Controller Design for Fractional Order Systems

A Thesis Submitted in Partial Fulfilment

of the Requirements for the Award of the Degree of

Master of Technology

in

Control & Automation

by

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CERTIFICATE

This is to certify that the thesis entitled "**Controller Design for Fractional Order Systems**" by **Ankush Kumar**, submitted to the National Institute of Technology (NIT), Rourkela for the award of Master of Technology in **Control and Automation**, is a record of bona fide research work carried out by him in the Department of Electrical Engineering, under our supervision and guidance.

I believe that this thesis fulfills part of the requirements for the award of degree of Master of Technology. The results embodied in the thesis have not been submitted for the award of any other degree elsewhere.

Place: Rourkela
Date:

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ABSTRACT

In recent time, the application of fractional derivatives has become quite apparent in modeling mechanical and electrical properties of real materials. Fractional integrals and derivatives has found wide application in the control of dynamical systems, when the controlled system or/and the controller is described by a set of fractional order differential equations. In the present work a fractional order system has been represented by a higher integer order system, which is further approximated by second order plus time delay (SOPTD) model. The approximation to a SOPTD model is carried out by the minimization of the two norm of the actual and approximated system. Further, the effectiveness of a fractional order controller in meeting a set of frequency domain specifications is determined based on the frequency response of an integer order PID and a fractional order PID (FOPID) controller, designed for the approximated SOPTD model. The advent of fuzzy logic has led to greater flexibility in designing controllers for systems with time varying and nonlinear characteristics by exploiting the system observations in a linguistic manner. In this regard, a fractional order fuzzy PID controller has been developed based on the minimization different optimal control based integral performance indices. The indices have been minimized using genetic algorithms. Simulation results show that the fuzzy fractional order PID controller is able to outperform the classical PID, fuzzy PID and FOPID controllers.

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FO	: Fractional Order
FOPTD	: First Order Plus Time Delay
SOPTD	: Second Order Plus Time Delay
PID	: Proportional-Integral-Derivative
FOPID	: Fractional Order Proportional-Integral-Derivative
IAE	: Integral Absolute Error
ISE	: Integral Square Error
ITAE	: Integral Time Absolute Error
ITSE	: Integral Time Square Error
FLC	: Fuzzy Logic Controller
MF	: Membership Function

INTRODUCTION

1.1 Introduction of Fractional Order System

Fractional calculus provides an excellent instrument for the description of memory and hereditary properties of various materials and processes. This is the primary advantage of fractional derivatives in comparison to classical integer order models, where such dynamics not taken into account. The advantages of fractional derivatives become more appealing in the modeling of mechanical, electrical and electro-mechanical properties of real materials, as well as in the description of rheological properties of rocks, and in many other fields. Recent times have wide application of field fractional integrals and derivatives also in the theory of control of dynamical systems, where the controlled system or/and the controller is described by a set of fractional differential equations. The mathematical modeling and simulation of systems and processes, based on the description of their properties in terms of fractional derivatives, naturally leads to differential equations of fractional order the necessity to solve such equations to obtain the response for a particular input. Thought in existence for more than 300 years, the idea of fractional derivatives and integrals has remained quite a strange topic, very hard to explain, due to absence of a specific tool for the solution of fractional order differential equations. For this reason, this mathematical tool could be judged "far from reality". But many physical phenomena have "intrinsic" fractional order description and so fractional order calculus is necessary to replicate their input-output characteristics.

Fractional order calculus allows us to describe and model a real object more accurately than the classical "integer" methods. Details of past and present progress in the analysis of dynamic systems modeled by FODEs can be found in [5–6]. PID (proportional integral derivative) controllers, which have been dominating industrial controllers, have been modified using the notion of a Fractional Order integrator and differentiator. It has been shown that the incorporation of two degrees of freedom from the use of a Fractional Order integrator and differentiator provides a greater degree of flexibility and hence makes it possible to further improve the performance of traditional PID controllers.

Fractional calculus have found wide in different unrelated topics such as: transmission line theory in Infinite line transmission system , chemical analysis of aqueous solutions, design of heat-flux meters, visco elasticity, dielectric polarization, electromagnetic wave, rheology of soils, growth of intergranular grooves on metal surfaces, quantum mechanical calculations, and dissemination of atmospheric pollutants. One of the prime application of fractional calculus in electrical engineering has been on the modeling an infinite length transmission line (Figure 1). Considering an infinite length transmission line with series and shunt impedance represented by Z_a and Z_b respectively, where

Infinite line transmission system:

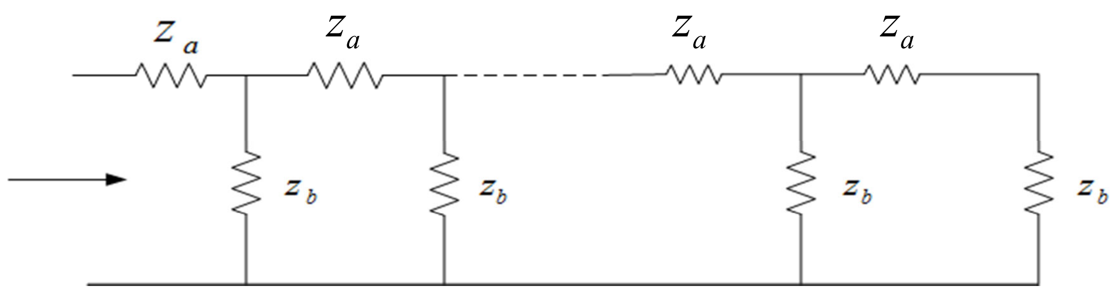

Figure 1.1: Infinite Line Transmission System

Equivalent impedance $$Z = \sqrt{z_a z_b}$$

When $z_a = R$ and $z_b = \frac{1}{sC}$

$$Z = \sqrt{\frac{R}{C}}\, s^{-1/2} \text{ (Fractional order system)}$$

1.2 Literature Review on Fractional order controller

This section provides a brief survey of the state of the art techniques in fractional order controller design. Podlubny given a more flexible structure $PI^{\lambda}D^{\mu}$ by extended in traditional notion of PID controllers [11] with the controller gains define the fractional differ-integrals as design variables. And givens several intelligent techniques for efficient tuning of such fractional order $PI^{\lambda}D^{\mu}$ controllers. To design $PI^{\lambda}D^{\mu}$ controllers' dominant pole placement based optimization problems have been attempted using Differential Evolution in Maiti *et al.* [16], Biswas *et al.* [17] and Invasive Weed Optimization with Stochastic Selection (IWOSS) in Kundu *et al.* [18]. Maiti *et al.* [19] also tuned a FOPID controller for stable minimum phase systems by minimizing an integral performance index i.e. ITAE criteria with Particle Swarm Optimization (PSO). A similar approach has been adopted for optimization of a weighted sum of Integral of Absolute Error (IAE) and ISCO to find out the controller parameters with GA by Cao, Liang & Cao [20] and with PSO by Cao & Cao [21]. Cai, Pan & Du [22] tuned a $PI^{\lambda}D^{\mu}$ controller by minimizing the ITAE criteria using multi-parent cross over evolutionary algorithm. Luo & Li [23] tuned a similar ITAE based $PI^{\lambda}D^{\mu}$ controller with Bacterial Foraging oriented by Particle Swarm Optimization (BF-PSO). Meng & Xue [24] designed a $PI^{\lambda}D^{\mu}$ controller using a multi-objective GA which minimizes the infinity-norm of the sensitivity (load disturbance suppression), and complementary sensitivity function (high frequency measurement noise rejection), rise time and percentage of maximum overshoot and additionally meets the specified gain cross-over frequency, phase margin and iso-damping property rather than minimizing these as a single objective with a controller tuning methodology using Self-Organizing Migrating Algorithm (SOMA), which is an extension of that proposed by Monje *et al.* [26] using constrained Nelder-Mead Simplex algorithm. Zhao *et al.* [27] tuned a controller for inter-area oscillations in power systems by minimizing a weighted sum of the weighted summation like Zamani *et al.* [29]. Kadiyala, Jatoth & Pothalaiah [28] designed PSO based optimization problem for minimizing a weighted sum of, steady-state error to design a $PI^{\lambda}D^{\mu}$ controller for aerofin control system. A PSO based similar approach can be found in Sadati, Zamani & Mohajerin [32] for SISO and MIMO systems. Sadati, Ghaffarkhah & Ostadabbas [30] designed a Neural Network based FOPID controller by minimizing the Mean Square Error (MSE) of the closed loop system while weights of the Neural Network and fractional orders are determined in the learning phase and the controller gains are adapted with change in the error. Ou, Song & Chang [31] designed a FOPID controller for First Order Plus Time Delay (FOPTD) systems using Radial Basis Function (RBF) neural network where the controller gains and differential-integral orders can be determined from the time constant and delay of the process after the neural network is trained with a large set of FOPID parameters and system parameters. Weighted sum of several time-domain and frequency-domain criteria based optimization approach has been used to tune a FOPID controller with PSO for an automatic voltage regulator by Ghartemani *et al.* [32] and Zamani *et al.* [25], [32]. The approach in [33] also proposes an H_∞-optimal FOPID controller by putting the infinity norm of the weighted sensitivity and complementary sensitivity functions as an inequality constraint to the objective function that in [25]. Lee & Chang [34-35] used Improved Electromagnetism with Genetic Algorithm (IEMGA) to minimize the Integral of Squared Error (ISE) while searching for optimal $PI^{\lambda}D^{\mu}$ parameters. Pan *et al.* [36] used evolutionary algorithms for time domain tuning of $PI^{\lambda}D^{\mu}$ controllers to cope with the network induced packet drops and stochastic delays in NCS applications.

Recent advent of few non-PID type intelligent fractional order controllers have been shown to be more effective over the existing technologies. Efe [37] used fractional order integration while designing an Adaptive Neuro-Fuzzy Inference System (ANFIS) based sliding mode control. Delavari *et al.* [38] proposed a fuzzy fractional sliding mode controller and tuned its parameters with GA. Barbosa *et al.* [39] incorporated fuzzy reasoning in fractional order PD controllers. Arena *et al.* [40-41] introduced a new Cellular Neural Network (CNN) with FO cells and studied existence of chaos in it. Valerio & Sa da Costa [42] studied fuzzy logic based approximation of variable complex and real order derivatives with and without memory.

In the present study, the tuning of a new fuzzy FOPID controller has been attempted with GA and the closed loop performances are compared with an optimal $PI^{\lambda}D^{\mu}$ controller. The input-output MFs(Membership function) and differ-integrals of the FO fuzzy PID controller are tuned while minimizing weighted sum of various error indices and control signal similar to that in Cao, Liang & Cao [21] and Cao & Cao [22] with a simple ISE criteria. While [37-38] focuses on fractional order fuzzy sliding mode controllers. The present work is concerned with the fuzzy analogue of the conventional PID controller, which is widely used in the process control industry. In Barbosa *et al.* [39], the fractional fuzzy PD controller is investigated in terms of digital implementation and robustness. However the tuning methodology is complex and might not always ensure optimal time domain performance. The performance improvement is even more for complicated and ill-behaved systems which have been enforced to obey a set of desired control objective with GA in the present formulation.

