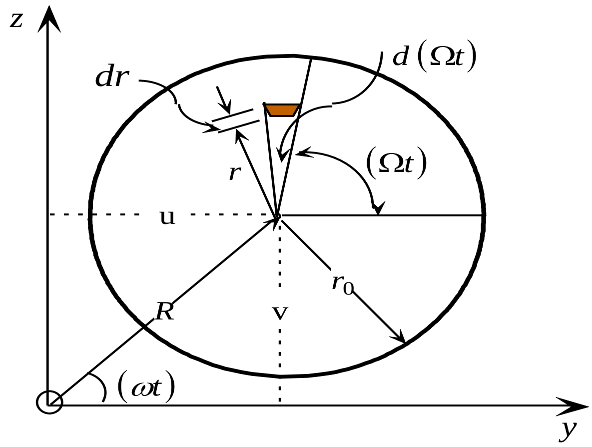


Figure 2.1 Displaced position of the shaft cross- section

Following Zorzi and Nelson [22] the bending moments at any instant about the *y* and *z*-axes are given as:

$$M_{zz} = \int_0^{2\pi}\int_0^{r_0} -\left(\mathrm{u} + r\cos(\Omega t)\right)\sigma_x r dr d(\Omega t)$$

$$M_{yy} = \int_0^{2\pi}\int_0^{r_0} \left(\mathrm{v} + r\sin(\Omega t)\right)\sigma_x r dr d(\Omega t) \qquad (2.2)$$

Put equation 2.1 into equation 2.2 and following zorzi and Nelson[22], the governing differential equation for one shaft element is given as:

$$\left([M_T]+[M_R]\right)\{\ddot{q}\}+\left(\eta_v[K_B]-\Omega[G]\right)\{\dot{q}\} + \left([K_B]+\eta_v\Omega[K_c]\right)\{q\}=\{f\} \qquad (2.3)$$

In the preceding equation $[M_T]_{(8\times8)}$, $[M_R]_{(8\times8)}$, $[G]_{(8\times8)}$, $[K_B]_{(8\times8)}$ and $[K_C]_{(8\times8)}$ are the translational mass matrix, rotary inertia matrix, gyroscopic matrix, bending stiffness matrix and skew symmetric circulatory matrix, respectively. The expressions for these matrices are given below. The full matrices are given in the Appendix.

where subscripts in the element describe the respective planes.

The equation of motion for full system is obtained by assembled the elements matrix into global matrix and it is written in the form as:

$$[M]\{\ddot{q}\}+[C]\{\dot{q}\}+[K]\{q\}=\{f\} \qquad (2.4)$$

Where $[M],[C]$ and $[K]$ are the global mass, damping and stiffness matrices, respectively and $\{f\}$ is the external force applied. Their expressions are written as:

$$[M]=[M_T]+[M_R];\ [C]=\eta_v[K_B]-\Omega[G];\ [K]=[K_B]+\eta_v\Omega[K_c]$$

The disc mass is added to the global mass matrix at a respective node. The global damping matrix contains the gyroscopic effects of shaft and disc, and effects of rotating and non-rotating damping.

Equation (2.4) once again is added by an identity equation to get the states space equation.

$$[\mathcal{A}]\{\dot{\mathcal{X}}\}+[\mathcal{B}]\{\mathcal{X}\}=\{Q\} \tag{2.5}$$

Where,

$$[\mathcal{A}]=\begin{bmatrix} C & M \\ M & 0 \end{bmatrix},\ [\mathcal{B}]=\begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix}$$

$$\{\mathcal{X}\}=\begin{Bmatrix} \{\dot{q}\} \\ \{q\} \end{Bmatrix}, \{Q\}=\begin{Bmatrix} \{0\} \\ \{f\} \end{Bmatrix}$$

Free vibration Equation of motion (2.4) for an eigenvalue problem can be written as by assuming, $\{u\}=e^{\lambda t}\{y\}$

$$\lambda[\mathcal{A}]\{\mathcal{X}\}+[\mathcal{B}]\{\mathcal{X}\}=\{0\} \tag{2.6}$$

λ is a complex Eigen values which represent the imaginary part and the real part indicates the natural frequency.

2.2 General Reduction Procedure

Model reduction is usually used to reduce large analytical model to develop a more effective model for further analytical studies, which is done by discarding few coordinates. Generally total degree of freedom or number of coordinate is classified in two categories, i) master coordinate and ii) slave coordinate. Master coordinate is also known as active or measured or retained coordinate and slave coordinate is also known as deleted or discarded or omitted coordinate. There are few methods for selecting those coordinates.

a) Slave coordinates, whose inertia force are insignificant compared to the elastic force. Thus it should be selected where inertia is small and stiffness is high.

b) Master coordinates, where inertia is high and stiffness are small.

c) Diagonal term in the ratio of stiffness and mass matrices, $\frac{K_{jj}}{M_{jj}}$, for the j^{th} coordinates. If $\frac{K_{jj}}{M_{jj}}$ is very small then there exist major inertia effects and associated coordinate is master coordinate.

d) If the diagonal term in the ratio $\frac{K_{jj}}{M_{jj}}$ is large then the j^{th} coordinates should be selected as a slave coordinate.

e) Another method to choose master and slave is that all the translational degree of freedom is chosen as master co-ordinates and all rotational co-ordinates are chosen as slave coordinate.

M_n, K_n, C_n are the full set of degrees of freedom and written in the form after the selection of master and slave co-ordinate.

$$M_n = \begin{bmatrix} M_{mm} & M_{ms} \\ M_{sm} & M_{ss} \end{bmatrix}$$

$$K_n = \begin{bmatrix} K_{mm} & K_{ms} \\ K_{sm} & K_{ss} \end{bmatrix}$$

$$C_n = \begin{bmatrix} C_{mm} & C_{ms} \\ C_{sm} & C_{ss} \end{bmatrix}$$

In general the relationship between the full set of analytical model and the reduced set of master degree of freedom as

$$\{q_n\} = \begin{Bmatrix} x_m \\ x_s \end{Bmatrix} = [T]\{x_m\} \tag{2.7}$$

Subscript 'n' represent the full set of analytical degree of freedom, 'm' represent the master degree of freedom and’s’ denotes the slave degree of freedom. ‘*T*’ represents transformation matrix between these two set of degree of freedom, which depends on the different reduction techniques used and will discuss in the following section.

Therefore, the reduced mass, stiffness and damping matrices are obtained by pre- and post multiplying the transformation matrix 'T' to the full set of degree of freedom matrices of (M_n, K_n, C_n).

$$[M_r] = [T]^T [M_n][T]$$

$$[K_r] = [T]^T [K_n][T]$$

$$[C_r] = [T]^T [C_n][T]$$

Where the size of the reduced matrices is ($r \times r$).

