Compressed Image Quality Measurement

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Compressed Image Quality Measurement

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Declaration

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- The work contained in this thesis is original and has been done by me under the guidance of my supervisor.
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Rourkela, May 2013

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C E R T I F I C A T E

This is to certify that the thesis entitled ***"Compressed Image Quality Measurement"*** *by* ***Mr. Sushant S. Sawant****, submitted to the National Institute of Technology, Rourkela (Deemed University) for the award of Master of Technology in* ***Electrical Engineering*** *with the specialization of* ***"Electronic Systems and Communication"****, is a record of bonafide research work carried out by him in the* ***Department of Electrical Engineering****, under my supervision. I believe that this thesis fulfills part of the requirements for the award of degree of Master of Technology. The results embodied in the thesis have not been submitted for the award of any other degree elsewhere.*

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Dedicated to

To My Loving parents, my sister Siddhi and friends Balaji, Prakash, Vipin

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Abstract

The strict requirement of the Nyquist criterion imposes acquiring large amount of data (redundant data). These data when converted to compressed domain can be represented by very few data points. Due to which most of the samples are ignored. So in any signal processing system efficient use of the sensors(if required), the memory requirements and the computational cost are not optimum. This give rise to increase in power requirements, computational complexity and over use of memory storage, which indirectly increases the cost of the system.

Generally the data is stored in compressed domain to reduce the memory requirements. The calculation of the compressed coefficients requires processing time, which is dependent on the number of samples acquired. In most of the Digital systems there is only requirement of estimation of parameter of signal. These parameters are generally computed in the spatial or time domain, which again requires calculation of the inverse of the compressed coefficient. Instead if we were to calculate the parameter in compressed domain itself then the time for inverse conversion would be avoided.

To further reduce the time and storage requirement one can make use of compressive measurement theory. The theory states that the compressed samples acquired can be used for certain parameter estimation. It also helps in reducing number of computations required, with less error in estimation.

One of such parameter to be estimated can be the quality of an image.

Quality estimation is required to provide an objective score to an image. Structural Similarity Index Measurement (SSIM) is one of the quality score under consideration of this thesis. The implementation of compressive measurement with SSIM is the main objective of this thesis. This incorporation will help in reducing the computation which will help in developing a real time system for estimation of quality for stream of data like HD video streaming. The thesis provides with statistical results in support of the developed quality estimation metric.

Keywords: Compressive Sampling(CS), Compressive Measurement(CM), L1-norm, sparsity, Wavelet Transform(WT), Structural Similarity Index Measurement(SSIM), Compressive Measurement SSIM (CM-SSIM).

Contents

List of Abbreviations

Abbreviation	Description
DSP	Digital Signal Processing
ADC	Analogue to Digital Converter
GUI	Graphical User Interface
CCD	Charge Coupled Devices
SSIM	Structural Similarity Index Measurement
CS	Compressive Sensing
CM	Compressive Measurement
RMS	Root Mean Square
MATLAB	Matrix Labratory
CM-SSIM	Compressive Measurement - Structural Similarity Index Measuremnt
FRIQ	Full Reference Image Quality Index
NRIQ	No Reference Image Quality Index
RRIQ	Reduced Reference Image Quality Index
MOS	Mean Opinion Score
DMOS	Differential Mean Opinion Score

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Chapter 1

INTRODUCTION

The sampling rate required for conversion of analog to digital signal, in a Digital Signal Processing (DSP) system, is governed by the Nyquist Sampling theorem[1] which states that 'the sampling frequency is to be twice the maximum frequency content of the analog signal'. This sampling rate heavily taxes the ADC system and hence has lead to limitation on the maximum analog signal that can be acquired in digital format. The rate of sampling has also taxed the computations and the storage requirement.

Due to the limits imposed by the sampling theorem, the signal has to be assumed bandlimited. This assumption usually fails in accurate representation of the data acquired or adds error in the computation process. The error or noise added in the signal has to be filtered out which requires an added designing procedure and computational time of the system. There is always need to compensate accuracy for faster speed in system, which is not always desirable.