1.3 Objective

Based on the literature survey and the scope outlined in the previous sub-section, the following objectives are framed for the present work:

- Representation fractional order linear system by an integer order system.
- Representation of a fractional order system by an approximated lower order (integer) system with time delay.
- Design of fractional order controllers based on frequency domain specifications.
- Design of fuzzy fractional order controllers based on the minimization of time domain based integral performance indices.

1.4 **Thesis Organization**

The thesis consists of five chapters organized as follows:

- Chapter 1 gives an introduction about fractional order system with specific stress on fractional order controllers.
- Chapter 2 provides a description on the time and frequency domain representation of fractional order system along with the representation of fractional order by it integer order equitant.
- Chapter 3 provides the design and the necessity of fractional order controllers.
- In chapter 4, design of fuzzy fractional order and fuzzy integer order controller are carried out and corresponding responses are compared.
- Chapter 5 provides conclusions and future work.

FRACTIONAL ORDER SYSTEM: Representation and Approximation

2.1 Introduction OF Fractional Order Calculus

Fractional order calculus is an area where the mathematicians deal with derivatives and integrals from non-integer orders. There are different definitions of Fractional Order differentiations and integrations. Some of the definitions extend directly from integer-order calculus. The well-established definitions include the Grünwald-Letnikov definition, the Cauchy integral formula, the Caputo definition and the Riemann-Liouville definition [1-4]. The definitions will be summarized first, and then their properties will be given.

2.1.1 Definitions of Fractional Order Calculus

Definition 2.1 (*Cauchy's Fractional Order integration formula*). This definition is a general extension of the integer-order Cauchy formula

$$\mathcal{D}^{\gamma} f(t) = \frac{\Gamma(\gamma+1)}{2\pi j}\int_C \frac{f(\tau)}{(\tau-t)^{\gamma+1}}\,d\tau \tag{2.1}$$

where C is the smooth curve encircling the single-valued function *f*(t).

Definition 2.2 (*Grünwald–Letnikov definition*). The definition is defined as [6]

$${}_a\mathcal{D}_t^{\alpha} f(t) = \lim_{h\to 0}\frac{1}{h^{\alpha}}\sum_{j=0}^{\left[\frac{t-a}{h}\right]}(-1)^j\binom{\alpha}{j} f(t-jh) \tag{2.2}$$

where $w_j^{\alpha} = (-1)^j\binom{\alpha}{j}$ represents the coefficients of the polynomial $(1-z)^{\alpha}$. The coefficients can also be obtained recursively from

$$w_0^{\alpha} = 1, w_j^{\alpha} = \left(1-\frac{\alpha+1}{j}\right) w_{j-1}^{\alpha} \quad j=1,2\ldots\ldots\ldots \tag{2.3}$$

Definition 8.3 (*Riemann–Liouville Fractional Order differentiation*). The Fractional Order integration is defined as [7]

$$ {}_a\mathcal{D}_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-\tau)^{\alpha-1} f(\tau) d\tau \tag{2.4}$$

where $0 < \alpha < 1$and a is the initial time instance, often assumed to be zero, i.e., $a = 0$.
The differentiation is then denoted as $\mathcal{D}_t^{-\alpha} f(t)$.
The Riemann–Liouville definition is the most widely used definition in fractional order calculus. The subscripts on both sides of D represent, respectively, the lower and upper bounds in the integration.

Definition 8.4 (*Caputo's definition of Fractional Order differentiation*). Caputo's definition is given by [2]

$$ {}_0\mathcal{D}_t^{\alpha} y(t) = \frac{1}{\Gamma(1-\gamma)} \int_0^t \frac{y^{(m+1)}(\tau)}{(t-\tau)^{\gamma}} d\tau \tag{2.5}$$

where $\alpha = m + \gamma$,m is a integer, and $0 < \gamma \le 1$. Similarly Caputo's Fractional Order integration is defined as

$$ {}_0\mathcal{D}_t^{\gamma} y(t) = \frac{1}{\Gamma(-\gamma)} \int_0^t \frac{y(\tau)}{(t-\tau)^{\gamma+1}} d\tau, \gamma < 0 \tag{2.6}$$

2.1.2 Properties of Fractional Order Differentiations

The Fractional Order differentiation has the following properties [8]:

1. The Fractional Order differentiation ${}_0D_t^{\alpha} f(t)$ with respect to t of an analytic function $f(t)$ is also analytical.
2. The Fractional Order differentiation is exactly the same with integer-order one, when $\alpha = n$ is an integer. Also
$$ {}_0D_t^{\alpha} f(t) = f(t). \tag{2.7}$$
3. The Fractional Order differentiation is linear; i.e., for any constants a,b one has
$$ {}_0D_t^{\alpha}[af(t) + bg(t)] = a\, {}_0D_t^{\alpha} f(t) + b\, {}_0D_t^{\alpha} g(t) \tag{2.8}$$

4. Fractional Order differentiation operators satisfy the commutative-law, and also satisfy

$$ {}_0D_t^{\alpha}[\,{}_0D_t^{\beta}f(t)] = {}_0D_t^{\beta}[\,{}_0D_t^{\alpha}f(t)] = {}_0D_t^{\alpha+\beta}f(t) \tag{2.9}$$

5. The Laplace transform of Fractional Order differentiation is defined as

$$\mathcal{L}\left[\,{}_0D_t^{\alpha}f(t)\right] = s^{\alpha}\mathcal{L}[f(t)] - \sum_{k=1}^{n-1} s^k\left[\,{}_0D_t^{\alpha-k-1}f(t)\right]_{t=0} \tag{2.10}$$

In particular, if the derivatives of the function *f (t)* are all equal to 0 at *t* = 0, one has $\mathcal{L}\left[\,{}_0D_t^{\alpha}f(t)\right] = s^{\alpha}\mathcal{L}[f(t)]$.

2.2 Frequency and Time Domain Analysis of Fractional Order Linear Systems

The Fractional Order system is the direct extension of classical integer-order systems. The Fractional Order system is established upon the Fractional Order differential equations, and the Fractional Order transfer function of a single variable system can be defined as

$$G(s) = \frac{b_1 s^{\gamma 1} + b_2 s^{\gamma 2} + \dots\dots\dots b_m s^{\gamma m}}{a_1 s^{\eta 1} + a_2 s^{\eta 2} + \dots\dots a_{n-1} s^{\eta n-1} + a_n s^{\eta n}} \tag{2.11}$$

where a_i, b_i are real numbers and the orders γ_i, η_i of the numerator and the denominator can also be real numbers. The analysis of the Fractional Order Laplace transformations and their inverse is very complicated. The closed-form solutions to the problems are not possible in general.

Fractional Order system transfer function give as

$$G_a(s) = \frac{-2s^{0.603} + 4}{s^{3.501} + 3.8s^{2.42} + 2.6s^{1.798} + 2.5s^{1.31} + 1.5} \tag{2.12}$$

2.2.1 Frequency Domain Analysis of Linear Fractional Order Systems

It can be seen that, when jω is used to substitute for the variable s in the Fractional Order transfer function model (2.12), the frequency domain response $G_a(jw)$ can be easily evaluated. Thus, Fractional Order Bode diagrams, Nyquist plots, and Nichols charts can be easily evaluated. In figure 2.1 frequency responses of Fractional Order system represented in (2.12) is given.

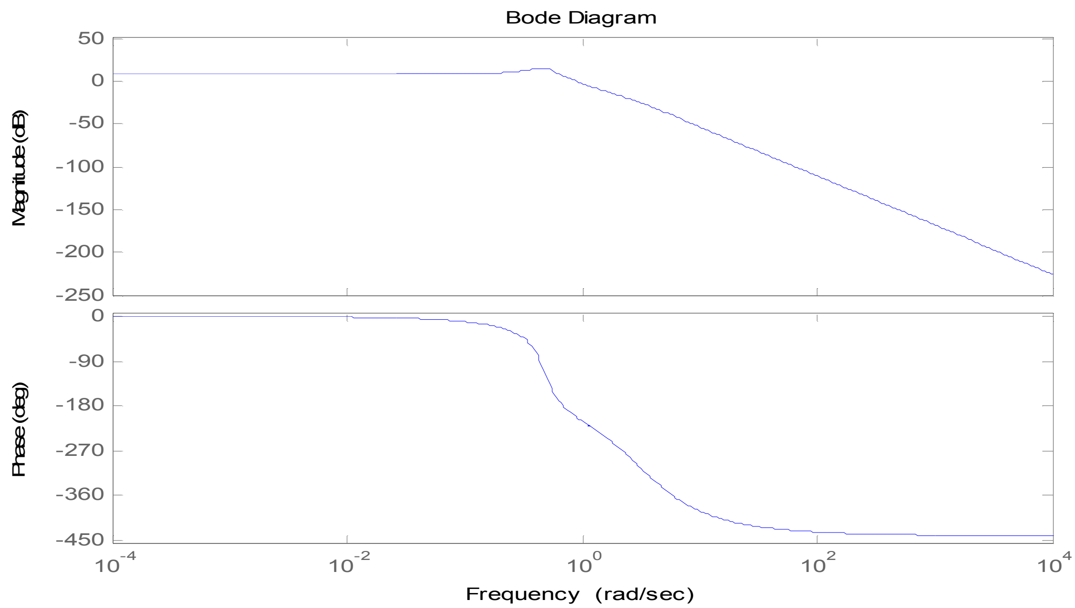

Figure 2.1 : Frequency response of Fractional Order system $G_a(s)$

2.2.2 Time Domain Analysis of Fractional Order Systems

The evaluation of the time domain response of a Fractional Order system is more complicated. Let us consider a special form of a Fractional Order differential equation [8]

$$a_1 D_t^{\eta 1} y(t) + \ a_2 D_t^{\eta 2} y(t) + \cdots + a_{n-1} D_t^{\eta n-1} y(t) + a_n D_t^{\eta n} y(t) = u(t) \tag{2.13}$$

where *u(t)* can be represented by a certain function and its Fractional Order derivatives. Assume also that the output function y*(t)* has zero initial conditions. The Laplace transform can be used to find the transfer function

$$G(s) = \frac{1}{a_1 s^{\eta 1} + a_2 s^{\eta 2} + \ldots\ldots a_{n-1} s^{\eta n-1} + a_n s^{\eta n}} \tag{2.14}$$

Consider the Grünwald–Letnikov definition in (2.2). The discrete form of it can be rewritten as

$$ {}_aD_t^{\eta i} y(t) \simeq \frac{1}{h^{\eta i}} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} w_j^{\eta i}\, y_{t-jh} \tag{2.15}$$

The numerical solution to the Fractional Order differential equation (2.12) is given as

$$y_t = \frac{1}{\sum_{i=1}^{n} \frac{a_i}{h^{\eta i}}} \left[u_t - \frac{a_i}{h^{\eta i}} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} w_j^{\eta i}\, y_{t-jh} \right] \tag{2.16}$$

For the general form of the Fractional Order transfer function in (2.12), the right-hand side can equivalently be evaluated first by using numerical methods. The final solution can be obtained from (2.16).Using equation (2.16) step response of fraction-order system $G_a(s)$ is obtained as in figure 2.2.

