Optimal Controller Design for Twin Rotor MIMO System

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Optimal Controller Design for Twin Rotor MIMO System

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CERTIFICATE

*This is to certify that the thesis titled "***Optimal Controller Design for Twin Rotor MIMO System***", by* ***Ankesh Kumar Agrawal****, submitted to the National Institute of Technology, Rourkela for the award of degree of Master of Technology with specialization in* ***Control & Automation*** *is a record of bona fide research work carried out by him in the Department of Electrical Engineering, under my supervision. I believe that this thesis fulfills part of the requirements for the award of degree of Master of Technology. The results embodied in this thesis have not been submitted in parts or full to any other University or Institute for the award of any other degree elsewhere to the best of my knowledge.*

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Finally, I dedicate this thesis to my family: my dear father, my dearest mother and my brother who supported me morally despite the distance that separates us. I thank them from the bottom of my heart for their motivation, inspiration, love they always give me. Without their support nothing would have been possible. I am greatly indebted to them for everything that I am.

ABSTRACT

Twin Rotor MIMO system (TRMS) is considered as a prototype model of Helicopter. The aim of studying the TRMS model and designing the controller for controlling the response of TRMS is that it provides a platform for controlling the flight of Helicopter. In this work, the non-linear model of Twin Rotor MIMO system has been linearized and expressed in state space form. For controlling action a Linear Quadratic Gaussian (LQG) compensator has been designed for a multi input multi output Twin Rotor system. Two degree of freedom dynamic model involving Pitch and Yaw motion has been considered for controller design. The two stage design process consists of the design of an optimal Linear Quadratic Regulator followed by the design of an observer (Kalman filter) for estimating the non-accessible state variable from noisy output measurement. LQR parameter i.e. Q and R are varied randomly to get the desired response. Later an evolutionary optimization technique i.e. Bacterial Foraging Optimization (BFO) algorithm has been used for optimizing the Q and R parameter of Linear Quadratic Gaussian compensator. Simulation studies reveal the appropriateness of the proposed controller in meeting the desired specifications.

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List of Abbreviations

Abbreviation	Description
UAV	Unmanned Air Vehicle
TRMS	Twin Rotor Multi input Multi output system
DC	Direct current
PID	Proportional Integrator Derivative
LPV	Linear parameter varying
QFT	Quantitative Feedback theory
LMI	Linear matrix inequality
GA	Genetic Algorithm
BFO	Bacterial Foraging Optimization
SISO	Single input Single output
LQ	Linear Quadratic
LQT	Linear Quadratic Tracking
DOF	Degree of Freedom
EP	Evolutionary Programming
ES	Evolutionary Strategies

INTRODUCTION

1.1 Background

Recent times have witnessed the development of several approaches for controlling the flight of air vehicle such as Helicopter and Unmanned Air Vehicle (UAV). The modeling of the air vehicle dynamics is a highly challenging task owing to the presence of high nonlinear interactions among the various variables and the non-accessibility of certain states. The twin rotor MIMO system (TRMS) is an experimental set-up that provides a replication of the flight dynamics. The TRMS has gained wide popularity among the control system community because of the difficulties involved in performing direct experiments with air vehicles. Aerodynamically TRMS consist of two types of rotor, main and tail rotor at both ends of the beam, which is driven by a DC motor and it is counter balanced by a arm with weight at its end connected at pivot. The system can move freely in both horizontal and vertical plane. The state of the beam is described by four process variables- horizontal and vertical angles which are measured by encoders fitted at pivot and two corresponding angular velocities. For measuring the angular velocities of rotors, speed sensors are coupled with DC motors.

The TRMS is basically a prototype model of Helicopter. However there is some significant difference in aerodynamically controlling of Helicopter and TRMS. In Helicopter, controlling is done by changing the angle of both rotors, while in TRMS it is done by varying the speed of rotors. Several works have been reported on dynamic modeling and control of TRMS. For instance, an intelligent control scheme for the design of hybrid PID controller has been proposed in [1]. Other notable works include LPV Modeling and Control [2], QFT based control [3], LMI based approach [4] and Single Neuron PID control [5]. Considering the unmodelled dynamics and the presence of noise in the output measurement, in the present work, a state feedback controller has been designed considering the effect of unmodelled dynamics and noisy output data. The design of a state feedback controller demands the availability of all the state variables in the output. However, for the TRMS since all the states are not accessible, an observer (Kalman filter) has been designed for estimating the unavailable state variables from the noisy output measurement. The Kalman filter has been coupled with an optimal controller *i.e.* Linear Quadratic Regulator (LQR) for tracking a desired trajectory. The combination of the Kalman filter and LQR is commonly referred to as Linear Quadratic Gaussian (LQG) Compensator. For an observer based state feedback control of a plant corrupted by state and measurement noise, the control action and the appropriateness of the estimated states is heavily dependent on the output and control weighting matrices. The selection of these parameters is not trivial problem and hence is carried out by trial and error method. This involves maintaining a trade-off between minimizing the control effort and improving the transient response. Thus an optimization technique, i.e. Genetic Algorithm (GA) [6-8], is used for the selection of weighting matrices of the LQR controller. But there are certain optimization problems for which GA is not preferred, because of the selection of large number of parameters and high computational cost. Thus Genetic Algorithm has limitation for use in real-time applications. Therefore in this work, a new evolutionary optimization technique, i.e. Bacterial Foraging Optimization (BFO) algorithm [9-11] is used for optimizing weighting matrices of LQR, which will overcome the limitation of Genetic Algorithm. BFO is a globally optimization technique for distributed optimization. Simulation results depict the appropriateness of the proposed controller in tracking a desired trajectory with minimum control effort.

1.2 Literature Review

This section reflects the brief review about optimal control of Twin Rotor MIMO system. S.M. Ahmad gives the dynamic modelling of TRMS [12]. The aerodynamic model and mathematical model of TRMS is explained in this paper. The paper shows that mathematical model of TRMS in non-linear, so linearization technique is explained by A. Bennoune and A. Kaddouri in Application of the Dynamic Linearization Technique to the Reduction of the Energy Consumption of Induction Motors [13] paper. The flight of TRMS is controlled by using various techniques like, by designing hybrid PID controller [1] given by J.G. Juang and W.K. Liu or by using LPV Modelling and Control [2] given by F. Nejjari and D. Rotondo. Some other controlling techniques include QFT based control [3], LMI based approach [4] and Single Neuron PID control [5]. The above discussed controllers are not optimal controllers. So in this work optimal controller, i.e. Linear Quadratic Regulator (LQR) which is proposed by L.S. Shieh in “Sequential design of Linear Quadratic state Regulator [15] is designed for controlling the flight of TRMS. While designing LQR all the states are assumed to be present. Since number of output is less than number of states, so all the states of system cannot be measured directly. For this Zhuoyi Chen has proposed a technique of designing Kalman Filter [19]. Kalman Filter is used for estimating states of system depending upon input-output combination. The technique of designing LQG compensator which is combination of LQR and Kalman Filter is proposed by R.N. Paschall in [21]. While designing LQR controller, the parameter of LQR, i.e. state and control weighting matrices are chosen by trial and error method. This involves maintaining a trade-off between minimizing the control effort and improving the transient response. Thus optimization technique is used for optimizing the LQR parameter. Several optimization techniques like Genetic Algorithm proposed by Subhojit Ghosh in [6] can be used for optimization. But due to its limitation another optimization technique proposed by Shiva Boroujeny Gholami in Active noise control using bacterial foraging optimization algorithm [9]. BFO algorithm is used for optimization the state and control weighting matrices.

1.3 Objective

The main objective of designing a controller for Twin Rotor MIMO system is to provide a platform through which flight of Helicopter can be controlled. The mathematical model of TRMS is non-linear so the first objective of this work is to linearize the non-linear model linear. The next objective is to design a optimal controller for TRMS system, which can control the response of system. So Linear Quadratic Regulator (LQR) is designed, which will control the response of system. The state of system is estimated by using Kalman Filter and it is combined with Linear Quadratic Regulator resulting in a Linear Quadratic Gaussian (LQG) controller. Bacterial foraging optimization (BFO) technique is used for optimizing the system.

1.4 TRMS Experimental set-up

Aerodynamic model of TRMS is shown in Figure-1.1. It consists of two propellers which are perpendicular to each other and joined by a beam pivoted on its base. The system can rotate freely in both vertical and horizontal direction. Both propellers are driven by DC motor and by changing the voltage supplied to beam, rotational speed of propellers can be controlled. For balancing the beam in steady state, counterweight is connected to the system. Both propellers are shielded so that the environmental effects can be minimized. The complete unit is attached to the tower which ensures safe helicopter control experiments. The electrical unit is placed under the tower which is responsible for communication between TRMS and PC. The electrical unit is responsible for transfer of measured signal by sensors to PC and transfer of control signal via I/O card.

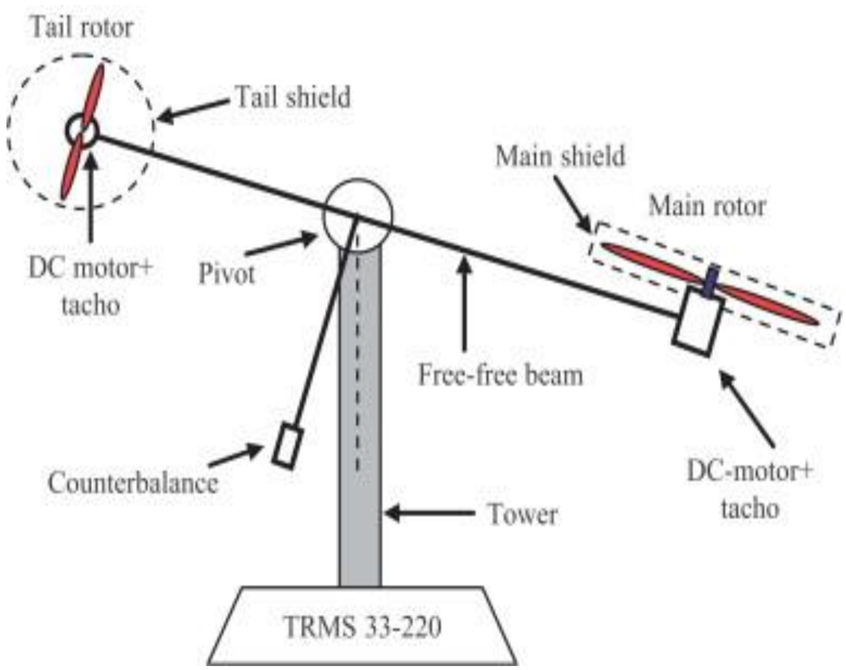

Figure-1.1 TRMS mechanical unit

Main rotor is responsible for controlling the flight of TRMS in vertical direction and Tail rotor is responsible for controlling the flight of TRMS in horizontal direction. There is cross-coupling between Main and Tail rotor.