The samples acquired from ADC are to be coded to reduce redundance present in the data. Most of the samples after coding are ignored and discarded. The ADC convertor are highly taxed to acquire this samples, but due to the redundancy included there is an inefficient use of the ADCs. To clarify this let use consider wavelet transform of an image and let use compare the inverse wavelet transform image with the original image. The comparison index used for image is SSIM[2] wherein 0 indicates worst image and 1 the best. Visually an SSIM index of 0.94 can be considered a good image in comparison with the original image.

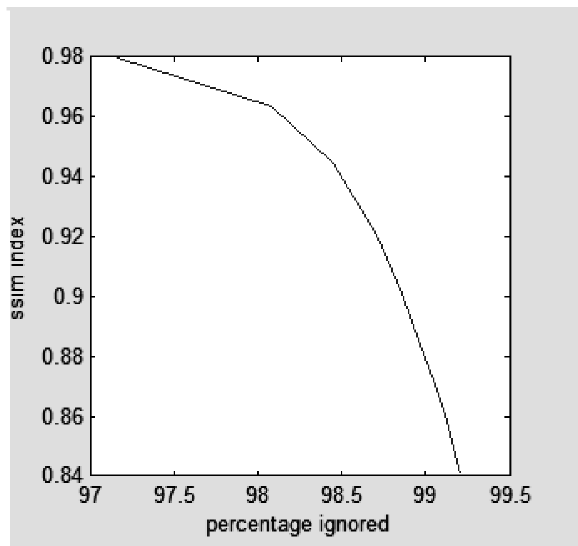

Figure 1.1: Wavelet coefficient ignored vs SSIM

The figure 1.1 shows a GUI output wherein an image is taken and then with decrease in number of wavelet coefficients taken for reconstruction of the image the SSIM is calculated for the reconstructed image with reference to the original image. The figure shows graph of percentage of wavelet coefficient ignored versus the SSIM index. The analysis of the graph shows that with 98% of the wavelet coefficient ignored we get image with SSIM index of 0.96. This can give an understanding of how much the sample acquired are redundant in nature.

The acquisition or sampling of any analog signal (like taking a digital photo) requires large number of sensors (CCD). These sensors have many constraints over the designing of the system like power requirement, acquisition time, physical dimensions, cost of acquisition etc. These criteria imposes many restriction on the system and makes no optimum use of the system.

In recent years a theory has been developed called as Compressive Sensing which tells that if a signal is compressible in a certain transform domain then the signal can be sampled with rate far lesser than that required by Sampling Theorem. There is criteria on the sampling method that has to be implemented, which has to satisfy the property of incoherence with the transform technique used. Reconstruction of the signal is possible with good amount of accuracy following a certain minimum constraint on number of samples taken and the sparsity of the signal. For further understanding of Compressive Sensing refer to chapter 4.

In signal processing, reconstruction of signal may not be the motive of the system designed. There are systems where estimation of parameters of the signals are needed. These estimation also require samples of the signal for estimation purposes. There is a theory based on the Compressive Sensing which says that 'if we sample the signal with use of Compressive Sensing technique then we can estimate the signal parameter from these samples itself', this theory is called as Compressive Measurement. This idea was published in 2006 [3] and is gaining grounds in many estimation problems.

To understand this idea more concretely let us consider a simple experiment, a signal is simulated in MATLAB and RMS value is estimated with use of CM theory, this estimates where compared with the exact RMS values.

The signal shown above was of 1000 samples length. The sampling method used is random sampling to satisfy the property of incoherence. Of the 1000 samples present in the signal, with use of calculation by CS only 90 random samples of the signal where taken. From these samples the RMS was estimated, and the error was calculated in estimation of RMS. This experiment

