MODELLING OF GEOTECHNICAL STRUCTURES USING MULTI-VARIATE ADAPTIVE REGRESSION SPLINE (MARS) AND GENETIC PROGRAMMING (GP)

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of

Master of Technology
In
Civil Engineering
(Geotechnical engineering)

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DEPARTMENT OF CIVIL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA

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Under the guidance and supervision of
Prof S.K.Das

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DEPARTMENT OF CIVIL ENGINEERING
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2013

National Institute of Technology
Rourkela

CERTIFICATE

This is to certify that the thesis entitled "Modeling of geotechnical structures using multi-variate adaptive regression spline (MARS) and genetic programming (GP)" being submitted by Swagatika Senapati in partial fulfillment of the requirements for the award of **Master of Technology** Degree in **Civil Engineering** with specialization in **GEOTECHNICAL ENGINEERING** at National Institute of Technology Rourkela, is an authentic work carried out by her under my guidance and supervision.

To the best of my knowledge, the matter embodied in this report has not been submitted to any other university/institute for the award of any degree or diploma.

Dr. SARAT Ku. DAS

Place: Rourkela
Associate Professor

Date: 28/05/2012
Department of Civil Engineering

NIT Rourkela

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The pleasant point of presenting a report is the opportunity to thank those who have contributed to build my knowledge. This is only possible due to God's grace, Co-operation of my guide, Parents support and their blessings.

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Swagatika Senapati

ABSTRACT

The evolution of computational geotechnical engineering analyses closely follows the development in computational methods. The soil is considered as a complex material produced by the weathering of solid rock. Due to its uncertain behavior, modeling the behavior of such materials is complex by using more traditional forms of mechanistic based engineering methods like analytical and finite element methods etc. Very often it is difficult to develop theoretical/statistical models due to the complex nature of the problem and uncertainty in soil parameters. These are situations where data driven approach has been found to more appropriate than model oriented approach. To take care of such problems in artificial intelligence (AI) techniques has been developed in the computational methods. Though AI techniques has proved to have the superior predictive ability than other traditional methods for modeling complex behavior of geotechnical engineering materials, still it is facing some criticism due to the lack of transparency, knowledge extraction and model uncertainty. To overcome this problem there are developments of improvised AI techniques. Different AI techniques as ‘black box’ i.e artificial neural network (ANN), ‘grey box’ i.e Genetic programming (GP) and ‘white box’ i.e multivariate adaptive regression spline (MARS) depending upon its transparency and knowledge extraction. Here, in this study of GP and MARS ‘grey box’ and ‘white box’ AI techniques are applied to some geotechnical problems such as prediction of lateral load capacity of piles in clay, pull-out capacity of ground anchor, factor of safety of slope stability analysis and ultimate bearing capacity of shallow foundations. Different statistical criteria are used to compare the developed GP and MARS models with other AI models like ANN and support vector machine (SVM) models. It was observed that for the problems considered in the present study, the MARS and GP model are found to be more efficient than ANN and SVM model and the model

equations are also found to be more comprehensive. But as every numerical method has its own advantages and disadvantages and are also problem specific, there is a need to apply these techniques to other Geotechnical engineering problems to draw final conclusions regarding its efficacy.

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CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Geotechnical engineering deals with the materials like soil and rock, which exhibit uncertain behavior due to the physical processes associated with the formation of these materials. The soil is considered as a complex material produced by the weathering of solid rock. Due to its uncertain behavior, modeling the behavior of such materials is complex by using more traditional forms of mechanistic based engineering methods like analytical and finite element methods etc..The evolution of computational Geotechnical engineering analyses closely follows the development in computational methods. At the early stage of geotechnical engineering, analytical methods and simple limit equilibrium method coupled with engineering judgment were used for development of physical models of geotechnical engineering problems. Over the years, analytical numerical and empirical methods are in use as shown in Figure 1.1. Though numerical methods like finite element methods, finite difference method and discrete element method are in use particularly for academic and sophisticated projects, experience based methods are useful for common and preliminary studies.

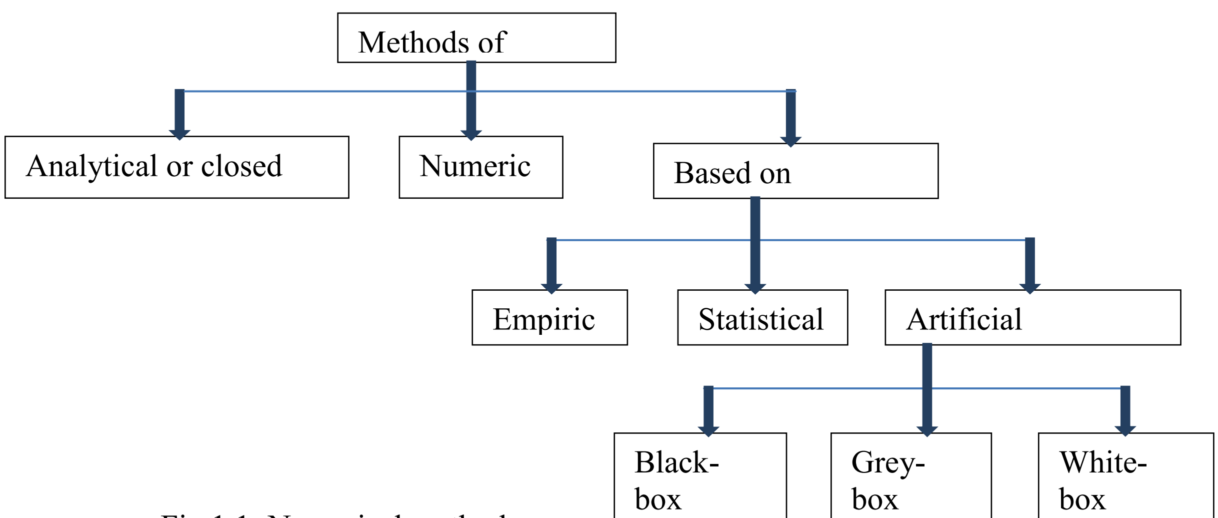


Fig 1.1: Numerical methods

Unlike other engineering materials the success of the above methods to applications in Geotechnical engineering is hindered due to difficulty in obtaining an accurate constitutive model and spatial variability of soil particularly for complex problems like liquefaction and pile capacity problems. Hence, based on case histories/field tests, statistically derived empirical methods, and semi-empirical methods based on analytical methods are more popular in such cases. The success of these empirical and semi-empirical methods depend to a great extent on the chosen statistical/theoretical model for the system to be analyzed matching the input output data and statistical methods used to find out the model parameters (Das and Basudhar, 2006). Very often it is difficult to develop theoretical/statistical models due to the complex nature of the problem and uncertainty in soil parameters. These are situations where data driven approach has been found to more appropriate than model oriented approach. To take care of such problems in artificial intelligence (AI) techniques have been developed in the computational methods. Within a short period it found wide applicability cutting across various disciplines. This has given a spurt in the research activities in the art of applying such methods to solve real life problems highlighting the latent capabilities and drawbacks of such methods. So, the researchers are encouraged to apply different AI techniques such as ANN is applied to predict modeling the axial and lateral load capacities of pile foundation in compression and uplift including driven piles (Ahmad et al., 2007; Ardalan et al., 2009; Das and Basudhar, 2006; Pal and Deswal, 2008; Shahin, 2010), drilled shafts (Goh et al., 2005; Shahin, 2010), and ground anchor piles (Shahin and Jaksa, 2005, 2006).

ANN is still considered as ‘black box’ system with poor generalization, though various attempts made for refinement and explanations. Recently support vector machine (SVM), based on statistical learning theory and structural risk minimization is being used as an alternate prediction model (Das et al. 2010; Das et al. 2011). The SVM uses structural constrained minimization penalizing the error margin during training (Vapnik 1998). The error function being a convex function better generalization used to observe in SVM (Das et al. 2010; Das et al. 2011) compared to ANN.

Though AI techniques has proved to have the superior predictive ability than other traditional methods for modeling complex behavior of geotechnical engineering materials, still it is facing some criticism due to the lack of transparency, knowledge extraction and model uncertainty. To overcome this there are a development of improvised AI techniques.

As shown in Figure 1.2, Giustolisi (2007) described different AI techniques as 'black box', 'grey box' and 'white box' depending upon its transparency and knowledge extraction. This figure shows the classification of modelling techniques based on colours which indicates that, with higher physical knowledge used during development, the physical interpretation of the phenomenon will be better that the model provides to the user.

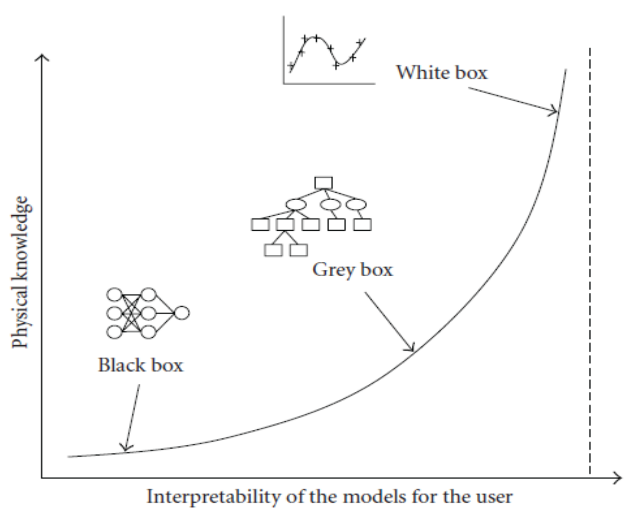


Fig-1.2: Graphical classification of modeling techniques

Other techniques, belonging to 'gray box' model (is genetic programming (GP) (Koza, 1992), mimics biological evolution of living organisms and makes use of principle of genetic algorithm (GA). Various attempts have been made in the recent past to use GP to some Geotechnical engineering problems (Gandomi and Alavi, 2011; 2012). GP helps in achieving greatly simplified model formula compared to ANN model, but a tradeoff is made between the complexity of the formula and accuracy of the model. Another class of model may be termed as 'white box' model is the multivariate adaptive regression spline (MARS) developed based on statistical model developed by Friedman (1991). MARS can adjust any functional form, hence suitable for exploratory data analysis. Samui et al. (2011) observed that the MARS model for uplift capacity of suction caisson has better statistical performance comparable to ANN and FEM model. Hence, more research is required in ANN regarding the generalization, control on the model parameters, extrapolation and depicting simplified model equation.

It may be mentioned here that , though above AI techniques are based on sound mathematical/numerical background, its application to different problems is an art. Hence, in this thesis an attempt has been made to apply two recent AI techniques, named GP and MARS to some foundation engineering problems to check its applicability and recommend guidelines for future application of these techniques to other Geotechnical engineering problems. The basic outline of this thesis is presented in Figure 1.3

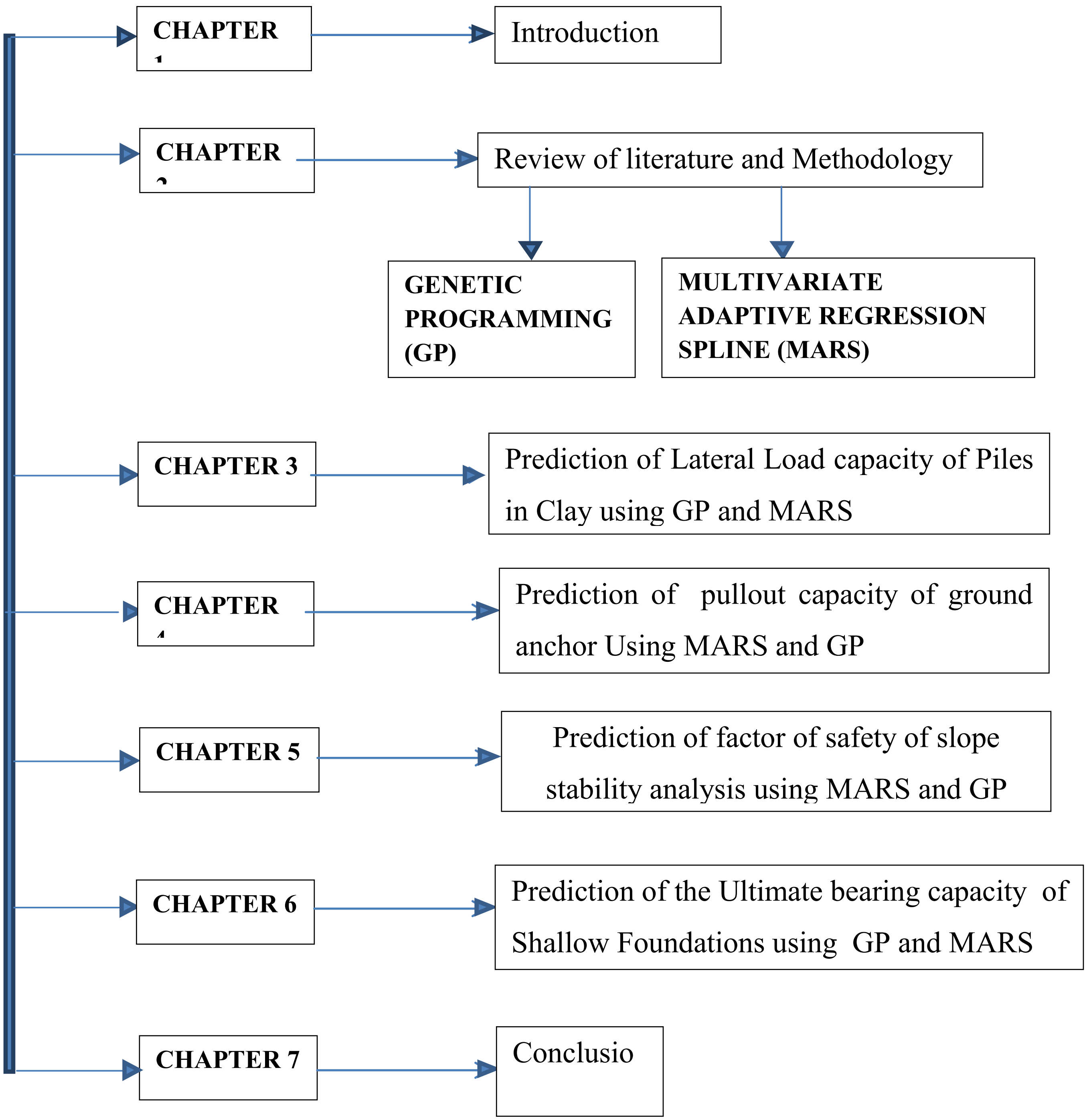


Fig- 1.3: Basic outline of thesis

1.2 Objective and scope

The objective of the present work is to develop Geotechnical modeling of some foundations using AI techniques, GP and MARS.

The scope of the present work includes :

- Prediction of Lateral Load capacity of Piles in Clay using GP and MARS
- Prediction of the pullout capacity of ground anchor Using MARS and GP
- Prediction of factor of safety of slope stability analysis using MARS and GP
- Prediction of the Ultimate bearing capacity of Shallow Foundations using GP and MARS

1.3 Thesis outline

After the brief introduction (Chapter 1), the review and methodology of Genetic programming (GP) and Multivariate adaptive regression spline(MARS) is described in chapter 2.

Chapter 3,4,5,6 describes the application of GP and MARS and comparison with ANN results in different geotechnical engineering problems such as Prediction of Lateral Load capacity of Piles in Clay, Prediction of pullout capacity of ground anchor , Prediction of factor of safety of slope stability analysis ,Prediction of the Ultimate bearing capacity of Shallow Foundations using GP and MARS respectively.

In chapter 6 conclusions drawn from various studies made in this thesis are presented. The general layout of the thesis work based on each chapter is shown in a flow diagram (Figure 1.3).

CHAPTER 2

REVIEW OF LITERATURE AND METHODOLOGY

2.1 INTRODUCTION

The AI techniques ANN has been extensively used in geotechnical engineering with limited use of SVM. But the application of GP and MARS is very limited. In the present Chapter a brief review about ANN and SVM and their application to different geotechnical problems are presented. But as the use of GP and MARS is very limited in geotechnical engineering problems, the details of GP and MARS are discussed

2.1.1 Artificial neural networks (ANN)

In the present study, the ANN models are trained with differential evolution and Bayesian regularization method and are defined as DENN and BRNN respectively. The use of DENN and BRNN are limited in geotechnical engineering (Das and Basudhar 2006,Das and Basudhar 2008, Goh *et al*. 2005, Das *et al*. 2011b). A brief description about the Bayesian regularization and differential evolution neural network is presented here for completeness.

Bayesian regularization neural network (BRNN)

In case of back propagation neural network (BPNN) the error function considered for minimization is the mean square error (MSE). This may lead to over-fitting due to unbounded values of the weights. The other method called as regularization, in which the performance function is changed by adding a term that consist of mean square error of weights and biases as given below

$$MSEREG = \gamma\, MSE + (1-\gamma)\ MSW \tag{2.1}$$

Where MSE is the mean square error of the network, γ is the regularization parameter and

$$MSW = \frac{1}{n}\sum_{j=1}^{n} w_j^2 \tag{2.2}$$

This performance function will cause the network to have smaller weights and biases thereby forcing networks less likely to be over-fit. The optimal regularization parameter γ is determined through the Bayesian framework (Demuth and Beale 2000) as the low value of γ will not adequately fit the training data and high value of it may result in over-fitting. The number of network parameters (weights and biases) are being effectively used by the network can be found out by the above algorithm. The effective number of parameters remains the same irrespective of the total number of parameters in the network.

Differential evolution neural network (DENN)

The differential evolution (DE) optimization is a population based heuristic global optimization method. Unlike other evolutionary optimization, in DE the vectors in current populations are randomly sampled and combined to create vectors for the next generation with real valued crossover factor and mutation factor. The detail of DENN is available in Ilonen*et al.* (2003).

2.2 *Genetic Programming*

Genetic Programming is a pattern recognition technique where the model is developed on the basis of adaptive learning over a number of cases of provided data, developed by Koza (1992). It mimics biological evolution of living organisms and makes use of principle of genetic algorithm (GA). In traditional regression analysis the user has to specify the structure of the model whereas in GP both structure and the parameters of the mathematical model are evolved automatically. It provides a solution in the form of tree structure or in the form of compact equation using the given dataset. A brief description about GP is presented for the completeness, but the details can be found in Koza (1992).

GP model is composed of nodes, which resembles to a tree structure and thus, it is also known as GP tree. Nodes are the elements either from a functional set or terminal set. A functional set may include arithmetic operators (+, ×, ÷, or -), mathematical functions (sin(.), cos(.), tanh(.) or ln(.)), Boolean operators (AND, OR, NOT etc), logical expressions (IF, or THEN) or any other suitable functions defined by the user. The terminal set includes variables (like x_1, x_2, x_3, etc) or constants (like 3, 5, 6, 9 etc) or both. The functions and terminals are randomly chosen to form a GP tree with a root node and the branches extending from each function nodes to end in terminal nodes as shown in Figure 1.3.

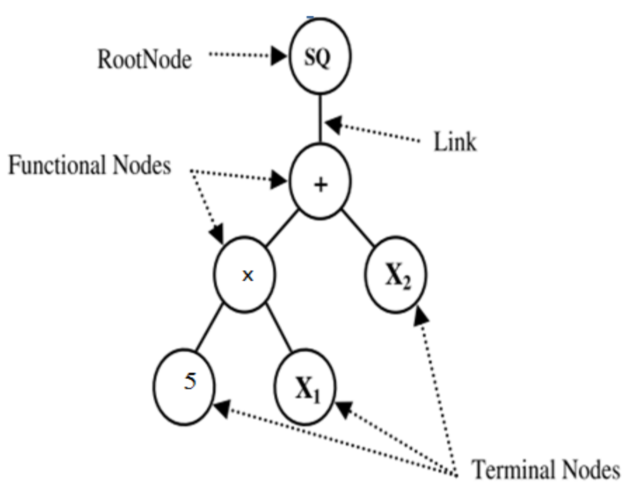


Fig 2.1. A GP tree expressing function: $(5X_1+X_2)^2$

Initially a set of GP trees, as per user defined population size, is randomly generated using various functions and terminals assigned by the user. The fitness criterion is calculated by the objective function and it determines the quality of the each individual in the population competing with the rest. At each generation a new population is created by selecting individuals as per the merit of their fitness from the initial population and then, implementing various evolutionary mechanisms like reproduction, crossover and mutation to the functions and terminals of the selected GP trees. The new population then replaces the existing population. This process is iterated until the termination criterion, which can be either a threshold fitness value or maximum number of generations, is satisfied. The best GP model, based on its fitness value that appeared in any generation, is selected as the result of genetic programming. A brief description on various evolutionary mechanisms in GP are presented below.

Initial Population

In the first step of genetic programming a number of GP trees are generated by randomly selecting user defined functions and terminals. These GP trees form initial population.

Reproduction

In the second stage of the GP, a proportion of the initial population is selected and copied to the next generation and this procedure is called reproduction. Roulette wheel selection, tournament selection, ranking selection etc. are the methods generally followed for the selection procedure.

Crossover

In crossover operation, two trees are selected randomly from the population in the mating pool. One node from each tree is selected randomly, the sub-trees under the selected nodes are swapped and two offspring is generated as shown in Figure 2.2.