2.2.1 Guyan Reduction

In Guyan reduction [7], the stiffness and mass matrices, are divided into separate quantities which relates the master and slave degrees of freedom. Assuming that no force is applied to the slave degrees of freedom and the damping is not considered, then the equation of motion becomes from equation (2.4)

$$\begin{bmatrix} M_{mm} & M_{ms} \\ M_{sm} & M_{ss} \end{bmatrix} \begin{Bmatrix} \ddot{x}_m \\ \ddot{x}_s \end{Bmatrix} + \begin{bmatrix} K_{mm} & K_{ms} \\ K_{sm} & K_{ss} \end{bmatrix} \begin{Bmatrix} x_m \\ x_s \end{Bmatrix} = \begin{Bmatrix} f \\ 0 \end{Bmatrix} \tag{2.8}$$

Neglect the inertia term

$$\begin{bmatrix} K_{mm} & K_{ms} \\ K_{sm} & K_{ss} \end{bmatrix} \begin{Bmatrix} x_m \\ x_s \end{Bmatrix} = \begin{Bmatrix} f \\ 0 \end{Bmatrix} \tag{2.9}$$

$$K_{sm} x_m + K_{ss} x_s = 0 \tag{2.10}$$

$$x_s = \frac{-[K_{sm}] x_m}{[K_{ss}]} = -[K_{ss}]^{-1} [K_{sm}] x_m$$

$$x_m = \frac{f}{[K_{mm}] - [K_{ms}][K_{ss}]^{-1}[K_{sm}]}$$

$$x_n = \begin{Bmatrix} x_m \\ x_s \end{Bmatrix} = \begin{Bmatrix} \dfrac{f}{[K_{mm}]-[K_{ms}][K_{ss}]^{-1}[K_{sm}]} \\ \dfrac{-[K_{ss}]^{-1}[K_{sm}]\times f}{[K_{mm}]-[K_{ms}][K_{ss}]^{-1}[K_{sm}]} \end{Bmatrix}$$

$$x_n = \begin{bmatrix} I \\ -[K_{ss}]^{-1}[K_{sm}] \end{bmatrix} \{x_m\} = T_s\{x_m\} \tag{2.11}$$

$[T_s]$ is the transformation matrix for Guyan reduction.

2.2.2 Improved Reduced System

O'Callahan [10] improves the Guyan reduction method by developing a new technique known as the Improved Reduced System (IRS) method. It is an extension of the Guyan reduction in which some additional effect of slave degree of freedom and inertia term is considered which causes distortion in the Guyan reduction techniques. The development is based on the circumstance that the static mechanical model containing distributed forces can be reduced.

For sinusoidal excitation, equation (2.8 is written as

$$\left[K_{ss} - \omega^2 M_{ss}\right] x_s = -\left[K_{sm} - \omega^2 M_{sm}\right] x_m \tag{2.12}$$

$$x_s = -\left[K_{ss} - \omega^2 M_{ss}\right]^{-1} \left[K_{sm} - \omega^2 M_{sm}\right] x_m$$

By using the binomial theorem

$$x_s = -K_{ss}^{-1}\left[K_{sm} + \omega^2\left(M_{ss}K_{ss}^{-1}K_{sm} - M_{sm}\right) + O\left(\omega^4\right)\right] x_m$$

Where, $O\left(\omega^4\right)$ is an error of order ω^4. The main aim is to improve the natural frequency from the reduced model is based on the Guyan reduction, to first order in ω^2.

$$\omega^2 M_R x_m = K_R x_m \quad \text{or} \quad \omega^2 x_m = M_R^{-1} K_R x_m$$

$$x_s = \left[-K_{ss}^{-1}K_{sm} + K_{ss}^{-1}\left(M_{sm} - M_{ss}K_{ss}^{-1}K_{sm}\right) M_R^{-1} K_R\right] x_m \tag{2.13}$$

$$T_i = T_s + SM_n T_s M_R^{-1} K_R \tag{2.14}$$

Where $t_s = -\left[K_{ss}\right]^{-1}\left[K_{sm}\right]$ from Guyan Reduction

$$T_s = \begin{bmatrix} [I] \\ [t_s] \end{bmatrix}$$

$$S = \begin{bmatrix} [0] & [0] \\ [0] & \left[K_{ss}^{-1}\right] \end{bmatrix}$$

M_R and K_R are reduced mass and stiffness matrix taken from Guyan reduction.

Reduced mass and stiffness matrices by Improved Reduction system are

$$\left[M_R\right] = \left[T_i\right]^T \left[M_n\right]\left[T_i\right]$$

$$\left[K_R\right] = \left[T_i\right]^T \left[K_n\right]\left[T_i\right]$$

$$\left[C_R\right] = \left[T_i\right]^T \left[C_n\right]\left[T_i\right]$$

$\left[T_i\right]$ is the transformation matrix for IRS.

2.2.3 System equivalent reduction expansion process

In Serep [15] reduction process there is a relationship between the master degree of freedom and the slave degree of freedom which can be written in general form as

$$\left\{x_n\right\} = \begin{Bmatrix} x_m \\ x_s \end{Bmatrix} = \left[T\right]\left\{x_m\right\} \tag{2.15}$$

The modal transformation can be written as:

$$\left\{x_n\right\} = \begin{Bmatrix} x_m \\ x_s \end{Bmatrix} = \begin{bmatrix} X_m \\ X_s \end{bmatrix}\left\{p\right\} \tag{2.16}$$

The modal matrix is obtained from Eigen vector and is partitioned into the 'm' active and’s’ slave or deleted set of degrees of freedom. The relationship for the active or master set of degrees of freedom is.

$$\left\{x_m\right\} = \left[X_m\right]\left\{p\right\} \tag{2.17}$$

$\left\{p\right\}$ is the modal participation vector obtained from least square solution.

The inverse specification of above equation contains a generalized inverse then the number of unknowns is not equal to the number of equations to be solved. There are two probable Solution.

1. When the number of equation 'm' is greater than or 'a' equal to the mode of interest.

2. When the number of equation 'm' is fewer than the number of solution variables 'a' means mode of interest.

Least Squares Solution – $m \geq a$ (mode of interest).

$$\left[X_m\right]^T \{x_m\} = \left[X_m\right]^T \left[X_m\right]\{P\}$$

$$\left(\left[X_m\right]^T\left[X_m\right]\right)^{-1}\left[X_m\right]^T\{x_m\} = \left(\left[X_m\right]^T\left[X_m\right]\right)^{-1}\left[X_m\right]^T\left[X_m\right]\{p\}$$

$$\{p\} = \left(\left[X_m\right]^T\left[X_m\right]\right)^{-1}\left[X_m\right]^T\{x_m\} = \left[X_m\right]^g\{x_m\} \tag{2.18}$$

$\left[X_m\right]^g =$ is also called pseudo-inverse .

Average solution- when $m < a$.

$$\{p\} = \left[X_m\right]^T\left(\left[\left[X_m\right]^T\left[X_m\right]\right]\right)^{-1}\{x_m\} = \left[X_m\right]^g\{x_m\} \tag{2.19}$$

$$\{x_n\} = \left[X_n\right]\left[X_m\right]^g\{x_m\} = \left[T_u\right]\{x_m\} \tag{2.20}$$

The Serep transformation matrix $\left[T_u\right]$ is used for the reduction of full original system. Serep is heavily relies on a "well developed" finite element model from which an 'n' dimensional Eigen solution of the problem are obtained for developing the mapping between the full set of n DOF and the reduced model of m DOF. The quality of the result obtained from most reduction techniques depend on the chosen of active degree of freedom, however it is not a concern when we use Serep techniques. In Serep techniques an arbitrary selection of active or master degree of freedom as well as an randomly selection of modes does not affect the natural frequencies which are conserved in the reduced system when using Serep technique.