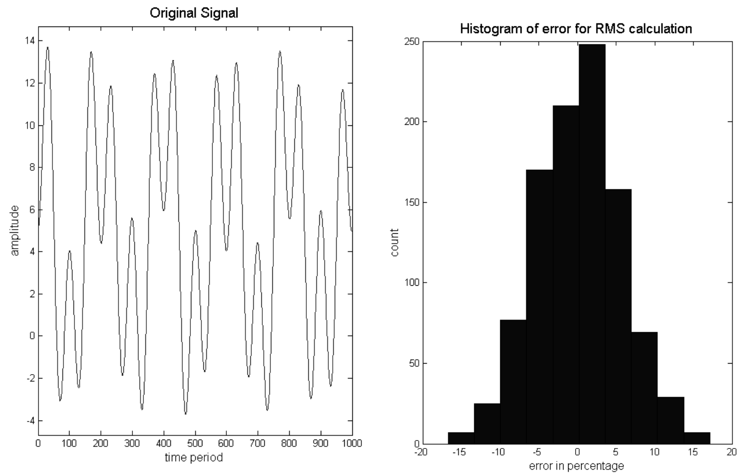

Figure 1.2: Calculation of RMS

was carried for 1000 times. After 1000 trials it was observed from the error histogram that the probability of maximum error is approximately 0.05, with max error being of only 20%. There can be improvement in the result if a more appropriate transform and sampling method is chosen.

From the above discussion we have just got the glimpse of Compressive Measurement Technique in estimation of parameters of the signal. This idea has been used in this paper to estimate the quality of an image.

1.1 Literature Review and Discussion

The following are the papers refereed for the literature survey. It includes the papers name author and brief description of the paper.

The author of the paper [4][5] has provided a good introduction to this theory in general, which can be understood by a novice easily. The paper will give clear understanding of why CS is needed, the requirements for the signal to be satisfied, condition on the sampling method, the performance criteria and a brief explanation of the application is being provided. The paper being

Table 1.1: Literature review

Motive	Reference	Highlights	
		Advantages	Disadvantages
Development of Compressive Sampling Theory	E. J. Candes, M. B. Wakin[4] M. A. Davenport[5]	Introduction to CS theory	Proofs Missing
	D. L. Donoho, P. B. Stark [6]	Initiator of theory	Undeveloped
	D. L. Donoho[7]	Use of L1 norm	Results not present
	E. J. Candes & T. Tao[8][9]	Noise considered	White noise only considered
Application of Compressive Sampling Theory	M. Lustig D. L. Donoho[10]	MRI	Time complexity unmentioned
	M. F. Duarte M. A. Davenport[11]	Single Pixel Camera	Better quality index req.
	R. Robucci[12]	Intelligent Sensor	Comparison with only DCT coeff.
	R. Marcia[13]	Super-resolution image	Better quality index req.
Compressive Measurement	M. A. Davenport P. T. Boufounos[3]	Introduction of CM	Generalisation required
SSIM	Z.Wang[14] A. Bovik[2]	SSIM Explanation with mathematical support	Explanation of some constant undefined

an introductory is not enough in understanding the whole theory but it gives just the glance at what can be achieved by the theory.

The paper[6] wherein the first sign of lesser sampling was raised. The paper was developed by use of uncertainty principle in signal processing to recovery a signal which is sparse in frequency or time domain. The manipulation of the signals property of being sparse was seen here and recovery of signal was sort after. But still here the main idea of CS did not shape up.

The prime need for the efficient working of the CS theory is the need to find a sparse solution to an under-determined system. The solution requires combinatorial optimization, making the problem NP-hard. The solution for such NP-hard have not yet been possible and they are intractable. The paper [7] give mathematical proves for the use of L1 norm instead of L0 norm used earlier. The use of L1-norm breaks down the problem to a linear problem making it easier to calculate. This paper only provide the mathematical part of the said problem, experimental results are being provided here.

The next paper [8] considers the problem if noise is added to signals sample and weather we can use L1 norm to recover the signal or not. Experimental analysis have been provided in this paper. Paper [9] also provide similar analysis for reconstruction of the signal. The paper also discusses about the various sampling schemes that can be applied practically.

The first application developed in this field is in MRI scan, [10] gives introduction to the working of MRI, then it explains why we need CS in it. Finally it gives idea about how we can incorporate CS into the MRI scanning technique. This paper can be considered as one of the references to application of CS theory. The next application developed is a single pixel camera [11]. This give the a new camera architecture using CS theory, where only one pixel is used to sense light intensity. This camera can be used for video processing also. The estimation in image like the direction of motion of an object can also be done using this camera with good accuracy. There have been many design suggestions give before and even after this paper. The analysis of these application can help to further increase the understanding of the theory in concern.