TRMS MODEL

2.1 TRMS Mathematical model

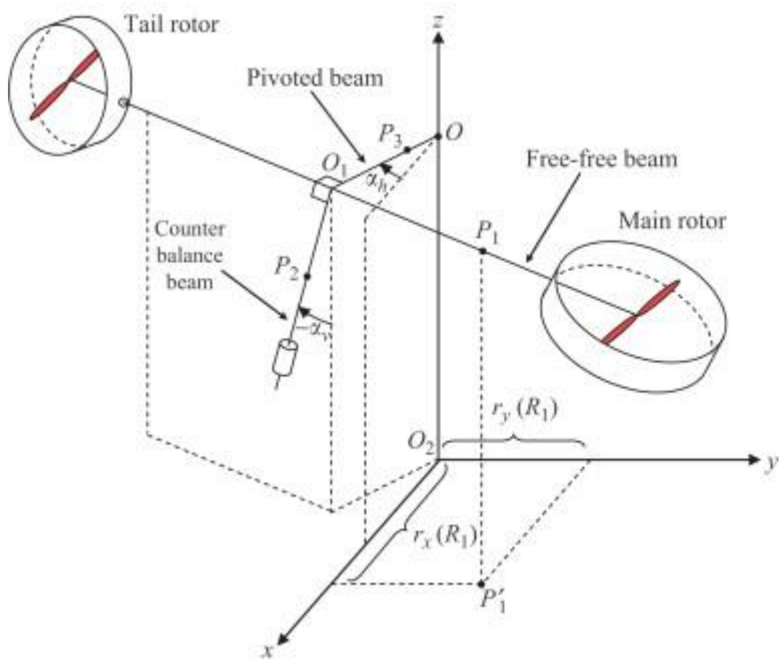

Figure-2.1 TRMS Phenomenological model

The mathematical model derived from phenomenological model shown in Figure-2.1 is non-linear in nature that means at least one of the states (rotor current or position) is an argument of non-linear function. In order to design the controller for controlling the flight of TRMS, the mathematical model should be linearized.

According to model represented in Figure-2.1, the non-linear mathematical model of TRMS can be represented as [12]-

Mathematical equation in vertical plane is given as-

$$I_1\left(\frac{d^2\alpha_v}{dt^2}\right) = M_1 - M_{FG} - M_{B\alpha_v} - M_G \tag{2.1}$$

where

$$M_1 = c_1\tau_1^2 + d_1\tau_1 \qquad - \text{nonlinear static characteristic} \tag{2.2}$$

$$M_{FG} = M_g \sin(\alpha_v) \qquad - \text{gravity momentum} \tag{2.3}$$

$$M_{B\alpha_v} = B_{1\alpha_v}\left(\frac{d\alpha_v}{dt}\right) + B_{2\alpha_v}\text{sign}\left(\frac{d\alpha_v}{dt}\right) \qquad - \text{friction forces momentum} \tag{2.4}$$

$$M_G = K_{gy}M_1\left(\frac{d\alpha_h}{dt}\right)\cos(\alpha_v) \qquad - \text{gyroscopic momentum} \tag{2.5}$$

The motor and the electrical control circuit is approximated as a first order transfer function, thus the rotor momentum in Laplace domain is described as-

$$\tau_1 = \left(\frac{k_1}{T_{11}s + T_{10}}\right)u_1 \tag{2.6}$$

Mathematical equation in horizontal plane is given as-

$$I_2\left(\frac{d^2\alpha_h}{dt^2}\right) = M_2 - M_{B\alpha_h} - M_R \tag{2.7}$$

where

$$M_2 = c_2\tau_2^2 + d_2\tau_2 \qquad - \text{nonlinear static characterstic} \tag{2.8}$$

$$M_{B\alpha_h} = B_{1\alpha_h}\left(\frac{d\alpha_y}{dt}\right) + B_{2\alpha_h}\text{sign}\left(\frac{d\alpha_h}{dt}\right) \qquad - \text{friction forces momentum} \tag{2.9}$$

$$M_R = \frac{k_c(T_0s + 1)}{(T_ps + 1)}\tau_1 \qquad - \text{cross reaction momentum} \tag{2.10}$$

Rotor momentum in Laplace domain is given as-

$$\tau_2 = \frac{k_2}{T_{21}s + T_{20}}u_2 \tag{2.11}$$

The model parameters used in above $(2.1) - (2.11)$ equations are chosen experimentally, which makes the TRMS nonlinear model a semi-phenomenological model.

The boundary for the control signal is set to [-2.5 to +2.5].

The following table gives the approximate value of parameter- [13].

Table- 2.1 TRMS system parameters

Parameter	Value
$I_1 -$ moment of inertia of vertical rotor	$6.8*10^{-2}$ kg.m^2
$I_2 -$ moment of inertia of horizontal rotor	$2*10^{-2}$ kg.m^2
$c_1 -$ static characteristic parameter	0.0135
$d_1 -$ static characteristic parameter	0.0924
$c_2 -$ static characteristic parameter	0.02
$d_2 -$ static characteristic parameter	0.09
$M_g -$ gravity momentum	0.32 N-m
$B_{1\alpha_v} -$ friction momentum function parameter	$6*10^{-3}$ N-m-s/rad
$B_{2\alpha_v} -$ friction momentum function parameter	$1*10^{-3}$ N-m-s^2/rad
$B_{1\alpha_h} -$ friction momentum function parameter	$1*10^{-1}$ N-m-s/rad
$B_{2\alpha_h} -$ friction momentum function parameter	$1*10^{-2}$ N-m-s^2/rad
$K_{gy} -$ gyroscopic momentum parameter	0.05 s/rad
$k_1 -$ motor 1 gain	1.1
$k_2 -$ motor 2 gain	0.8
$T_{11} -$ motor 1 denominator parameter	1.1
$T_{10} -$ motor 1 denominator parameter	1
$T_{21} -$ motor 2 denominator parameter	1
$T_{20} -$ motor 2 denominator parameter	1
$T_p -$ cross reaction momentum parameter	2
$T_0 -$ cross reaction momentum parameter	3.5
$k_c -$ cross reaction momentum gain	-0.2

2.2 Linearized model

The mathematical model given in equation $(2.1) - (2.11)$ are non-linear and in order to design controller for system, the model should be linearized. The first step in linearization technique [14-15] is to find equilibrium point.

Equations $(2.1) - (2.11)$ are combined to represent alternate model of TRMS. The alternate model is given as-

$$\frac{d^2\alpha_v}{dt^2} = \frac{\left(c_1\tau_1^2 + d_1\tau_1 - M_g \sin(\alpha_v) - B_{1\alpha_v}\left(\frac{d\alpha_v}{dt}\right) - B_{2\alpha_v}\mathrm{sign}\left(\frac{d\alpha_v}{dt}\right) - K_{gy}(c_1\tau_1^2 + d_1\tau_1)\frac{d\alpha_h}{dt}\cos(\alpha_v)\right)}{I_1} \tag{2.12}$$

$$\frac{d\tau_1}{dt} = \frac{(k_1u_1 - \tau_1T_{10})}{T_{11}} \tag{2.13}$$

$$\frac{d^2\alpha_h}{dt^2} = \frac{\left(c_2\tau_2^2 + b_2\tau_2 - B_{1\alpha_h}\frac{d\alpha_v}{dt} - B_{2\alpha_h}\mathrm{sign}\left(\frac{d\alpha_h}{dt}\right) - M_R\right)}{I_2} \tag{2.14}$$

$$\frac{d\tau_2}{dt} = \frac{(k_2u_2 - \tau_2T_{20})}{T_{21}} \tag{2.15}$$

$$\frac{dM_R}{dt} = \frac{\left(\left(k_c - \frac{k_cT_0T_{10}}{T_{11}}\right)\tau_1 + \frac{k_cT_0k_1}{T_{11}}u_1 - M_R\right)}{T_p} \tag{2.16}$$

Now let us assume - $\alpha_v = x_1$

$\alpha_h = x_2$

$\tau_1 = x_3$

$\tau_2 = x_4$

$M_R = x_5$

$\frac{d\alpha_v}{dt} = x_6$

$$\frac{d\alpha_h}{dt} = x_7$$

Equations $(2.12) - (2.16)$ can be represented with state space variable as-

$$\frac{dx_1}{dt} = x_6 \tag{2.17}$$

$$\frac{dx_2}{dt} = x_7 \tag{2.18}$$

$$\frac{dx_3}{dt} = -\frac{T_{10}}{T_{11}} x_3 + \frac{k_1}{T_{11}} u_1 \tag{2.19}$$

$$\frac{dx_4}{dt} = -\frac{T_{20}}{T_{21}} x_4 + \frac{k_2}{T_{21}} u_2 \tag{2.20}$$

$$\frac{dx_5}{dt} = \frac{\left(k_c - \frac{k_c T_0 T_{10}}{T_{11}}\right) x_3}{T_p} - \frac{x_5}{T_p} + \frac{k_c T_0 k_1}{T_p T_{11}} u_1 \tag{2.21}$$

$$\frac{dx_6}{dt} = \frac{\left(c_1 x_3^2 + d_1 x_3 - M_g \sin(x_1) - B_{1x_1} x_6 - B_{2x_1} \operatorname{sign}(x_6) - K_{gy}(c_1 x_3^2 + d_1 x_3) x_7 \cos(x_1)\right)}{I_1} \tag{2.22}$$

$$\frac{dx_7}{dt} = \frac{\left(c_2 x_4^2 + d_2 x_4 - B_{1x_2} x_6 - B_{2x_2} \operatorname{sign}(x_7) - x_5\right)}{I_2} \tag{2.23}$$

Now Taylor series is applied to find equilibrium point. For this make all the derivative term of equations $(2.17) - (2.23)$ equal to zero and find equilibrium point, take $u_1 = 0$ and $u_2 = 0$.