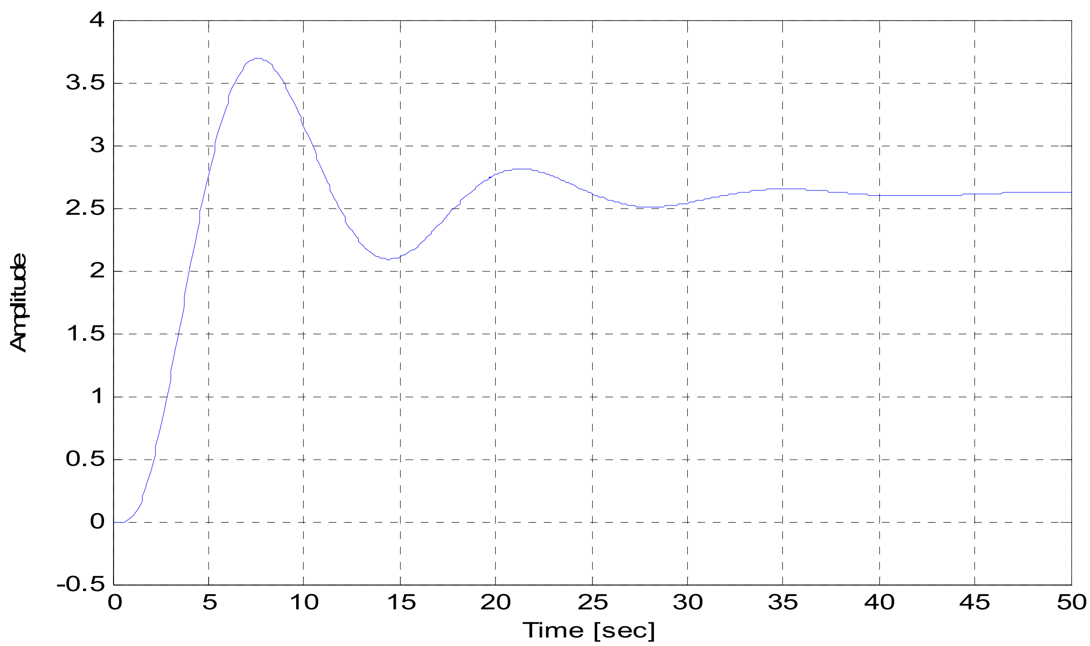

Figure 2.2 : Step response of Fractional Order model $G_a(s)$

2.3 Integer order Approximation of Fractional Order System

A Fractional order linear time invariant (LTI) system is mathematically equivalent to an infinite dimensional LTI filter. Thus a fractional order system can be approximated using higher order polynomials having integer order differ-integral operators. The realization of FO differ-integrators in integer order can be carried in two ways [8].

- Continuous time realization
- Discrete time realization

Here we are used continuous time realization to realize our Fractional Order system into integer order system by using Oustaloup's recursive filter [9].

2.3.1 Oustaloup's recursive filter

Oustaloup's recursive filter gives a very good fitting to the Fractional Order elements within a chosen frequency band. Let us assume that the expected fitting range is (ω_b,ω_h). The filter can be written as

$$G_f(s) = K\prod_{-N}^{N}\frac{s+w'_k}{s+w_k} \tag{2.17}$$

where the poles, zeros, and gain of the filter can be evaluated from as

$$w'_k = w_b\left(\frac{w_h}{w_b}\right)^{\frac{k+N+1/2(1-\gamma)}{2N+1}}\ ;\ K = w_h^{\gamma}\ ;\ w_k = w_b\left(\frac{w_h}{w_b}\right)^{\frac{k+N+1/2(1+\gamma)}{2N+1}} \tag{2.18}$$

where γ is the order of the differentiation, $2N+1$ is the order of the filter, and the frequency fitting range is given by $(\omega_b,\ \omega_h)$. The filter can be designed such that it may fit very well within the frequency range of the fractional order differentiator.

Integer order transfer function of fractional terms 0.603, 0.501, 0.242, 0.789, 0.131 of Fractional Order system are respectively given as

$$g_1(s)=\frac{64.42s^9+23990s^8+1584000s^7+21700000s^6+63500000s^5+39900000s^4+5412000s^3+156600s^2+939.8s+1}{s^9+939.8s^8+156600.0s^7+5142000.0s^6+39990000.0s^5+6350000.0s^4+21700000.0s^3+1584000s^2+23990s+64.42} \quad (2.19)$$

$$g_2(s)=\frac{31.84s^9+12820s^8+915500s^7+13560000s^6+42950000s^5+29240000s^4+4270000s^3+133900s^2+869s+1}{s^9+869.0s^8+133900.0s^7+4279000.0s^6+29240000.1s^5+42950000s^4+13560000s^3+915500s^2+12820s+31.84} \quad (2.20)$$

$$g_3(s)=\frac{18.2s^9+7799s^8+592500s^7+9340000s^6+3147000s^5+28200000s^4+3551000s^3+182000s^2+816.7s+1}{s^9+816.7s^8+118200s^7+3557000s^6+23800000.5s^5+31470000s^4+9340000s^3+592000s^2+7799s+18.2} \quad (2.21)$$

$$g_4(s)=\frac{247.7s^9+79440s^8+4515000s^7+53260000s^6+134310000s^5+72770000s^4+8479s^3+211200s^2+1092s+1}{s^9+1092.5s^8+211200.4s^7+8479000s^6+7270000.9s^5+134300000s^4+53260000s^3+4515000s^2+79440s+247.7} \quad (2.22)$$

$$g_5(s)=\frac{8.511s^9+3969s^8+328100s^7+5628000s^6+2036000s^5+16270000s^4+2575000s^3+99850s^2+1092s+1}{s^9+750.55s^8+99850.5s^7+2757000.3s^6+16270000s^5+20630000s^4+5628000s^3+328100s^2+3969s+8.511} \quad (2.23)$$

Equivalent total integer order transfer function G(s) of Fractional Order system (2.12) using the Oustaloup’s Recursive Filter is given as

$$G(s)=\frac{\begin{array}{l}-1248s^{45}-.4846\times10^{6}s^{44}-8.073\times10^{8}s^{43}-7.601\times10^{11}s^{42}-4.49\times10^{14}s^{41}-1.762\times10^{17}s^{40}-4.732\times10^{19}s^{39}-\\ 8.88\times10^{21}s^{38}-1.183\times10^{24}s^{37}-1.125\times10^{26}s^{36}-7.68\times10^{28}s^{35}-3.783\times10^{29}s^{33}-1.34\times10^{31}s^{32}-3.43\times10^{34}s^{31}\\ -6.31\times10^{33}s^{30}-8.267\times10^{36}s^{29}-7.61\times10^{37}s^{28}-4.79\times10^{28}s^{27}-1.93\times10^{39}s^{26}-3.92\times10^{39}s^{25}+4.08\times10^{40}s^{24}\\ +5.42\times10^{40}s^{23}+1.84\times10^{40}s^{22}+3.682\times10^{40}s^{21}+4.89\times10^{40}s^{20}+4.529\times10^{40}s^{19}+2.97\times10^{39}s^{18}+1.40\times10^{39}s^{17}\\ +4.78\times10^{38}s^{16}+1.18\times10^{37}s^{15}+2.129\times10^{36}s^{14}+2.79\times10^{35}s^{13}+2.66\times10^{34}s^{12}+1.859\times10^{33}s^{11}+9.43\times10^{31}s^{10}\\ +3.485\times10^{30}s^{9}+9.331\times10^{28}s^{8}+1.805\times10^{26}s^{7}+2.502\times10^{24}s^{6}+2.48\times10^{22}s^{5}+1.725\times10^{20}s^{4}+8.25\times10^{17}s^{3}+\\ 2.63\times10^{14}s^{2}+6.205\times10^{12}s+3.14\times10^{8}\end{array}}{\begin{array}{l}31.84s^{48}+1.275\times10^{4}s^{47}+2.198\times10^{7}s^{46}+2.143\times10^{10}s^{45}+1.318\times10^{13}s^{44}+5.393\times10^{16}s^{43}+1.518\times10^{18}s^{42}+\\ 3.0\times10^{21}s^{41}+4.228\times10^{23}s^{40}+4.291\times10^{25}s^{39}+3.158\times10^{27}s^{38}+1.69\times10^{29}s^{37}+6.68\times10^{36}s^{36}+1.93\times10^{32}s^{35}\\ +4.1701\times10^{33}s^{33}+6.674\times10^{34}s^{32}+7.991\times10^{35}s^{31}+7.2191\times10^{37}s^{30}+4.9451\times10^{38}s^{29}+2.5875\times10^{39}s^{28}+\\ 1.31\times10^{39}s^{27}+3.219\times10^{40}s^{26}+7.70\times10^{40}s^{25}+1.428\times10^{40}s^{24}+2.06\times10^{40}s^{23}+2.29\times10^{40}s^{22}+2.01\times10^{40}s^{21}\\ +1.392\times10^{40}s^{20}+1.8\times10^{39}s^{19}+7.92\times10^{39}s^{18}+3.90\times10^{38}s^{17}+1.41\times10^{38}s^{16}+3.72\times10^{37}s^{15}+7.101\times10^{36}s^{14}\\ +9.771\times10^{35}s^{13}+6.63\times10^{34}s^{12}+3.15\times10^{32}s^{11}+1.06\times10^{31}s^{10}+7.6\times10^{29}s^{9}+1.51\times10^{28}s^{8}+2.10\times10^{26}s^{7}\\ +2.12\times10^{24}s^{6}+1.5\times10^{22}s^{5}+7.35\times10^{19}s^{4}+2.4\times10^{17}s^{3}+2.033\times10^{14}s^{2}+2.35\times10^{11}s+1.181\times10^{8}\end{array}} \quad (2.24)$$

The bode diagram of Fractional Order system and integer order system is depicted in figure 2.3. It is observed that bode diagram Fractional Order system and integer order systems (higher order) converge as. And the corresponding step response of Fractional Order derivative system and integer order approximated system is shown in figure 2.4.