2.2.3.1 Main point to be remember using System Equivalent reduction expansion process.

1. When the number of mode of interest used is less then active degree of freedom in reduced system model (m>a). The size of the reduced matrices are 'm' by 'm', then rank of the reduced system model is only 'a'. Hence, the reduced stiffness and mass matrices are rank deficient; therefore precaution must be taken for these reduced matrices like mass, stiffness and damping matrix. Due to this rank deficiency result.

2. The Serep process produce an exact solution when active degree of freedom is equal to the mode of interest means (m=a).

2.3 Numerical problem

A rotor bearing system with three disk on the shaft and discretised into 13 beam element having same length and cross-sectional radius of shaft is 0.05m. It is supported with two orthotropic bearing. The shaft and disk material is steel and unbalance mass 200gm is situated on disk2.

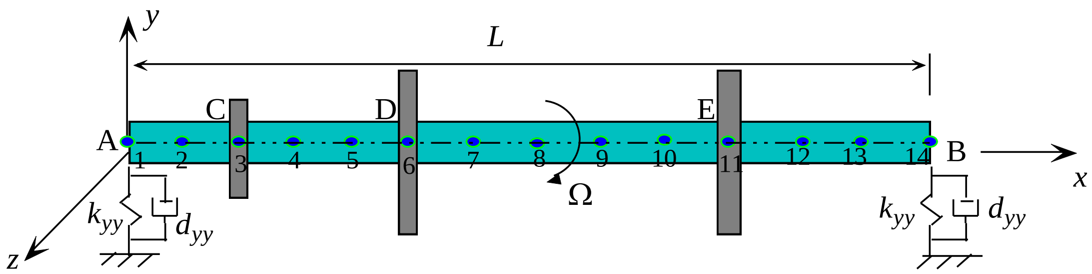


Figure 2.2: Schematic Diagram of the Rotor

Table 2.1: Material property and shaft data

Material	**Density (Kg/m^3)**	**Young Modulus (GPa)**	**Length**	**Diameter**	**Damping Coefficient (N-s/m)**
Mild steel	7800	200	1.3	0.1	0

Table 2.2: Bearing data

Bearing properties	**Stiffness**	**Damping**
Plane xx	5e7	5e2
Plane zz	7e7	7e2

Table 2.3: Disc data

Disk	**Disk 1**	**Disk 2**	**Disk 3**
Inner radius (m)	0.05	0.05	0.05
Outer radius (m)	0.12	0.2	0.2
Thickness (m)	0.05	0.05	0.06

2.3.1 Mass unbalance response

The mass unbalance of 200gm is situated at disk 2 at node 6. The response amplitude is plotted for three different reduced model viz. Guyan Reduction, System Equivalent Reduction Expansion Process, Improved Reduced System. Before comparing these techniques global matrices is divided into two parts master and slave coordinate and then applying different techniques to plot unbalance response and effectiveness of these techniques is noted by compared it with original plot.

2.3.1.1 Comparison between Full system, IRS and Guyan Reduction

The full system have 56 degree of freedom by applying reduction process system is reduced to 24 degree of freedom. And unbalance response is plotted from Guyan reduction and Improved reduced system techniques and compare it with original system with 56 degree of freedom. We noted from the figure 2.3 Guyan reduction, IRS and full system plot of unbalance response show the effectiveness of three techniques. It is seen from figure 2.3 the Guyan reduction is very close to the original system but its highest peak is not coinciding with the original system. Same nature is observed for IRS also when compared with the full system. In IRS techniques some inertia term is included to get the effect of mass.

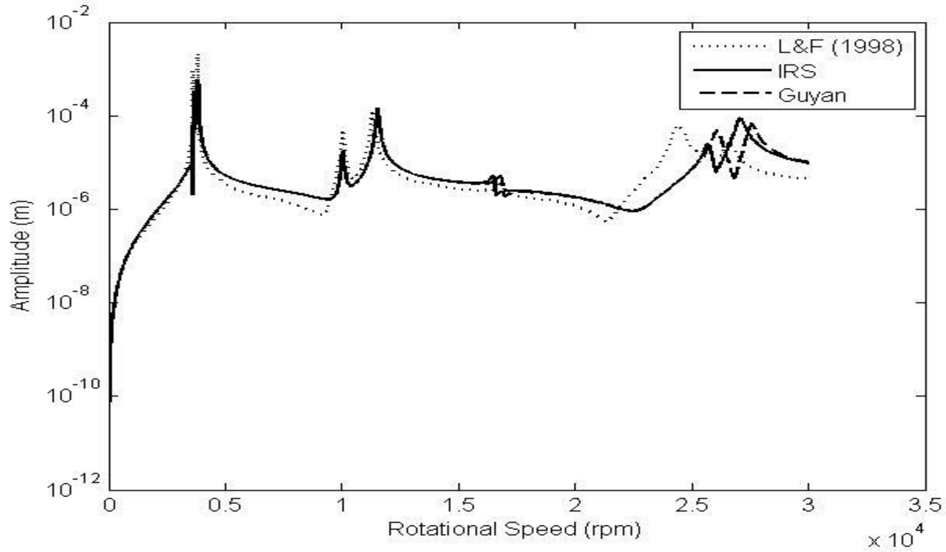


Figure 2.3: Comparison of unbalance response between full system, IRS and Guyan Reduction

2.3.1.2 Comparison between Full system, Serep and Guyan Reduction

In figure 2.4 Serep process shows effectiveness than the Guyan reduction because we see from the plot highest peak is same as the original one and after that it is same as the Guyan reduction. In Serep process we reduce the state space equation with the help of eigenvectors, and arbitrary selection of master co-ordinate and arbitrary selection of mode of interest does not affect its accuracy when compared with the original system. So, from the comparison between three techniques we see that Serep is close to the actual or original full system model