Thus equilibrium point will be- $x_{10} = 0, \pi$

$$x_{30} = 0$$

$$x_{40} = 0$$

$$x_{50} = 0$$

$$x_{60} = 0$$

$$x_{70} = 0$$

The non-linear equations $(2.17) - (2.23)$ can be represented in state space form given as –

$$\dot{x} = Ax + Bu \tag{2.24}$$

$$y = Cx \tag{2.25}$$

where A, B, C can be found by applying Jacobean matrix method. Thus A, B, C are given as –

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -0.909 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0.218181 & 0 & -0.5 & 0 & 0 \\ -4.70588 & 0 & 1.358823 & 0 & 0 & -0.088235 & 0 \\ 0 & 0 & 0 & 4.5 & -50 & -5 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0.8 \\ -0.35 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

By using A, B, C matrix TRMS system can be represented in state space form by using equation (2.24) and (2.25).

After representing the system in state space form, the next approach is to design controller for the system to achieve desired output.

CONTROLLER DESIGN FOR TRMS

3.1 Controllers

Controller is a device, in the form of analogue circuits or digital circuits which monitors and alters the parameter of system to attain desired output. Controllers are basically used if the system does not meet desired performance specification, i.e. both stability and accuracy. Controllers can be connected either in series with plant or parallel to the plant depending upon requirement.

A simple feedback control along with controller is shown in Figure – 3.1.

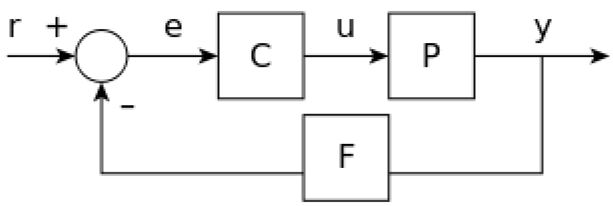

Figure-3.1 Feedback control Loop

As shown in Figure-3.1 error signal ‘e’ is generated, which is difference between reference signal ‘r’ and output signal ‘y’. The error signal decides the magnitude by which output signal deviates from reference value. Depending upon error signal value parameter of controller ‘C’ will get changed and control input ‘u’ is applied to plant which will give satisfactory output.

For a plant with multiple input and multiple output, it requires multiple controllers. If the system is SISO system with single input and single output than it requires single controller for controlling purpose. Depending on the set-up of the physical (or non-physical) system, adjusting the system's input variable (assuming it is MIMO) will affect the operating parameter, otherwise known as the controlled output variable. The notion of controllers can be extended to more complex systems. Natural systems and human made systems both requires controller for proper operation.

3.2 Types of Controllers

There are various types of controller which can be used for improving performance specification of system. Basically all the controllers can be broadly classified in two categories, feedback and feed forward controller. The input to a feedback controller is the same as what it is trying to control - the controlled variable is "feedback" into the controller. However, feedback control usually results in intermediate periods where the controlled variable is not at the desired set-point. Feed-forward control can avoid the slowness of feedback control. By using feed-forward control, the disturbances are measured and accounted for before they have time to affect the system.

Controllers can be broadly classified as-

a) Proportional controller
b) Proportional – integral controller
c) Proportional – derivative controller
d) Proportional – integral- derivative controller
e) Pole placement controller
f) Optimal controller

The first four controllers are feedback controller and the fifth one is full state feedback controller. Pole placement controller is a feedback controller which is used for placing the closed loop poles to desired location in s plane. But pole placement can be used only for SISO system. For MIMO system, problem of over-abundance of design parameters are faced. For such systems, we did not know how to determine all the design parameters, because only a limited number of them could be found from the closed loop pole locations. Optimal control provides the technique by which all the design parameters can be found even for multi-input, multi-output system. Also in pole placement technique some trial and error procedure with pole locations was required because we don’t know priori which pole location will give satisfactory performance. Optimal control allows us to directly formulate the performance objective of a control system and get desired response. Also optimal control minimizes the time and cost required for designing the system.

3.3 Linear Quadratic Regulator (LQR)

3.3.1 Overview

The principle of optimal control is basically concerned with operating a dynamic system at minimal cost. The system whose dynamics are given by set of linear differential equations and cost by Quadratic function is called Linear Quadratic (LQ) problem. The setting of a controller which governs either a machine or process are basically found by mathematical algorithm that minimizes cost function consist of weighing factors. Mathematical algorithms are basically objective function that must be minimized in design process.

The cost objective function for optimal control must be time integral of sum of control energy and transient energy expressed as function of time. If system transient energy can be defined as total energy of system when it is undergoing transient response, then control system should have transient energy which decays to zero quickly. Maximum overshoot is defined by maximum value of transient energy and time taken by transient response to decay to zero represent the settling time. Thus acceptable value of settling time and maximum overshoot can be specified by including transient energy in objective function. In same way, control energy should also be included in objective function to minimize the control energy of system. Figure-3.2 shows the block diagram of plant along with Linear Quadratic Regulator (LQR) [15-17]. Here output of plant is controlled by varying the gain K of Linear Quadratic Regulator.

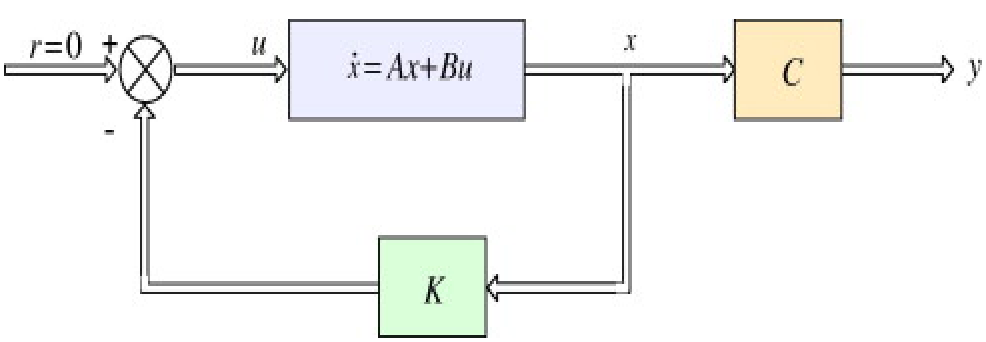

Figure-3.2 Block diagram of Linear Quadratic Regulator

Consider a linear plant given by the following state equations –

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{3.1}$$

Here time-varying plant in equation (3.1) are taken because optimal control problem is formulated for time-varying system. Control input vector for full state feedback regulator of the plant is given by –

$$u(t) = -K(t)x(t) \tag{3.2}$$

The control input given by equation (3.2) is linear, because the plant is also linear. The control energy is given by $u^T(t)R(t)u(t)$, where $R(t)$ is a square and symmetric matrix called control cost matrix. The expression for control energy is in quadratic form because the function $u^T(t)R(t)u(t)$ contains quadratic function of $u(t)$. The transient energy can be expressed as $x^T(t)Q(t)x(t)$, where $Q(t)$ is square and symmetric matrix called state weighing matrix. Thus objective function can be represented as –

$$J(t, t_f) = \int_t^{t_f} \left(x^T(\tau)Q(\tau)x(\tau) + u^T(\tau)R(\tau)u(\tau) \right) d\tau \tag{3.3}$$

where t and t_f are initial and final time values respectively, where controlling process begins at $\tau = t$ and ends at $\tau = t_f$. The main objective of optimal control problem is to find matrix $K(t)$ such that objective function $J(t, t_f)$ given in equation (3.3) is minimized. The minimization process is done in a way such that solution of plant's state-equation (3.1) is given by state vector $x(t)$. The main objective of design is to bring $x(t)$ to zero at time $t = t_f$.

3.3.2 Estimating Optimal control gain K

The closed loop state equation is given by substituting equation (3.2) into equation (3.1), which is given as –

$$\dot{x}(t) = \left(A - BK(t)\right)x(t) \tag{3.4}$$

$$\dot{x}(t) = A_C x(t) \tag{3.5}$$

where $A_C = (A - BK(t))$ is closed loop state dynamics matrix. The solution of equation (3.5) is given as –

$$x(t) = \theta_C(t, t_0)x(t_0) \tag{3.6}$$

where $\theta_C(t, t_0)$ is state transition matrix of closed loop system given by equation (3.5).