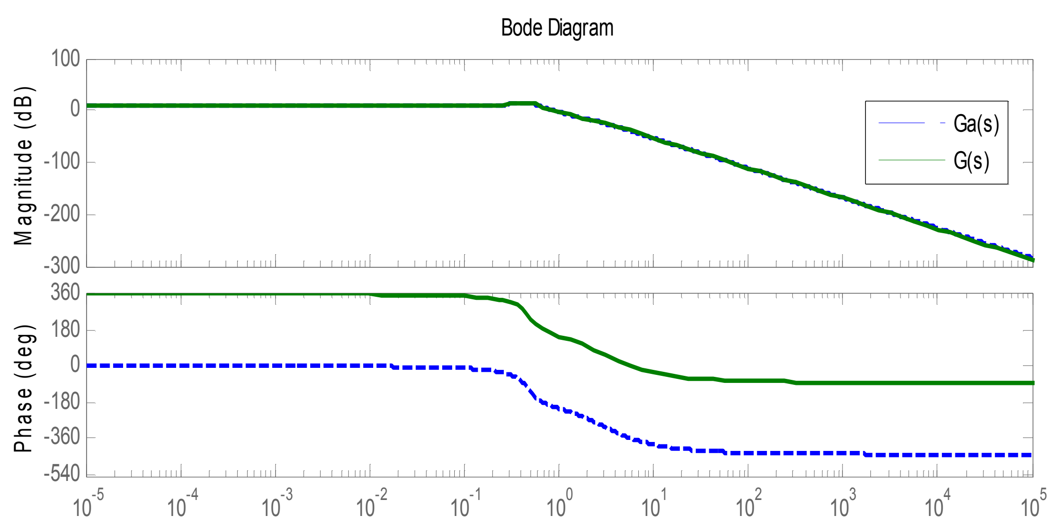

Figure 2.3: Frequency response FO model $G_a(s)$ and higher integer order G(s)

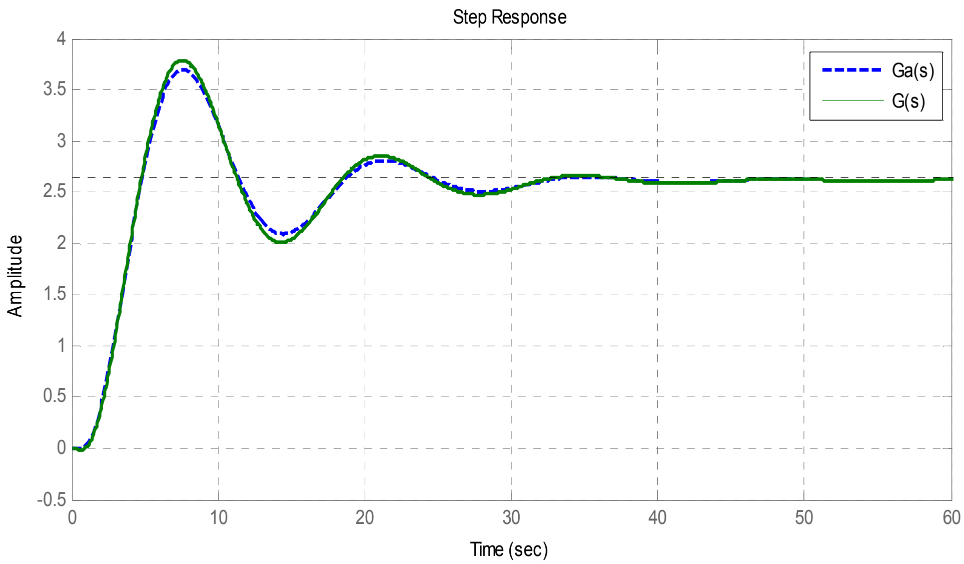

Figure 2.4: Step response FO model $G_a(s)$ and higher integer order G(s)

2.4 Model Reduction Techniques for Fractional Order Systems

As observed in the previous section if the integer-order approximation is used to fit the Fractional Order transfer function models with the use of the Oustaloup recursive filter, the order of the resulting integer order system could be extremely high. Thus, a low-order approximation to the original problem can be found using the optimal model reduction method. Recall that the expected reduced-order model given by [12].

$$G_{r/m,\tau}(s) = \frac{\beta_1 s^r + \ldots\ldots\ldots + \beta_r s + \beta_{r+1}}{s^m + \alpha_1 s^{m-1} + \ldots\ldots\ldots \alpha_{m-1} s + \alpha_m} e^{-\tau s} \tag{2.25}$$

An objective functions for minimizing the H_2-norm of the deviation between the transfer functions of higher order and approximated system can be defined

$$J = \min_{\phi} \left\| \hat{G}(s) - G_{r/m,\tau}(s) \right\|_2 \tag{2.26}$$

Where ϕ is the set of parameters to be optimized

$$\phi = [\beta_1, \beta_2 \ldots\ldots \beta_r, \alpha_1, \alpha_2 \ldots\ldots \alpha_{m,} \tau] \tag{2.27}$$

For an easy evaluation of the criterion *J,* the delayed term in the reduced-order model can be further approximated by a rational function $G_{r/m}(s)$ using the Pade approximation technique. Thus, the revised criterion can then be defined by

$$J = \min_{\phi} \left\| \hat{G}(s) - G_{r/m}(s) \right\|_2 \tag{2.28}$$

The H_2-norm computation can be evaluated recursively using an optimization algorithm [10].

2.5 Genetic Algorithm

This section provides a brief description about genetic algorithm (GA) [14-15] and its application in the minimization of J (2.28).

Genetic algorithm is a stochastic optimization process inspired by natural evolution. During the initialization phase, a random population of solution vectors with uniform distribution is created over the whole solution domain. The population is encoded as a double vector.

Fitness evaluation: Since the purpose of using genetic algorithm is to determine a reduce order model with minimizing objective function J (2.23) from the search space.

Reproduction: Individual strings are copied based on the fitness and sent to the mating pool. The reproduction operation is implemented using roulette wheel arrangement.

Crossover: During crossover operation, two strings selected at random from the mating pool undergo crossover with a certain probability at a randomly selected crossover point to generate two new strings.

Mutation: Depending on whether a randomly generated number is larger than a predefined mutation probability or not, each bit in the string obtained after crossover is altered (changing 0 to 1 and 1 to 0).

In each generation, the fittest member's fitness function value is compared with that of the previous fittest one. If a very insignificant improvement is seen for some successive generations then the algorithm is stopped, otherwise all the operations described above are carried out till a model is obtained with a desired objective function.

Table 2.1 : Approximated reduced order models of (2.12) obtained using GA.

Type of system	Reduced- order Model $G_{r,m,\iota}(s)$	Objective Function value
FO System	$G_{0,1,0}(s) = \frac{1.023}{s+0.3866}$	1.6918
FOPTD System	$G_{0,1,1}(s) = \frac{3.155}{s+1.177} e^{-2.41s}$	1.8009
SO System	$G_{0,2,0}(s) = \frac{.5059}{s^2 + 0.2338s + 1.1912}$	0.4958
SOPTD System	$G_{0,2,1}(s) = \frac{0.6167}{s^2 + 0.2343s + 0.2331} e^{-.866s}$	0.056
Third Order System	$G_{2,3,0}(s) = \frac{-0.4159s^2 + 0.5681s + 0.007992}{s^3 + 0.2439s^2 + 0.2297s + 0.00302}$	0.2161
Fourth Order System	$G_{2,4,0}(s) = \frac{-8.838s^2 + 12.58s + 0.1764}{s^4 + 22.34s^3 + 5.619s^2 + 5.017s + 0.0660}$	0.1849

In the present works obtain a lower order integer equivalent of fractional order system by, Genetic Algorithm used to minimizing the objective function *J* different type models obtained integer order approximations are considered. The corresponding approximated models are given in Table 2.1. It is observed that objective function value is minimum for SOPTD model, as expected step response (figure 2.5) and bode plot (figure 2.6) of SOPTD model close to higher order integer order system (2.19) with minimum error.