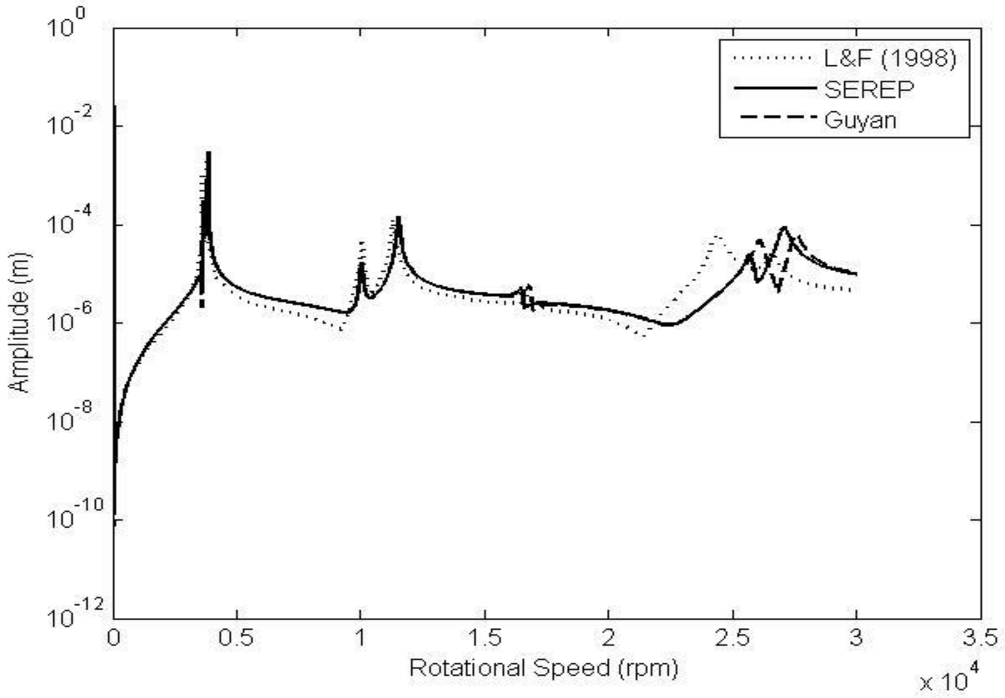


Figure 2.4: Comparison of unbalance response between full system, SEREP and Guyan Reduction

2.3.2 Campbell Diagram

Campbell diagram is plot between imaginary part of eigenvalues or whirl line (WL) vs. spin speed. The Campbell diagram is used to find out the critical speed (Ωcr). A line having inclination of 45 ° is known as synchronous whirl line (SWL). Intersection between SWL and WL indicates the critical speed. The Campbell diagram for full system drawn in figure 2.5 with 6 natural frequencies is considered. It shows the forward and backward whirl. The full system having 56 degree of freedom is reduced to 24 degree of freedom by applying different techniques like Guyan, IRS, and Serep reduction. The Campbell diagram is plotted from the reduced model and compares it with the full system. Table 2.4 shows the comparison of eigenvalues for various techniques.

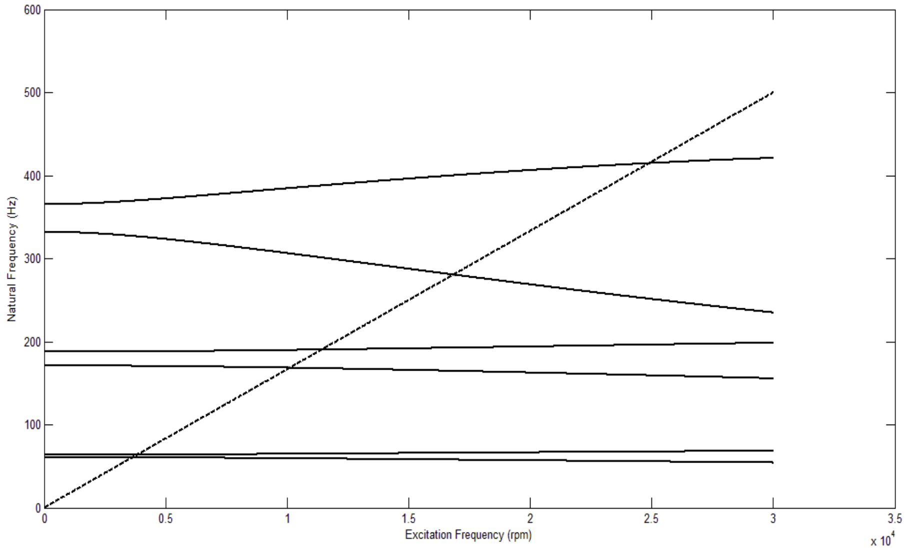
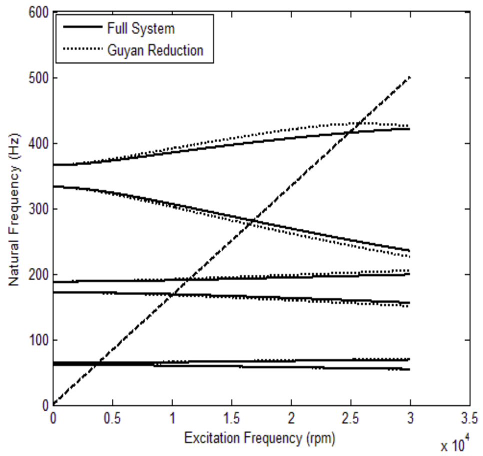


Figure 2.5: Campbell diagram with full system

Table 2.4: Comparison of eigenvalues between full systems, Guyan, IRS, Serep reduction techniques

Mode	Full system	Guyan reduction	IRS	Serep
1	55	53.9	53.7	53.8
2	68	69.6	69.5	69.5
3	157	151.6	150.3	150.3
4	197	205.1	204.5	204.5
5	238	235.4	225.9	225.9
6	415	435.8	424.8	424.7
7	456	459.6	454.5	454.5
8	616	682.4	598.0	597.8
9	738	801.6	797.0	796.7
10	1130	1196.8	1067.4	1067.2
11	1144	1223.4	1188.5	1187.8
12	1494	1602.2	1454.2	1453.9

From the figure 2.6(a) and figure 2.6(b) we see that the Campbell diagram for reduced system is close to the full system. The natural frequencies obtained from full system are close enough as compared with Guyan reduction for lower modes. IRS and Serep techniques have same values of natural frequencies for lower modes as well as for higher modes also and close to the full system model. But Guyan gives better result than IRS and SEREP in lower mode.



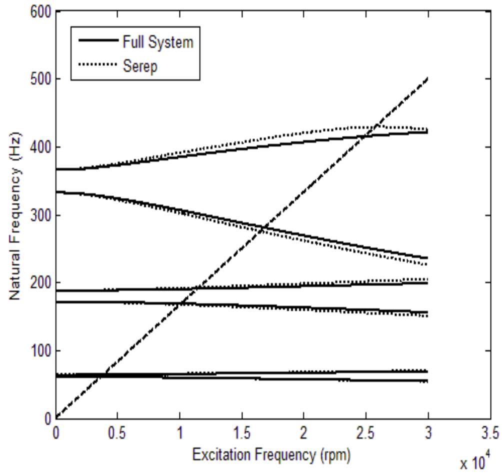


Figure 2.6(a): Campbell diagram for Full system and Guyan reduction model	Figure 2.6(b): Campbell diagram for Full system and Serep model

Chapter 3

Modal Analysis

In this chapter modal analysis of rotor is done because it is an important mathematical tool to get the idea of dynamic behavior and modal identification parameter of the system for example the mode shape, modal damping factor and Campbell diagram, frequency response function, directional frequency response function.