1.2 Motivation:

Huge amount of data is constantly being handled by our communication channels. The data being received is reconstructed for user understanding. The reconstruction of data taxes the processor and if the received data is faulty then data must be discarded or processed upon. If it would be possible to estimate the quality of the signal in the decoded state itself then the unnecessary use of reconstruction of data and discarding it would be saved. If the concept of Compressive Measurement is used then the estimate will require less processor hardware.

HD videos or Digital images are huge amount of data which are in compressed state when being transmitted through data channel or when are stored as a file in the memory. Reconstruction of these data is required when the quality is to be determined. So the use of Compressive Measurement can be possible to estimate the quality of the video or images. The idea to estimate the SSIM index in compressible state is discussed in this paper.

1.3 Objectives

The salient objectives of the thesis are:

i. Building the background for Compressive Measurement and the SSIM quality measurement indexing parameter.

ii. Explaining the developed Compressive Measurement SSIM

iii. Analysis and statistical backing for the developed CM-SSIM

1.4 Thesis Organisation

The thesis is organized as follows.

- Chapter 1, introduction of the thesis.
- Chapter 2, short discussion on FRIQ algorithms and SSIM index.
- Chapter 3, analysis of SSIM.
- Chapter 4, brief description of the developed algorithm.
- Chapter 5, results of the developed quality measure discussed.

Chapter 2

FULL REFERENCE IMAGE QUALITY MEASUREMENT

2.1 Introduction: Full Reference Image Quality Index

Digital images undergo various types distortion during acquisition, processing, storage, compression and reproduction. For application where the processed images are to be perceived by human eye there is requirement of a subjective analysis of the processed image to provide the quality of the image. This might not be possible always as it requires time, money and is highly inconvenient. To replace this highly cumbersome process we can model an algorithm that can approximately behave like a human eye. The algorithm will give an objective quality score for the image under test and the score will tell us how poor the quality of the image is, without the requirement of a subjective analysis.

There are many algorithms developed to provide an index for the image quality analysis. These algorithms have been divided into three parts namely Full Reference Image Quality Index(FRIQ), No Reference Image Quality Index(NRIQ) and Reduced Reference Image Quality Index(RRIQ). Of these the interest of the thesis lies in FRIQ. In FRIQ index the quality of the test image is measured by comparing the reference image or undistorted image with the test image. The algorithm using certain parameters of the image estimates the quality score of the test image with reference to the undistorted image and give the quality score of the image. The algorithm that has been used in this project for analysis is SSIM. The explanation of the same is being provided in the next subsection.

2.2 Structural Similarity Index Measurement: SSIM

SSIM [2, 14] is a FRIQ indexing algorithm which is why it requires a reference image to estimate the quality of the test image. The parameter that are considered for comparison are the luminance, contrast and structure of the images. These three factors are estimated from the images and a relative score is being provided to the test image. The factors mentioned are some of the important factors used by the human eye to provide a subjective analysis of the images. These physical factor are simulated with use of the basic statistical parameters like mean, variance and covariance.

The mathematical formulation of the physical parameters being calculated have been provided below.

2.2.1 Luminance Comparison

$$l(x,y) = \frac{2\mu_x\mu_y + C_1}{\mu_x^2 + \mu_y^2 + C_1} \tag{2.1}$$

$$C_1 = (K_1 L)^2 \tag{2.2}$$

Equation 2.1 gives the statistical analysis of the luminance comparison of two images. μ indicates the mean of the image pixel. The C_1 factor is added to provided stability to the equation in case of zero in the denominator. In the equation 2.2 the constant L defines the range of pixel intensity values in an image which is generally 255, hence considered a constant for most of the case. As L is considered constant for most of the cases, the constant C_1 is then governed totally by the constant K_1. The constant $K_1 \ll 1$, where the value is decided such that it should provide stability to the equation as well as should not dominate the comparison factor.

