

APPLICATION OF FUNCTIONAL NETWORK IN GEOTECHNICAL ENGINEERING

**A THESIS SUBMITTED IN PARTIAL FULFILMENT OF THE
REQUIREMENTS FOR THE AWARD OF THE DEGREE OF**

**Master of Technology
In
Civil Engineering
(Geotechnical Engineering)**

By

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Roll No-212ce1026



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National Institute of Technology Rourkela
Rourkela-769008,
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Under the Guidance of

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CERTIFICATE

This is to certify that the Thesis Report entitled “**Application of functional networks in geotechnical engineering**”, submitted by **Ms. Pavani.M** Roll no. **212CE1026** in partial fulfilment of the requirements for the award of **Master of Technology in civil Engineering** with specialization in “**geotechnical**” during session 2012-2014 at National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other university/institute for the award of any Degree or Diploma.

Date:

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Pavani.M

ABSTRACT

In nature spatial variability of the soil is inevitable. The analysis of such unpredictable material only on the basis of experimental, finite element method and other traditionally available methods is reliable, but overall modeling based on these methods makes it more complex and this problem necessitated the usage of statistical models to develop some empirical and semi empirical methods with the obtained input and output data.

Many statistical methods came from the past outperforming one another. Since the efficiency of certain tool also depends on the data chosen, the developed models though showed good results poor generalization was observed for some of the complex problems.

Functional networks introduced by Castillo as an alternative to artificial neural network (ANN), in which functions are learned instead of weights, and also the functions are random chosen, unlike ANN they are constrained to certain functions. The selection of topology depends on both domain knowledge i.e. associativity, commutativity and others, where as ANN is a black box which blindly access the data by increasing the weights (trial and error process)

The objective of this study is to show how functional network can be effectively used to model certain problems in geotechnical engineering. In this thesis four examples are considered under study (1) Prediction of lateral load capacity of piles in clay, (2) Prediction of factor of safety of slope, (3) Uplift capacity of suction caisson in clay, (4) Swelling pressure in clays, and the results are analyzed based on certain criterion like correlation coefficient, root mean square error, efficiency, cumulative probability distribution function.

The observed results are also compared with other statistical methods like ANN, SVM, MGGP, etc and it was observed that FN almost added a rung over all those methods and this shows that this method can be better used in every aspect of geotechnical engineering.

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CHAPTER 1

INTRODUCTION

1.1 GENERAL

Soil in nature formed due to weathering and decaying process. The mode of transportation, temperature and many factors affect the soil existence and hence its properties vary from place to place. Most of the conventional methods in analyzing the concepts of soil mechanics include few assumptions in order to simplify the solution and come to a conclusion. All these traditional methods excel its performance in the practical applications. But still because of the limitations exercised in all these cases many theories came into existence like numerical methods in geotechnical engineering, Theoretical soil mechanics, Finite element methods etc.

The traditional methods in analyzing the problems in civil engineering mostly follow the concepts of mechanics, empirical correlations, experimental analysis, finite element method etc.,

Because of its uncertain behavior, spatial variability, the constitutive modeling of soil is difficult compared to other engineering materials. Depending on case histories/field tests, statistically derived empirical methods and semi empirical methods based on analytical methods are more famous in such cases. The success and failure of these empirical and semi empirical methods used, mainly depends on statistical/theoretical model chosen for the system to be analyzed matching the input output data and statistical methods used to find out model parameters (Das and Basudhar 2006).

Very often it is difficult to develop theoretical/statistical models due to complex nature of the problem and uncertainty in soil parameters. These are situations where data driven approach have been found to more appropriate than model oriented approach. To overcome such problems Functional Network a neural based paradigm has been used as an application in Geotechnical Engineering problems. Within a short period of time it has cut its significance in the various fields in engineering and sciences. This has given a spurt in the research activities in the art of applying such methods to solve real life problems highlighting the latent capabilities and drawbacks of such methods.

Functional Network is an extension over artificial neural network (ANN) introduced by Enrique Castillo (1998). ANN is inspired by behavior of the brain, which consist of one or several layers of neurons, or computing units, connected by links. ANN has been identified as a powerful tool to learn and reproduce systems in various fields of applications. Most important property of neural networks is their ability to learn from data. The process of selecting the number of hidden layers and neuron in the hidden layer is by trial and error until good fit of the data is obtained. The poor generalization and the constraints of ANN lead to develop FN, which was a mathematical based analysis. The analysis of problem in functional network is based on the data; topology selection. It takes into account the real world problems, and uses both domain knowledge and data knowledge to produce the output more accurately.

This idea of functional network was inoculated by Castillo from the fact that everything in this world was a function of something (Castillo 1998).(Castillo1998; Castillo et al. 2000a) Gomez (Castillo and Ruiz-Cobo1992 and Castillo et al). (Castillo et al. 1998, 2000b) developed functional network into a powerful tool as an alternative to Artificial Neural Network (ANN). FN is coined as a novel generalization of neural network, this is because of the fact that it takes into account both domain knowledge and data knowledge to estimate the unknown neuron functions. The modeling of the initial topology of the network is based on the properties of the real world. This is further simplified using the functional equations, and further by using suitable basis function the network is learned and thus one can come up with a simplified equation.

The functional network: a neural based paradigm. The key features which make it look unique to that of the standard neural network are explained below:

1. In FN, the selection of topology can be done on the basis of the data knowledge, domain knowledge or both, whereas in ANN only the data is used.
2. Unlike in the Neural Network where the functions are known and the weights associated with the functions are learned, in FN the functions are learned both in the stage of structural learning (it involves obtaining the simplified network) and parametric learning (for obtaining optimal neuron functions)

3. In FN, arbitrary functions can be used for neurons, where as in ANN they are fixed sigmoidal functions.

4. The weights in the ANN is incorporated with the neural functions in FN.

5. The neural functions in functional networks can be multi argument and multivariate.

In addition to the above, to get effective results with neural network the data has to be normalized in the range off 0-1, in FN there is no such restriction, instead one can say that the Neural Networks are the special case of functional network.

1.2 Scope and Objective:

The objective of this study is to provide an improved intelligent approach via the use of functional network to develop some compact models in the field of Geotechnical engineering. Because of the versatility and flexibility of the FN, any problem can be easily modeled in the functional network. Functional network is becoming more reliable than statistical method due to their special attributes of identifying complex system when the input and output are known from either laboratory and field experimentation. However, there is not any comprehensive literature on critical evaluation of application of modeling aspects of ANN in geotechnical engineering. Since, the efficiency of all numerical methods in general problem dependent and the techniques used in solving the problem depend on the efficiency one can use the tool.

The scope of this project is to model the following problems in functional network:

1. Prediction of lateral load capacity of piles in clay
2. Prediction of factor of safety of slope
3. Uplift capacity of suction caisson in clay
4. Swelling pressure in clays

CHAPTER 2

LITERATURE REVIEW

The ideology of solving a problem is an art; in this context nature is the ideal for any task to be accomplished. The two main tools used by the nature in such circumstances are:

1. Learning
2. Reduction in disorder

The nature doesn't use any Mathematics, but it is flexible, adaptive and clever. It is tolerant of imprecision, uncertainty, partial truth and approximation. It has a family of problem solving methods which uses biological reasoning.

The real world problems are pervasively imprecise and uncertain. Most of the difficult problems in the engineering are inverse problems. Very effective methods in solving these are by learning from nature's problem solving strategies, so scientists inspired from human mind and introduced Soft Computing techniques, which analogous to nature shows gradual improvement by random search.

Soft computing techniques (SCT), though this term can't be precisely defined, these are used to solve many complex problems in this world. One way to define it is mimicking the natural creatures: plants, animals, human beings, which act as a chain and tackle the situation. The real world problems are pervasively imprecise and uncertain, since the precision and accuracy carry a cost, the SCT exploit the tolerance of imprecision, uncertainty, partial truth and approximation to achieve tractability, robustness and low solution cost. It uses nature's strategy of reduction in disorder and learning. SCT are different from conventional mathematically based methods. The principal constituents i.e. tools of SCT are Fuzzy logic, Neural Networks, Support Vector machine, Machine learning and Probabilistic learning. They operate with imprecise tolerance, Non-universality, Functional non-uniqueness.

Earlier, standard computing methods are used for information processing, in which computations are performed in sequential order. Some of the characteristics while handling the information adopted by them were as follows:

1. The knowledge was explicitly represented using rules, probabilistic models and semantic nets etc...
2. The human logical reasoning was imitated for problem solving, focusing on actions and underlying motives (rule changing, probabilistic reference), and
3. The information was sequentially processed.

The development of some artificial intelligence fields such as pattern recognition, where there is necessity of logical reasoning rather than, explicit representation of knowledge. Therefore, the standard algorithm approach and computational structures were inappropriate

in solving these problems. These kinds of problems which are hard, intractable and difficult to formulate using conventional computing techniques, gave place to new computing paradigms like parallel computation and neural networks. The key element of these paradigms is a novel computational structure which composed of several interconnected elements for processing and operating simultaneously unlike traditional serial processing computations.

Neural Networks took its role in late 1950's but due to lack of technology and breakthroughs in other artificial intelligence techniques little progress were made. The increasing power of available computers in the 1970's and the development of efficient parallel computing techniques renewed the interest in this field among computer and artificial intelligence scientists.

Artificial neural networks were introduced as alternative computational structures, created with the aim of reproducing the functions of human brain. The human brain is composed of several interconnected neurons which receive electrochemical signals from other neurons through synaptic junctions which connect the axon of the emitting and dendrites of the receiving neurons. Based on the received input the neuron computes and sends its own signal. The emission process is computed by the internal potential associated with the neuron. If this potential is a threshold, an electrical pulse is sent down the axon; otherwise no signal is sent.

Unlike conventional computing methods neural networks follow rigid programmed rules. Rather they use a learn-by-analogy learning process. i.e. connection weights are automatically adjusted to produce a representative set of training patterns with the aim of capturing the structure of the problem. This is also inspired in the way learning occurs in neurons, changing effectiveness of the synapses, so the influence of one neuron on the other changes.

Functional Networks was introduced by Castillo (1998) to overcome the draw backs in neural networks.

Castillo et al. 2000 have applied FNs to two structural engineering in deriving prediction of shear, moment, slope, and deflection of a beam.

S Rajsekaran researched in Functional Network in Structural engineering with the by using associativity functional network and analyzed five problems in structural engineering, and he modeled the slope, moment, deflection of the beam done by Castillo using Orthogonal equations.

Ahmed Adeniran et.al also used associativity functional network in Softsensor for formationporosity and water saturation in oil Wells.

Many works are carried out by Castillo et.al and established its wide variety of applications in engineering and sciences. The scope of this project is to effectively use functional network in modeling problems in geotechnical engineering. In this thesis a narrow field in functional network was used in modeling problems, but still some of the problems in geotechnical engineering can't be solved with this narrow field analysis. An overview of knowledge in functional network was presented here. According to the data present, by supplementing with the proper domain knowledge functional network can be effectively used to solve any problem in any field.

In Geotechnical engineering the application of functional network was very less only one literature was available for determination of permeability in a carbonate reservoir (El-sebakhy et.al). To this extent no more research much more observed in the geotechnical engineering and this research provides it application in some of the fields in geotechnical engineering.

CHAPTER 3

METHODOLOGY

3.1 INTRODUCTION

Functional Networks, a novel generalization of neural networks that bring together domain knowledge and data knowledge in order to determine the structure of the network and to estimate the unknown neuron functions. In functional networks the arbitrary neural functions are allowed, and also they are assumed to be multiargument and vector input length principle is used to propose a quality measure to be used in the selection procedure.

Depending on the topology of the neural network the output units are written in several different forms and leads to a system of functional equations, which are further simplified, which leads the multidimensional arguments reduce to fewer arguments. The crucial step in functional network lies in the learning, which deals with both domain and data knowledge. The two types of learning are as follows:

1. Structural Learning:

- (a) It is done based on the initial topology and some properties available according to the design
- (b) The posterior simplification using functional equations, leading to a simpler equivalent structure

2. Parametric Learning:

It concerns about the estimation of the neuron functions, and their associated parameters from the available data.

3.2 DEFINITION OF FUNCTIONAL NETWORK:

A functional network is a pair (X, U) , where X is a set of nodes and $U = \{(Y_i, F_i, Z_i) / i=1, \dots, n\}$ is a set of functional units over X , which satisfies the following conditions: Every $X_i \in X$ must be either an input or an output node of at least one functional unit in Y .

3.3 ELEMENTS OF FUNCTIONAL NETWORK:

The functional Network consists of the following elements:

1. Storing Units

- One layer of Input storing units for the input data x_1, x_2, x_3, \dots
- One layer of output storing units which contains output data f_4, f_5 .
- One or several layers of processing units evaluate inputs from the previous layer and delivers to the next layer, f_6 .

2. Layer of computing units, f_1, f_2, f_3 : A neuron in the computing unit evaluates a set of input values coming from a previous layer and delivers a set of output values of the next layer.

3. A set of directed links: The intermediate functions are not arbitrary but depend on the structure of the network. Such as $x_7 = f_4(x_4, x_5, x_6)$ as in Fig. 3.1

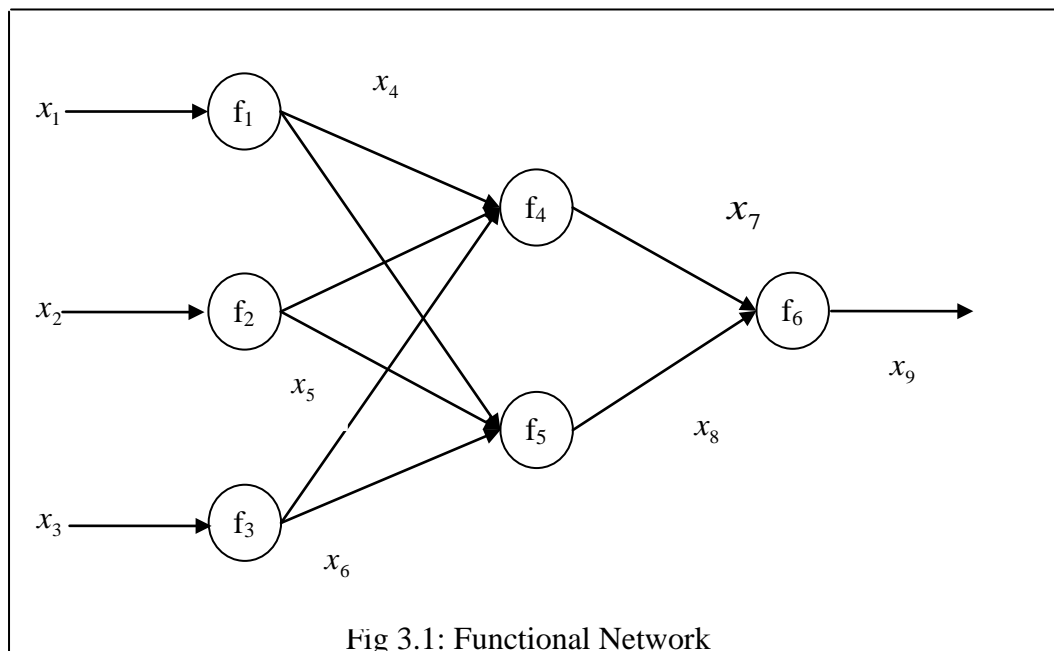


Fig 3.1: Functional Network

In addition to the data, information about the other properties of the functional network like the associative, commutative and invariance are used in the selection of the final network of the model.

3.4 WORKING WITH FUNCTIONAL NETWORK:

The sequence of steps to be followed while working with the functional network was presented as a flow chart in fig 3.2 and the detailed description of each step was explained in the later section followed.

