STUDY OF IMAGE DENOISING USING CURVELET TRANSFORM

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STUDY OF IMAGE DENOISING USING CURVELET TRANSFORM

A Thesis submitted in partial fulfillment of the requirements for the degree

Of

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By

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CERTIFICATE

This is to certify that the thesis entitled "**Image de-noising using curvelet transform**", submitted by **Rajat Singh (Roll No. 109CS0345) and Devendra Singh Meena (Roll No. 109CS0633)** in partial fulfillment of the requirements for the award of **Bachelor of Technology** in **Computer Science and Engineering** during session 2012-2013 at National Institute of Technology, Rourkela. A bonafide record of research work carried out by them under my supervision and guidance.

The candidates have fulfilled all the prescribed requirements.

The thesis which is based on candidates' own work, have not submitted elsewhere for a degree/diploma.

In my opinion, the thesis is of standard required for the award of a bachelor of technology degree in Computer Science and Engineering.

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Dedicated to

Our beloved parents

ABSTRACT

The images usually bring different kinds of noise in the process of receiving, coding and Transmission. In our implementation the Curvelet transform is used for de-noising of image. Two digital implementations of the Curvelet transform the Unequally Spaced Fast Fourier Transform (USFFT) and the Wrapping Algorithm are used to de-noise images degraded by different types of noises such as Gaussian, Salt and Pepper, Random, Speckle and Poisson noise. This thesis aims at the effect the Curvelet transform has in Curve-let shrinkage assuming different types of noise models. A signal to noise ratios a measure of the quality of de-noising was preferred. The experimental results show that the normal Curvelet shrinkage approach fails to remove Poisson noise in medical images.

Keywords

Curvelet transform, wrapping algorithm, USFFT

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CHAPTER **1**

Introduction

Introduction

A very vast portion of digital image processing is concerned with image de-noising. This includes research in algorithm and routine goal oriented image Processing. Image restoration is the removal or reduction of degraded images that are Incurred while the image is being obtained. Degradation comes from blurring as Well as noise due to various sources. Blurring is a form of bandwidth reduction in the image caused by the imperfect image formation process like relative motion between the camera & the object or by an optical system which is out of the focus. When aerial photographs are taken for remote sensing purposes, atmospheric turbulence introduces blurs, optical system aberration and relative motion between camera and the ground. With these blurring effects, the recorded image can also be corrupted by noises. A noise can be introduced in the transmission medium due to a noisy channel, errors during the measurement process and during quantization of the data for digital storage. Each element in the imaging chain such as film, lenses, digitizer, etc. contribute to the degradation. Image de-noising is often used in the field of photography or publishing where an image is somehow degraded but it needs to be improved before it can be printed. For this type of application we need to know about the degradation process in order to design a model for it. When a model for the degradation process is designed, the inverse process can be applied to the image to de-noise it back to its original form. It is the type of image de-noise often used in space exploration to help eliminate artifacts generated by mechanical jitter in a spacecraft or to reduce distortion in the optical system of a telescope. Image de-noising finds applications in fields such as astronomy where the resolution limitations are high, in medical imaging where the physical requirements for high quality imaging are needed for analyzing the images of unique events and in the forensic science where potentially useful photographic information is sometimes of extremely bad quality.

Digital images are 2-D matrices in image processing and important task is to adjust values of these matrices in order to get clear features of images. The adjusting of values obeys a certain mathematical model. The main challenge is to build suitable mathematical models for practical requirements. Taking image de-noising example many mathematical models are based on a frequency partition of the image, where components having high frequency are interpreted as noise which have to be removed while those with low frequency can be seen as features to be remained. Curve-lets, which we are using, can be seen as an effective model that not only considers a multi scale time-frequency local-partition but also makes use of the direction of features.

Applications of wavelets are increasingly being used in scientific and engineering fields, traditional wavelets do well only at representing point singularities, as they ignore the geometric properties of structures and do not exploit the regularity of edges. Thus, de-noising, wavelet based compression or structure extraction become computationally inefficient for geometric features with line and surface singularities. For ex, when we download compressed image or video, we mostly find a mosaic phenomenon. The mosaic phenomenon comes from the poor ability of wavelets to handle line singularities. In fluid mechanics, discrete wavelet thresholding mostly leads to oscillations along edges of the coherent eddies, and to the deterioration of the vortex tube structures, which later can cause an unphysical leak of energy into neighboring scales producing an artificial “cascade" of energy.

