VIBRATION ANALYSIS OF LAMINATED COMPOSITE CURVED SHELLS

A Thesis submitted in partial fulfilment of the requirements for the degree of

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in

Civil Engineering

by

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CERTIFICATE

This is to certify that the thesis entitled **"Vibrational Analysis of Laminated Composite Curved Shells by Experimental and Numerical Methods"** submitted by **Ankur Mittal** bearing roll no. **110CE0045** in partial fulfillment for the requirements for the award of **Bachelor of Technology Degree** in **Civil Engineering** at **National Institute of Technology, Rourkela** is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter contained in this report has not been submitted or deposited to any other University/Institute for award of any Degree or Diploma.

Date: 8th May,2014

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ABSTRACT

Composite Materials and structures are becoming very popular and are used in a variety of applications in aeronautical, marine and automotive industries primarily because of its specific strength i.e strength to weight ratio and stiffness. So it becomes very important to determine the vibrational characteristics of shells. Very little research has been done related to determination of natural frequencies of shells by experimental methods so this work is primarily inclined to determination of natural frequency of shells from experimental methods and validate it by using a MATLAB program. In the present research cylindrical shells of varying thickness are cast by the use of glass fibre. The effects of radius of curvature, thickness of shells, number of layers, different support conditions, ply orientation and Aspect ratio on the natural vibration frequency of shells are studied. A finite element package, MATLAB is used to obtain the numerical results and these numerical results are checked against the experimental results obtained.

The composite shells are casted by using woven roving glass fiber by hand lay-up technique. Other materials needed for fabrication include epoxy as resin, hardener, polivinyl alcohol which is used a releasing agent. The fiber and matrix are used in the ratio of 50:50 proportion by weight. The vibration characteristics are determined using an FFT analyzer, accelerometer, modal hammer and a display unit. The frequency response spectrum is obtained on display unit when a small impact is given on shell using the modal hammer. The Frequency Response Function (FRF) is studied using Pulse Lab shop. A single composite plate is initially casted and is cut in to sizes of 25mm×250mm for tensile test using INSTRON 1195. The effective length of the specimen is kept as 150mm.

The results showed that the frequencies of vibration increases with increase in thickness and number of layers of shell. For different boundary conditions, the natural frquencies were found to be greater for 4 sides free (FFFF) condition as compared to 2 sides simply supported and 2 sides free (SFSF) condition. It was seen that natural frequencies increases with the increase in aspect ratio due to higher stiffness. It was seen that natural frequencies decreases with increase in radius of curvature of shell. It was also observed that vibration frequencies increases with the increase in ply orientation and was found to be maximum at an angle of 45° owing to bidirectional nature of glass fiber.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Laminated composite single/doubly curved shell panels are the mainstay of aerospace and marine engineering. Earlier the body parts of aeroplanes were made of aluminium but composite fibres has taken their place in today's life. The main reason for this trend are the unique properties of composites such as large strength to weight ratio, very good corrosion resistance and low coefficient of thermal expansion. That is why, many structural outfits in engineering structures are rapidly replaced by the fiber reinforced composite specimens because it is easy to deal with and also long lasting.

1.2 Importance of present study

The earlier studies carried out in this field is mostly related to the use of numerical methods for the computation of natural frquencies of unidirectional composite plates, impact studies on shells and development of failure cracks. All structures are subjected to dynamic loads i.e subjected to external loads hence it becomes very important to find the natural frequency of the structure so as to prevent the phenomenon of resonance. The method used in this study is a non-destructive one. The results obtained by experimental methods are compared with the numerical ones and any deviation from the actual values indicate local failure or cracks in the laminated shell. The major objective of finding out the natural frequency is to avoid the phenomenon of resonance in case of large structures where the natural frequency of the structure matches with the applied frequency which results in large damage and destruction.

1.3 Outline of present study

The present study deals with vibration characteristics of laminated composite curved shells. During this study results were obtained by varying a number of parameters which include Radius of curvature, number of layers of glass fiber, thickness of shell, support conditions, ply orientation and aspect ratio. Tests were conducted to find the natural frequency of these shells using experimental approach and numerically using a finite element program using MATLAB.

This thesis comprises of five chapters. The first chapter introduces the topic and provides a brief insight to the importance and application of this research work.

The second chapter deals with all the literature reviewed during the various stages of work to have a proper understanding of the concepts behind the research work. It also includes the aims and scope of present study.

The third chapter deals with the mathematical formulation for the solution to vibrational problems subjected to an external load. A subsequent computer program is developed in MATLAB which provides us with the natural frequencies when suitable parameters are provided.

The fourth chapter deals with the experimental procedure employed to cast specimen starting from materials used to casting technique and various precautionary measures adopted while casting. After casting of specimens the shells are cut in to regular sizes for testing. The specimens are subjected to different support conditions and tested with the help of modal hammer, accelerometer, FFT analyzer and a display unit. The Frequency Response Function (FRF) obtained gives us the natural frequencies of shells.

The fifth chapter presents the results obtained from present investigation and are discussed in detail. The effects of several parameters like radius of curvature, support conditions, number oflayers, thickness of shell, ply orientation and Aspect ratio on vibrational frequency are studied and discussed.

In the sixth chapter the conclusions drawn from various test results are discussed in brief and furthur scope of the research work is described.

The final chapter lists all the references which were used to develop the understanding of the topic and to make this project a successful one.

CHAPTER 2

REVIEW OF LITERATURE

2.1 Introduction

Natural frequencies are computer are computed in order to determine the behavior of structures subjected to dynamic loads. The modal analysis is avery efficient technique for the assessment of stiffness of structures. Any deviation if found in the experimental and numerical frequency indicate some cracks, failure plane or any sort of weakness in the composite shell. The literature was thus critically reviewed so as to have a detailed knowledge of the problem statement and how to deal with any problems which might crop during the execution of the project. Most of the research is related to theoretical methods of finding out the natural frequency and very less literature is found related to experimental determination of frequency analysis.

2.2 Vibration of Laminated Composite curved shells

In the last few decades the demand for composite materials have gone up drastically and has resulted in extensive research in the field of composites and their vibrational characteristics. Szechenyi [1] devised simple approximate formulae to determine the natural frequencies of the modes of vibration for singly curved shells with in-plane stresses. Chao and Reddy [2] determined numerical results for free vibration and non-linear bending in case of laminated curved composite panels subjected to several boundary conditions. They used a three-dimensional element including the geometric non-linearity. Reddy found the exact solutions for free vibration and bending of thick laminated composite shells subjected to simply supported condition. Chandrasekhara [3] presented the free vibration characteristics of doubly curved panels by using nine-node isoparametric elements. Cawley and Adams [4] found the natural frequencies of aluminium plates with different ply orientations subjected to free-free support condition. They also used dynamic methods to detect any damage or location of cracks, failure in the casted composite specimen. Crawley [5] experimentally determined the natural frequencies and mode shapes of cylindrical composite shells for various laminates and altering aspect ratio using electro-magnetic shaker and compared the obatined results by finite element method.

Qatu and Leissa [6] investigated the free vibartion characteristics of doubly curved cylindical panels. They used the Ritz procedure in addition to an algebraic function. Chai [7] used the same Rayleigh- Ritz technique to determine the natural frequencies of Composite curved shells for different boundary conditions and different ply orientations. Balamurugan and Narayanan [8] developed the mechanics for the analysis of piezolaminated curved shells are studied using finite element approach and their vibration control performance was considered. Chia and Chia [9] found the non-linear vibrations in case of thick laminated shallow curved panels subjected to simply supported boundary conditions. The solution was formulated in fourier series with coefficient which were dependent on time to satisfy the boundary conditions.