Equation (3.6) indicates at any time ‘t’ state $x(t)$ can be obtained by post multiplying the state at some initial time, $x(t_0)$ with $\theta_C(t, t_0)$. On substituting equation (3.6) into equation (3.3), the expression for objective function is given as –

$$J(t, t_f) = \int_t^{t_f} x^T(t)\theta_C^T(\tau, t)\Big(Q(\tau) + K^T(\tau)R(\tau)K(\tau)\Big)\theta_C(\tau, t)x(t)d\tau \tag{3.7}$$

Equation (3.7) can be written as –

$$J(t, t_f) = x^T(t)M(t, t_f)x(t) \tag{3.8}$$

where

$$M(t, t_f) = \int_t^{t_f} \theta_C^T(\tau, t)\Big(Q(\tau) + K^T(\tau)R(\tau)K(\tau)\Big)\theta_C(\tau, t)d\tau \tag{3.9}$$

Linear optimal regulator problem given by equation $(3.1) - (3.3)$ also called Linear Quadratic Regulator problem because the objective function shown in equation (3.3.8) is a quadratic function of initial state. By using the equation (3.6) and (3.7), it is given as –

$$J(t, t_f) = \int_t^{t_f} x^T(\tau)\Big(Q(\tau) + K^T(\tau)R(\tau)K(\tau)\Big)x(\tau)d\tau \tag{3.10}$$

Now on differentiating equation (3.10) partially with respect to time‘t’, we get –

$$\frac{\partial J(t, t_f)}{\partial t} = -x^T(t)\Big(Q(t) + K^T(t)R(t)K(t)\Big)x(t) \tag{3.11}$$

Also partial differentiating equation (3.8) with respect to‘t’ we get –

$$\frac{\partial J(t, t_f)}{\partial t} = \big(\dot{x}(t)\big)^T M(t, t_f)x(t) + x^T(t)\left(\frac{\partial M(t, t_f)}{\partial t}\right)x(t) + x^T(t)M(t, t_f)\dot{x}(t) \tag{3.12}$$

On combining equation (3.5) and equation (3.12) we get –

$$\frac{\partial J(t,t_f)}{\partial t} = x^T(t)\left(A_C^T(t)M(t,t_f) + \frac{\partial M(t,t_f)}{\partial t} + M(t,t_f)A_C(t)\right)x(t) \tag{3.13}$$

Equating equations (3.11) and (3.13) following matrix differential equation is obtained, which is given as –

$$-\frac{\partial M(t,t_f)}{\partial t} = A_C^T(t)M(t,t_f) + M(t,t_f)A_C(t) + \left(Q(t) + K^T(t)R(t)K(t)\right) \tag{3.14}$$

The matrix Riccati equation for finite time duration is given by equation(3.14).

By solving the Riccati equation (3.14) optimal feedback gain matrix $K(t)$ is given by –

$$K(t) = R^{-1}(t)B^T(t)M \tag{3.15}$$

There are large number of control problem where control time interval is infinite. By considering infinite time interval optimal control problem gets simplified. The quadratic objective function for infinite final time is given as –

$$J_\infty(t) = \int_t^\infty \left(x^T(\tau)Q(\tau)x(\tau) + u^T(\tau)R(\tau)u(\tau)\right)d\tau \tag{3.16}$$

where $J_\infty(t)$ is the objective function of the optimal control problem for infinite time. For infinite final time, $M(t,\infty)$ is either constant or does not gives any energy to any limit. Thus

$$\frac{\partial M}{\partial t} = 0$$

Thus Riccati equation for infinite final time is given by –

$$0 = A^TM + MA - MBR^{-1}(t)B^TM + Q(t) \tag{3.17}$$

Since equation (3.17) is an algebraic equation, thus it is called Algebraic Riccati equation. The condition for the solution of Riccati equation (3.17) to exist is either the system is asymptotically stable or the system is controllable and observable with output $y(t) = C(t)x(t)$, where $Q(t) = C^T(t)C(t)$ and $R(t)$ is positive definite matrix and symmetric. If the system is stabilizable and output $y(t) = C(t)x(t)$ is detectable then also solution to Riccati equation will exists with $Q(t) = C^T(t)C(t)$ and $R(t)$ is positive definite matrix and symmetric.

In this system positive definite matrix $Q(t)$ and positive semi definite matrix $R(t)$ are time independent and are randomly chosen. While designing LQR value of Q and R are varied until the output of system decays to zero at steady state.

For the present work Q and R matrix are given as –

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.001 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.4 \end{bmatrix} \quad \text{And} \quad R = \begin{bmatrix} 0.0395 & 0 \\ 0 & 1 \end{bmatrix}$$

In matrix Q, the element $q_5 = 0.001$ represents cross-coupling coefficient which needs to be minimized, so its weight is taken to be minimum.

By applying LQR technique on system by using Q and R given above we calculate the optimal control gain K of system.

The optimal control gain calculated is given as –

$$K = \begin{bmatrix} 22.7462 & 1.5716 & -7.2280 & 3.9397 & -52.6533 & 0.1295 & 3.6697 \\ 2.5257 & 0.0494 & -1.0458 & 0.6041 & -3.3438 & -0.6577 & 0.1323 \end{bmatrix}$$

Now by using value of optimal control gain K in equation (3.2), optimal control input 'u' is calculated. With the control input 'u', output of TRMS is regulated and response decays to zero at steady state. Here optimal gain K is obtained by randomly varying Q and R matrix. This involves maintaining a trade-off between minimizing the control effort and improving the transient response. To overcome this, a optimization technique is used to optimize the value of Q and R. Thus in this work, optimization algorithm, i.e. BFO algorithm is used for optimizing the state and control weighting matrices.

3.3.3 Linear Quadratic Tracking Problem

In Linear Quadratic Regulator problem the output of system decays to zero at steady state. In this case no reference signal is applied to system. But if reference signal is applied then Linear Quadratic Regulator problem becomes Linear Quadratic Tracking (LQT) problem. In Linear Quadratic Tracking problem reference signal is applied to the system and output of system tracks the reference signal.

Consider linear, time invariant plant given by equation(3.1). Now our aim is to design a tracking system for plant (3.1) if desired state vector is given by $x_d(t)$, which is solution of equation –

$$\dot{x}_d(t) = A_d(t)x_d(t) \tag{3.18}$$

The desired state dynamics is given by homogeneous state equation, because $x_d(t)$ is unaffected by the input signal $u(t)$. Now by solving equations (3.1) and (3.18) we get the state equation for tracking error $e(t) = x_d(t) - x(t)$.

$$\dot{e}(t) = Ae(t) + (A_d - A)x_d(t) - Bu(t) \tag{3.19}$$

The main objective is to find control input $u(t)$, which makes the tracking error given by $e(t)$ equal to zero in steady state. To achieve this by optimal control, our first aim is to find objective function which is to be minimized. In tracking problem control input will depend on state vector $x_d(t)$. Now combining equations (3.1) and (3.19) and taking the state vector as $x_c(t) = [e(t)^T; x_d(t)^T]^T$, thus control input is given by following linear control law –

$$u(t) = -K_c(t)x_c(t) = -K_c(t)[e(t)^T; x_d(t)^T]^T \tag{3.20}$$

where $K_c(t)$ is combined feedback gain matrix. The equations (3.3.18) and (3.3.19) can be written as following combined state equation –

$$\dot{x}_c(t) = A_c x_c(t) + B_c u(t) \tag{3.21}$$

$$\text{where } A_c(t) = \begin{bmatrix} A & [A_d(t) - A(t)] \\ 0 & A_d(t) \end{bmatrix},\ B_c(t) = \begin{bmatrix} -B(t) \\ 0 \end{bmatrix} \tag{3.22}$$

So now objective function can be expressed as –

$$J(t,t_f) = \int_t^{t_f} \left(x_c^T(\tau)Q_c(\tau)x_c(\tau) + u^T(\tau)R(\tau)u(\tau) \right) d\tau \tag{3.23}$$

In tracking error problem final time t_f cannot be taken as infinite, because the desired state vector $x_d(t)$ will not go to zero in steady state, thus non-zero control input $u(t)$ will be required in steady state. The system represented by equation (3.21) is uncontrollable, because desired state dynamics given by equation (3.18) is unaffected by input $u(t)$. Since the system represented by equation (3.21) is uncontrollable, thus unique solution of system is not guaranteed. Thus for having a guaranteed positive definite and unique solution of the optimal control problem, we have to exclude the uncontrollable desired state vector from objective function by choosing combined state weighting matrix as follows –

$$Q_c(t) = \begin{bmatrix} Q(t) & 0 \\ 0 & 0 \end{bmatrix}$$

Thus changed objective function will be –

$$J(t,t_f) = \int_t^{t_f} \left(e^T(\tau)Q(\tau)e(\tau) + u^T(\tau)R(\tau)u(\tau) \right) d\tau \tag{3.24}$$

Here in equation (3.24) $u(t)$ is given by equation (3.20). Thus for existence of unique and positive definite solution of optimal control problem, we choose $Q(t)$ and $R(t)$ to be positive semi definite and definite respectively. The optimal gain $K_c(t)$ is given by –

$$K_C(t) = R^{-1}(t)B_C^T(t)M_C \tag{3.25}$$

where M_C is solution of the following equation –

$$-\frac{\partial M_C}{\partial t} = A_C^T M_C + M_C A_C - M_C B_C R^{-1}(t) B_C^T(t) M_C + Q_C(t) \tag{3.26}$$

M_C is symmetric matrix which can be represented as –

$$M_C = \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix} \tag{3.27}$$

where M_1 and M_2 corresponds to plant and desired state dynamics. Now substitute equation (3.27) and (3.22) into equation (3.25), the optimal feedback gain matrix is given as –

$$K_C(t) = -[R^{-1}(t)B^T M_1; R^{-1}(t)B^T M_2] \tag{3.28}$$

and optimal control input is given by –

$$u(t) = R^{-1}(t)B^T M_1 e(t) + R^{-1}(t)B^T M_2 x_d(t) \tag{3.29}$$

Now substitute equation (3.22) and (3.27) into equation (3.26) we get –

$$-\frac{\partial M_1}{\partial t} = A^T M_1 + M_1 A - M_1 B R^{-1}(t) B^T M_1 + Q(t) \tag{3.30}$$

$$-\frac{\partial M_2}{\partial t} = M_2 A_d + M_1(A_d - A) + (A^T - M_1 B R^{-1} B^T) M_2 \tag{3.31}$$

Optimal matrix M_1 can be obtained by solving equation (3.30) and this value is used in equation (3.31). Thus equation (3.31) can be written as –

$$-\frac{\partial M_2}{\partial t} = M_2 A_d + M_1(A_d - A) + A_C^T M_2 \tag{3.32}$$

Where $A_C = A - BR^{-1}(t)B^T M_1$ (3.33)

Most of the time it is required to track a constant desired state vector given as, $x_d(t) = x_d^C$, which corresponds to $A_d = 0$. Thus both M_1 and M_2 are constants in the steady state. Thus equations (3.30) and (3.31) can be written as –

$$0 = A^T M_1 + M_1 A - M_1 B R^{-1}(t) B^T M_1 + Q(t) \tag{3.34}$$

$$0 = -M_1 A + A_C^T M_2 \tag{3.35}$$

The equation (3.34) is the algebraic Riccati equation. From equation (3.35) we get –

$$M_2 = \left[A_C^T\right]^{-1} M_1 A \tag{3.36}$$

Now substituting equation (3.36) in equation (3.29) we get –

$$u(t) = R^{-1}(t)B^T M_1 e(t) + R^{-1}(t)B^T \left[A_C^T\right]^{-1} M_1 A x_d^C \tag{3.37}$$