CHAPTER **2**

Additive and Multiplicative Noises

Noise is undesired information that degrades the image. In the image de-noising process, information of the type of noise present in the original image plays a significant role. Mostly images can be corrupted with noise modeled with either a uniform, Gaussian, or salt and pepper distribution. Another type of noise is a speckle noise which is multiplicative in nature. Noise is present in image either in an additive or multiplicative form.

Rule for additive noise

$w(x, y) = s(x, y) + n(x, y)$, ……………(1)

Rule for multiplicative noise

$w(x, y) = s(x, y) \times n(x, y)$, ……………(2)

where (x,y) is original signal, $n(x,y)$ is the noise introduced into the signal to produce a noisy image $w(x,y)$, and (x,y) is the pixel location. The above image algebra is done at pixel level. Image addition also has applications in image morphing. Image multiplication means the brightness of the image is varied.

The digital image acquisition process transforms an optical image into a continuous electrical signal that is, sampled. In every step of the process there are fluctuations caused by natural phenomena, adding random value to the exact brightness value for a given pixel.

2.1 Gaussian Noise

A gaussian noise is evenly distributed in the signal. That means every pixel in the noisy image is the sum of the a random Gaussian distributed noise value and true pixel value. This type of noise has a Gaussian distribution, which has a probability distribution function given by,

$$F(g) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(g-m)^2/2\sigma^2}, \qquad \text{……….(3)}$$

Where g represents the gray level, *m* is the mean of the function and σ is standard deviation in the noise. Graphically, it is represented as shown in Figure below.

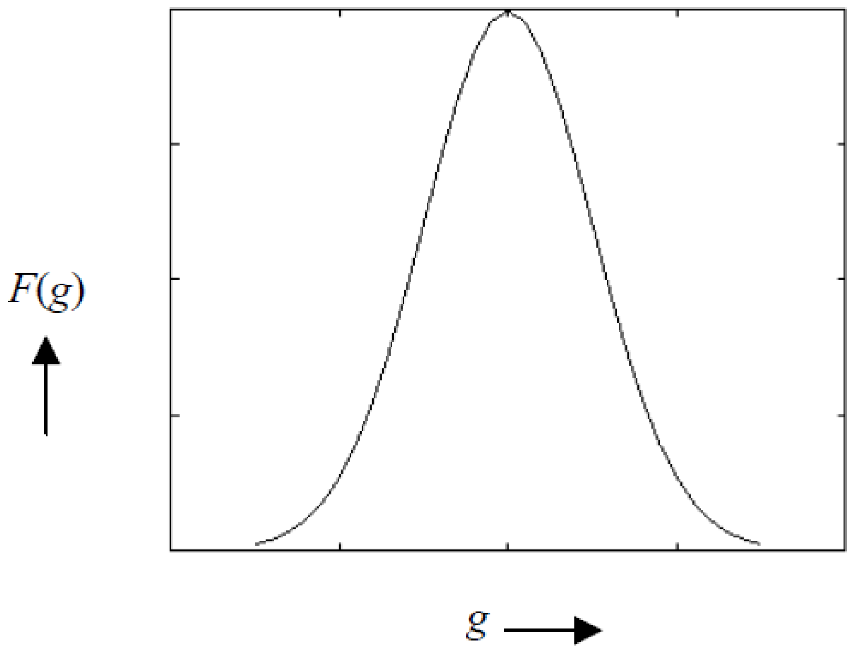
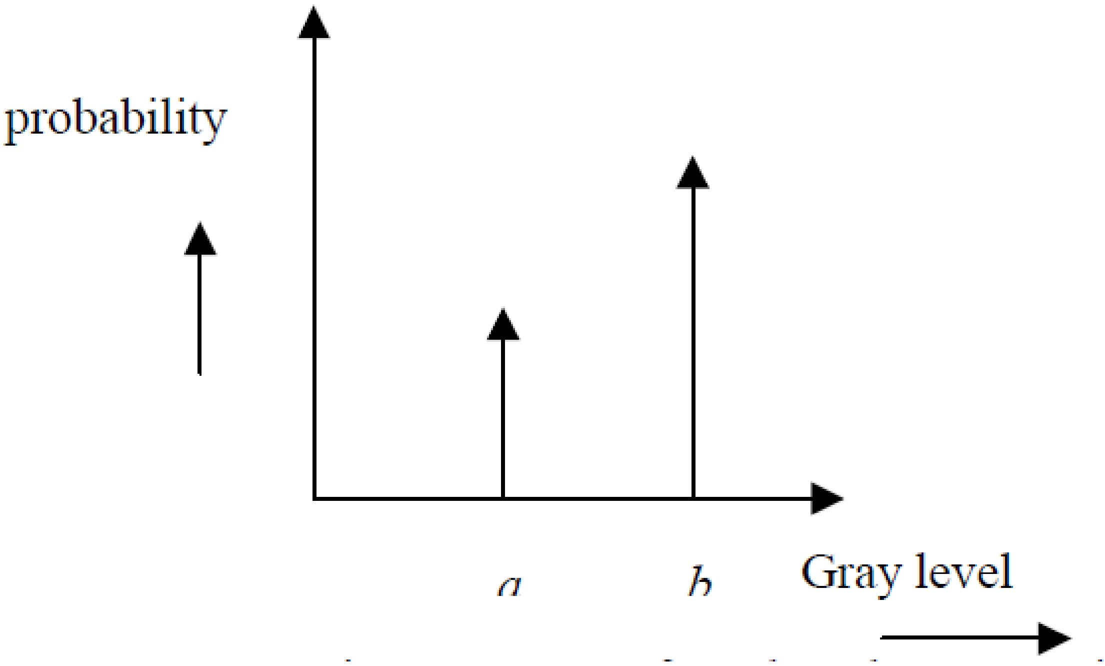