Now reduce order integer order model of higher order model is rewritten as

$$G(s) = \frac{0.6167}{s^2 + 0.2343s + 0.2331} e^{-.866s} \tag{2.30}$$

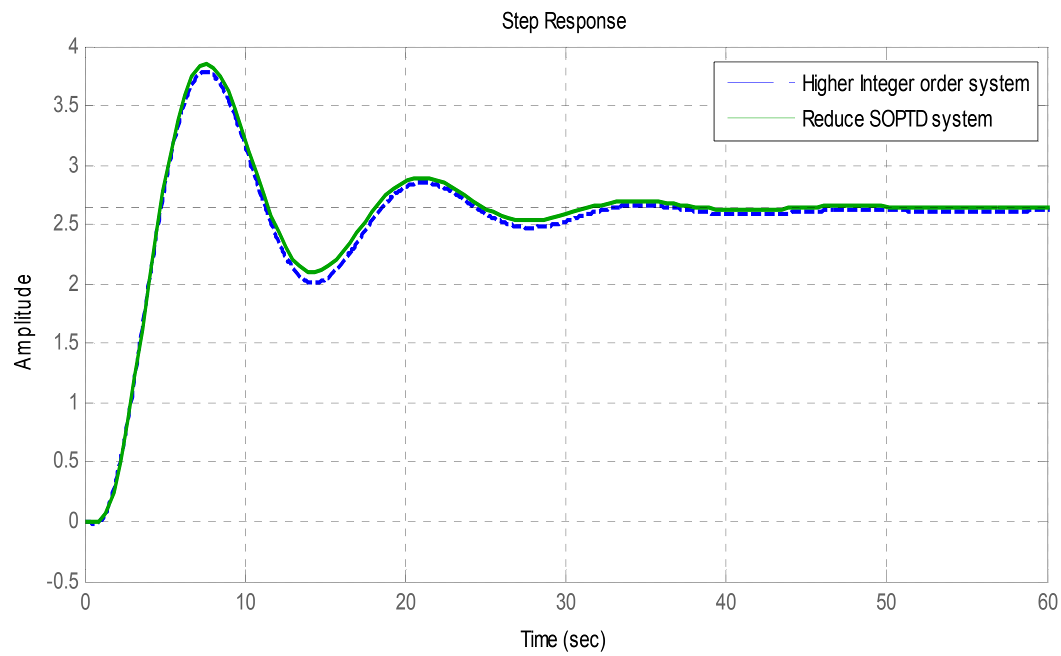

Figure 2.5: Step response of higher order system and reduce SOPTD system

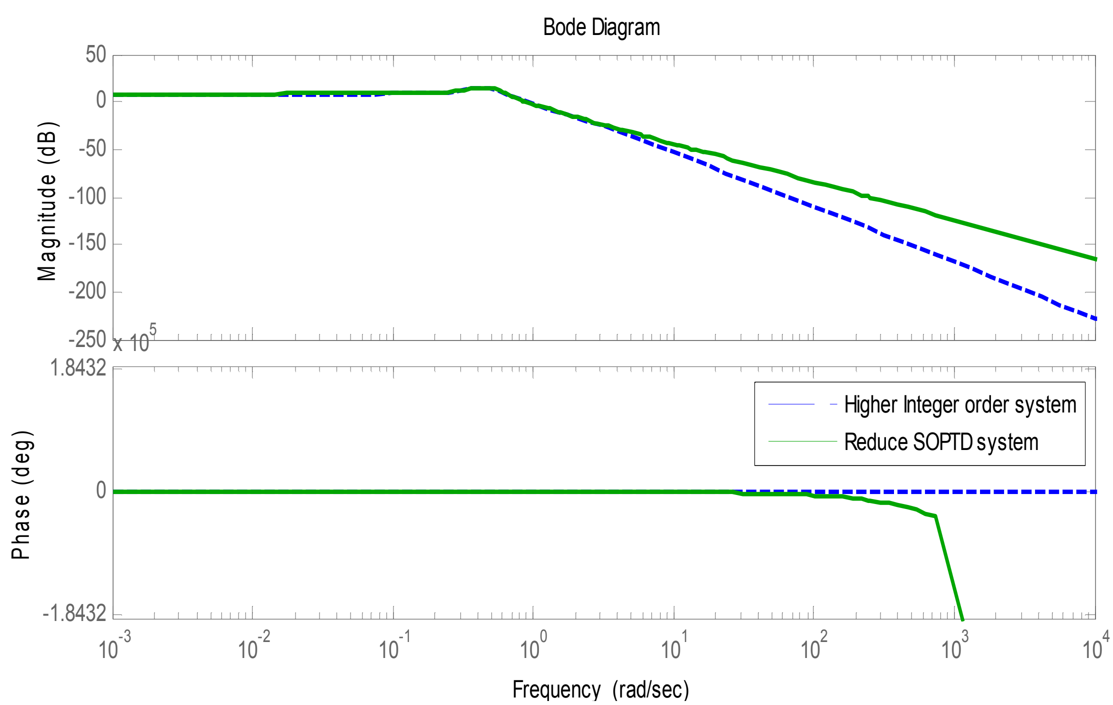

Figure 2.6: Frequency response of higher order system and reduce SOPTD system

FRACTIONAL ORDER CONTROLLER DESIGN

3.1 Introduction to Fractional Order Controller

Controlling industrial plants requires satisfaction of wide range of specification. So, wide ranges of techniques are needed. Mostly for industrial applications, integer order controllers are used for controlling purpose. Now day's fractional order (FOPID) controller is used for industrial application to improve the system control performances. The most common form of a fractional order PID controller is the $PI^{\lambda}D^{\mu}$ controller [11]. FOPID controller provides extra degree of freedom for not only the need of design controller gains (k_p, k_i, k_d) but also design orders of integral and derivative. The orders of integral and derivative are not necessarily integer, but any real numbers. As shown in Fig. 3.1, The FOPID controller generalizes the conventional integer order PID controller and expands it from point to plane. This expansion could provide much more flexibility in PID control design. The transfer function of such a controller has the following form [17]

$$G_c(s) = K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu} \tag{3.1}$$

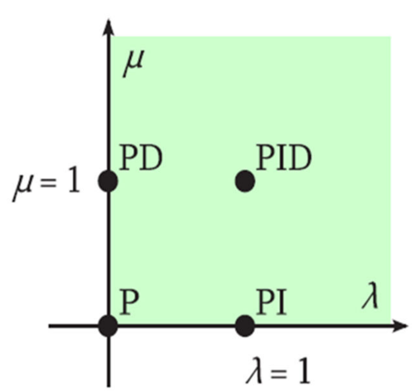

Figure 3.1 : General form of a fractional order PID controller

It is Clear, by selecting $\lambda = 1$ and $\mu = 1$, a classical PID controller can be recovered. Using $\lambda = 1$, $\mu = 0$, and $\lambda = 0$, $\mu = 1$, respectively corresponds to the conventional PI & PD controllers. All these classical types of PID controllers are special cases of the $PI^{\lambda}D^{\mu}$ controller.

3.2 Advantage of Fractional Order controller

As compared to an integer order controller, a fractional order is supposed to offer the following advantages [11]

- If the parameter of a controlled system changes, a fractional order controller is less sensitive than a classical PID controller.
- FOC have two extra variables to tune. This provides extra degrees of freedom to the dynamic properties of fractional order system.

3.3 Fractional order PID Controller tuning

Based on the derived SOPTD model (2.30), the design of PID controller with integer order and Fractional Order dynamics carried out. In the present work Fractional Order PID controllers are tuned based on

1) Frequency domain specifications

2) Time domain based optimial control tuning

3.3.1 Frequency domain Analysis

"Monje-Vinagre" proposed an optimization method fractional controller tuning for tuning of FOPID controllers [13]. In this method, a proposed tuning rule is based on a specified desirable behavior of the controlled system related to specified value of the following objectives.

1. No steady-state error
2. Specified gain crossover frequency

$$\left|C(jw_{cg})G(jw_{cg})\right|_{dB} = 0dB \tag{3.2}$$

3. Specified phase margin ϕ_m represented as

$$-\pi + \phi_m = arg\left(C(jw_{cg})G(jw_{cg})\right) \tag{3.3}$$

4. Robustness against variations of gains of the plant, so around the gain cross over frequency

phase of the open loop transfer function must be constant.

$$\left(\frac{darg\big(C(jw_{cg})G(jw_{cg})\big)}{dw}\right)_{w=w_{cg}} = 0 \tag{3.4}$$

5. For rejecting high-frequency noise, at high frequencies the closed loop transfer function must have small magnitude. Thus it is required that at some specified frequency its magnitude be less than some specified gain.

$$\left|T(jw) = \frac{C(jw)G(jw)}{1+c(jw)G(jw)}\right|_{dB} \leq A\ dB \ \text{ at frequency } w \geq w_t \text{ rad/s} \tag{3.5}$$

6. The sensitivity function must have a small magnitude at low frequencies. To reject output disturbances and tack references it should satisfy the following

$$\left|S(jw) = \frac{1}{1+c(jw)G(jw)}\right|_{dB} \leq B\ dB \ \text{ at frequency } w \leq w_s \text{ rad/s} \tag{3.6}$$

The closed-loop system is required to meet the above define six specifications by properly tuning five parameter of FOPID. All five nonlinear equation (3.2-3.6) needs to be solved simultaneously, to find out FOPID unknown five parameter (k_p, k_i, k_d, λ, μ).

In this section, an FOPID controller is designed for the approximated reduce integer order SOPTD model, obtained in previous chapter i.e.

$$G(s) = \frac{0.6167}{s^2+0.2343s+0.2331}e^{-0.866s} \tag{3.7}$$

The fallowing designing specifications are considered for designing FOPID and integer order PID controller.

- Phase Margin =80 degree
- Gain crossover frequency=0.3 rad/s
- Robustness to variations in the gain of the plant must be fulfilled
- A=-20 dB at $w \geq w_t$ =10rad/s
- B=-20 dB at $w \leq w_s$=.01 rad/s

By solving five design criteria (3.2-3.6)

Fractional order PID controller is obtained as.

$$C_{FOPID}(s) = 0.4805 + \frac{0.0427}{s^{1.4455}} + 0.2243s^{1.6151} \quad (3.8)$$

Putting λ=1 and μ=1 and solving upper five equation (3.2-3.6) corresponding integer order PID controller is obtained as

$$C_{PID}(s) = 0.2960 + \frac{0.0856}{s} + 0.3931s \quad (3.9)$$

3.3.2 Time domain Analysis

For designing controllers based on time domain, controllers aim at minimization of different integral performance indices namely

1) Integral square error $ISE = \int_0^t e^2(t)dt$
2) Integral absolute error $IAE = \int_0^t |e(t)|dt$
3) Integral time-square error $ITSE = \int_0^t te^2(t)dt$
4) Integral time-absolute error $ITAE = \int_0^t t|e(t)|dt$

Starting from random initialized parameters, GA progressively minimizes different integral performance indices iteratively while finding optimal set of parameters for the FOPID and PID controller. The algorithm terminates if the value of the objective function does not change appreciably over some successive iterations.

For model (3.7) the parameters of FOPID and PID are calculated for different performance indices using GA and the results are depicted in Table 3.1. Step response specifications for different FOPID and PID are given in Table 3.2.

Table 3.1: Design of a fractional order PID and Integer order PID controllers of plant with different performance indices.

Type of controller	Performance Index	Minima of performance indices	K_P	K_i	K_d	λ	μ
FOPID	IAE	3.6574	0.6363	0.09869	0.94797	1.4995	1.3358
PID	IAE	3.5552	0.2200	0.1075	0.9227	1	1
FOPID	ISE	1.5206	0.3179	0.20531	0.96305	0.9994	1.1362
PID	ISE	1.7745	0.22496	0.1171	0.91576	1	1
FOPID	ITAE	8.5770	0.26606	0.26179	1.1360	0.94913	1.0559
PID	ITAE	8.7677	0.21974	0.20375	1.0982	1	1
FOPID	ITSE	4.0621	0.22389	0.17318	1.0773	0.9809	1.2784
PID	ITSE	4.1256	0.23349	0.16342	0.94079	1	1

Table 3.2: Comparison of closed loop performance of plant with fractional order PID and integer order PID controller for different performance indices.

Type of controller	Performance Index	Rise time (sec)	Peak time (sec)	Peak overshoot (%)	Setting time (sec)
FOPID	IAE	1.8241	3.5552	18.5823	27.00
PID	IAE	9.4423	17.1450	1.1964	12.980
FOPID	ISE	2.1619	5.8568	1.7525	12.1195
PID	ISE	1.6963	5.8142	8.2923	24.5183
FOPID	ITAE	1.3890	12.0156	4.2754	14.986
PID	ITAE	1.2644	3.4643	0.8964	17.6724
FOPID	ITSE	1.5685	13.1141	2.7964	21.8917
PID	ITSE	1.7199	13.4711	3.4940	22.9981

3.4 Simulation Results and Discussions

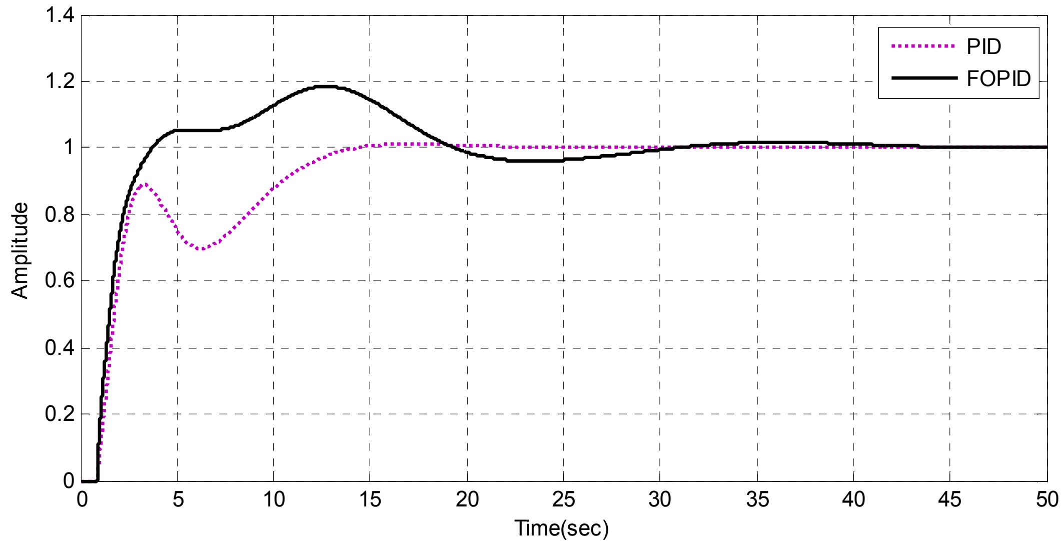

Figure 3.2 :Step Response of FOPID and PID Controller using GA,while considering IAE as objective function.

