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NONLINEAR STATIC ANALYSIS OF MAGNETOSTRICTIVE LAMINATED COMPOSITE PLATE

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NONLINEAR STATIC ANALYSIS OF MAGNETOSTRICTIVE LAMINATED COMPOSITE PLATE

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[MECHANICAL ENGINEERING]

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Dedicated to my parents & Guide

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CERTIFICATE

This is to certify that the thesis entitled "NONLINEAR STATIC ANALYSIS OF MAGNETOSTRICTIVE LAMINATED COMPOSITE PLATE" which is being submitted by PRADEEP KUMAR MISHRA as partial fulfillment of Master of Technology degree in MACHINE DESIGN AND ANALYSIS (Mechanical Engineering) during the academic year 2011-2013 in the Department of Mechanical Engineering, National Institute of Technology, Rourkela.

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PRADEEP KUMAR MISHRA

DECLARATION

I hereby declare that the thesis entitled "NONLINEAR STATIC ANALYSIS OF MAGNETOSTRICTIVE LAMINATED COMPOSITE PLATE" is a bonafied record of work done by me, as a functional part towards the fulfillment of Master of Technology degree in Machine Design and Analysis specialization (Mechanical) from National Institute of Technology, Rourkela during the academic year 2011-2113.

This is purely academic in nature and it has not formed the basis, for the award of any Degree/ Diploma/Ascertain ship/ fellowship or similar title to any Candidate.

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ABSTRACT

A third order shear deformation theory is used to study the nonlinear static behavior of laminated smart composite plate with magnetostrictive layer. In this study, geometric nonlinearity is taken in Green-Lagrange sense and Terfenol-D is used as a magnetostrictive material. In addition to that, the stresses are obtained using coupled equation through the constitutive relation by considering the effect of magnetic field induction. A C^0 finite element formulation is proposed to discretize the present model and the governing equations are obtained using the minimization of the total potential energy theorem. Non-dimensionalized displacements and in plane stresses are computed for the laminated plate with and without magnetostrictive layer. The results are compared with these available literatures.

An ANSYS model has also been developed for the said problem and few results are obtained and compared with available exact/numerical results.

Keywords: Smart material; Magnetostrictive material; Third order shear deformation theory; Geometrical nonlinearity; Green-Lagrange; Finite element analysis; Nonlinear static analysis; Laminated plate; ANSYS 14.0;

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Chapter - 1

1.1. Introduction

A composite is a structural material that consists of two or more combined constituents are combined at a macroscopic level. One constituent is called the reinforcing phase and the other one is called the matrix. A general classification of different composite materials can be seen in Fig 1. A schematic presentation of all composites can be seen in Fig. 2 to have a clear visualization. Composites are utilized in a wide range of fields like mechanical, aerospace, marine, automotive, biomedical and MEMS due to their light weight, high specific strength, high specific stiffness, and excellent fatigue and corrosion resistance in comparison to their conventional counterpart.

As discussed in the above lines the composites have number of tailoring properties and due to that many structural components are fast replaced by composites. Even though composites have distinct features over conventional materials, they do have few limitations or drawbacks. In general, composites are flexible in nature as compared to conventional material and exposed to combined loading condition which in turn affects their structural behavior like vibration, bending and buckling responses considerably. They may suffer from large amplitude vibration and/or large deformation early than the other conventional material. To overcome the above short comings many functional (smart) materials (piezoelectric and electrostrictive materials, shape memory alloys, magnetostrictive materials, electro and magneto rheological fluids etc.) are developed in recent years. Each smart material has a unique advantage and disadvantage of its own in sensing, control, and actuation. In the present analysis, out of different functional materials magnetostrictive material is taken due to its unique property and wide applicability in different industries. A brief discussion on this material has been given in the following paragraph.

1.2 Magnetostrictive materials and the working principle

Magnetostrictive materials are probably the most popular active material used in both actuator and sensor applications because of its low cost, low power consumption, low weight, high frequency response and ease in embedding or bonding with the structure. According to

James P.Joule (1842) magntostrictive material is the smart material which changes its magnetic state in response to applied stresses when exposed to a magnetic field.

There are different magnetostrictive (Terfenol-D, Galfenol etc.) materials are available based on the required application. In this present study, Terfenol-D is considered to be the magnetosrictive smart material due to its relatively low strains and moderate forces over a wide frequency range serves as best commercial magnetostrictive material available in the market. The Terfenol-D has some dominant advantages as actuators and sensors over other materials. The coupled mechanical and magnetic properties of magnetostrictive smart make them well suited for use as actuators and sensors in smart structures.

The direct and converse magnetic effect governs the interaction between the mechanical and magnetic behavior of this type of material. The direct magnetic effect states that a strain applied to the material is applied converted to magnetic field intensity. On the other hand the converse of magnetic effect states that a magnetic intensity applied to the material is converted to strain. The design and fabrication of large complicated structures with integrated magnetostrictive materials requires the accurate modeling and analysis as beforehand by using available analytical and/or numerical method. Today design engineers/engineering firms show confidence on results of finite element modeling and analysis either by the commercial finite element package and/or analysis of structures using customize code using different computer language. Terfenol-D is an alloy of terbium, iron, and dysprosium and their application in today's engineering is given in Fig 3 and 4. It can serve both as actuator and sensor and produce strains up to 2500µm, which is 10 times more than a piezoceramic material. It also has high energy density, negligible weight, and point excitation with a wide frequency bandwidth.

As discussed aforementioned paragraph, many research works have been performed successfully to simulate the various linear/nonlinear responses of conventional and composite materials using ANSYS finite element software in recent years. These studies show that ANSYS can precisely simulate all sorts of material and geometrical (linear/nonlinear) modeling of laminated composite with and without functional material. All types of nonlinearities are allowed large deformations, plasticity, creep, stress stiffening, contact (gap) elements, hyper elastic elements, and so on.

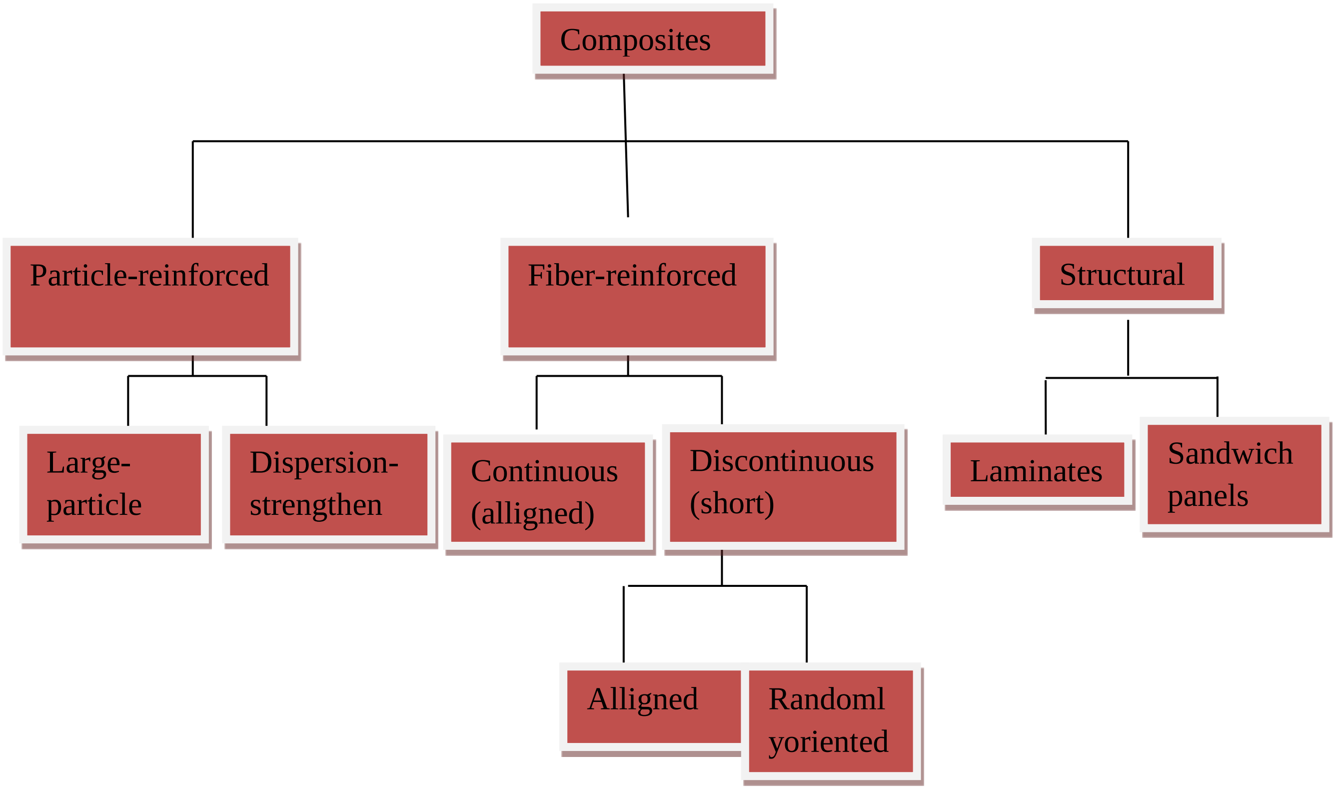

Fig 1 Classification of composites

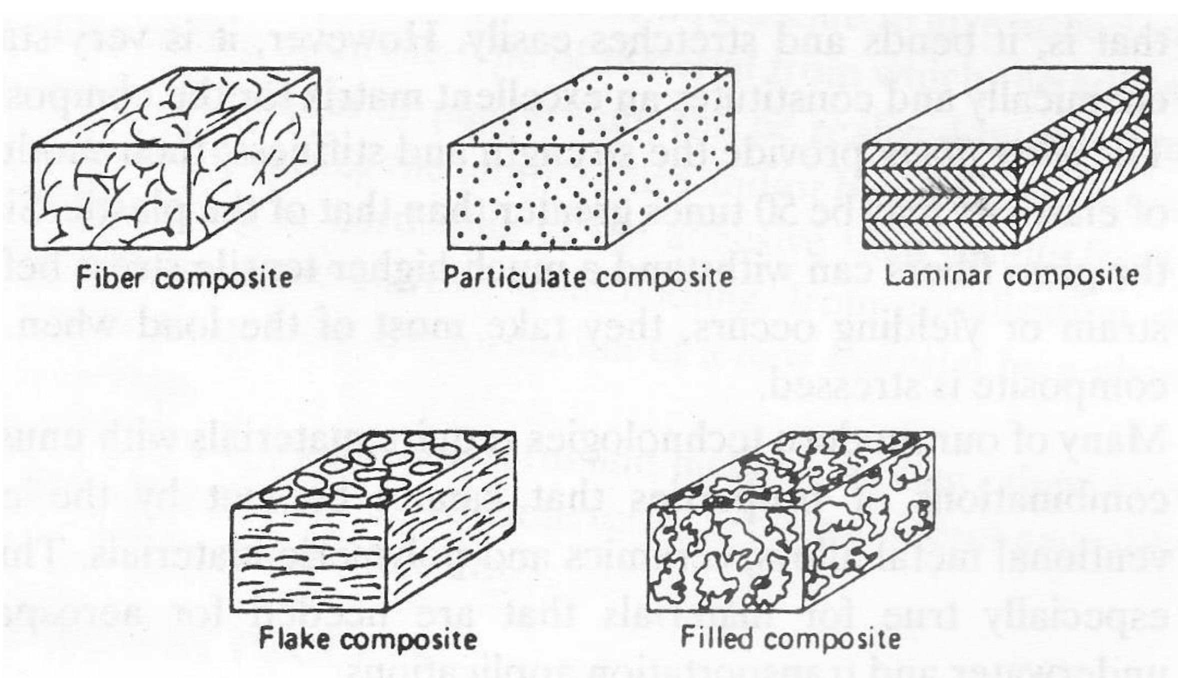

Fig.2 Schematic presentation of composite materials (www.mechlook.com)