Figure 1 : Gaussian distribution

2.2 Salt and Pepper Noise

Salt and pepper noise is impulse type of noise, that is also referred to as intensity spikes. It is caused generally due to the errors in the data transmission. It has only two possible values that is *a* and *b*. The probability of each is typically less than 0.1. Corrupted pixels can be set alternatively to the minimum or to the maximum value, giving image a “salt and pepper” like appearance. Pixels remain unchanged for unaffected. For a 8-bit image, the value of pepper noise is 0 and for salt noise 255. Salt and pepper noise is mainly caused by malfunctioning of pixel elements in the sensors of cameras, faulty memory locations, or timing errors of the digitization process.

The probability density function for this type of noise is shown in the Figure below.



. **Figure 2: PDF for salt and pepper noise**

2.3 Speckle Noise

Speckle noise is multiplicative noise. This type of noise occurs mostly in all coherent imaging systems such as acoustics, laser, acoustics and SAR (Synthetic Aperture Radar) imagery. Source of this noise is attributed to the random interference between the coherent returns. A fully developed speckle noise has the characteristic of multiplicative noise. Speckle noise follows a gamma distribution and given as

$$F(g) = \frac{g^{\alpha-1}}{(\alpha-1)!a^{\alpha}} e^{-\frac{g}{a}} \quad \text{..................(4)}$$

where variance is $a^2 \alpha$ and g is the gray level.

2.4 Brownian Noise

Brownian noise is under the category of fractal or 1/*f* noises. The mathematical model for 1/*f* noise is the fractional Brownian motion. Brownian motion is non-stationary stochastic process which follows a normal distribution. Brownian noise is a special case of 1/*f* noise. It can be obtained by integrating white noise. It can be graphically represented as shown in Figure below.

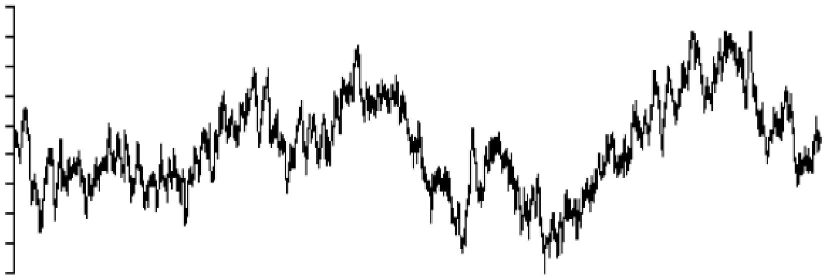


Figure 3 : Brownian noise distribution

2.5 Poisson Noise

Many images like as those from radiography, have noise that satisfies a poisson distribution. Magnitude of Poisson noise varies across an image and it depends on the image intensity, thatmakes removing such noise very hard. Poisson images occur in situations where the image acquisition is performed using the detection of particles (e.g) counting photons being emitted from a radioactive source is applied in medical imaging like SPECT and PET , therefore Poisson noise reduction is an essential problem.Poisson noise is generated from the data in place of adding artificial noise to the data. For ex. if a pixel in an unsigned integer input has the value 10, then corresponding output pixel will be generated from Poisson distribution with a mean 10.