Maiti and Sinha [10] used shear deformation theory which deals with the free vibration, impact response and bending in case of thick laminated composite panels. The study was extended for delaminated plates and shells by Ju *et al*. [11] who studied the effects of delamination on composite structures. Sheinman and Reichman [12] dealt with the vibration and buckling characteristics of laminated curved shells. Chakravorty and Bandopadhyay [13] developed a finite element method for determination of the natural frequencies of doubly curved shells. The effects of parameters including sequence of lamination, ply orientation and aspect ratio was studied for doubly curved cylindrical shells. Civalek [14] studied the free vibration anlysis of symmetric laminated composite shells using first order shear deformation theory by discrete convolution method. Amabili [15] studied the non-linear vibrations for doubly curved shallow shells subjected to four sides simply supported end condition. The solution was obtained by lagrangian technique. Kumar *et al*. [16] developed a finite element formula for the static and dynamic analysis of smart cylindrical shells. The formulation is closely realted to first order shear deformation technique and Hamilton's principle.

Messina [17] studied the free vibrations of doubly curved panels based on mix- variational and vibrational approach and piecewise-smooth functions. This paper explains the dynamics behind vibration of curved shells. Alijani [18] found the accuracy of the vibrations for doubly curved shallow shells. Chakraborty *et al.*[19] found the frequencies using experimental approach for glass fiber plates and checked their validity using a commercial finite element package (NISA). Shen-Shen and Xiang [20] investigated the non-linear vibrations of nanotube reinforced cylindrical composite shells. Hatami *et al.* [21] studied the free vibrations of symmetrically moving laminated plates subjected to planar forces.

Qatu *et al.* [22] discussed the recent advancements in the field of dynamic analysis of cylindrical shells. This paper reviews all the recent researches done putting emphasis on the type of testing, support conditions employed and various shell geometrics taken in to the research. Ribeiro and Akhavan [23] discussed the non-linear vibrations of composite shells with varying stiffness. Yasin and kapuria [24] dealt with the effective layerwise finite element method for finding out the natural frequencies in case of shallow cylindrical shells. Jin *et.al* [25] developed an exact solution for the vibrational analysis of composite curved shells subjected to elastic boundary conditions. All the shell displacements are taken as standard cosine series along with auxillary functions. Rayleigh Ritz procedure is used to find out the energy functions of the cylindrical shell.

Soutis [26] described all the uses of composite shells especially in the use of aircraft construction. It explains the advantages of composite shells that it being very light is able to bear high loads and has a high stiffness coefficient to go with it. It also compares the data elated to fuel efficiency of aircrafts made by composites and also discusses the use of CFRP for aircraft construction. Lam and Qian [27] studied the free vibration characteristics of thick laminated composite curved panels. In this research frequency variation with change in H/R and L/R ratios are studied in comparison with frequencies in case of symmetric laminates. Tornabene et al. [28] presented a general formulation for higher order free vibrations for laminated cylindrical composite shells. Maheri [29] used theoretical relations for square shaped composite panels to study the effects of modal damping in case of various boundary conditions. The effect of fiber orientation on modal damping is also discussed in this paper. Lei et al. [30] discussed the effect of different woven structures for glass fiber on the dynamic properties of the whole structure.

The present study deals with the modal testing of Glass Fiber Reinforced Polymer (GFRP). During our experiment composite shells of varying radius of curvature were casted and variation in natural frequency of these shells were studied subjected to different boundary conditions. The other parameters which were varied during the study includes ply orientation, thickness of layers, number of layers of glass fiber and aspect ratio. The frequencies so obtained by experimental approach is compared to the numerical results obtained using Finite element method in MATLAB.

2.3 Scope of Present Study

An extensive review of study of literature depicts that a large amount of work has been done in the field of laminated composite shells to determine the natural frequencies. But most of the above studies are related to numerical approach to find out the natural frequencies and very less work is done in the field of determination of natural frequencies of singly curved composite shells. Therefore this research work is an effort to fill in the voids which remain in field of vibrational characteristics in case of curved composite shells. The various parameters varied while this study are :

- Radius of Curvature of the shell
- Support Conditions
- Number of layers of Glass fiber
- Ply orientation
- Thickness of shells
- Aspect ratio

After we get the experimental frequencies by varying these parameters numerical values are determined by using finite element technique using MATLAB.

CHAPTER 3

MATHEMATICAL FORMULATION

3.1 Introduction

The vibration analysis is implemented by finite element technique (FEM). An eight node element is selected in this analysis with five degree of freedom for each node. Mass matrix [M] and stiffness matrix [K] are determined with the help of minm potential energy concept. The overall mass and stiffness matrix are obtained by lumping the individual values using the skyline technique.

3.2 Governing Equations

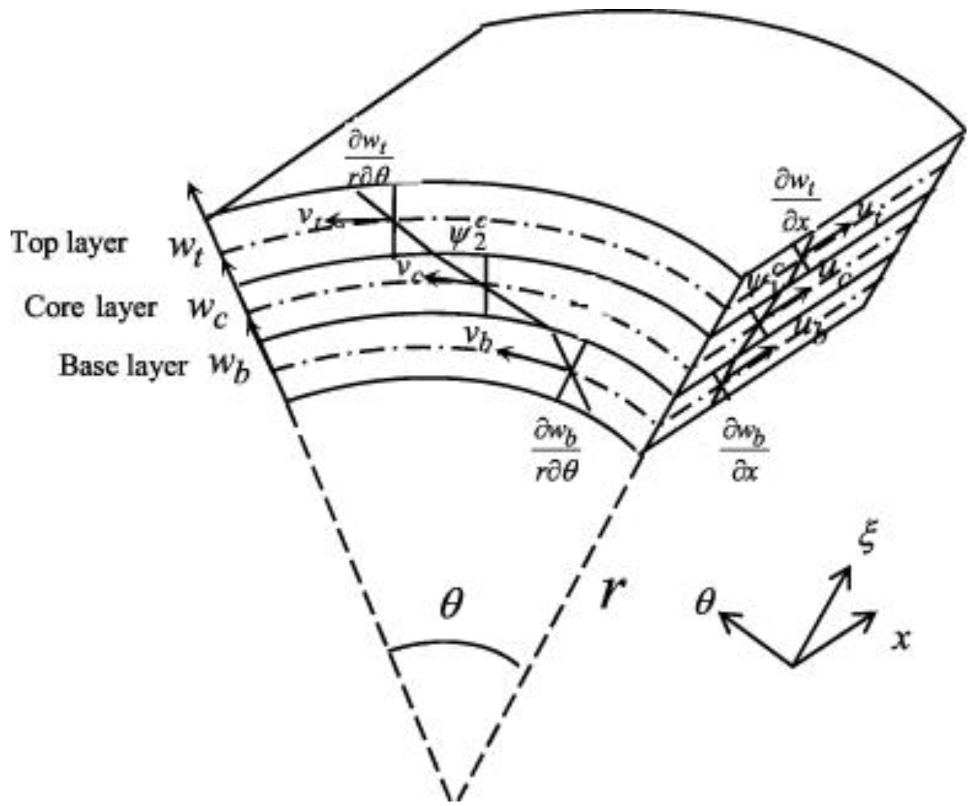

Figure 1: Laminated composite singly curved shell

We consider a singly curved composite shell with uniform thickness ‘h’ and radius of curvature R. Each laminae is oriented at an angle θ to the x- axis. The governing equations to completely describe the vibration characteristics of shell are :

$$\{\boldsymbol{F}\} = [\boldsymbol{D}]\{\in\} \qquad (1)$$

Where,

$$\{F\} = \{N_x, N_y, N_{xy}, M_x, M_y, M_{xy}, Q_x, Q_y\}^{\mathbf{T}}$$

$$\{\epsilon\} = \{\epsilon_x^0, \epsilon_y^0, \gamma_{xy}^0, k_x, k_y, k_{xy}, \gamma_{xz}^0, \gamma_{yz}^0\}^{\mathrm{T}}$$

$$[\mathrm{D}] = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{44} & S_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{45} & S_{55} \end{bmatrix} \qquad (2)$$