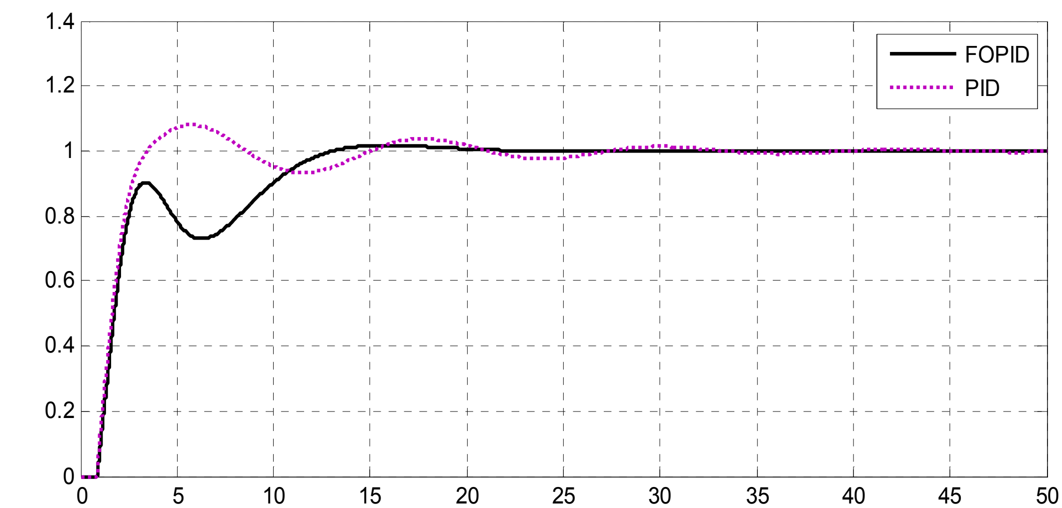

Figure 3.3 :Step Response of FOPID and PID Controller using GA,while considering ISE as objective function.

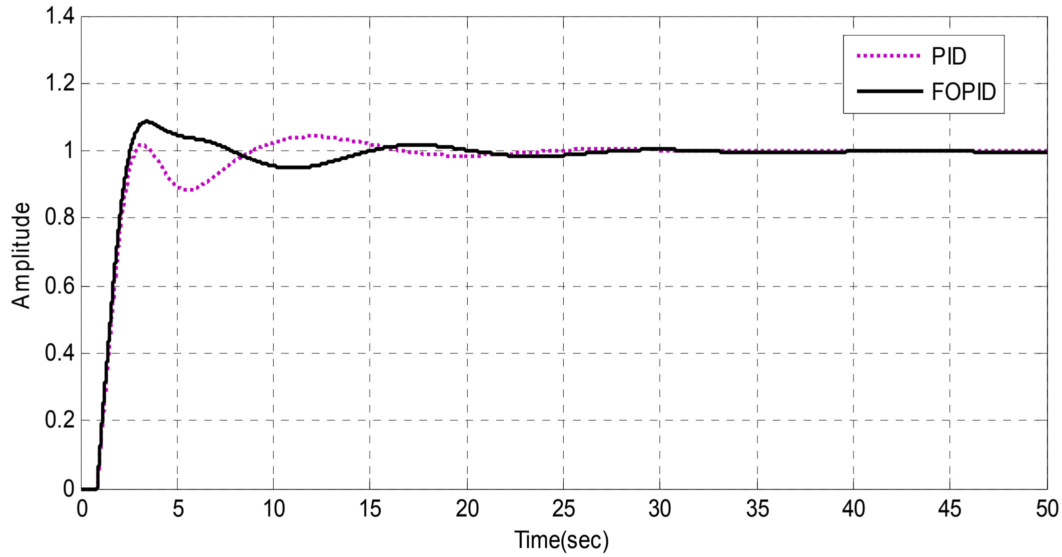

Figure 3.4 :Step Response of FOPID and PID Controller using GA,while considering ITAE as objective function.

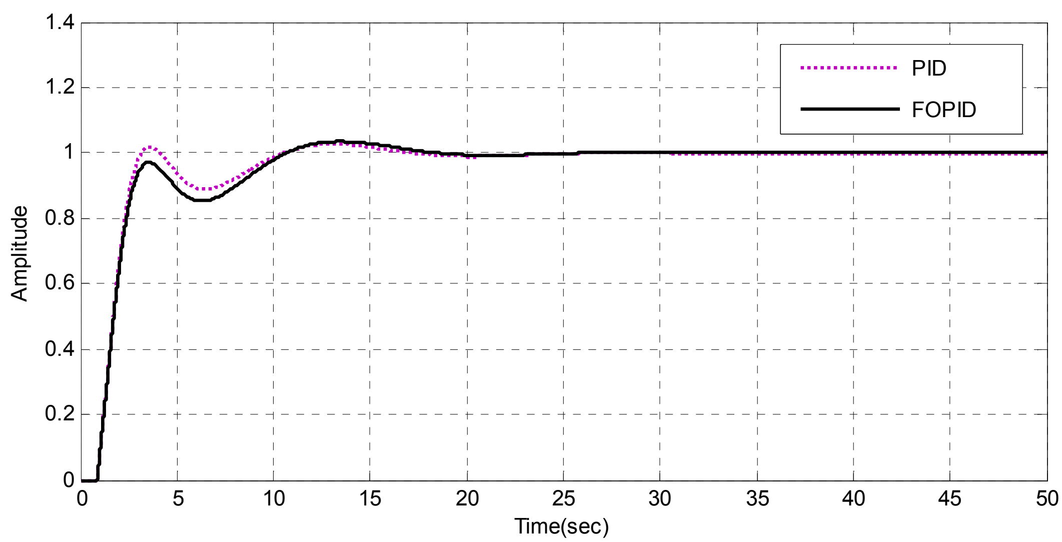

Figure 3.5 :Step Response of FOPID and PID Controller using GA,while considering ITSE as objective function.

Figure 3.2 shows step response of PID and FOPID with IAE performance index result shows that FOPID gives faster response than PID but settling time is more. Figure 3.3 shows step response of PID and FOPID with ISE performance index. FOPID reaches steady state faster with low peak overshoot. Figure 3.4 shows step response of PID and FOPID with ITAE performance index, steady state is reached at earlier for FOPID. Figure 3.5 step response of PID and FOPID with ITSE performance index FOPID gives better performance. In next chapter Fuzzy FOPID and Fuzzy PID are designed for same performance indices and results are compared.

FUZZY CONTROLLER DESIGN

4.1 Introduction of Fuzzy controller

In the present study, the extra of freedom provided by fractional rate of error in the design of conventional FLC based PID controllers [43]. It is logical that the fractional rate of error introduces some extra degree of flexibility in the input variables of FLC and can be tuned also like the input-output scaling factors as the FLC gain and shape of the membership functions (MF) to get enhanced closed loop performance. The present study tests the effectiveness of the proposed fuzzy FOPID controller at producing better performance compared to classical PID, fuzzy PID and even FOPID controllers.

4.2 Fuzzy Fractional order Controller

The structure of the fuzzy PID considered here is a combination of fuzzy PI and fuzzy PD controllers (Figure 4.1). In integer order fuzzy PID controller, the inputs are the error and the derivative of error and the FLC output is multiplied by scaling factor *a* and its integral multiplied with *b* and then summed to give the total controller output [44]. But in the present case the integer order rate of the error at the input to the FLC is replaced by its fractional order counterpart (μ). Also the order of the integral is replaced by a fractional order (λ) at the output of the FLC represents a fractional order summation (integration) of the FLC outputs.

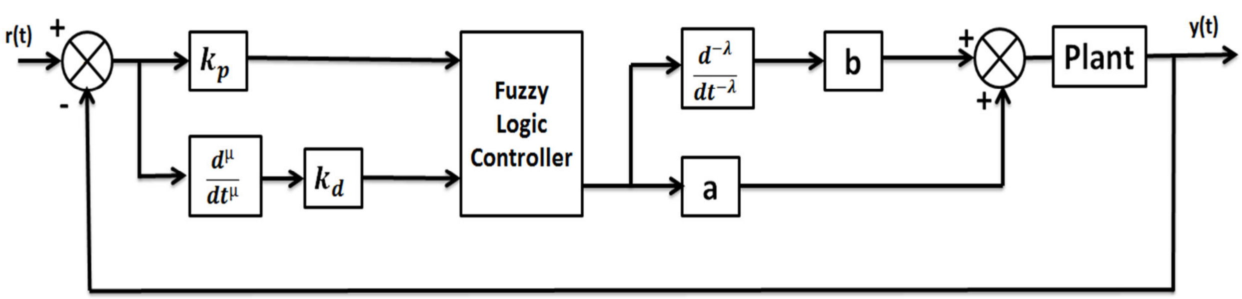

Figure 4.1: Structure of the Fuzzy Fractional Order PID controller

4.3 Fuzzy membership function and Rule base

The proposed FLC based FOPID controller uses a two dimensional linear rule base (Table:4.1) for the error, and fractional rate of change of error and the FLC output with standard triangular membership functions and Mamdani type inferencing [43]. The triangular membership function is chosen over the other types like Gaussian, trapezoidal, bell-shaped, π-shaped etc. as it is easier to implement in practical hardware. In Fig.4.2, the fuzzy linguistic variables NB, NS, Z, PS, and PB with range [-1 1] represent negative big, negative small, zero, positive small and positive big respectively. The FLC output is determined by using centre of gravity method by defuzzification.