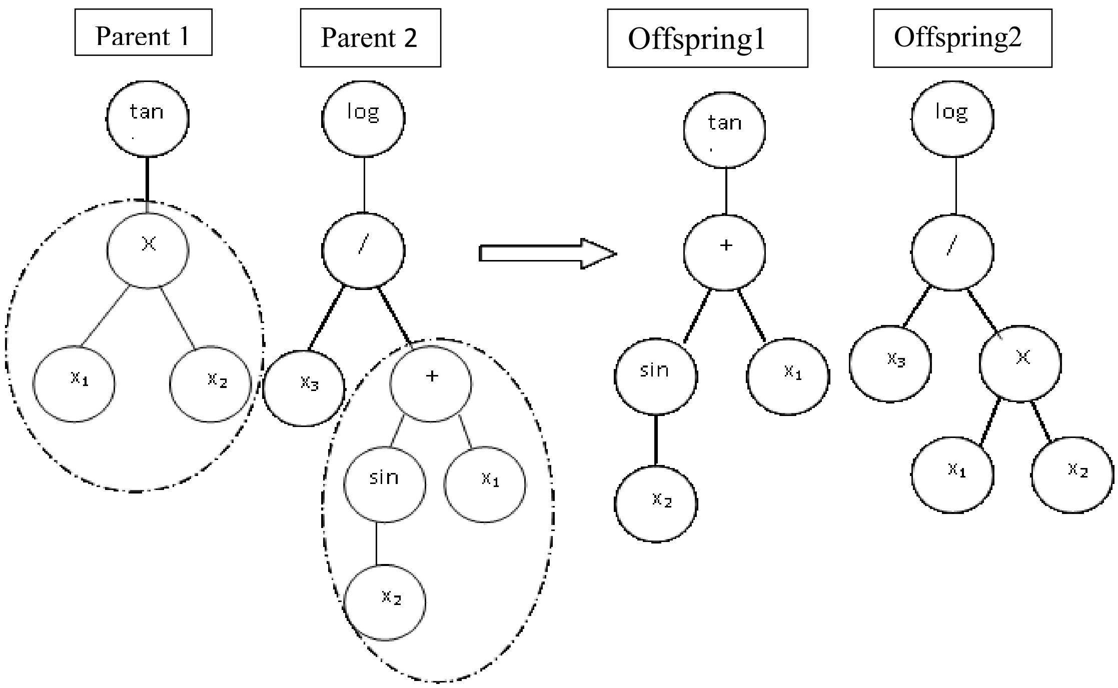


Fig 2.2.A typical crossover mechanism in GP.

Mutation

A GP tree is first selected randomly from the population in the mating pool and any node of the tree is replaced by any other node from the same function or terminal set as shown in Figure 2.3. A function node can replace only a function node and the same principle is applicable for the terminal nodes.

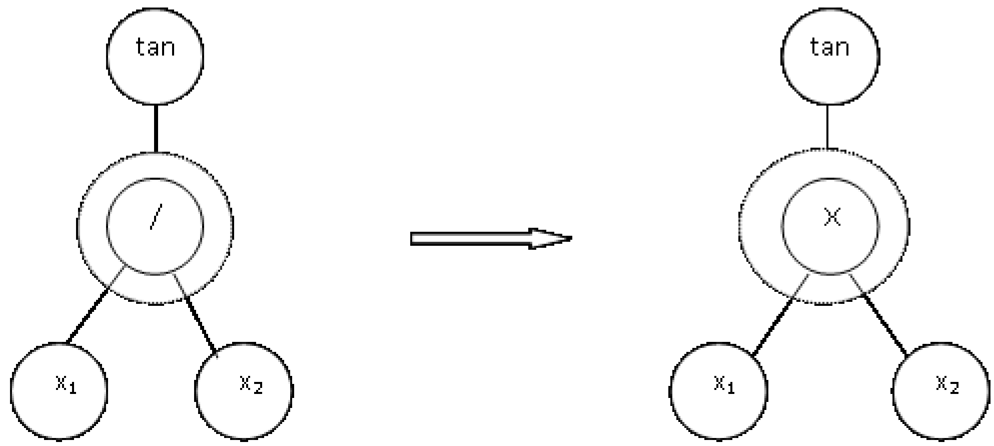


Figure 2.3. A typical mutation mechanism in GP

The general form of proposed GP model can be presented as:

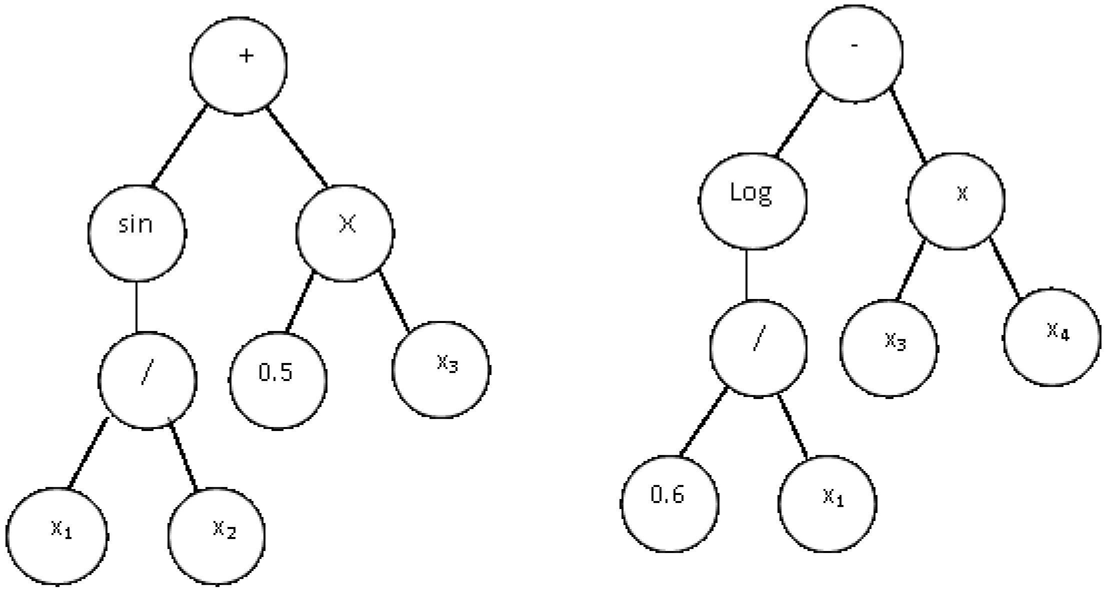
$$Q_p = \sum_{i=1}^{n} F[X, f(X), b_i] + b_0 \quad (2.3)$$

where, F= the function created by the GP referred herein as pile load function, X = vector of input variables = { D, L, e, S_u} ,where D = diameter of pile , L= depth of pile embedment, e = eccentricity of load, S_u = un-drained shear strength of soil, b_i is constant, f is the function defined by the user and n is the number of terms of target expression and b_0= bias. The GP as per Searson*et al.* (2010) is used and the present model is developed and implemented using Matlab (Math Work Inc. 2005).

2.2.1 ***Multi-gene genetic programming***

In traditional genetic programming where input output relationship is presented in terms of empirical mathematical model of data acquired from a system is referred as symbolic regression. MGGP based symbolic regression is a weighted linear combination of outputs of a number of GP trees. Here each tree is considered as a gene. Figure 1.6 shows an example of MGGP model where the output is predicted using 4 input variables (x_1, x_2, x_3, x_4). This is a non linear model as it contains nonlinear terms (sin (.) and log(.)) but it is linear in the parameters with respect to weights c_1 and c_2. The complexity of the generated model depends on two MGGP parameters: maximum allowable number of genes (G_{max}) and maximum depth of GP tree (d_{max}). Thus, the user specifies the values of G_{max} and d_{max} to have a control over the complexity of MGGP based Models.

Gene-1 Gene-2



$$y = c_0 + c_1\left(\sin\left(x_1 / x_2\right) + 0.5x_3\right) + c_2\left(Log\left(0.6 / x_1\right) - x_3x_4\right)$$

Fig 2.4. An example of typical multi-gene GP model

The Nash-Sutcliff coefficient of efficiency (E) of the model based on training data generally increases with increasing values G_{max} and d_{max}, but it results in increasing the complexity of the evolved model. Thus, there are optimum values of G_{max} and d_{max}, which produces a relatively compact model which is a linear combination of lower order non-linear transformations of input variables (Searsons*et al*. 2010). The linear coefficients (c_1 and c_2) and the bias (c_0) of the model are obtained from the training data using statistical regression analysis (ordinary least square method). In MGGP procedure the user defined initial population is generated by creating individuals that contain randomly evolved GP trees (genes) varying from 1 to G_{max} (Searsons*et al*.2010). In addition to the standard GP evolution mechanisms as discussed earlier there are some special MGGP crossover mechanisms which allow the exchange of genes between individuals and brief descriptions of them arepresented below.

Two point high level cross over

Two point high level crossover operation allows swapping of genes between two parent individuals in the mating pool and can be explained through an example where the first parent individual is having four genes [G_1, G_2, G_3, G_4] and the second contains three genes [G_5, G_6, G_7] with G_{max} = 5. Two crossover points are selected randomly for each parent and genes enclosed by crossover points are denoted by {...}.

[G_1,{G_2, G_3, G_4}], [G_5, G_6,{G_7}]

The genes enclosed by the cross over points are swapped and thus, two offspring individuals are created as shown below.

[G_1, {G_7}], [G_5, G_6,{G_2, G_3, G_4}]

By this operation both parent individuals acquire new genes as well as genes are deleted from either individual. If swapping of genes results in an individual containing more genes than G_{max} then genes are randomly selected and removed till the individual contain G_{max} genes.

Two point low level crossover

Standard GP sub-tree crossover is referred as two point low level crossover. In this operation first a gene is randomly selected from each of the parent individuals (any two) in the mating pool and then swapping of sub- trees under arbitrarily selected nodes of each gene is performed. The resulting trees replace the parent trees in otherwise unchanged parent individual producing offspring individuals for the next generation.

Similarly MGGP also provides six methods of mutation of GP trees (Gandomi and Alavi 2012a). The probabilities of the various evolutionary mechanisms can be set by the user for achieving best MGGP model. These mechanisms are grouped into categories referred as events. Therefore, the probability of crossover event, mutation event and direct reproduction event are to be specified by the user in such a way that the sum of these probabilities is 1.

The general form of MGGP model can be presented as:

$$LI_p = \sum_{i=1}^{n} F\left[X, f(X), b_i\right] + b_0 \qquad (2.4)$$

where, LI_p= predicted value of *LI*, *F*= the function created by the MGGP referred herein as liquefaction index function, for Model-I *X* = vector of input variables = {q_{c1N}, I_c, σ_v', $CSR_{7.5}$} (Juang *et.al*. 2003), q_{c1N}= normalized cone tip resistance (Juang *et al*. 2003), I_c= soil type index (Juang *et al*. 2003), σ'_v= vertical effective stress of soil at the depth studied, $CSR_{7.5}$ is the cyclic stress ratio adjusted to the benchmark earthquake of moment magnitude (M_w) of 7.5 as presented by Youd *et al*. (2001); for Model-II *X*== {q_c, R_f, σ_v, σ_v', a_{max}/g, M_w} (Goh and Goh 2007), q_c= measured cone tip resistance, R_f = friction ratio, σ_v= vertical total stress of soil at the depth studied, a_{max}= peak horizontal ground surface acceleration, g = acceleration due to gravity, b_i is a constant, *f* is the function defined by the user and *n* is the number of terms of target expression and b_0= bias. In the present study MGGP models are developed using GPTIPS toolbox (Searson 2009)in combination with the functions coded in Matlab (Math Work Inc 2005).

2.3 MULTIVARIATE ADAPTIVE REGRESSION SPLINE(MARS)

MARS is basically a nonparametric regression procedure that does not assume any functional relationship between independent and dependent variables. Instead, MARS uses the regression data to construct this relation and forms some sets of coefficients and basis functions. In other words it can be said that this method is based on "divide and conquer" strategy, which divides the input parameters into groups or say regions, each having its own regression equation. So this makes MARS particularly suitable for problems with higher input dimensions (i.e., with more number of variables), whereas other techniques face problem of dimensionality with large number of input variables.

2.3.1 ABOUT MARS MODELLING

Multivariate Adaptive Regression Spline (MARS) is a method to estimate general functions of high dimensional arguments given sparse data it has an increasing number of applications in many areas of science, economy and technology. At the same time it is a research challenge, to which this present project wishes to contribute, especially, by means of various geotechnical regression problems. We shall mostly refer to a regression formulation, but also classification will become addressed. The finitely many data underlying may base on different types of experiments, records or they may be obtained with different kinds of technologies.

MARS is an adaptive procedure because the selection of basis functions is data-based and specific to the problem at hand. This algorithm is a nonparametric regression procedure that makes no specific assumption about the underlying functional relationship between the dependent and independent variables. It is very useful for high dimensional problems and shows a great promise for fitting nonlinear multivariate functions. A special advantage of MARS lies in its ability to estimate the contributions of the basis functions so that both the additive and the interactive effects of the predictors are allowed to determine the response variable.

For this model an algorithm was proposed by Friedman (1991) [4] as a flexible approach to high dimensional nonparametric regression, based on a modified recursive partitioning methodology. MARS uses expansions in piecewise linear basis functions of the form

$$c^{+}(x,\tau)=[+(x-\tau)]_{+},\, c^{-}(x,\tau)=[-(x-\tau)]_{+} \quad (2.5)$$

where $[q]_{+} := \max\{0, q\}$ and τ is an univariate knot. Each function is piecewise linear, with a knot at the value τ, and it is called a reflected pair. For visualization see Figure 2.5:

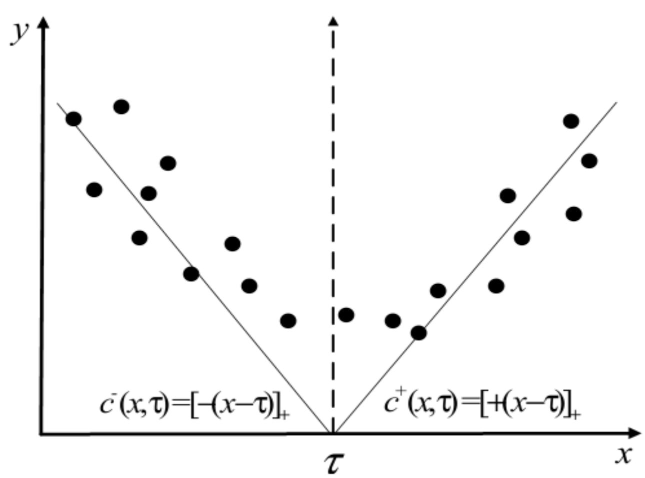


Fig 2.5: A basic element in regression with MARS

The points in this figure illustrate the data (xi , yi) (i = 1, 2,...N), composed by a p-dimensional input specification of the variable x and the corresponding 1-dimensional responses which specify the variable y. The following general model is considered on the the relation between input and response:

$$Y = f(X) + \varepsilon \qquad (2.6)$$

Where, Y is a response variable, X=(X1 ,X2,…………Xn)^T is a vector of predictors and ε is an additive stochastic component which is assumed to have zero mean and finite variance.

The goal is to construct reflected pairs for each input x_j (j=1,2………p) with p-dimensional knots $\tau_i = (\tau_{i,1}, \tau_{i2},\ldots \tau_{i,p})^T$. Actually, the knots $\tau_{i,j}$ more far away from the input values $x_{i,j}$can even be chosen, if any such a position promises a better data fitting.

After these preparations, the set of basis functions is:

$$\delta := \{(X_j - \tau)_+, (\tau - X_j)_+ \mid \tau \in \{x_{1,j}, x_{2,j}, \ldots\ldots., x_{N,j}\}, j \in \{1,2,\ldots\ldots., p\}\} \qquad (2.7)$$

If all of the input values are distinct, there are 2Np basis functions altogether. Thus, f (X) can be represented by a linear combination which is successively built up by the set above and with the intercept θ_0 , such that (2.6) takes the form:

$$Y = \theta_0 + \sum_{m=1}^{M} \theta_m \psi_m(X) + \varepsilon. \qquad (2.8)$$

The MARS algorithm for estimating the model function f(x) consists of two algorithms (Friedman 1991):

(i) The forward stepwise algorithm: Here, forward stepwise search for the basis function takes place with the constant basis function, the only one present initially. At each step, the split that minimized some “lack of fit” criterion from all the possible splits on each basis function is chosen. The process stops when a user-specified value max M is reached. At the end of this process we have a large expression. This model typically overfits the data and so a backward deletion procedure is applied.

(ii) The backward stepwise algorithm: The purpose of this algorithm is to prevent from over-fitting by decreasing the complexity of the model without degrading the fit to the data. Therefore, the backward stepwise algorithm involves removing from the model basis functions that contribute to the smallest increase in the residual squared error at each stage, producing an optimally estimated model f^{α} with respect to each number of terms, called α. We note that α expresses some complexity of our estimation. To estimate the optimal value of α, generalized cross-validation can be used which shows the lack of fit when using MARS. This criterion is defined by

$$GCV := \frac{\sum_{i=1}^{N}(y_i - f_{\alpha}(x_i))^2}{(1 - M(\alpha)/N)^2}, \qquad (2.9)$$

Where (M (α) := u + d M). Here, N is the number of sample observations, M is the number of linearly independent basis functions, M (α) is the number of knots selected in the forward process, and d is a cost for basis-function optimization as well as a smoothing parameter for the procedure. We do not employ the backward stepwise algorithm to estimate the function f (x) . At its place, as an alternative we propose to use penalty terms in addition to the least-squares estimation in order to control the lack of fit from the viewpoint of the complexity of the estimation. Because of this new treatment offered, we do not need to run the backward stepwise algorithm.

Flow chart explaining MARS.

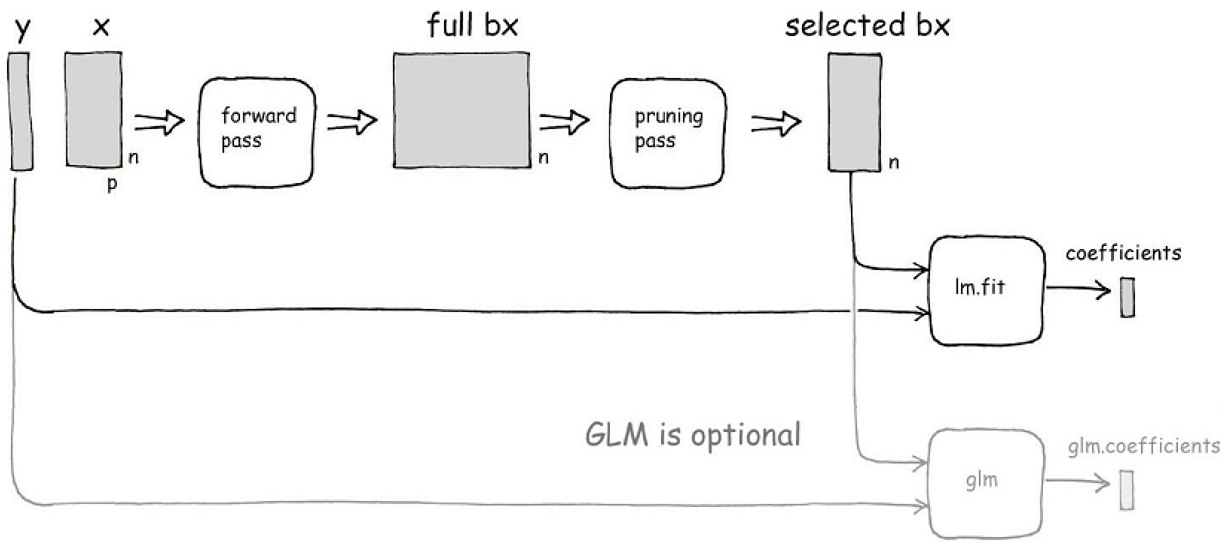


Fig 2.6: Flow chart showing MARS modelling.

Y: Dependent variable matrix

X: Independent variable matrix

Full bx: Model matrix including all the basis functions

Selected bx: Model matrix containing the selected functions after pruning pass.

2.3.2 MARS APPLICATION USING EARTH

Here in the present study 'EARTH' package of R to predict the model of some geotechnical problems. R is a system for statistical computation and graphics. Nowadays it is used in various statistical problems related to engineering, medical, economics etc. Moreover it can also be used for regression problems such as linear, nonlinear, and single or multivariate. The advantage of using R is that, it is very easy to work on R. We don't have to write long syntax, each and every function of R consists of small syntax. Also data from excel can be directly entered into R from clipboard.

R is a system for statistical computation and graphics. It consists of a language plus a run-time environment with graphics, a debugger, access to certain system functions, and the ability to run programs stored in script files.

The design of R has been heavily influenced by two existing languages: Becker, Chambers &Wilks' and Sussman's Scheme. Whereas the resulting language is very similar in appearance to S, the underlying implementation and semantics are derived from Scheme.

The core of R is an interpreted computer language which allows branching and looping as well as modular programming using functions. Most of the user-visible functions in R are written in R. It is possible for the user to interface to procedures written in the C, C++, or FORTRAN languages for efficiency. The R distribution contains functionality for a large number of statistical procedures. Among these are: linear and generalized linear models, nonlinear regression models, time series analysis, classical parametric and nonparametric tests, clustering and smoothing. There is also a large set of functions which provide a flexible graphical environment for creating various kinds of data presentations. Additional modules ("add-on packages") are available for a variety of specific purposes.

R was initially written by Ross Ihaka and Robert Gentleman at the Department of Statistics of the University of Auckland in Auckland, New Zealand. In addition, a large group of individuals has contributed to R by sending code and bug reports.

Since mid-1997 there has been a core group (the "R Core Team") who can modify the R source code archive. The group currently consists of Doug Bates, John Chambers, Peter Dalgaard, Seth Falcon, Robert Gentleman, Kurt Hornik, Stefano Iacus, Ross Ihaka, Friedrich Leisch, UweLigges, Thomas Lumley, Martin Maechler, Duncan Murdoch, Paul Murrell, Martyn Plummer, Brian Ripley, DeepayanSarkar, Duncan Temple Lang, Luke Tierney, and Simon Urbanek.

We used R version 2.15.0 (2012-03-30) for our MARS modelling. We have used EARTH package for this purpose. This package is also free-ware and readily available.