3.1 Modal Analysis in Rotor

The equation of motion of rotor from equation (2.4) with internal material damping is considered and once again equation is written as following Mesquita [28].

$$[M]\begin{Bmatrix}\{\ddot{y}\} \\ \{\ddot{z}\}\end{Bmatrix}+[C]\begin{Bmatrix}\{\dot{y}\} \\ \{\dot{z}\}\end{Bmatrix}+[K]\begin{Bmatrix}\{y\} \\ \{z\}\end{Bmatrix}=\begin{Bmatrix}\{f_y\} \\ \{f_z\}\end{Bmatrix} \tag{3.1}$$

The equation of motion can be written in state space form are

$$[\mathcal{A}]\{\dot{X}\}+[\mathcal{B}]\{X\}=\{Q\} \tag{3.2}$$

Where,

$$[\mathcal{A}]=\begin{bmatrix} C & M \\ M & 0 \end{bmatrix},\ [\mathcal{B}]=\begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix},\ \{X\}=\begin{Bmatrix}\{\dot{q}\} \\ \{q\}\end{Bmatrix},$$

$$\{Q\}=\begin{Bmatrix}\{0\} \\ \{f\}\end{Bmatrix},\ \{f\}=\begin{Bmatrix}\{f_y\} \\ \{f_z\}\end{Bmatrix},\{q\}=\begin{Bmatrix}\{y\} \\ \{z\}\end{Bmatrix}$$

The matrices [A] and [B] are real, non symmetrical, and indefinite in general, causing a non-self-adjoint eigenvalue problem. The eigenvalue problems related with equation (3.2) are

$$\left(\lambda[A]-[B]\right)\{\psi\}=\{0\} \text{ and } \left(\lambda[A]^T-[B]^T\right)\{l\}=\{0\} \tag{3.3}$$

Eigenvalues of the above problem are the same. The eigenvalues and eigenvector appear in complex conjugate pairs. The eigenvectors of the Eigen problems (3.2) are the vectors known as right and left eigenvectors, and given as

$$\{\psi\} = \begin{Bmatrix} \lambda\{u\} \\ \{u\} \end{Bmatrix} \text{ and } \{l\} = \begin{Bmatrix} \lambda\{v\} \\ \{v\} \end{Bmatrix} \tag{3.4}$$

The vectors {u} and {v} are the eigenvectors of the Eigen problems. The right and left eigenvectors may be biorthonormalised as

$$\begin{aligned} \{l\}^T [A]\{\psi\}_r &= \delta_{ir} \\ \{l\}_i^T [B]\{\psi\}_r &= \lambda_r \delta_{ir} \end{aligned} \tag{3.5}$$

To uncouple the equation (3.2), the following coordinate transformation is done

$$\{w\} = [\psi]\{\eta\} = \sum_{r=1}^{4N} \{\psi\}_r \eta_r \tag{3.6}$$

$$\dot{\eta}_r - \lambda_r \eta_r = \{l\}_r^T \{Q\}; \qquad \eta_r = \frac{\{l\}_r^T \{Q\}}{(j\omega - \lambda_r)} \tag{3.7}$$

Substituting the response in (3.7) into equation (3.6) leads to

$$\{w\} = \sum_{r=1}^{4N} \frac{\{\psi\}_r \{l\}_r^T}{(j\omega - \lambda_r)} \{Q\} \quad \text{or} \quad \{q\} = \sum_{r=1}^{4N} \frac{\{u\}_r \{v\}_r^T}{(j\omega - \lambda_r)} \{F\} \tag{3.8}$$

Then

$$\begin{Bmatrix} \{Y(\omega)\} \\ \{Z(\omega)\} \end{Bmatrix} = \sum_{r=1}^{4N} \frac{\{u\}_r \{v\}_r^T}{(j\omega - \lambda_r)} \begin{Bmatrix} \{F_y(\omega)\} \\ \{F_z(\omega)\} \end{Bmatrix} = \left[H(\omega)\right] \begin{Bmatrix} \{F_y(\omega)\} \\ \{F_z(\omega)\} \end{Bmatrix} \tag{3.9}$$

Thus we can define the frequency response function matrix as

$$\left[H(\omega)\right] = \sum_{r=1}^{4N} \frac{\{u\}_r \{v\}_r^T}{(j\omega - \lambda_r)} = \sum_{r=1}^{2N} \left(\frac{\{u\}_r \{v\}_r^T}{(j\omega - \lambda_r)} + \frac{\{\bar{u}\}_r \{\bar{v}\}_r^T}{(j\omega - \bar{\lambda}_r)} \right) \tag{3.10}$$

$$H_{ik}(\omega) = \sum_{r=1}^{4N} \frac{u_{ir} v_{kr}}{(j\omega - \lambda_r)} = \sum_{r=1}^{2N} \left(\frac{u_{ir} v_{kr}}{(j\omega - \lambda_r)} + \frac{\bar{u}_{ir} \bar{v}_{ir}}{(j\omega - \lambda_r)} \right)$$

$$\left[H(\omega)\right] = \begin{bmatrix} \left[H_{yy}(\omega)\right] & \left[H_{yz}(\omega)\right] \\ \left[H_{zy}(\omega)\right] & \left[H_{zz}(\omega)\right] \end{bmatrix} \tag{3.11}$$

Therefore, Directional Frequency Response Function or Complex Frequency Response Function (dFRF) by Lee [26]

$$\boldsymbol{H}_{pg} = \left(\frac{1}{2}\right)\left(\boldsymbol{H}_{yy} + \boldsymbol{H}_{zz} - i\left(\boldsymbol{H}_{yz} - \boldsymbol{H}_{zy}\right)\right)$$

$$H_{p\hat{g}} = \left(\frac{1}{2}\right)\left(H_{yy} - H_{zz} + i\left(H_{yz} + H_{zy}\right)\right)$$
$$H_{\hat{p}g} = \left(\frac{1}{2}\right)\left(H_{yy} - H_{zz} - i\left(H_{yz} + H_{zy}\right)\right)$$
$$H_{\hat{p}\hat{g}} = \left(\frac{1}{2}\right)\left(H_{yy} + H_{zz} + i\left(H_{yz} - H_{zy}\right)\right) \tag{3.12}$$

From above equation we can conclude as:

$$H_{\hat{p}g}(i\omega) = H_{p\hat{g}}(-i\omega); \quad H_{\hat{p}\hat{g}}(i\omega) = H_{pg}(i\omega)$$

3.2 Numerical Problem

A rotor shaft system with flexible supports at its ends and having one offset circular disc, as shown in figure 3.1. In all the considerations, bearing anisotropy and cross coupled stiffness is not considered.