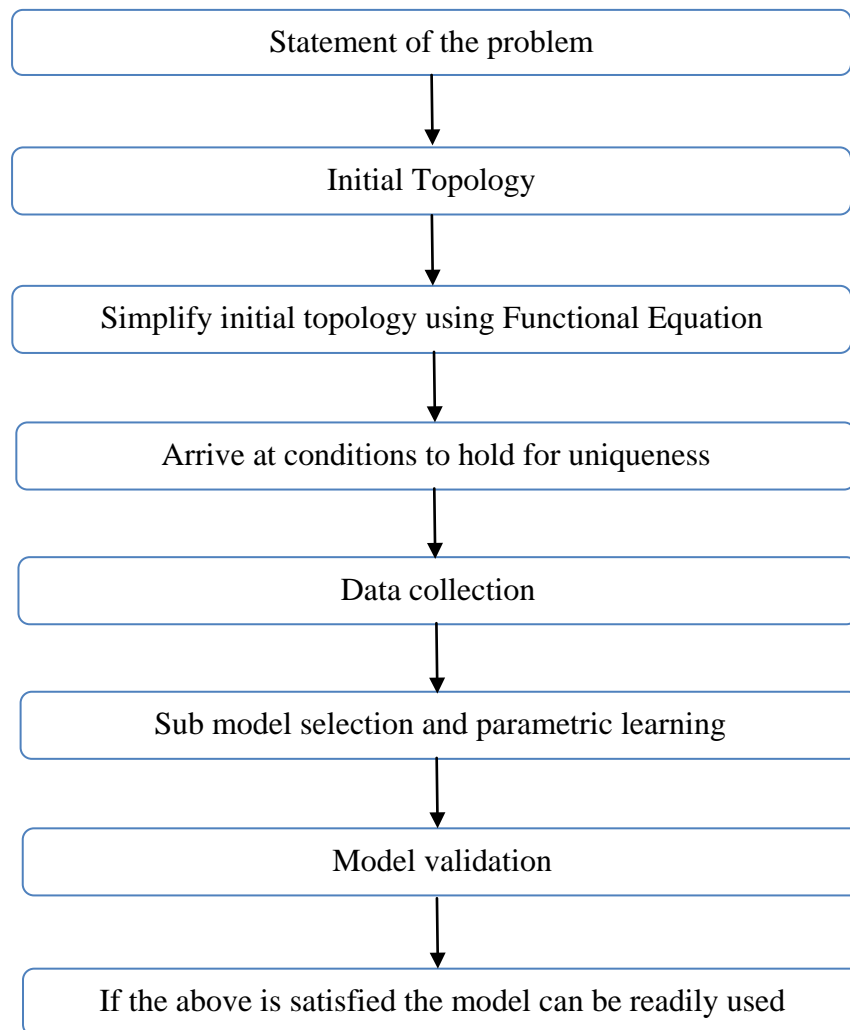


Fig 3.2 Working with Functional Network

Step1: Statement of the problem: This is the first crucial step in functional network, the problem taken to be understood carefully.

Step2: Initial Topology: Based on the knowledge of the problem the initial functional network is selected. Based on the properties and the initial topology has to be selected which lead to a clear and single network structure.

Step 3: Simplification or Structural learning: In these functional equations play main tool for simplifying the functional networks.

Step 4: Uniqueness of representation: Before learning a functional network, a unique representation of the structure is to be obtained.

Step 5: Data collection: For learning, the data is to be collected

Step 6: Parametric learning: The neural functions are estimated based in the given data. This is done by considering the linear combinations of appropriate functional families and using some minimization methods to obtain the optimal coefficients.

Step 7: Model validation: the test for the cross validation of the model is performed, and the error check is important in this.

Step 8: If the validation process is satisfactory the model is ready to be used.

3.5 FUNCTIONAL EQUATIONS:

While working with functional Network some knowledge of functional equations is necessary without which one will be constrained to the fixed number of functional network and well known topologies, hence this theory helps in two ways:

1. While simplification the functional equations are to be solved to obtain the simplified network.
2. In the Uniqueness representation step, to know the number of degrees of freedom

This aims to establish a connection between the physical, economical or engineering problem and the resulting equation. The definition of the functional equation and the general solutions of the most important functional equations is explained.

3.5.1 Definition of Functional Equation (Castillo 1998)

A Functional equation is an equation in which the unknowns are functions, excluding differential and integral equations.

3.5.2 Examples of Functional Equations:

3.5.2.1 Cauchy's Equation

The four most common Cauchy's functional equation was given and the proofs of the theorem can be found in Aczel(1966) and Castillo and Ruiz-Cobo (1992)

Theorem 3.1: If the equation

$$f(x+y) = f(x) + f(y) ; x, y \in R \quad (3.10)$$

is satisfied for all real x, y and if the function $f(x)$ is (a) continuous at a point, or (b) nonnegative for small x , or (c) bounded in interval or (d) integrable or (e) measurable, then

$$f(x) = cx, \quad x \in R \quad (3.11)$$

where c is an arbitrary constant.

Theorem 3.2: The most general solutions of the functional equation

$$f(x+y) = f(x)f(y), \quad x, y \in R \text{ or } x, y \in R_{++} \quad (3.12)$$

which are continuous at a point are

$$f(x) = \exp(cx) \text{ and } f(x) = 0. \quad (3.13)$$

Theorem 3.3: The most general solution of the functional equation

$$f(xy) = f(x) + f(y), \quad x, y \in T, \quad (3.14)$$

which are continuous at a point, are

$$f(x) = \begin{cases} c \log(x), & \text{if } T = R_{++}, \\ c \log(|x|), & \text{if } T = R - \{0\}, \\ 0, & \text{if } T = R.. \end{cases} \quad (3.15)$$

Theorem 3.4: The most general solutions of the functional equation

$$f(xy) = f(x)f(y), \quad x, y \in T, \quad (3.16)$$

which are continuous at a point, are

$$\left. \begin{aligned}
 f(x) &= 1, \\
 f(x) &= \begin{cases} |x|^c, & x \neq 0, \\ 0, & x = 0, \end{cases} \\
 f(x) &= \begin{cases} |x|^c \operatorname{sgn}(x), & x \neq 0 \\ 0, & x = 0 \end{cases}
 \end{aligned} \right\}, \text{ if } T = R \quad (3.17)$$

$$\left. \begin{aligned}
 f(x) &= |x|^c \\
 f(x) &= |x|^c \operatorname{sgn}(x)
 \end{aligned} \right\}, \text{ if } T = R - \{0\}$$

$$f(x) = x^c, \text{ if } T = R_{++},$$

where c is an arbitrary real number, together with

$$f(x) = 0; f(x) = \begin{cases} 0, & |x| \neq 1, \\ x, & |x| = 1, \end{cases}; f(x) = \begin{cases} 0, & |x| \neq 1, \\ 1, & |x| = 1, \end{cases}$$

which are common to the three domains.

3.5.2.2 The Pexider's Equations:

Here, the solutions of the four most common pexider's functional equations are described

Theorem 3.5: Pexider's main equation

$$f(x+y) = g(x)+h(y) ; x,y \in R \text{ or } [a,b] \text{ with } a,b \in R \quad (3.18)$$

with f : (a) continuous at a point , or (b) non- negative for small x , or (c) bounded in an interval, is

$$f(x) = Ax + B + C ; g(x) = Ax+ B ; h(x) = Ax + C , \quad (3.19)$$

where A,B and C are arbitrary constants.

Theorem 3.6: The most general system of solutions of

$$f(xy) = g(x) + h(x) ; x,y \in R \text{ or } R_{++} \text{ or } R - \{0\}, \quad (3.20)$$

with f continuous at a point is

$$\begin{aligned}
 & \left. \begin{aligned} f(x) &= A \log(BCx) \\ g(x) &= A \log(Bx) \\ h(x) &= A \log(Cx) \end{aligned} \right\} ; x, y \in \mathbb{R}_{++} \\
 & \left. \begin{aligned} f(x) &= A \log(BC|x|) \\ g(x) &= A \log(B|x|) \\ h(x) &= A \log(C|x|) \end{aligned} \right\} ; x, y \in \mathbb{R} - \{0\} \\
 (3.21)
 \end{aligned}$$

$$f(x) = A + B ; g(x) = A ; h(x) = B ;$$

if $x, y \in \mathbb{R}$ or $\mathbb{R} - \{0\}$ or \mathbb{R}_{++} .

Theorem 3.7: The most general system of solutions of

$$f(x+y) = g(x) + h(y) ; x, y \in \mathbb{R} \quad (3.22)$$

with f continuous at a point is

$$f(x) = ABe^{Cx} ; g(x) = Ae^{Cx} ; h(x) = Be^{Cx}, \quad (3.23)$$

where A, B and C are arbitrary non-zero constants, together with the trivial solutions.

$$f(x) = g(x) = 0 ; h(x) \text{ arbitrary}, \quad (3.24)$$

$$f(x) = h(x) = 0 ; g(x) \text{ arbitrary}.$$

Theorem 3.8: The most general system of solutions of

$$f(xy) = g(x)h(x) ; x, y \in \mathbb{R} \text{ or } \mathbb{R}_{++} \text{ or } \mathbb{R} - \{0\}, \quad (3.25)$$

with f continuous at a point is

$$f(x) = AB ; g(x) = A ; h(x) = B$$

$$\text{if } x,y \in \mathbb{R} \text{ or } \mathbb{R}_{++} \text{ or } \mathbb{R} - \{0\}, \quad (3.26)$$

$$\left. \begin{aligned} f(x) &= AB x^C \\ g(x) &= A x^C \end{aligned} \right\} \\ \text{if } x,y \in \mathbb{R}_{++} \quad (3.27)$$

$$h(x) = B x^C$$

$$f(x) = g(x) = 0; h(x) \text{ arbitrary}, (3.28)$$

$$f(x) = h(x) = 0; g(x) \text{ arbitrary.}$$

$$\left. \begin{aligned} f(x) &= A B |x|^C \\ g(x) &= A |x|^C \\ h(x) &= B |x|^C \end{aligned} \right\} \text{ or } \left. \begin{aligned} f(x) &= A B |x|^C \operatorname{sgn}(x) \\ g(x) &= A |x|^C \operatorname{sgn}(x) \\ h(x) &= B |x|^C \operatorname{sgn}(x) \end{aligned} \right\} \text{ if } x,y \in \mathbb{R} - \{0\},$$

$$\left. \begin{aligned} f(x) &= \begin{cases} AB|x|^C & x \neq 0 \\ 0 & x=0 \end{cases} \\ g(x) &= \begin{cases} A|x|^C & x \neq 0 \\ 0 & x=0 \end{cases} \\ h(x) &= \begin{cases} B|x|^C & x \neq 0 \\ 0 & x=0 \end{cases} \end{aligned} \right\} \text{ or} \quad (3.29)$$

$$\left. \begin{aligned} f(x) &= \begin{cases} AB|x|^C \operatorname{sgn}(x) & x \neq 0 \\ 0 & x=0 \end{cases} \\ g(x) &= \begin{cases} A|x|^C \operatorname{sgn}(x) & x \neq 0 \end{cases} \end{aligned} \right\}$$

$$h(x) = \begin{cases} 0 & x=0 \\ B|x|^C \operatorname{sgn}(x) & x \neq 0 \\ 0 & x=0 \end{cases} \quad \text{if } x, y \in \mathbb{R},$$

Where A, B and C are arbitrary constants.

3.5.2.3: Uniqueness

Theorem 3.9: Two solutions $\{ f_1, g_1, h_1 \}$ and $\{ f_2, g_2, h_2 \}$ of the functional equation (3.28) are related by :

$$f_2(x) = f_1(x); g_2(x) = g_1(x + \alpha); h_2(x) = h_1(x) + \beta, \quad (3.30)$$

where α and β are arbitrary constants.

3.5.2.4 The Uniqueness Model Functional Equation:

Theorem 3.10: (Uniqueness of representation of $F(x, y) = k(f(x) = g(y))$) if the function $F(x, y)$

has the following two representations.

$$F(x, y) = f_3^{-1}[f_1(x) + f_2(y)] = g_3^{-1}[g_1(x) + g_2(y)],$$

$$x, y \in \mathbb{R} \text{ or } [\alpha, \beta] \text{ with } \alpha, \beta \in \mathbb{R}, \quad (3.31)$$

where the functions f_i, g_i ($i=1, 3$) are continuous and strictly monotonic functions. Then, we must have

$$\begin{aligned} f_1(x) &= ag_1(x) + b \\ f_2(y) &= ag_2(y) + c \\ f_3^{-1}(u) &= g_3^{-1}((u-b-c)/a), \end{aligned} \quad (3.32)$$

where a, b and c are arbitrary constants.

3.5.3: Functional Equations in Functions of Several Variables

For the functions involving several unknown input variables with a single output is described here, this is the most important generalization of functional equations mostly one come out while dealing with the general problems.

3.5.3.1 The Generalized Associativity Functional Equation

Theorem 3.11: (Generalized associativity equation)

The general continuous on a real rectangle of the functional equation

$$F[G(x,y),z]=K[x,N(y,z)] \quad (3.33)$$

With G invertible in both variables, F invertible in the first variable for a fixed value of the second variable and K and N invertible in the second variable for a fixed value of the first , is

$$\begin{aligned} F(x,y) &= k[f(x) + r(y)], G(x,y) = f^{-1}[p(x) + q(y)], \\ K(x,y) &= k[p(x)+n(y)], N(x,y) = n^{-1}[q(x) + r(y)], \end{aligned} \quad (3.34)$$

Where f,r,k,n,p and q are arbitrary continuous and strictly monotonic functions. The two sides of (3.33) can be written as

$$K[p(x)+q(y)+r(z)]. \quad (3.35)$$

Theorem 3.12: (Generalized associativity equation (uniqueness))

Assume there are two sets of functions $\{k_1,p_1,r_1,q_1\}$ and $\{k_2,p_2,r_2,q_2\} \exists$

$$k_1[p_1(x)+q_1(y)+r_1(z)]= k_2[p_2(x) + q_2(y) + r_2(z)] \quad (3.36)$$

then we must have

$$k_2(u) = k_1((u-b-c-d)/a)$$

$$p_2(x) = ap_1(x) +b,$$

$$q_2(y) = aq_1(y) +c,$$

$$r_2(z) = ar_1(z) +d,$$

where a,b,c and d are arbitrary constants.

In addition to the above several other functional equations can be used. All the above are the overview of the kind of functions to be used when dealing with different aspects in functional networks. For further study refer (Castillo 1998)

3.6: MODELS IN FUNCTIONAL NETWORK:

In analyzing the problems in Functional Network initially, the structure of the network has to be chosen. This is followed by simplifying the structure of the network by using the functional equations and reducing the number of arguments in the neuron functions. After simplification of the functional network, uniqueness of the problem leading to important conditions is to be satisfied for the estimation of the model to be correct. In all these aspects functional equations play an important role.

The important functional network models, detailed analyses of the simplification and the uniqueness of representation of the problem are presented in the following section.

Some important functional network models are as follows:

1. The Uniqueness model
2. The Associative Model
3. The Generalizes Associative Model
4. The Separable Model
5. The Generalized Bisymmetry Model
6. Serial Functional Model
7. Independent Multiple Output Models
8. Dependent Multiple Output Network
9. One Layer Functional Network

Though all these models are used in the analyses relevant to the problem under consideration, only the above first three are explained here, and description of the other models are not much significant and also beyond the scope of the, thesis and for further knowledge refer (Castillo 1998).

1. THE UNIQUENESS MODEL:

The uniqueness model is an extension of the Associative model. The architecture is shown in Fig 3.6, and the output z can be written as a function of the input x and y as follows:

$$z = f_3^{-1}(f_1(x) + f_2(y)) \quad (3.37)$$

Simplification of the Model:

Since no arrows are convergent to the storing units included in the network, further simplification was not possible.