What is Earth?

The earth package is an implementation of Jerome Friedman's Multivariate Adaptive Regression Splines, commonly known as "MARS". The earth source code is licensed under the GPL and runs in an R environment, or can be used as a stand-alone C library. Earth is derived from the mda:mars library written by Trevor Hastie and Rob Tibshirani.

2.3.3 How to load EARTH on R?

Open R and click on package tab. Then choose load package option. Load package window will open from there choose ‘earth’ option and click ‘ok’ button.

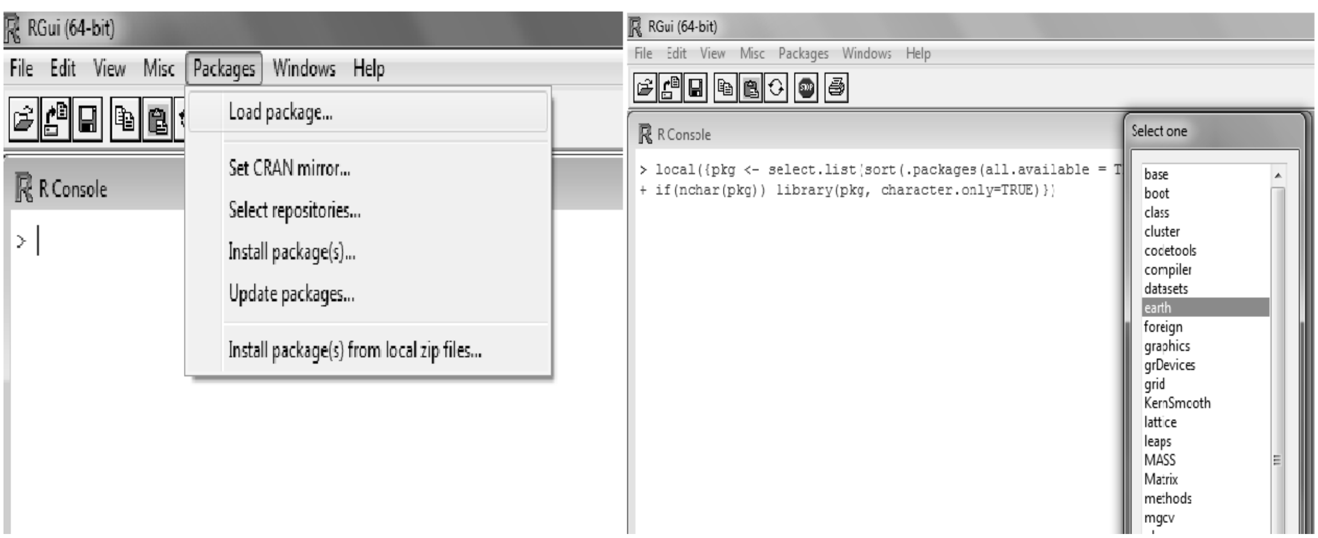


Fig 2.7: How to load earth package in R.

Steps for MARS modelling in R

Step 1: Read data from excel worksheet by copying the required data in clipboard and run the following command in R console:

<data<-read.table(“clipboard”)

and press enter. Here data is name of data matrix (can be anything).

To see the data stored type <data and press enter.

Step 2: Now use the earth command to run MARS for model generation using the following command:

<a<-earth(V5~.,data)

Here ‘a’ is an object of earth and ‘V5’ is the dependent variable name as mentioned in the data matrix, ‘~.’ means that we want to predict V5 from all the independent variables present in the data matrix.

If u want to predict V5 using any particular variable use the name of variable instead of ‘.’.

Step 3: For getting the predicted data as per the generated model use following command:

<summary(a)

This command will show the general details of the model. It gives all the basis functions used in the model and its coefficient. It also shows the number of terms generated in the full ‘bx’ matrix after forward pass and how many are selected after pruning pass. The number of predictors utilized in the model is also mentioned. Next information shown is the importance of variables (variables are arranged in decreasing order of importance).

Step 4: Using plot( ) function we can get various plots.

<plot(a) gives different types of graphs.

CHAPTER 3

PREDICTION OF LATERAL LOAD CAPACITY OF PILES IN CLAY USING GENETIC PROGRAMMING AND MULTIVARIATE ADAPTIVE REGRESSION SPLINE

3.1 INTRODUCTION

The design of pile foundation has drawn more attention than other foundation structures. The axial loaded pile is more frequently used and can be designed using equations of static equilibrium and other dynamic equations (Poulos and Davis1980). However, the lateral loaded piles are used in more difficult conditions, particularly in tall and offshore structures. The design of laterally loaded piles is more difficult and requires the solution of nonlinear differential equations. The elastic analysis adopting Winkler soil model (Poulos and Davis 1980) is not suitable for the nonlinear soil behavior. Matlock and Reese (1962) adopted an elastic analysis using a nonlinear lateral load capacity – deflection (p-y) curves. Portugal and Seco e Pinto (1993) used nonlinear p-y curves and finite element method for prediction of the behavior of laterally loaded piles. The above two methods are more accurate and widely used. But, spatial variability of soil is inevitable. Thus, developing a sufficiently accurate site model for FEM analysis requires extensive site characterization effort and desired constitutive modeling of clayey soil is also very difficult, even with considerable laboratory testing. So methods based on field data (Hansen 1961, Broms 1964, Meyerhof 1976) have become very much popular for the above study, particularly for the preliminary estimate of pile load capacity. These methods are based on pile load test case histories and involve statistically derived empirical equations for estimation of expected lateral load capacity.

Artificial intelligence (AI) techniques such as artificial neural networks (ANNs) and support vector machine (SVM) are considered as alternate statistical methods and are found to be more efficient compared to statistical methods (Das and Basudhar, 2006, Das *et al*., 2011a). ANN method has been found to be efficient in predicting the pile load capacity in both cohesion-less soil and clayey soil compared to traditional empirical methods (Goh 1995, Chan *et al*. 1995, Goh 1996,Lee and Lee 1996, Teh *et al*. 1997, Abu-Kiefa 1998). The performance of SVM model was found to be better than that of ANN model for prediction of frictional resistance of pile in clay (Samui 2008). Similar studies have also been made for prediction of lateral load capacity of piles in clay using ANN (Das and Basudhar 2006). Based on various statistical performance criteria, Das and Basudhar (2006) observed that ANN model is better compared to Brom's and Hansen's method. Using the same dataset, Pal and Deswal (2010) developed Gaussian process regression (GPR) and SVM models. They observed that GPR model is better compared to SVM model. However, they have compared the GPR model with the ANN model of Das and Basudhar (2006) only in terms of correlation coefficient (*R*) and root mean square error (*RMSE*). *R is* a biased estimate (Das and Sivakugan, 2010) and it is difficult to assess the prediction of the model in terms of under prediction or over prediction on the basis of *R* value only. The *RMSE* explains the overall error of the dataset instead of the maximum deviation in the prediction of individual case.

The most important problem associated with efficient implementation of ANN is generalization for some complex problems. The developed model needs to be equally efficient for new data during testing or validation, which is called as generalization. There are different methods for generalization like early stopping and cross validation (Basheer 2001, Das and Basudhar 2006). The magnitude of weight is one of the reasons for poor generalization (Bartlett 1998). The methods like Bayesian regularization neural network (BRNN) (Das and Basudhar 2008) have been used to consider the magnitude of weights as the part of the error function. Another reason for the poor generalization is due to the optimization of error function of ANN. The error function associated with weights and sigmoid function is a highly nonlinear optimization problem with many local minima (Das and Basudhar 2008). As the characteristic of traditional nonlinear programming based optimization method are the initial point dependent, the use of global optimization algorithms like genetic algorithm and simulated annealing are being widely used in training the ANN model (Morshed and Kaluarachchi 1998, Goh *et al*. 2005). The training of the feed-forward neural network using differential evolution optimization is known as differential evolution neural network (DENN) (Ilonen *et al*. 2003, Das *et al*. 2011a). Das *et al*. (2011b) observed that the performance of DENN is better than BRNN and traditionally used Levenberg-Marquardt neural network (LMNN) for the slope stability analysis. The ANN is termed as a 'black box' system unable to explain the input output relationships and in SVM error parameter 'C' and sensitive function 'e' are to be found out by trial and error. However, now it is possible to write down a model equation based on the trained ANN model (Goh*et al.* 2005, Das and Basudhar 2006, Das and Basudhar 2008) and SVM model (Das *et al*. 2010, Das *et al*. 2011a), still the developed model particularly SVM model is not comprehensive. The modified artificial intelligence techniques in the class of 'grey box' and 'white box' are now a day being popular (Giustolisi*et al.* 2007). The genetic programming (GP) is defined as next generation AI technique and also called as 'grey box' model (Giustolisi *et al*. 2007) in which the mathematical structure of the model can be derived, allowing further information of the system behaviour. GP models have been applied to few difficult geotechnical engineering problems (Yang *et al*. 2004, Javadi*et al*. 2006, Rezania and Javadi 2007) with success. A modified statistical technique called multivariate adaptive regression spline (MARS) is popularized by Friedman(1991) for solving regression-type problems. MARS is also called as 'white box' system of predictive model, as it is based on physical laws and underlying physical relationships of the system can be explained. The MARS technique is very popular in the area of data mining because it does not assume or impose any particular type or class of relationship (e.g., linear, logistic, etc.) between the predictor variables and the dependent (outcome) variables of interest. This makes MARS particularly suitable for problems with a number of variables. It has an increasing number of applications in many areas of economy, science and technology. However, its use in geotechnical engineering is very limited (Samui 2011).It needs to compare the efficacy of the present GP and MARS models vi's-à-via ANN, and other empirical models in terms of different statistical performance criteria.

In the present study prediction models for lateral load capacity of piles in clay under un-drained condition have been developed using GP, MARS and ANN (BRNN, DENN). Different statistical criteria like correlation coefficient (R), Nash-Sutcliff coefficient of efficiency (E) (Das and Basudhar 2008) and root mean square error ($RMSE$) are used to compare the GP and MARS models with ANN (DENN, BRNN) models and existing empirical models (Broms and Hansen's). A ranking system (Abu-Farsakh and Titi 2004) using rank index (RI) has also been followed to compare the different models basing on four criteria: (i) the best fit calculations (R and E) for predicted lateral load capacity (Q_p) and measured lateral load capacity (Q_m), (ii) arithmetic calculations (mean, μ and standard deviation, σ) of the ratio, Q_p/Q_m (iii) 50% and 90% cumulative probabilities (P_{50} and P_{90}) of the ratio, $Q_p/Q_{m.}$and (iv) the probability of pile load capacity within 20% accuracy level in percentage using histogram and lognormal probability distribution of Q_p/Q_m.

3.2 Database and Preprocessing

In the present study the experimental database of Rao and Suresh Kumar (1996) has been considered. Das and Basudhar (2006) have developed ANN model and Pal and Deswal (2010) have developed GPR and SVM models using the above database. The data consist of *D*, *L*, *e*, S_u as the inputs and $Q_m$as output. Out of the mentioned 38 data, 29 data are selected for training and remaining 09 data are used for testing the developed model as per Das and Basudhar (2006). The data were normalized in the range 0 to 1 to avoid the dimensional effect of input parameters. The data are normalized in the range [0, 1] and [-1, 1] for MARS and ANN (DENN, BRNN) models respectively to avoid the dimensional effect of input parameters.In the GP modeling normalization or scaling of the data is not required.

Table 3.1: Data table of prediction of lateral load pile.(Training data)

ng data					
D	L	e	C_u	Q_m	Q_p(Predicted using MARS)
6.35	146.1	19.1	38.8	69.5	82.98
13	260	0	24	225	225.92
12.5	130	0	24	106	100.21
13.5	300	50	3.4	30	25.7
13.5	300	50	4	36	32.6
13.5	300	50	5.5	50	49.82
13.5	300	50	7.2	64	69.35
18	300	50	10	89	93.23
18	300	50	3.4	39	41.19
20.4	300	50	4	46	53.42
12.3	300	50	5.5	44	45.69
18.4	300	50	4	51	48.97

18	300	50	10	116.5	93.23
33.3	300	50	3.4	78.5	75.21
33.3	300	50	5.5	110.5	99.33
12.3	300	50	3.4	29.5	21.58
6.35	139.7	25.4	38.8	65.5	58.47
12.3	300	50	7.2	58	65.22
12.3	300	50	10	81	73.61
18.4	300	50	5.5	65.5	66.2
18.4	300	50	7.2	86.5	85.72
18.4	300	50	10	114	94.12
20.4	300	50	5.5	59.5	70.64
20.4	300	50	7.2	76.5	90.17
20.4	300	50	10	87	98.56
25.4	300	50	7.2	90	101.29
25.4	300	50	10	118.5	109.68
25.4	300	50	3.4	50	57.65
25.4	300	50	5.5	75	81.76
			Correlation coefficient		0.970

Table 3.2: Data table of prediction of lateral load pile.(testing data).

Testing data					
D	**L**	**e**	**Cu**	**Qm**	**Qp**
13.5	190	0	24	128	160.88
20.4	300	50	3.4	38	46.53
18.4	300	50	3.4	42.5	42.08
25.4	300	50	4	58	64.54
13	132.1	33.8	38.8	53	49.57

18	300	50	4	49	48.08
18	300	50	5.5	65	65.31
18	300	50	7.2	87	84.83
12.3	300	50	4	35	28.47
			Correlation coefficient		0.980

Commond used for preparing MARS model of above data:

a<-earth(V5~.,data)

Dependent variable was predicted using all the independent variables.

Table 3.3: Basis functions and their coefficients for predicting Qm.

	coefficients
(Intercept)	68.76
h(D-18)	2.22
h(18-D)	-3.44
h(L-130)	0.95
h(e-0)	-2.92
h(C_u-7.2)	3
h(7.2-C_u)	-11.48

<plot(a) gives following types of graphs

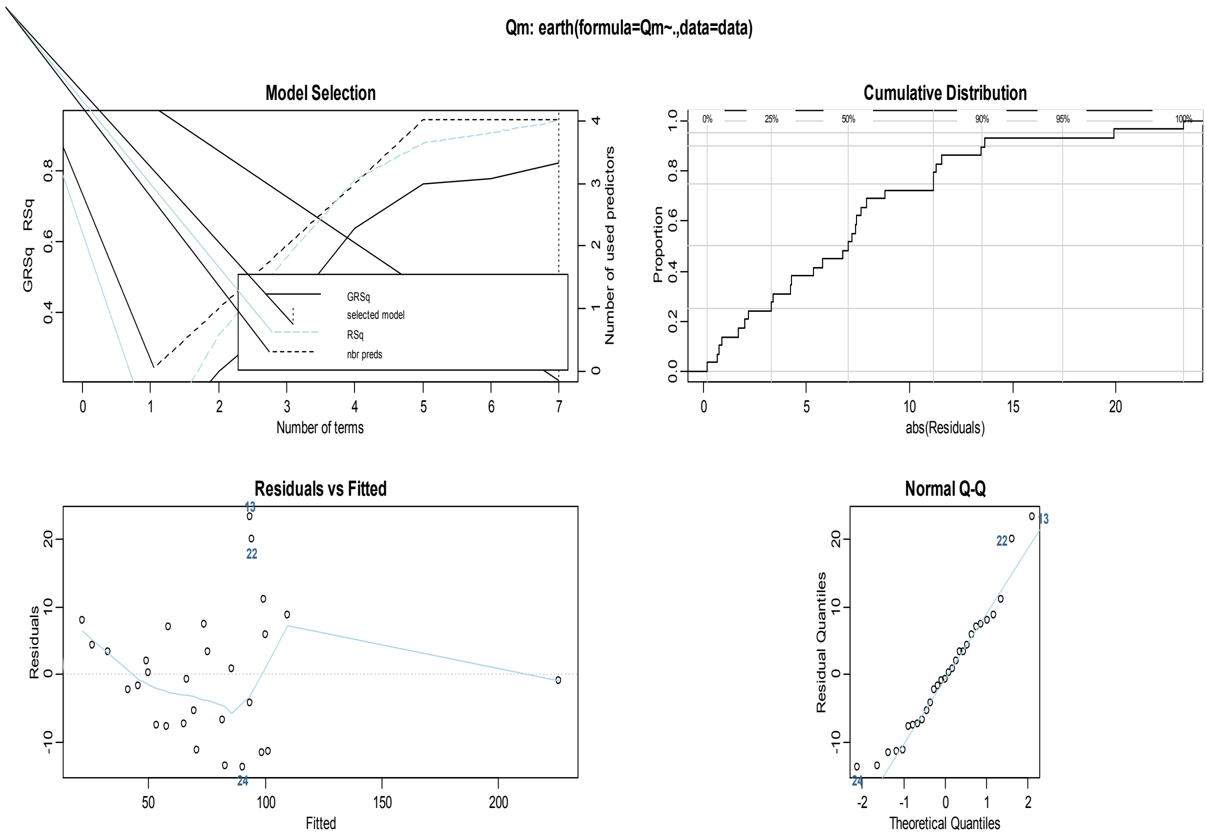


Fig 3.1: Model selection graphs Q_m

<plotmo(a)

It gives the variation of the dependent variable with each independent variable

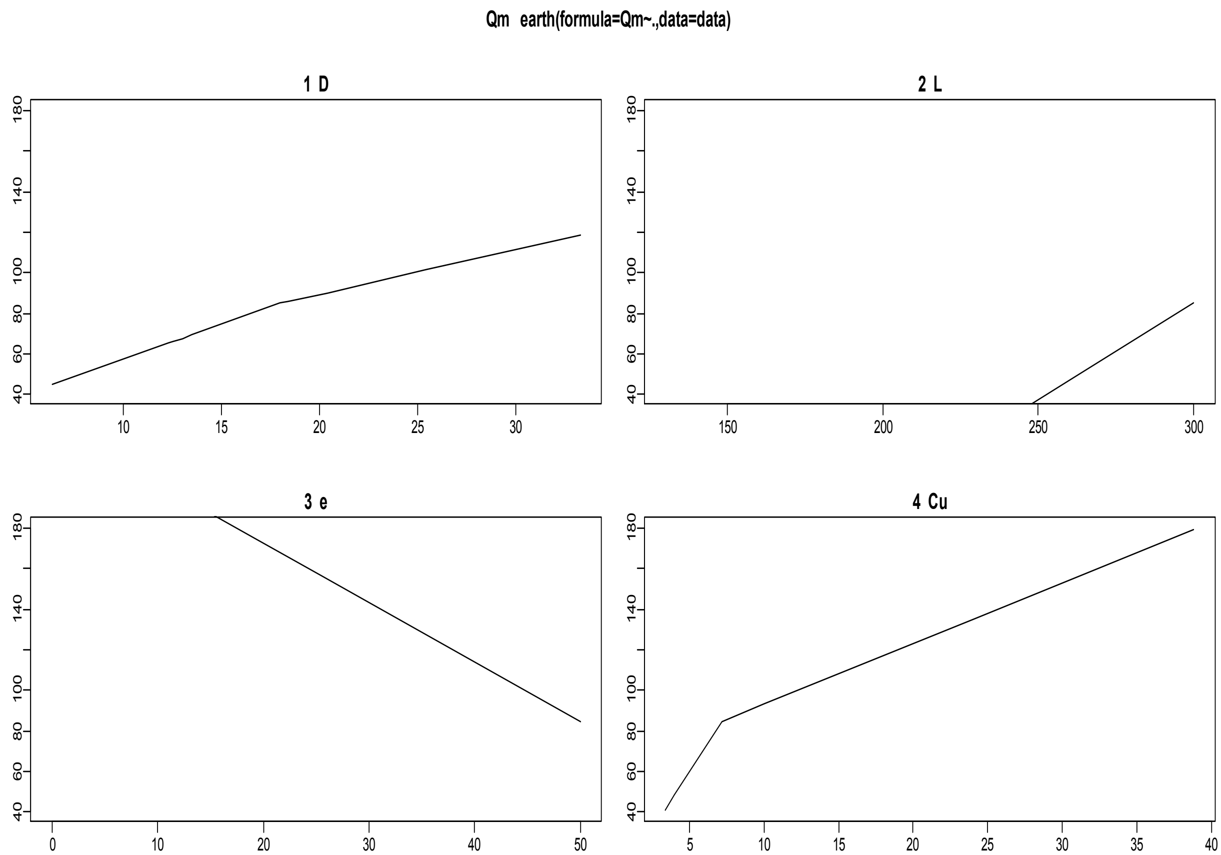


Fig 3.2: Variation graphs of Q_m ~ D, L, e and C_u respectively.

Table 3.4:Variables and their importance in the MARS model.

Importance chart			
	nsubsets	GCV	RSS
C_u	5	100	100
L	5	94.7	91.4
e	5	94.7	91.4
D	3	53.2	48

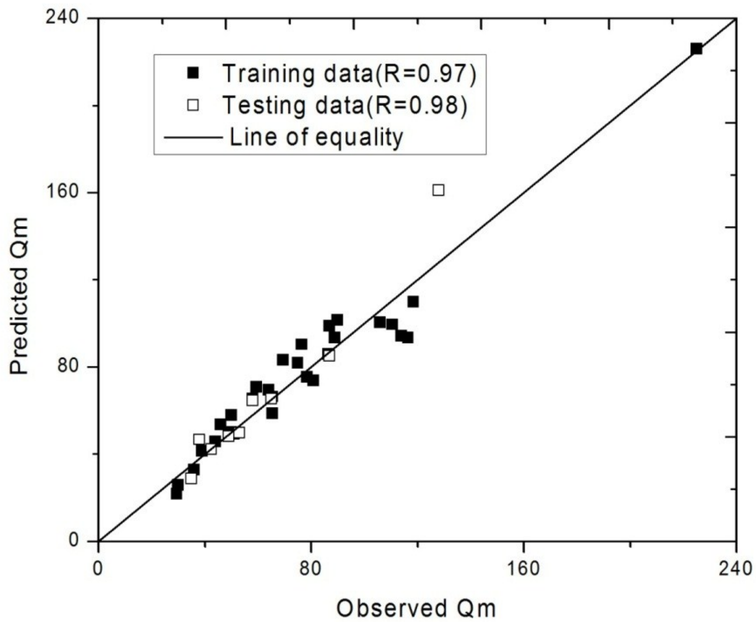


Fig 3.3: Comparisons of predicted and measured load capacity of piles by MARS for training data.