Substituting equation (3.37) into equation (3.19) we get –

$$\dot{e}(t) = A_C e(t) - \left[A + BR^{-1}(t)B^T\left(A_C^T\right)^{-1}M_1 A\right] x_d^C \tag{3.38}$$

Thus from equation (3.38) it is clear that tracking error can become zero in the steady state for any non-zero constant desired state x_d^C. The final optimal control input is given as –

$$u(t) = R^{-1}(t)B^T M_1 e(t) - K_d(t)x_d^C \tag{3.39}$$

where $K_d(t)$ is feed forward gain matrix which will make $e(t)$ zero in steady state for some value of x_d^C. By substituting equation (3.39) into equation (3.19), the state equation for tracking is calculated as –

$$\dot{e}(t) = A_C e(t) - [A - BK_d(t)]x_d^C \tag{3.40}$$

Thus by using the same value of positive semi definite matrix Q and positive definite matrix R as used in Linear Quadratic Regulator problem, optimal control gain K is calculated. Now by taking specific reference value x_d^C optimal control input is calculated by using equation (3.39). In this particular TRMS system there are two output i.e. pitch and yaw. So two reference signal are taken, which are –

$$x_{d1}^C = 1$$

$$x_{d1}^C = 2$$

Thus output of TRMS, pitch will track x_{d1}^C and yaw will track x_{d1}^C at steady state.

3.4 Kalman Filter

3.4.1 Overview

The Kalman Filter, which is also known as Linear Quadratic Estimation (LQE), is basically an algorithm which uses series of measurements observed over time, comprises of noise and other inaccuracies and it produces estimates of unknown variables that seems to be more precise than those based on single measurement. Kalman Filter [18-20] has large number of application in technology. Some of applications are navigation and control of vehicles, guidance. Kalman Filter is widely applied concept in time analysis used in fields like signal processing and econometrics.

3.4.2 Requirement of Kalman Filter

TRMS model is a stochastic system because due to the presence of process noise and measurement noise, it cannot be modeled by using deterministic model. Thus a noisy plant is a stochastic system, which can be modeled by passing white noise through appropriate linear system. Consider a linear plant –

$$\dot{x}(t) = Ax(t) + Bu(t) + F(t)w(t) \tag{3.41}$$

$$y(t) = Cx(t) + Du(t) + v(t) \tag{3.42}$$

where $v(t)$ is measurement noise vector and $w(t)$ is process noise vector and this may arise due to modelling error such as neglecting high frequency and nonlinear dynamics. The correlation matrices of non-stationary white noise, $w(t)$ and $v(t)$, and can be expressed as –

$$R_v(t,\tau) = W(t)\delta(t-\tau) \tag{3.43}$$

$$R_z(t,\tau) = V(t)\delta(t-\tau) \tag{3.44}$$

where $W(t)$ and $V(t)$ are time-varying power spectral density matrices of $w(t)$ and $v(t)$.

while designing the control system for stochastic plant, we cannot depend on full state feedback, because state vector $x(t)$ cannot be predicted. Thus for stochastic plant observer is for predicting the state vector based upon measurement of output $y(t)$ given in equation (3.42) and input $u(t)$. State observer cannot be used, because it would not take into account power spectral density of process noise and measurement noise. And also there is designing problem for multi-input multi-output plants, thus it is used only for single output case. Thus we require the observer that takes into account process and measurement noise into consideration and estimate the state vector $x(t)$ of plant based upon statistical value of vector output and plant. Such observer is called Kalman Filter.

3.4.3 Mathematical model of Kalman Filter

Kalman filter which is an optimal observer, minimizes the statistical error of estimation error, $e_0(t) = x(t) - x_0(t)$, where $x_0(t)$ is estimated state vector. The state equation of Kalman Filter is given as –

$$\dot{x_0}(t) = Ax_0(t) + Bu(t) + L(t)[y(t) - Cx_0(t) - Du(t)] \tag{3.45}$$

where L is Kalman Filter gain matrix. As optimal regulator minimizes the objective function comprises of transient and steady state response and control energy, in the same way Kalman Filter minimizes covariance of estimation error,$R_e(t,t) = E[e_0(t)e_0^T(t)]$. Subtracting equation (3.45) from (3.41) we get –

$$\dot{e_0}(t) = [A - L(t)C]e_0(t) + F(t)w(t) - L(t)v(t) \tag{3.46}$$

Thus after minimizing the covariance of estimation error $R_e(t,t)$, algebraic Riccati equation results for optimal covariance matrix, R_0^e -

$$0 = A_G R_e^0 + R_e^0 A_G^T - R_e^0 C^T V^{-1} C R_e^0 + F W_G F^T \tag{3.47}$$

where,

$$A_G = A - F(t)\varphi(t)V^{-1}(t)C(t) \tag{3.48}$$

$$W_G(t) = W(t) - \varphi(t)V^{-1}(t)\varphi^T(t) \tag{3.49}$$

and $\varphi(t)$ is cross spectral density matrix between $w(t)$ and $v(t)$.

Kalman Filter gain matrix is given as –

$$L = R_e^0 C^T V^{-1} \tag{3.50}$$

where R_e^0 is calculated by solving algebraic Riccati equation (3.47). The necessary and sufficient condition for existence of a positive and semi-definite solution for L is that, $[A, F]$ is stabilizable and $[A, C]$ is detectable.

3.4.4 Design of Kalman Filter

While designing the Kalman Filter, the process noise spectral density matrix V and measurement noise spectral density matrix Z are randomly chosen. These density matrices are varied until we get desired response. The condition for checking the desired response is that the ratio between the elements of the returned optimal covariance matrix of estimation error P and covariance of simulated estimation error cov(e) should be same.

For system TRMS the value of process spectral density V and measurement noise spectral density matrix Z are taken as –

$W = 10^{-6}F^{T}F$ and $V = 10^{-6}CC^{T}$

where, F=B

Thus the Kalman gain L, Returned optimal covariance matrix of estimation error P, eigen value of Kalman Filte E of TRMS system is given as –

$$L = \begin{bmatrix} 1.0412 & -0.0534 \\ -.0.0534 & 7.1475 \\ 0.1080 & -0.0946 \\ 0.0047 & 0.0375 \\ -0.0839 & -0.7523 \\ 0.0435 & -0.6881 \\ 0.2505 & 25.0448 \end{bmatrix} \qquad E = \begin{bmatrix} -3.7371 \\ -1.8623+3.1630i \\ -1.8623-3.1630i \\ -0.6253+1.9775i \\ -0.6253-1.9775i \\ -0.9830 \\ -0.9908 \end{bmatrix}$$

$$P = \begin{bmatrix} 0.0010 & -0.0001 & 0.0001 & 0 & -0.0001 & 0 & 0.0003 \\ -0.0001 & 0.0071 & -0.0001 & 0 & -0.0008 & -0.0007 & 0.0250 \\ 0.0001 & -0.0001 & 0.0005 & 0 & 0 & 0.0002 & -0.0008 \\ 0 & 0 & 0 & 0.0005 & 0 & 0 & 0.0003 \\ -0.0001 & -0.0008 & 0 & 0 & 0.0004 & -0.0001 & -0.0057 \\ 0 & -0.0007 & 0.0002 & 0 & -0.0001 & 0.0043 & -0.0051 \\ 0.0003 & 0.0250 & -0.0008 & 0.0003 & -0.0057 & -0.0051 & 0.1378 \end{bmatrix} * 10^{3}$$

3.5 Linear Quadratic Gaussian (LQG)

3.5.1 Overview

Linear Quadratic Gaussian (LQG) [21-23] controller is an optimal controller. It deals with linear system with additive white Gaussian noise and having incomplete state information and undergoing control to quadratic cost. The solution of LQG control problem is unique and consists of Linear dynamic feedback control law that can be easily implemented. Linear Quadratic Gaussian controller is combination of Kalman Filter and Linear Quadratic Regulator. LQG works on separation principle, it means that Kalman Filter and Linear Quadratic Regulator can be designed and computed independently.

LQG controller application can be applied to Linear time invariant system along with Linear time varying system. Here in this work Linear time invariant system is being considered. Designing of system with LQG controller does not guarantee Robustness of system. The robustness of system should be checked once the LQG controller has been designed. Figure-3.3 shows block diagram of LQG controller.

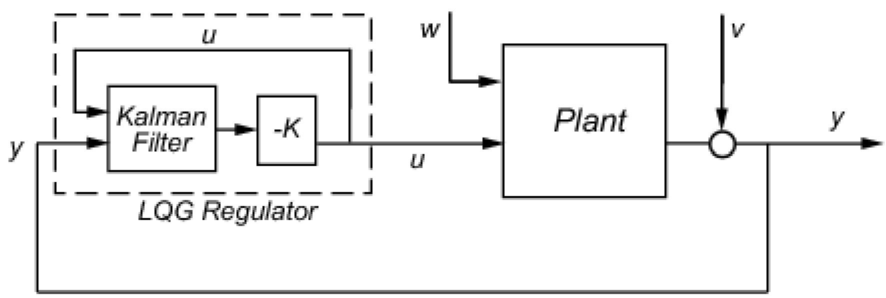

Figure-3.3 Block diagram of LQG controller along with plant

Here in Figure-3.3 it can be seen that Linear Quadratic Gaussian (LQG) controller composed of Kalman Filter (which will estimate all the state of system), followed by Linear Quadratic Regulator (LQR) (which is responsible for controlling the response of system). Along with control input ‘u’ process noise ‘w’ is also applied to system. External white Gaussian noise is added to plant because plant is stochastic with some unknown noise. Measurement noise ‘v’ is also added to system and finally we get response as ‘y’.