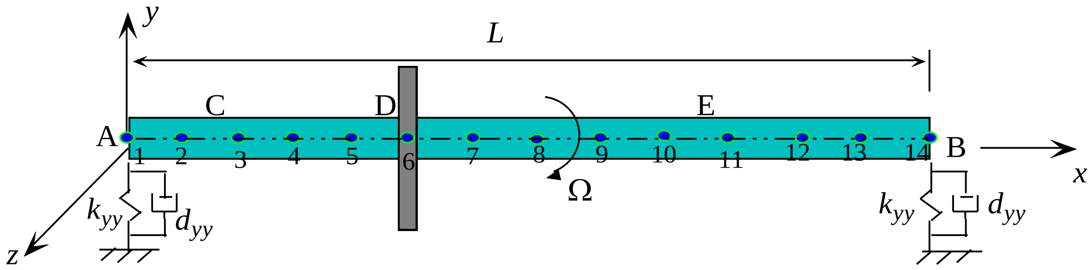


Figure 3.1 Schematic diagram of rotor

Table 3.1: Material properties and shaft parameters

Material	**Density (Kg/m^3)**	**Young Modulus (GPa)**	**Length**	**Diameter**	**Damping Coefficient (N-s/m)**
Mild steel	7800	200	1.4	0.1	0.0002

Table 3.2: Disc Parameter

Disc	**Diameter (m)**	**Thickness (m)**
1	0.40	0.05

Table 3.3: Bearing data

Bearing properties	**Stiffness**	**Damping**
Plane yy	1.75e7	7e2
Plane zz	1.75e7	7e2

$$K_{yy} = K_{zz} = 1.75e7\,N/m,\; C_{yy} = C_{zz} = 700\,N-\sec/m,\; \eta_v = 0.0002$$

3.2.1 Campbell Diagram

The Campbell diagram for damped system is plotted in figure 3.2 with four natural frequencies which give first forward and first backward for first mode and also for second mode. At first forward and first backward at 8949rpm and 9729rpm at which the system is at resonant.

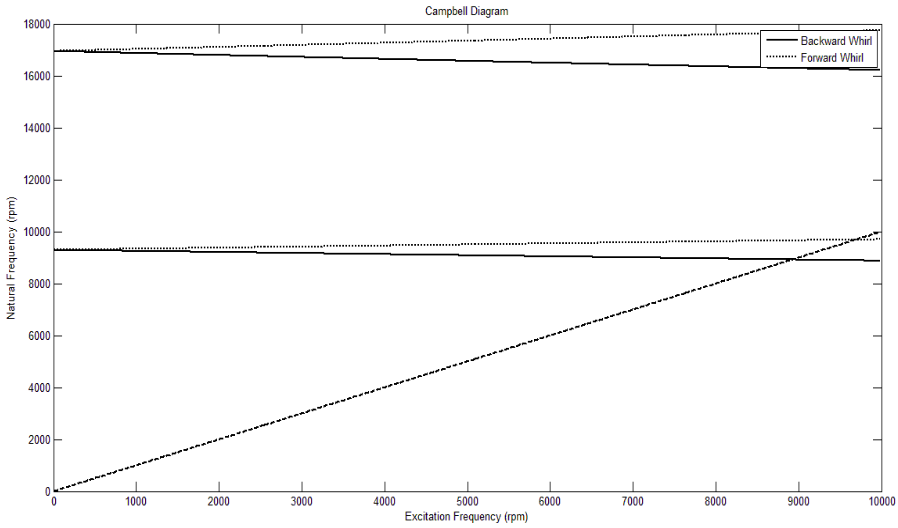


Figure 3.2: Campbell diagram

3.2.2 Modal Damping factor

The modal damping factor of two consecutive modes is plotted for undamped and damped system in figure 3.3(a) and 3.3(b) respectively. In 3.3(a) it is a straight line because no internal or external damping is considered. In figure 3.3(b), backward whirl has incremental nature with spin speed and decremental nature for forward whirl. The positive modal damping factor indicates stability and negative modal damping factor indicates instability because vibrational energy is dissipates and rotational energy support rotor whirl due to the addition of energy. Thus the system may become unstable due to forward whirl.

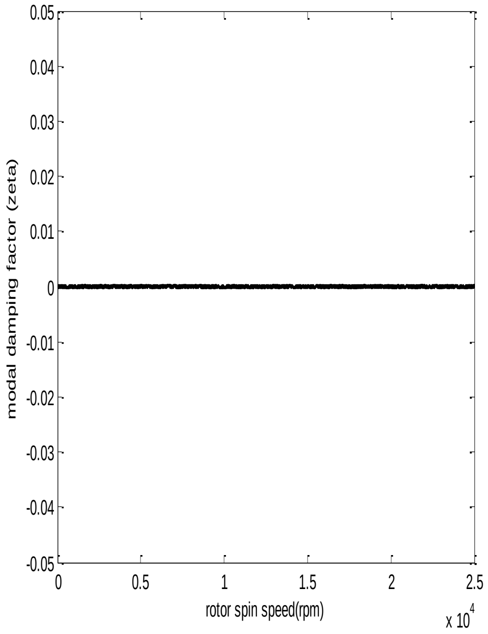


Figure 3.3(a): Modal damping factor Vs. rotor spin speed (without support damping)

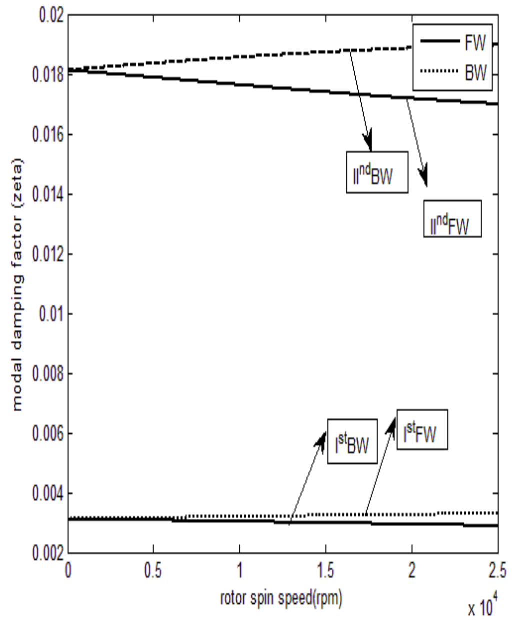


Figure 3.3(b): Modal damping factor Vs. rotor spin speed (with support damping)

3.2.3 Three dimensional mode shape for simply supported rotor system

Mode shape is plotted for simply supported rotor in which eigenvector is used to plot these modes. The two consecutive modes for forward and backward whirl are presented here. The clockwise rotation is considered as backward whirl and counter clockwise is reflected as forward whirl. The stating of the locus is marked with star and the locus is left incomplete at the end to measure the direction of whirl. The mode shape for damped rotor is unsymmetric due to the addition of skew symmetric matrix in the equation of motion.