The coefficients of different basis functions produced for the developed MARS model, and the coefficient of intercept generated is presented in Table 3.3.Hence, model equations can be written using the obtained coefficients and basis functions as presented in Equation (3.1) as follows:

$$Qp = 68.758707 + 2.223738\, h(D - 18) - 3.441136\, h(18 - D) + 0.953778\, h(L - 130) - 2.921405\, h(e - 0) + 2.998536\, h(C_u - 7.2) - 11.484484\, h(7.2 - C_u) \quad (3.1)$$

Where,

$$h(D - 18) = \max(0, D - 18) \quad (3.1.1)$$

$h(18 - D) = \max(0, 18 - D)$ (3.1.2)

$h(L - 130) = \max(0, L - 130)$ (3.1.3)

$h(e - 0) = \max(0, e - 0)$ (3.1.4)

$h(C_u - 7.2) = \max(0, C_u - 7.2)$ (3.1.5)

$h(7.2 - C_u) = \max(0, 7.2 - C_u)$ (3.1.6)

In the present study each individual in the population consists of more than one gene and each gene is a traditional GP tree. Here, function set used include: +, ×, ÷, -, sin(.), cos(.), tanh(.) and exp(.). As discussed earlier in GP procedure first a number of potential models are evolved at random. Each model is trained and tested using the training and testing cases respectively. The fitness of each model is determined by minimizing *RMSE* between the predicted (Q_p) and actual (Q_m) value of the output variable as the objective function,

$$RMSE = f = \sqrt{\frac{\sum_{i=1}^{n}\left(Q_m - Q_p\right)^2}{n}} \quad (3.2)$$

where$n$ = number of cases in the fitness group. If the errors calculated by using Equation (3.2) for all the models in the existing population do not satisfy the termination criteria, the generation of new population continues till ‘best’ model is developed as per the earlier discussion. The ‘best’ $Q_p$model was obtained with population size of 2000 individuals and 150 generations with reproduction probability of 0.05, crossover probability of 0.85, mutation probability of 0.1 and with tournament selection. In GP model development it is important to make a tradeoff between accuracy in prediction of Q_p and complexity of the model equation which is achieved by proper selection of number of genes and depth of GP tree. In this study optimum result was obtained with maximum number of genes as two and maximum depth of GP tree as four. The developed GP model can be described as Equation (3.3) and shown below.

$$Q_p = \exp\left[0.037(D-6.35)\right]\begin{bmatrix}0.032(L-130)(C_u-3.4)\times \\ \left[0.000035(L-130)^2+\sin\left(0.028(C_u-3.4)\right)\right] \\ -19.259\left(0.02e-3.625\right)\left[0.037\left(D-6.35\right)-0.02e\right]\end{bmatrix}+81.307 \quad (3.3)$$

Based on the DENN and BRNN analysis best models were developed with 3 and 2 hidden layer neurons respectively. Model equations for above two models can be written using the obtained weights and biases following Das and Basudhar (2006, 2008).

As it is important that the efficiency of model should be compared in terms of testing data than that with training data (Das and Basudhar 2008), in this study the comparison of the methods are done on the basis of testing data only. Figure 3.4 shows the performance of predicted and observed values of lateral load capacity of piles for GP, MARS and ANN (DENN, BRNN) models.

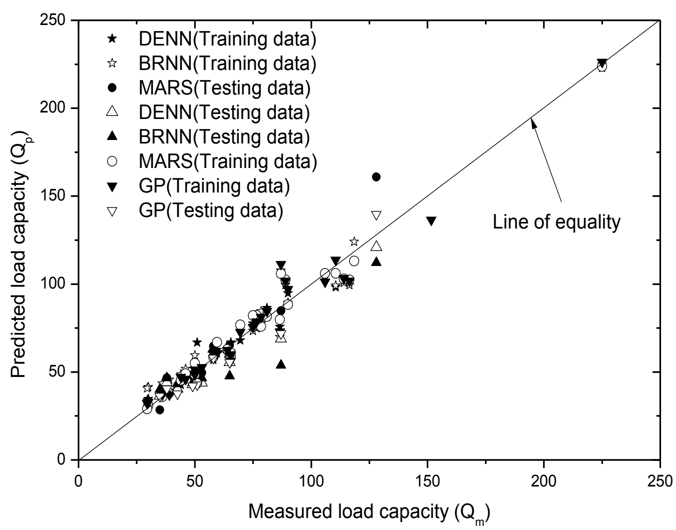


Fig-3.4:Comparisons of predicted and measured load capacity of piles by methods.

There is less scatter of data for the GP and MARS models compared to the other models. Table 3.5 shows the statistical performance in terms of *R, E*, *AAE*, *MAE* and *RMSE* for the GP and MARS model along with the results of ANN (DENN and BRNN), Broms and Hansen's models for both training and testing data set. The developed GP, MARS and DENN models shows good generalization in terms of close values of *R* and *E* for training and testing data. It also indicates that GP model is better than other models as it outperforms all other models in terms of the most of the statistical parameters under consideration.

Table 3.5.Comparison of statistical performances of different models.

Models		Statistical Performances				
		R	*E*	*AAE*	*MAE*	*RMSE*
GP	Training	0.980	0.961	5.337	24.378	7.831
	Testing	0.972	0.913	6.702	15.070	8.194
MARS	Training	0.970	0.940	7.258	23.273	9.108
	Testing	0.98	0.900	6.858	32.875	11.815
DENN	Training	0.980	0.959	5.647	18.705	7.667
	Testing	0.967	0.905	7.170	18.110	8.549
BRNN	Training	0.975	0.949	6.609	20.680	8.582
	Testing	0.899	0.734	10.800	33.169	14.312
Hansen	Training	0.950	0.209	30.712	65.360	33.825
	Testing	0.919	0.119	23.650	49.480	26.066
Broms	Training	0.967	0.807	12.391	48.660	16.703
	Testing	0.985	0.574	12.082	46.380	18.127

While describing prediction of pile load capacity based on cone penetration test (CPT) Briaud and Tucker (1988) have emphasized that other statistical criteria should be used along with the correlation coefficient. Abu-Farsakh and Titi (2004) and Das and Basudhar (2006) have used the mean (μ) and standard deviation (σ) of ratio of predicted pile load capacity (Q_p) to the measured pile load capacity *(Q_m)* as important parameters in evaluating different models. The mean (μ) and standard deviation (σ) of $Q_p/Q_m$are important indicators of the accuracy and precision of the prediction method. Under ideal condition an accurate and precise method gives the mean value as 1.0 and the standard deviation to be 0. The μ value greater than 1.0 indicates over prediction and under prediction otherwise. In present study the μ (1.006, 1.032) and σ (0.125, 0.141) of $Q_p/Q_m$for the MARS model is very close to those of GP [(1.007,0.94), (0.090, 0.107)] and DENN [μ (1.018,0.948), σ (0.106,0.125)] for training and testing data. The values for BRNN (μ (1.042, 0.942), σ(0.143,0.196) and other models as also presented in Table 3.6.

Table 3.6. Evaluation of performance of different prediction models considered in this study.

Pile apacity ethods		Best fit calculations			Arithmetic calculations of Q_p/Q_m			Cumulative probability			± 20% Accuracy (%)			Overall rank	
		R	E	R1	μ	σ	R2	Q_p/Q_m at P_{50}	Q_p/Q_m at P_{90}	R3	Log-normal	Histo-gram	R4	RI	Final rank
GP	Training	0.980	0.961	1	1.007	0.090	1	1.020	1.096	2	94	96	1	5	1
	Testing	0.972	0.913		0.940	0.107		0.890	1.092		80	100			
MARS	Training	0.970	0.940	3	1.006	0.125	2	1.004	1.178	1	92	90	2	8	2
	Testing	0.980	0.900		1.032	0.141		0.990	1.256		84	100			
DENN	Training	0.980	0.959	2	1.018	0.106	3	1.012	1.156	3	90	92	3	11	3
	Testing	0.967	0.905		0.948	0.125		0.945	1.161		84	88			
BRNN	Training	0.975	0.949	4	1.042	0.143	4	1.005	1.238	4	86	86	4	16	4
	Testing	0.899	0.734		0.942	0.196		0.896	1.226		62	66			
Hansen	Training	0.950	0.209	6	0.580	0.111	6	0.542	0.741	6	0	0	6	24	6
	Testing	0.919	0.119		0.590	0.149		0.523	0.838		8	22			
Broms	Training	0.967	0.807	5	1.143	0.144	5	1.112	1.382	5	64	72	5	20	5
	Testing	0.985	0.574		1.166	0.136		1.140	1.392		50	66			

The other criterion like cumulative probability of Q_p/Q_m (Das and Basudhar 2006, Abu-Farsakh and Titi 2004) should also be considered for the evaluation of performance of different models. The ratio $Q_p/Q_m$is arranged as per their values in an ascending order and the cumulative probability is calculated from the following equation(3.4):

$$P = \frac{i}{n+1} \tag{3.4}$$

where$i$= order number given to the Q_p/Q_mratio; n is the number of data points. If the computed value of 50% cumulative probability (P_{50}) is less than unity, under prediction is implied; values greater than unity means over prediction. The 'best' model is corresponding to the P_{50} value close to unity. The 90% cumulative probability (P_{90}) reflects the variation in the ratio of Q_p/Q_m for the total observations. The model with P_{90} for Q_p/Q_mclose to 1.0 is a better model.

Figure 3.5 shows the cumulative probability plots of Q_p / Q_m for different methods for both training and testing data.

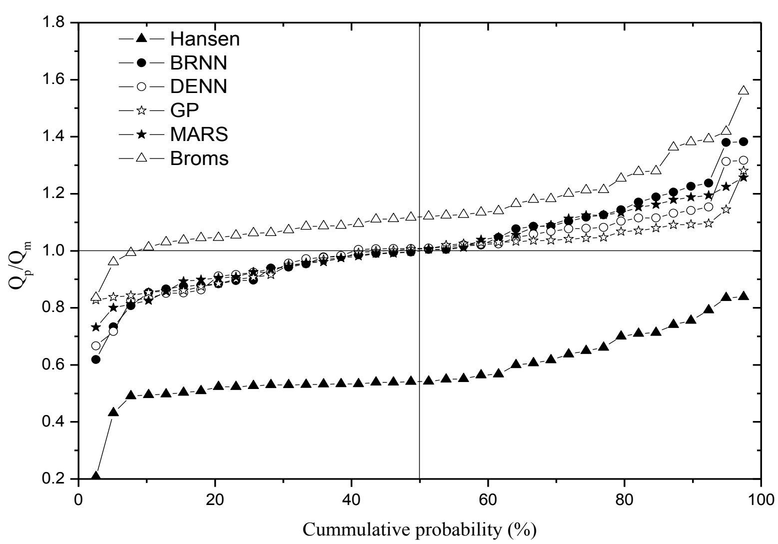


Figure 3.5. Cumulative probability plots of $Q_p/Q_m$for different methods.

Based on the figure(3.5) it can be seen that GP, MARS, DENN and BRNN models are very closely following each other. It can also be seen from Table 3.6 that P_{50} values of MARS (1.004,0.990), DENN (1.012,0.945) , BRNN(1.005, 0.896) and GP(1.020, 0.885) models for training and testing data are comparable whereas the Hansen method (P_{50} =0.542,0.523) under predicts the pile load capacity and Broms method (P_{50}=1.112, 1.140) over-predicts the same. However based on the P_{90}value GP (1.096, 1.092) model is found to be close to MARS (1.178, 1.256) and DENN (1.156, 1.161) models and better than other models. The lognormal distributions of the $Qp/Q_m$for different models are shown in Figure 3.6.

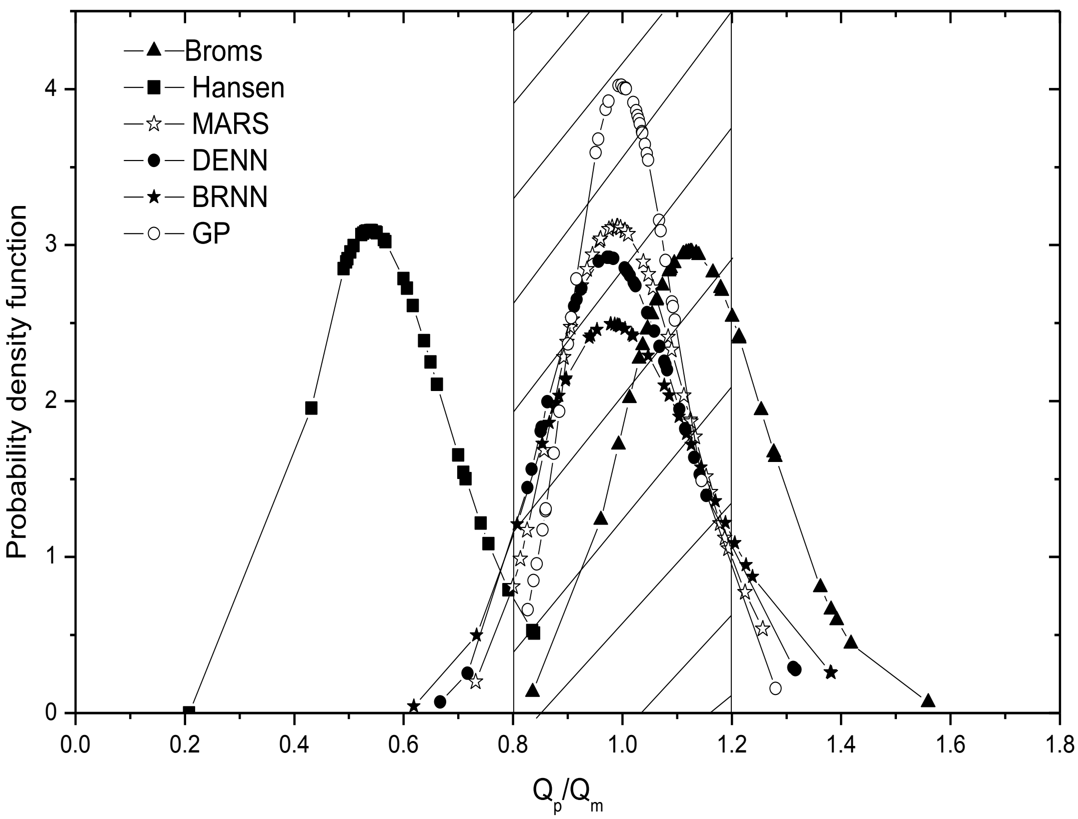


Figure 3.6. Log normal distribution of Q_p/Q_m for different methods

Based on the figure(3.6) it can be seen that GP model is predicting the lateral load capacity of the pile within 20% accuracy level (i.e. Q_p / Q_m = 0.8 to 1.2) better than MARS, DENN, BRNN and other statistical models as the shaded area under the lognormal distribution plot of GP model is more than those of the other models within the above limit.

As per the best fit calculations (*R*, *E*) (*R1*), arithmetic calculations of $Q_p/Q_m(\mu, \sigma)$ (*R2*), cumulative probability of Q_p/Q_m (P_{50}, P_{90}) (R3)and prediction of pile load capacity within 20% accuracy level (*R4*), a ranking system is made among different models and presented in Table 3.7. The overall performance of the various models under present study is evaluated using *RI* as per Abu-Farsakh and Titi (2004). The *RI* is the sum of the ranks of different models as per the above four criteria (*RI=R1+R2+R3+R4*). Lower the value of *RI* indicates better performance of the particular method. It can be seen from the Table 3.7 that GP model (*RI=5*) is 'best' among various models used in the present study and is closely followed by MARS model (*RI*=8) and other models [DENN (*RI*=11), BRNN(*RI*=16) ,Broom's (*RI*=20) and Hansen's (*RI*=24)].

The results of present developed models are also compared with the results of SVM and GPR models as given by Pal and Deswal (2010). However, the SVM and the GPR results are available for the testing data in terms of *R* and *RMSE* only. The *R* values of SVM and GPR models are 0.920 and 0.980 respectively. Similarly, the values of *RMSE* are 11.47 and 6.32 for SVM and GPR models respectively. Hence, the present GP (0.972, 8.194) and DENN (0.967, 8.549) models are found to be better than the SVM model as per *R* and *RMSE* values. The *R* value of GP (0.972) and MARS (0.980) models are comparable to GPR model, though GPR model is better than above two models in terms of *RMSE* value. However, due to absence of other data, performance of these two models based on other criteria as discussed in the above paragraph could not be made to make an elaborate comparison using *RI*.

CHAPTER 4

PREDICTION OF THE PULLOUT CAPACITY OF GROUND ANCHOR USING MULTIVARIATE ADAPTIVE REGRESSION SPLINE AND GP

4.1 INTRODUCTION

Small anchors, which are often installed vertically, are used to connect the marquees and other temporary light structures to the ground. The anchors resist uplift imposed by wind and other forces acting on the structures and also resist the tensile forces from the structure by means of shear resistance of the surrounding soil, which provide the structural stability. These anchors consist of steel rod of length less than 1m, that are driven into the ground. Due to the short term nature of temporary light structures , Geotechnical investigations examining the pullout capacity of the anchors used to secure these structures. According to Lau and Simons(1986) , very little published information exists regarding the uplift capacity of small ground anchors. Das(1990) also stated that the studies available to estimate the uplift capacity of anchor piles are limited.

A series of 119insitu anchor pull out tests were conducted at six different locations within Adelaide, South Australia, and to compare the results with the predictions from two different methods of pile pull out capacity that use direct cone penetration test(CPT) data. The capacities obtained from these CPT based methods are compared with predictions from an artificial neural network(ANN) model that was recently developed by Shahin and Jaksa(2005). To determine the influence of factors such as soil type, anchor diameter, embedment depth, installation technique and natural variability on the pull out capacity of anchors, a number of comparative tests are carried out. Statistical analysis , which compare the measured pullout loads with those obtained using the CPT methods and the ANN model , is carried out and these are used to evaluate and rank the performance of the pullout capacity prediction methods used.

Samui et al. (2011) observed that the MARS model for uplift capacity of suction caisson has better statistical performance compared to ANN and FEM model. Hence, more research is required in ANN regarding the generalization, control on the model parameters, extrapolation and depicting simplified model equation. Here the prediction of pull out capacity of ground anchor is done by using MARS and GP model.

Table 4.1: Data table of prediction of field pull capacity.(Training data)

training data					
Deq(mm)	L(mm)	qc	fc	Installation technique	Qu(KN)
25	600	1.65	52.1	1	2.47
25	600	1.89	46.83	1	2.01
33.5	600	1.89	46.83	1	2.08
25	400	1.05	55.68	1	1.16
25	600	1.28	64.64	1	3.2
44.6	600	1.89	46.83	1	1.9
25	600	1.89	46.83	1	1.76
25	800	2.02	53.73	1	2.3
25	600	1.74	35.93	1	2.03
44.6	600	1.28	64.64	1	2.49
25	600	1.28	64.64	1	2.15
25	600	2.76	20.82	1	0.92
25	800	1.27	70.91	1	1.69
25	600	2.76	20.82	1	1.19
44.6	400	1.05	55.68	1	1.34
44.6	600	1.89	46.83	1	2.23
25	400	1.05	55.68	1	1.06
44.6	600	1.89	46.83	1	1.85
33.5	600	1.89	46.83	1	2.3
25	600	1.89	46.83	1	1.87
25	800	1.27	70.91	1	2.48
33.5	600	1.28	64.64	1	2.79
33.5	400	1.14	32.52	1	1.18
25	600	2.02	53.73	1	3.02
44.6	400	1.14	32.52	1	1.45
25	600	3.03	178.26	1	2.09
25	600	2.67	13.99	1	0.9
25	600	2.67	13.99	1	0.87
33.5	600	1.89	46.83	1	2.39

25	400	1.14	32.52	1	1.24
44.6	600	1.28	64.64	1	2.29
33.5	600	1.28	64.64	1	3.11
33.5	400	1.05	55.68	1	1.44
25	600	1.89	46.83	1	1.99
25	600	1.65	52.1	1	1.7
25	600	1.89	46.83	1	2.16
25	600	2.2	87.93	1	2.99
33.5	600	1.89	46.83	1	1.9
44.6	600	1.89	46.83	1	1.95
25	600	1.89	46.83	1	2.39
44.6	800	1.27	70.91	1	3.47
33.5	600	1.89	46.83	1	2.24
25	600	1.28	64.64	1	1.29
25	600	1.28	64.64	1	2.09
44.6	800	2.02	53.73	1	2.45
25	600	1.28	64.64	1	2.25
25	600	1.65	52.1	1	0.9
25	800	1.27	70.91	1	3.06
25	600	1.89	46.83	1	2
25	800	3.55	26.01	2	1.11
25	800	1.68	54.35	2	2.19
33	400	2.28	179.71	2	1.76
33	800	2.24	105.1	2	2.95
33	800	3.55	26.01	2	1.71
44.6	400	1.66	40.94	2	1.96
25	600	2.67	13.99	2	0.35
33	600	1.65	52.1	2	0.63
25	600	1.65	52.1	2	1.52
25	600	2.67	13.99	2	0.53
25	600	1.65	52.1	2	0.94
44.6	600	1.65	52.1	2	1.73
25	600	1.65	52.1	2	1.63
25	600	2.2	87.93	2	2.18
25	600	2.67	13.99	2	0.6
25	600	2.2	87.93	2	2.09
44.6	600	1.74	35.93	2	2.95
44.6	400	2.28	179.71	2	2.55
33	400	0.95	12.22	2	0.29
33	600	2.2	87.93	2	2.39
33	400	2.21	70.33	2	1.81
33	600	1.65	52.1	2	1.7
25	600	2.76	20.82	2	0.73
25	800	1.68	54.35	2	1.33
33	800	1.49	41.23	2	2.63

25	600	1.74	35.93	2	1.73
25	400	1.63	44.46	2	1.36
44.6	600	3.03	178.26	2	3.44
25	600	2.67	13.99	2	0.48
25	600	1.65	52.1	2	1.76
25	400	1.66	40.94	2	1.25
25	400	2.21	70.33	2	1.37
25	600	2.2	87.93	2	1.79
25	600	2.76	20.82	2	0.94
44.6	400	2.21	70.33	2	1.65
				Correlation coefficient	0.887

Table 4.2: Data table of prediction of pullout capacity.(testing data).