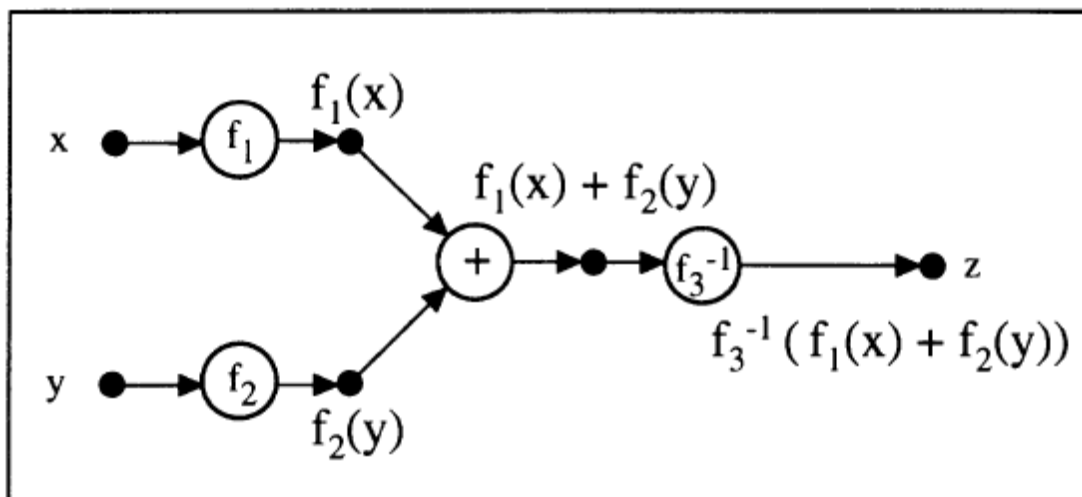


Fig 3.3: Uniqueness Functional Network

Uniqueness of Representation:

Consider two different triplets of functions $\{f_1, f_2, f_3\}$ and $\{g_1, g_2, g_3\}$, \exists their equivalent associated functional Network was as follows:

$$F(x, y) = f^{-1}[f_1(x) + f_2(y)] = g_3^{-1}[g_1(x) + g_2(y)] \quad (3.38)$$

$$f_1(x) = ag_1(x) + b$$

$$f_2(x) = ag_2(y) + c \tag{3.39}$$

$$f_3(x) = g_3^{-1}\left(\frac{u-b-c}{a}\right)$$

Where a, b and c are arbitrary constants i.e. if they are replaced in the Eq. 4.2 we obtain the same F(x,y), however might be the values of a, b and c are chosen..

Learning the Model:

Learning the functional Eq. 4.2 is equivalent to learning the functions $f_1(x)$, $f_2(x)$ and $f_3(x)$. thus, learning reduces to estimating these functions from the triplet data set $\{(x_{1i}, x_{2i}, x_{3i}) | x_{3i} = F(x_{1i}, x_{2i}); i = 1, 2, \dots, n\}$

Minimization can be done using sum of square errors, given by the equation

$$Q = \sum_{i=1}^n e_j^2$$

$$\sum_{i=1}^n e_j^2 = \sum_{i=1}^n \left(x_{3i} - \sum_{l=1}^{m_3} a_{3l} \phi_{3l} \left(\sum_{s=1}^2 \sum_{i=1}^{m_s} a_{sj} \phi_{sj}(x_{si}) \right) \right)^2, \tag{3.39}$$

it is constrained to

$$f_l(x_0) \equiv \sum_{j=1}^{m_l} a_{lj} \phi_{lj}(x_0) = \alpha_l; l = 1, 2, 3, \tag{3.40}$$

The learning can be done either using linear model or non linear model (Castillo 1998), where α_l and x_0 are constants, these are the conditions to achieve the uniqueness of solution.

Solving Eq. 4.6 and replacing them in Eq. 4.5, the equation of the coefficient take the form

$$a_{l1} = \frac{\alpha_l - \sum_{j=2}^{m_l} a_{lj} \phi_{lj}(x_0)}{\phi_{l1}(x_0)}; \quad l = 1, 2, 3 \quad (3.41)$$

2. ASSOCIATIVITY FUNCTIONAL NETWORK:

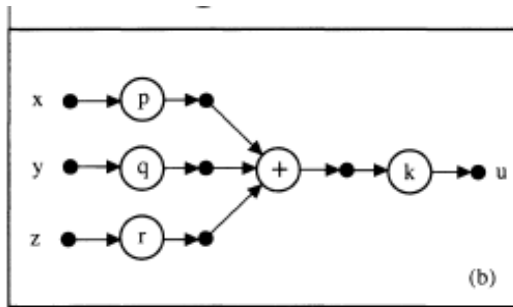


Fig 3.4: Associativity Functional Network

Assume two inputs x_1, x_2 and the output x_3 is given. The obtained functional network for this model is as follows:

$$f_s(x_s) = \sum_{i=1}^m a_{si} \phi_{si} \quad (3.42)$$

Where $s=1, 2, \dots, m_s$

ϕ_{si} , shape function which can be any polynomial, trigonometric, exponential or any admissible function, or simply called shape functions. The function f_3 can be expressed as:

$$f_3(x_3) = \sum_{i=1}^m a_{3i} \phi_{3i} \quad (3.43)$$

From the input functions, error function obtained can be written as:

$$e_j = f_1(x_1) + f_2(x_2) - f_3(x_3) \quad (3.44)$$

Minimizing the equation using the sum of least square error

$$Q = \sum_{j=1}^n \left(\sum_{i=1}^n a_i [\phi_i(x_{1j}) + (\phi_i(x_{2j}) - \phi_i(x_{3j}))] \right)^2 \quad (3.45)$$

Subject to constraints

$$f(x_0) \equiv \sum_{i=1}^m a_i \phi_i(x_0) = \alpha, \quad (3.46)$$

α is a real constant. Thus using Lagrangean minimization technique the equation is minimized and the constants are obtained.

The minimum corresponds to

$$Q_\lambda = \sum_{j=1}^n \left(\sum_{i=1}^m a_i b_{ij} \right)^2 + \lambda \left(\sum_{i=1}^m a_i \phi_i(x_0) - \alpha \right),$$

where

$$b_{ij} = \phi_i(x_{1j}) + \phi_i(x_{2j}) - \phi_i(x_{3j}) \quad (3.47)$$

$$\frac{\partial Q_\lambda}{\partial a_r} = 2 \sum_{j=1}^n \left(\sum_{i=1}^m a_i b_{ij} \right) b_{rj} + \lambda \phi_r(x_0) = 0; \quad r = 1, 2, \dots, m,$$

$$\frac{\partial Q_\lambda}{\partial \lambda} = \sum_{i=1}^m a_i \phi_i(x_0) - \alpha = 0,$$

In matrix form the above can be represented as

$$\begin{pmatrix} BB^T & \phi_0 \\ \phi_0^T & 0 \end{pmatrix} \begin{pmatrix} a^T \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix},$$

Where,

B- matrix of coefficients b_{ij} , $a = a_1, a_2, \dots, a_m$.

3. SEPERABILITY FUNCTIONAL NETWORK:

The general separable model is represented as shown in the figure

Simplification of the model:

Let $f_1(x), f_2(x), \dots, f_n(x)$ and $g_1(x), g_2(x), \dots, g_n(x)$ are two sets of linearly independent functions, the general solution is given by:

This can be written as:

$$z = F(x, y) = \sum_{i=1}^n f_i(x)r_i(y)$$

which can be written as

$$\sum_{i=1}^k f_i(x)g_i(y) = 0$$

$$f_j(x) = \sum_{j=1}^r a_{jk} f_k(x); \quad j = r + 1, \dots, k$$

$$g_s(y) = -\sum_{j=1}^{k-r} a_{js} g_{r+j}(y); \quad s = 1, \dots, r. \tag{3.49}$$

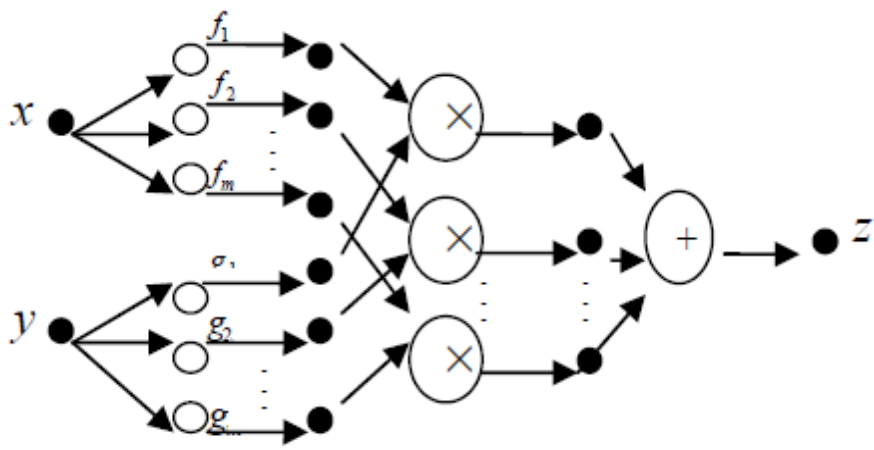


Fig: 3.5 Separability Functional Network with two inputs and one output

Learning the Model:

For learning the functional conditions least square method technique was obtained, and the error can be measured as:

$$e = x_{oi} - \sum_{i=1}^r \sum_{j=1}^{s-r} c_{ij} f_i(x_{1k}) g_j(x_{2k}) \quad i = 1, \dots, n$$

For optimum coefficients it is minimised as :

$$\frac{\partial Q}{\partial c_{pq}} = 2 \sum_{k=1}^n e_k f_p(x_{1k}) g_q(x_{2k}) = 0; \quad p = 1, \dots, r; q = 1, \dots, r - s \quad (3.50)$$

4. GENERALIZED ASSOCIATIVITY MODEL:

The output of the Generalize Associative functional Network can be represented as the function of $G(x,y)$ which is a function of inputs x,y and the input z , or as a function of $N(z,y)$ (a summary of inputs z,y) and input x , this can be represented with the following equation

$$F[G(x,y),z]=K[x,N(y,z)] \quad (3.51)$$

4.3.1 Simplification of the model

The general solution of (4.21), as shown in the Theorem 3.11, is:

$$\begin{aligned} F(x,y) = k[f(x)+r(y)] \quad ; \quad G(x,y) = f^{-1}[p(x)+q(y)]; \\ K(x,y) = k[p(x)+n(y)] \quad ; \quad N(x,y) = n^{-1}[q(x)+r(y)]; \end{aligned} \quad (3.52)$$

Where f, r, k, n, p and q are arbitrary continuous and strictly monotonic functions. Substituting (4.22) in (4.21) we get

$$F [G(x,y),z] = K[x, N(y ,z)] = u = k(p(x) + q(y) + r(z)). \quad (3.53)$$

Thus, the functional network in Figure 4.3(a) is equivalent to the functional network in Figure 4.3(b).

4.3.2 Uniqueness of Representation

In Theorem 3.12 we have seen that if there are two sets of functions $\{k_1, p_1, q_1, r_1\}$ and $\{k_2, p_2, q_2, r_2\}$ such that

$$k_1 [p_1(x) + q_1(y) + r_1(z)] = k_2 [p_2(x) + q_2(y) + r_2(z)]; \quad (3.54)$$

Then we must have

$$(3.55)$$

$$k_2(u) = \left(\frac{u - b - c - d}{a} \right),$$

$$p_2(x) = ap_1(x) + b,$$

$$q_2(y) = aq_1(y) + c,$$

$$r_2(z) = ar_1(z) + d,$$

Where a, b, c, d are arbitrary constants. Thus, uniqueness of solution requires fixing the functions k, p, q and r at a point.

4.3.3 Learning the Model

The problem of learning from data, the functional network in Figure 4.3(b) involves estimating the functions k, p, q, r in (4.23). To this end, we write (4.23) in the form

$$(u) = p(x) + q(y) + r(z), \quad (3.56)$$

And define the error

$$e_i = \hat{p}(x_{1i}) + \hat{q}(x_{2i}) + \hat{r}(x_{3i}) - \hat{k}(x_{4i}); \quad i = 1, \dots, n, \quad (3.57)$$

Where n is the sample size and $\{(x_{1i}, x_{2i}, x_{3i}, x_{4i}) \mid i = 1, \dots, n\}$ is the observed sample. We have used x_1, x_2, x_3, x_4 to denote x, y, z, u respectively.

Suppose that each of the functions is a linear combination of known functions from given families, i.e. :

$$\begin{aligned} \hat{p}(x_1) &= \sum_{j=1}^{m_1} a_{1j} \phi_{1j}(x_1) ; \hat{q}(x_2) = \sum_{j=1}^{m_2} a_{2j} \phi_{2j}(x_2), \\ \hat{q}(x_3) &= \sum_{j=1}^{m_3} a_{3j} \phi_{3j}(x_3) ; \widehat{k}^{-1}(x_4) = \sum_{j=1}^{m_4} a_{4j} \phi_{4j}(x_4), \end{aligned} \quad (3.58)$$

Where the coefficients a_{kj} are the parameters of the functional network. Then , the sum of square errors becomes a linear function of the coefficients a_{kj} :

$$Q = \sum_{i=1}^n e_i^2 = \sum_{i=1}^n \left(\sum_{k=1}^4 \sum_{j=1}^{m_k} a_{kj} \phi_{kj}(x_{ki}) \right)^2 \quad (3.59)$$

To guarantee the uniqueness of representation we specify the values of the four functions p, q, r, k^{-1} each at a point, that is :

$$\begin{aligned} \hat{p}(\alpha_1) &= \sum_{j=1}^{m_1} a_{1j} \phi_{1j}(\alpha_1) = \beta_1 ; \\ \hat{q}(\alpha_2) &= \sum_{j=1}^{m_2} a_{2j} \phi_{2j}(\alpha_2) = \beta_2 ; \\ \hat{r}(\alpha_3) &= \sum_{j=1}^{m_3} a_{3j} \phi_{3j}(\alpha_3) = \beta_3 ; \\ & \quad (3.60) \end{aligned}$$

$$\widehat{k}^{-1}(\alpha_4) = - \sum_{j=1}^{m_4} a_{4j} \phi_{4j}(\alpha_4) = \beta_4 ;$$

Where (α_k, β_k) ; $k = 1,2,3,4$ are the selected points. Then ,the Lagrange multipliers technique leads to the auxiliary function

$$Q_\lambda = \sum_{i=1}^n \left(\sum_{k=1}^4 \sum_{j=1}^{m_k} a_{kj} \phi_{kj}(x_{ki}) \right)^2 + \sum_{k=1}^4 \lambda_k \left(\sum_{j=1}^{m_k} a_{kj} \phi_{kj}(\alpha_k) - \beta_k \right) \quad (4.31)$$

The minimum can be obtained by solving the following system of linear equations, where the unknowns are the coefficients in the set $\{ a_{kj} | j = 1, \dots, m_k; k = 1,2,3,4 \}$ and the multipliers

$$\lambda_1, \dots, \lambda_4 .$$

$$\frac{\partial Q_\lambda}{\partial a_{sr}} = 2 \sum_{i=1}^n e_i \phi_{sr}(x_{si}) + \lambda_s \phi_{sr}(\alpha_s) = 0;$$

$$\frac{\partial Q_{\lambda}}{\partial \lambda_s} = \sum_{j=1}^{m_s} a_{sj} \phi_{sj}(\alpha_s) - \beta_s = 0 ; \quad (3.61)$$

Then, solving this linear system of equations we get the optimal neuron functions for a given problem.

Alternatively, the non-linear method can also be used, but the linear method seems more convenient.

3.7 MODEL SELECTION:

3.7.1 Introduction:

For the given data, selection of the topology of the network, simplifying the network and achieving the uniqueness of the network by using the functional equations was described in the above section. While learning, to approximate the neuron functions we chose different sets of linearly independent functions. Hence, the significance in solving the problem consists of selecting the optimal model using the Minimum Description Length Principle (MDLP). The description length is a measure that computes not only different functional networks but also the quality of different approximations. Using this principle, for a given problem the best functional network is approximated.

3.7.2 Necessity of Model Selection:

For the given data if we assume $\{(x_{1i}, x_{2i}, y_i)\} | i \in I$ are the 2 inputs and one output of the data, and the functional network of the model is as follows

$$y_i = f^{-1}[f(x_{1i}) + f(x_{2i})]$$

This step as described above to be followed by approximating with the functional neurons f by some of the basis function models as Polynomial , Fourier, Exponential, Trigonometric approach as follows:

1. Polynomial approximation:

$$f(x) = \sum_{i=1}^n a_i x^i, \quad (3.62)$$

The approximation of the above is based on the set of linearly independent function as $\{1, x, x^2, \dots, x^n\}$

2. Fourier approach:

$$f(x) = a_0 + \sum_{i=1}^n \{a_{2i-1} \sin(ix) + a_{2i} \cos(ix)\},$$

This can be further approximated in simplified method on the basis of set of linearly independent functions:

$$\{1, \sin(x), \dots, \sin(nx), \cos(x), \dots, \cos(rx)\}.$$

The same approach is applied for the any type of basis function selected. After approximating the neurons with one of the basis function, the best model of the above all to be chosen, and determine the best value 'n' for the selected model. Inorder to approximate this, the least squared sum of square errors developed in the later sections becomes more complex, hence the necessity of measuring the quality of fit is required, as we look forward the MDLP supplements in solving the problem without much complexity.