CHAPTER **3**

The Wavelet Transforms and De-noising

3.1. Discrete Wavelet Transform (DWT) - Principles:

Wavelets are the mathematical functions which analyze data according to the scale or resolution. They help in studying a signal in different windows or in different resolutions. For example, if the signal is viewed in the large window, gross feature can be noticed, and if viewed in a small window, only the small features can be noticed. The wavelets provide some advantages over Fourier transforms. For instance, they do a great job in approximating signals with sharp spikes and signals having discontinuities. Wavelets can also model music, speech, video and non-stationary stochastic signals. The wavelets can be used in applications such as turbulence, image compression, human vision, earthquake prediction, etc.

The term "wavelets" is refered to a set of orthonormal basis functions generated by translation and dilation of scaling function ϕ and a mother wavelet ϕ. A finite scale multi resolution representation of a discrete function is called as a discrete wavelet transform. DWT is a fast linear operation on the data vector, whose length is an integeral power of 2. This transform is orthogonal and invertible where the inverse transform expressed as the matrix is the transpose of the transform matrix. The wavelet basis or function, unlike sines and cosines in Fourier transform, is localized in space. Similar to sines and cosines the individual wavelet functions are localized in frequency.

The orthonormal basis or wavelet basis is defined as

$$\psi_{(j,k)}(x) = 2^{j/2}\psi(2^j x - k) \quad \text{...............(5)}$$

The scaling function is given as

$$\phi_{(j,k)}(x) = 2^{j/2}\phi(2^j x - k) \quad \text{............ (6)}$$

Where ϕ is the wavelet function and j and k are integers that scale and dilate the wavelet function. Factor 'j' in Equations is called as the scale index, which indicates the width of the wavelet. The location index k provides the position. The wavelet function is dilated by powers of two and is translated by the integer k. In terms of the wavelet coefficients, the wavelet equation is

$$\psi(x) = \sum_{k}^{N-1} g_k \sqrt{2\phi(2x-k)} \quad \text{............(7)}$$

Where g_0, g_1, g_2.... are high pass wavelet coefficients. The scaling equation in terms of scaling coefficients as given below

$$\phi(x) = \sum_{k}^{N-1} h_k \sqrt{2\phi(2x-k)} \quad \text{............(8)}$$

The function ϕ *(x)* is the scaling function and the coefficients h_0, h_1,..... are low pass scaling coefficients. The wavelet and scaling coefficients are related by the quadrature mirror relationship, which is

$$g_n = (-1)^n h_{1-n+N} \quad \text{............(9)}$$

Where N is the number of vanishing moments.

3.2 Properties of DWT

Properties of a discrete wavelet transforms are given below.

- DWT is a fast linear operation, that can be applied on data vectors having length as integeral power of 2.
- DWT is invertible and orthogonal. The scaling function ϕ and the wavelet function ϕ are orthogonal to each other in $L_2(0, 1)$, i.e., $< \phi , \phi >= 0$.
- The wavelet basis is localized in the space and frequency.
- The coefficients satisfies some constraints

$$\sum_{i=0}^{2N-1} h_i = \sqrt{2}$$

$$\sum_{i=0}^{2N-1} h_i h_{i+2l} = \delta_{1,0}$$

………………………(10) & (11)

Here δ is the delta function and *l* is the location index.

$$\sum_{i=0}^{2N-1} (-1)^i i^k h_i = 0$$

…………….(12)

3.3 Wavelet Thresholding

The term wavelet thresholding is defined as decomposition of the data of image into wavelet coefficients, comparing the detailed coefficients having a given threshold value, and minimizing these coefficients close to zero to remove the effect of noise in the data. Then image is reconstructed from modified coefficients. This is also known as inverse discrete wavelet transform. At the time of thresholding, a wavelet coefficient is compared to the given threshold and is set to zero if its magnitude is less than the threshold otherwise, it is then retained or modified depending on the thresholding rule. Thresholding distinguishes between coefficients due to noise and the ones consisting of important signal information. The selection of a threshold is an important point of interest. It plays an important role in the removal of noise in the images because de-noising most frequently produces smoothed images, by reducing the sharpness of the image. Care should be taken to preserve the edges of the de-noised image. Various methods for wavelet thresholding exists, which rely on the choice of a threshold value. Typically used methods for image noise removal include Sureshrink, VisuShrink and BayesShrink. It is necessary to know about the two generic categories of thresholding. These are hard thresholding and soft thresholding. The hard-thresholding TH is given as