The force resultants are expressed as

$$\{F\} = \{N_x, N_y, N_{xy}, M_x, M_y, M_{xy}, Q_x, Q_y\}^{\mathbf{T}}$$

$$= \left[\int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}, \sigma_x z, \sigma_y z, \tau_{xy} z, \tau_{xz}, \tau_{yz}) dz\right]^T \qquad (3)$$

The stiffness matrix elements are defined as:

$$A_{ij} = \sum_{k=1}^{n} (Q_{ij})(z_k - z_{k-1})$$

$$B_{ij} = 1/2 \sum_{k=1}^{n} (Q_{ij})_k (z_k^2 - z_{k-1}^2)$$

$$D_{ij} = 1/3 \sum_{k=1}^{n} (Q_{ij})_k \, (z_k^3 - z_{k-1}^3) \quad \text{i, j = 1,2,6}$$

and

$$S_{ij} = \sum_{k=1}^{n} F_i F_j (G_{ij})_k (z_k - z_{k-1}) \quad \text{i, j = 4,5} \qquad (4)$$

where Q_{ij} are the elements of the off-axis elastic constant matrix which is given by,

$$\left[Q_{ij}\right]_{off} = [T_1]^{-1} \left[Q_{ij}\right]_{on} [T_1]^{-1} \quad \text{i, j =1,2,6} \qquad (5)$$

$$\left[G_{ij}\right]_{off} = [T_2]^{-1} \left[Q_{ij}\right]_{on} [T_2], \text{ i, j = 4,5} \qquad (6)$$

$$[T_1] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix}, [T_2] = \begin{bmatrix} m & -n \\ n & m \end{bmatrix}$$

in which m = cos θ and n = sin θ

$$\left[Q_{ij}\right]_{on} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \text{ i, j = 1, 2, 6}$$

$$\left[Q_{ij}\right]_{on} = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \text{ i, j = 4, 5}$$

in which,

$$Q_{11} = (1 - \upsilon_{12}\upsilon_{21})^{-1} E_{11}, \;\; Q_{22} = (1 - \upsilon_{12}\upsilon_{21})^{-1} E_{22}$$

$$Q_{12} = (1 - \upsilon_{12}\upsilon_{21})^{-1} E_{11}\upsilon_{21,} \, Q_{66} = G_{12}, Q_{44} = G_{13}, Q_{55} = G_{23}$$

The strain displacement relations are as follows :

$$\left[\epsilon_x, \epsilon_y, \gamma_{xy,} \gamma_{xz,} \gamma_{yz}\right]^T = \left[\epsilon_x^0, \epsilon_y^0, \gamma_{xy}^0, \gamma_{xz}^0, \gamma_{yz}^0\right]^T + z\left[k_x, k_y, k_{xy}, k_{xz}, k_{yz}\right]^T \quad (7)$$

where,

$$\begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \\ \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} = \begin{Bmatrix} \frac{du}{dx} - w/R_{xx} \\ \frac{dv}{dy} - w/R_{yy} \\ \frac{du}{dy} + \frac{dv}{dx} - 2w/R_{xy} \\ \alpha + dw/dx \\ \beta + dw/dy \end{Bmatrix}, \quad \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \\ k_{xz} \\ k_{yz} \end{Bmatrix} = \begin{Bmatrix} d\alpha/dx \\ d\beta/dy \\ \frac{d\alpha}{dy} + d\beta/dx \\ 0 \\ 0 \end{Bmatrix} \qquad (8)$$

3.3 Finite Element Formulation

An eight node singly curved shell is used for the present description. Degree of freedom is taken to be five in account with two rotations namely α and β. The mass matrix and element stiffness matrix is obtained using the principle of minimum potential energy. The displacement and rotation are represented in the form of nodal values as :

$$u = \sum_{i=1}^{8} N_i\, u_i, \quad v = \sum_{i=1}^{8} N_i v_i\,, \quad w = \sum_{i=1}^{8} N_i\, w_i$$

$$\alpha = \sum_{i=1}^{8} N_i\, \alpha_i, \qquad \beta = \sum_{i=1}^{8} N_i\, \beta_i \qquad (9)$$

The shape functions are as follows :

$$N_i = 1/4(1+\xi\xi_i)(1+\eta\eta_i)(\xi\xi_i + \eta\eta_i - 1) \text{ for i = 1, 2, 3, 4}$$

$$N_i = 1/2(1+\xi\xi_i)(1-\eta^2) \text{ for i = 5, 7}$$

$$N_i = 1/2(1+\eta\eta_i)(1-\xi^2) \text{ for i = 6, 8} \qquad (10)$$

The relation for strain displacement is given as :

$$\{\epsilon\} = [B]\{\delta\}, \qquad (11)$$

where,

$$\{\delta\} = \left[u_1, v_{1,} w_{1,} \alpha_{1,} \beta_1, \dots, u_{8,} v_{8,} w_8, \alpha_{8,} \beta_8\right]^T$$

and

(12)

The element stiffness matrix is formulated as :

$$[K_e] = \int \int [B]^T \, [D][B] dx \, dy, \tag{13}$$

The element mass matrix is formulated as :

$$[M_e] = \int \int [N]^T \, [P][N] dx \, dy, \tag{14}$$

where,

$$[N] = \sum_{i=1}^{8} \begin{bmatrix} N_i & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix}$$

and

$$[P] = \begin{bmatrix} P & 0 & 0 & 0 & 0 \\ 0 & P & 0 & 0 & 0 \\ 0 & 0 & P & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}$$

with

$$p = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \rho \, dz \, , \quad and \quad I = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} z^2 \, \rho dz \tag{15}$$

The mass and stiffness matrices are first evaluated using the integrals in their local co-ordinates and thereby performing the integration using the method stated above. The element matrix are then assembled correctly to obtain global mass matrix [M] and global stiffness matrix [K].

The free vibration analysis calculates the natural frequencies using the relation :

$$|[K] - \omega_n^2[M]| = 0 \qquad (16)$$

This is the general equation followed for computation of natural frequencies.

CHAPTER 4

EXPERIMENTAL PROGRAMME

The experimental programme describes the details regarding the casting of specimens for tensile test, the procedure for tensile test and the casting of composite shells for vibration testing.

4.1 Fabrication Technique

Composite specimens were casted using hand layup method. The hand layup technique is well explained in figure 2. In hand layup technique glass or carbon fiber is placed along with liquid resin to give a finished and smooth surface on open mould. As with many other composite materials, the two materials are combined to overcome each others' deficits. The plastic resins are very strong in compression and relatively weak in tension and on the other hand glass fiber is very strong in tension. Thus this combination acts as a very efficient composite to resist both tension and compression. The composition of fiber and matrix is taken as 50:50 by weight where matrix includes the epoxy and hardener. The distribution of epoxy and hardener is 42% and 8%. The hardener generally used for this method are Araldite HY556, Hardener HY591 and Ciba-Geigy. The weight of glass fiber used for manufacturing composite specimens is 360 g/m². The glass fiber is manufactured by Owens corning. To prepare the specimen the matrix is uniformly applied over the glass fiber after each layer. Steel rollers are used to expel out any air which might be present between the layers. A thin plastic sheet is provided both at top and bottom of the composite shell to give it a smooth surface and also protect it from environment attack. While providing the plastic sheet a spray of polyvinyl alcohol is applied which acts as a releasing agent. After the shell has been casted a heavy conical covering with sand completely filled is put in top of the casted to shell which acts as a curing medium for the same. The shells are cured for a minimum of 48 hours after which they are cut in to prescribed dimensions for vibrational testing.