3.7.3 The Minimum Description Length Principle:

Based on the data of the model, the parameters θ , parametric space, model space M , the model $f_m(x/\theta)$ ($m \in M$) associated probability $\pi_m(\theta)$, the way of encoding the model, i.e. minimization criteria. The minimization criteria can be done in three ways,

1. Based on m
2. Based on θ
3. Based on δ_j

Rissanen (1989) gave a brief description leading to an approximate solution, and came up with a general equation to perform the MDLP. While applying it to Functional Networks, Castillo(1998), has used this equation directly, and it is as follows:

If we consider all the data used to estimate the parameters θ , i.e. $n_j=n \forall j$, and the error is as follows:

$$e_j(\theta) = f_m(y_j | \theta) - f_m(x_{1j} | \theta) - f_m(x_{2j} | \theta) \quad (3.63)$$

are normal $N(0, \sigma^2)$, we have

$$-\log f_m(x | \theta) = \frac{n}{2} \log \left(\frac{1}{n} \sum_{j=1}^n e_j(\theta)^2 \right) \quad (3.64)$$

The above equation leads to

$$L(x) = -\log \pi_m(\theta) + \frac{k \log n}{2} + \frac{n}{2} \log \left(\frac{1}{n} \sum_{j=1}^n e_j(\theta)^2 \right) \quad (3.65)$$

With the above equation different approximations and different functional networks can also be compared using several different methods. Out of those few important models are described here.

Exhaustive method: In this method $L(x)$ of all possible functional networks and different subsets of approximating functions is calculated and the one leading to smallest $L(x)$ is selected, but the disadvantage is that it requires lot of computational power.

Forward method: For all possible models of functional networks starting with single parameter selection process continues which outputs with minimum value of $L(x)$. Next by selecting new model and incorporating another parameter, leading to smallest value of $L(x)$, all this process comes to end until no improvement in $L(x)$ is obtained.

Backward method: In this method, it starts the model with all parameters and removes the parameter which gives smallest $L(x)$, this elimination continues until the no improvement in the model was observed.

Forward-Backward method: For all possible models of functional networks starting with single parameter selection process continues which outputs with minimum value of $L(x)$. Next by selecting new model and incorporating another parameter, leading to smallest value of $L(x)$, all this process comes to end until no improvement in $L(x)$ is obtained. After this the reverse process is applied i.e. it sequentially omits the parameter leading to the smallest value of $L(x)$. Without adding or eliminating the parameters this double process continues until no improvement in $L(x)$ is observed.

Backward-forward method: In this method the backward method is followed as explained above leading to smallest $L(x)$, and next it is followed by the forward method but starting with the obtained model and this double process continues until no improvement in $L(x)$ is observed neither by adding or deleting the model.

3.8 NOTES ON THE PRESENT STUDY:

The development of present model follows the following steps:

Selection of topology:

The first step in the functional network is selection of initial topology. Since the selection of topology depends on the problem chosen, the generalized equation is represented as follows:

$$y = \sum_{r=1}^m \sum_{rk=1}^n C_{r1} \dots C_{rk} \phi_{r1}(x_1) \dots \phi_{rk}(x_k) \quad (3.66)$$

where, y is the model parameter and $C_{r1} \dots C_{rk}$ are the weights to be estimated and $\phi_{r1} \dots \phi_{rk}$ are the family of linearly independent functions such as polynomial $(1, x, x^2, \dots, x^n)$, Fourier functions $(1, \sin(x), \cos(x), \sin(2x), \cos(2x), \dots)$, or exponential functions. Any of these can be selected and can be learned accordingly.

Simplification of the model:

In the above network assuming all the coefficients in between the functions are zero and the simplified model can be written as follows:

$$y = \sum_{r1=1}^{m1} C_{r1} \phi_{r1}(x_1) + \sum_{r2=1}^{m2} C_{r2} \phi_{r2}(x_2) + \dots + \sum_{rk=1}^{m3} C_{rk} \phi_{rk}(x_k) \quad (3.67)$$

The above equation can be represented in matrix form as

$$y = \langle c \rangle \{ \phi \} \quad (3.68)$$

Where c is a row matrix ($c = c_1, c_2, \dots, c_n$), ϕ is a column matrix ($\phi = \phi_1, \phi_2, \dots, \phi_m$)

Learning:

Now the next step includes learning the simplified network. The aim of learning includes the estimation of neural functions based on set of data, $\{D=I_i, O_i\}$, $i=1,2,\dots,n$, based on the set of Euclidean norm error E , which is given as follows:

$$E = \frac{1}{2} \sum_{i=1}^{m_i} \{O_i - f_i\}^2 \quad (3.69)$$

the associated optimization function may lead to a system of linear or non linear algebraic equations, and with reference to equation (3.69) the equation (3.68) can be represented as following:

$$E = y \cdot y^t = (y - \langle c \rangle \{ \phi \})^t \cdot (y - \langle c \rangle \{ \phi \}) \quad (3.70)$$

The coefficients can be estimated with the following equation:

$$c = (\phi^t \phi)^{-1} \phi^t y \quad (3.71)$$

Thus with the obtained coefficients the output is obtained and validation and testing is done and if the obtained results is satisfactory it the model can be used to estimate other functions.

The flow chart represented in fig 3.6 shows the sequence of steps to be followed in solving problems in functional network

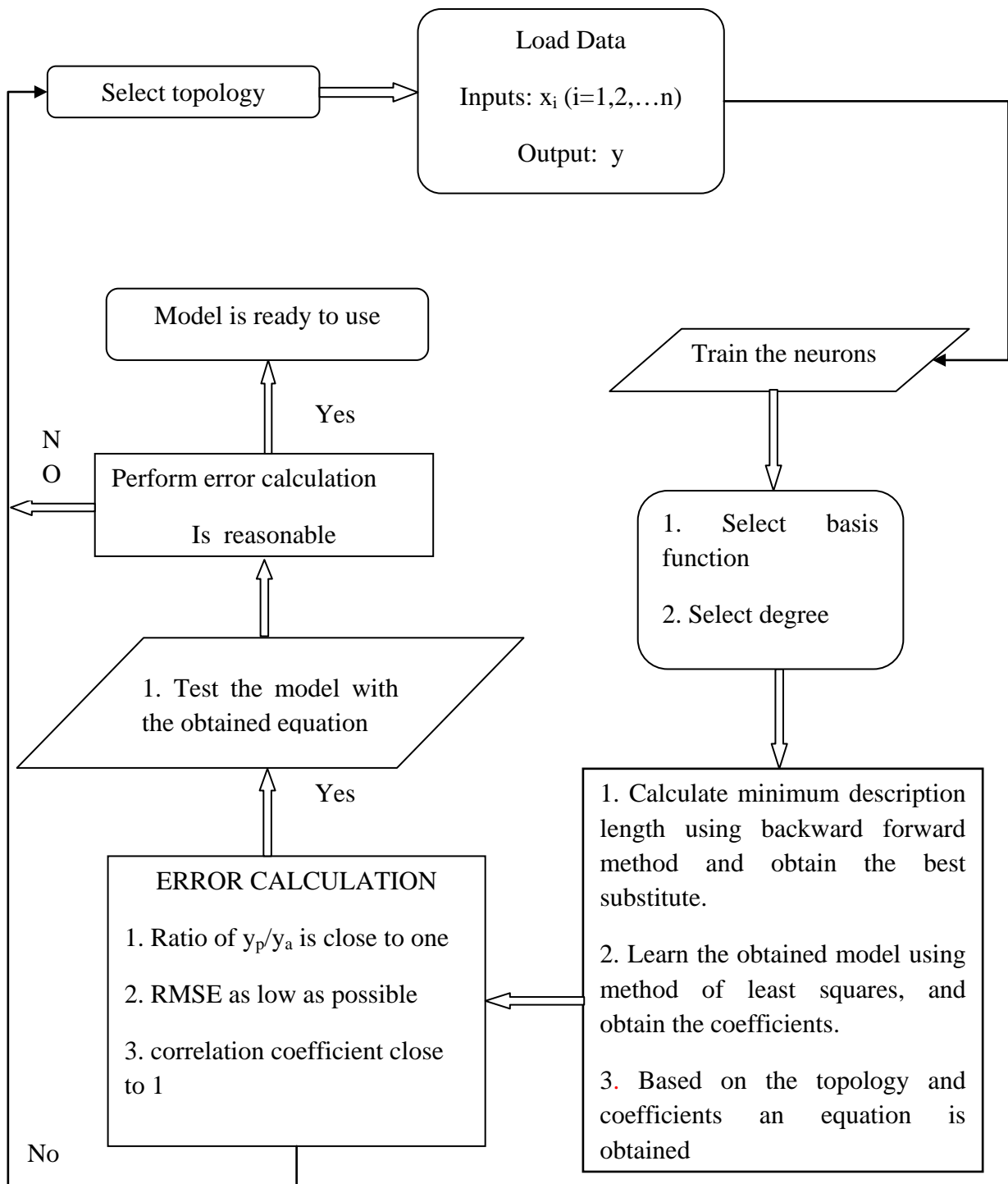


Fig 3.6 Flow chart for performing in Functional Network

CHAPTER 4

PREDICTION OF LATERAL LOAD CAPACITY OF PILES IN CLAY USING FUNCTIONAL NETWORK

4.1 INTRODUCTION

Pile foundations have its own significance in the present day construction field because of the increase in demand of the various requirements of the people. Significant research work went on in the design of piles taking into account various factors like location, load coming to the structures etc. (Poulos and Davis 1980), designed the axially loaded piles based on the static and dynamic equilibrium equations. Considering the structures like in that of tall buildings, off shore structures where lateral forces are significant, lateral loaded piles came to the design field. Analysis of this type of structures involves non linear techniques. Winkler's elastic model of soil, (Poulos and Davis 1980), equations is not suitable for the nonlinear soil behavior. A non linear analysis was carried out on, lateral load (p) capacity- deflection(y) i.e. (p-y) curves based on the theory of elasticity by Matlock and Reese (1962). Portugal and Secoe Pinto (1993) examined the behavior of laterally loaded piles using the non linear p-y curves and the finite element method.

These two methods most widely used in the design of laterally loaded piles. Though these methods gave good platform for design, but performing FEM analysis require extensive field characterization, and also it is complex to build constitutive models in clayey soils even with suitable laboratory testing methods. So for the initial estimation of load capacity of pile, field based methods (Hansen 1961, Broms 1964, Meyerhof 1976), has become much popular. All these methods require pile load test case histories and involve statistically derive empirical equations for determination of lateral load capacity of piles. But later came the computational methods like Artificial neural Networks (ANN), Support Vector machine (SVM) which proved to develop a good correlation technique for the above research field, (Das and Basudhar, 2006, Das *et al.*, 2011a). (Goh 1995, Chan *et al.* 1995, Goh 1996, Lee and Lee 1996, Teh *et al.* 1997, Abu-Kiefa 1998), has found that ANN is very effective in predicting the pile load capacity in both clayey soils and cohesion less soils. (Samui 2008), worked with the Support Vector machine and improved the efficiency of prediction than that of ANN, especially for the frictional resistance of the piles in clay. Similar studies done by (Das and Basudhar 2006), based on their studies and examining it with various Statistical methods they

observed that ANN is more effective than that of the studies conducted by Brom's and Hansen's method. With the same data, SVM and Gaussian Process Regression (GPR) models are developed and observed to get better than SVM model. They also compared the GPR model with the ANN model in terms of correlation coefficient (R) and Root mean square error (RMSE). Since R is only a biased estimate (Das and Sivakugan, 2010), it is difficult to predict the model in terms of under prediction or over prediction in terms of R value. RMSE gives the overall error in the data set but doesn't give the maximum deviation from the prediction of individual case. Generalization is an important aspect while dealing with the computational methods, i.e. the developed model should efficiently present itself during testing and validation. In the process of implementation of ANN for complex problems generalization had become a problem, therefore (2001, Das and Basudhar 2006) developed different methods for generalization like early stopping and cross validation. One of the reasons for this poor generalization is because of the magnitude of weights, in order to compensate with it Bayesian regularization neural network (BRNN) (Das and Basudhar 2008) have been used, which served as a platform to reduce the error due to magnitude of weights. The optimization process for error function can also be taken into account as the reason for poor generalization; the error function connected with weights and sigmoid functions is a highly nonlinear optimization problem and ends up with many local minima (Das and Basudhar 2008). The traditional non linear optimization methods are the initial point dependent, hence global optimization techniques like simulated annealing and genetic algorithm are widely used in training the ANN model (Morshed and Kaluarachchi 1998, Goh *et al.* 2005). Differential evolution neural network (DENN) (Ilonen *et al.* 2003, Das *et al.* 2011a), in which differential evolution optimization is used in training the feed-forward neural network. Das *et al.* (2011b) observed that the DENN performance is better than that of BRNN and it traditionally used Levenberg-Marquardt neural network (LMNN) in the analysis of slope stability. The ANN is coined as 'black box' because of its inconsistency in relating between the input and output. To have a brief description about SVM, in this the error parameter 'C' and sensitivity function 'e' are found out by trial and error process. As it is not possible to write down an equation using trained SVM model (Das *et al.* 2010, Das *et al.* 2011a) and developed ANN model, but now (Goh *et al.* 2005, Das and Basudhar 2006, Das and Basudhar 2008) with the trained ANN model and SVM model an equation was developed, but in that model SVM was not comprehensive. Later with the same data points many researches have come up with their own models with the tools like Genetic Programming (GP) and Multivariate Adaptive Regression Spline (MARS), every tool aims to

contribute its part to develop a best model and FN which was developed as powerful tool over ANN, added a small rung to the ladder over all this models.

The developed model was compared with the other models based on the following criteria.

1. Best fit calculation i.e. correlation coefficient (R) and Error (E) for predicted lateral load capacity (Q_p) and measured lateral load capacity (Q_m)
2. Mean μ and Standard deviation σ of the ratio Q_p/Q_m .
3. 50% and 90% cumulative probabilities (P_{50}, P_{90}) of the ratio Q_p/Q_m .

4.2 Database and Preprocessing

The present study takes the experimental database of Rao and Suresh kumar (1996). ANN model was developed by Das and Basudhar (2006), GPR and SVM models was developed by Pal and Deswal (2010) using the above database. The inputs of the data are depth (D), Length (L), eccentricity (e), Undrained Shear strength (S_u) and Q_{mas} as output are taken. Out of which 80% is used for training and remaining used for testing. FN doesn't show much difference in the output though data is normalized or not, but inorder to reduce the scaling effects this can be done.