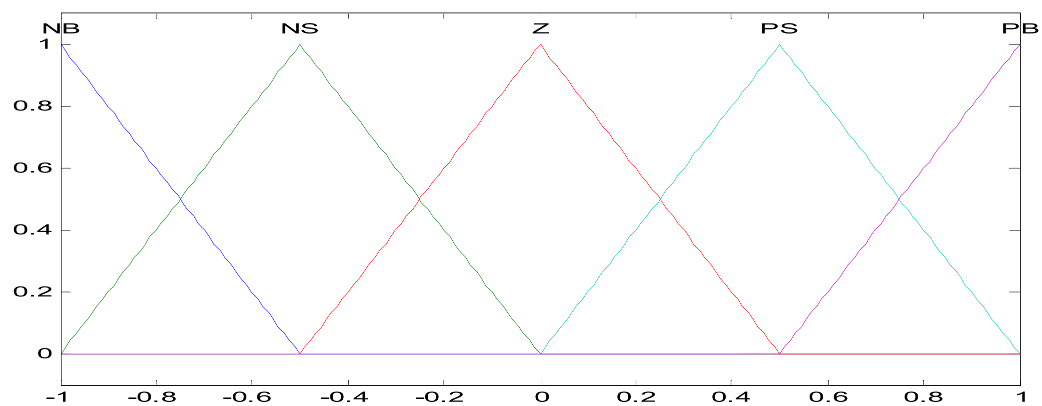

Figure 4.2: Membership functions for error, fractional rate of error and FLC output

Table 4.1: Rule base

DE \ E	NB	NS	Z	PS	PB
NB			NB	NS	
NS		NB	NS	Z	
Z	NB	NS	Z	PS	PB
PS		Z	PS	PB	
PB		PS			

Genetic Algorithm has been used here to find the optimum set of values for the controller parameters. The variables that constitute the search space for the fractional fuzzy PID controller are $\{a, b, K_p, K_d, \lambda, \mu\}$. The intervals of the search space for these variables are $\{a, b, K_p, K_d\} \in [0,10]$ and $\{\lambda, \mu\} \in [0,2]$.

For tuning the GA, a large number of selection and crossover strategies were tested through pilot runs to determine the most suitable ones. Single point crossover is used in conjunction with elitist strategy based Roulette wheel selection. The elitist strategy, which is able to preserve superior strings, is incorporated in the present work by replacing the 10 worst strings of a particular generation with the 10 best strings of the previous generation. The number of chromosomes in the initial population and the maximum number of generations is set at 30 and 150 in each run. The crossover and mutation probability was fixed at 0.8 and 0.1, respectively.

Table 4.2: Design of a Fuzzy FOPID and PID controllers of plant with different performance Indices.

Type of controller	Performance Index	Minima of performance indices	Controller parameters					
			a	b	K_p	K_d	λ	μ
Fuzzy FOPID	IAE	3.1845	0.8171	0.2261	0.9156	2.3820	1.7636	1.0001
Fuzzy PID	IAE	4.4517	2.1832	0.6161	0.2978	0.6022	1	1
Fuzzy FOPID	ISE	2.0505	1.9360	0.5213	0.4996	1.1544	1.7946	1.0003
Fuzzy PID	ISE	2.5313	2.9718	0.7875	0.2504	0.4934	1	1
Fuzzy FOPID	ITAE	7.9320	0.9453	0.2645	0.4976	1.3964	1.7407	0.9213
Fuzzy PID	ITAE	9.2237	1.4990	0.4691	0.4039	0.8549	1	1
Fuzzy FOPID	ITSE	3.3807	2.5679	0.4000	0.7072	0.9281	1.2859	1.2963
Fuzzy PID	ITSE	6.4875	1.4562	0.4050	0.4843	1.0049	1	1

Table 4.3: Comparison of closed loop performance of plant with Fuzzy FOPID and Fuzzy PID controller for different performance indices.

Type of controller	Performance Index	Rise time (sec)	Peak time (sec)	Peak overshoot (%)	Setting time (sec)
Fuzzy FOPID	IAE	1.9481	4.8105	14.522	10.226
Fuzzy PID	IAE	1.9541	4.4517	3.678	28.525
Fuzzy FOPID	ISE	1.4928	4.0974	16.977	47.782
Fuzzy PID	ISE	1.6936	2.5313	8.926	35.263
Fuzzy FOPID	ITAE	2.4760	10.848	2.6260	11.682
Fuzzy PID	ITAE	2.1157	10.867	2.6824	29.202
Fuzzy FOPID	ITSE	1.5720	4.5470	16.840	21.881
Fuzzy PID	ITSE	1.7831	4.1920	3.2531	27.927

4.4 Simulation Results

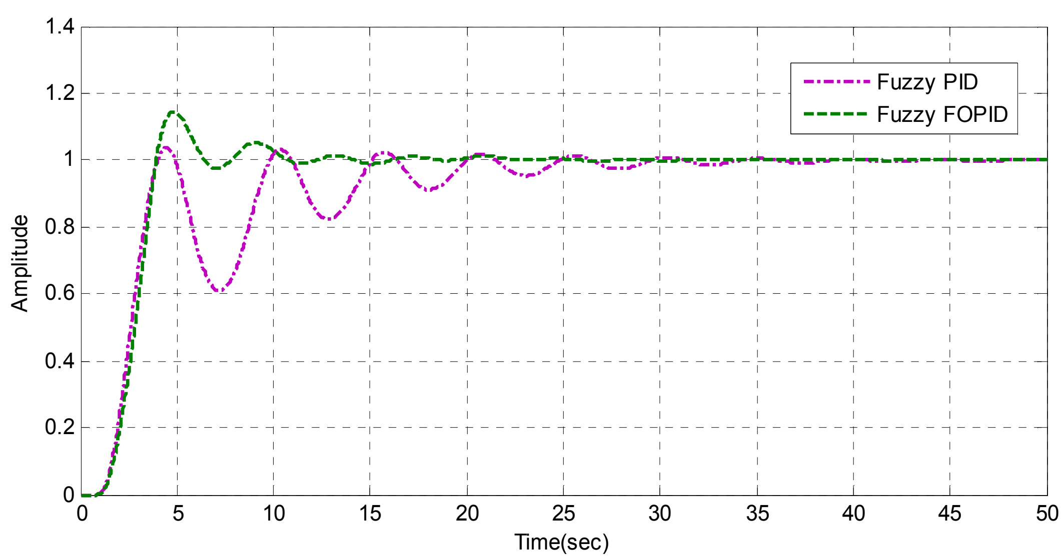

Figure 4.3 :Step Response of Fuzzy FOPID and Fuzzy PID Controller using GA,while considering IAE as objective function.

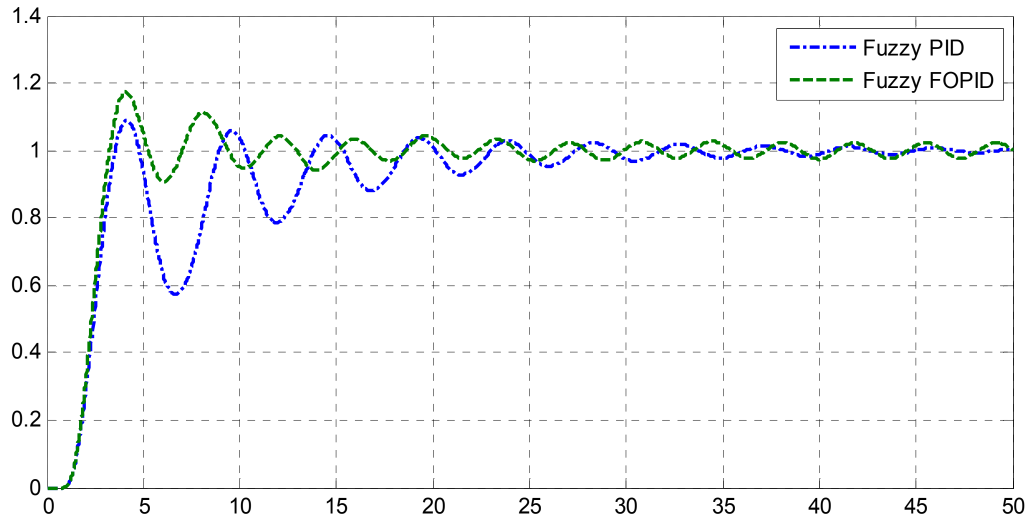

Figure 4.4 :Step Response of Fuzzy FOPID and Fuzzy PID Controller using GA,while considering ISE as objective function.

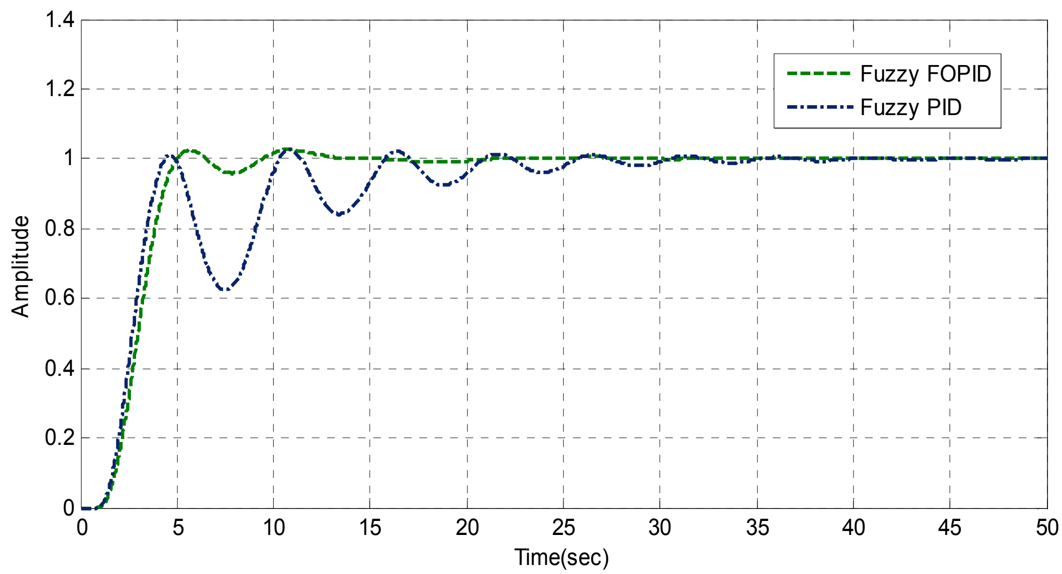

Figure 4.5 :Step Response of Fuzzy FOPID and Fuzzy PID Controller using GA,while considering ITAE as objective function.