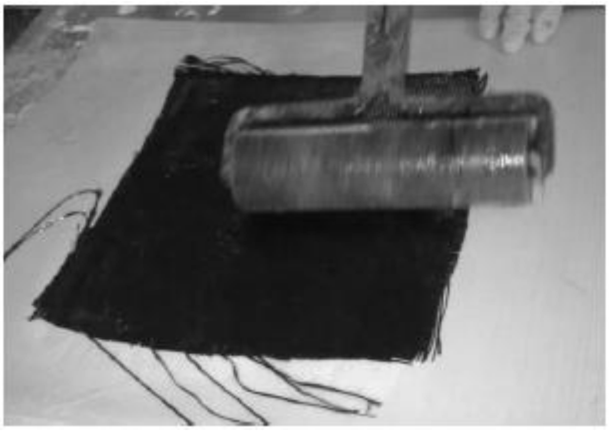

Figure 2: Hand Layup technique

4.2 Determination of Material Constants

The Young's Modulus of the composite fiber is determined using tensile test of specimens. For this test initially a composite plate of size 30cm×30cm is cast using the hand layup technique. After the plate is casted and cured it is cut in to definite shapes of size 25cm × 5cm. Diamond cutter was used in order to have a sharp finish to the tensile testing specimens. A figure of the diamond cutter is shown in figure 3 and the tensile specimens are showed in figure 4.

Figure 3: Diamont cutter

Figure 4:Tensile testing specimens

After the specimens are cut in to prescribed sizes their elastic modulus is found by tensile test method. The overall procedure for tensile test is explained in ASTM-D3039M-08[31]. The specimens are tested using INSTON 1195 Universal testing Machine. The rate of loading/extension is kept constant at 2mm/min. Specimens are fixed in upper jaw first and then in the lower jaw. 50mm length was considered on each side for gripping which gave 150mm as the effective length of the specimen. The load and extension was closely observed and measured using load cell and extensometer respectively.

Figure 5: Tensile test setup in INSTRON 1195 UTM machine

Figure 6: Failure pattern of tensile test specimen

The machine parameters and data obtained from tensile test of composite specimens for zero degree ply orientation are as follows :

4.2.1 Machine parameters of test

Sample rate (Pts per second) – 4.552

Crosshead Speed (mm/min) – 1.000

Full Scale Load Range (KN) – 50.00

Sample Type – ASTM

Humidity – 50%

Temperature (in F) – 73

4.2.2 Observations from tensile test

Specimen number	Displacement at peak (mm)	Load at peak (KN)	Stress at peak (MPa)	% strain at peak (%)	Young's Modulus (MPa)
1	4.972	13.33	280.6	3.315	11170
2	5.250	14.18	298.5	3.500	11400
3	4.683	13.20	278.0	3.122	11580
4	5.149	13.37	281.4	3.433	11010
Avg. Value	**5.048**	**13.45**	**284.4**	**3.343**	**11240**

From the above test we found the value of Young's Modulus **E_1= E_2 =11.24 GPa**. The ratio of transverse to longitudinal strain gives the Poisson's ratio. Here the value of Poisson's ratio is taken to be 0.25.

The shear modulus is determined using the following formula from Jones [32] as:

$$G_{12} = \frac{1}{\frac{4}{E_{45}} - \frac{1}{E_1} - \frac{1}{E_2} + 2\vartheta_{12}/E_1}$$

4.3 Experimental Programme for vibration study

The fabrication procedure is then carried out for various curved composite shells with varying radius of curvature. The specimens are casted using hand layup technique and curing is carried out at room temperature. A typical structure of Mould kept during curing process is shown in figure 7 and 8. The radius of curvature for shells were taken to be 0.91m, 1.2625m and 2.09m.

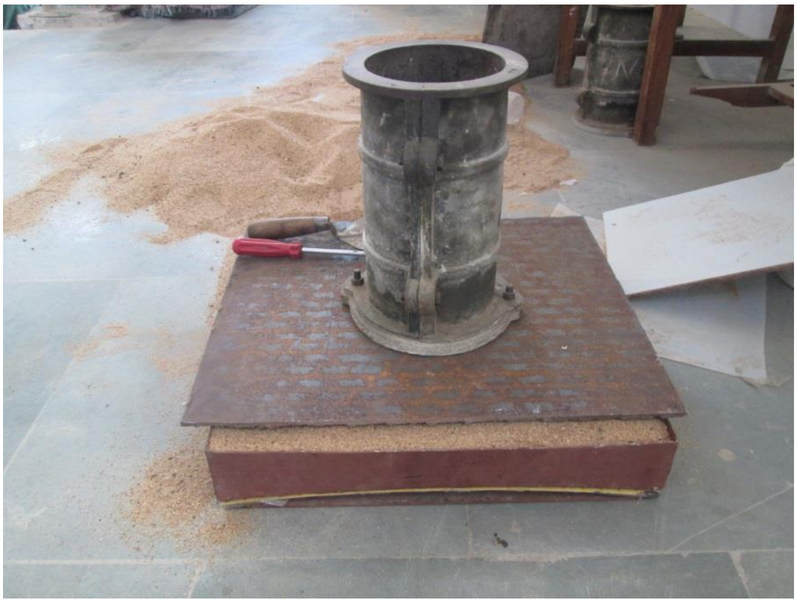

Figure 7: Curing of curved composite shells

Figure 8: Parallel view of shell and mould during curing

The curing is carried out for 48 hours at room temperature. A heavy iron sheet is put on mould filled with sand so as to have a uniform pressure exerted on the curved composite shell. The specimens are casted with various effects including the change in ply orientation and aspect ratio.

Figure 9: Woven glass fiber

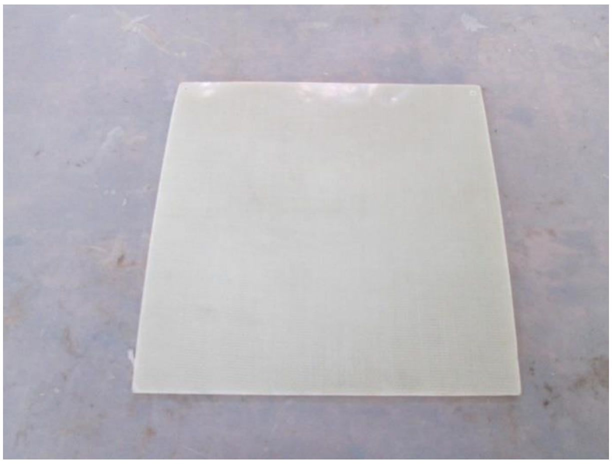

Figure 10: Curved composite shell

4.3.1 Equipments Required for Vibration Test

- Modal hammer
- Accelerometer
- FFT Analyzer
- Pulse software

Modal Hammer : The shell is excited by means of a modal hammer (Model 2302-5). It is basically used to excite the specimen so that the shell vibrates with its natural frequency which can be further recorded by the accelerometer

Accelerometer : (B&K Type 4507) Acclerometer is the device connected to sensor i.e it senses the excitation provided by means of Impact hammer and conveys it to FFT Analyzer. It is mounted on the specimen by means of bees wax.

FFT Analyzer : FFT analyzer processes the signals received from accelerometer and is the primary component of this testing outfit. It further transfers the signals to the computer which shows the output in the form of FRF (Frequency Response Function).