Table 4.1: Data set used for prediction of lateral pile (Training)

D	L	e	C_u	Q_m	Q_p (FN)
18	300	50	10	89	98.23483
12.5	130	0	24	106	106
13	260	0	24	225	225
13	132.1	33.8	38.8	53	53
33.3	300	50	3.4	78.5	83.20318
12.3	300	50	3.4	29.5	22.92844
6.35	146.1	19.1	38.8	69.5	69.5
13.5	190	0	24	128	128
25.4	300	50	3.4	50	50.33178
25.4	300	50	10	118.5	110.5017
18.4	300	50	4	51	46.28954

12.3	300	50	10	81	83.09833
33.3	300	50	5.5	110.5	106.8979
25.4	300	50	7.2	90	88.9587
12.3	300	50	7.2	58	61.55536
18	300	50	10	116.5	98.23483
20.4	300	50	7.2	76.5	80.4745
6.35	139.7	25.4	38.8	65.5	65.5
20.4	300	50	3.4	38	41.84758
20.4	300	50	5.5	59.5	65.54229
25.4	300	50	5.5	75	74.02649
20.4	300	50	10	87	102.0175
12.3	300	50	5.5	44	46.62315
18	300	50	5.5	65	61.75965
18.4	300	50	7.2	86.5	77.37319
18	300	50	3.4	39	38.06494
13.5	300	50	4	36	34.80341
13.5	300	50	7.2	64	65.88706
13.5	300	50	5.5	50	50.95485
20.4	300	50	4	46	49.39085

Table 4.2set used for prediction of lateral pile (Testing)

Depth(D)	Length(L)	e	C _u	Q _a	Q _p (FN)
13.5	300	50	3.4	30	27.26013
18.4	300	50	5.5	65.5	62.44098
18.4	300	50	10	114	98.91616
18.4	300	50	3.4	42.5	38.74627
25.4	300	50	4	58	57.87505
18	300	50	4	49	45.60821
18	300	50	7.2	87	76.69186
12.3	300	50	4	35	30.47171

4.3 Results and Discussion

The methodology adopted by the FN in predicting the Lateral load capacity of piles was discussed in Sec 3.8. With Polynomial function as the basis function, and degree 3 the workability of functional network and the equation generated was discussed below:

$$y = a_0 + \sum_{i=1}^s \sum_{j=1}^n f_i(x_j) \quad (4.1)$$

Where, s = no of input variables

n= degree of the function

$$a_0 = -23.77$$

$$f_1(x_1) = 0.208x_1 - 0.061x_1^2 + 0.286x_1^3$$

$$f_2(x_2) = 0.286x_2 + 20.6406x_2^2 + 26.1716x_2^3$$

$$f_3(x_3) = 5.355x_3 + 3.3564x_3^2 - 20.8241x_3^3$$

$$f_4(x_4) = 6.5032x_4 + 7.4179x_4^2 + 3.5723x_4^3$$

The scatter of the predicted vs. actual values for training and testing data is drawn. From the graph we can depict the less scattering of the data proving the efficiency of functional network.

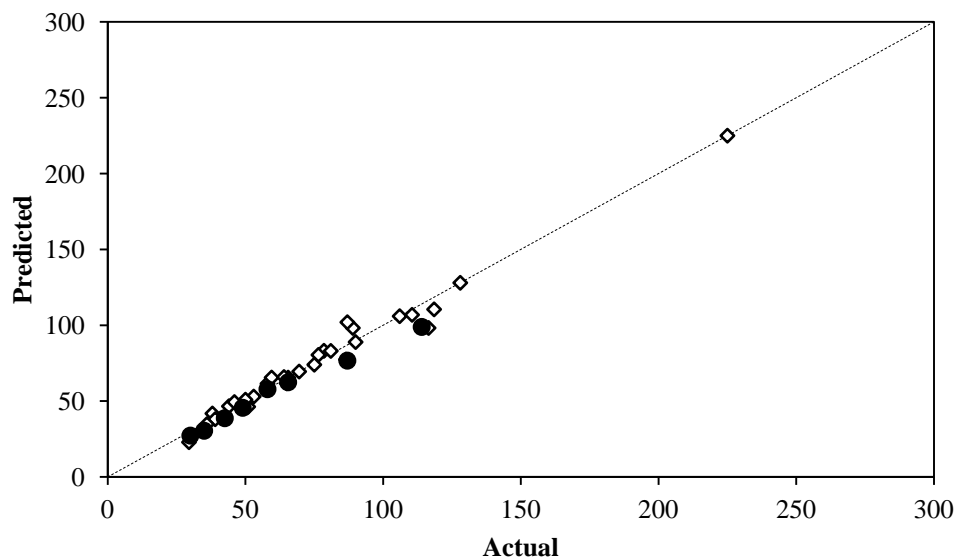


Fig: 4.1 Comparison of predicted and measured load capacity of piles for training and testing

From Fig: 4.1 we can observe that there is less scatter of data in the FN. Table 3.5 shows the statistical performance of various methods on the basis of R,E, AAE, RMSE, MAE. It was

observed that FN outperform the other models in the analysis of the present study, for both training and testing cases and also gives a very compact equation.

Table: 3.3 statistical performances

Models		Statistical Performances				
		R	E	AAE	MAE	RMSE
FN	Training	0.988	0.976	3.844	18.265	5.8141
	Testing	0.944	0.930	5.374	15.084	7.0424
GP	Training	0.980	0.961	5.337	24.378	7.381
	Testing	0.972	0.913	6.702	15.070	8.194
DENN	Training	0.980	0.959	5.647	18.705	7.667
	Testing	0.967	0.905	7.170	18.110	8.549
BRNN	Training	0.975	0.949	6.609	20.680	8.582
	Testing	0.899	0.734	10.800	33.169	14.312
Hansen	Training	0.950	0.209	30.712	65.360	33.825
	Testing	0.919	0.119	23.650	49.480	26.066
Broms	Training	0.967	0.807	12.391	48.660	16.703
	Testing	0.985	0.574	12.082	46.380	18.127
MARS	Training	0.970	0.940	7.258	32.875	9.108
	Testing	0.98	0.900	6.858	18.705	11.815

Briaud and Tucker (1988) , in mean while predicting the pile load capacity based on cone penetration test (CPT) have emphasized that statistical methods have to be carried out along with correlation coefficient. Abu-Farsakh and Titi (2004) and Das and Basudhar (2006) have used the

mean (μ) and standard deviation (σ) of ratio of predicted pile load capacity (Q_p) to the measured pile load capacity (Q_m) as important parameters in evaluating different models. These parameters are the important indicators of the precision and accuracy of the predicted model. under ideal conditions the mean is 1.0 and standard deviation is 0. The value if greater than 1 is an indication of over prediction, else under prediction. In the present study the μ (1.002,0.910) and σ are (0.080, 0.045) , shows the values are almost close to 1 and 0 respectively.

The other criterion like cumulative probability of Q_p/Q_m (Das and Basudhar 2006, Abu-Farsakh and Titi 2004) should also be considered for the evaluation of performance of different models.

The ratio Q_p/Q_m is arranged as per their values in an ascending order and the cumulative probability is calculated from the following equation:

$$p = \frac{i}{n+1} \tag{4.2}$$

Where i = order number given to the Q_p/Q_m ratio; n is the number of data points. If the computed value of 50% cumulative probability ($P50$) is less than unity, under prediction is implied; else over prediction. The ‘best’ model is that the obtained $P50$ value close to unity. The 90% cumulative probability ($P90$) reflects the variation in the ratio of Q_p/Q_m for the total observations. The model with $P90$ for Q_p/Q_m close to 1.0 is a better model.

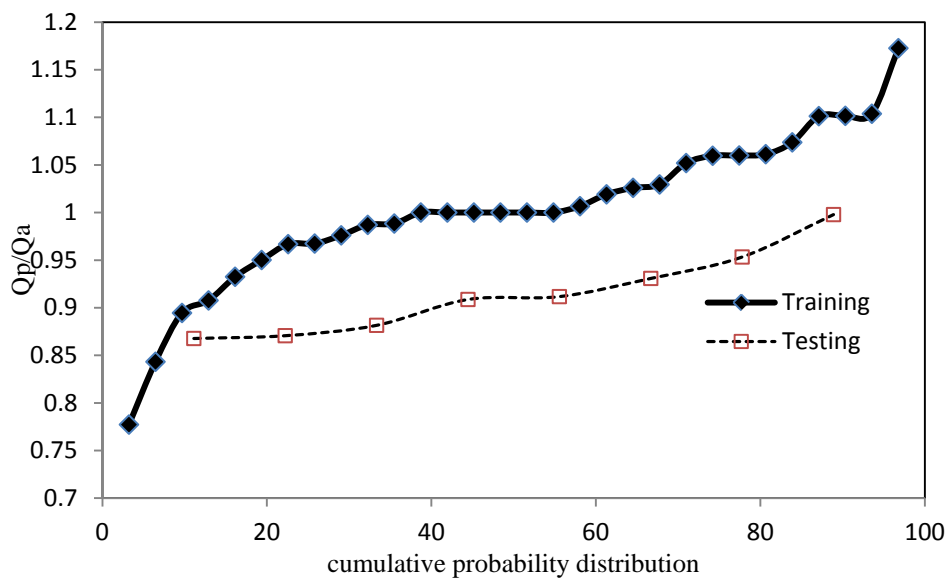


Fig 4.5: Cumulative probability plot of Q_p/Q_a for functional Network for training and testing

Table 4.5: Results of the cumulative probability distribution

	P ₅₀	P ₉₀
Training	1.0	0.9
Testing	1.1	1.0

5.4 CONCLUSION:

With the selected tool the developed models showed good correlation with the desired output and their performance is estimated based on following criteria obtained in testing.

1. Correlation coefficient
2. Root mean square error
3. Efficiency and
4. Cumulative probability distribution function (P₅₀ and P₉₀)

Table : Comparison criterion

Criterion	Value
R	0.944
RMSE	7.0424
Efficiency	0.976
P ₅₀ and P ₉₀	1.0 & 1.1

From the above table we can conclude that the error was almost minimum and the efficiency is 0.956, the obtained results are good and the also from the literature of the results of other statistic models one can observe that the functional network has performed better w.r.t. other tool with the selected model

CHAPTER 5

PREDICTION OF FACTOR OF SAFETY OF SLOPE USING FUNCTIONAL NETWORK

5.1 INTRODUCTION

The analysis of Factor of safety for both manmade and natural slopes is a great challenge for the civil engineers, in order to avoid catastrophic failures, save life, property etc. Generally limit equilibrium method is used for analyzing the factor of safety because of the ease of calculation and also of the accuracy obtained compared to that of the rigorous methods like finite difference, finite element and variational approach. Though limit equilibrium methods are always available for the determination of factor of safety, statistical methods also supplemented along with it in the analysis. Considering some of the case studies, Sha *et al.* (1994) initiated the application of statistical approach in the prediction of factor of safety in slope stability analysis. They took almost 46 case studies (29 failed and 17 stable), out of which 14 cases were for circular slope analysis (8 failed and 6 stable) and remaining wedge failure, for which they proposed separate regression equations for circular and wedge failure slopes using maximum likelihood method, and observed to have a strong correlation with the obtained results and the results of the LEM of about 0.911 for circular and 0.954 for wedge failure analysis. But, the results are not checked with the new set of data.

Using back propagation neural network (ANN) Sakellariou and Ferentinou (2005) predicted FOS and compared the results in terms of Mean Squared Error (MSE) with different number of training data set. To predict the stability number of layered slopes Samui and Kumar (2006) used ANN as an alternate statistical method to upper bound limit analysis. To predict the FOS of Yudonghe landslide (China) Wang *et al.* (2005) used four layers BPNN, with five input nodes, two hidden layers and two output nodes and found that the FOS is close to 1.1.

An alternate statistical method ANN was proposed by Samui and Kumar (2006), which include upper bound limit analysis to predict the stability number of layered slopes. Wang *et al.* (2005) used Back propagation neural network to predict the FS of landslide occurred in

China (Yudonghe) and found FS using four-layered BPNN model with 5 input nodes, 2 hidden layers and 2 output nodes.

5.2 DATABASE AND PROCESSING

Table 5.1: Training data set for prediction of Factor of safety

γ	C	ϕ	β	H	r_u	FS _a	
19.06	11.71	28	35	21	0.11	1.09	1.1817
28.44	150.05	45	53	214	0.5	2.05	2.0647
26	150.05	45	50	200	0	1.2	1.1852
12	0	0	16	4	0	0.625	0.7089
28.44	39.23	38	35	100	0	1.99	2.0394
23.47	0	32	37	214	0	1.08	1.0746
21.43	0	20	20	61	0.5	1.03	0.9724
22.4	100	45	45	15	0.25	1.8	1.8187
18.84	57.46	20	20	30.5	0	2.045	2.0666
28.44	57.46	40	45	100	0.5	2.045	1.8806
20	0	36	45	50	0.5	0.67	0.6401
18	5	30	20	8	0.3	2.05	2.0536
20.41	33.52	11	16	45.72	0.2	1.28	1.2026
16	70	20	40	115	0	1.11	1.2084
18.5	25	0	30	6	0	1.09	0.8951
24	0	40	33	8	0.3	1.58	1.6375
14	11.97	26	30	88	0.45	0.625	0.4929
12	0	30	45	8	0	0.8	0.8892
22.4	10	35	30	10	0	2	1.891
14.8	0	17	20	50	0	1.13	1.049
12	0	30	35	4	0	1.46	1.3149
20	0	36	45	50	0.25	0.79	0.7585
22	0	36	45	50	0	0.89	0.9364
20	0	24.5	20	8	0.35	1.37	1.457
23	0	20	20	100	0.3	1.2	1.134
21.51	6.94	30	31	76.81	0.38	1.01	1.095
25	120	45	53	120	0	1.3	1.2992
14	11.97	26	30	88	0	1.02	1.1312
21.82	8.62	32	28	12.8	0.49	1.03	1.298
18.68	26.34	15	35	8.23	0	1.11	1.2346
22.4	10	35	45	10	0.4	0.9	0.8278
18.84	14.36	25	20	30.5	0	1.875	1.7915
18.84	0	20	20	7.62	0.45	1.05	1.13
20.6	16.28	26.5	30	40	0	1.25	1.2754
20.41	24.9	13	22	10.67	0.35	1.4	1.4524

18	24	30.15	45	20	0.12	1.12	1.0046
18.84	15.32	30	25	10.67	0.38	1.63	1.6885
21.4	10	30.34	30	20	0	1.7	1.4812
18.5	12	0	30	6	0	0.78	0.9149
18.84	14.36	25	20	30.5	0.45	1.11	1.1532
19.63	11.97	20	22	12.19	0.41	1.35	1.2201
22	0	40	33	8	0.35	1.45	1.4419
20	20	36	45	50	0.25	0.96	0.9886
28.44	29.42	35	35	100	0	1.78	1.8185
20	20	36	45	50	0.5	0.83	0.8702
14	0	0	20	3.66	0	0.67	0.7423
16.5	11.49	0	30	3.66	0	1	0.9064
22	20	36	45	50	0	1.02	1.1665
12	0	30	45	8	0	0.86	0.8892
12	0	30	35	4	0	1.44	1.3149
22.4	10	35	45	10	0.4	0.9	0.8278
20	20	36	45	50	0.25	0.96	0.9886

Table 5.3: Testing data for prediction of factor of safety of slope

γ	C	ϕ	β	H	r_u	FS _a	
28.44	29.42	35	35	100	0	1.78	1.8185
20	0	36	45	50	0.25	0.79	0.7585
20	0	36	45	50	0.5	0.67	0.6401
22	0	40	33	8	0.35	1.45	1.4419
20	0	24.5	20	8	0.35	1.37	1.457
18.84	57.46	20	20	30.5	0	2.045	2.0666
16.5	11.49	0	30	3.66	0	1	0.9064
14	11.97	26	30	88	0	1.02	1.1312
22	20	36	45	50	0	1.02	1.1665
19.63	11.97	20	22	12.19	0.41	1.35	1.2201
18.84	0	20	20	7.62	0.45	1.05	1.13
24	0	40	33	8	0.3	1.58	1.6375