2.2.2 Contrast Comparison

$$l(x, y) = \frac{2\sigma_x\sigma_y + C_2}{\sigma_x^2 + \sigma_y^2 + C_2} \tag{2.3}$$

$$C_2 = (K_2L)^2 \tag{2.4}$$

Equation 2.3 gives the statistical analysis of the contrast comparison of two images. σ indicates the standard deviation of the image pixel. The C_2 factor is added to provide stability to the equation in case of zero in the denominator. In the equation 2.4 the constant L defines the range of pixel intensity values in an image which is generally 255, hence considered a constant for most of the case. As L is considered constant for most of the cases, the constant C_2 is then governed totally by the constant K_2. The constant $K_2 \ll 1$, where the value is decided such that it should provide stability to the equation as well as should not dominate the comparison factor.

2.2.3 Structure Comparison

$$l(x, y) = \frac{2\sigma_{xy} + C_3}{\sigma_x\sigma_y + C_3} \tag{2.5}$$

Equation 2.5 gives the statistical analysis of the structure comparison of two images. σ_{xy} indicates the covariance between the image pixels of two images under consideration. The constant C_3 is same as that in previous two cases.

These comparison parameters are then combined together to for a unique Index called as SSIM index. The contribution of the parameters is generally taken as equal and so the resultant equation for SSIM calculation after some simplifications and assumption is

$$SSIM(x, y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)} \tag{2.6}$$

The values of the constants used, which where defined above, are as follows,

- $K_1 = 0.01$
- $K_2 = 0.03$
- L = 255

SSIM is calculated locally instead of globally, this approach is undertaken to extract details from the image locally to estimate the comparison parameters. The use of a window is done to calculate the SSIM locally. The window which is considered in the project work is of size 11×11. The distribution of the window can be rectangular or gaussian distribution. The gaussian distribution is preferred to avoid blocking effect which is predominant in rectangular window. The project considers gaussian distribution with parameters μ=0 and σ=1.5.

For more understanding of how SSIM is calculated one can refer to [2] and [14].

2.3 Chapter Summary

This chapter has given the description of the SSIM algorithm, being studied and modified for calculation in the transform domain. The chapter forms the mathematical base for the coming chapters. The mathematical formulation will be used in the development of the new algorithm for estimation of the quality of the image.

Chapter 3

ANALYSIS OF SSIM

3.1 Introduction

The earlier chapter describes about the SSIM indexing parameter. There are certain pointers about SSIM stated below,

- Being a FRIQ method it always requires a reference image which many times is not possible to produce.
- The constant used in the formulation are not defined.
- For reduction in time complexity down-sampling of image has been carried out.
- For calculation of the image quality index images should be in the spatial domain. This requires conversion of the image from sparse domain to spatial domain which will be problematic in time critical systems.

Of these the fourth point has been addressed in these thesis and will be dealt in coming chapters. This chapter will deal with the second and third points.

3.2 Effect of variable Constants

The calculation of SSIM as seen in the equation 2.6 includes two constants,

C_1 & C_2, who's use have been explained in the section 2.2. The constants are being defined by another set of constants namely K_1 & K_2, which provides stability to the SSIM equation. These constants have not been defined, meaning there is no fixed value to be considered while calculating SSIM, this can lead to faulty interpretation of the quality of the index. Let us consider an original image and its distorted gaussian noise ($\sigma = 0.005$ and $\mu = 0$) as shown below,

Figure 3.1: Original Image and Distorted Image (Gaussian Noise $\sigma = 0.005$ & $\mu = 0$)

The SSIM value where calculated in MATLAB for which the code has been provided in the link[15]. There where three different SSIM being calculated with use of different K_1 & K_2 values. The values have been tabulated below;

Table 3.1: SSIM Variation

K_1	K_2	SSIM VALUE
0.01	0.03	0.4612
0.05	0.05	0.5725
0.01	0.01	0.3753

Of the values stated the $K_1 = 0.01$ & $K_2 = 0.03$ is considered as default in the SSIM code. If we calculate the percentage error in calculation of SSIM, (assuming the default values as the reference), then the percentage error for values $K_1 = 0.05$ & $K_2 = 0.05$ is almost -25% and for values $K_1 = 0.01$ & $K_2 = 0.01$ is almost 20%. These error in estimation of the quality of the image can lead to faulty decisions. This is a rough analysis on the problem at hand future explanation is being provided in the preceding sections.