Figure 11: Modal Hammer

Figure 12: Accelerometer

Figure 13: FFT Analyzer

Figure 14: Display Unit

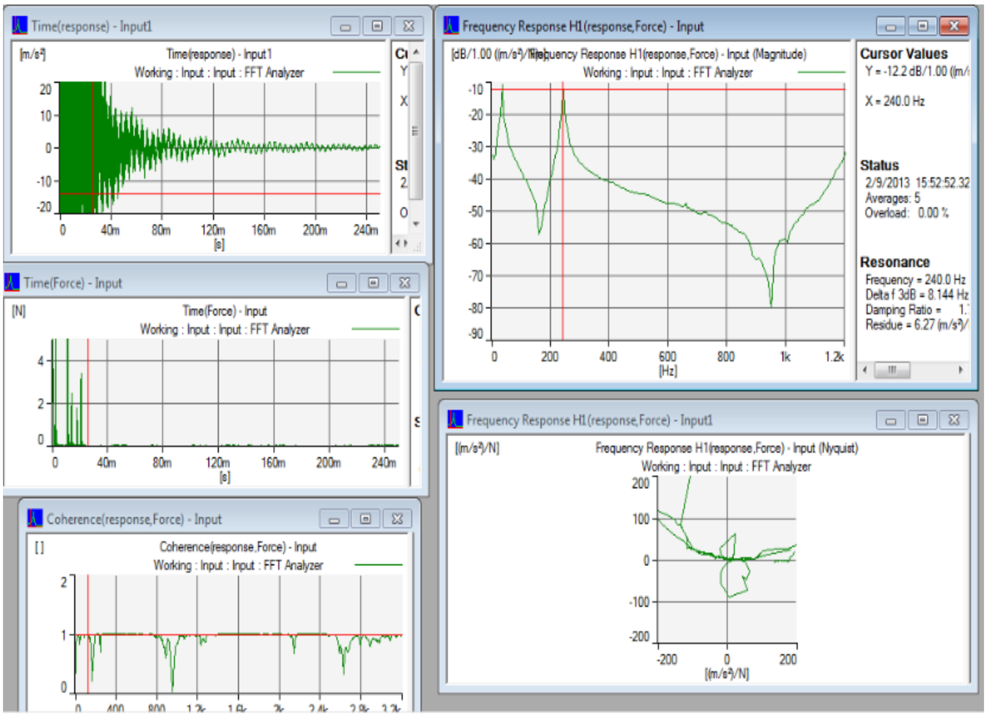

Fig 15: A typical output in display unit

4.3.2 Setup and Test procedure for vibration test

Initial boundary for the test was taken as free- free. The test specimens were fitted properly to the iron frame. The connections of FFT analyzer, transducers, modal hammer and cables to the system were done. The plate is excited slowly by means of an Impact hammer. The resulting vibrations of the specimen on the selected point were measured using an accelerometer, mounted on the specimen with the help of bees wax. For FRF, at every singular point the impact hammer was struck five times and the average value of the response was displayed on the screen of the display unit. When striking with the impact hammer precautions were taken to hit the stroke perpendicular to the plates. Then by pointing to the peaks of FRF the frequencies are measured.

Figure 16: FFFF condition for vibration testing

Figure 17: SFSF condition for vibration testing.

CHAPTER 5

RESULTS AND DISCUSSIONS

5.1 Determination of material properties

The elastic modulus of specimens with fibre orientation in 0, 30 and 45 degrees is shown in table 2. Because the fibre is bidirectional in nature the longitudinal elastic modulus is same as transverse modulus. The shear modulus of the specimen is determined by the following formula :

$$G_{12} = \frac{1}{\frac{4}{E_{45}} - \frac{1}{E_1} - \frac{1}{E_2} + 2\vartheta_{12}/E_1} \tag{17}$$

Where,

G_{12} is the shear modulus

E_{45} is the tensile modulus when the fibre orientation is at 45°

E_1 and E_2 are tensile modulus in longitudinal and transverse direction for 0° ply orientation

ϑ_{12} is the poisson's ratio, which is determined by taking a set of specimens and strain gauge which are bonded over the gauge length in two mutually perpendicular direction.

Using equation (17) the values of $G_{12}$was found to be 3.04 GPa. The value of ϑ was taken to be 0.25.

Sl.No	Ply orientation (in degrees)	Young's Modulus, E_{12}
1	0	11.24 Gpa
2	30	8.868 Gpa
3	45	7.253 Gpa

Table 2 : Variation of Young's Modulus with ply orientation

The non dimensional frequencies are determined using the formula:

$$\omega_1 = \omega a^2 \sqrt{\frac{\rho}{E_{11} h^2}}$$

Where, $'\omega_1'$ is the non-dimensional frequency

$'\omega'$ is the natural frequency of the system

$'\rho'$ is the density of the plate

$'E_{11}'$ is the tensile modulus obtained in longitudinal direction

$'h'$ is the thickness of shell

5.2 Modal testing of Glass fiber composite plates

1. Effect of Radius of Curvature of shell on natural frequency

Modal Frequencies obtained for the first five modes in case of composite shells is shown in table 3. The frequencies are shown in case of four sides free (FFFF) condition and 8 layers.

R=0.91m	**R=1.2625m**	**R=2.09m**
80	76	72
124	116	104
204	188	156
240	212	180
276	256	232

Table 3: Variation of Frequency with radius of curvature of shell for FFFF condition

It can be seen from the observations of table 3 that the natural frequency of vibration for shells decreases with the increase in radius of curvature of shell. This result is with close proximity to the results obtained by Askraba *et al* [33] which depicted the same results that the frequency decreases with the increase in radius of curvature of shells. The decrease in frequency is found to be **7.44%** as frequency increase from 0.91m to 1.2625m and **18.85%** as it increases from 0.91m to 2.09m.

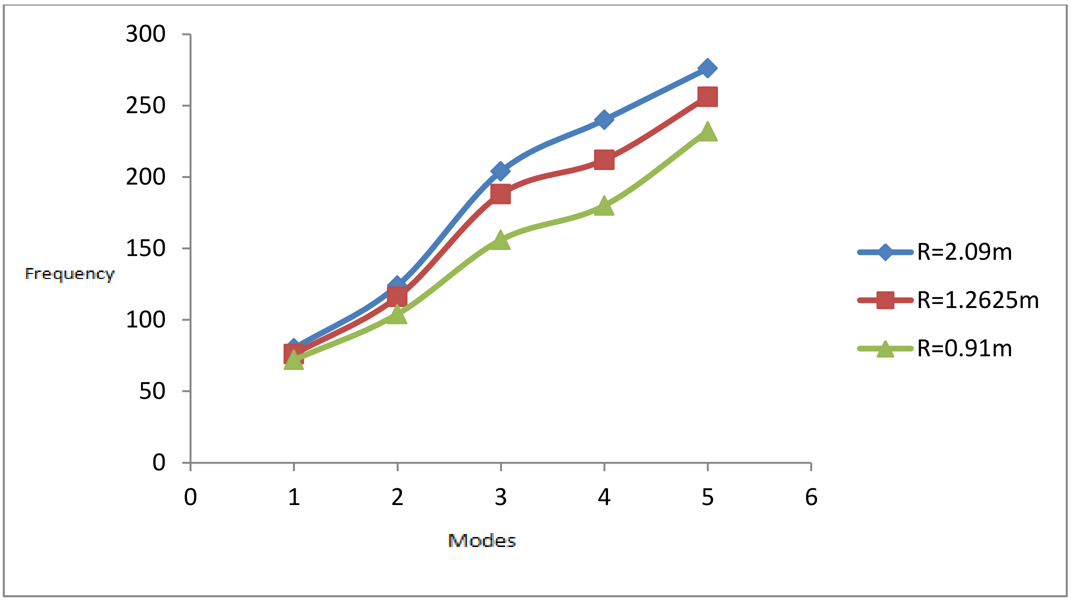

Figure 18: Variation of Frequency with radius of curvature for FFFF condition

When we compare the frequencies of different radius of curvature for two sides simply supported and two sides free condition (SFSF) it is found that the frequency actually decreases with the increase in radius of curvature of shell.