5.3 FUNCTIONAL NETWORK MODELING RESULTS:

Using prescribed basis function and degree of the selected function. The model in functional network was developed. Though by increase in degree the obtained results would be accurate at the same time the complexity in the problem also increases, hence a trade cut off is made in the present study, and the best FN model was obtained with degree 5 and polynomial basis

function

The equation predicted with the FN was presented below:

$$y = a_0 + \sum_{i=1}^n \sum_{j=1}^m f_i(x_j) \quad (5.1)$$

where, n= no. of variables

m=degree of variable

Here, n=5 and m=5, and the coefficient of the above equation 5.1 was written below:

$$a_0 = 0.1055$$

$$f_1(x_1) = -0.8624x_1 + 5.2381x_1^3 - 4.11x_1^5$$

$$f_2(x_2) = -8.8463x_2^2 - 8.8155x_2^3 + 7.095x_2^4 + 8.3969x_2^5$$

$$f_3(x_3) = 1.1646x_3 + 2.0845x_3^2 - 3.1044x_3^3 - 1.4989x_3^4 + 3.5769x_3^5$$

$$f_4(x_4) = -1.2395 + 0.7001x_4^2 + 1.2705x_4^5$$

$$f_5(x_5) = -2.0556x_5 - 4.3702x_5^2 + 1.7681x_5^3 + 4.6891x_5^4$$

$$f_6(x_6) = -2.4041x_6^3 + 0.1211x_6^4 + 2.1169x_6^5$$

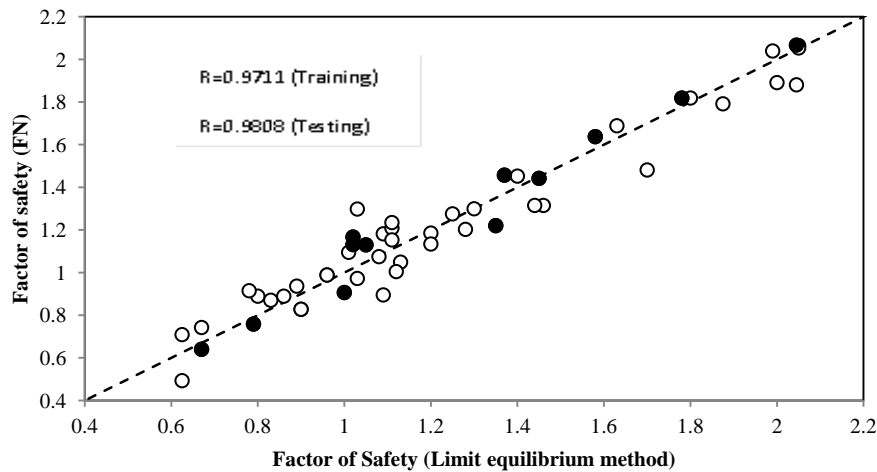


Fig: 4.1 Comparison of predicted and measured factor of safety for training and testing

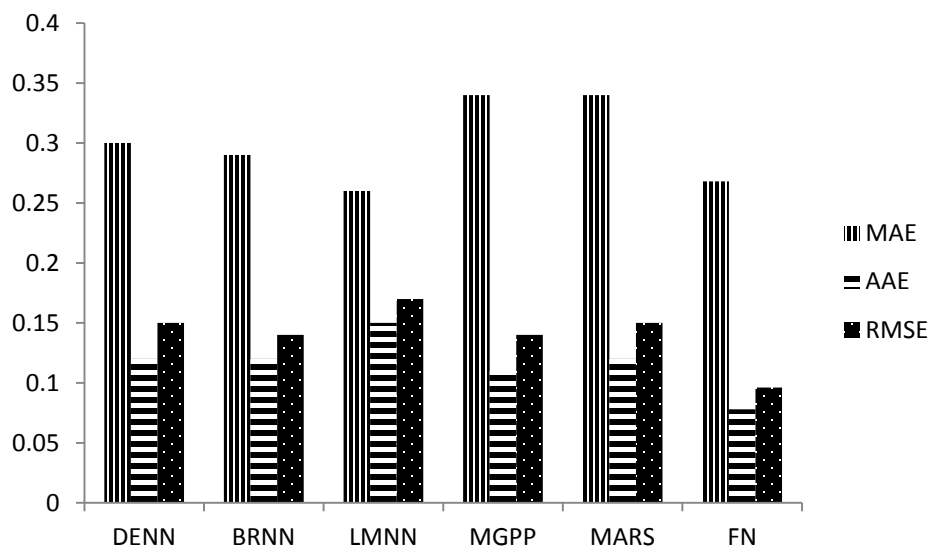


Fig: 5.2 Comparison of errors of DENN, BRNN, LMNN, MGPP, MARS, FN for training data

Table 5.3 Comparison of errors of various statistical methods for training FS

error	DENN	BRNN	LMNN	MGPP	MARS	FN
MAE	0.3	0.29	0.26	0.34	0.34	0.268
AAE	0.12	0.12	0.15	0.11	0.12	0.078
RMSE	0.15	0.14	0.17	0.14	0.15	0.0961

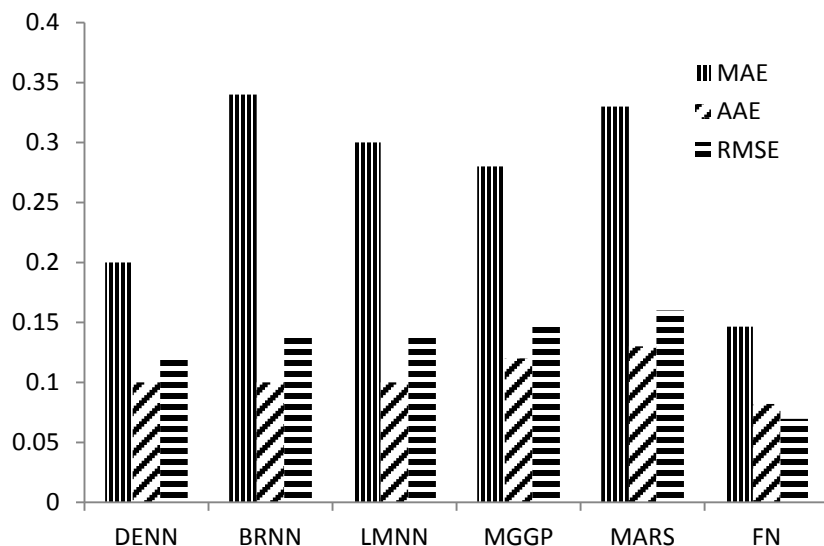


Fig: 5.3 Comparison of error results of DENN, BRNN, LMNN, MGPP, MARS, FN for prediction of FS for test data

Table 5.4 Comparison of errors of various statistical methods for training FS

error	DENN	BRNN	LMNN	MGPP	MARS	FN
-------	------	------	------	------	------	----

MAE	0.2	0.34	0.3	0.28	0.33	0.1465
AAE	0.1	0.1	0.1	0.12	0.13	0.082
RMSE	0.12	0.14	0.14	0.15	0.16	0.0696

Table 5.5: Statistical performance of ANN, SVM, MGGP, MARS AND FN model

Reference	Models	Coefficient of Correlation (R)		Coefficient of efficiency (E)	
		Training	Testing	Training	Testing
ANN (Das et al. 2011)	BRNN	0.937	0.920	0.871	0.885
	LMNN	0.902	0.923	0.807	0.846
	DENN	0.922	0.950	0.848	0.842
SVM (Samui 2008)	SVM-G	0.922	0.922	-	-
	SVM-P	0.983	0.844	-	-
	SVM-P	0.995	0.918	-	-
MGGP		0.924	0.929	0.852	0.851
MARS		0.917	0.915	0.842	0.825
FN		0.972	0.981	0.945	0.956

The performance of FN is compared with other statistical model in terms of R, E in table 5.5, from the above table it was observed that the performance of FN was increased.

Table 5.6: Cumulative probabilities depending on sorted Fp/Fu for ANN, SVM, MGGP, MARS and FN models

	P ₅₀		P ₉₀	
	Training	Testing	Training	Testing
FN	1.04	0.985	1.1	1.08
LMNN	0.97	1.016	1.22	1.25
BRNN	0.99	0.991	1.16	1.202
DENN	0.957	0.979	1.16	1.202
MGGP	1.02	1.04	1.23	1.129
MARS	0.976	1.055	1.22	1.179

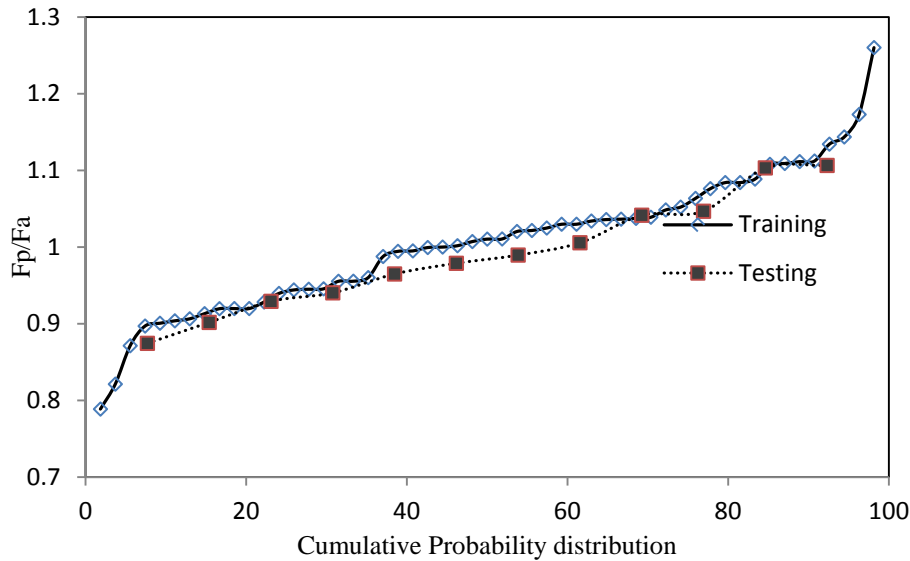


Fig 5.4: Cumulative Probability distribution for Training and testing data for FS_a/FS_p

5.4 CONCLUSION:

With the selected tool the developed models showed good correlation with the desired output and their performance is estimated based on following criteria obtained in testing.

1. Correlation coefficient
2. Root mean square error
3. Efficiency and
4. Cumulative probability distribution function (P_{50} and P_{90})

Table : Comparison criterion

Criterion	value
R	0.981
RMSE	0.0696
Efficiency	0.956
P_{50} and P_{90}	0.985 & 1.05

From the above table we can conclude that the error was almost minimum and the efficiency is 0.956, the obtained results are good and the also from the literature of the results of other statistic models one can observe that the functional network has performed better w.r.t. other tool with the selected model

CHAPTER 6

UPLIFT CAPACITY OF SUCTION CAISSON IN CLAY USING FUNCTIONAL NETWORK

6.1 INTRODUCTION

Caissons are used in the offshore structures to serve as anchors because they demand less construction time and give greater effectiveness. The caissons are designed for both static and cyclic load due to sustain both wind and loop current. The horizontal and inclined loads acting on it transfers the uplift force to the caisson anchors. The total uplift capacity of the caisson depends upon passive suction under caisson-sealed cap, self weight of caisson, frictional resistance along the soil-caisson interface, submerged weight of soil plug inside the caisson and uplift soil (reverse end bearing) bearing pressure (Albert et al. 1987). Hence, particularly in clayey soils suction pressure become more effective. Various methods used in analyzing the suction pressure in clayey soils. Upper bound analysis (Clukey et al. 1995), finite element method (Whittle and Kavvadas 1994; El-Gharbawy and Olson 2000; Zdravkovic et al. 2001; Cao et al. 2001, 2002a, 2002b), laboratory model (Goodman et al. 1961; Larsen 1989; Steensen-Bach 1992; Datta and Kumar 1996; Singh et al. 1996; Rao et al. 1997a, 1997b), centrifuge model (Clukey and Morrison 1993; Clukey et al. 1995) and prototype model tests (Hogervorst 1980; Tjelta et al. 1986; Dyvik et al. 1993; Cho et al. 2002) have been attempted to understand the axial and lateral load capacity of suction caisson for static and cyclic load and under different soil conditions. Therefore, development of sufficient accurate site model for a detailed numerical analysis requires extensive site characterization effort. The constitutive modeling of clay is very difficult even with considerable laboratory testing. Field tests are also expensive but various tests have been conducted to find out the feasibility of suction caisson in various soil types (Cho et al. 2002). However, several issues and uncertainties related to capacity estimation and failure mechanisms are still unresolved.

Rahman et al. (2001) used an ANN model to predict the uplift capacity of suction caisson in clay. The performance of the ANN model is found to be better than the FEM model in terms of correlation coefficient (R). Pai (2005) observed that FEM model is better compared to genetic algorithm based neural network model using the same database. (Goh et al. 2005; Das

and Basudhar 2006; Das and Basudhar 2008), developed an equation using ANN. But, ANN is associated with poor generalization for some complex problems (Das et al. 2012). Another group of artificial intelligence techniques; support vector machine (SVM) and relevance vector machine (RVM) are based on statistical learning theory (Vapnik 1998), are also used. The performances of SVM and RVM are found to be better than ANN models for some geotechnical engineering problems (Das et al. 2010; Das et al. 2011). Using GP (Muduli et al.) used GP to model the present study.

In the present study FN-based prediction model for uplift capacity (Q) of suction caisson in clay is developed using the database from literature (Rahman et al. 2001). Different statistical criteria like correlation coefficient (R), Nash-Sutcliffe coefficient of efficiency (E), root mean square error (RMSE), average absolute error (AAE), maximum absolute error (MAE) and normalized mean biased error (NMBE) are used to compare the FN model with FEM, ANN, SVM and RVM models.

6.2 DATABASE AND PROCESSING

Table 6.1: Data table for Prediction of uplift capacity of suction pressure in clay (Training)

L/d	S_u	T_k	θ	D/L	Q_a	Q_b
1.84	11	0	90	0	88.2	96.4021
1.84	1.84	1.84	1.84	1.84	1.84	1.8481
1.32	38	0	0	0.1	149	149.0155
1.4	5.5	0	10	0.56	71.8	71.8001
2.31	23.9	0	15	0.69	387.2	387.202
4	5.2	0	75	0.47	48.1	51.5135
2	25	0	90	0	244.1	244.9694
0.68	24	0	0	0	21.3	21.3114
0.43	4.2	0	80	0	48.7	48.7113
0.23	31	0	0	0.05	128.3	128.3119
1.4	9	0	0	0	37	37.0113
0.7	13.7	0	90	0	135	135.0228
1.5	1.8	0.0001	90	0	12.9	12.9043
4	5.2	0	90	0	48.8	48.8142

1.84	15.8	0	90	0	160.5	158.0023
2	5.8	0.0001	90	0	46.4	41.3759
1	5.8	0.0001	90	0	35.6	34.0786
0.23	24	0	0	0	72	72.0157
2	20.5	0	90	0	209.4	207.2601
1.4	9	0	0	0.5	70.5	70.5117
1.32	14.3	0	90	0	144.6	160.472
0.4	6.8	0	90	0	75	75.0219
2	1.8	0.0001	90	0	15.6	17.6495
0.75	2.5	0.04	90	0	10.1	6.7713
1.5	5.8	0.0001	90	0	38.1	36.6306
2	8.3	0	90	0	71.7	78.4442
2.31	21.6	0	11	0.68	370.4	370.4186
1.32	14.3	0	90	0	176.3	160.472
0.75	6	0.04	90	0	21.5	24.8645
1	1.8	0.0001	90	0	11.1	10.3523
1.32	38	0	0	0	133.1	133.122
2	3.6	0.0001	90	0	33.6	36.119
1.5	3.6	0.0001	90	0	28.8	31.3679
1	3.6	0.0001	90	0	26.4	28.8218
2	22.5	0	90	0	214.9	211.5938
2	6	0	90	0	66.3	65.4903
2	9	0	90	0	90.1	83.1542
1.84	11	0	90	0	105.8	96.4021
1.5	6	0.04	90	0	23	23.0181
2	7	0	90	0	80.2	69.6358
2	10.5	0	90	0	90.4	90.0339
0.75	6	0.0004	90	0	31	32.4145
2	24	0	90	0	245.3	242.7232
0.75	2.5	0.0004	90	0	15.7	14.3214
2	22.5	0	90	0	204.9	211.5938
2	2.4	0.0001	90	0	21.9	22.4301
2	7.8	0	90	0	64.5	74.8632

1.5	2.4	0.0001	90	0	18.7	17.6789
1.5	6	0.004	90	0	26.6	26.6226
1	2.4	0.0001	90	0	15.2	15.127
4	5.2	0	75	0.47	54.9	51.5135
2	7.5	0	90	0	70.5	72.7757
1.84	15.8	0	90	0	154.3	158.0023

Table 6.2: Data table for Prediction of uplift capacity of suction pressure in clay (Testing)

L/d	S _u	T _k	θ	D/L	Q _a	Q _b
1.32	38	0	0	0	134.9	133.122
0.75	2.5	0.004	90	0	13.2	10.3758
0.75	6	0.004	90	0	26	28.469
1.32	38	0	0	0.1	145.5	149.0155
1.32	14.3	0	90	0	149.9	160.472
1.5	6	0.0004	90	0	32.2	30.5623
1.84	11	0	90	0	86.4	96.4021
1.84	11	0	90	0	92.6	96.4021
2	8.5	0	90	0	75.3	79.8495
2	6	0	90	0	62.7	65.4903

6.3 RESULTS AND DISCUSSION

The present problem was modeled using Functional Network with exponential function as the basis function with degree 10 and the equation was represented below

$$y = a_0 + \sum_{i=1}^n \sum_{j=1}^m a_{ij} e^{x_i^j} \quad (6.)$$

where, i= no. of variables

j= degree of variable

The coefficients are presented in the table 6.3

Table 6.3: Coefficients of the obtained equation

A	1	2	3	4	5	6	7	8	9	10
1	0.000	0.000	- 0.0001	0.0011	- 0.0073	0.0292	- 0.0729	0.1089	-0.088	0.0291
2	-	0	0	0	0	0	- 0.0001	0.0001	- 0.0001	0
3	0	0.0005	- 0.2394	9.8934	-	- 9.7417	-	-	-	-
4	0.0022	- 0.0347	0.1206	-	-	-	-	-3.364	6.324	-3.048
5	0	-	0	0.0871	-	-	-	-	-	-

All the above coefficient are to multiplied with 1*E10.