3.3 Mathematical and Experimental analysis

This section will deal with the experimental and mathematical analysis on the effect of the K_1 & K_2 values on the estimation of SSIM index.

3.3.1 Experimental Analysis

To show the effect of K_1 & K_2 variations an experiment was carried out in which an image and its fixed degraded image was taken. The image is taken from a standardized database (LIVE Database-buildings.bmp) and its degraded image was also taken from the same database (fastfading-img67.bmp). The SSIM for default values of K_1 & K_2 is 0.6632. The two figure are shown in figure 3.2. For the degraded image SSIM was calculated for varying K_1 & K_2 values. The range in which K_1 & K_2 were varied is 0 to 1 with increment of 0.01. The result of the simulation are shown in figure 3.3.

Figure 3.2: Original Image and Distorted Image

The figure 3.3 shows plot of variations of SSIM with respect to K_1 & K_2. From the plot it can be inferred that for fixed value of K_1, SSIM varies with K_2 and for fixed value of K_2, SSIM remains almost constant with variation of K_1.

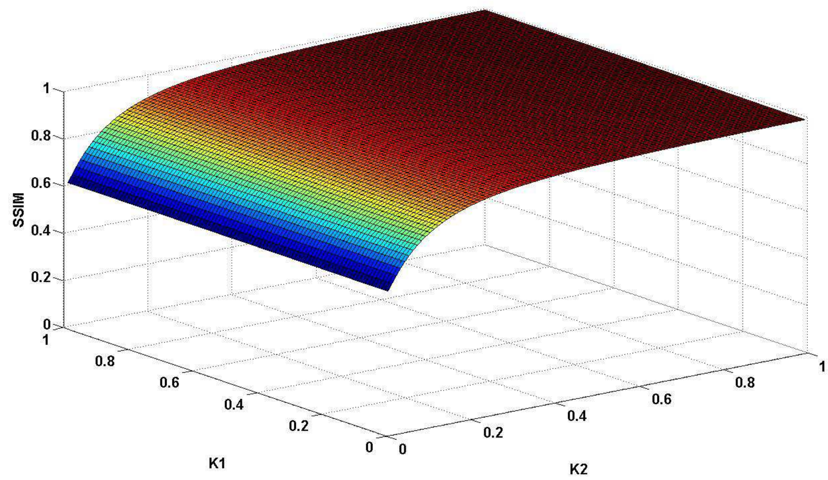

Figure 3.3: Surface plot of SSIM and K_1 & K_2

The next experimental setup indulge in analysing Pearson correlation variation with the change in the values of the constants. The setup includes calculation of SSIM on a standardized database and calculation of Person correlation of the SSIM value with the corresponding DMOS values of the distorted image (more on this in chapter 5). The database used for this purpose is the CSIQ database (information on this is provided in chapter 5). Figure 3.4 shows the result of this experimental setup. The figure shows plot of Pearson correlation values with respect to varying values of K_1 & K_2. The range of values in which the constants were varied were 0 to 1, with increment of 0.01.

From the plot it can be observed that as the value of K_2 increases keeping the value of K_1 constant, the Pearson correlation decreases by almost 10%. But this is not true for the case of variation of K_1 keeping K_2 constant. The Pearson correlation is constant for variation of K_1 but vary with changes in value of K_2.

From these set of results we can see that the constant K_2 has a major effect on the estimation of the SSIM index. The effect can be seen both in