R=0.91	**R=1.2625**	**R=2.09**
56	52	48
88	80	76
220	212	200
260	252	236

Table 4: Variation of frequency with radius of curavature for SFSF condition

The frequencies were found to decrease by **5.73%** when radius of curvature increased from 0.91m to 1.2625m and decreased by **11.55%** when radius of curvature increased from 0.91m to 2.09m.

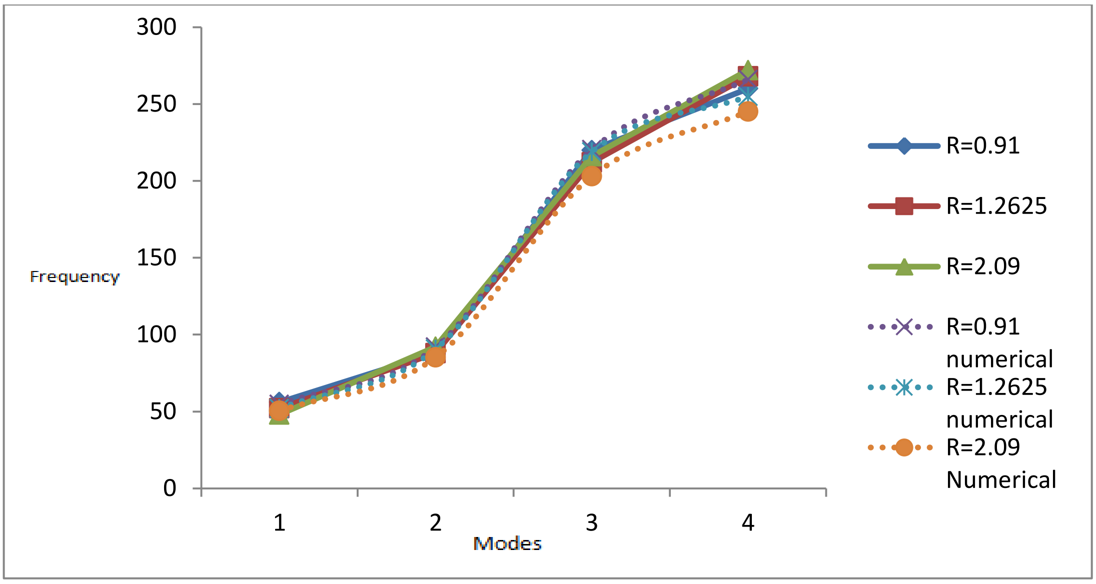

Fig 19: Variation of frequency with radius of curvature for SFSF condition

2. Variation of Natural frequency with support conditions

The observations obtained when shells with radius of curvature 0.91m and 2.09 m were subjected to four sides free (FFFF) condition and two sides simply supported and two sides free (SFSF) condition is shown in table 5.

R=0.91 FFFF Condition	**R=0.91 SFSF Condition**	**R=2.09 FFFF Condition**	**R=2.09 SFSF Condition**
80	56	72	48
124	88	104	92
204	192	156	144
240	228	180	168

Table 5: Frequencies for different support conditions

It can be seen from the above table that the frequency of vibration is more for shell subjected to FFFF condition as compared to SFSF condition. It is in close analogy with the results obtained from Finite element program in MATLAB.

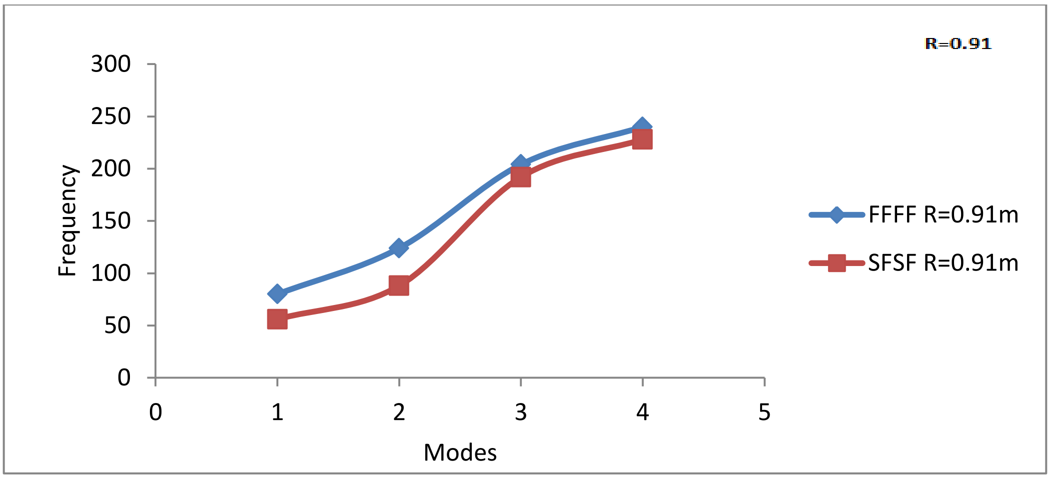

Figure 20: Variation of frequency with support condition for different modes for R=0.91m

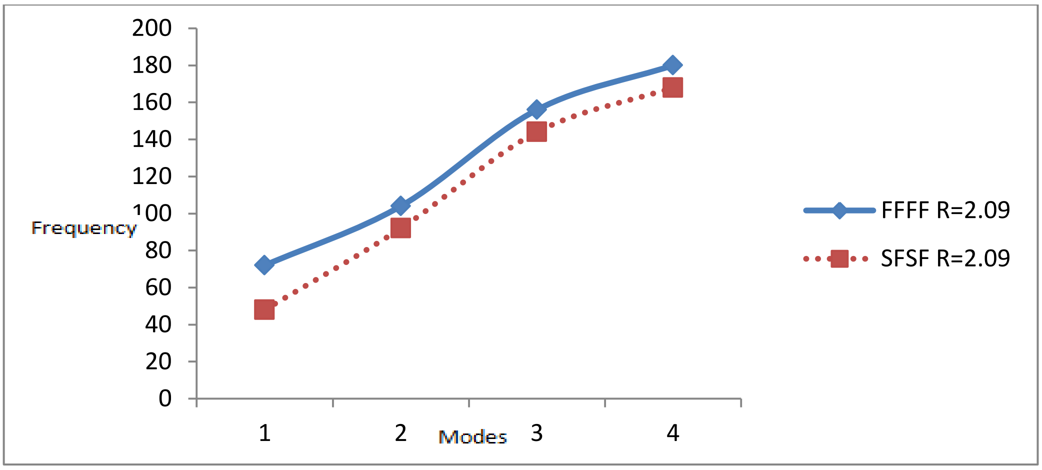

Figure 21: Variation of frequency with support condition for different modes for R=2.09m

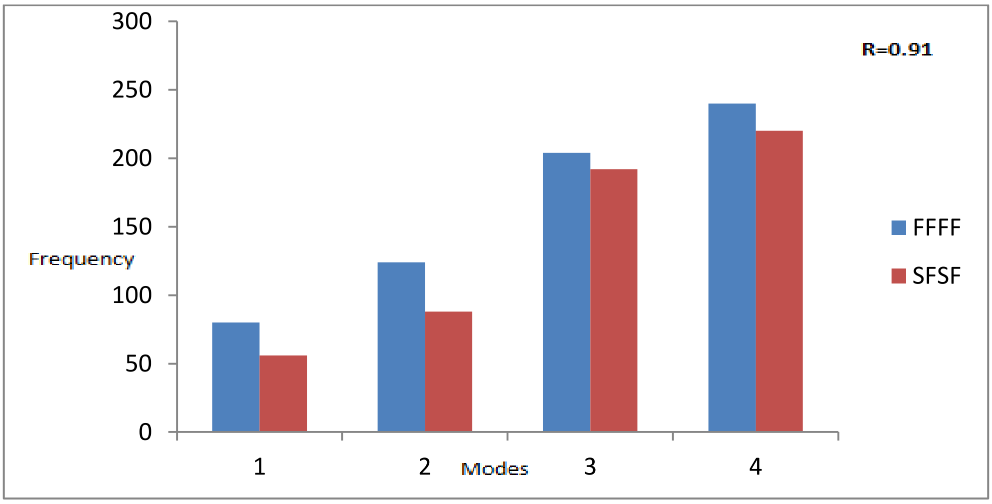

Figure 22: Plot showing difference in frequency with support condition for every mode