Testing					
D_{eq}(mm)	L(mm)	q_c	f_c	Installation technique	Q_u(KN)
44.6	600	2.2	87.93	2	2.31
44.6	600	2.67	13.99	2	1.1
33	600	1.65	52.1	2	1.57
25	600	3.03	178.26	2	2.19
33	800	2.92	166.57	2	3.8
25	800	1.49	41.23	2	3.11
25	600	2.76	20.82	2	0.61
25	600	2.2	87.93	2	1.88
25	400	1.63	44.46	2	1.05
44.6	400	2.28	179.71	2	2.39
33	600	2.67	13.99	2	0.89
25	400	2.12	17.21	2	0.43
25	600	2.2	87.93	2	1.98
33	800	1.68	54.35	2	2.22
44.6	400	1.63	44.46	2	0.78
33	400	2.12	17.21	2	0.56
25	600	1.65	52.1	2	2.42
25	400	2.28	179.71	2	1.52
25	600	2.2	87.93	2	2
33	400	1.63	44.46	2	1.44
25	400	1.66	40.94	2	1.35
25	600	3.03	178.26	2	1.96
44.6	600	1.65	52.1	2	1.9
44.6	400	0.95	13.99	2	0.63
33	600	2.76	20.82	2	0.96
33	600	1.74	35.93	2	1.51

44.6	600	2.76	20.82	2	1.1
25	600	1.65	52.1	2	2
25	600	1.74	35.93	2	1.66
25	600	1.74	35.93	2	1.54
44.6	400	2.12	17.21	2	0.66
25	800	2.24	105.1	2	3.17
33	400	1.66	40.94	2	1.19
25	400	0.95	12.22	2	0.35
25	600	2.67	13.99	2	0.44
				Correlation coefficient	0.899

Command used for preparing MARS model :

```
a<-earth(V5~.,data)
```

The dependent variable was predicted using all the independent variables.

Table 4.3: Basis functions and their coefficients for predicting Qm.

	Coefficients		
(Intercept)	2.405447		
h(V1-33.5)	0.009157		
h(33.5-V1)	0.047694		
h(V2-600)	0.001429		
h(600-V2)	-0.0033		
h(V4-40.94)	-0.05805		
h(40.94-V4)	-0.04546		
h(V4-52.1)	0.144774		
h(V4-64.64)	-0.06606		
h(33.5-V1)	*	h(V3-2.2)	-0.06918
h(33.5-V1)	*	h(2.2-V3)	-0.06768
h(33.5-V1)	*	h(V4-54.35)	-0.00311
h(33.5-V1)	*	h(54.35-V4)	-0.00169
h(33.5-V1)	*	h(V5-1)	-0.02345
h(V3-2.21)	*	h(V4-40.94)	0.003048
h(2.21-V3)	*	h(V4-40.94)	-0.00856
h(V4-40.94)	*	h(V5-1)	-0.01672

Where, V1- D_{eq}, V2- L, V3- qc, V4- fc, V5- Installation technique

<plot(a) gives following types of graphs

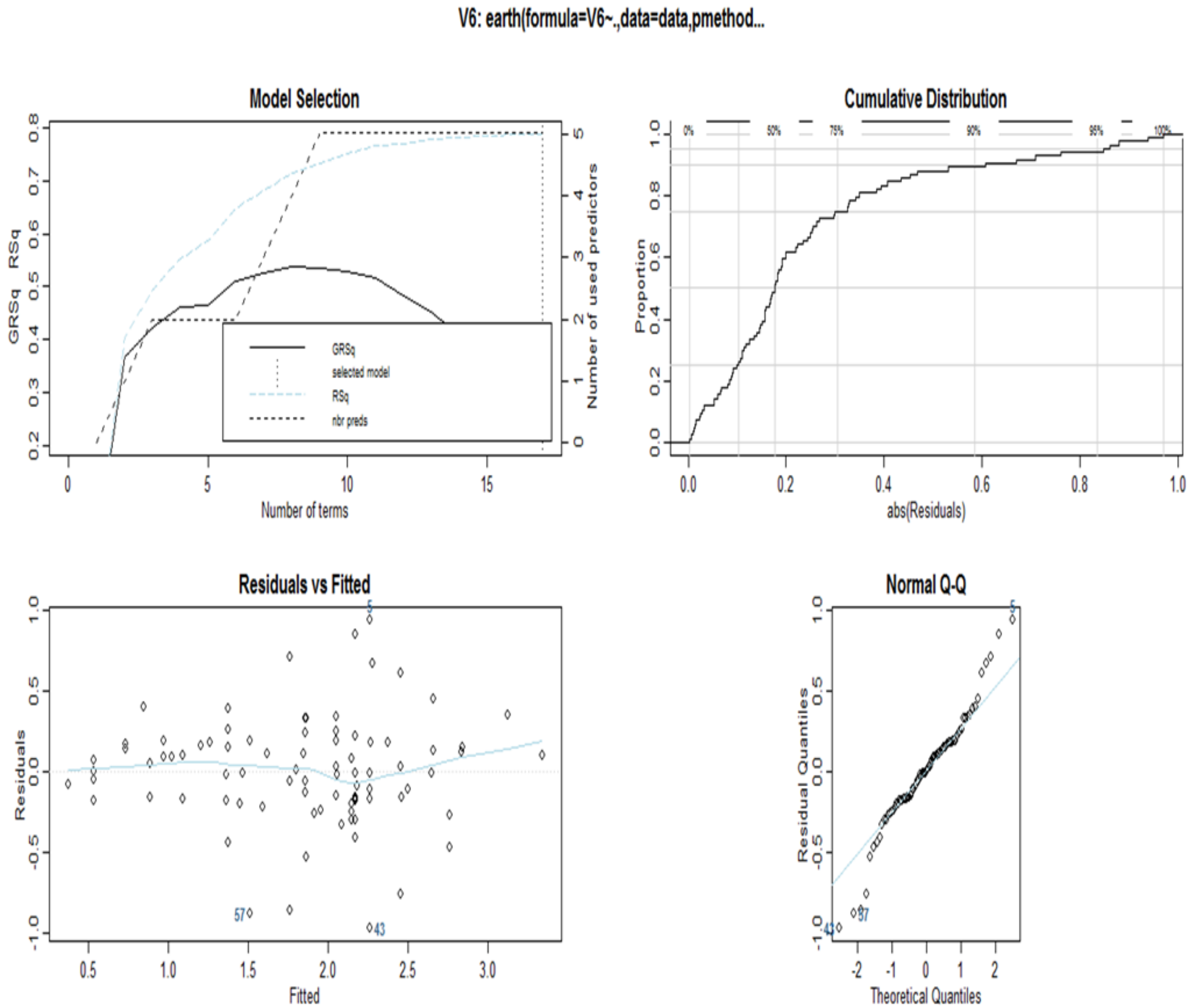


Fig 4.1: Type of model selection graph by ‘EARTH’.

<plotmo(a)

It gives the variation of dependent variable with each independent variable

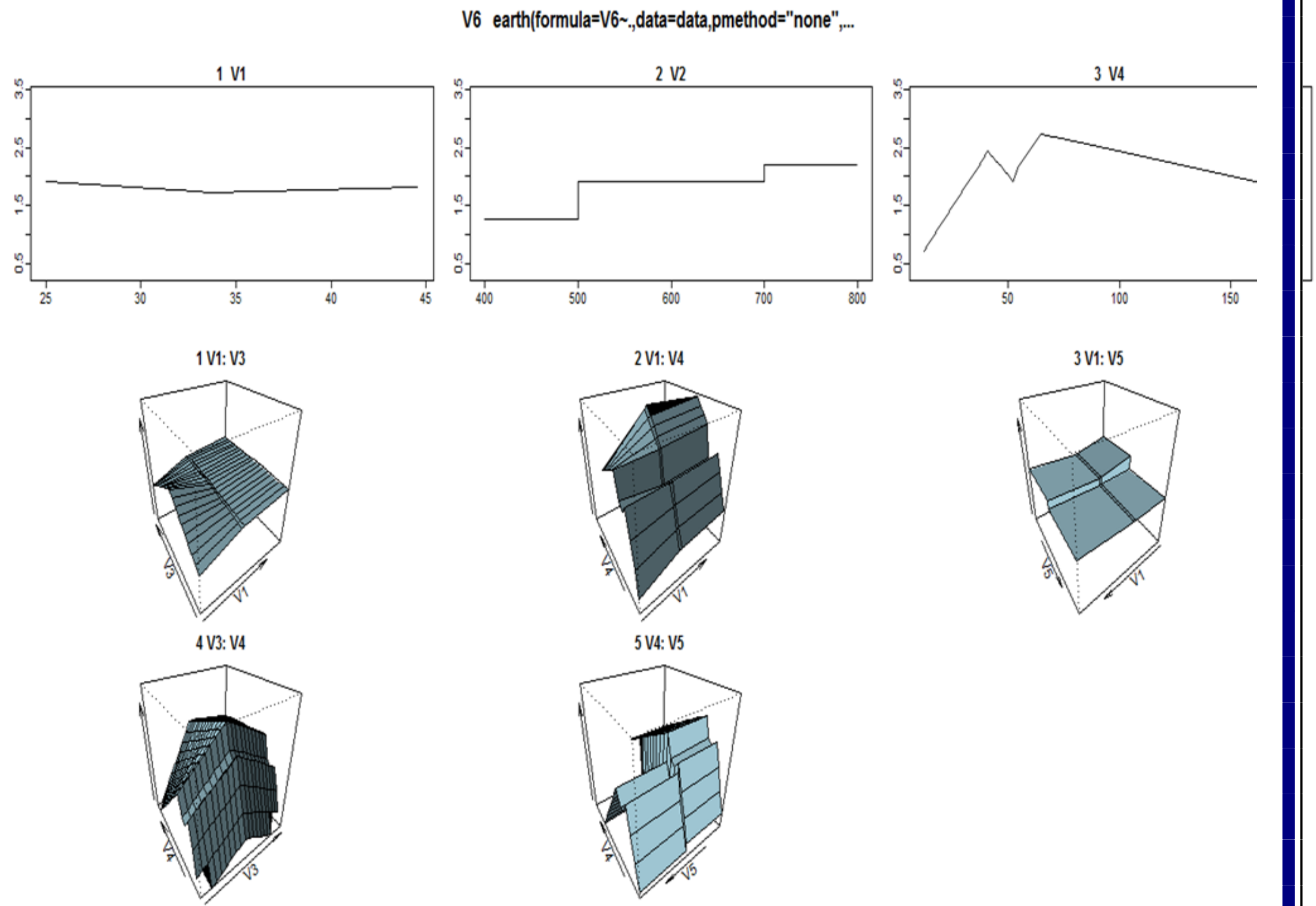


Fig 4.2: Variation graphs of $Q_m \sim$ D, L, e and C_u respectively.

Table 4.4: Variables and their importance in the MARS model.

	nsubsets	gcv	rss
V4	16	78.9	100
V2	15	-69.3	69.8
V1	11	-95.7	42.1
V5	10	-97.8	36.8
V3	9	-100	30.7

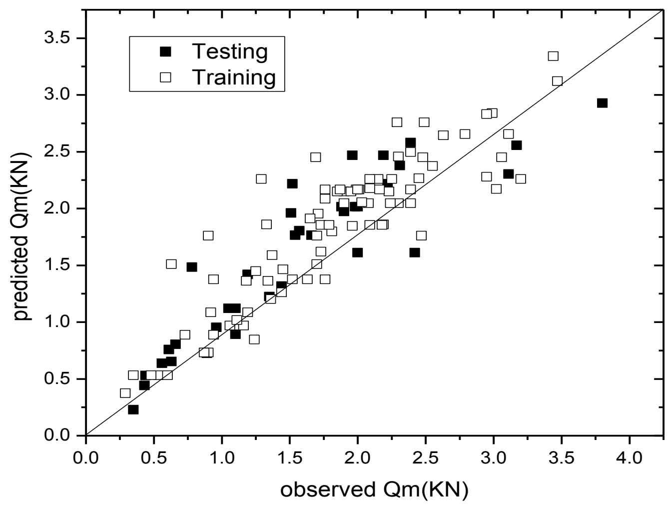


Fig4.3: Comparisons of predicted and measured load capacity of piles by MARS for training data.

As discussed in the methodology, the performance of GP model depends upon the population size, number of generation, reproduction, crossover and mutation probability, tree depth (d_{max}) and the number of genes (G_{max}). In the present study, the best $Q_p$model was obtained with population size of 1000 individuals at 100 generations with reproduction probability of 0.05, crossover probability of 0.85, mutation probability of 0.1 and with tournament selection (tournament size of 2). The optimum result was obtained with G_{max} as 4 and d_{max} as 4.

The developed model is presented below as Equation(4.1)

$$Q_u = 0.003999 \times L - \frac{0.03167 \times L}{f_c} + \frac{1.486 \times L \times {f_c}^2}{10^7} - \frac{\frac{2.084 \times L \times {f_c}^3}{10^8}}{D_{eq}} - \frac{0.00067 \times L^2 \times I}{{D_{eq}}^2} + 0.1671 \tag{4.1}$$

I=Installation technique

Figure 4.4 shows the variation of predicted with that of measured pull out capacity value . It can be seen that data points are close to line of equality for both training and testing data. This shows the performance of the GP model for the prediction of pull out capacity. Table 4.5 shows different statistical criteria for training and testing data of the GP model and the results have been compared with the ANN and MARS model. Fig. 4.5 and 4.6 shows the comparison of errors of MGGP, MARS and ANN models for prediction of pull out capacity for training and testing data.

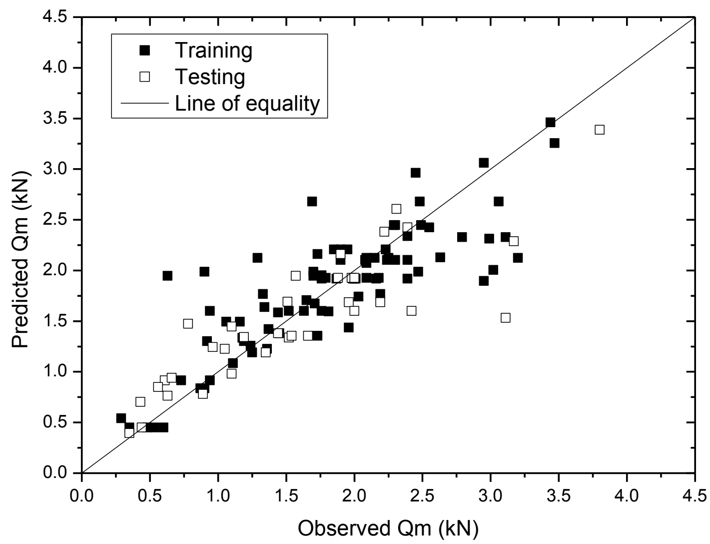


Fig- 4.4: Comparisons of predicted and measured load capacity of pile by GP for training and testing data.

Table-4.5: Statistical performance

Models		Statistical Performances				
		R	*E*	*AAE*	*MAE*	*RMSE*
MARS	Training	0.887	0.787	0.113	0.942	0.336
	Testing	0.899	0.808	0.129	0.761	0.36
ANN	Training	0.805	0.642	0.189	1.392	0.435
	Testing	0.866	0.738	0.177	1.537	0.42
GP	Training	0.832	0.693	0.280	1.316	0.403
	Testing	0.864	0.735	0.299	1.578	0.423

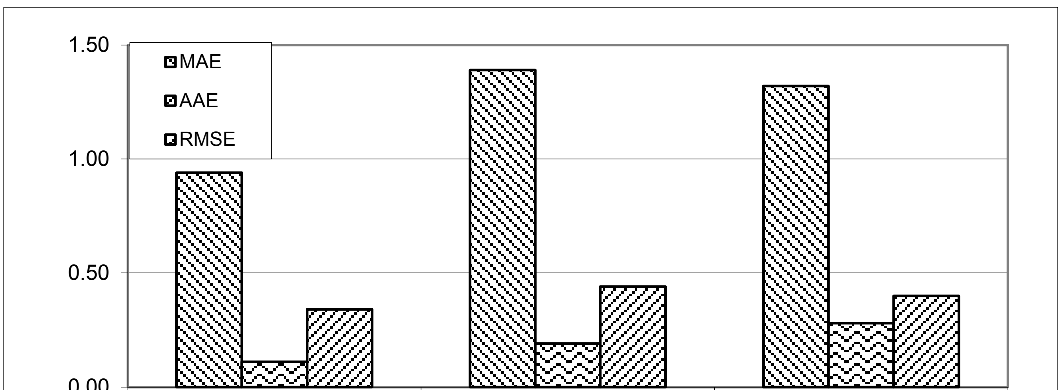
R- Correlation coefficient

E-Coefficient of efficiency

AAE-Average absolute error

MAE-Maximum absolute error

RMSE-Root mean square error



	MARS	ANN	GP
MAE	0.94	1.39	1.32
AAE	0.11	0.19	0.28
RMSE	0.34	0.44	0.4

Fig-4.5: Comparison of errors of MGGP, MARS and ANN models for prediction of pull out capacity for training data

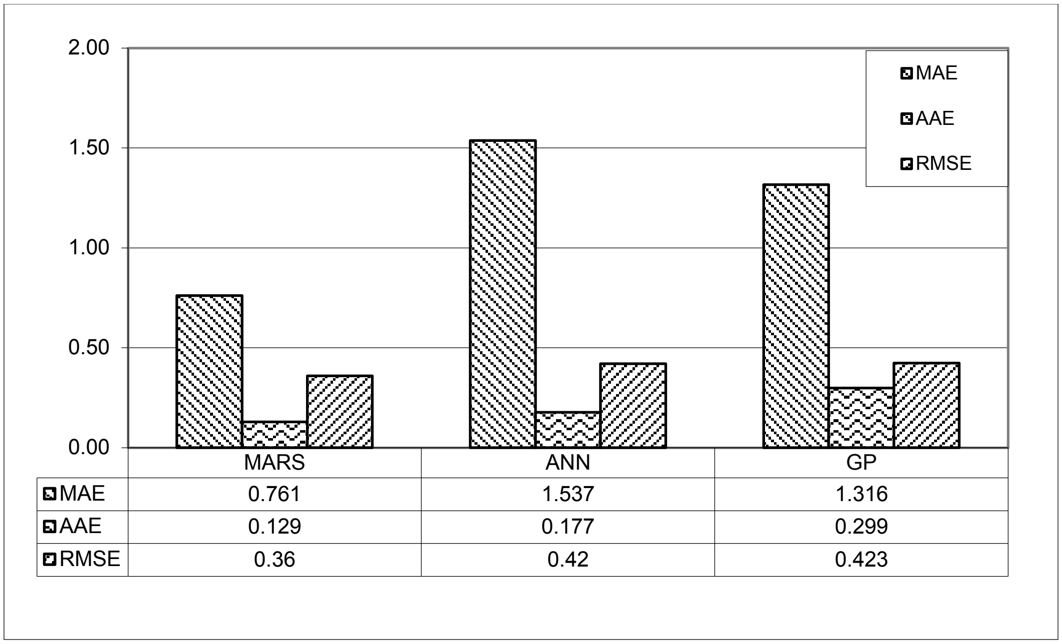


Fig-4.6: Comparison of errors of MGGP, MARS and ANN models for prediction of pull out capacity for testing data

The rank index (RI) proposed by Abu-Farsakh and Titi (2004) is used to evaluate and rank the CPT –based methods and the ANN model ,used in this study .

$$RI=R_1+R_2+R_3+R_4$$

R_1, R_2 ,R_3 ,the rank criterion, is described as follows and are given in table 4.6 for each pullout prediction method used in the present work. A low value of RI indicates the optimal performance of a pullout prediction method.

Regression analysis is carried out to obtain the best fit-line of Q_p/Q_u of the available 119 anchor tests for each pullout capacity prediction method. The relationship of the best fit line of $Q_{fit}/Q_u$and the corresponding coefficient of correlation, R, are calculated.The first criterion(R_1) depends on the better performance that is indicated by the prediction method that has both the ratio Q_{fit}/Q_uandR closer to unity.The results are shown in table 4.6 and from this it is found that

R_1 of the GP model is 2 and hence, it ranks second , ANN model ranks third and for MARS model it is 1.

For the second criterion (R_2) arithmetic mean value,μ, and the corresponding standard deviation,σ, of Q_p/Q_u is calculated. Based on this criterion, optimal performance is obtained when μ (Q_p/Q_u) approaches unity with $\sigma(Q_p/Q_u)$ approaching zero. The results in the table shows that MARS model ranks first, GP model ranks second and ANN model ranks third.