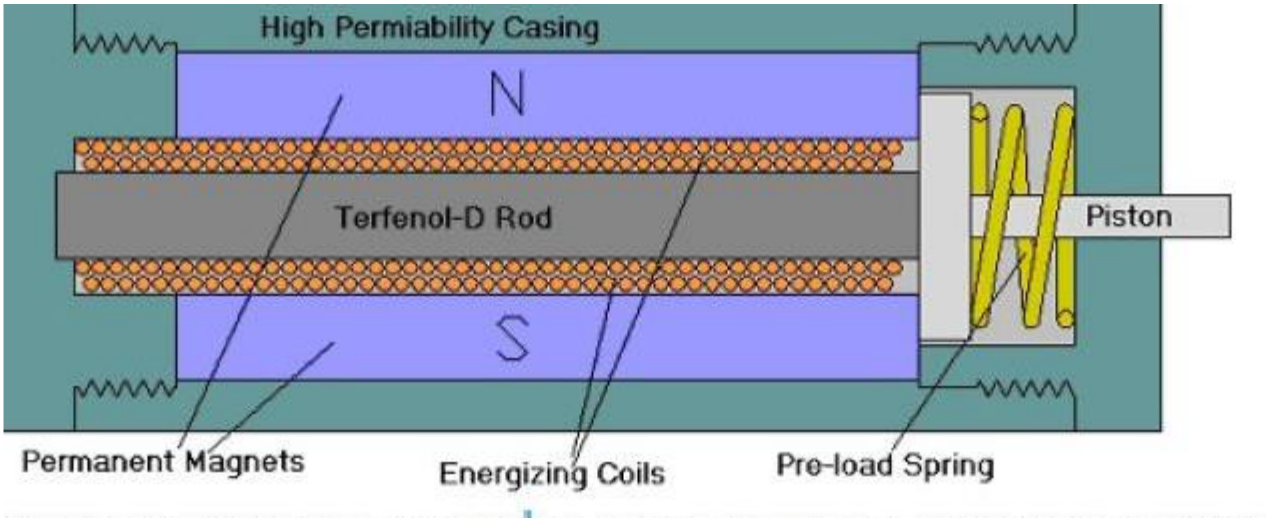

Fig 3 Detailed view of Magnetostrictive Actuator (Google Image)

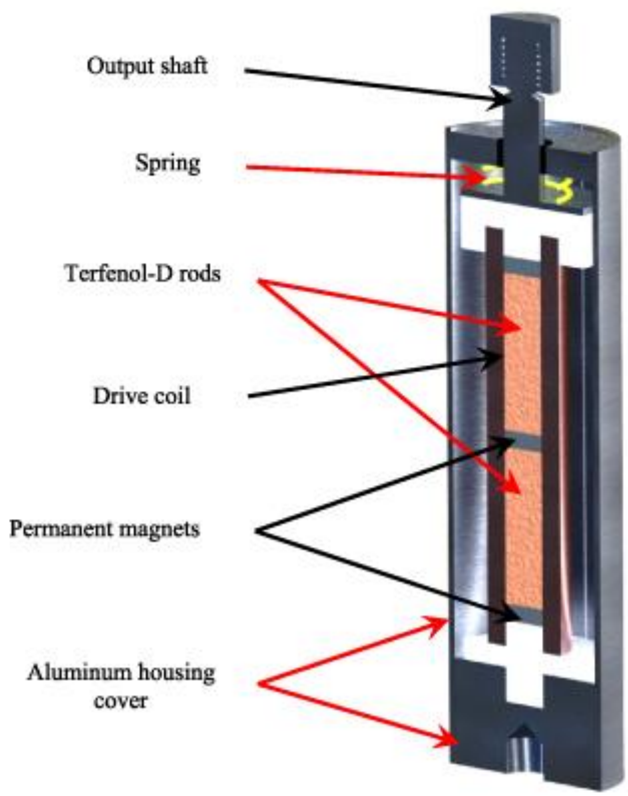

Fig 4 Magnetostrictive actuator (iopscience.iop.org)

1.3. Scope of the work

- ❖ Development of a general mathematical formulation of magnetostrictive smart composite plate by taking Green-Lagrange geometrical nonlinearity.
- ❖ Development of nonlinear finite element model.
- ❖ Development of an ANSYS model.
- ❖ Development of MATLAB code and its comprehensive testing.
- ❖ Nonlinear static response of smart composite plate with and without magnetostrictive material.
- ❖ Applications to various problems.

1.4. Thesis outline

The present chapter discusses the introduction of the problem and a short discussion on the composite and magnetostrictive materials. Subsequently, the scope of the present research has been discussed point wise. The remainder of this thesis is organized in five more chapters. Chapter 2 includes literature review to provide a summary of the base of knowledge already available involving the issues of interest. Chapter 3 discussed the general mathematical model development and their solution steps. Next to that Chapter 4 discusses various responses obtained using the present developed model. Finally, Chapter 5 presents the closure of the work based on the output and future scope of the work.

1.5. Conclusions

Present chapter highlights the importance of the present work and the next chapter discussed the literature review on the said problem through extensive study of recent and past available literatures.

Chapter - 2

2.1. Introduction

Mechanical, aerospace, civil engineering structures, sport equipment and medical prosthetics are the broad areas where smart composite components are being used. This is well known that the composite materials are very much flexible as compared to the conventional material and they suffer from large deformation under combined loading. Hence, for designing of high performance components, simulating the true material behavior and to have a better understanding of physical phenomena, nonlinear static analysis is very much essential. Many studies have already been completed on the smart composite structure by taking the magnetostrictive material as a smart material. In this regard some of the recent and earlier literatures are discussed in the following paragraph.

A considerable literature is available on the nonlinear static analysis of the smart laminated composite plates in Green-Lagrange sense with and without taking into account the transverse shear effects using various theories. A brief review of the available literature in this field is discussed for the sake of continuity. The responses like static, dynamic, stability and vibration of laminated structures for different geometry and materials are discussed in Reddy [1]. Wang [2] presented the finite element formulation of large-scale geometrically nonlinear laminated composite shell structures. Vuksanovic [3] obtained the numerical solution of static, dynamic, free vibration and buckling behavior using finite element method based on various plate theories (classical plate theory and first order shear deformation theory). A 3-D, 27-node hybrid-interface element is used to analyzed the thick laminated plate based on the minimum potential energy principle is presented by Desai and Bambole [4]. Nonlinear free vibration and transient behavior of laminated composite shells under hygrothermal loading are reported by Naidu and Sinha [5-6]. They have developed the nonlinear model using Green-Lagrange nonlinearity based on first order shear deformation theory (FSDT) in conjunction with nonlinear finite element method (FEM). Kundu and Han [7] studied geometrical nonlinear bending behavior of laminated composite spherical, cylindrical and conical shell panels subjected to hygrothermal loading using FEM steps. Kant and Swaminathan [8] derived the equations of equilibrium using the principle of minimum potential energy (PMPE) and Navier’s technique to solve the boundary value problem of composite plate. Swaminathan and Ragounadin [9] studied the static analysis of antisymmetric angle-ply laminated composite and sandwich plates using a higher-order refined theory. Ghugal and Shimpi [10] presented a comprehensive review of refined theories for shear deformable isotropic and anisotropic laminated plates based on the various plate theories such as classical plate theory, FSDT, second order shear deformation theory and higher order shear deformation theory (HSDT). Litewka and Sygulski [11] solved static problems for plates of intermediate thickness using a direct formulation of the boundary element method and modified Gauss integration method. Samanta and Mukhopadhyay [12] analyzed a stiffened shell element for the geometric nonlinear static analysis of shallow and deep shells. Baltacıoglu *et al.* [13] derived nonlinear static response of laminated rectangular plates using the FSDT. Luiz *et al.* [14] studied the efficiency and the robustness of an one-point quadrature eight-node hexahedral element for the analysis of shells, plates and beams undergoing large displacements and rotations. Kumar *et al*. [15] explored the shape control and active vibration suppression of a laminated composite shell with integrated piezo-electric sensors and actuators. Bogdanovich and Pastore [16] investigated the structural behavior of textile reinforced composites by using smart material approach. Yuan *et al.* [17] studied the magnetostrictive static force sensor with a giant magnetostrictive material rod. Ueno and Higuchi [18] investigated the improvements of Terfenol and PZT actuator embedded composite against conventional material using the magnetic force control principle. Linnemann *et al*. [19] studied the phenomenological behavior of magnetostrictive and piezoelectric materials using a thermodynamic constitutive model. Zheng *et al.* [20] developed a giant magnetostrictive device based on the Jiles–Atherton (JA) magnetomechanical hysteresis model using the theory of the magnetomechanical effect. Dash and Singh [21] studied of the nonlinear free vibration of the laminated composite plate with embedded and/or surface bonded piezoelectric layers in the framework of the HSDT. They have derived the model using Green-Lagrange type nonlinearity and the model is consisting of all nonlinear higher order terms to have a general approach. Panda and Singh [22] find out the nonlinear free vibration analysis of the laminated composite spherical shell panel using Green-Lagrange nonlinear strains. Lacarbonara and Pasquali [23] derived a geometrically exact theory of thin multilayered composite plates with general stacking sequences which accounts for mid-plane stretching, flexure, and transverse shear strains. Carrera [24] reformulated mixed theory originally proposed by Toledano and Murakami and extended to dynamic analyses of plates and doubly curved shells. Lee and Kim [25] investigated the nonlinear vibration of hybrid laminated plates with aluminum, glass fibre reinforced plastic, carbon fibre reinforced plastic and boron fibre reinforced plastic by considering the extension-bending coupling effect in the laminated plates using the Lagrangian equation. Mechab *et al*. [26] explored the analytical solutions of cross-ply laminated plates under thermo-mechanical loading based on the HSDT. Detwiler *et al*. [27] derived a new finite element formulation to analyze the mechanical-electrical behavior of laminated composite structures containing distributed piezoelectric actuators and sensors. Dash and Singh [28] studied the nonlinear bending analysis of the laminated composite plates in the framework of the HSDT by taking the geometric nonlinearity in Green–Lagrange sense. Angulo *et al*. [29] analyzed the influence of resin load on magnetic properties of Terfenol-D material. Lim *et al.* [30] fabricated Terfenol-D composites with good magnetic (including magnetostrictive) and mechanical properties. Seung [31] presented analytical and finite element solutions of laminated composite plate and shell structures with smart material lamina based on the theoretical formulations. Carman [32] developed a nonlinear constitutive relation for magntostrictive materials that includes nonlinear coupling effects arising between temperature/preload and magnetic field strengths. Lee and Reddy [33] derived the governing equations of the third order shear deformation theory (TSDT) including thermal effects and von-Karman non-linear strains. Kishore *et al*. [34] reported the nonlinear static responses of laminated composite plate embedded with magnetostrictive materials based on the TSDT by taking the geometric nonlinearity in von-Karman sense. Dapino *et al*. [35] determine the trends and inherent uncertainties in the functional dependence of fundamental elasto-magnetic properties of Terfenol-D on changes in operating conditions in a controlled transducer environment. Pratt *et al*. [36] presented active vibration control and saturated phenomenon of a cantilever beam embedded with Terfenol-D actuator. Civalek [37] derived an approximate numerical solution of doubly curved shallow shells resting on Winkler Pasternek elastic foundations using the von Karman–Donnel nonlinear kinematics. Ganapathi *et al*. [38] developed a C^0 eight-nodded quadrilateral serendipity plate element with thirteen degrees of freedom to analyze nonlinear free and forced vibration.