The developed model in FN was compared with statistical performance

Table 6.4: Comparison of Statistical performances off different models

Models	Statistical Performances				
	R	E	AAE	MAE	RMSE
FN	0.997	0.997	5.357	10.572	5.3574
GP	0.997	0.988	8.065	27.055	11.155
ANN	0.991	0.975	12.204	32.820	16.031
SVM	0.989	0.955	15.640	42.020	21.310
RVM	0.992	0.964	14.960	35.980	19.040
SVM	0.989	0.955	15.640	42.020	21.310

The other criterion like cumulative probability of Q_p/Q_m (Das and Basudhar 2006, Abu-Farsakh and Titi 2004) should also be considered for the evaluation of performance of different models.

The ratio Q_p/Q_m is arranged as per their values in an ascending order and the cumulative probability is calculated from the following equation:

$$p = \frac{i}{n+1} \tag{6.2}$$

Where i = order number given to the Q_p / Q_m ratio; n is the number of data points. If the computed value of 50% cumulative probability ($P50$) is less than unity, under prediction is implied; else over prediction. The ‘best’ model is that the obtained $P50$ value close to unity. The 90% cumulative probability ($P90$) reflects the variation in the ratio of Q_p / Q_m for the total observations. The model with $P90$ for Q_p / Q_m close to 1.0 is a better model.

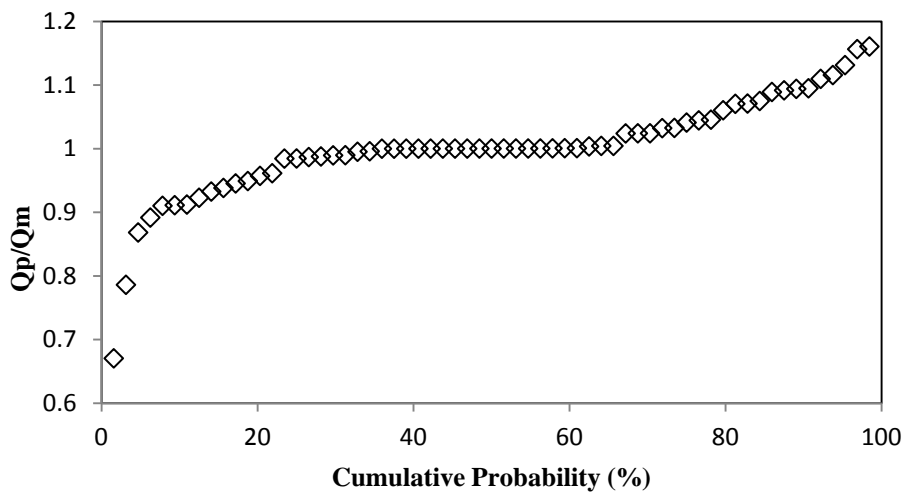


Fig 6.2: Cumulative probability distribution function for overall training and testing data

Probability @	P ₅₀	P ₉₀
Overall performance	1.0	1.1

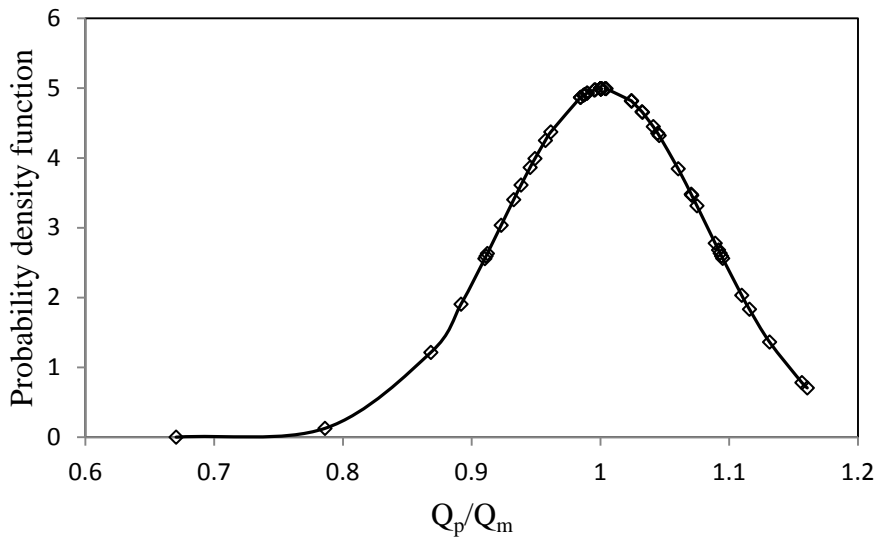


Fig 6.5: Probability density function for overall data

The probability density function shows that 90% of the data lies in the range of 0.9-1.1, and from this we can say the deviation of predicted values from actual and so thus the reliability of the functional network can be assured. From this we can say that the functional network is an effective tool in solving any problem.

6.4 CONCLUSION:

With the selected tool the developed models showed good correlation with the desired output and their performance is estimated based on following criteria obtained in testing.

1. Correlation coefficient
2. Root mean square error
3. Efficiency and
4. Cumulative probability distribution function (P₅₀ and P₉₀)

Table : Comparison criterion

Criterion	value
R	0.997
RMSE	5.3574
Efficiency	0.997
P ₅₀ and P ₉₀	1.0 & 1.1

From the above table we can conclude that the error was almost minimum and the efficiency is close to 1 shows that the functional networks gave good and accurate results and the reliability of the functional network can be assured and can be extended to wide variety of application

CHAPTER 7

SWELLING PRESSURE IN CLAYS

7.1 INTRODUCTION

Expansive soil and bedrock underlie more than one-third of world's land surface. Each year, damage to buildings, roads, pipelines, and other structures by expansive soils is much higher than the damage caused by floods, hurricanes, tornadoes, and earthquakes combined (Jones and Holtz 1973). The estimated annual cost of damage due to expansive soils is \$1,000 million in the USA, £150 million in the UK, and many billions of pounds worldwide (Gourley et al. 1993).

However, as the hazards due to expansive soils develop gradually and seldom present a threat to life, they have received limited attention, despite their severe effects on the economy. Much of the damage related to expansive soils is not due to a lack of appropriate engineering solutions but to the non-recognition of expansive soils and expected magnitude of expansion early in land use and project planning. The damage to foundation on expansive soil can be avoided/minimized by proper identification, classification, quantification of swell pressure, and provision of an appropriate design procedure. Swelling potential of clayey soil is a measure of the ability and degree to which such a soil might swell if its environments were changed in a definite way. Hence, the expansive soil is classified based on its potential for swelling. Though factors like clay content, Atterberg's limits, and mineral types are found to affect the swelling potential, the available literature presents contradicting results. McCormack and Wilding (1975) found clay content to be reliable in predicting swelling potential for soil dominated by illite, whereas according to Yule and Ritchie (1980) and Gray and Allbrook (2002), there is no relationship between clay percentage and soil swelling.

The cation exchange capacity (CEC), saturation moisture, and plasticity index (PI) are also important indices for estimation of swelling potential (Gill and Reaves 1957). Parker et al. (1977) concluded swell index and PI as superior to other indices for swelling potential. El-Sohby and El-Sayed (1981) observed that parameters like initial water content, type of clay mineral, initial dry density, clay content, and type of coarse grained fraction are major controlling factors for the swelling pressure of soil. The swelling pressure depends upon

various soil parameters such as mineralogy, clay content, Atterberg's limits, dry density, moisture content, initial degree of saturation, etc. along with structural and environmental factors. The parameters are interrelated in a complex manner, and it is difficult to model and analyze effectively taking all the above aspects into consideration. However, it can be measured easily with relevant data pertaining to soil, structure, and environment. So various statistical/empirical methods have been attempted for predicting the swelling pressure based on index properties of soil (Mowafy and Bauer 1985; Mallikarjuna 1988; Das 2002). Recently, Erzin and Erol (2004) presented regression equation for prediction of swelling pressure of Bentonite–Kaolinite clay mixture. However, these regression methods are developed based on the total available data and have not been tested with new data set.

Das et.al (2009) analyzed the above model using ANN. Kayadelen et al. (2009) presented a neuro-fuzzy model for prediction of swelling potential of compacted soil. Whereas the biggest challenge in successful application of ANN is when to stop training. If training is insufficient then the network will not be fully trained, whereas if training is excessive then it will memorize the training pattern or learn noise. When the numbers of data points are scanty the training set is driven to a very small value, but when new data are presented to the network the error is too large, which is known as overfitting. The network needs to be equally efficient for new data during testing or validation, which is called as generalization. There are different methods for generalization like early stopping and cross validation (Basheer 2001; Shahin et al.). The FN are becoming more reliable than any other statistical method due to their special attributes of identifying complex system when the input and output are known from either laboratory or field experimentation. The draw backs in ANN are come up with FN and the results of the present study are presented as follows: (A= actual) ,(p=predicted)

7.2 DATABASE AND PREPROCESSING:

Table 7.1: Data set for swelling pressure in clays (Training)

w	γ_d	LL	PI	CLAY	$\log S_{pa}$	$\log S_{pp}$	A/P
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20	13.5	75	43	45	1.8325	1.8976	1.035525
5	15.4	193	165	38	2.415	2.437	1.00911
63.9	10.1534	111	76	77.1	0.4771	0.477	0.99979
5	20.7972	28	14	42	1.7782	1.8053	1.01524
17.8	17.1381	122	81	73	2.9058	2.9162	1.003579
38.7	11.3796	105	73	97	1.7924	1.8188	1.014729
30	12.5	193	165	38	1.7404	1.6357	0.939841
30	12.1	47	27	28	0.8451	0.8458	1.000828
5	14	115	74	47	2.5682	2.5753	1.002765
4	16.1865	85	41	19	1.8932	1.8355	0.969523
34.6	12.3606	102	75	66.4	2.1303	2.1782	1.022485
41.9	12.3116	74	48	51.6	1	1.113	1.113
14.6	18.7862	54	33	60	2.0792	1.8293	0.87981
16.3	18.0014	37	22	27.7	1.3802	1.641	1.188958
5	13.4	47	27	28	1.6532	1.43	0.864989
20.2	16.677	53	35	31.4	2.1553	1.8007	0.835475
27.7	15.0584	76	47	60	2.4472	1.9969	0.815994
12	16.1865	105	55	54	1.9557	1.9868	1.015902
25	15.7	115	74	47	2.6284	2.5744	0.979455
51.7	10.9136	94	62	76.9	1.7404	1.7367	0.997874
10	12.5	193	165	38	1.9542	1.9124	0.97861
25	15.4	193	165	38	2.2742	2.2409	0.985357
20	17.1185	59	37	52	1.4771	1.9487	1.319274
10	15.1	75	43	45	2.4393	2.3186	0.950519
25	13.4	59	35	25	1.4914	1.4888	0.998257
2.7	14.3226	73	47	60	1.9243	1.9719	1.024736
20	15.696	100	50	27	1.9047	1.8486	0.970547
20	12.7	115	74	47	2.243	2.1163	0.943513
27.7	12.6549	69	44	58.9	1.3802	1.5149	1.097595
5	12.2	75	43	45	1.7559	1.7552	0.999601
14.9	15.7941	85	61	53	2.8865	2.9205	1.011779
19.3	14.4698	45	27	50	1.5051	1.4611	0.970766
10	12.1	47	27	28	1.1139	1.1225	1.007721

10.7	18.0504	33	19	27.9	2.1072	1.8281	0.867549
4.8	16.7261	28	15	32	1.6021	1.6484	1.0289
32.5	11.9682	96	64	95	2.2923	2.2524	0.982594
15	14.9	59	35	25	1.8062	1.835	1.015945
37.3	13.1945	53	28	51	1.301	1.1549	0.887702
36.5	13.1945	86	57	60.8	1.6435	1.8334	1.115546
26.3	11.6249	85	52	40	1.7324	1.6452	0.949665
23.4	15.1074	63	40	52.6	2.0607	1.9672	0.954627
30.9	13.2926	85	56	72.1	2.0128	2.1088	1.047695
12	18.1485	45	27	50	1.301	1.6834	1.293928
15.3	18.0504	50	30	38	1.9031	1.8817	0.988755
22.8	16.1375	97	57	59	2.6201	2.3712	0.905004
20	13.8	193	165	38	2.0212	2.0546	1.016525
20	15.696	50	27	35	1.6454	1.8849	1.145557
10	15.7	115	74	47	2.7634	2.7677	1.001556
20	12.1	59	35	25	1.1461	1.2165	1.061426
10	13.5	75	43	45	2.0682	2.0585	0.99531
22.2	16.5789	49	27	59	1.7782	1.6503	0.928073
13.2	15.696	46	27	46	1.7559	1.7402	0.991059
18.2	17.3147	103	66	65	2.9058	2.8952	0.996352
15	14.2	47	27	28	1.4624	1.4811	1.012787
5	14.9	59	35	25	1.9956	1.9478	0.976047
30	13.3	115	74	47	2.1139	2.145	1.014712
20	15.696	85	41	19	1.6609	1.7138	1.03185
28.4	13.685	66	41	46.1	1.6021	1.828	1.141002
38	12.0173	62	40	44.1	1.3802	1.1023	0.798652
25	15	47	27	28	1.3979	1.5072	1.078189
8	16.1865	100	50	27	1.9926	2.0595	1.033574
15	14	115	74	47	2.4728	2.4624	0.995794
10	12.7	59	35	25	1.415	1.5143	1.070177
25.4	14.0774	77	49	59	2.0569	1.9364	0.941417
5	13.8	193	165	38	2.1399	2.2184	1.036684
15	15.4	193	165	38	2.4116	2.3242	0.963759

30	13.4	47	27	28	1.1139	1.1504	1.032768
20.8	16.5887	116	71	68	2.658	2.766	1.040632
29.2	14.0283	69	45	58	1.6021	1.8067	1.127707
10	12.7	115	74	47	2.3181	2.2772	0.982356
25	14.9	59	35	25	1.6902	1.7517	1.036386
30	12.2	75	43	45	1.5563	1.4756	0.948146
23.4	16.2846	72	44	58	2.1761	2.0828	0.957125
30	13.8	193	165	38	1.9542	1.9388	0.99212
25	14	115	74	47	2.3674	2.3791	1.004942
20	12.2	75	43	45	1.6232	1.5914	0.980409
11.9	19.5219	35	22	31.4	2.1038	2.0677	0.982841
20	15.696	105	55	54	1.8663	1.8595	0.996356
20.6	15.4998	54	35	49.9	1.6812	1.7516	1.041875
15	13.4	59	35	25	1.6532	1.5721	0.950944
20	12.5	193	165	38	1.8325	1.7515	0.955798
20	13.4	47	27	28	1.2553	1.2662	1.008683
8.6	21.0425	35	19	49	1.9031	1.8952	0.995849
12	16.1865	85	41	19	1.8407	1.8411	1.000217
4	16.1865	105	55	54	2.0017	1.9812	0.989759
30	12.7	59	35	25	1.2553	1.2375	0.98582
25	15.1	75	43	45	2.1072	2.1253	1.00859
11.6	18.2466	34	19	29.2	1.8062	1.7883	0.99009
5	15	47	27	28	1.8325	1.7034	0.92955
5	14.3	75	43	45	2.1732	2.2194	1.021259
5	13.4	59	35	25	1.8129	1.6849	0.929395
10	14.6	193	165	38	2.3118	2.3539	1.018211
30	12.1	59	35	25	1	1.1007	1.1007
10	13.4	47	27	28	1.3802	1.4272	1.034053
20	12.1	47	27	28	1	0.9616	0.9616
20.6	14.2736	68	45	49.5	1.9777	2.0028	1.012692
15	14.3	75	43	45	2.1239	2.1065	0.991808
4	16.1865	100	50	27	1.7731	1.9703	1.111218
20	14.6	193	165	38	2.2175	2.193	0.988952