$$T_H = \begin{cases} x & \text{for } |x| \geq t \\ 0 & \text{in all other regions.} \end{cases} \qquad \text{...............(13)}$$

where t is the threshold value. A plot of T_H is shown in Figure below

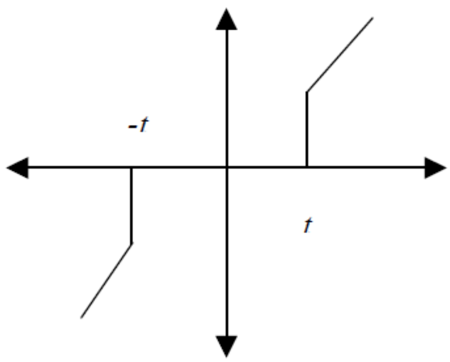


Figure 4 : Hard thresholding

Therefore, all coefficients whose magnitude is greater than the selected threshold value *T* remains same and the others with magnitudes smaller than *t* are set to zero. It creates a region around 0 where the coefficients are considered to be negligible.

Soft thresholding is that where the coefficients with greater than the threshold are shrunk towards zero after comparing them to the threshold value. It is defined as below

$$T_s = \begin{cases} \text{sign}(x)(|x| - t) & \text{for } |x| > t \\ 0 & \text{in all other regions.} \end{cases}$$

……………..(14)

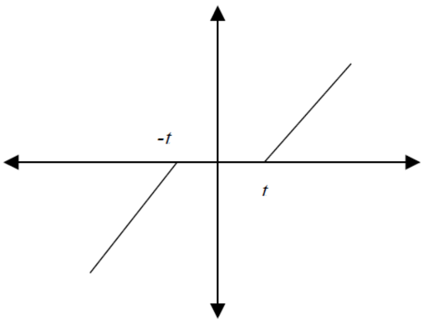


Figure 5 : Soft Thresholding

Practically, it can be seen that the soft method is much better and yields more visually pleasant images. This is because the hard method is discontinuous and yields abrupt artifacts in the images recovered. Also, the soft method yields a smaller MSE (minimum mean squared error) compared to hard form of thresholding.

CHAPTER **4**

The Curvelet Transform

4.1 Introduction

The Curvelet transform is a higher dimensional generalization of the wavelet transform designed to represent images at different scales and different angles. Curvelet transform is a special member of the multi scale geometric transforms. It is a transform with a multi scale pyramid with many directions at each length scale. Curvelets will be superior over wavelets in following cases:

- Optimal sparse representation in object with edges
- Optimal image reconstruction of severely ill-posed problems
- Optimal sparse representation in wave propagators

The idea of the Curve-let transform is first to decompose the image into sub-bands, i.e. to separate the object into a series of disjoint scales. Curve-lets are initially introduced by Candes and Donoho. The Discrete Curvelet transform (DCT) takes as input a Cartesian grid of the form $f(n_1, n_2)$, $0 \leq n_1, n_2 < n$

, 0 , and outputs a collection of coefficients cD (j, l, k)

defined by

$$C^D(j,k,l) = \sum_{n_1,n_2} f(n_1,n_2)\, \overline{\emptyset^D_{j,l,k}(n_1,n_2)} \qquad \textbf{................(15)}$$

where $\emptyset^D_{j,l,k}(n_1,n_2)$ are digital curvelet waveforms which preserve the listed properties of the continuous curve-let. DCT can be implemented in two ways. First method is based on unequally-spaced fast Fourier transform (USFFT) and the second is based on the Wrapping of specially selected Fourier samples. The two implementations are differ by spatial grid used to translate curve-lets at each scale and angle.