Swaminathan and Ragounadin [39] presented the analytical solutions static behavior of anti-symmetric angle-ply composite and sandwich plates using a higher-order refined theory. Zhang and Kim [40] developed a displacement based flat triangular plate element (3 nodded and 18 degrees of freedom) to analyze linear and nonlinear behavior of thin to moderately thick laminated plates. Lakshminarayana and Murthy [41] developed a higher-order triangular plate element (3-node and 15 degrees of freedom per node) for linear analysis of laminated (isotropic and anisotropic) plates. Aagaah *et al*. [42] reported finite element based static behavior of a rectangular multi-layered composite plates by taking the mid plane kinematics in the framework of the TSDT. Setoodeh and Karami [43] analyzed static, free vibration and buckling behavior of anisotropic thick laminated composite plates under different supports (distributed and point elastic support) using a 3-D layer-wise FEM. Argyris and Tenek [44, 45] developed a 3-noded multilayered triangular facet element of 18-dof and analyzed linear/nonlinear bending behavior of isotropic, sandwich, laminated composite and hybrid plates. Vuksanovic [46] proposed a C^0 isoperimetric single layered finite element model based on higher-order theory and checked its applicability to static, dynamic, free vibration and buckling analysis of symmetric cross-ply and angle-ply laminated composite and sandwich plates. Yu [47] presented higher-order finite element analysis using a 6-nodded triangular layered shell element. Ibrahimbegovic [48] used Timoshenko's beam function method to analyze thick and thin plates. Soh *et al*. [49] introduced a new nine degree of freedom triangular element for analysis of thick and thin plates using Timoshenko's beam function method. Soh *et al*. [50] developed twelve degrees of freedom (DOF) quadrilateral element for thick and thin laminated plates. Two displacements based quadrilateral elements for the linear and nonlinear static analysis of laminated plates are developed by [51, 52]. Auricchio and Sacco [53] analyzed laminated composite plates using mixed-enhanced finite element. Wilt *et al*. [54] presented a mixed elemental analysis for laminated plates and shells. Whitney [55, 56] studied the effect of bending-extensional coupling and support conditions on the responses of laminated plates under transverse load. Somashekar *et al*. [57] introduced a field consistent four-nodded (five degrees of freedom per node) anisotropic plate/shell element to analyze the degree of anisotropy and their effect on in-extensional bending of corresponding shapes. Zaghloul and Kennedy [58] studied linear and non-linear behavior of symmetrically laminated plates under different boundary conditions using finite difference method. Putcha and Reddy [59] presented a refined mixed shear flexible finite element analysis for non-linear analysis of laminated composite plates by taking eleven degrees of freedom per node (three displacements, two rotations and six moment resultants). Cheng *et al*. [60] performed geometrically non-linear analysis of composite laminates using the perturbation finite element method (PFEM) by taking the discrete-layer shear deformation theory.

This can be understood from the above survey that many studies related to nonlinear static behavior of laminated plates have been reported in the past but the study embedding smart magnetostrictive layer in composite plate are less in number. In this present study an effort has been made to model the laminated composite plates with and without magnetostrictive layer by considering geometrical nonlinearity in Green-Lagrange sense in the framework of the HSDT to investigate the nonlinear static behavior. A nonlinear finite element model is proposed to discretize using an isoperimetric eight nodded serendipity element. The sets of nonlinear equations are obtained through minimum potential energy. In addition to that an ANSYS model also has been developed for laminated structure with the magnetostrictive material as the functional material. In present analysis, non-dimensionalized displacements and in plane stresses are computed for the laminated plate with and without magnetostrictive laminated composite plate. The responses are obtained using a computer code developed in MATLAB and ANSYS 14.0 for different parameters such as loading, boundary conditions thickness ratio, aspect ratio and different angle lay-up effect.

A detail discussion on finite element modeling, governing differential equation and ANSYS model are given in the subsequent chapter.

Chapter - 3

3.1. Assumptions

1. The composite plate considered in the present investigation is orthotropic in nature.
2. The laminated plates problems are taken in this study are of equal thickness.
3. The transverse shear strains vanishes on top and bottom surfaces of the plate:
4. The number of layers with or without magnetostrictive layers is perfectly bonded.

3.2. Displacement field

The displacement field within the laminate is assumed to be based on the Reddy's TSDT as discussed earlier. The in plane displacements are expanded as cubic functions of thickness coordinate to maintain parabolic shear stress and strain profile, while the transverse displacement varies linearly through the plate thickness.

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\phi_x(x,y) + z^2\theta_x(x,y) + z^3\lambda_x(x,y) \\ v(x,y,z) &= v_0(x,y) + z\phi_y(x,y) + z^2\theta_y(x,y) + z^3\lambda_y(x,y) \\ w(x,y,z) &= w_0(x,y) + zw_1(x,y) \end{aligned} \quad \ldots (1)$$

where *u*, *v* and *w* denote the displacements of a point along the (*x, y, z*) coordinates. u_0, v_0 and w_0 are corresponding displacements of a point on the mid plane. Similarly, $\phi_x, and \phi_y$ are the rotations of normal to the mid plane about the y-axis and x-axis, respectively. The functions $\theta_x, \theta_y, \lambda_x, \lambda_y$ and w_1 are the higher order terms in the Taylor series expansion defined in the mid plane of the plate to maintain cubic.

The above displacement field as given in Eq. (1) can be rewritten in the following form after incorporating the assumptions that the transverse shear strains vanishes on top and bottom surfaces of the plate:

$$
\begin{aligned}
u &= u_0 + f_1(z)\phi_x + f_2(z)\beta_x + f_3(z)\eta_x; \\
v &= v_0 + f_1(z)\phi_y + f_2(z)\beta_y + f_3(z)\eta_y; \\
w &= w_0 + f_4(z)w_1;
\end{aligned}
\qquad \dots (2)
$$

3.3. Strain displacement relations

The following equations define the nonlinear strain displacement relation by taking Green-Lagrange type nonlinearity in geometry for any general material continuum:

$$
\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \varepsilon_{yz} \\ \varepsilon_{xz} \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix}
\frac{\partial u}{\partial x} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right] \\
\frac{\partial v}{\partial y} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] \\
\frac{\partial w}{\partial z} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] \\
\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} + \left[\left(\frac{\partial u}{\partial y}\frac{\partial u}{\partial z}\right) + \left(\frac{\partial v}{\partial y}\frac{\partial v}{\partial z}\right) + \left(\frac{\partial w}{\partial y}\frac{\partial w}{\partial z}\right)\right] \\
\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} + \left[\left(\frac{\partial u}{\partial x}\frac{\partial u}{\partial z}\right) + \left(\frac{\partial v}{\partial x}\frac{\partial v}{\partial z}\right) + \left(\frac{\partial w}{\partial x}\frac{\partial w}{\partial z}\right)\right] \\
\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \left[\left(\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}\right) + \left(\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}\right) + \left(\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)\right]
\end{Bmatrix} \qquad \dots (3)
$$

or, $\{\varepsilon\} = \{\varepsilon_L\} + \{\varepsilon_{NL}\}$

The linear strain vector corresponding to the displacement field is written as

$$
\begin{aligned}
\{\varepsilon_L\}^T &= \{\varepsilon_1 \varepsilon_2 \varepsilon_3 \varepsilon_4 \varepsilon_5 \varepsilon_6\}^T \\
&= \left\{ \frac{\partial u}{\partial x}\ \frac{\partial v}{\partial y}\ \frac{\partial w}{\partial z}\ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right\}^T
\end{aligned}
\qquad \dots (4)
$$

The terms of the linear strain vector are expressed as thickness and in plane coordinate in following lines:

$$
\begin{aligned}
\varepsilon_1 &= \varepsilon_1^{\ 0} + zk_1^{\ 1} + z^2 k_1^{\ 2} + z^3 k_1^{\ 3} \\
\varepsilon_2 &= \varepsilon_2^{\ 0} + zk_2^{\ 1} + z^2 k_2^{\ 2} + z^3 k_2^{\ 3} \\
\varepsilon_3 &= \varepsilon_3^{\ 0} \\
\varepsilon_4 &= \varepsilon_4^{\ 0} + zk_4^{\ 1} + z^2 k_4^{\ 2} \\
\varepsilon_5 &= \varepsilon_5^{\ 0} + zk_5^{\ 1} + z^2 k_5^{\ 2} \\
\varepsilon_6 &= \varepsilon_6^{\ 0} + zk_6^{\ 1} + z^2 k_6^{\ 2} + z^3 k_6^{\ 3}
\end{aligned}
\qquad \dots (5)
$$

The linear strain vector as given in Eq.(5) can also be written in matrix form as

$$\{\varepsilon_L\}_{6\times1} = \left[T^L\right]_{6\times19} \left\{\overline{\varepsilon_L}\right\}_{19\times1} \qquad \dots (6)$$

where, $\left\{\overline{\varepsilon_L}\right\} = \left\{\varepsilon_1{}^0 \varepsilon_2{}^0 \varepsilon_3{}^0 \varepsilon_4{}^0 \varepsilon_5{}^0 \varepsilon_6{}^0 k_1{}^1 k_2{}^1 k_4{}^1 k_5{}^1 k_6{}^1 k_1{}^2 k_2{}^2 k_4{}^2 k_5{}^2 k_6{}^2 k_1{}^3 k_2{}^3 k_6{}^3\right\}^T$ and is the function of thickness coordinate. The terms in $\left\{\overline{\varepsilon_L}\right\}$ having superscripts '0','1','2 and 3' are membrane, curvature and higher order strain terms, respectively. The individual terms of linear strain vectors are provided in Appendix A.

The vector $\left\{\overline{\varepsilon_L}\right\}$ has been expressed in operator and field variable and conceded as

$$\left\{\overline{\varepsilon_L}\right\} = [L]_{19\times10} \{q\}_{10\times1} \qquad \dots (7)$$

where $[L]$ is differential operator matrix and $\{q\}$ is the displacement field vector.

The nonlinear strain vector $\{\varepsilon_{NL}\}$ is obtained from Eq.(3) are obtained as following the same steps in linear case

$$\{\varepsilon_{NL}\} = \begin{Bmatrix} \varepsilon_1{}^{NL} \\ \varepsilon_2{}^{NL} \\ \varepsilon_3{}^{NL} \\ \varepsilon_4{}^{NL} \\ \varepsilon_5{}^{NL} \\ \varepsilon_6{}^{NL} \end{Bmatrix} = \begin{Bmatrix} \frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right] \\ \frac{1}{2}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] \\ \frac{1}{2}\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] \\ \left(\frac{\partial u}{\partial y}\cdot\frac{\partial u}{\partial z}\right) + \left(\frac{\partial v}{\partial y}\cdot\frac{\partial v}{\partial z}\right) + \left(\frac{\partial w}{\partial y}\cdot\frac{\partial w}{\partial z}\right) \\ \left(\frac{\partial u}{\partial x}\cdot\frac{\partial u}{\partial z}\right) + \left(\frac{\partial v}{\partial x}\cdot\frac{\partial v}{\partial z}\right) + \left(\frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial z}\right) \\ \left(\frac{\partial u}{\partial x}\cdot\frac{\partial u}{\partial y}\right) + \left(\frac{\partial v}{\partial x}\cdot\frac{\partial v}{\partial y}\right) + \left(\frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial y}\right) \end{Bmatrix} \qquad \dots (8)$$