For, the third criterion (R_3) , the ratios of Q_p/Q_u is sorted in ascending order and the cumulative probability is calculated as follows.(Long and Wysockey 1999):

$$P=i/(n+1)$$

Where i is the order number given for the considered ratio, and n is the number of anchors. Then the 50% and 90% cumulative probabilities (i.e P_{50} and P_{90}) of $Q_p/Q_u$are then obtained. Based on this criterion , optimal performance is indicated by values of P_{50} and P_{90} approaching unity. From the results shown in the table it is found that MARS model ranks first, GP model ranks second and ANN model ranks last.

For, the fourth criterion (R_4); the histogram and logarithm normal distributions of the ratio of Q_p/Q_u are plotted. The probability of predicting the pullout capacity within ±20% accuracy is obtained by calculating the difference of CDF value at $1.2Q_u$ and $0.8Q_u$. Based on this criterion the higher the probability of ±20% accuracy , the better the performance of the prediction method. The histogram and logarithm- normal distributions of the methods used are shown in fig. 4.8 and 4.7 and the corresponding probabilities and the rank of the ±20% accuracy are given in table. Results in the table shows that the MARS model ranks first.

The results of the overall rank , RI is calculated by the above mentioned formula and shown in table 3.6 . According to the evaluation criteria it is found that the MARS model performs best than the GP and ANN model.

Table 4.6: Evaluation of performance of different prediction models considered in this study

Pile Capacity methods	Best fit calculations				Arithmetic calculations of Q_p/Q_m			Cumulative probability			± 20% Accuracy (%)			Overall rank	
		R	E	R1	μ	σ	R2	Q_p/Q_m at P_{50}	Q_p/Q_m at P_{90}	R3	Log-normal	Histo-gram	R4	RI	Final rank
MARS	Training	0.887	0.787	1	1.043	0.258	1	1.0034	1.2898	1	0.66	0.79	1	4	1
	Testing	0.899	0.808		1.048	0.236		1.0352	1.2984		0.64	0.67			
ANN	Training	0.805	0.642	3	1.130	0.387	3	1.0428	1.597	3	0.51	0.68	3	12	3
	Testing	0.866	0.738		1.121	0.343		1.0857	1.8636		0.49	0.5			
GP	Training	0.832	0.693	2	1.065	0.338	2	0.987	1.4	2	0.58	0.6	2	8	2
	Testing	0.864	0.735		1.065	0.288		1.023	1.5		0.56	0.6			

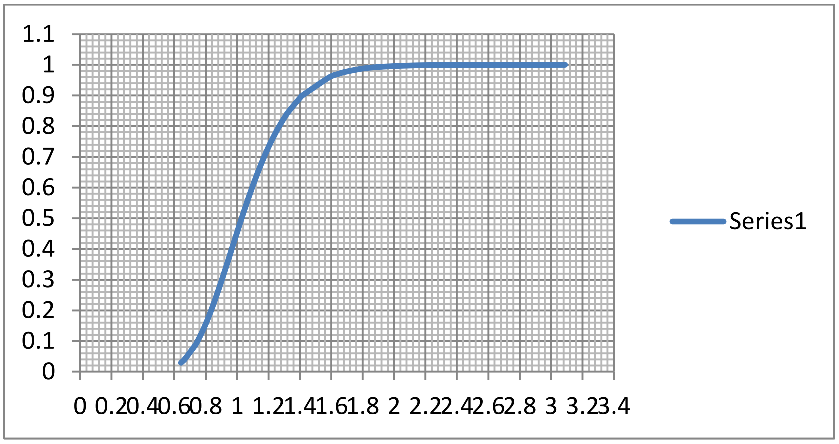


Fig-4.7a: Log normal graph for training dataset(MGGP)

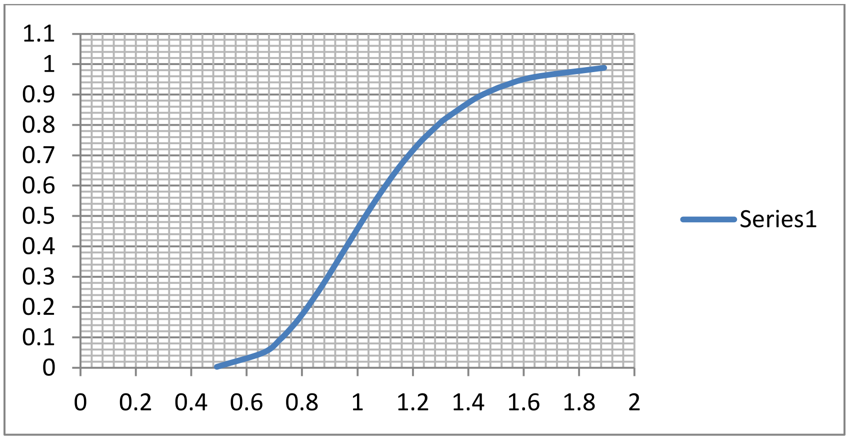


Fig-4.7b: Log normal graph for testing dataset(MGGP)

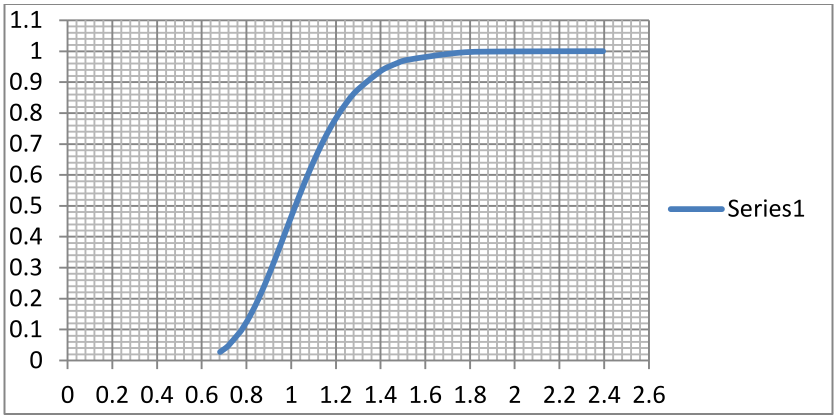


Fig-4.7c:Log normal graph for training dataset(MARS)

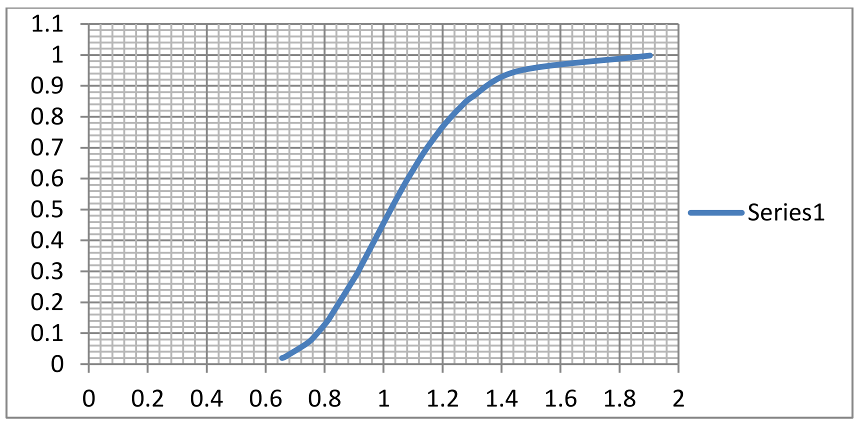


Fig-4.7d: Log normal graph for testing dataset(MARS)

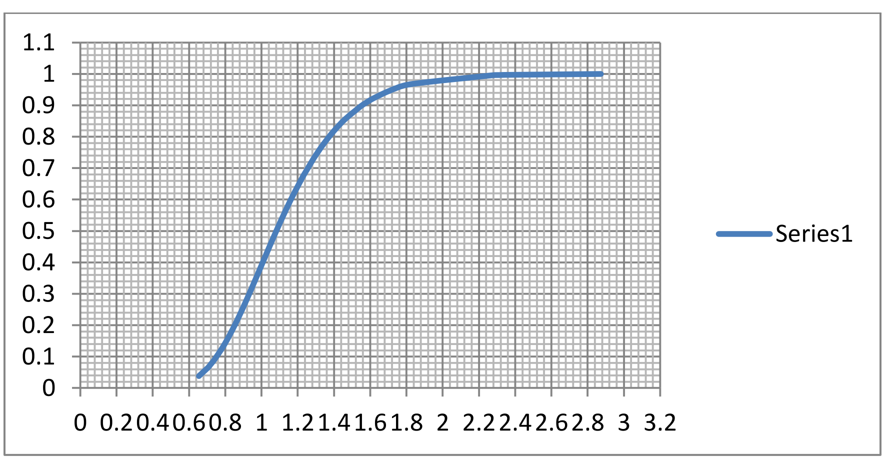


Fig-4.7e: Log normal graph for training dataset(ANN)

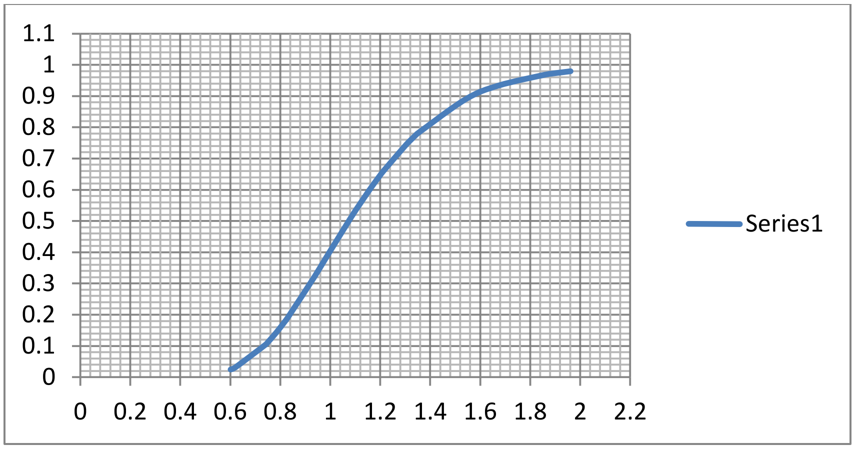


Fig-4.7.f: Log normal graph for testing dataset(ANN)

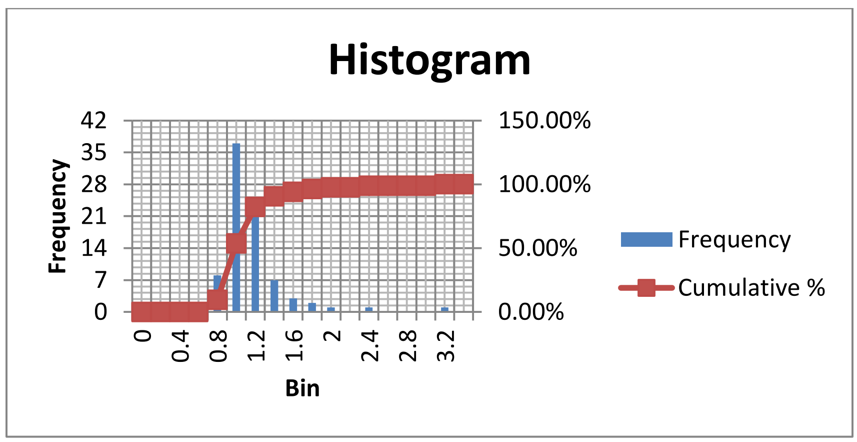


Fig-4.8.a: Histogram graph for training dataset(GP)

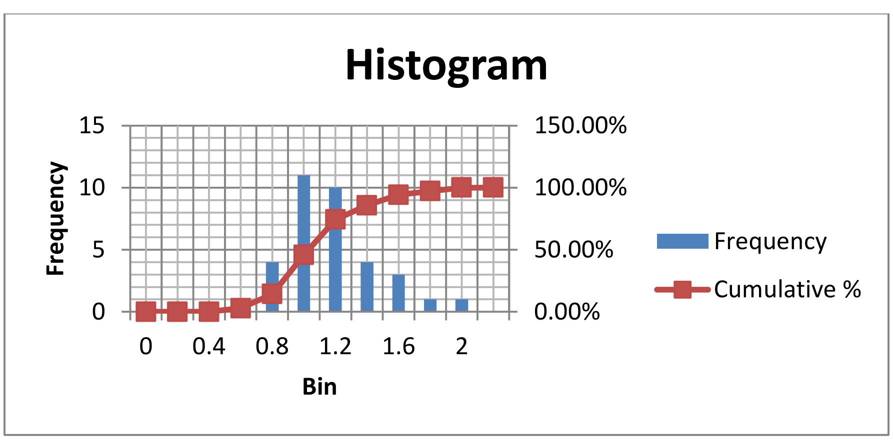


Fig-4.8.b: Histogram graph for testing dataset(GP)

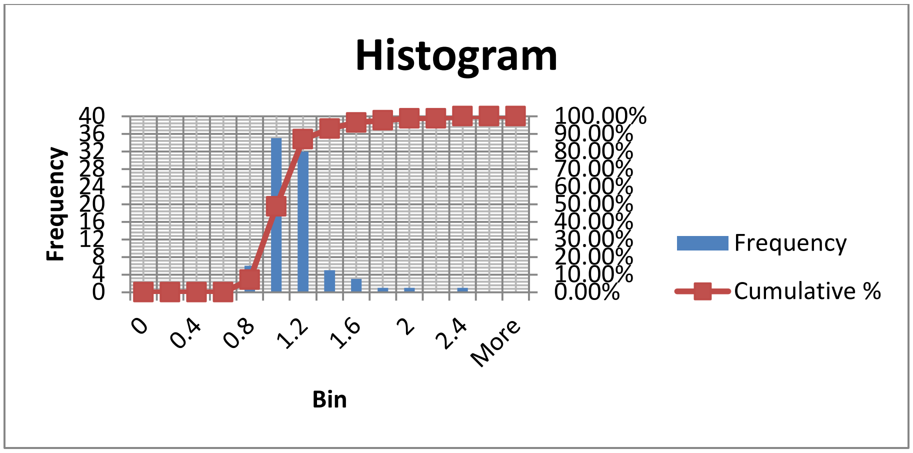


Fig-4.8.c: Histogram graph for training dataset(MARS)

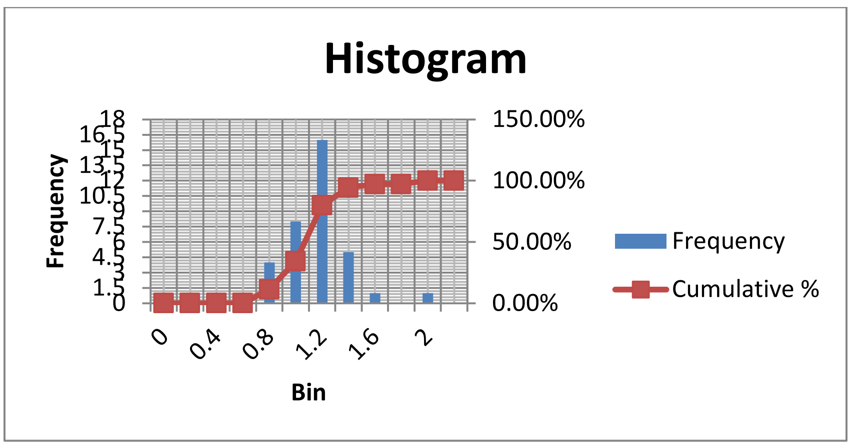


Fig-4.8.d: Histogram graph for testing dataset(MARS)

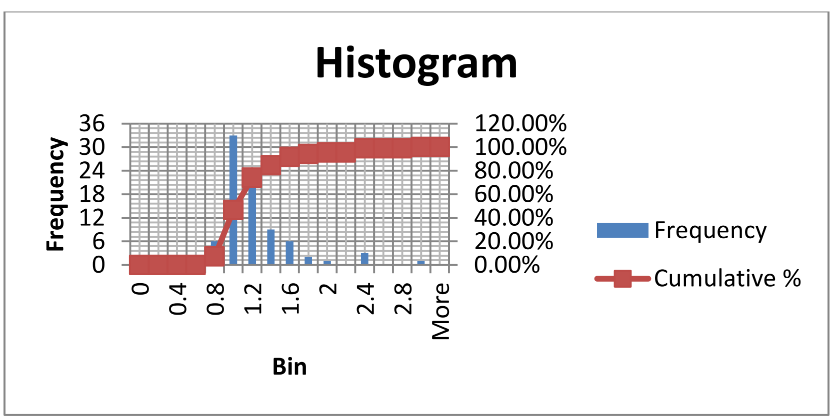


Fig-4.8.e: Histogram graph for training dataset(ANN)

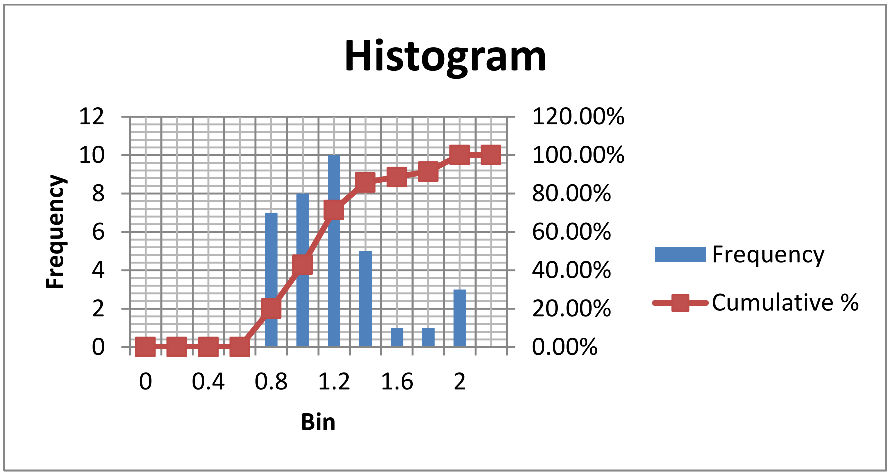


Fig-4.8.f: Histogram graph for testing dataset(ANN)

CHAPTER 5

PREDICTION OF FACTOR OF SAFETY OF SLOPE STABILITY ANALYSIS USING GP AND MARS

5.1 INTRODUCTION

The stability analysis of natural and man-made slopes is one of the most important studies of Geotechnical engineering. The limit equilibrium method (LEM) is the most common method of slope stability analysis due to ease of calculation with accuracy comparable to rigorous methods like finite element, finite difference and variational approach. Though, the limit equilibrium method is the most widely used methods for the slope stability analysis, statistical methods also have been investigated for the slope stability analysis. Sha *et al*. (1994) initiated the application of statistical method in the prediction of factor of safety in slope stability analysis considering some case studies. They proposed separate regression equations for circular and wedge failure surface of considering 46 case studies (29 failed and 17 stable) for circular slope failure and 14 cases (8 failed and 6 stable) of wedge failure slopes using maximum likelihood method. It was observed that the results of regression model and FOS obtained using LEM have strong correlation value with a correlation coefficient (R) varying from 0.911 (circular slip surface) to 0.954 (wedge failure surface). However, the results have not been verified using a new set of data (testing data set). Sakellariou and Ferentinou (2005) used back propagation neural network (ANN) to predict the FOS and compared the results in terms of MSE with different number of training data set. Samui and Kumar (2006) used ANN as an alternate statistical method to upper bound limit analysis to predict the stability number of layered slopes. Wang *et al*. (2005) used BPNN to predict the factor of safety of Yudonghe landslide (China) and found that the FOS is close to 1.1 using a four-layer BPNN model with five input nodes, two hidden layers, and two output nodes. With 26 data points as the input data it is quite possible of overfitting (Das and Basudhar 2006). Samui (2008) used support vector machine (SVM) to classify the soil as stable (1) and failed (0) and to predict the FOS for the data set used by Sah*et al*. (1994) and to new data set (Sakellariou and Ferentinou 2005). The SVM is based on statistical learning theory unlike the ANN, which is biological inspired. The SVM is found to 100% efficient in classifying the slope as stable or failed for the training set, but for testing data set the efficiency varies from 75.57 to 85.71% for different kernel function. Similarly for prediction problem, the correlation coefficient(R) value found to vary from from0.884 to 0.922 for different kernel function. Das et al. (2011) used different types of ANN models like Bayesian regularization neural network (BRNN), Levenberg-Marquardt neural net work (LMNN) and differential evolution neural network (DENN). They observed that DENN is better compared to other ANN and SVM models.

Yang *et al*. (2004) proposed a two stepped algorithm of genetic programming(GP) and GA to propose a statistical equation for the FOS based on parameters unit weight (γ), cohesion (C) and friction angle ($\emptyset$) of soil, height of slope (H), slope angle (β) and pore pressure parameter (r_u). Yang *et al*. (2004) divide the data set as training (40 data) and testing (6 data) set and proposed a model equation to present an equation for the FOS. The results of GP found to be better than that of maximum likelihood estimation of Sah*et al*. (1994). However, the number of data set used for testing set is less and results for testing data are found to be not that efficient like training data set.The most important problem associated with efficient implementation of data driven approach is generalization. The model needs to be equally efficient for new data during testing or validation, which is called as generalization. Recently Gondami and Alavi

(2011, 2012) proposed a variant of GP called multi gene genetic programming (MGGP) and found to efficient to some test problems in structural geotechnical engineering.

However, engineering application of numerical methods is a science as well as an art. Though the developed algorithms are based on scientific logic and belong to the special branch of applied mathematics, their successful application to new problems is problem oriented and is an art. As no method can be the panacea to solve all problems to the last details, their application to new areas needs critical evaluation. With above in view, in the present study two recent modeling techniques, MGGP and MARS are used to develop model equations for the FOS of slope stability problems. Different statistical criteria like correlation coefficient (*R*), Nash-Sutcliff coefficient of efficiency (*E*) (Das and Basudhar, 2008),maximum absolute error (MAE), average absolute error (AAE) and root mean square error (*RMSE*) are used to compare the developed MGGP and MARS models with available ANN and SVMmodels.