3.5.2 Requirement of LQG compensator

For TRMS system, while designing Linear Quadratic Regulator or Linear Quadratic Tracking controller, we have assumed full state feedback. It means we have assumed that all the states of system are available and can be measured directly. But since in our TRMS system, number of output is less than number of states, thus all the state of system cannot be measured directly. Thus for that type of system, an observer is designed, that will estimate all the state of system based on input and output combination of system. Thus Kalman Filter is designed which will estimate all the state of system, i.e. seven states from two output measurement and it is combined with Linear Quadratic Regulator and combination will give Linear Quadratic Gaussian controller.

3.5.3 Steps in the Design of LQG Compensator

a. Design optimal regulator (LQG) for linear plant assuming full state feedback, with a quadratic objective function. Here we have assumed that all the state of system can be measured directly. The regulator will generate the control input $u(t)$ based upon state vector $x(t)$.
b. Design a Kalman Filter for the linear plant with control input $u(t)$, measured output $y(t)$ and combined with white noise $v(t)$ and $z(t)$. The Kalman Filter will give optimal estimate of state vector $x_0(t)$. The Kalman Filter designed in this work is full order Kalman Filter.
c. Now combine Linear Quadratic Regulator (LQR) with Kalman Filter and the combination will give Linear Quadratic Gaussian (LQG) controller, that will be responsible for controlling the response of plant. This compensator will generate control input $u(t)$ based upon state estimated by Kalman Filter.

3.5.4 Compensator for TRMS system

The state-space representation of optimal compensator (LQG), for regulating the noisy plant with state-space model is given by following state and output equation –

$$\dot{x}_0(t) = (A - BK - LC + LDK)x_0(t) + Ly(t) \tag{3.51}$$

$$u(t) = -Kx_0(t) \tag{3.52}$$

where L and K are Kalman Filter and optimal regulator gain matrices respectively.

Here optimal regulator gain matrix K is obtained by using following command –

$$K = lqr(A, B, Q, R) \tag{3.53}$$

where

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 100000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.1 \end{bmatrix} \qquad R = \begin{bmatrix} 0.0395 & 0 \\ 0 & 1 \end{bmatrix}$$

$$K = \begin{bmatrix} 0.0008 & 0.0002 & -0.0004 & 0.0007 & -1.5892 & 0.0034 & 0.0003 \\ 0.0039 & 0.0003 & -0.0010 & 0.0021 & -0.0030 & -0.0006 & 0.0007 \end{bmatrix} * 10^3 s$$

Kalman Filter gain parameter L can be obtained by using –

$$[L, P, E] = lqe(A, F, C, 10^{-6} * F^T F, 10^{-6} * CC^T) \tag{3.54}$$

The value of L,P,E is given in section 3.4.5.

The Eigen values of Linear Quadratic Gaussian (LQG) compensator, consists of Eigen values of Linear Quadratic Regulator (LQR) and Eigen values of Kalman Filter. For system to be stable Eigen values of Linear Quadratic Gaussian (LQG) compensator should be on left hand side of imaginary axis. Ideally response of Linear Quadratic Gaussian (LQG) compensator should be same that of Linear Quadratic Regulator (LQR). For that Eigen values Linear Quadratic Regulator (LQR) should be dominating compared to Eigen values of Kalman Filter. It means that

Eigen value of Kalman Filter should be far from imaginary axis as compared to Eigen values of Linear Quadratic Regulator (LQG).

As Kalman Filter does not require control input signal, thus its Eigen values can be shifted deeper into left half plane without requirement of large control input and it can be achieved free of cost. But in some cases it is not possible to shift the Eigen value of Kalman Filter deeper into left half plane by simply varying the noise spectral densities, so in that case proper choice of Kalman Filter spectral densities will yield best recovery of full state feedback dynamics.

3.6 Results and Discussion

In this work two reference inputs signal $u_1 = 0.5$ and $u_2 = 0.3$ are applied for tracking the path of TRMS as shown in Fig.3.4. The output of TRMS will track this corresponding reference signal. The value of reference signal u_1 and u_2 can also be changed, depending upon requirement, i.e. it can be either sinusoidal or it can be step or ramp input.

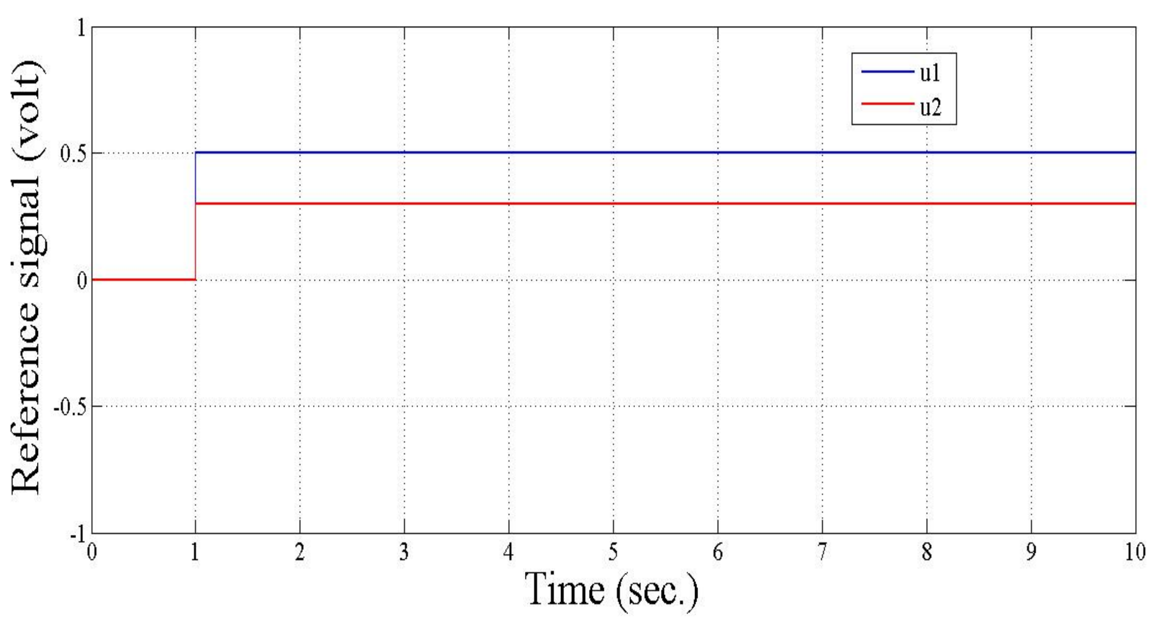

Figure-3.4 Reference signal applied to TRMS

Now firstly TRMS response is controlled by Linear Quadratic Regulator. The output of TRMS in 2-DOF using LQR controller is shown in Fig.3.5.

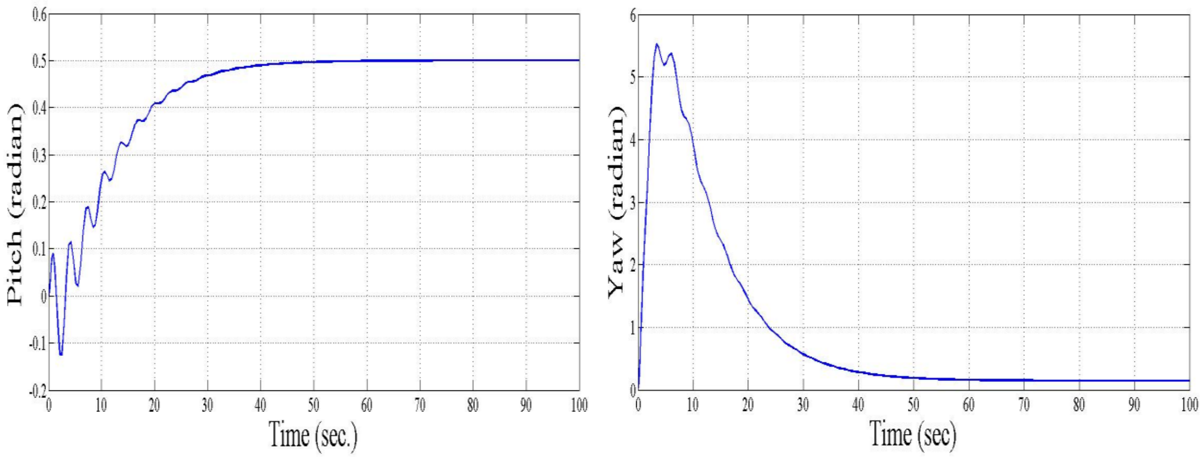

Figure-3.5 Output of TRMS using LQR

The output of TRMS, i.e. pitch is able to track the desired reference $u_1 = 0.5$ with no steady state error and yaw is able to track the desired reference $u_2 = 0.3$ with 4.6% steady state error. The response of Yaw shows large Maximum peak overshoot because of cross coupling nature between vertical plane and horizontal plane.

In using Linear Quadratic Regulator (LQR) as controller it has been assumed that all states are directly measurable. But here in our case number of output is less than number of states, i.e. number of output is two and number of states is seven. So it is not possible to measure all the state of system directly. Thus for measuring all the state of system Kalman Filter is used.

Kalman Filter basically describes the states of plant. It shows the variation of plant parameter with time. In this work full state Kalman Filter has been designed, which estimates all the state of system depending upon input-output combination. The Fig.3.6 shows variation of TRMS state using Kalman Filter.