The terms of the nonlinear strain vector as given in Eq. (8) can be expressed as

$$\begin{aligned}
\varepsilon_1{}^{NL} &= \frac{1}{2}\left\{\varepsilon_1{}^{NL0} + zk_1{}^{NL1} + z^2k_1{}^{NL2} + z^3k_1{}^{NL3} + z^4k_1{}^{NL4} + z^5k_1{}^{NL5} + z^6k_1{}^{NL6}\right\} \\
\varepsilon_2{}^{NL} &= \frac{1}{2}\left\{\varepsilon_2{}^{0} + zk_2{}^{NL1} + z^2k_2{}^{NL2} + z^3k_2{}^{NL3} + z^4k_2{}^{NL4} + z^5k_2{}^{NL5} + z^6k_2{}^{NL6}\right\} \\
\varepsilon_3{}^{NL} &= \frac{1}{2}\left\{\varepsilon_3{}^{0} + zk_3{}^{NL1} + z^2k_3{}^{NL2} + z^3k_3{}^{NL3} + z^4k_3{}^{NL4}\right\} \qquad \ldots (9) \\
\varepsilon_4{}^{NL} &= \varepsilon_4{}^{0} + zk_4{}^{NL1} + z^2k_4{}^{NL2} + z^3k_4{}^{NL3} + z^4k_4{}^{NL4} + z^5k_4{}^{NL5} \\
\varepsilon_5{}^{NL} &= \varepsilon_5{}^{0} + zk_5{}^{NL1} + z^2k_5{}^{NL2} + z^3k_5{}^{NL3} + z^4k_5{}^{NL4} + z^5k_5{}^{NL5} \\
\varepsilon_6{}^{NL} &= \varepsilon_6{}^{0} + zk_6{}^{NL1} + z^2k_6{}^{NL2} + z^3k_6{}^{NL3} + z^4k_6{}^{NL4} + z^5k_6{}^{NL5} + z^6k_6{}^{NL6}
\end{aligned}$$

The nonlinear strain vector $\{\varepsilon_{NL}\}$ as expressed in Eq. (9) can be written in terms of mid plane nonlinear strain as

$$\{\varepsilon_{NL}\}_{6\times 1} = \left[T^{NL}\right]_{6\times 38}\{\overline{\varepsilon_{NL}}\}_{38\times 1} \qquad \ldots (10)$$

where,

$$\{\overline{\varepsilon_{NL}}\} = \begin{Bmatrix} \varepsilon_1{}^{NL0} & \varepsilon_2{}^{NL0} & \varepsilon_3{}^{NL0} & \varepsilon_4{}^{NL0} & \varepsilon_5{}^{NL0} & \varepsilon_6{}^{NL0} & k_1{}^{NL1} & k_2{}^{NL1} & k_3{}^{NL1} & k_4{}^{NL1} & k_5{}^{NL1} & k_6{}^{NL1} & k_1{}^{NL2} & k_2{}^{NL2} & k_3{}^{NL2} & k_4{}^{NL2} & k_5{}^{NL2} & k_6{}^{NL2} & k_1{}^{NL3} \\ k_2{}^{NL3} & k_3{}^{NL3} & k_4{}^{NL3} & k_5{}^{NL3} & k_6{}^{NL3} & k_1{}^{NL4} & k_2{}^{NL4} & k_3{}^{NL4} & k_4{}^{NL4} & k_5{}^{NL4} & k_6{}^{NL4} & k_1{}^{NL5} & k_2{}^{NL5} & k_4{}^{NL5} & k_5{}^{NL5} & k_6{}^{NL5} & k_1{}^{NL6} & k_2{}^{NL6} & k_6{}^{NL6} \end{Bmatrix}^T$$

and $\left[T^{NL}\right]$ is the function of thickness co–ordinate. The terms in $\{\overline{\varepsilon_{NL}}\}$ having superscripts'0','1','2-3' are nonlinear membrane ,curvature and higher order strain terms, respectively.

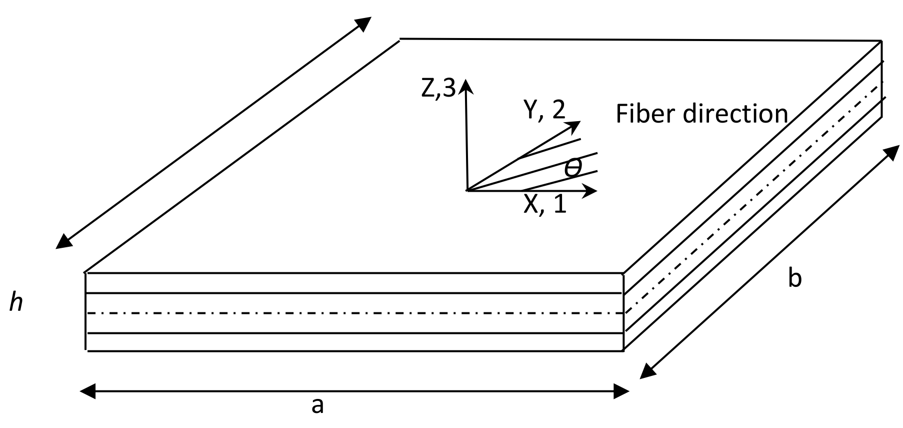

Fig 5 Laminated composite plate

3.4. Lamina constitutive relation

It is assumed that each lamina behaves as an orthotropic material with its material axes oriented arbitrarily with respect to the laminate coordinates. The coupled constitutive equations (composite and magnetostrictive) of each layer with respect to the laminate coordinates (*x*, *y*, *z*) are shown in (Fig.5) having dimensions ($a \times b \times h$).

$$\{\sigma\}^k = \left[\bar{Q}\right]^k - \left[\bar{e}\right]^k \{H\}, \quad \text{Or}$$

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{Bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{13} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{23} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{13} & \bar{Q}_{23} & \bar{Q}_{33} & \bar{Q}_{36} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{36} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{Bmatrix} - \begin{bmatrix} 0 & 0 & \bar{e}_{31} \\ 0 & 0 & \bar{e}_{32} \\ 0 & 0 & \bar{e}_{33} \\ 0 & 0 & \bar{e}_{36} \\ \bar{e}_{14} & \bar{e}_{24} & 0 \\ \bar{e}_{15} & \bar{e}_{25} & 0 \end{bmatrix} \quad \ldots (11a)$$

$$\{B\}^k = \left[\bar{e}\right]^k \{\varepsilon\} + \left[\mu\right]^k \{H\}, \quad \text{Or}$$

$$\begin{Bmatrix} B_x \\ B_y \\ B_z \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & \bar{e}_{14} & \bar{e}_{15} \\ 0 & 0 & 0 & 0 & \bar{e}_{24} & \bar{e}_{25} \\ \bar{e}_{31} & \bar{e}_{32} & \bar{e}_{33} & \bar{e}_{36} & 0 & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \end{Bmatrix} + \begin{bmatrix} \mu_{11} & \mu_{12} & 0 \\ \mu_{12} & \mu_{22} & 0 \\ 0 & 0 & \mu_{33} \end{bmatrix} \begin{Bmatrix} H_x \\ H_y \\ H_z \end{Bmatrix} \quad \ldots (11b)$$

where $\{\sigma\}$ = stress vector, $\{B\}$= magnetic induction, $\{\varepsilon\}$ =strain vector, $\{H\}$ =magnetic field intensity, $\bar{e}_{ij}$ =transformed magnetostrictive stress coefficients, μ_{ij} =strain permeability of magntostrictive coefficients, $\bar{Q}_{ij}$ =transformed constitutive matrix with respect to fiber orientations (Fig 5).

3.5. Finite element model

The displacement fields for different assumed displacement model are expressed in terms of desired field variables. In the present study the requirements of c^1 continuity has been reduced to c^0 by assuming the first derivatives of the transverse displacement as independent field variables.

Model: $\{X\} = \left[u_0 v_0 w_0 \phi_x \phi_y w_1 \theta_x \theta_y \lambda_x \lambda_y\right]^T$ … (12)

For finite element approximation, the displacement field in the domain of the plate may be expressed in terms of nodal field variables with the help of shape functions as given below:

$$\{X\} = \sum_{i=1}^{9} [N_i]\{\bar{X}_i\}, \quad \text{… (13a)}$$

$$\{\psi\} = \sum_{i=1}^{9} [N_i]\{\bar{\psi}_i\}, \quad \text{… (13b)}$$

where $\bar{X}_i, \bar{\psi}_i$ and N_i denote nodal displacement, magnetic potential and shape function for nine nodded serendipity element, respectively.

The mid-plane strain vector $\{\bar{\varepsilon}\}$ and magnetic field vector $\{\bar{H}\}$ can further be expressed using finite element approximation as

$$\{\bar{\varepsilon}\} = [L_{CFRP}]\sum_{i=1}^{9}[N_i]\{\bar{X}_i\} = [B_{CFRP}]\{\bar{X}\} \quad \text{… (14a)}$$

$$\{\bar{H}\} = [L_M]\sum_{i=1}^{9}[N_i]\{\bar{\psi}_i\} = [B_M]\{\bar{\psi}\} \quad \text{… (14b)}$$

where, Eq.(14a)and (14b) represents mid-plane strain and magnetic field vectors for CFRP and Magnetostrictive layers respectively.

$[L_{CFRP}]$ and $[L_M]$ are the operator matrices for composite laminate and magnetostrictive laminate respectively, $[B_{CFRP}] = [L_{CFRP}][N_i]$ and $[B_M] = [L_M][N_i]$.

3.6. Governing equations

The equations of equilibrium for the static analysis are obtained using the principle of minimum potential energy which can be written in analytical form as

$$\delta(\Pi) = \delta(U + V) = 0 \qquad \dots (15)$$

where, U is the total strain energy due to deformations is the potential of external loads, and Π is the total potential energy and δ denotes the variational symbol.

The expression for strain energy is given by

$$U = \frac{1}{2}\int_A \int_{Z_{K-1}}^{Z_K} \{\sigma_i\}\{\varepsilon_L + \varepsilon_{NL}\} \qquad \dots (16)$$

and work done by external forces is given by

$$V = \int_V \left(\frac{1}{2}[H]^T \{B_i\} \right) dV - \int_S [U]^T \{\Phi\}\, dS \qquad \dots (17)$$

The total potential energy (Π) is obtained by adding up the above two terms (Eqs. (16) and (17)). Total potential energy is expressed in terms of nodal degrees of freedom.

The governing equations are derived from Eq. (12) and may be expressed as

$$\begin{bmatrix} [K_{xx}] & [K_{x\psi}] \\ [K_{\psi x}] & [K_{\psi\psi}] \end{bmatrix} \begin{Bmatrix} \{\bar{X}\} \\ \{\bar{\psi}\} \end{Bmatrix} = \begin{Bmatrix} \{q_s\} \\ \{0\} \end{Bmatrix} \qquad \dots (18)$$

where, $[K_{x\psi}]$, $[K_{\psi x}]$ and $[K_{\psi\psi}]$ are the global stiffness matrices, $\{\bar{X}\}$ and $\{\bar{\psi}\}$ are the global displacement vector and potential vector, respectively and $\{q_s\}$ is the global load vector. The above equation may be written in the following decoupled form as

$$[K]\{\bar{X}\} = \{q_s\}, \qquad \dots (19)$$

with $[K] = [K_{xx}] - [K_{x\psi}][K_{\psi\psi}]^{-1}[K_{\psi x}]$.

3.7. Boundary conditions

Boundary conditions are very much essential to minimize the number of constants in the governing equation and establish a relation between physical and mathematical model.

To solve the above said governing equation following boundary conditions are taken in the present study:

All edges simply supported edges (SSSS)

$u = w = \phi_x = w_1 = \theta_x = \lambda_x = 0$ at y=0,b

$v = w = \phi_y = w_1 = \theta_y = \lambda_y = 0$ at x=0,a

All edges clamped (CCCC).

$u = v = w = \phi_x = \phi_y = w_1 = \theta_x = \theta_y = \lambda_x = \lambda_y = 0$ at x=0,a and y=$0,b$

3.8. Solution steps

Nonlinear terms of the governing equation are represented in $\bar{X}$ of Eq. (19). The nonlinear solution is carried out by direct iterative and Newton Raphson iteration method for composite plate with and without magnetostrictive material and the steps are followed from Ref. [21] and [34].