In the present study data base available in Shah *et al*. (1994) have been considered.The data base consist of case studies of 23 dry and 23 wet slopes with 29 failed and 17 stable slopes. The input data consist of parameters like height of slope H(m), unit weightγ (kN/m3), cohesion C (kPa), internal friction angle(ϕ^0), slope angle $\beta^0$and pore pressure parametersr_u.The output data baseconsists of quantitative information (factor of safety as per limit equilibrium method). Samui (2008) and Das et al. (2011) have used the above database for development of SVM and ANN model, respectively. Following Das et al. (2011), out of 46 data points, 32 were used for training and 14 data points were used for testing. In the MGGP and MARS modelling the data points are not normalized unlike ANN and SVM model. This is an added advantage of MGGP and MARS techniques over ANN and SVM.

Table 5.1: Data table of prediction ofFoS.(Training data)

γ(kN/m3)	C (kPa)	ϕ^0	β^0	H(m)	r_u	FOS
18.68	26.34	15.00	35.00	8.23	0.00	1.11
18.84	14.36	25.00	20.00	30.50	0.00	1.875
18.00	5.00	30.00	20.00	8.00	0.30	2.05
20.00	20.00	36.00	45.00	50.00	0.50	0.83
28.44	39.23	38.00	35.00	100.00	0.00	1.99
20.60	16.28	26.50	30.00	40.00	0.00	1.25
14.80	0.00	17.00	20.00	50.00	0.00	1.13
26.00	150.05	45.00	50.00	200.00	0.00	1.2
25.00	120.00	45.00	53.00	120.00	0.00	1.3
18.50	25.00	0.00	30.00	6.00	0.00	1.09
18.50	12.00	0.00	30.00	6.00	0.00	0.78
22.40	10.00	35.00	30.00	10.00	0.00	2
21.40	10.00	30.34	30.00	20.00	0.00	1.7
22.00	0.00	36.00	45.00	50.00	0.00	0.89
12.00	0.00	30.00	35.00	4.00	0.00	1.46
12.00	0.00	30.00	45.00	8.00	0.00	0.8
12.00	0.00	30.00	35.00	4.00	0.00	1.44
12.00	0.00	30.00	45.00	8.00	0.00	0.86
23.47	0.00	32.00	37.00	214.00	0.00	1.08
16.00	70.00	20.00	40.00	115.00	0.00	1.11
20.41	24.90	13.00	22.00	10.67	0.35	1.4
21.82	8.62	32.00	28.00	12.80	0.49	1.03
20.41	33.52	11.00	16.00	45.72	0.20	1.28
18.84	15.32	30.00	25.00	10.67	0.38	1.63
21.43	0.00	20.00	20.00	61.00	0.50	1.03
19.06	11.71	28.00	35.00	21.00	0.11	1.09
18.84	14.36	25.00	20.00	30.50	0.45	1.11
21.51	6.94	30.00	31.00	76.81	0.38	1.01
14.00	11.97	26.00	30.00	88.00	0.45	0.625
18.00	24.00	30.15	45.00	20.00	0.12	1.12
23.00	0.00	20.00	20.00	100.00	0.30	1.2
22.40	100.00	45.00	45.00	15.00	0.25	1.8

Table 5.2: Data table of prediction of FoS.(testing data).

γ(kN/m3)	C (kPa)	ϕ^0	β^0	H(m)	r_u	FOS
22.40	10.00	35.00	45.00	10.00	0.40	0.9
20.00	20.00	36.00	45.00	50.00	0.25	0.96
28.44	29.42	35.00	35.00	100.00	0.00	1.78
20.00	0.00	36.00	45.00	50.00	0.25	0.79
20.00	0.00	36.00	45.00	50.00	0.50	0.67
22.00	0.00	40.00	33.00	8.00	0.35	1.45
20.00	0.00	24.50	20.00	8.00	0.35	1.37
18.84	57.46	20.00	20.00	30.50	0.00	2.045
16.50	11.49	0.00	30.00	3.66	0.00	1
14.00	11.97	26.00	30.00	88.00	0.00	1.02
22.00	20.00	36.00	45.00	50.00	0.00	1.02
19.63	11.97	20.00	22.00	12.19	0.41	1.35
18.84	0.00	20.00	20.00	7.62	0.45	1.05
24.00	0.00	40.00	33.00	8.00	0.30	1.58

MGGP modelling results

As discussed in the methodology, the performance of the GP model depends upon the population size, number of generations, reproduction, crossover and mutation probability, tree depth (d_{max}) and the number of genes (G_{max}). In the present study, the best FOS_pmodel was obtained with population size of 1000 individuals at 100 generations with reproduction probability of 0.05, crossover probability of 0.85, mutation probability of 0.1 and with tournament selection (tournament size of 2). The optimum result was obtained with G_{max} as 3 and d_{max} as 4.

The developed model is presented below as Equation (5.1)

$$FOS = 0.3931 \times \phi - 0.039 \times \beta + \frac{0.0786 \times \gamma}{H} + \frac{0.6577 \times c}{c + H \times \exp(r_u)} - \frac{3.894 \times \phi \times (\tanh(r_u))^2}{\gamma + \beta} + 1.326 \quad (5.1)$$

The variation of predicted FOS as per MGGP model with at obtaining using LEM is present in Figure 5.1 along with results of different ANN models as per Das et al. (2011|). It can be seen that the results are comparable in terms of scatterness in data. In terms of correlation coefficient (|R) value for a new set of data (testing data), MGGP model is found to better than SVM and LMNN and BRNN models. The model equation as per MGGP is also very compact in comparison to that ANN model given by Das et al. (2011). It is also well known that R is a biased estimate (Das and Sivakugan 2010), hence the results are also compared in terms of Nash-Sutcliff coefficient of efficiency (*E*) (Das and Basudhar, 2008) and shown in Table 5.3.It can be seen that MGGP model has a better generalization in terms of close E values for training and testing data. The E value for MGGP model is also found to be better than BRNN and DENN models. The MGGP model is also compared in terms of other statistical criteria like maximum absolute error (MAE), average absolute error (AAE) and root mean square error (*RMSE*) and the results are presented in Figure 5.2 and Figure 5.3 for training and testing data, respectively. It can be seen that the results of MGGP are better or comparable to that of ANN models.

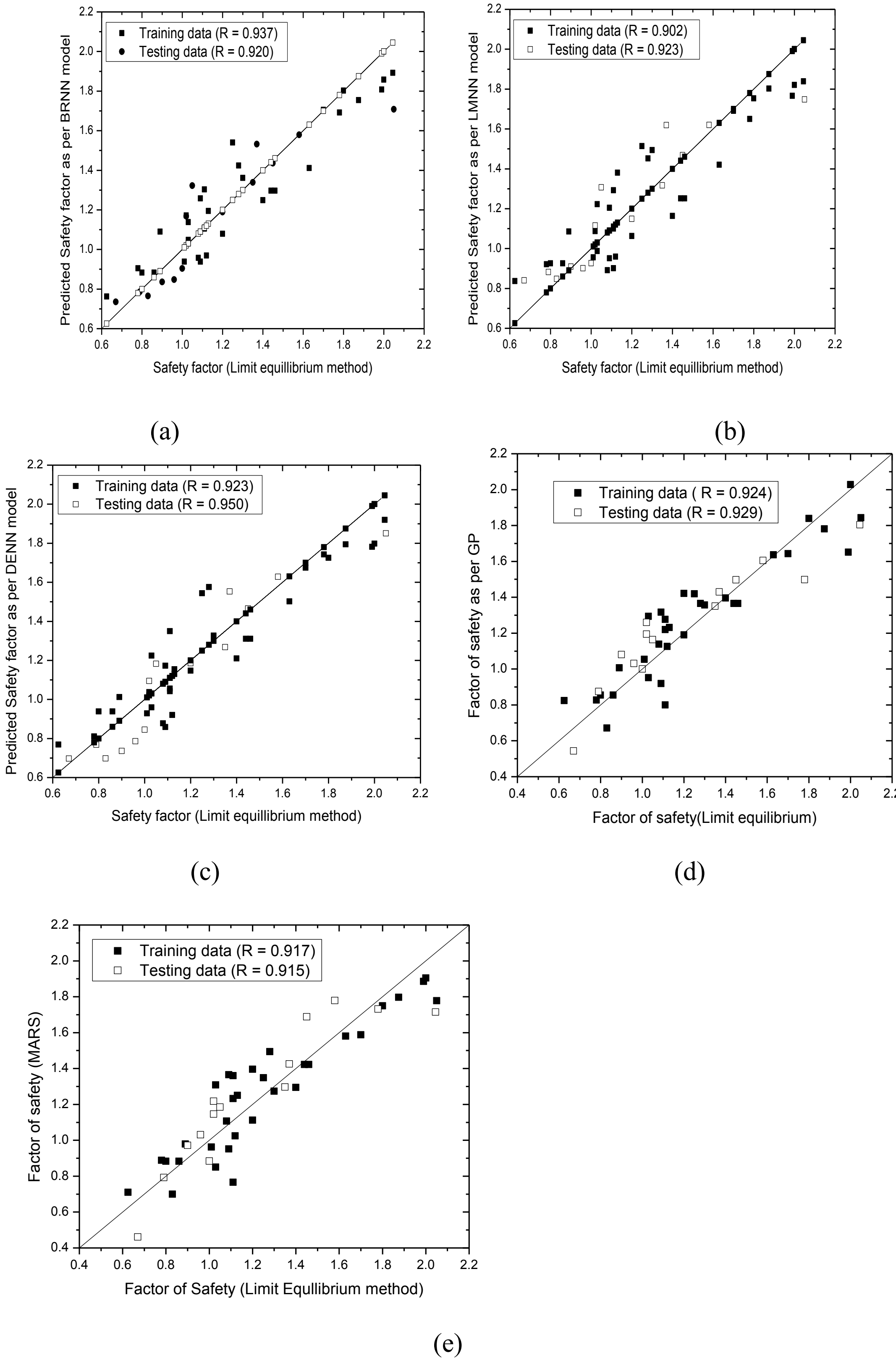
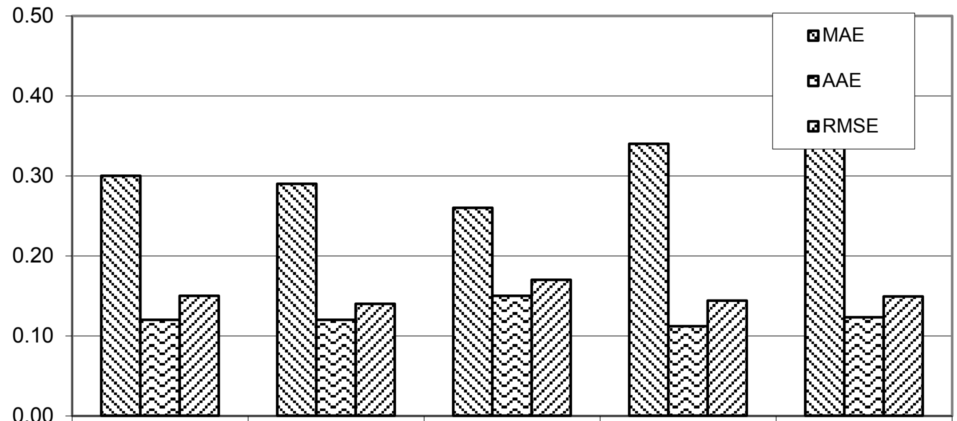
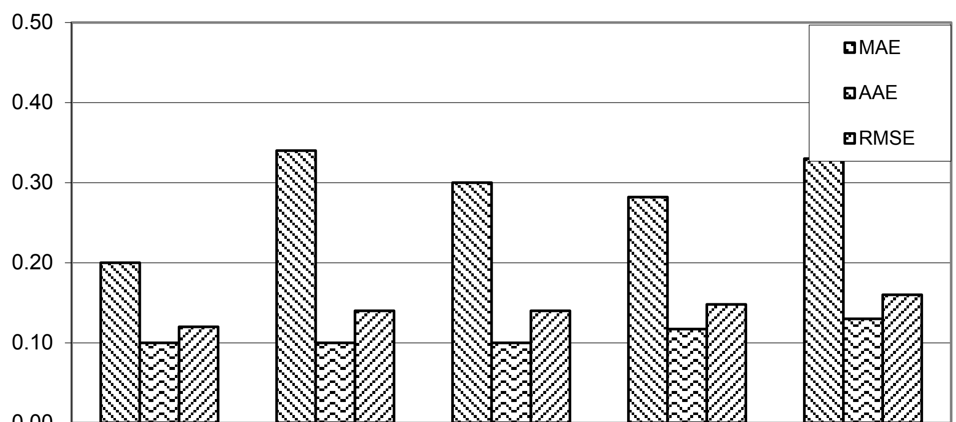


Fig-5.1: Performance of different models for prediction of FOS as per (a) BRNN (b) LMNN, (c) DENN, (d) MGGP and (e) MARS



	DENN	BRNN	LMNN	MGGP	MARS
MAE	0.30	0.29	0.26	0.34	0.34
AAE	0.12	0.12	0.15	0.11	0.12
RMSE	0.15	0.14	0.17	0.14	0.15

Fig-5.2: Comparison of errors of MGGP, MARS and ANN models for prediction of FOS for training data



	DENN	BRNN	LMNN	MGGP	MARS
MAE	0.20	0.34	0.30	0.28	0.33
AAE	0.10	0.10	0.10	0.12	0.13
RMSE	0.12	0.14	0.14	0.15	0.16

Fig- 5.3: Comparison of errors of MGGP, MARS and ANN models for prediction of FOS for testing data

MARS modelling results

The best MARS model has been developed with a six basis functions after several trials with different number of basis functions. Each set of basis functions was used to predict the factor of safety (F_p) and their correlation coefficient (R) was calculated. The performance of MARS model depends upon the number of basis function used. As the number of basis function increased, performances increases but at the same time complexity of model also increased. Hence, tradeoff is to be made, keeping this in mind in the present study best MARS model was obtained with six basis function.

The coefficients of different basis functions produced for the developed MARS model can be written using the obtained coefficients and basis functions as presented in Eq.5.2 as follows:

$$\begin{aligned} FOS = 0.57 &+ 0.012 \times h(\gamma - 21.4) - 0.006 \times h(21.4 - \gamma) + 0.000097 \times h(c - 16.28) - 0.015 \\ &\times h(16.28 - c) + 0.023 \times \phi + 0.028 \times h(\phi - 26.5) - 0.01 \times h(\beta - 30) + 0.046 \\ &\times h(30 - \beta) - 0.041 \times h(\beta - 35) - 0.0005 \times h(H - 50) + 0.0065 \times h(50 - H) \\ &- 1.32 \times h(r_u - 0.2) + 0.524 \times h(0.2 - r_u) \qquad (5.2) \end{aligned}$$

Similar to MGGP model, the performance of MARS model was also compared with ANN, SVM and MGGP model. As shown in Figure 5.1, the close values of R for training and testing data show good generalization of the model. However, the R value is less compared to that of SVM models but less efficient compared to ANN and MGGP model. As per Table 5.3, in terms of Nash-Sutcliff coefficient of efficiency (E) the MARS model is found to less efficient compared to that of MGGP model. Similarly in terms of MAE, AAE and RMSE, MGGP model is also found to efficient in comparison to MARS model.

The developed MGGP and MARS models are also compared in terms of cumulative probabilities of F_p/F_u at 50% and 90% (i.e P_{50} and P_{90}) as per Das and Basudhar (2006). The ratios of predicted FOS (F_p) as per developed model (MGGP and MARS) and FOS (F_u) as per Limit equilibrium method is sorted in ascending order and the cumulative probability is calculated as following Equation- 5.3(Das and Basudhar 2006):

$$P=i/(n+1) \tag{5.3}$$

Where,*i*is the order number given for the considered ratio, and n is the number of anchors. Then the 50% and 90% cumulative probabilities (i.e P_{50} and P_{90}) of F_p/F_u are then obtained which is shown in the Table 5.4.

Based on this criterion, optimal performance is indicated by values of P_{50} and P_{90} approaching unity. From the below table it is found that MGGP model outperforms both ANN and MARS model. It can be seen that based on P_{50} and P_{90}values for both training and testing data MGGP model is found to more efficient compared to ANN and MARS models.

Table 5.3: Statistical performance of ANN, SVM, MGGP and MARS model

Reference	**Models**	**Coefficient of Correlation (R)**		**Coefficient of efficiency (E)**	
		Training	**Testing**	**Training**	**Testing**
ANN (Das et al. 2011)	BRNN	0.937	0.920	0.871	0.885
	LMNN	0.902	0.923	0.807	0.846
	DENN	0.922	0.950	0.848	0.842
SVM (Samui 2008)	SVM-G	0.992	0.922	-	-
	SVM-P	0.983	0.844	-	-
	SVM-S	0.995	0.918	-	-
MGGP Model	Present study	0.924	0.929	0.852	0.851
MARS Model	Present study	0.917	0.915	0.842	0.825

Table 5.4: Cumulative probabilities depending on sorted Fp/Fu for ANN, SVM, MGGP and MARS models

	P_{50}		P_{90}	
	Training	Testing	Training	Testing
LMNN	0.970	1.016	1.220	1.250
BRNN	0.990	0.991	1.160	1.202
DENN	0.957	0.979	1.230	1.129
MGGP	1.020	1.040	1.200	1.220
MARS	0.976	1.055	1.220	1.179

CHAPTER 6

PREDICTION OF THE ULTIMATE BEARING CAPACITY OF SHALLOW FOUNDATIONS USING GP AND MARS

6.1 INTRODUCTION

The lowest part of a structure that transmits its weight to the underlying soil or rockis the foundation. Foundations can be classified into two major types—*shallow foundations* and *deep foundations*. Individual footings, square or rectangular in plan, that support columns and strip footings that support walls and other similar structures are generally referred to as shallow foundations. Shallow foundations have become an cost-effective (and sometimes the only practical)alternative to deep pile foundations (Barari and Ibsen, 2012).For determining the bearing capacity of shallow foundations, analytical methods has drawn a great deal of consideration following initial work by Terzaghi. There has also been considerable recent interest in the development of innovative solutions for shallow foundations based on experimental, numerical, and soft computing techniques. Because of the uncertain nature of soils and the difficulties inherent in laboratory and *in situ* testing, there has been an increasing trend toward development of bearing capacity prediction methods using nontraditional computing techniques to develop accuracy. The great complexity encountered in geotechnical engineering such as slope stability, liquefaction, and shallow foundation and pile capacity prediction have inspired researchers to employ powerful new optimization algorithms and methods.Here GP and MARS method was employed to develop modified expressions for predicting the bearing capacity of shallow foundations founded on granular material.

Terzaghi (1943) was the first to present a theory for evaluating the ultimate bearing capacity of rough shallow foundations. He expressed the ultimate bearing capacity of a strip footing using a semi-empirical equation 6.1

$$q_u = cN_c + qN_q + \frac{1}{2}\gamma B N_\gamma \qquad (6.1)$$

However, in this equation, the shearing resistance along the failure surface in the soil above the bottom of the foundation was not taken into account, and he did not consider rectangular or inclined footings.

The data sets used in this study were obtained from previously published experimental studies (Muhs and Weiß, 1973; Briaud and Gibbens, 1999; Gandhi, 2003) (Table 6.1 and 6.2). The data used for calibrating and validating the model included load tests on full scale models to determine the uniaxial limit states. A series of data sets comprising geometries such as square, rectangular, and strip footings and installed on sands of various densities were separately tested.To enhance the performance of the model 90 datasets are taken as training data and 16 data are taken as testing data.