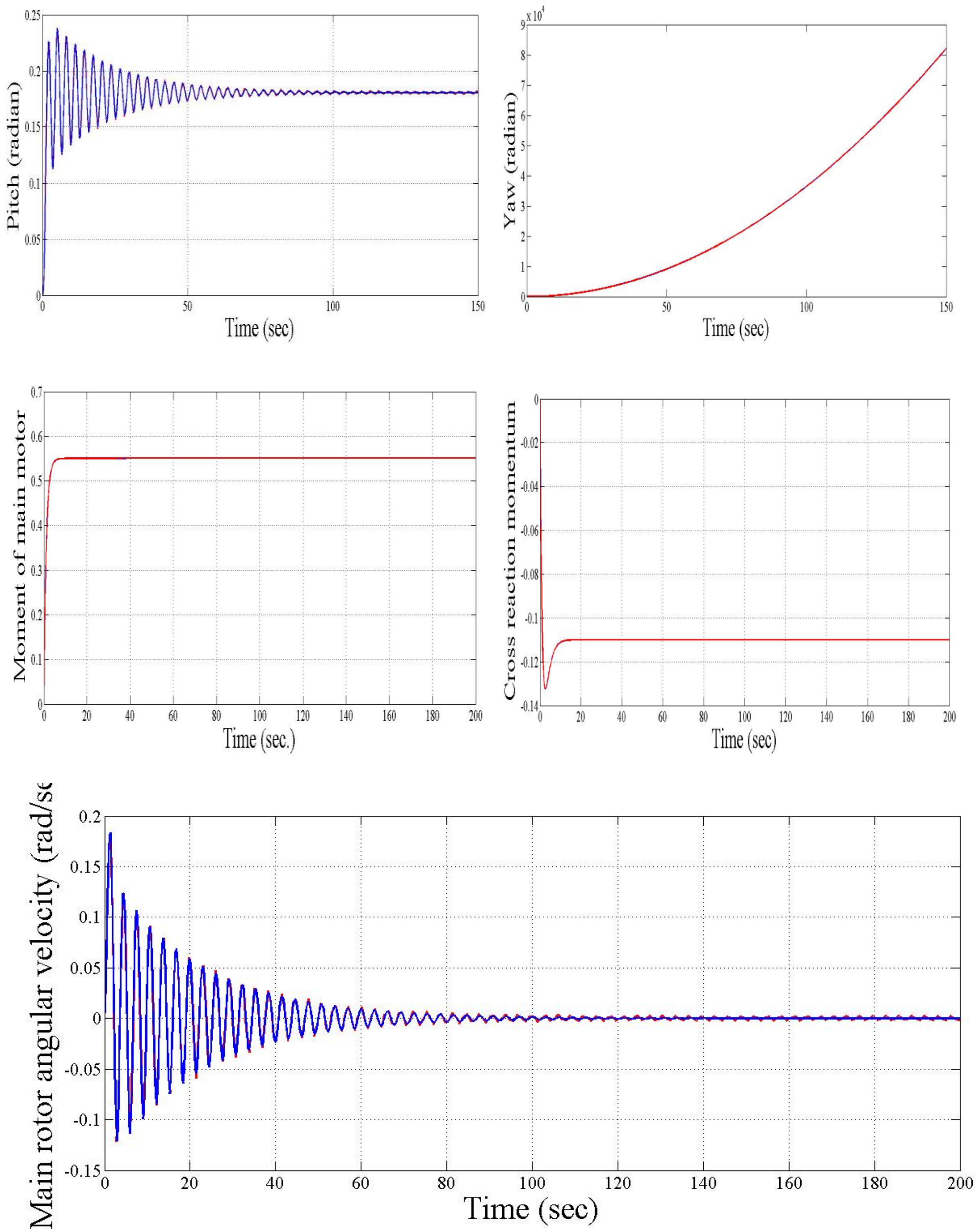

Figure-3.6 Output of Kalman Filter

Fig.3.6 shows that some of the states of system, including one output i.e. yaw is unstable. Thus with one output unstable, the complete plant is considered to be unstable, it means that output of is system unbounded for bounded input. Thus to make the plant stable Linear Quadratic Regulator (LQG) controller is combined with Kalman filter. Thus in this work, Linear Quadratic Gaussian (LQG) controller is used for controlling the performance of TRMS. For arriving at the simulated results of the standard LQG compensator, different combinations of spectral densities are tried. The answers reported here correspond to the best combination. The Fig.3.7 shows the output of TRMS using LQG controller.

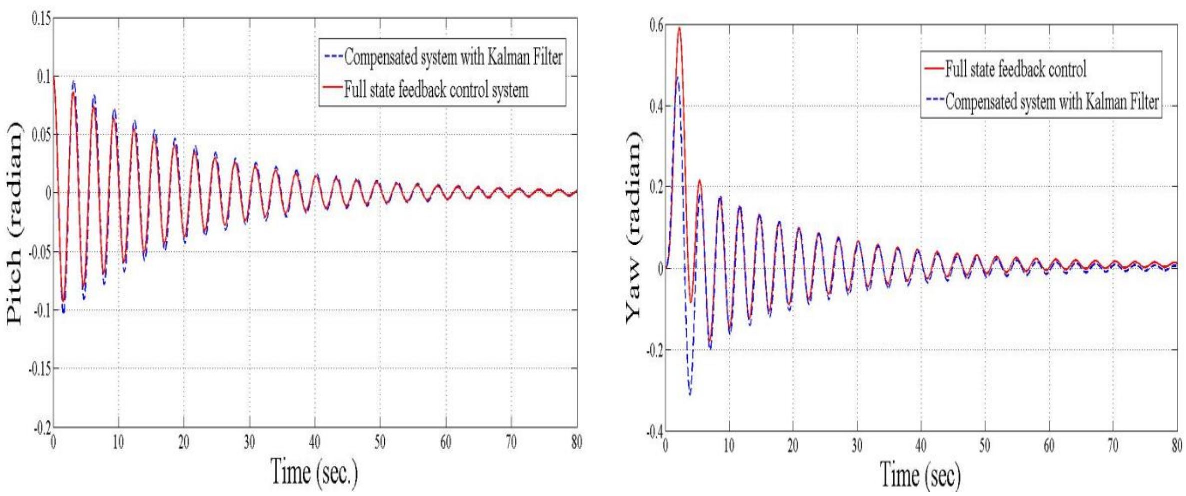

Figure-3.7 Output of TRMS using LQG

In the Fig.3.7, there is no reference signal applied to system, so it is a regulating problem whose steady state value is zero. Here, output of Linear Quadratic Regulator (LQR) is compared with output of Linear Quadratic Gaussian (LQG) controller. As shown in Fig.3.7, the response of LQR and LQG overlap to each other, and finally there steady state value is zero.

Fig.3.7 shows that Twin Rotor MIMO system can be made to give stable and accurate response by using Linear Quadratic Gaussian (LQG) controller.

BACTERIAL OPTIMIZATION ALGORITHM BASED CONTROLLER DESIGN

4.1 Bacterial Foraging Optimization Algorithm

4.1.1 Overview

Optimization techniques are used to minimize the input effort required and to maximize the desired benefit. For over the last five decades, optimization algorithms like Genetic Algorithms (GAs), Evolutionary Programming (EP), Evolutionary Strategies (ES), which draw their inspiration from evolution and natural genetics, have been dominating the realm of optimization algorithms. Genetic Algorithm has certain limitations like, it gives poor fitness function which will generate bad chromosome blocks and the optimization response time obtained may not be constant. In addition to this, it is not sure that this optimization technique will give global optimum value. Thus BFO Algorithm is used as an optimization technique which overcomes all limitations of GA [6-8]. Bacterial foraging optimization algorithm (BFO) is widely accepted optimization algorithm for distributed optimization and control. BFO algorithm works on the principle of behaviour of Escherichia coli bacteria. BFO algorithm is capable of optimizing real-time problems, which arise in several application domains. Bacterial foraging optimization algorithm is based on nature inspired optimization algorithm. The key idea behind BFOA is grouping foraging strategy of Escherichia coli bacteria in multi optimal function optimization. Bacterial search takes place in a manner that it maximizes the energy intake per unit time. Each bacterium communicates with other bacteria by sending signals.

4.1.2 Steps involved in BFO

The complete Bacterial foraging optimization algorithm is divided into 4 basic steps. They are –

a) **Chemotaxis** – In this step, movement of an Escherichia coli bacterium through swimming and tumbling via flagella is simulated. Escherichia coli bacteria basically can move in two ways. The bacteria can swim for a period of time in one direction or may tumble and alternate between these two modes for entire life time.

b) **Swarming –** In this step, some of the bacterium will attract each other and move with higher density.

c) **Reproduction –** In this step, the least healthy bacteria (bacteria with highest cost function) will die, and while other healthier bacteria will split into two and will take their place.

d) **Elimination and Dispersal –** In this step, there is sudden or gradual changes takes place due to which some of the bacteria die due to various reasons, like, rise of temperature may kill bacteria that are in region with high concentration of nutrient gradients. In this, events take place in such a way that either group of bacteria gets killed or dispersed to new position. To simulate this in BFOA some amount of bacteria are liquidated with small probability at random time.

4.1.3 BFO Algorithm

For a given objective function, BFO algorithm involves the execution of the following steps –

1) Let S be the number of bacteria used for the searching algorithm. Here each bacterium represents a string of filter weights.
2) The number of parameters to be optimized is represented as ‘p’.
3) Swimming length represented as N_s, after each tumbling of bacteria it is taken in Chemotaxis loop.
4) Number of iterations which is to be undertaken in Chemotaxis loop is given as N_c.
5) The maximum amount of reproduction is given as N_{re}.
6) N_{ed} represent the maximum amount of Elimination dispersal iteration that bacteria undergoes.
7) P_{ed} represent the probability by which elimination dispersal process will take place.
8) P gives the position of each bacterium in bacteria population.
9) $C(i)$ represent the size of step taken for each bacterium in random direction.

4.1.4 Problem Formulation

In this section, the controller design process has been framed as an optimization problem, with controller parameters as the process variables. Modelling of bacterial population is carried out through Chemotaxis, Reproduction and Elimination and Dispersal steps.

Let us assume j=k=l=0, where j,k,l represents Chemotaxis, Reproduction, Elimination-dispersal iteration respectively. By updating θ^i, P is automatically updated.