3.9. Conclusion

This chapter provides the insight into the basic behavior of strain displacement relation, finite element modeling, and the governing equations for nonlinear static analysis of composite plate embedded with and without smart magnetostrictive layers considering geometrical nonlinearity in Green-Lagrange sense. The subsequent chapter deals extensively various responses obtained using the present developed model and solution steps described above.

Chapter - 4

4.1 Introduction

A nonlinear finite element model has been developed by taking the Green-Lagrange type geometric nonlinearity based on the TSDT. A suitable nonlinear FEM model has been developed and the nonlinear stiffness matrices are obtained numerically using the Gauss quadrature integrations. The nonlinear static responses of laminated plates are obtained using direct iterative method and magnetostrictive embedded plate responses are obtained using the Newton-Raphson steps. In order to demonstrate the accuracy of the present developed model several numerical examples have been solved. The results are compared with those published results. Based on convergence study, a (10×10) mesh has been used throughout the study for the computation of the responses. The material properties and geometrical properties are taken same as the references (Putcha and Reddy [59] and Lee & Reddy [33]) for nonlinear static responses of laminated composite. The composite and magnetostrictive material properties are given in Table 1.

The deflection and the load are nondimensionized as below.

$$\bar{w} = w/h \quad \text{and} \quad \bar{p} = \frac{q_0 a^4}{E_{22} h^4}.$$

Graphite/Epoxy			Magnetostrictive (Terfenol-D)		
$E_{11}=138.6\times10^9$	$G_{12}=4.96\times10^9$	$\upsilon_{12}=0.26$	$E_{11}=26.5\times10^9$	$G_{12}=13.25\times10^9$	$\upsilon_{12}=0$
$E_{22}=8.3\times10^9$	$G_{23}=4.12\times10^9$	$\upsilon_{23}=0.26$	$E_{22}=26.5\times10^9$	$G_{23}=13.25\times10^9$	$\upsilon_{23}=0$
$E_{33}=8.3\times10^9$	$G_{13}=4.96\times10^9$	$\upsilon_{13}=0.26$	$E_{33}=26.5\times10^9$	$G_{13}=13.25\times10^9$	$\upsilon_{13}=0$

Table 1 Material properties of Graphite/Epoxy and magnetostrictive material

4.2 Convergence and validation study of laminated composite plate without magnetostrictive material

As discussed in the above paragraph, here, in this section the nonlinear static responses of laminated composite plates with/without magnetostrictive materials are obtained using the developed mathematical model. The results are compared with the references and the ANSYS results.

Fig 6 presents nondimensionalized central deflections of four different symmetric angle ply (±15°, ± 25°, ±35°, and ± 45°) composite plates. The responses are obtained using ANSYS parametric design language (APDL) code. The results are computed for a simple supported square laminated plate with the geometric properties are a=20 and h=0. 002. It can be seen from the figure that the results are converging well with the mesh refinement and a (10×10) is sufficient to compute the further results. The figure clearly shows that the performance of the present ANSYS model is very good compared to the TRIPLT [41].

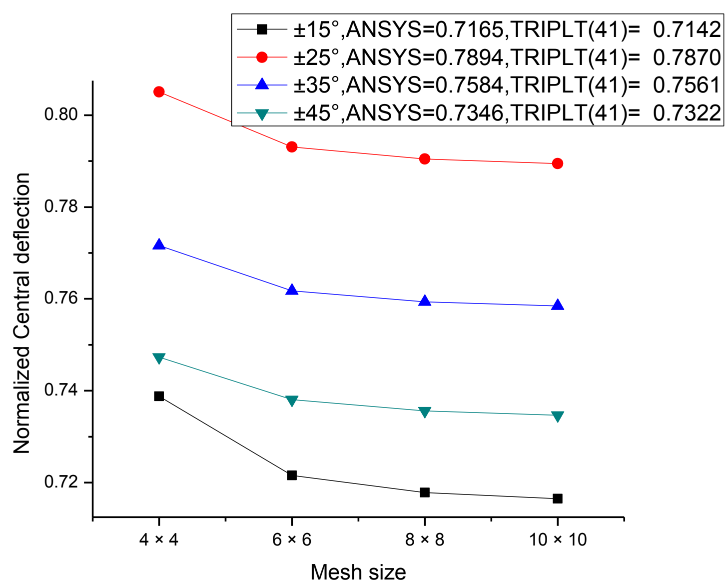

Fig 6 Normalized central deflection of a simple supported 2-layer square plate under uniform transverse load

In this example the nondimensionalized nonlinear central deflections of a clamped symmetric cross-ply $(0°/90°)_s$ square plate with side length a = 12 in. and thickness h = 0.096 in. subjected to uniformly distributed load is analyzed and plotted in Fig 7 and compared with the Zhang and Kim[40] and Putcha and Reddy [59]. The material properties used for the computation of results are same as the Zhang and Kim [40]. (E_1 = 1.8282 $\times 10^6$ psi, E_2 = 1.8315 $\times$ 10^6 psi, G_{12} = G_{13} = G_{23} = 3.125$\times 10^5$ psi, μ_{12} = 0.23949). It is evident from the figure that the nonlinear central deflection of present model is good agreement with that of Putcha and Reddy [59] and Zhang and Kim [40]. It can also be noted that nonlinear central deflection increases as load increases.

Further one more example has been solved to prove the efficacy of the developed model. Linear and nonlinear normalized central deflections of a clamped square plate is obtained using the same material properties of Putcha & Reddy [59] in both ANSYS APDL and MATLAB code and plotted in Fig 8. It can be seen that the linear and nonlinear results obtained using the present developed model is showing higher value as compared to the responses obtained in ANSYS and Agyris and Tenek [44]. It is because of the fact that the present model has been developed using Green-Lagrange nonlinearity based on the TSDT which makes the model more flexible to achieve a general case. This is also visualized that linear and nonlinear central deflections increases with increase in load values.

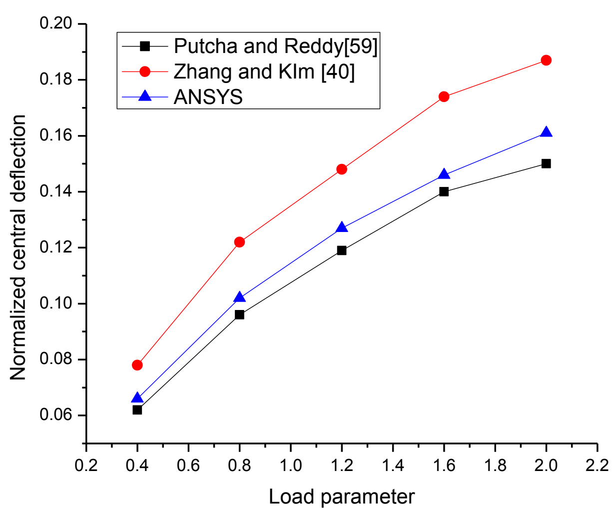

Fig 7 Normalized central deflection of a clamped symmetric cross-ply $[0°/90°]_s$ square plate subjected to a uniformly distributed load

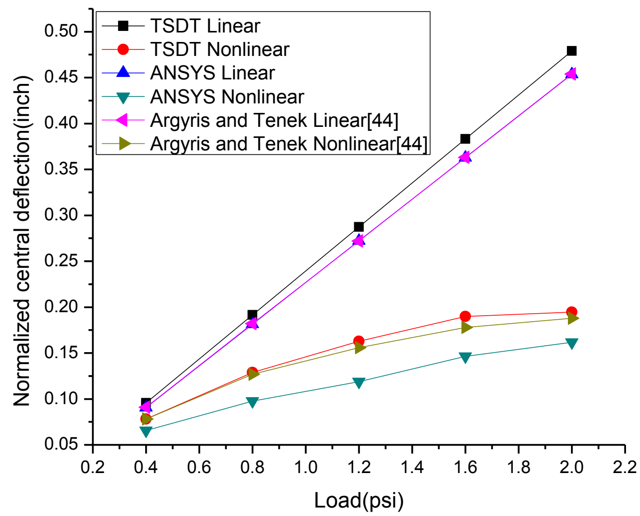

Fig 8 Normalized central deflection of a clamped symmetric cross-ply [0°/90°]$_s$ square plate subjected to a uniformly distributed load

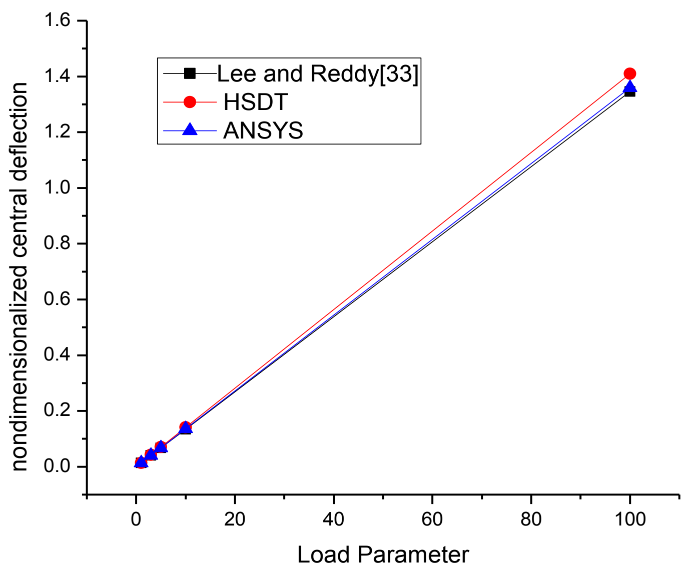

Fig 9 Variation of nondimensionalized central deflections with different load parameter of simply supported symmetric cross-ply laminate under distributed load with magnetostrictive layer

4.3 Comparison study of magnetostrictive embedded laminated composite plate

The comparison of nondimensionalized central deflection and load parameter of simply supported symmetric cross-ply laminate under distributed load with magnetostrictive layer is presented in Fig 9. In the present problem, a graphite epoxy laminated composite plate made of 10-layer (m, $90^0/0^0/90^0/0^0$)s of plate size 1m length & equal thickness of each layer is considered and the central transverse deflections are non-dimensionalized as discussed earlier.

The convergence and comparison illustrated in the Fig. 9 states that the variation in the nondimensionalized central deflections of present model show flexibility in comparison to ANSYS and Lee & Reddy results due to introduction of higher order shear deformation theory. It can also be conceded that the ANSYS result is showing good converging rate with the Lee & Reddy and the error is within one percent.

4.4 Parametric Study

In this section, some numerical examples are presented by considering different parameters to bring out complete quantitative understanding of the nonlinear bending behavior of laminated plates for symmetric/un-symmetric lamination, cross/angle-ply layup and two different supports (simply supported and clamped). In addition to that, the effects of number of layers, thickness ratios and aspect ratios on the central deflections are computed and discussed in the following section.

Normalized central deflections for a clamped symmetric cross ply $[0^0/90^0]_s$ square plate subjected to a uniformly distributed load in taking account of TSDT (MATLAB code) and FSDT (ANSYS APDL code) have been tabulated in Table 2. The result obtained in present analysis is found to be more in compared to ANSYS APDL code which shows the efficacy of considering higher order shear deformation instead of first order shear deformation one.