The curve-let de-noising method consists of the following steps:

- Estimate the noise standard deviation ‘σ’ in the input image
- Calculate the Curve-let transform of the input image. We get a set of bands wj, each band wj contains Nj coefficients and corresponds to a given resolution level.
- Calculate noise standard deviation’ σ_j ’ for each band *j* of the Curve-let transform.
- For each band *j* do : Calculate the maximum of the band and multiply each curve-let coefficient.
- Reconstruct the image from the modified curvelet Coefficient

4.2 Image de-noising using curve-lets

Image De-noising is implemented to produce good estimates of the original image from noisy observations. The restored images hould contain less noise than the observations while still keep sharp transitions (i.e edges). Suppose an image f(m,n) is corrupted by the additive noise

$$g(m,n) = f(m,n) + \eta(m,n) \qquad \text{……………}\mathbf{(16)}$$

where $\eta(m,n)$ are independent identically distributed Gaussian random variable with zero mean and variance $\sigma 2$. Image de-noising algorithms vary from simple thresholding to complicate model based methods. However simple thresholding methods can remove most of the noise.

4.3 Algorithm

1. Apply the Forward Curvelet transform to the noisy image.

2. Threshold the Curvelet co-efficients to remove some insignificant curve-let co-efficients by using a thresholding function in the curve-let domain.

3. Inverse Curvelet transform of the thresholded co-efficients to reconstruct a function.

4.4 Curve-let Shrinkage/Thresholding

Shrinkage/thresholding plays an important role incurve-let application. Various thresholding techniques have been applied on the curve-let co-efficient of the observed image. The small co-efficients are dominated by noise, while co-efficient with large absolute value carry more signal information than noise. As a result noisy co-efficients (small co-efficients below acertain threshold value) are replaced by zero.

The curve-let shrinkage is taken as

$$P_{\sigma}u = T^{-1}S_{\sigma}T(u) \qquad \textbf{............(17)}$$

T denotes the Curve-let transform, T-1 the inverse transform and Sσ is the thresholding function.

CHAPTER 5

Results and Discussion

5.1 Results

The experiments are performed on several types of gray scale images of size 256 x 256 in MATLAB platform and the Curve-let transform via USFFT and Wrapping technique was Implemented based on curve-let software package. The effectiveness of the Curve-let shrinkage methods as well as partial reconstruction of Curve-let coefficients are demonstrated for various gray scale images with a noise factor of σ = 20 for Random noise, noise density of 0.04 for Salt & Pepper noise, a multiplicative noise factor of 0.05 for Speckle noise and for Gaussian white noise a mean of 0 and 0.01 variance is used.

To compare the results of different curvelet thresholding techniques, PSNR measure is used

$$\mathrm{PSNR} = 10\log_{10}\left(\frac{f_{max}^2}{\mathrm{MSE}}\right) \qquad \text{…………..(18)}$$

here fmax is the maximum value of the image intensities.

The average PSNR measurements of various gray scale images degraded by Random noise, Speckle noise, Salt & Pepper noise, Gaussian noise and Poisson noise with the two digital implementations of Curvelet transform are listed in tables.

Table1: De-noising results (PSNR in dB) with curve-let via USFFT for the four de-noising methods for CT images

Noise	Noisy image	Soft Thresholding	Hard Thresholding	Partial reconstruction
Random	21.09	26.89	28.90	24.28
Salt and pepper	18.67	24.70	20.87	19.82
Guassian	19.77	27.70	28.12	22.40
Speckle	21.20	27.12	28.90	22.50
Poisson	27.55	27.28	31.24	28.10

Table2: De-noising results (PSNR in dB) with curve-let via USFFT for the four de-noising methods for MRI images

Noise	Noisy image	Soft Thresholding	Hard Thresholding	Partial reconstruction
Random	22.29	26.67	28.30	24.50
Salt and pepper	16.47	22.74	19.87	17.34
Guassian	20.77	25.76	25.12	22.23
Speckle	27.20	26.42	28.39	27.28
Poisson	33.56	27.55	28.54	29.10

Table3: De-noising results (PSNR in dB) with curve-let via USFFT for the four de-noising methods for satellite images

Noise	Noisy image	Soft Thresholding	Hard Thresholding	Partial reconstruction
Random	22.70	29.67	33.50	23.30
Salt and pepper	18.56	23.82	19.05	18.10
Guassian	21.80	26.36	26.62	23.23
Speckle	25.40	30.83	33.45	27.76
Poisson	32.90	30.55	33.67	32.34

Table4: De-noising results (PSNR in dB) with curve-let via Wrapping for the four de-noising methods for CT images