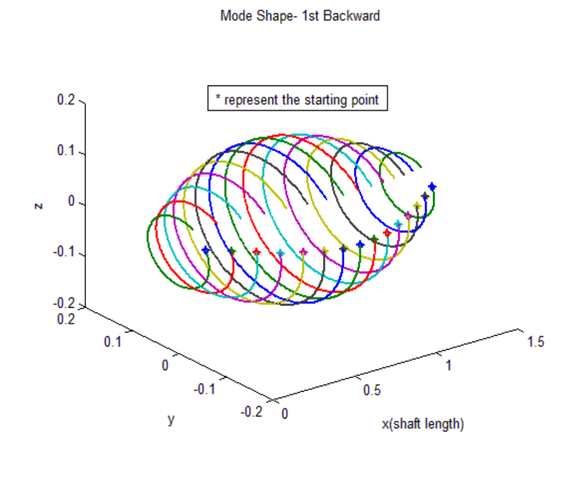


Figure 3.4(a): First Backward whirl

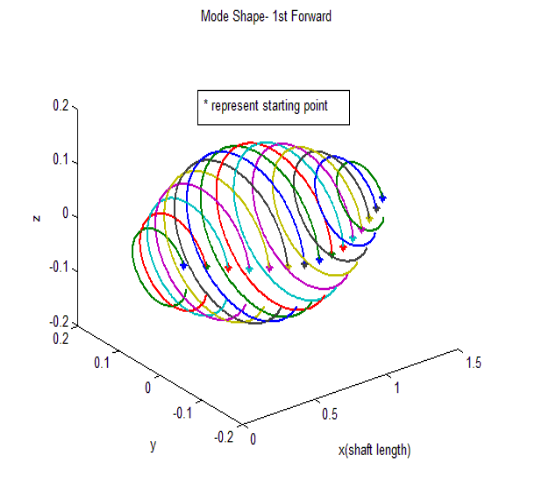


Figure 3.4(b): First Forward whirl

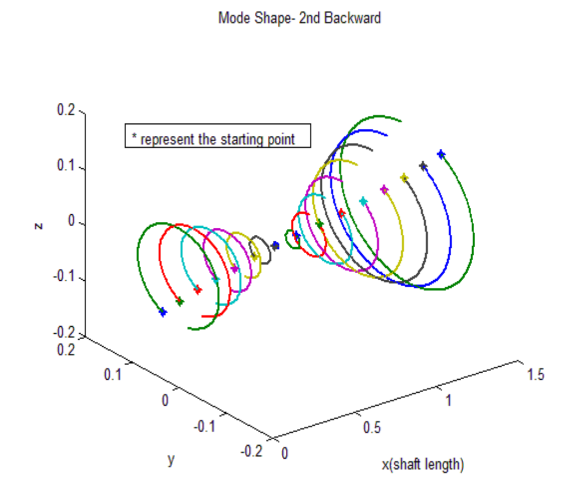


Figure 3.4(c): Second Backward whirl

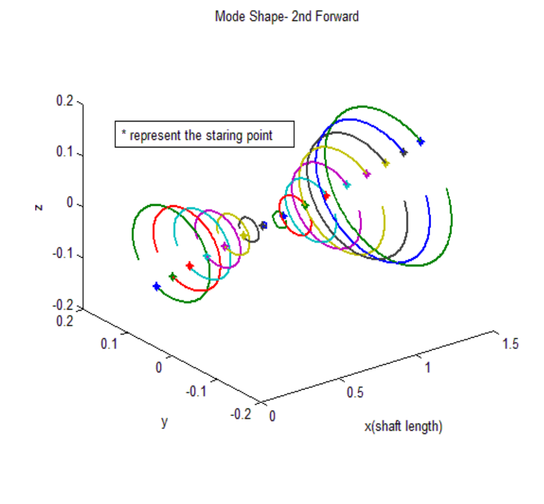


Figure 3.4(d): Second Forward whirl

3.2.4 Frequency response function (FRF) and Directional Frequency response function (dFRF).

From the figure 3.5(a) and 3.5(b), Hyy and Hyz we see that there are four modes but no information of directivity of Frequency means forward and backward modes in frequency response function (FRF). In figure 3.6, Hpg we see directly from the plot backward modes, 152Hz and 55.2Hz, appears in the negative frequency zone and forward modes, 55.6Hz and 157.6Hz, appears in the positive frequency zone. So, we clearly noted that directional frequency response function (dFRF) has the capability to separates backward and forward modes, which is mixed in FRF plot.

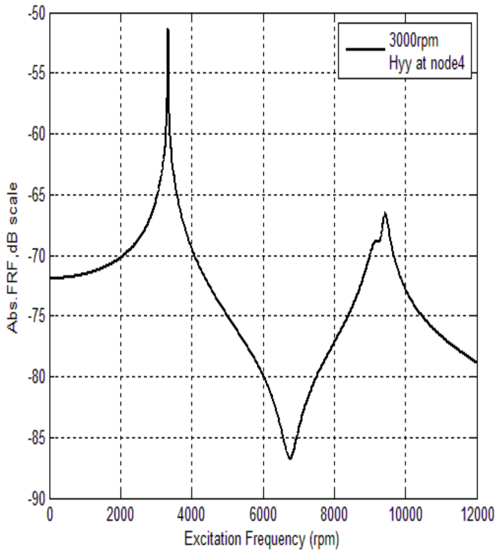


Figure3.5(a): Frequency Response Function (Hyy)

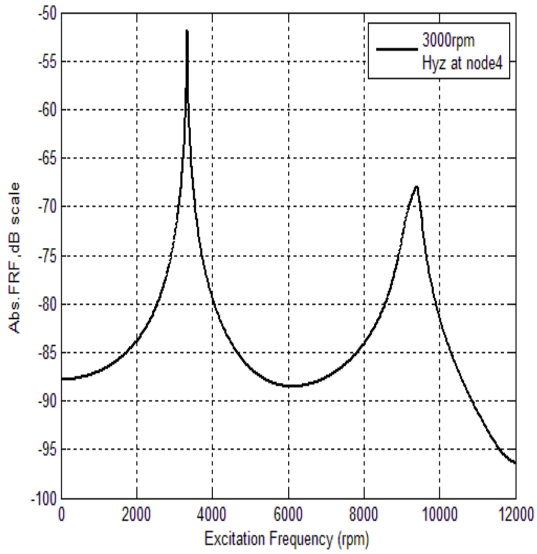


Figure3.5(b): Frequency Response Function (Hyz)

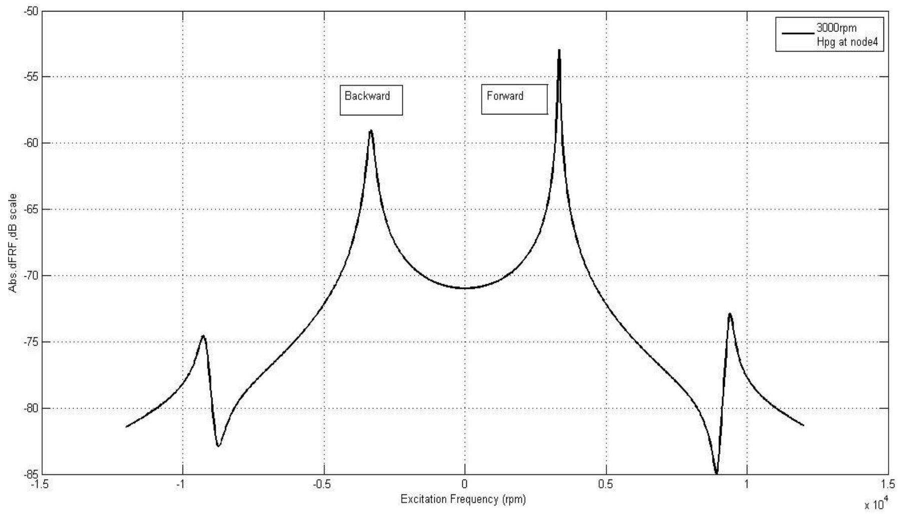


Figure 3.6: Directional Frequency Response Function (Hpg)