Load	TSDT Linear	TSDT Nonlinear	ANSYS linear	ANSYS nonlinear
0.4	0.297	0.2435	0.2734	0.2269
0.8	0.5939	0.4869	0.5464	0.4426
1.6	1.1878	0.9734	1.0808	0.8646
2	1.14848	1.2175	1.3214	1.0571

Table 2 Normalized central deflections of a clamped symmetric cross ply square plate $[0^0/90^0]_s$ subjected to a uniformly distributed load

4.4.1 Effect of boundary condition

The effect of two support conditions (simply supported and clamped) on the nondimensionalized nonlinear central deflections are examined for symmetric cross ply laminate under uniformly distributed load and the responses are shown in Table 3 & 4. It is observed that, the nondimensionalized central deflections (linear and nonlinear) increases with increase in load parameter and it is comparatively less for the clamped support.

a/h	**Load Parameter**	**SSSS**	**CCCC**
10	10	0.1359	0.0286
	30	0.1525	0.0857
	100	1.3597	0.2855

Table 3 Nondimensionalized central deflections (*w*/*h*) for symmetric cross- ply laminate under different load and boundary conditions (linear)

a/h	**Load Parameter**	**SSSS**	**CCCC**
100	10	0.1159	0.0418
	30	0.3478	0.1174
	100	1.1593	0.4018

Table 4 Nondimensionalized central deflections (*w*/*h*) for symmetric cross- ply laminate under different load and boundary conditions (nonlinear)

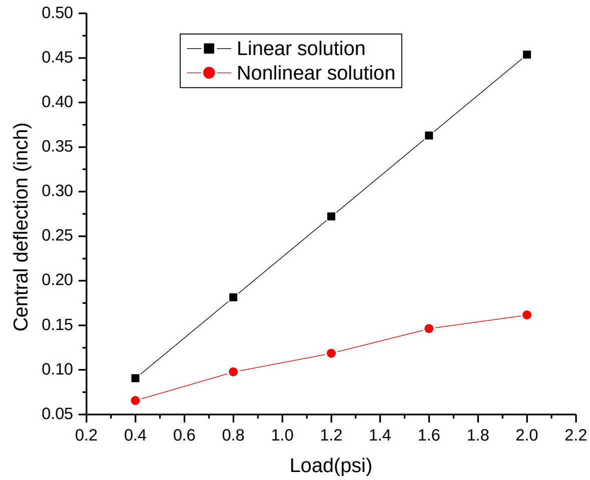

Fig 10. The comparison between linear and nonlinear central deflection for simply supported 8-layer unidirectional $[0°]_8$ square plate subjected to uniformly distributed load

4.4.2 Effect of number of layers

The effect of number of layers on the center deflection is examined for 8-layer unidirectional $[0°]_8$ square plate subjected to uniformly distributed load with clamped boundary condition is presented in Fig 10. It can be seen that difference of linear and nonlinear results for 8 layer unidirectional laminate of layers increases as the load increases and is prominent.

4.4.3 Effect of thickness ratio

Fig 11 shows the effect of thickness (a/h) ratio on central deflection. In this part of study a $[0°/90°]_s$ square plate (a/b=1) is taken. The results portray that as the thickness ratio increases the central deflection decreases.

4.4.4 Effect of aspect ratio

The effect of aspect ratio on central deflection under different boundary and loading conditions for a $(0°/90°)_s$ lamina having a/h ratio 10 are shown in Fig 12. It can be seen that central deflection shows decreasing trend as aspect ratio increases and difference is less after aspect ratio 5.

4.4.5 Effect of angle lay up

The influence of angle lay up on central deflections are represented in Fig 13. For this analysis a composite plate with a/b=1 & a/h=10 is taken. It is concluded from Fig 13 that central deflections decrease for cross ply to angle ply laminate.

4.4.6 Effect of variation of in plane normal stress

The variation of the nondimensionalized in plane normal stress σ_x with load parameter of a symmetric cross-ply (m, 90^0, 0^0, 90^0, 0^0)$_s$ simply supported square plate is plotted in Fig 14.It is observed that the in-plane normal stress σ_x varies linearly in FSDT and parabolically with TSDT and within the expected region.

Finally some contour plots i.e., lamina lay up, boundary , load distribution, deflection and stress in ANSYS APDL code are given in Fig 15, 16,17 and 18.

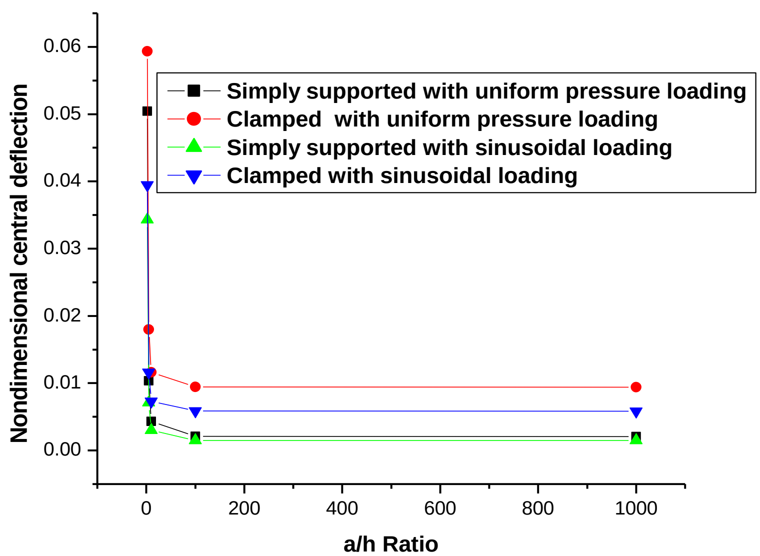

Fig 11. Effect of thickness ratio on central deflection under different boundary and loading condition

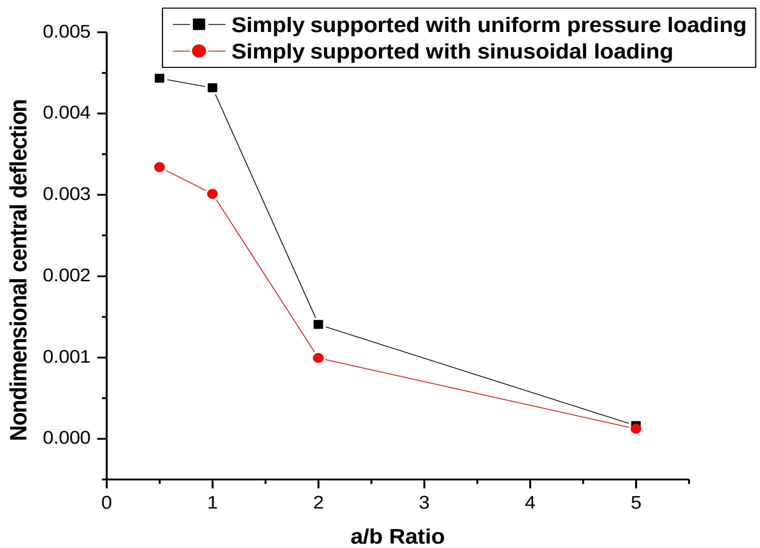

Fig 12. Effect of aspect ratio on central deflection under different boundary and loading condition

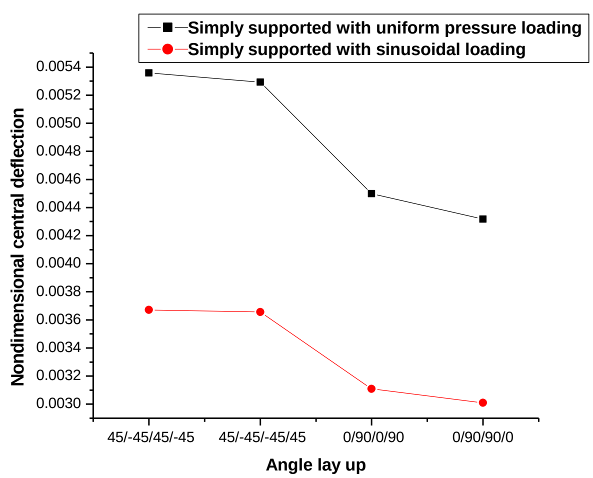

Fig 13. Effect of angle lay-up ratio on central deflection under different boundary and loading condition

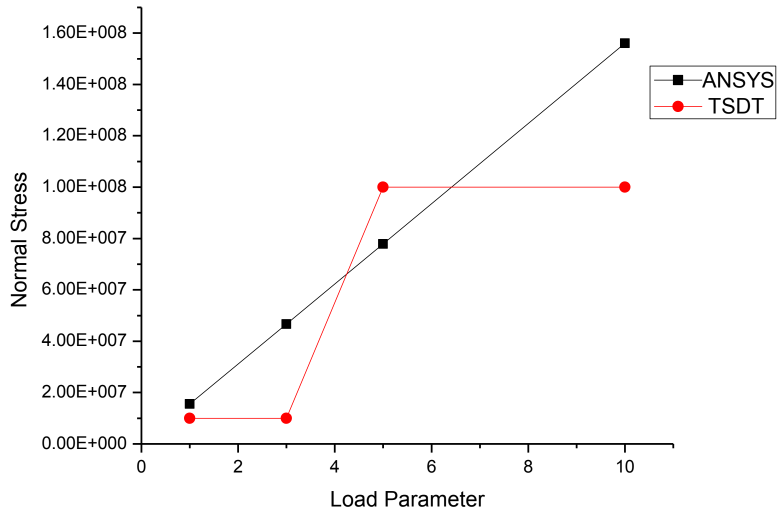

Fig 14 Variation of nondimensionalized in-plane normal stress σ_x with load parameter of a symmetric cross-ply (m, 90^0, 0^0, 90^0, 0^0)$_s$ simply supported square plate