3. Effect of number of layers of glass fiber on natural frequency

The number of layers of glass fiber were varied from 4 layers to 8 layers to 12 layers. The observation were very accurate when compared with the numerical data. It is found that the frequency of vibration increases with the increase in number of layers. This is mainly because of the increase in stiffness of plates. The average increase in frequency for **All sides free (FFFF)** condition was found to be **113.3%** and **270%** as the thickness increases from 4 layers to 8 and 12 layers respectively. The experimental values and their graphs are shown.

Frequency (4 layers)	Frequency (8 layers)	Frequency (12 layers)
28	72	104
40	104	156
92	156	292
108	180	432

Table 6: Frequency values for different layers of glass fiber

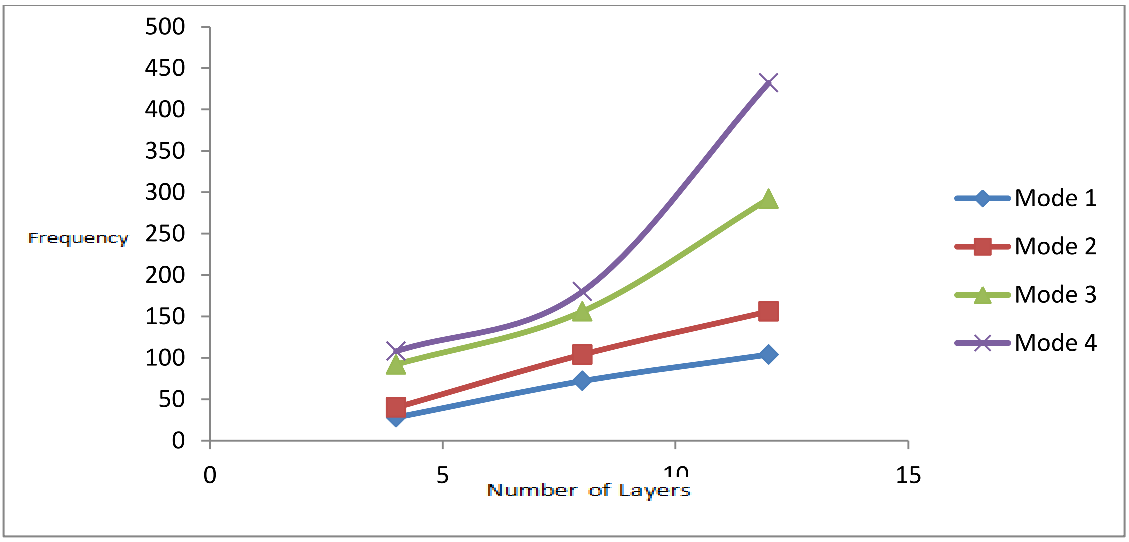

Figure 23: Variation of frequency with number of layers for FFFF condition

The same trend was seen when frequencies were compared for shells with different number of layers for two sides simply supported and two sides free (SFSF) condition. The average increase in frequency for SFSF condition was found to be **154.5%** and **257.2%** as the thickness increases from 4 layers to 8 and 12 layers respectively. The experimental values and their graphs are shown.

Frequency (4 layers)	Frequency (8 layers)	Frequency (12 layers)
24	48	84
32	92	116
84	216	296
100	272	364

Table 7: Frequencies for different number of layers for SFSF condition

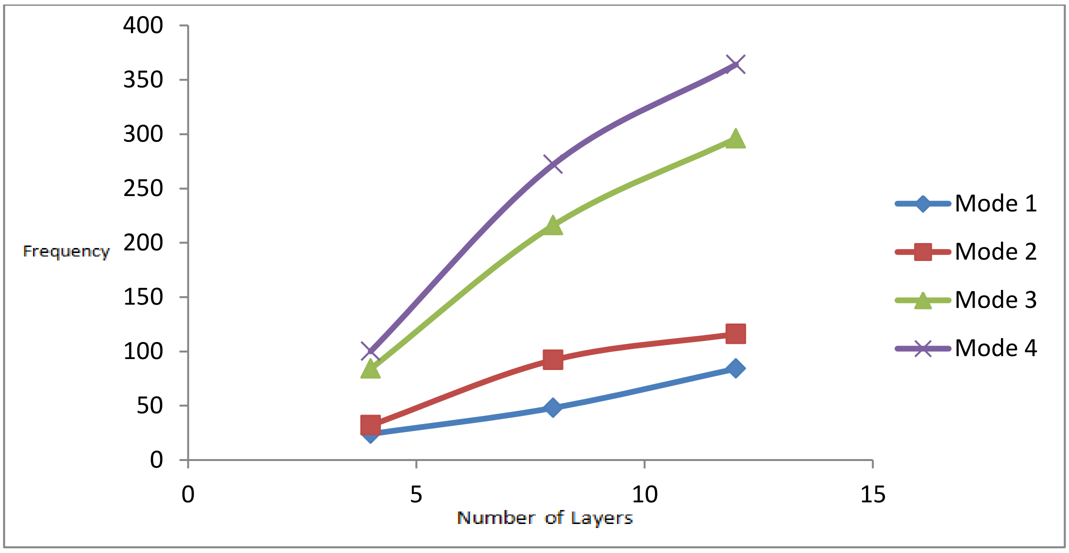

Figure 24: Variation of frequency with number of layers for SFSF condition

4. Effect of ply orientation on natural frequency

This is a very important parameter for testing of natural frequency and was carried out with extreme care. The ply was oriented at 3 different angles i.e 0°, 30° and 45°. From the observations it was observed that the frequency of vibration increase with the increase in ply orientation and was found to be maximum at an angle of 45°. This is because of the fact that the fibre used for casting is a bi-directional fibre and orientation at an angle θ is equal to (90- θ).