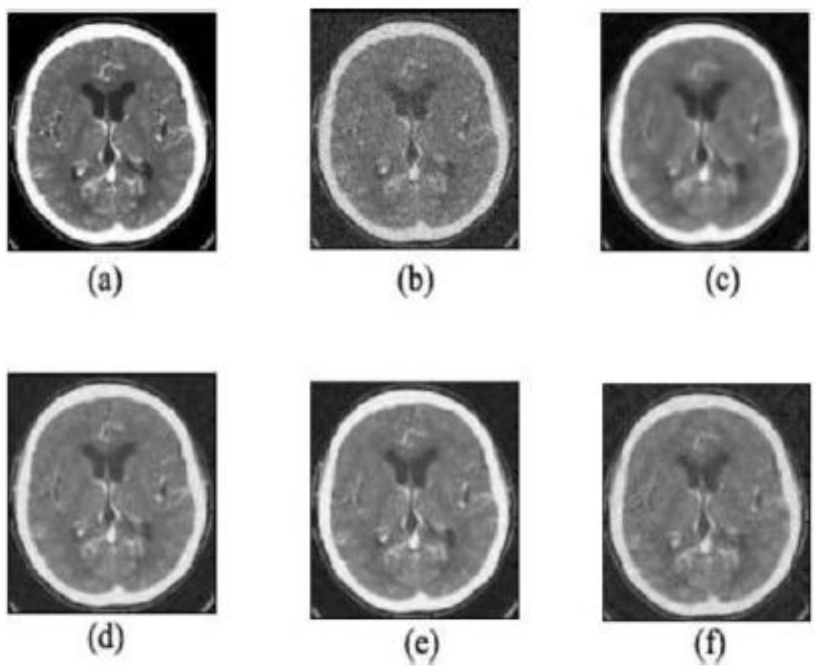
Noise	Noisy image	Soft Thresholding	Hard Thresholding	Partial reconstruction
Random	22.05	27.56	29.54	22.30
Salt and pepper	17.36	22.80	19.05	17.26
Guassian	21.96	24.46	27.12	21.24
Speckle	22.51	24.53	27.67	22.53
Poisson	29.85	25.68	30.10	30.05

Table5: De-noising results (PSNR in dB) with curve-let via Wrapping for the four de-noising methods for MRI images

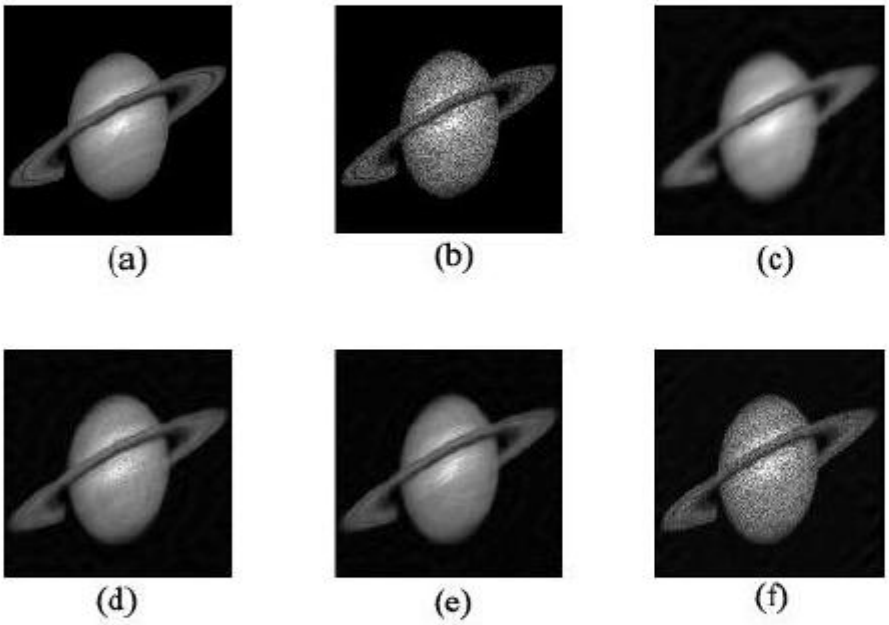
Noise	Noisy image	Soft Thresholding	Hard Thresholding	Partial reconstruction
Random	22.66	26.78	28.54	22.30
Salt and pepper	16.46	21.64	18.67	16.26
Guassian	21.63	24.56	27.33	23.54
Speckle	28.24	28.98	28.69	28.33
Poisson	34.50	26.23	30.16	34.25