10	12.9	75	43	45	1.8451	1.9164	1.038643
20	14.1	59	35	25	1.6902	1.6673	0.986451
10	14.1	59	35	25	1.8451	1.8282	0.990841
20	12.7	59	35	25	1.3617	1.3534	0.993905
25	12.8	47	27	28	1.0414	1.0902	1.04686
15	15	47	27	28	1.7404	1.5905	0.91387
10	14.8	115	74	47	2.6232	2.6965	1.027943
15	15.1	75	43	45	2.2967	2.2086	0.961641
30	13.5	75	43	45	1.7853	1.7818	0.99804
18.1	15.0093	49	26	49	1.7709	1.6911	0.954938
20	15.2055	65	41	52	1.8692	2.0392	1.090948
10	12.2	75	43	45	1.6335	1.7524	1.072788
25	14.3	75	43	45	1.9638	2.0232	1.030247
25	14.6	193	165	38	2.1139	2.1606	1.022092
5	15.1	75	43	45	2.3444	2.3215	0.990232
20.2	15.696	68	42	59	2.0828	2.0718	0.994719
5	15.7	115	74	47	2.8426	2.7706	0.974671
15	13.5	75	43	45	1.9031	1.9485	1.023856
10	14.2	47	27	28	1.5315	1.5911	1.038916
20	14.2	47	27	28	1.3617	1.4302	1.050305
24.9	12.3606	85	52	40	1.8451	1.806	0.978809
20	13.4	59	35	25	1.5441	1.5212	0.985169
10	13.8	193	165	38	2.1903	2.2155	1.011505
19.4	14.5188	65	41	52	2.1335	1.9654	0.921209
15	13.4	47	27	28	1.301	1.3171	1.012375
10	13.4	59	35	25	1.699	1.6821	0.990053
15	14.6	193	165	38	2.2672	2.2438	0.989679
25	14.2	47	27	28	1.301	1.3978	1.074404
20	14	115	74	47	2.3243	2.4115	1.037517
15	13.8	193	165	38	2.0531	2.1055	1.025522
15	15.7	115	74	47	2.7324	2.6577	0.972661
5	13.5	75	43	45	2.0374	2.0614	1.01178
25	13.8	193	165	38	1.9777	2.0222	1.022501

5	14.6	193	165	38	2.29	2.3567	1.029127
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Table 7.2: Data set for swelling pressure in clays (Testing)

w	γ_d	LL	PI	CLAY	$\log S_{pa}$	$\log S_{pp}$	A/P
5	14.8	115	74	47	2.6551	2.6994	1.016685
10	14	115	74	47	2.5378	2.5725	1.013673
10	13.3	115	74	47	2.4314	2.4217	0.996011
15	14.8	115	74	47	2.5185	2.5865	1.027
20	13.3	115	74	47	2.2833	2.2608	0.990146
25	14.8	115	74	47	2.4048	2.5032	1.040918
30	12.7	115	74	47	2.1072	2.0004	0.949317
15	12.9	75	43	45	1.7782	1.8064	1.015859
20	12.9	75	43	45	1.716	1.7555	1.023019
25	13.5	75	43	45	1.8325	1.8652	1.017844
30	12.9	75	43	45	1.6812	1.6396	0.975256
10	13.1	193	165	38	2.0128	2.0569	1.02191
20	13.1	193	165	38	1.9031	1.896	0.996269
30	13.1	193	165	38	1.8633	1.7802	0.955402
5	14.2	47	27	28	1.7076	1.5939	0.933415
10	12.8	47	27	28	1.2304	1.2836	1.043238
20	12.8	47	27	28	1.1139	1.1226	1.00781
25	13.4	47	27	28	1.2304	1.2338	1.002763
30	12.8	47	27	28	1	1.0068	1.0068
5	14.1	59	35	25	1.8751	1.8311	0.976535
10	12.1	59	35	25	1.2788	1.3774	1.077104
15	14.1	59	35	25	1.7243	1.7182	0.996462
15	12.7	59	35	25	1.3802	1.4043	1.017461

25	14.1	59	35	25	1.6812	1.6349	0.97246
30	13.4	59	35	25	1.3222	1.4053	1.06285
8	16.1865	85	41	19	1.8791	1.9246	1.024214
16	16.1865	85	41	19	1.7903	1.7636	0.985086
4	16.1865	105	55	54	1.9877	1.9812	0.99673
8	16.1865	105	55	54	1.9791	2.0704	1.046132
16	16.1865	105	55	54	1.9101	1.9094	0.999634
12	16.1865	100	50	27	1.9605	1.9759	1.007855
16	16.1865	100	50	27	1.9315	1.8985	0.982915
21.5	15.4017	67	44	50	2.0414	2.1019	1.029637

7.3 RESULTS AND DISCUSSION

The methodology adopted by the FN in predicting the Lateral load capacity of piles was discussed in Sec 3.8. with exponential function as the basis function, and degree 3 the workability of functional network and the equation generated was discussed below:

$$y = a_0 + \sum_{i=1}^s \sum_{j=1}^n a_{ij} \exp(x_{ij}) \quad (4.1)$$

Where, s = no of input variables

n= degree of the function

Table 7.3: Coefficients of the obtained equation

a	1	2	3	4	5	6	7
1	3	-38	166	134	314	-111	0
2	0	0	27	-95	115	-46	0
3	0	-158	2364	-13396	37049	-50452	27075
4	15	-191	986	-2594	3463	0	-4160
5	0	20	-117	255	-235	78	0

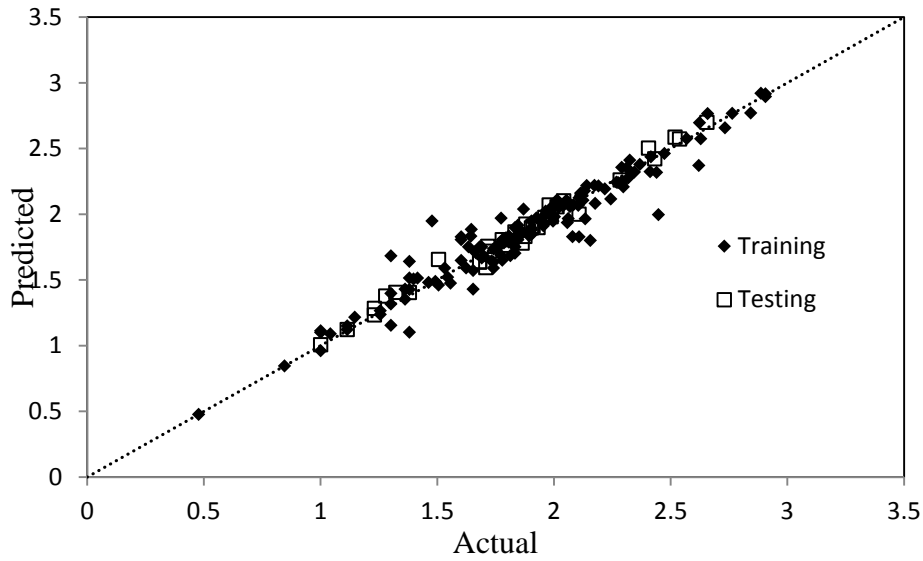


Fig: 7.1 Comparison of predicted and measured swelling pressure of clays for training and testing

Table 7.4: Error calculation for training and testing

Error	Training	Testing
AAE	0.0787	0.0464
MAE	0.4715	0.1507
RMSE	0.1177	0.0593
R	0.9652	0.9901

The comparison of results with other criterion like cumulative probability distribution, and efficiency also to be considered for the evaluation of the best performance of the model.

The ratio Q_p/Q_m is arranged as per their values in an ascending order and the cumulative probability is calculated from the following equation:

$$p = \frac{i}{n+1}$$

Where i = order number given to the Q_p/Q_m ratio; n is the number of data points. If

the computed value of 50% cumulative probability (P_{50}) is less than unity, under prediction is implied; else over prediction. The ‘best’ model is that the obtained P_{50} value close to unity. The 90% cumulative probability (P_{90}) reflects the variation in the ratio of Q_p/Q_m for the total observations. The model with P_{90} for Q_p/Q_m close to 1.0 is a better model. The results are presented in the table below:

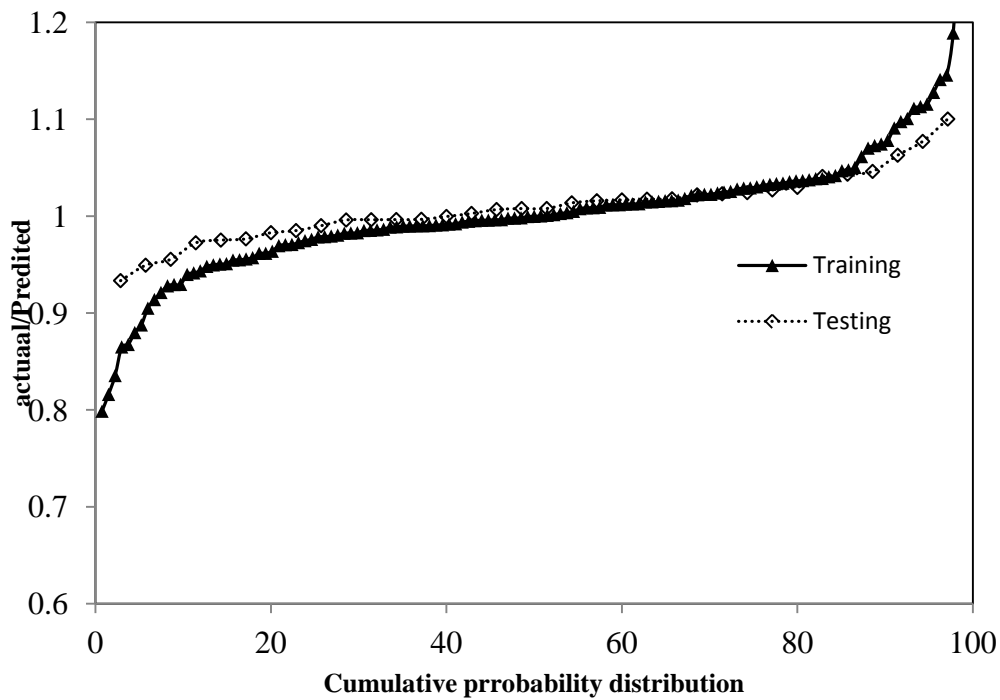


Fig 7.2: Cumulative probability distribution plot for swelling pressure in clay for training and testing

Table 7.4: Cumulative probability distribution functions

Cumulative probability	P_{50}	P_{90}
Training	1.00	1.08
Testing	1.00	1.10

7.4 CONCLUSION:

With the selected tool the developed models showed good correlation with the desired output and their performance is estimated based on following criteria obtained in testing.

1. Correlation coefficient
2. Root mean square error
3. Efficiency and
4. Cumulative probability distribution function (P_{50} and P_{90})

Criterion	value
R	0.9901
RMSE	0.0593
Efficiency	1
P_{50} and P_{90}	1.0 & 1.1

From the above table we can conclude that the error was almost minimum and the efficiency is close to 1 shows that the functional networks gave good and accurate results and the reliability of the functional network can be assured and can be extended to wide variety of applications

CHAPTER 8

CONCLUSIONS

In this study, we propose an alternative approach, functional networks, which provide a satisfactory prediction for

1. Prediction of lateral load capacity of piles in clay
2. Prediction of factor of safety of slope

3. Uplift capacity of suction caisson in clay

4. Swelling pressure in clays

A simplified generalized functional network model is learned and tested with combination of data sets from two wells. Different basis functions are used in the model and minimum description length was used to determine the best basis function to use for the problem. The results show that functional networks successfully predicted all the problems discussed.

A clear advantage of this technique over neural networks is the quick and unique solution obtained from the model. Another important advantage is that it discovers the relationship that exists between the predictor variables and the output. This provides valuable information about the variables, making it easy to know their significance as well as to compare with existing empirical or theoretical models.

In the present study the functional network was applied to solve some of the problems in geotechnical engineering. To the extent applied functional network performed better than the other statistical models like SVM, MGGP, ANN and others.

REFERENCES

Abu-Farsakh, M. Y. and Titi, H. H., 2004. Assessment of direct cone penetration test methods for predicting the ultimate capacity of friction driven piles. *J. Geotech. Geoenv Engg.*, 130 (9), pp. 935–944.

Abu-Kiefa, M. A. ,1998. General regression neural networks for driven piles in cohesionless soils. *J. Geotech. Geoenv Engg., ASCE* , 124(12), pp. 1177–1185.

Ahmed Adeniran,(2009) Functional Network Softsensor for formation porosity and water saturation in oil Wells

Castillo et al(1998) Functional network a neural based paradigm

Castillo, E. (1998) "Functional Networks", *Neural Processing Letters*, 7, 151-159

Castillo, E., Cobo, A., Gutierrez, J.M., and Pruneda E. (1998), "Working with Differential, Functional and Difference Equations using Functional Networks", *Applied Mathematical Modelling*, in press.

Castillo, E. and Gutierrez, J. M. (1998) "Nonlinear Time Series Modeling and Prediction Using Functional Networks. Extracting Information Masked by Chaos", *Physics Letters A*, 244, 71-84.

Castillo, E., Gutierrez, J. M., Cobo, A., and Castillo, C. ~2000b!. "Some learning methods in functional networks." *Comput. Aided Civ. Infrastruct. Eng.*, 1, 427–439.

Das, S. K. and Basudhar P. K. ,2008. Prediction of residual friction angle of clays using artificial neural network. *Engineering Geology*,100 (3-4), pp. 142- 145.

Das, S.K., Basudhar, P.K. (2006). Undrained lateral load capacity of piles in clay using artificial neural network. *Comput Geotech* 33(8):454–459.

Das, S.K., Biswal, R.K., Sivakugan, N., Das, B. (2011). Classification of slopes and prediction of factor of safety using differential evolution neural networks. *Environ Earth Science* , 64:201–210.

Das, S.K.(2005). Applications of genetic algorithm and artificial neural network to some geotechnical engineering problems. *Ph.D. Thesis, Indian Institute of Technology Kanpur*, Kanpur, India.

Das, S. K. and Sivakugan, N. ,2010. Discussion of Intelligent computing for modeling axial capacity of pile foundations. *Canadian Geotechnical Journal*, 2010, 37 (8), pp. 928-930.

Das, S. K. , Samui, P. and Sabat, A. K. ,2011a. Application of Artificial Intelligence to Maximum Dry Density and Unconfined Compressive Strength of Cement Stabilized Soil. *Geotechnical and Geological Journal*, 29(3), pp. 329-342.

Das, S.K., Samui, P., Sabat, A.K.(2011). “Prediction of field hydraulic conductivity of clay liners using artificial neural network and support vector machine”. *International Journal of Geomechanics*. doi: [http://dx.doi.org/10.1061/\(ASCE\)GM.1943-562](http://dx.doi.org/10.1061/(ASCE)GM.1943-562)

Das, S. K. Samui, P. Sabat, A. K. and Sitharam, T. G. ,2010. Prediction of swelling pressure of soil using artificial intelligence techniques. *Environmental Earth science*, 61(2), pp. 393-403.

E. Castillo, J. M. Guteirrez, A. S. Hadi and B. Lacruz, “Some applications of functional networks in statistics and engineering” *Technometrics*,vol. 43, no. 1 February 2001.

S Rajasekaran (2000) *Functional Networks in structural engineering*.