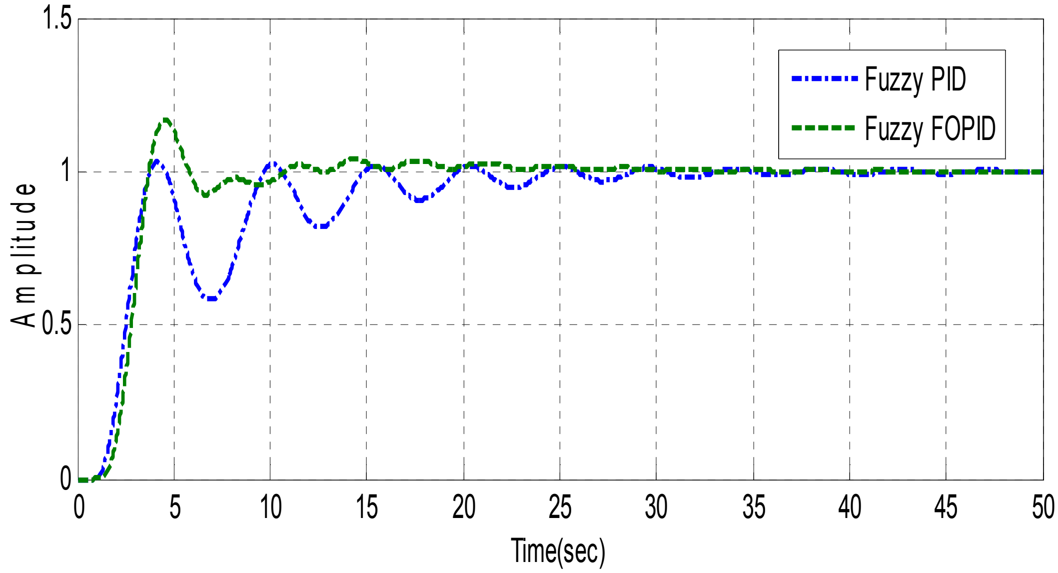

Figure 4.6 :Step Response of Fuzzy FOPID and Fuzzy PID Controller using GA,while considering ITSE as objective function.

4.5 Comparison and Study of Different Controllers

The parameters of fractional order fuzzy logic controller are optimally tuned with GA to handle the model (2.30). Time domain performances of controllers (PID, FOPID, fuzzy PID and fuzzy FOPID) with various integral error indices are compared in this sectioned.

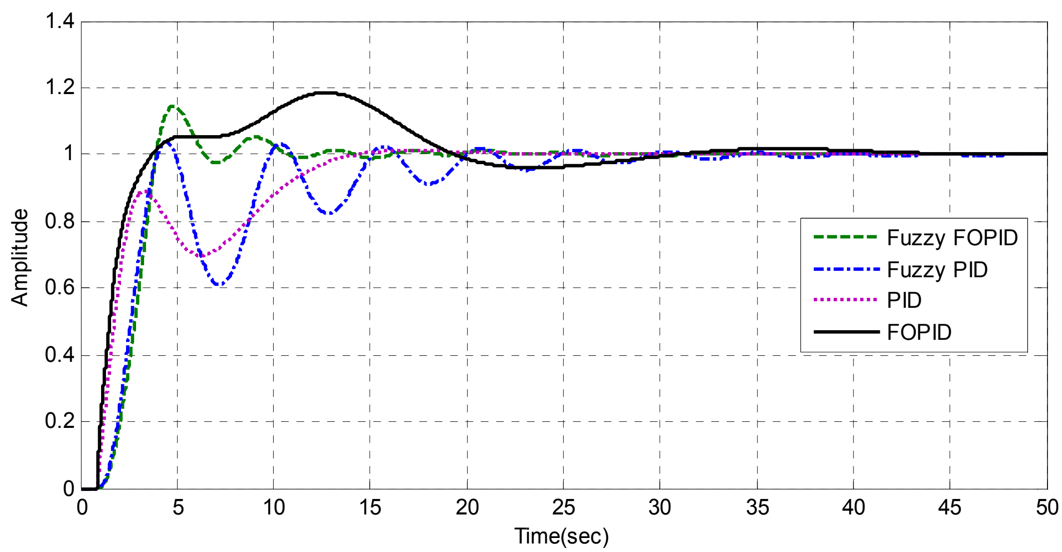

Figure 4.7: Step Response of Controller Design using GA, while considering IAE as objective function.

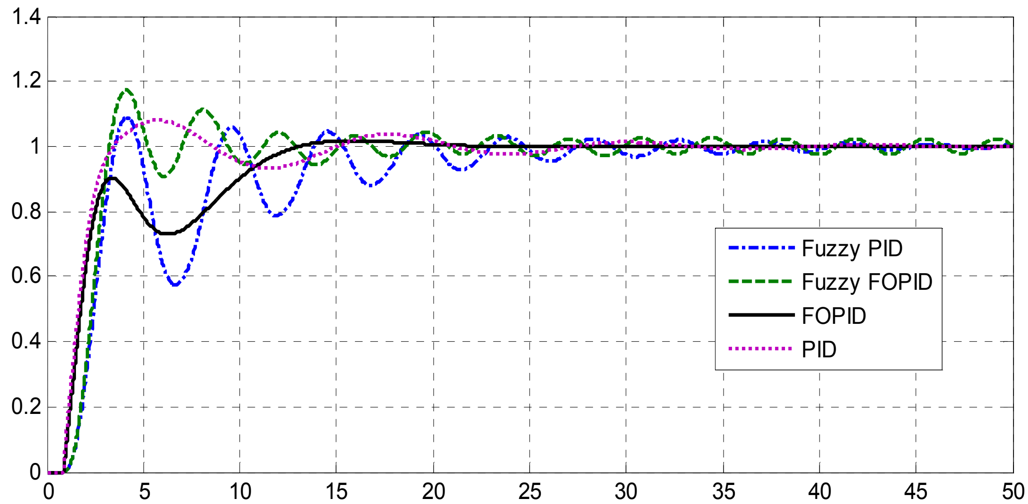

Figure 4.8: Step Response of Controller Design using GA, while considering ISE as objective function.

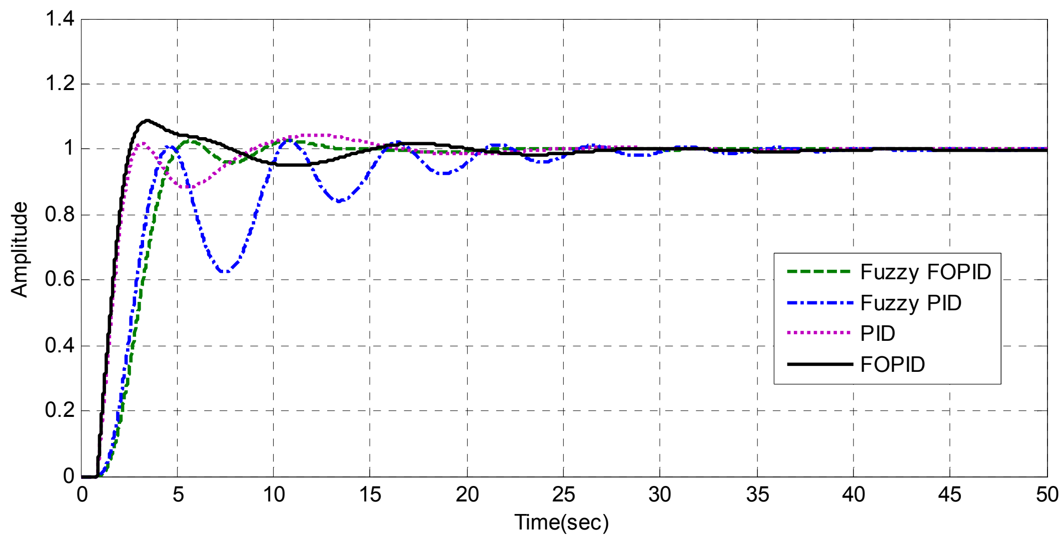

Figure 4.9: Step Response of Controller Design using GA, while considering ITAE as objective function.

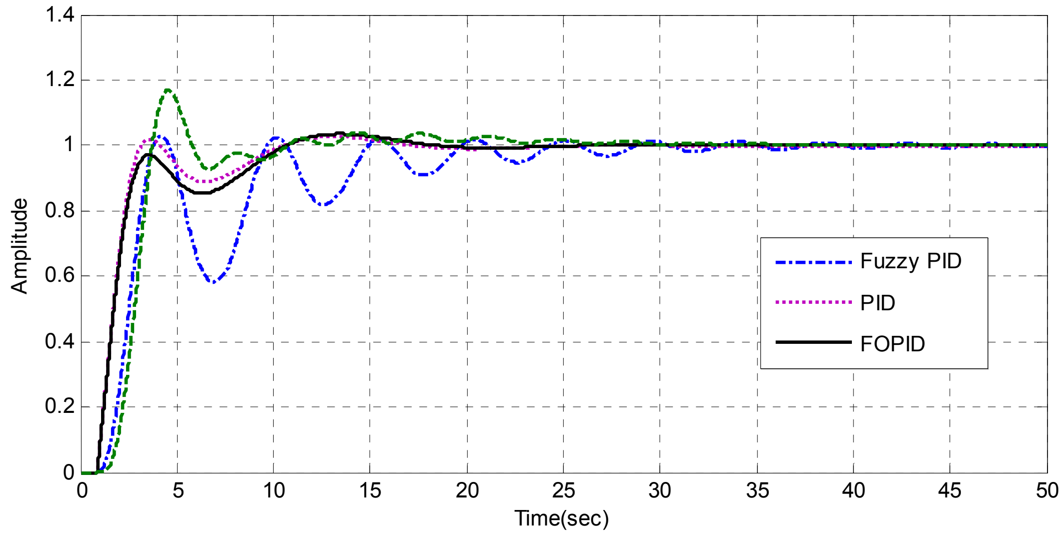

Figure 4.10: Step Response of Controller Design using GA, while considering ITSE as objective function.

For IAE performance index fuzzy FOPID gives faster response as compared to others. Fuzzy FOPID and fuzzy PID both have steady state error for ISE performance index. In ITAE performance index fuzzy PID gives faster response with low overshoot and in case of ITSE all controller performance almost same.

CONCLUSIONS AND FUTURE WORK

5.1 Conclusions

In this thesis work, a fractional order system is represented by a higher integer order system, which is further approximated by second order plus time delay (SOPTD) model. The represented SOPTD model of fractional order system is verified both frequency and time domain gives the approximate representation of fractional-order system. Further, the optimal time domain tuning of fractional order PID and classical PID controller based on Genetic algorithm. Genetic algorithm is used to minimizing various integral performance indices. It is observed that the controller performance depends on the type of process to be controlled and also on the choice of integral performance indices. Considering the uncurtaining in system dynamics, a fuzzy fractional order PID and fuzzy PID is designed. Simulation results show that the fuzzy fractional order PID controller is able to outperform the classical PID, fuzzy PID and FOPID controllers.

5.2 Future work

Future work in this direction will be aimed at improving the performance of fuzzy FOPID controller. It is expected that a fuzzy FOPID controller performance can be achieved by the proper tuning of fuzzy membership functions and the rule base. In this regard, work is planned on the application of evolutionary optimization techniques for the optimal configuration of fuzzy fractional order PID controller.

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