1) Elimination dispersal iterations: $l = l + 1$
2) Reproduction iterations: k=k+1
3) Chemotaxis iterations: j=j+1
   a) For i=1,2,3,…S, perform the bacterium iterations as follows –
   b) Determine cost function –

$$J(i,j,k,l) = \int_{t_0}^{t_f} \left(\left(\frac{y_1}{y_{1max}}\right)^2 + \left(\frac{y_2}{y_{2max}}\right)^2 + \left(\frac{u_1}{u_{1max}}\right)^2 + \left(\frac{u_2}{u_{2max}}\right)^2 \right) dt \qquad (4.1)$$

   where y_1 and y_2 are TRMS outputs, i.e. pitch and yaw, and u_1 and u_2 are control inputs.
   c) Let $J_{last} = J(i,j,k,l)$, save value of cost function, because we may find any better value.
   d) Tumble. Now generate a random vector $\Delta(i)$, such that each element of $\Delta m(i)$, m=1,2….p, is between [-1,1].
   e) The next position is given by –

$$\theta^i(j+1,k,l) = \theta^i(j,k,l) + C(i)\frac{\Delta(i)}{\sqrt{\Delta^T(i)\Delta(i)}} \qquad (4.2)$$

   This will give the step of size $C(i)$ for bacterium i, in direction of tumble.
   f) Compute the next cost function $J(i,j+1,k,l)$ corresponding to position $\theta^i(j+1,k,l)$.
   g) Swimming
      I. Let counter for swim length is defined as m=0
      II. While $m < N_s$ following condition takes place –
         i. Let m=m+1

ii. Now check cost function condition that if $J(i, j+1, k, l) < J_{last}$ then $J_{last} = J(i, j+1, k, l)$, this will give another step of size $C(i)$ in the direction given in equation (4.1).

iii. Use $\theta^i(j+1, k, l)$ to find new cost function $J(i, j+1, k, l)$.

iv. Else if $m = N_s$, while statement will terminate.

h) Now switch to next bacterium $(i+1)$ and check if $i \neq S$ then go to step 'b' to process the next bacterium.

4) Now check if $j < N_c$ switch to step 3. Now since the life of bacteria is not over hence continue Chemotaxis iteration.

5) Reproduction

a) For the given value of k and l and i=1,2,…S , let

$$J^i_{health} = \min\big(J(i, j, k, l)\big) \text{ for } j = 1,2,\ldots N_c + 1 \qquad (4.3)$$

Equation (4.2) gives the health of bacteria i. now sort the bacteria in ascending order of J_{health}.

b) $S_r = S/2$ bacteria which have highest J_{health} will die out and rest of S_r will split out and takes their place.

6) If $k < N_{re}$, switch to step 2, because the number of given reproduction step has not yet completed, so start the Chemotaxis step.

7) Now Elimination-dispersal: for each i=1,2… S with probability P_{ed} elimination and dispersal of each bacterium takes place. Thus it keeps the number of total bacteria in population constant. To perform this operation simply place one to random location in optimization domain.

For the present work, the parameter θ^i consists of the tuning parameters for the LQR compensator *i.e.*, the elements of the matrices Q and R.

4.2 Results and Discussion

For the present work, the application of Bacterial foraging optimization algorithm (BFO), aims at obtaining an optimal compensator for the TRMS system. For this, the positive semi-definite state weighting matrix Q and positive definite control input weighting matrix R is given by –

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & q2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & q6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{And} \quad R = \begin{bmatrix} r1 & 0 \\ 0 & 1 \end{bmatrix}$$

Here q2, q6, r1 are assumed to be varying and needs to be optimized by using Bacterial foraging optimization algorithm (BFO). Values of q2, q6, r1 are given in Table-4.1.

Table-4.1

Parameter	Initial Value	Final Value
q2	0.05	0.1
q6	0.08	0.4
r1	0.12	0.395

Figure-4.1 depicts the iterative variation of cost function given by equation (4.1), cost function decreases with number of iteration and attain minimum value of 1.1023e+5 at twelfth iteration.

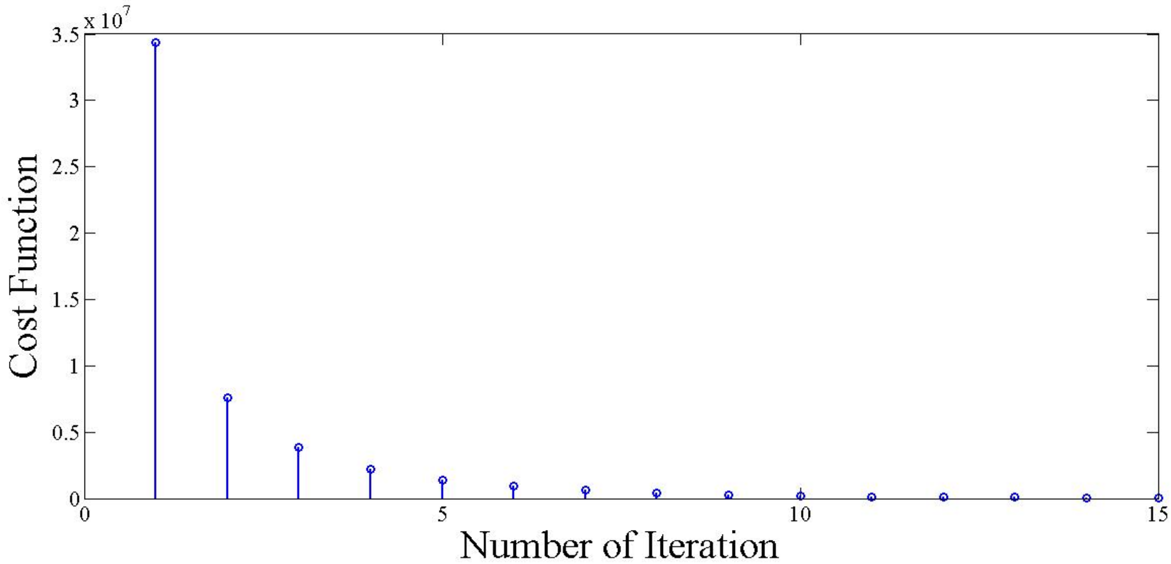

Figure 4.1 Variation of cost function $J(i, j, k, l)$ with respect to number of iteration.

Figure-4.2 shows comparison of output of TRMS of BFO based LQG with standard LQG.

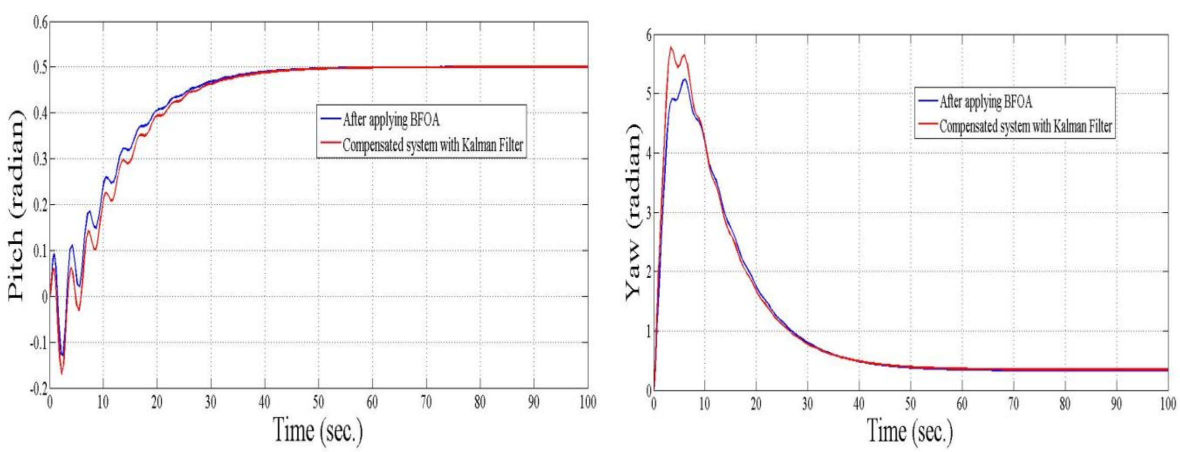

Figure-4.2 Comparison between output of TRMS using with and without BFOA

As shown in Figure-4.2, response of system i.e. pitch and yaw, after using Bacterial Foraging Optimization (BFO) Algorithm is better than the standard LQG. Table-4.2 displays the performance specification of LQG compensator with random selection of spectral densities (standard LQG) along with BFO based selection.

Table – 4.2 Comparison of TRMS responses with and without using BFO

Performance Specification	BFO based LQG		Standard LQG	
	Pitch	**Yaw**	**Pitch**	**Yaw**
Rise Time	25.4113	0.1579	27.1669	0.1658
Settling Time	38.2387	46.8260	38.7667	46.8942
Maximum Peak Overshoot	0.0035	1.6464e+003	0.0034	1.8249e+003
Phase Margin	∞	∞	∞	∞
Gain Margin	57.4db	36db	54db	33.3db

As given in Table-4.2 the performance specification of system improves after applying Bacterial foraging optimization algorithm (BFO). The response of system becomes faster with decrease in rise time, system response settles to steady state value in less time and with decrease in maximum peak overshoot, stability of system gets improved.

Phase margin and Gain margin of system is calculated by using the mat lab command,

```
A = a-b*k-l*c;
sys = c*inv(s*eye(7)-(A))*b;
bode (sys);
```

where a, b, c is plant matrix, k is LQR gain and l is Kalman gain.

The improvement in performance specification of system takes place because after using Bacterial foraging optimization algorithm (BFO), we obtain optimal value of state and control weighting matrix. However, this comes at the cost of high bandwidth, which ultimately leads to low noise attenuation at high frequencies.

CONCLUSIONS

5.1 Conclusions

In this work, a state feedback controller (LQG) based on noisy output measurement has been designed for an experimental set-up (TRMS), that replicates the aircraft flight dynamics, i.e. pitch and yaw. The simulation results corresponding the cases related to full state feedback and observer (Kalman Filter) based feedback with state estimation reflects the appropriateness of the proposed approach in meeting the desired specifications. In using Linear Quadratic Gaussian (LQG) compensator for controlling the flight dynamics of TRMS, the elements of the positive semi definite matrix ‘Q’ and positive definite matrix ‘R’ are randomly chosen. To overcome the hit and trial approach, an evolutionary optimization technique i.e. Bacterial Foraging Optimization (BFO) algorithm has been applied and their results has been compared with Linear Quadratic Gaussian (LQG) controller result.

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