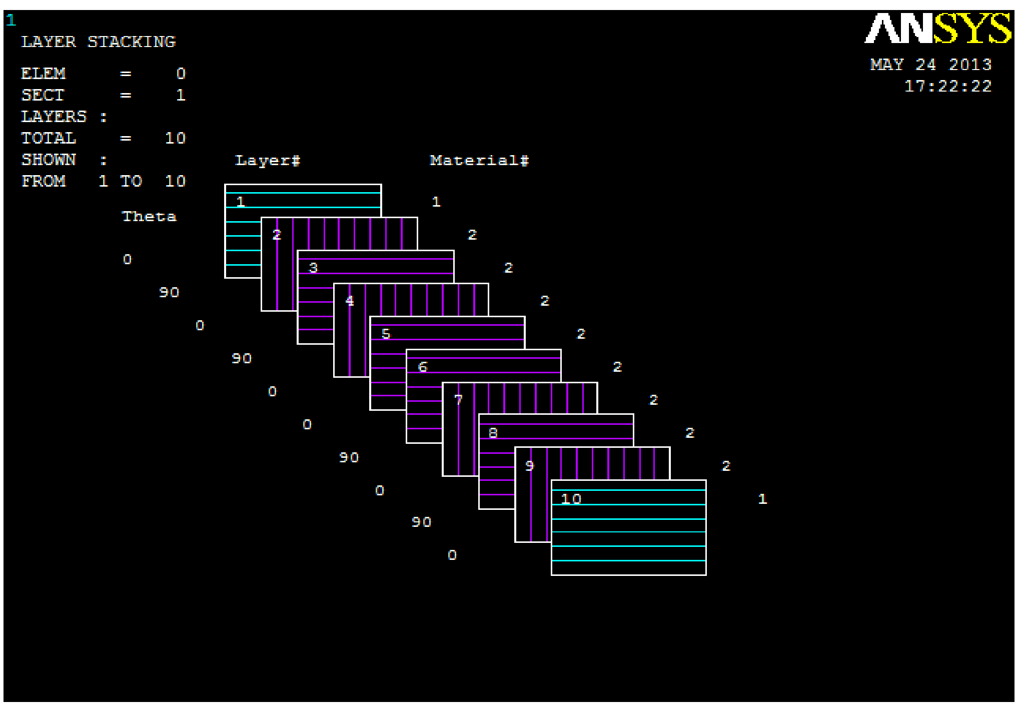

Fig 15 Lay-up of Laminate

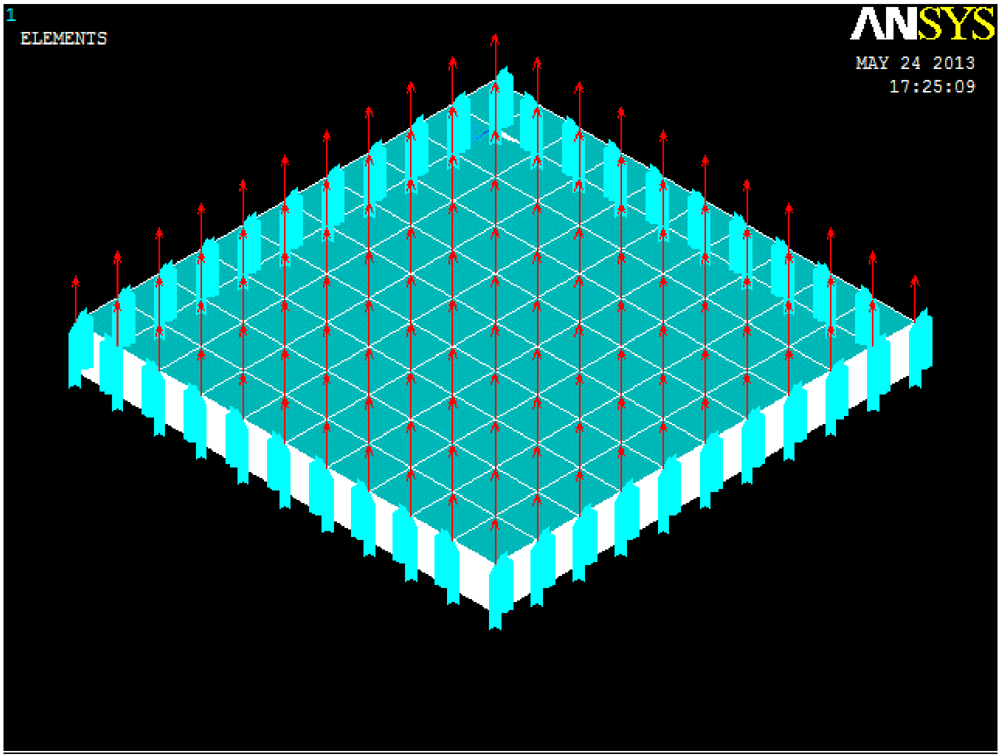

Fig 16 Boundary and Load distribution plot

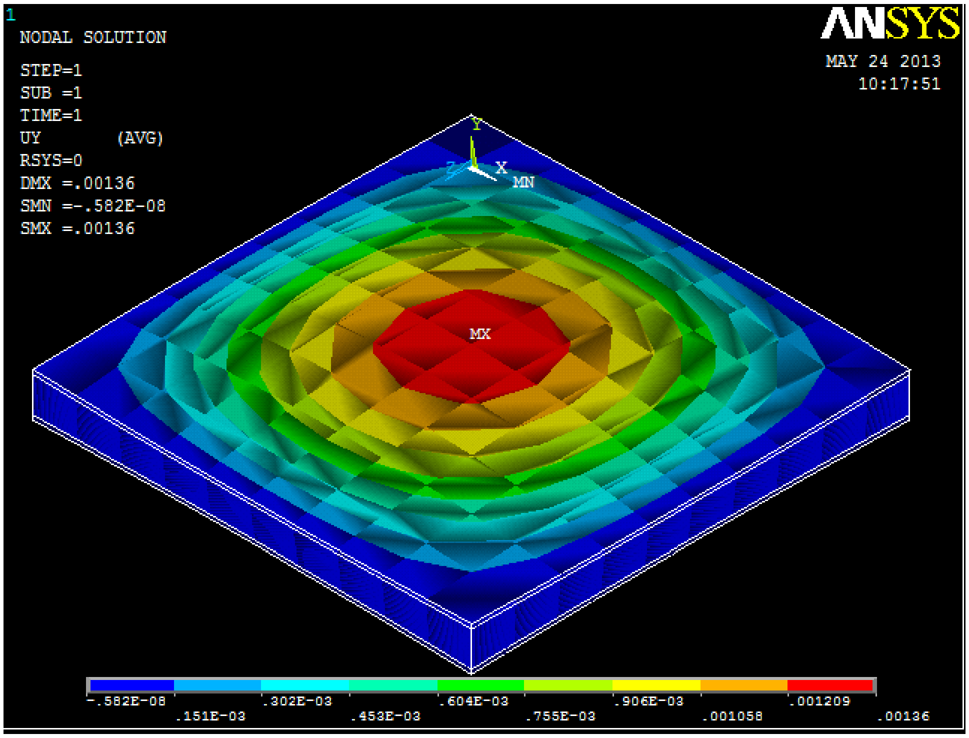

Fig 17 Contour plot with load parameter 1(with magneto)

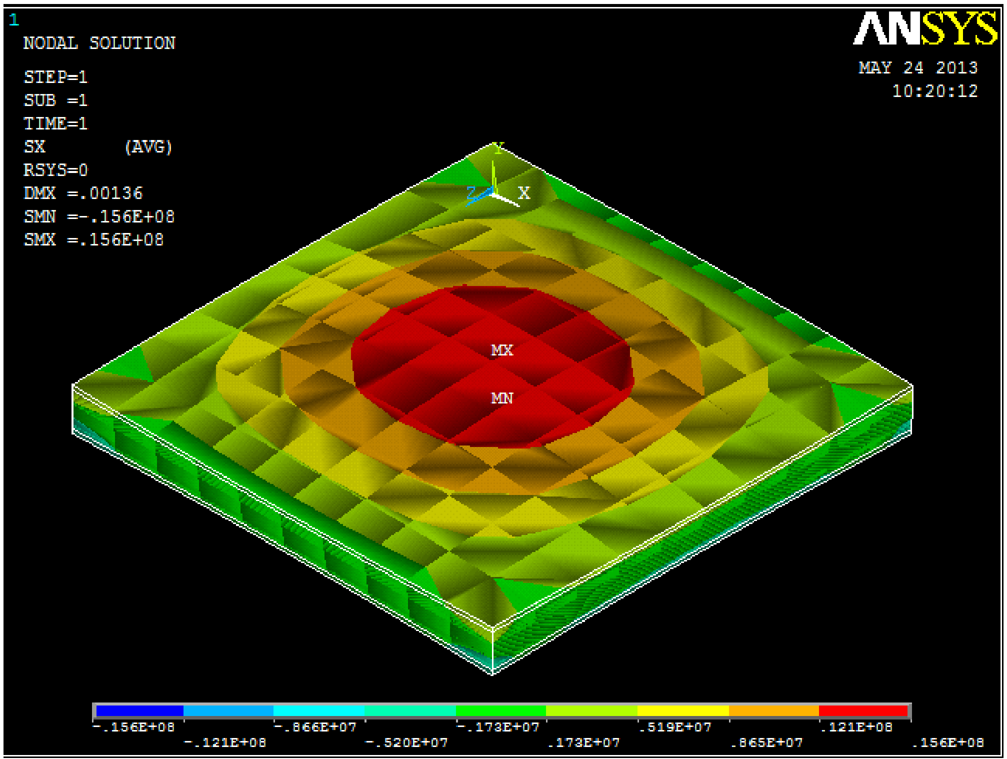

Fig 18 Stress contour plot with load parameter 1(with magneto)

Chapter - 5

5.1. Introduction

In the present analysis a C^0 finite element formulation based on higher order shear deformation theory is developed for the nonlinear static analysis of laminated composite with and without magnetostrictive layer. The geometric nonlinearity is considered in Green-Lagrange sense. Non-dimensionlized displacements and in plane stresses are computed with and without magnetostrictive layers using the assumed displacement model. The following findings are concluded from the present parametric study:

1. The nondimensionalized central deflection is less for nonlinear cases as compared to linear.
2. The nondimensionalized central deflections (linear and nonlinear) increases as load parameter increases and deflections are showing relatively smaller value for clamped support.
3. It can be seen that the differences of linear and nonlinear results increases as the load increases and it is noticeable when number of layer increases.
4. It is observed that the in-plane normal stress σ_x varies linearly and parabolically when evaluated using the FSDT and the TSDT, respectively.
5. Finally, the angle layups, the boundary conditions, the thickness ratios, and the aspect ratios have significant effect on the nonlinear static responses of the laminated composite plate.

5.2. Limitation of the study

In spite of several advantages obtained through proposed study, the following few points mays be treated as limitations of the present study.

1. In this study the nonlinear higher order terms are considered up to third order.
2. In this analysis, the geometrical nonlinearity is only taken into account.
3. The laminated plates problems are taken in this study are of equal thickness.

5.3. Future scope

1. The present study can be extended for the dynamic analysis of composite plate with and without magnetostrictive layer.
2. Effect of magnetic field can be considered to analyze the linear and nonlinear responses of the composite plate.
3. Material and boundary nonlinearities can be considered for the better understanding of the real life situation.

Appendix A. Linear Terms of HSDT

$$\left(\varepsilon_1{}^0\right)=\frac{\partial u_0}{\partial x},\quad \left(k_1{}^1\right)=\frac{\partial \phi_x}{\partial x},\quad \left(k_1{}^2\right)=-\frac{1}{2}\frac{\partial \beta_x}{\partial x},\quad \left(k_1{}^3\right)=-\frac{4}{3h^2}\left(\frac{\partial \phi_x}{\partial x}+\frac{\partial \eta_x}{\partial x}\right)$$

$$\left(\varepsilon_2{}^0\right)=\frac{\partial v_0}{\partial y},\quad \left(k_2{}^1\right)=\frac{\partial \phi_y}{\partial y},\quad \left(k_2{}^2\right)=-\frac{1}{2}\frac{\partial \beta_y}{\partial y},\quad \left(k_2{}^3\right)=-\frac{4}{3h^2}\left(\frac{\partial \phi_y}{\partial y}+\frac{\partial \eta_y}{\partial y}\right)$$

$$\left(\varepsilon_3{}^0\right)=-w_1,$$

$$\left(\varepsilon_4{}^0\right)=\phi_y+\frac{\partial w_0}{\partial y},\quad \left(k_4{}^1\right)=-\beta_y-\frac{\partial w_1}{\partial y},\qquad \left(k_4{}^2\right)=-\frac{4}{h^2}\left(\phi_y+\eta_y\right)$$

$$\left(\varepsilon_5{}^0\right)=\phi_x+\frac{\partial w_0}{\partial x},\quad \left(k_5{}^1\right)=-\beta_x-\frac{\partial w_1}{\partial x},\qquad \left(k_5{}^2\right)=-\frac{4}{h^2}\left(\phi_x+\eta_x\right)$$

$$\left(\varepsilon_6{}^0\right)=\frac{\partial v_0}{\partial x}+\frac{\partial u_0}{\partial y},\left(k_6{}^1\right)=\frac{\partial \phi_x}{\partial y}+\frac{\partial \phi_y}{\partial x},\left(k_6{}^2\right)=-\frac{1}{2}\left(\frac{\partial \beta_x}{\partial y}+\frac{\partial \beta_y}{\partial x}\right),\left(k_6{}^3\right)=-\frac{4}{3h^2}\left(\frac{\partial \phi_x}{\partial y}+\frac{\partial \eta_x}{\partial y}+\frac{\partial \phi_y}{\partial x}+\frac{\partial \eta_y}{\partial x}\right)$$