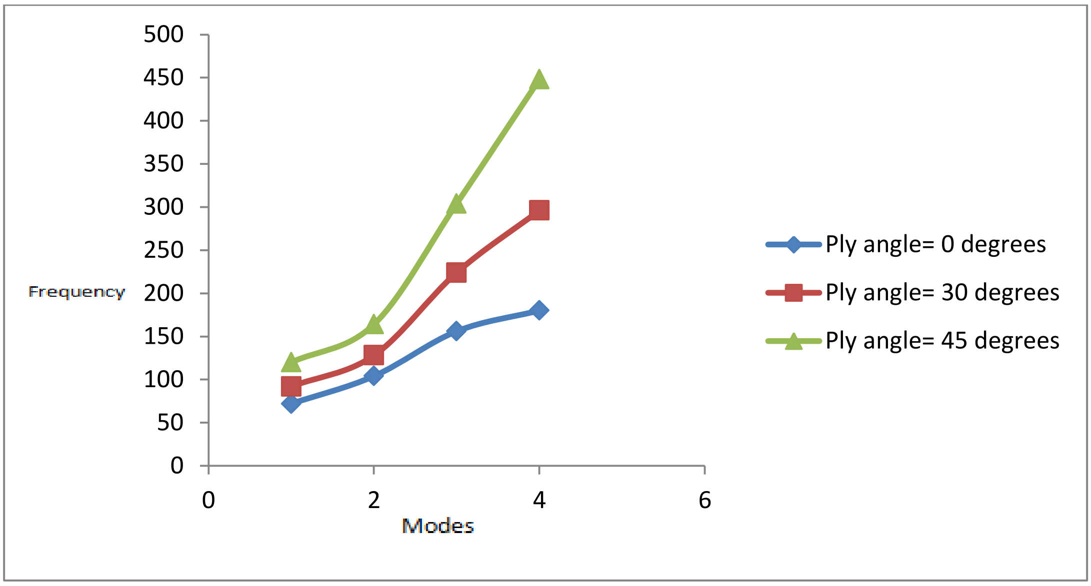

Figure 25: Variation of frequency with ply orientation

5. Effect of thickness of shell on vibration frequency

The variation of thickness becomes an important parameter as this is an experimental approach to determine the frequency of curved shells. As mentioned earlier the shells used for vibration testing is earlier cast using hand layup technique. Because the specimens are fabricated in laboratory it is not possible to have a shell of uniform thickness throughout the crossection and the thickness of casted shell varies slightly with every cast. So specimens were cast on the same mould and the effect of thickness of shell on vibrational frequency was observed. It is found that **frequency increases by 5.9% when 't' changes from 2.35mm to 2.56mm and by 10.75% when it changes from 2.56mm to 3.04mm**.

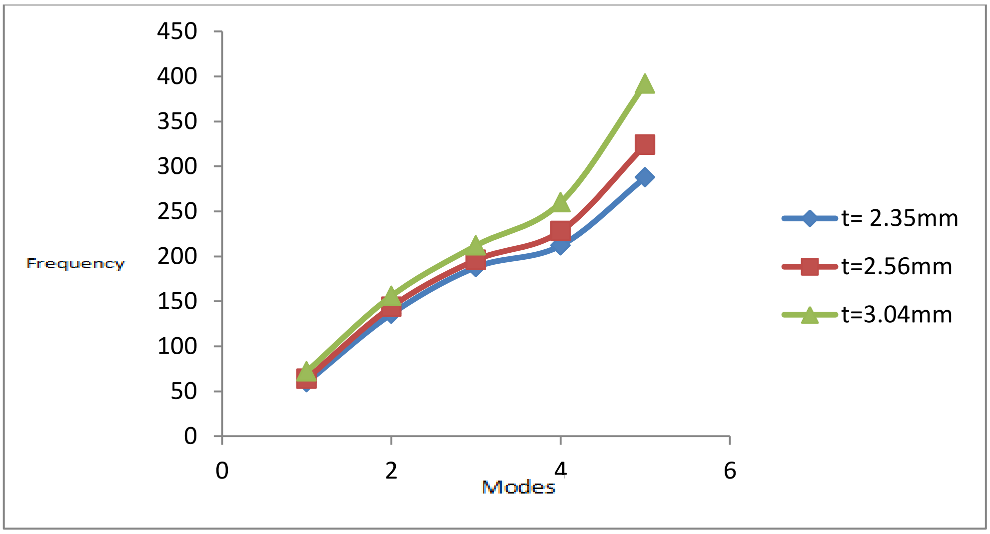

Figure 26: Variation of vibrational frequency with thickness of shell

6. Effect of aspect ratio on natural frequency of shell

Aspect ratio or l/b ratio is an important parameter for study of vibrational frequency. The aspect ratio was varied as 1, 1.5 and 2. It is found that the frequency of vibration increases with the increase in aspect ratio. This increase in frequency is due to the increase in stiffness of shell.

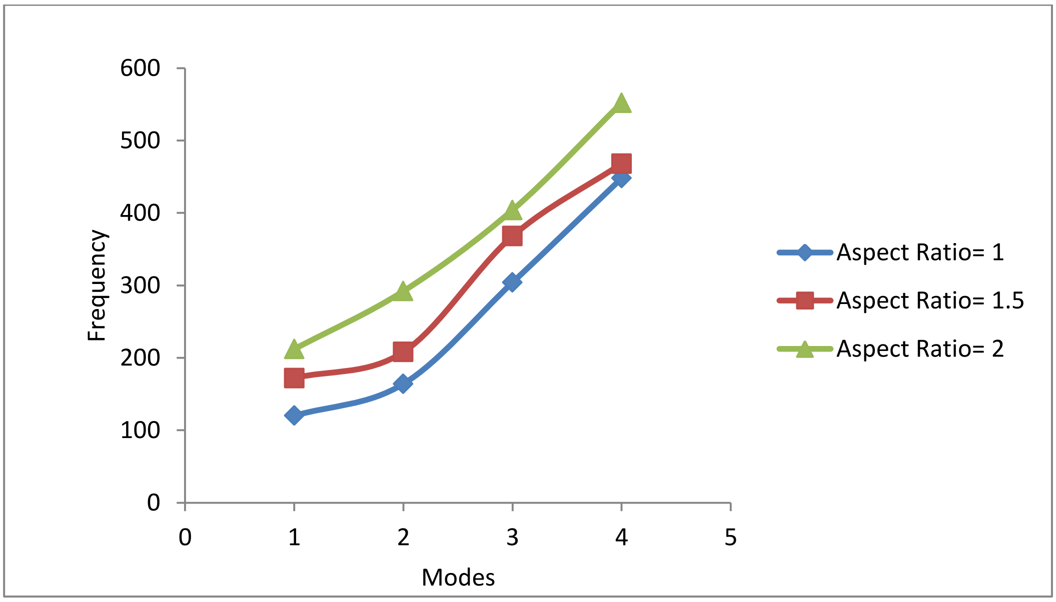

Figure 27: Variation of frequency with aspect ratio

CHAPTER 6

CONCLUSION

The present study deals with the vibrational testing of curved composite shells where composite shells were initially casted using hand layup technique and then cured for 48 hours. After that it is cut in to sizes of 23.5 cm × 23.5 cm for vibration testing using FFT analyzer. The natural frequency of vibration was observed in the form of Frequency Response Function (FRF) by varying the various parameters which includes radius of curvature of shell, support conditions, number of layers, thickness of shell, ply orientation and aspect ratio. The results so obtained are compared with the results obtained by using MATLAB program.

The various conclusions which can be drawn out are:

- The vibrational frequency decreases with the increase in radius of curvature of shell.
- The frequency of vibration is more for four sides free (FFFF) condition as compared to two sides simply supported and two sides free (SFSF) condition.
- The frequency of vibration increases with the number of layers of glass fiber due to increase in stiffness.
- The frequency of vibration increase with the increase in ply orientation and was found to be maximum at an angle of 45 degrees.
- The frequency of vibration was found to increase with the thickness of shells.
- The frequency of vibration also increases with the increase in aspect ratio of the shell.

CHAPTER 7

FUTURE SCOPE OF RESEARCH

The present study on vibrational study of laminated curved composite shells can be extended to the following areas :

- Study of vibration characteristics on curved shells made of carbon fiber.
- Study of Hygrothermal effects on the vibration frequency of shells.
- Study of buckling characteristics on the curved composite plates and study of Hygrothermal effects on buckling characteristics.
- Effect of joints on the vibrational behavior of composite shells
- Study of vibrational characteristics on delaminated specimens of curved composite shell.
- Development of a computer code to design the composite specimens based on the data obtained during this research.

CHAPTER 8

PUBLICATION

- Mittal, A. and Sahu, S.K., Vibration Analysis of Laminated Composite Curved Panels, Structural Engineering Convention, Department of Civil Engineering, Indian Institute of Technology (IIT) Delhi, 22nd – 24th December, 2014 (Selected for presentation)

CHAPTER 9

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