Table 6.1: Data table of prediction of ultimate bearing capacity.(Training data)

Training					
φ(◦)	γ(kN/m^3)	L/B	D(m)	B(m)	qu(kPa)
32	15.8	1	0.711	0.991	1773.7
32	15.8	1	0.762	3.004	1019.4
32	15.8	1	0.762	2.489	1158
32	15.8	1	0.889	3.016	1161.2
32	13.2	1	0	0.06	14
34	15.7	5.95	0.029	0.0585	58.5
34	15.7	5.95	0.058	0.0585	70.91
34	15.7	6	0.047	0.094	74.7
34	15.7	6	0.094	0.094	91.5
34	15.7	5.95	0.075	0.152	98.2
34	15.7	1	0.047	0.094	67.7
34	15.7	1	0.094	0.094	90.5
34	15.7	1	0.075	0.152	91.2

34	15.7	1	0.15	0.152	124.4
34.9	9.85	2	0.3	0.6	270
37	11.7	1	0	0.5	111
37	11.7	2	0	0.5	143
37	11.7	4	0.029	0.5	109
37	11.7	4	0.127	0.5	187
37	11.7	1	0.3	0.5	446
37	11.7	4	0.3	0.5	322
37	11.7	2	0.5	0.5	565
37	11.7	4	0.5	0.5	425
37	11.77	2	0	0.5	134
37	11.77	1	0.3	0.5	370
37	11.77	2	0.5	0.5	464
37	16.1	5.95	0.029	0.0585	82.5
37	16.1	5.95	0.058	0.0585	98.93
37	16.1	6	0.094	0.094	127.5
37	16.1	5.95	0.075	0.152	143.3
37	16.1	5.95	0.15	0.152	176.4
37	16.1	1	0.094	0.094	131.5
37	16.1	1	0.075	0.152	135.2
37	16.1	1	0.15	0.152	182.4
37.7	10.2	2	0	0.6	200
37.7	10.2	2	0.3	0.6	570
37.7	10.2	1	0	0.5	165
37.7	10.2	3	0	0.5	214
37.7	10.2	3.85	0	0.52	186
37.7	10.2	1	0.3	0.5	681
37.7	10.2	2	0.3	0.5	542
37.7	10.2	3.85	0.3	0.52	413
39	11.97	3	0.2	1	710
39.5	16.5	5.95	0.029	0.0585	121.5
39.5	16.5	5.95	0.058	0.0585	142.9
39.5	16.5	6	0.047	0.094	155.8
39.5	16.5	6	0.094	0.094	185.6
39.5	16.5	5.95	0.075	0.152	211.2
39.5	16.5	5.95	0.15	0.152	254.5
39.5	16.5	1	0.094	0.094	191.6
39.5	16.5	1	0.075	0.152	201.2
39.5	16.5	1	0.15	0.152	264.5
40	12	4	0	0.5	461
40	11.93	3	0	1	630
41.5	16.8	5.95	0.029	0.0585	157.5
41.5	16.8	5.95	0.058	0.0585	184.9
41.5	16.8	6	0.047	0.094	206.8
41.5	16.8	6	0.094	0.094	244.6
41.5	16.8	5.95	0.15	0.152	342.5
41.5	16.8	1	0.047	0.094	196.8

41.5	16.8	1	0.094	0.094	253.6
41.5	16.8	1	0.075	0.152	276.3
41.5	16.8	1	0.15	0.152	361.5
42	12.27	4	0.49	0.5	1492
42	15.89	1	0	0.03	52
42	15.89	1	0	0.04	92
42	14.8	1	0	0.06	72
42	15.4	1	0	0.06	106
42.5	17.1	5.95	0.029	0.0585	180.5
42.5	17.1	5.95	0.058	0.0585	211
42.5	17.1	6	0.047	0.094	235.6
42.5	17.1	5.95	0.075	0.152	335.3
42.5	17.1	5.95	0.15	0.152	400.6
42.5	17.1	1	0.047	0.094	228.8
42.5	17.1	1	0.094	0.094	295.6
42.5	17.1	1	0.15	0.152	423.6
42.8	17.2	1	0	0.08	133
42.8	17.2	1	0	0.15	246
42.8	17.2	1	0	0.05	109
42.8	17.1	1	0	0.08	130
42.8	17.1	1	0	0.15	214
42.8	17.1	1	0	0.2	266
42.8	17.1	1	0	0.25	333
42.8	17.1	1	0	0.3	404
44	12.41	1	0	0.5	782
44	12.41	1	0.3	0.5	2266
44	12.41	2	0.5	0.5	2847
44	12.41	4	0.5	0.5	2033
44.8	10.85	2	0	0.6	860
44.8	10.85	2	0.3	0.6	1760

Table 6.2: Data table of prediction of ultimate bearing capacity.(Testing data)

Testing					
φ(◦)	γ(kN/m^3)	L/B	D(m)	B(m)	qu(kPa)
32	15.8	1	0.762	1.492	1540
34	15.7	5.95	0.15	0.152	122.3
37	11.7	1	0.013	0.5	137
37	11.77	1	0	0.5	123
37.7	10.2	2	0	0.5	203
37.7	10.2	3	0.3	0.5	402
39.5	16.5	1	0.047	0.094	147.8
40	12	4	0.5	0.5	1140
41.5	16.8	5.95	0.075	0.152	285.3
42	15.89	1	0	0.05	95

42.5	17.1	6	0.094	0.094	279.6
42.8	17.1	1	0	0.1	152
44	12.41	4	0	0.5	797
42.5	17.1	1	0.075	0.152	325.3
37	16.1	6	0.047	0.094	104.8
37	16.1	1	0.047	0.094	98.8

MGGP modelling results

The 'best' $Q_p$model was obtained with population size of 1000 individuals and 100 generations with reproduction probability of 0.05, crossover probability of 0.85, mutation probability of 0.1 and with tournament selection seven. In GP model development it is important to make a tradeoff between accuracy in prediction of Q_p and complexity of the model equation which is achieved by proper selection of number of genes and depth of GP tree. In this study optimum result was obtained with maximum number of genes as four and maximum depth of GP tree as three. The developed GP model can be described as Equation (6.2) and shown below.

GP eqn.

$$q_u = 21.26 \times \emptyset - 42.52 \times \gamma - 56800 \times D + \frac{56800 \times D}{\emptyset} + \frac{3.652 \times 10^7 \times D}{\emptyset \times (\emptyset - D)} + 21.26 \times \emptyset \times D \times (\emptyset - B) - 43.31 \quad (6.2)$$

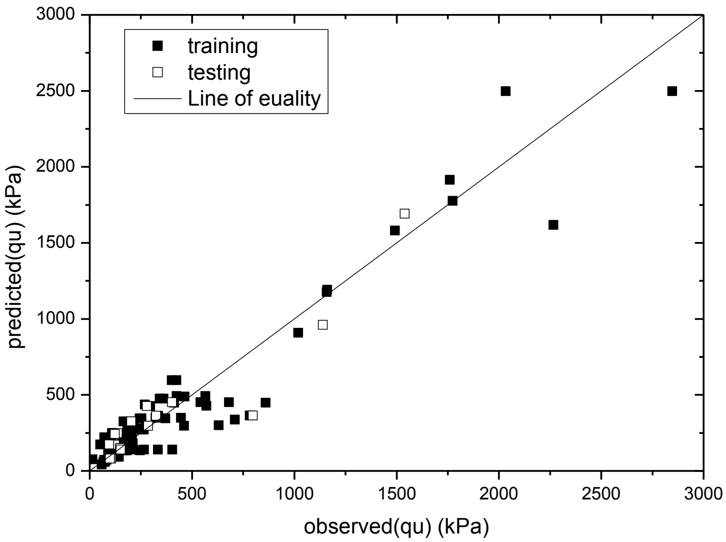


Fig-6.1: Comparisons of predicted and measured load capacity of piles by GP

MARS modelling result

The best MARS model has been developed with a fourteen basis functions after several trials with different number of basis functions. Each set of basis functions were used to predict the factor of safety (F_p) and their correlation coefficient (R) was calculated. The performance of MARS model depends upon the number of basis function used. As the number of basis function increased, performances increases but at the same time complexity of model also increased.

Command used for preparing MARS model :

a<-earth(V5~.,data)

Dependent variable was predicted using all the independent variables

Table 6.3: Basis functions and their coefficients for predicting Qu.

	coefficients		
(Intercept)	1100.601		
h(42.8-V1)	-155.731		
h(V4-0.058)	916.9641		
h(0.058-V4)	-1510.74		
h(0.6-V5)	-1669.41		
h(V1-42.8)	*	h(V3-2)	-329.347
h(V1-42.8)	*	h(2-V3)	-457.131
h(V1-37)	*	h(V4-0.058)	242.765
h(37-V1)	*	h(V4-0.058)	404.1557
h(V1-42.5)	*	h(0.058-V4)	-2160.26
h(V1-42.8)	*	h(0.6-V5)	7548.099
h(42.8-V1)	*	h(0.6-V5)	270.7011
h(2-V3)	*	h(V4-0.058)	792.5457
h(V4-0)	*	h(V5-0.6)	-605.941
h(V4-0.047)	*	h(0.6-V5)	-3285.75

<plot(a) gives following types of graphs

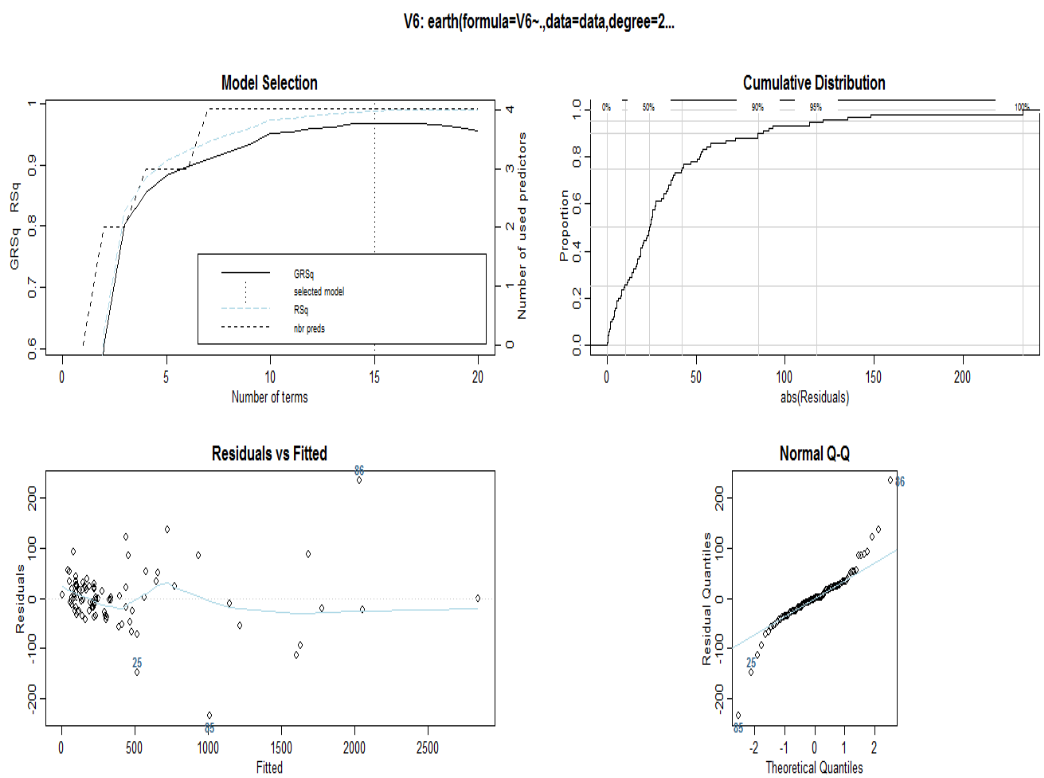


Fig 6.2: Type of model selection graph by ‘EARTH’.

<plotmo(a)

It gives the variation of dependent variable with each independent variable

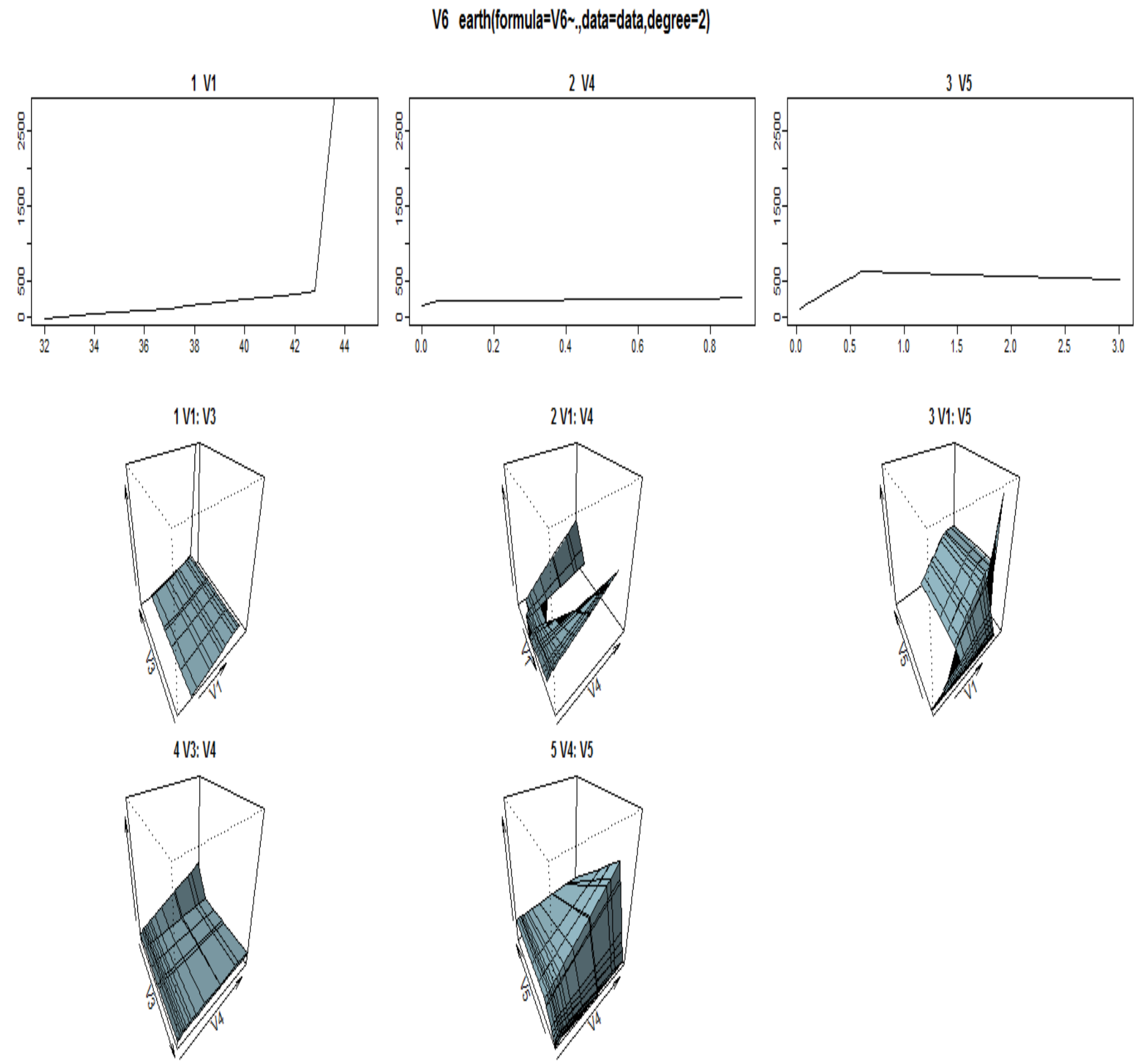


Fig 6.3: Variation graphs of $Q_m \sim D$, L, e and C_u respectively.

Table 6.4: Variables and their importance in the MARS model.

	nsubsets	gcv	rss
V1	14	100	100
V4	14	100	100
V5	12	41.3	40.7
V3	9	27	25.4

MARS equn

$$q_u = 1100.601 - 155.731 \times h(42.8 - \emptyset) + 916.964 \times h(D - 0.058) - 1510.74 \times h(0.058 - D) - 1669.41 \times h(0.6 - B) - 329.347 \times h(\emptyset - 42.8) \times h\left(\frac{L}{B} - 2\right) - 457.131 \times h(\emptyset - 42.8) \times h\left(2 - \frac{L}{B}\right) + 246.765 \times h(\emptyset - 37) \times h(D - 0.054) + 404.1557 \times h(37 - \emptyset) \times h(D - 0.054) - 2160.26 \times h(\emptyset - 42.5) \times h(0.058 - D) + 7548.099 \times h(\emptyset - 42.8) \times h(0.6 - B) + 270.7011 \times h(42.8 - \emptyset) \times h(0.6 - B) + 792.5457 \times h\left(2 - \frac{L}{B}\right) \times h(D - 0.054) - 605.941 \times h(D - 0) \times h(B - 0.6) - 3285.75 \times h(D - 0.04) \times h(0.6 - B)$$

(6.3)

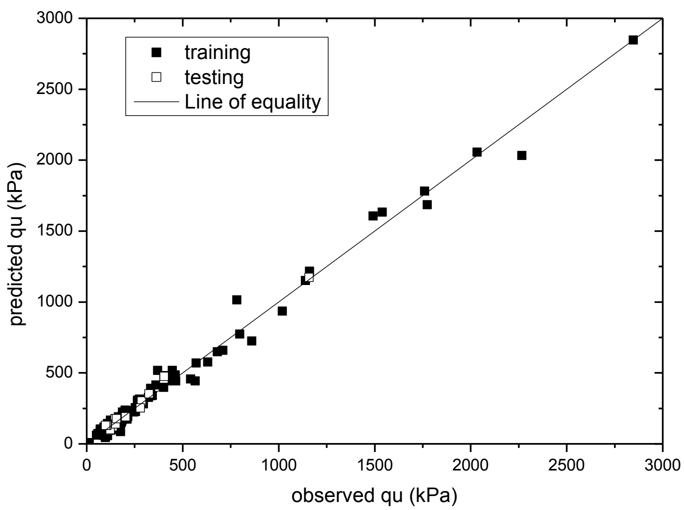


Fig-6.4:Comparisons of predicted and measuredultimate bearing capacity by GP for training and testing data

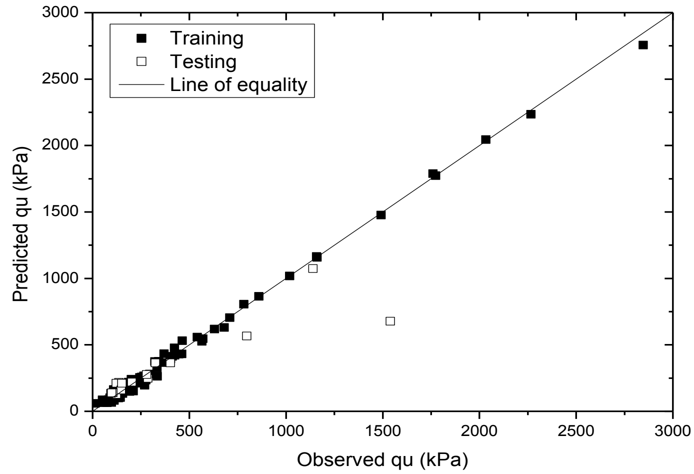


Fig-6.5: Comparisons of predicted and measured ultimate bearing capacity by ANN for training and testing data

Table-6.5: Statistical performance

Models		Statistical Performances				
		R	*E*	*AAE*	*MAE*	*RMSE*
MARS	Training	0.994	0.988	36.05	233.3	55.87
	Testing	0.994	0.985	25.84	70.56	30.54
GP	Training	0.954	0.91	98.93	648.2	151.64
	Testing	0.939	0.88	96.71	432.5	141.28
ANN	Training	0.998	0.996	23.33	90.6	30.74
	Testing	0.888	0.69	102.99	862.1	227.4

The rank index (RI) proposed by Abu-Farsakh and Titi (2004) is used to evaluate and rank the CPT –based methods and the ANN model ,used in this study .

$$RI=R_1+R_2+R_3+R_4$$

R_1, R_2, R_3, R_4 ,the rank criterion, are described in previous chapter and are given in table for each ultimate bearing capacity prediction method used in present work. A low value of RI indicates the optimal performance of a pullout prediction method. So, in this study MARS method performs best of ANN and GP model for prediction of ultimate bearing capacity of shallow foundation as MARS model is having low RI

Table-6.6: Evaluation of performance of different prediction models considered in this study

Bearing Capacity methods	Best fit calculations				Arithmetic calculations of Q_p/Q_m			Cumulative probability			± 20% Accuracy (%)			Overall rank	
		R	E	R1	μ	σ	R2	Q_p/Q_m at P_{50}	Q_p/Q_m at P_{90}	R3	Log-normal	Histo-gram	R4	RI	Final rank
MARS	Training	0.994	0.988	1	0.998	0.186	1	1.002	1.236	1	0.68	0.76	1	4	1
	Testing	0.994	0.985		1.037	0.16		1.033	1.27		0.78	0.72			
GP	Training	0.954	0.91	2	1.185	0.674	3	1.073	1.81	3	0.36	0.42	3	11	3
	Testing	0.939	0.88		1.209	0.435		1.106	1.902		0.37	0.52			
ANN	Training	0.998	0.996	3	1.034	0.379	2	0.997	1.193	2	0.82	0.8	2	9	2
	Testing	0.888	0.69		1.125	0.332		1.08	1.64		0.44	0.48			

CHAPTER 7

GENERAL OBSERVATIONS, CONCLUSIONS, AND SCOPE OF FUTURE STUDIES

7.1 SUMMARY

The application of AI techniques like GP and MARS in different branches of science and engineering discipline is phenomenal. However, the applications of the above techniques in geotechnical engineering are very limited. The primary focus of this research was to explore some applications of the artificial intelligence techniques, GP and MARS in geotechnical engineering.

Based on above study for different geotechnical engineering problems following conclusions can be made

(1) Chapter 3, discussed about lateral load pile capacity penetrating clay. The proposed GP model is found to be effective and efficient than available MARS, ANN (DENN, BRNN), SVM and other statistical models in predicting the lateral load capacity of piles in clay. Using a ranking method based on different statistical criteria (statistical performances for predicted load capacity (Q_p) and measured capacity (Q_m), the mean and standard deviation of the ratio Q_p/Q_m , the cumulative probability for $Q_p/Q_{m.}$ and prediction of load capacity within 20% accuracy level) it has also been found that the developed GP model is more efficient compared to other AI and statistical models.

The developed model equation is found to more compact compared to the MARS and other AI models and can easily be used by the professionals with the help of a spreadsheet without going into the complexity of model development.

(2) In Chapter 4, for the prediction capacity of ground anchors, the MARS model performs best than the ANN and GP model based on evaluation criteria.

(3) The application of MGGP and MARS for prediction of factor of safety of slopes based on available field data bases of slopes is discussed in Chapter 5. The results have been compared with available ANN and SVM model. The model equations as per MGGP and MARS are found to more compact and compressive compared to that of ANN model equation. Based on different statistical criteria like, correlation coefficient, coefficient of efficiency, maximum absolute error, average absolute error, root mean square error and cumulative probability P_{50} and P_{90} values, MGGP model is found to be the best model in comparison to ANN, SVM and MARS models.

(4) Based on different statistical criteria MARS model found to be 'best' model in comparison to GP and ANN model for the prediction of ultimate bearing capacity for shallow foundation which is discussed in Chapter 6.

7.2 SCOPE OF FUTURE STUDIES

Scope of application of GP and MARS to geotechnical engineering problems is very promising and can be applied to a variety of problems related to decision making. Some of the following problems are recognized for further studies.

1. Application of the methods to other geotechnical engineering problems like liquefaction analysis, land slides etc. with real time monitoring using GIS and other data.
2. Application of the above parameter estimation technique to develop limit state function for reliability analysis.
3. Development of sophisticated Geotechnical instruments calibrated using GP / MARS correlations .

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