Appendix B. Nonlinear Terms of HSDT

$$\left(k_1{}^{NL1}\right)=2\left(\frac{\partial u_0}{\partial x}\cdot\frac{\partial \phi_x}{\partial x}+\frac{\partial v_0}{\partial x}\cdot\frac{\partial \phi_y}{\partial x}-\frac{\partial w_0}{\partial x}\cdot\frac{\partial w_1}{\partial x}\right)$$

$$\left(k_2{}^{NL1}\right)=2\left(\frac{\partial u_0}{\partial y}\cdot\frac{\partial \phi_x}{\partial y}+\frac{\partial v_0}{\partial y}\cdot\frac{\partial \phi_y}{\partial y}-\frac{\partial w_0}{\partial y}\cdot\frac{\partial w_1}{\partial y}\right)$$

$$\left(k_3{}^{NL1}\right)=-2\phi_x\beta_x$$

$$\left(k_4{}^{NL1}\right)=\left(-\beta_x\frac{\partial u_0}{\partial y}-\beta_y\frac{\partial v_0}{\partial y}+w_1\frac{\partial w_1}{\partial y}+\phi_x\frac{\partial \phi_x}{\partial y}+\phi_y\frac{\partial \phi_y}{\partial y}\right)$$

$$\left(k_5{}^{NL1}\right)=\left(-\beta_x\frac{\partial u_0}{\partial x}-\beta_y\frac{\partial v_0}{\partial x}+w_1\frac{\partial w_1}{\partial x}+\phi_x\frac{\partial \phi_x}{\partial x}+\phi_y\frac{\partial \phi_y}{\partial x}\right)$$

$$\left(k_6{}^{NL1}\right)=2\left(\frac{\partial u_0}{\partial x}\cdot\frac{\partial \phi_x}{\partial y}+\frac{\partial v_0}{\partial x}\cdot\frac{\partial \phi_y}{\partial y}-\frac{\partial w_0}{\partial x}\cdot\frac{\partial w_1}{\partial y}\right)$$

$$\left(k_1^{NL2}\right)=\left(\frac{\partial\phi_x}{\partial x}\right)^2+\left(\frac{\partial\phi_y}{\partial x}\right)^2+\left(\frac{\partial w_1}{\partial x}\right)^2-\frac{\partial u_0}{\partial x}\cdot\frac{\partial\beta_x}{\partial x}-\frac{\partial v_0}{\partial x}\cdot\frac{\partial\beta_y}{\partial x}$$

$$\left(k_2^{NL2}\right)=\left(\frac{\partial\phi_x}{\partial y}\right)^2+\left(\frac{\partial\phi_y}{\partial y}\right)^2+\left(\frac{\partial w_1}{\partial y}\right)^2-\frac{\partial u_0}{\partial y}\cdot\frac{\partial\beta_x}{\partial y}-\frac{\partial v_0}{\partial y}\cdot\frac{\partial\beta_y}{\partial y}$$

$$\left(k_3^{NL2}\right)=\beta_x^{\ 2}+\beta_y^{\ 2}-\frac{8}{h^2}\left[\phi_x^{\ 2}+\phi_y^{\ 2}+\phi_x.\eta_x+\phi_y.\eta_y\right]$$

$$\left(k_4^{NL2}\right)=-\frac{4}{h^2}\left[\left(\phi_x+\eta_x\right)\frac{\partial u_0}{\partial y}+\left(\phi_y+\eta_y\right)\frac{\partial v_0}{\partial y}\right]-\left[\beta_x\cdot\frac{\partial\phi_x}{\partial y}+\beta_y\cdot\frac{\partial\phi_y}{\partial y}\right]-\frac{1}{2}\left[\phi_x\cdot\frac{\partial\beta_x}{\partial y}+\phi_y\cdot\frac{\partial\beta_y}{\partial y}\right]$$

$$\left(k_5^{NL2}\right)=-\frac{4}{h^2}\left[\left(\phi_x+\eta_x\right)\frac{\partial u_0}{\partial x}+\left(\phi_y+\eta_y\right)\frac{\partial v_0}{\partial x}\right]-\left[\beta_x\cdot\frac{\partial\phi_x}{\partial x}+\beta_y\cdot\frac{\partial\phi_y}{\partial x}\right]-\frac{1}{2}\left[\phi_x\cdot\frac{\partial\beta_x}{\partial x}+\phi_y\cdot\frac{\partial\beta_y}{\partial x}\right]$$

$$\left(k_6^{NL2}\right)=\left[-\frac{\partial u_0}{\partial x}\cdot\frac{\partial\beta_x}{\partial y}-\frac{\partial v_0}{\partial x}\cdot\frac{\partial\beta_y}{\partial y}+\frac{\partial^2\phi_x}{\partial x\partial y}+\frac{\partial^2\phi_y}{\partial x\partial y}+\frac{\partial^2 w_1}{\partial x\partial y}\right]$$

$$\left(k_1^{NL4}\right)=\frac{1}{4}\left[\left(\frac{\partial\beta_x}{\partial x}\right)^2+\left(\frac{\partial\beta_y}{\partial x}\right)^2\right]-\frac{8}{3h^2}\left[\frac{\partial^2\phi_x}{\partial x^2}+\frac{\partial^2\phi_y}{\partial x^2}+\frac{\partial\phi_x}{\partial x}\cdot\frac{\partial\eta_x}{\partial x}+\frac{\partial\phi_y}{\partial x}\cdot\frac{\partial\eta_y}{\partial x}\right]$$

$$\left(k_2^{NL4}\right)=\frac{1}{4}\left[\left(\frac{\partial\beta_x}{\partial y}\right)^2+\left(\frac{\partial\beta_y}{\partial y}\right)^2\right]-\frac{8}{3h^2}\left[\frac{\partial^2\phi_x}{\partial y^2}+\frac{\partial^2\phi_y}{\partial y^2}+\frac{\partial\phi_x}{\partial y}\cdot\frac{\partial\eta_x}{\partial y}+\frac{\partial\phi_y}{\partial y}\cdot\frac{\partial\eta_y}{\partial y}\right]$$

$$\left(k_3^{NL4}\right)=\frac{16}{h^4}\left[\phi_x^{\ 2}+\eta_x^{\ 2}+\phi_y^{\ 2}+\eta_y^{\ 2}+\left(\phi_x\cdot\eta_x+\phi_y\cdot\eta_y\right)\right]$$

$$\left(k_4^{NL4}\right)=\frac{2}{h^2}\left[\left(\phi_x+\eta_x\right)\frac{\partial\beta_x}{\partial y}+\left(\phi_y+\eta_y\right)\frac{\partial\beta_y}{\partial y}\right]+\frac{4}{h^2}\left[\left(\frac{\partial\phi_x}{\partial y}+\frac{\partial\eta_x}{\partial y}\right)\beta_x+\left(\frac{\partial\phi_y}{\partial y}+\frac{\partial\eta_y}{\partial y}\right)\beta_y\right]$$

$$\left(k_5^{NL4}\right)=\frac{2}{h^2}\left[\left(\phi_x+\eta_x\right)\frac{\partial\beta_x}{\partial x}+\left(\phi_y+\eta_y\right)\frac{\partial\beta_y}{\partial x}\right]+\frac{4}{h^2}\left[\left(\frac{\partial\phi_x}{\partial x}+\frac{\partial\eta_x}{\partial x}\right)\beta_x+\left(\frac{\partial\phi_y}{\partial x}+\frac{\partial\eta_y}{\partial x}\right)\beta_y\right]$$

$$\left(k_6^{NL3}\right)=-\frac{4}{3h^2}\left[\left(\frac{\partial\phi_x}{\partial y}+\frac{\partial\eta_x}{\partial y}\right)\frac{\partial\phi_x}{\partial x}+\left(\frac{\partial\phi_y}{\partial y}+\frac{\partial\eta_y}{\partial y}\right)\frac{\partial\phi_y}{\partial x}+\left(\frac{\partial\phi_x}{\partial x}+\frac{\partial\eta_x}{\partial x}\right)\frac{\partial\phi_x}{\partial y}+\left(\frac{\partial\phi_y}{\partial x}+\frac{\partial\eta_y}{\partial x}\right)\frac{\partial\phi_y}{\partial y}\right]+\frac{1}{4}\left[\frac{\partial^2\beta_x}{\partial x\partial y}+\frac{\partial^2\beta_y}{\partial x\partial y}\right]$$

$$\left(k_1^{NL5}\right)=\frac{4}{3h^2}\left[\frac{\partial\phi_x}{\partial x}\cdot\frac{\partial\beta_x}{\partial x}+\frac{\partial\phi_y}{\partial x}\cdot\frac{\partial\beta_y}{\partial x}+\frac{\partial\beta_x}{\partial x}\cdot\frac{\partial\eta_x}{\partial x}+\frac{\partial\beta_y}{\partial x}\cdot\frac{\partial\eta_y}{\partial x}\right]$$

$$\left(k_2^{NL5}\right)=\frac{4}{3h^2}\left[\frac{\partial\phi_x}{\partial y}\cdot\frac{\partial\beta_x}{\partial y}+\frac{\partial\phi_y}{\partial y}\cdot\frac{\partial\beta_y}{\partial y}+\frac{\partial\beta_x}{\partial y}\cdot\frac{\partial\eta_x}{\partial y}+\frac{\partial\beta_y}{\partial y}\cdot\frac{\partial\eta_y}{\partial y}\right]$$

$$\left(k_4^{NL5}\right)=\frac{16}{3h^4}\left[\left(\frac{\partial\phi_x}{\partial y}+\frac{\partial\eta_x}{\partial y}\right)\left(\phi_x+\eta_x\right)+\left(\frac{\partial\phi_y}{\partial y}+\frac{\partial\eta_y}{\partial y}\right)+\left(\phi_y+\eta_y\right)\right]$$

$$\left(k_5^{NL5}\right)=\frac{16}{3h^4}\left[\left(\frac{\partial\phi_x}{\partial x}+\frac{\partial\eta_x}{\partial x}\right)\left(\phi_x+\eta_x\right)+\left(\frac{\partial\phi_y}{\partial x}+\frac{\partial\eta_y}{\partial x}\right)+\left(\phi_y+\eta_y\right)\right]$$

$$\left(k_6^{NL3}\right)=\frac{2}{3h^2}\left[\left(\frac{\partial\phi_x}{\partial y}+\frac{\partial\eta_x}{\partial y}\right)\frac{\partial\beta_x}{\partial x}+\left(\frac{\partial\phi_y}{\partial y}+\frac{\partial\eta_y}{\partial y}\right)\frac{\partial\beta_y}{\partial x}+\left(\frac{\partial\phi_x}{\partial x}+\frac{\partial\eta_x}{\partial x}\right)\frac{\partial\beta_x}{\partial y}+\left(\frac{\partial\phi_y}{\partial x}+\frac{\partial\eta_y}{\partial x}\right)\frac{\partial\beta_y}{\partial y}\right]$$

$$\left(k_1^{NL6}\right)=\frac{16}{9h^4}\left[\left(\frac{\partial\phi_x}{\partial x}\right)^2+\left(\frac{\partial\eta_x}{\partial x}\right)^2+\left(\frac{\partial\phi_y}{\partial x}\right)^2+\left(\frac{\partial\eta_y}{\partial x}\right)^2+2\left(\frac{\partial\phi_x}{\partial x}\cdot\frac{\partial\eta_x}{\partial x}+\frac{\partial\phi_y}{\partial x}\cdot\frac{\partial\eta_y}{\partial x}\right)\right]$$

$$\left(k_2^{NL5}\right)=\frac{16}{9h^4}\left[\left(\frac{\partial\phi_x}{\partial y}\right)^2+\left(\frac{\partial\eta_x}{\partial y}\right)^2+\left(\frac{\partial\phi_y}{\partial y}\right)^2+\left(\frac{\partial\eta_y}{\partial y}\right)^2+2\left(\frac{\partial\phi_x}{\partial y}\cdot\frac{\partial\eta_x}{\partial y}+\frac{\partial\phi_y}{\partial y}\cdot\frac{\partial\eta_y}{\partial y}\right)\right]$$

$$\left(k_6^{NL3}\right)=\frac{16}{9h^4}\left[\left(\frac{\partial\phi_x}{\partial x}+\frac{\partial\eta_x}{\partial x}\right)\left(\frac{\partial\phi_x}{\partial y}+\frac{\partial\eta_x}{\partial y}\right)+\left(\frac{\partial\phi_y}{\partial x}+\frac{\partial\eta_y}{\partial x}\right)\left(\frac{\partial\phi_y}{\partial y}+\frac{\partial\eta_y}{\partial y}\right)\right]$$

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