Table6: De-noising results (PSNR in dB) with curve-let via Wrapping for the four de-noising methods for satellite images

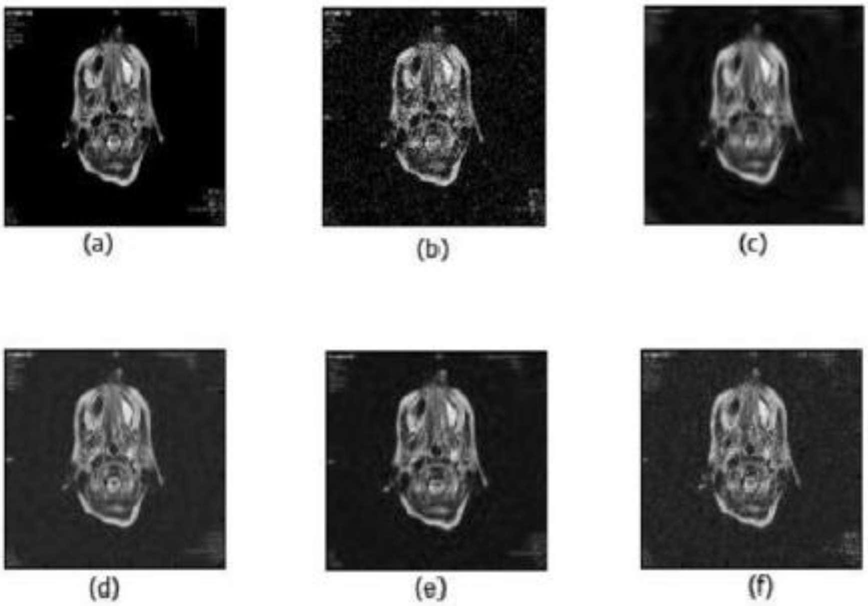
Noise	Noisy image	Soft Thresholding	Hard Thresholding	Partial reconstruction
Random	22.23	30.48	28.55	22.36
Salt and pepper	16.24	23.40	19.33	16.25
Guassian	21.47	26.55	26.35	22.19
Speckle	25.45	30.90	28.70	28.99
Poisson	32.66	30.60	28.15	34.40



De-noising results for CT image (Random Noise) via USFFT (a) Original (b) Noisy (PSNR=22.05) (c) Soft (PSNR=26.48) (d) Hard (PSNR=29.94) (e) Garrote (PSNR=29.79) (f) Partial Reconstruction (PSNR=24.42)



De-noising results for Satellite image (Speckle Noise) via USFFT (a) Original (b) Noisy (PSNR=26.82) (c) Soft (PSNR=33.77) (d) Hard (PSNR=36.67) (e) Garrote (PSNR=36.48) (f) Partial Reconstruction (PSNR=27.75)



De-noising results for MRI image (Gaussian) via Wrapping (a) **Original (b) Noisy (PSNR=21.69) (c) Soft (PSNR=23.32) (d) Hard (PSNR=25.27) (e) Garrote (PSNR=24.79) (f) Partial Reconstruction (PSNR=22.17)**

CHAPTER 7

Conclusion

7.1 Conclusions and Discussion:

A strategy for digitally implementing the Curve-let transform via USSFT and Wrapping technique is presented. The Curve-let transform is capable of resolving 2D singularities and represents edges more efficiently in images. The resulting implementations have the exact reconstruction property and gives table reconstruction. The experimental results showed that the Curve-let transform implemented with USFFT & Wrapping Algorithm using Hard &Garrote thresholding consistently produces the highest PSNR for Bio medical images, satellite images & natural images degraded by Random noise, Gaussian noise and Speckle noise, while soft thresholding offers best results for de-noising Salt & Pepper noise for all the test images used.

Alhough best results are obtained while de-noising the test images corrupted by Random noise, Speckle noise, Gaussian noise and Salt & Pepper noise in terms of PSNR gain it is noticed in the result that lowest PSNR gain is obtained for Bio-medical images when compared to satellite images.

The results clearly indicates that Curve-let transform using thresholding techniques proves to be inferior in de-noising Bio-medical images corrupted by Random noise, Gaussian noise, Speckle noise and Salt and Pepper noise. Curve-let thresholding techniques proves to be a failure in the removal of Poisson noise in Bio-medical images. Partial reconstruction of curve-let coefficients proves to be a failure for all types of noises tested with various images. However the visual quality of images is preserved.

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