From the plot we see clearly that forward and backward are separated in their frequency zone in figure 3.7. The backward modes are 154.4Hz and 55.2Hz appear in the negative frequency zone and forward modes are 55.6Hz and 158.4Hz appear in the positive frequency zone at node 6 with internal material damping is considered in figure 3.8 backward modes, 144.8Hz and 55.2Hz appear in the negative frequency zone and forward modes are 56.0Hz and 162.8Hz appear in the positive frequency zone at node 4 with no internal material damping.

dFRF with internal and external damping at node 6	dFRF with support damping at node 4

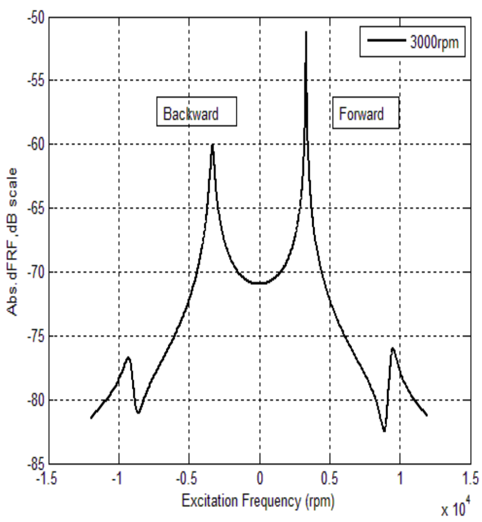
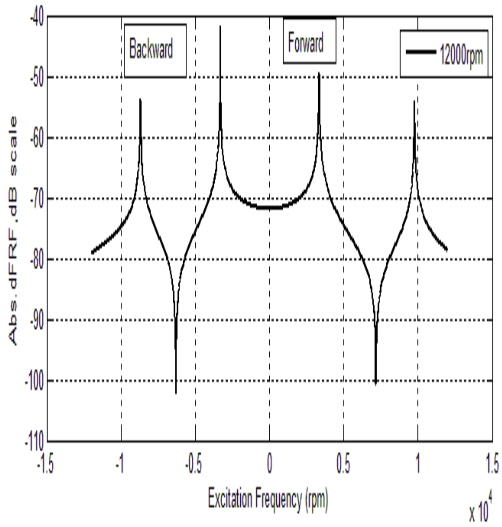

	

Figure 3.7: Directional frequency response function (Hpg) at node 6	Figure 3.8: Directional frequency response function (Hpg) at node 4

Chapter 4

Conclusions and Future scope

4.1 Conclusions

This work includes the model reduction and modal analysis of a rotor system with simply supported at its end by considering the external damping and internal material damping. For this Finite element formulation for the rotor shaft system is first obtained. From that following conclusion can be made.

1. From the reduced system the nature of unbalance response is close agreement as compared to the full system.
2. The reduction techniques like Guyan reduction produce the natural frequencies close to the natural frequencies of the full system for lower modes. At higher modes IRS and Serep produces the natural frequencies close to the natural frequencies of the full system.
3. The reduction process like Serep is effective in reproducing that natural frequencies of the full system whose mode is include in transformation matrix.
4. From dFRF we separate forward and backward modes in a frequency zone which is mixed in FRF plot.
5. From the mode shape we can easily visualized the forward and backward modes.
6. From the modal damping factor we can conclude that the positive modal damping factor indicates the stable zone and negative modal damping factor indicate the unstable zone.

4.2 Future Scope

Following are the point for the future research in this area is

1. No one model reduction techniques is fully exact to the full system model so search is continue for suitable second order model as well as for higher order model.
2. Modal analysis for higher order model of viscoelastic rotor and also the work is extended to nonlinear problem.
3. The modal analysis is also done for unsymmetric rotor for symmetric rotor it is straight forward but unsymmetric rotor required complex modal analysis of rotor shaft system.

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4.4 Appendix

$$
[M_T] = \frac{\rho A l}{420}\begin{pmatrix}
156 & 22l & 0 & 0 & 54 & -13l & 0 & 0 \\
 & 4l^2 & 0 & 0 & 13l & -3l^2 & 0 & 0 \\
 & & 156 & -22l & 0 & 0 & 54 & 13l \\
 & & & 4l^2 & 0 & 0 & -13l & -3l^2 \\
 & & & & 156 & -22l & 0 & 0 \\
\mathit{Symmetric} & & & & & 4l^2 & 0 & 0 \\
 & & & & & & 156 & 22l \\
 & & & & & & & 4l^2
\end{pmatrix}
$$

$$
[M_R] = \frac{\rho I_D}{36l}\begin{pmatrix}
36 & 3l & 0 & 0 & -3l & 3l & 0 & 0 \\
 & 4l^2 & 0 & 0 & -3l & l^2 & 0 & 0 \\
 & & 36 & -3l & 0 & 0 & -36 & -3l \\
 & & & 4l^2 & 0 & 0 & 3l & -l^2 \\
 & & & & 36 & -3l & 0 & 0 \\
 & & & & & 4l^2 & 0 & 0 \\
\mathit{Symmetric} & & & & & & 36 & 3l \\
 & & & & & & & 4l^2
\end{pmatrix}
$$

$$
[G] = \frac{2\Omega\rho I_D}{36l}\begin{pmatrix}
0 & 0 & 36 & 3l & 0 & 0 & -36 & -3l \\
 & 0 & 3l & -4l^2 & 0 & 0 & -3l & l^2 \\
 & & 0 & 0 & 36 & -3l & 0 & 0 \\
 & & & 0 & -3l & -4l^2 & 0 & 0 \\
 & & & & 0 & 0 & 36 & 3l \\
 & & & & & 0 & -3l & 4l^2 \\
\mathit{Skew}\ \text{symmetric} & & & & & & 0 & 0 \\
 & & & & & & & 0
\end{pmatrix}
$$

$$\left[K_B\right] = \frac{EI}{l^3}\begin{pmatrix} 12 & 6l & 0 & 0 & -12 & 6l & 0 & 0 \\ & 4l^2 & 0 & 0 & -6l & 2l^2 & 0 & 0 \\ & & 12 & -6l & 0 & 0 & -12 & -6l \\ & & & 4l^2 & 0 & 0 & 6l & 2l^2 \\ & & & & 12 & -6l & 0 & 0 \\ & & & & & 4l^2 & 0 & 0 \\ \textit{Symmetric} & & & & & & 12 & 6l \\ & & & & & & & 4l^2 \end{pmatrix}$$

$$\left[K_c\right] = \frac{EI}{l^3}\begin{pmatrix} 0 & 0 & 12 & -6l & 0 & 0 & -12 & -6l \\ & 0 & 6l & -4l^2 & 0 & 0 & -6l & -2l^2 \\ & & 0 & 0 & 12 & -6l & 0 & 0 \\ & & & 0 & -6l & 2l^2 & 0 & 0 \\ & & & & 0 & 0 & 12 & 6l \\ & & & & & 0 & -6l & -4l^2 \\ \textit{Skew}\ \text{symmetric} & & & & & & 0 & 0 \\ & & & & & & & 0 \end{pmatrix}$$