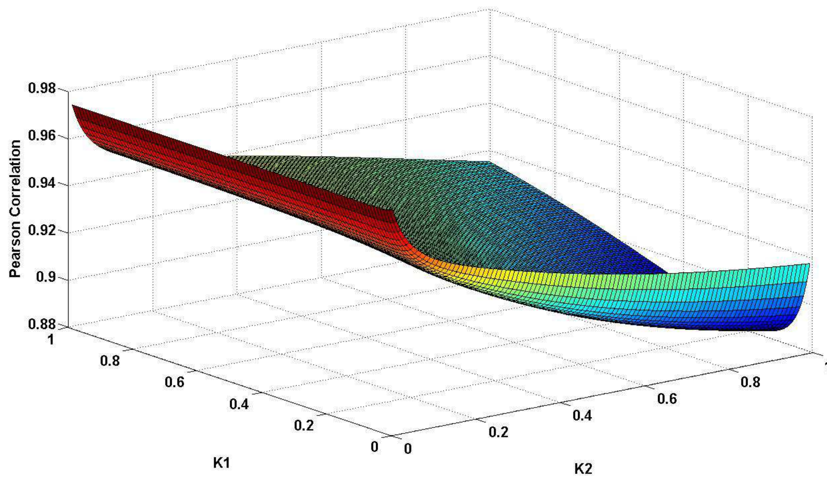

Figure 3.4: Surface plot of Pearson Correlation and K_1 & K_2

the changes in values of SSIM as well as changes in the Pearson Correlation values of SSIM. This effect the accuracy in estimation of the SSIM index which can lead to a faulty interpretation of the index, which indirectly indicates the quality of the image.

3.3.2 Mathematical Analysis

This section will deal with calculation of sensitivity of SSIM with both K_1 & K_2. The set of equations are as follows:

From equation 2.6 we have,

$$SSIM(x,y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)} \tag{3.1}$$

Now from the explanation provided in 2.2 $C_1 = (K_1L)^2$ and $C_2 = (K_2L)^2$, equation 3.1 gets converted to,

$$SSIM(x,y) = \frac{(2\mu_x\mu_y + (K_1L)^2)(2\sigma_{xy} + (K_2L)^2)}{(\mu_x^2 + \mu_y^2 + (K_1L)^2)(\sigma_x^2 + \sigma_y^2 + (K_2L)^2)} \tag{3.2}$$

Equation 3.2 is going to be the basis equation for calculation of sensitivity of SSIM with respect to the constants K_1 & K_2.

Sensitivity of SSIM with respect to K_1 will be calculated by keeping K_2 constant. The other parameters like the μ_x , μ_y, σ_{xy}, σ_x, σ_y & L are also to be held constant(we are assuming that SSIM is being calculated on same set of images, original and distorted image). The equation 3.2 will be brought down to,

$$SSIM(x,y) = \mathcal{C}_1 \frac{(2\mu_x\mu_y + (K_1L)^2)}{(\mu_x^2 + \mu_y^2 + (K_1L)^2)} \tag{3.3}$$

where $\mathcal{C}_1 = \frac{(2\sigma_{xy}+(K_2L)^2)}{(\sigma_x^2+\sigma_y^2+(K_2L)^2)}$.

Taking a partial derivative of equation 3.3 with respect to K_1 we get,

$$\frac{\partial SSIM}{\partial K_1} = \mathcal{C}_1 \frac{(2K_1L^2)(\mu_x - \mu_y)^2}{(\mu_x^2 + \mu_y^2 + (K_1L)^2)^2} = \mathcal{C}_2 \frac{K_1}{(1 + K_1^2(\frac{L^2}{\mu_x^2+\mu_y^2}))^2} \tag{3.4}$$

where $\mathcal{C}_2 = \mathcal{C}_1 \frac{2L^2(\mu_x-\mu_y)^2}{(\mu_x^2+\mu_y^2)^2}$.

Similarly while calculating sensitivity of SSIM with respect to K_2, K_1 is to kept constant. The other factors mentioned above are also to be assumed constant (we are assuming the SSIM is being calculated on same set of images, original and distorted image). The equation 3.2 will be,

$$SSIM(x,y) = \mathcal{C}_1' \frac{(2\sigma_{xy} + (K_2L)^2)}{(\sigma_x^2 + \sigma_y^2 + (K_2L)^2)} \tag{3.5}$$

where $\mathcal{C}_1' = \frac{(2\mu_x\mu_y+(K_1L)^2)}{(\mu_x^2+\mu_y^2+(K_1L)^2)}$.

Taking a partial derivative of equation 3.5 with respect to K_2 we get,

$$\frac{\partial SSIM}{\partial K_2} = \mathcal{C}_1' \frac{(2K_2L^2)(\sigma_x - \sigma_y)^2}{(\sigma_x^2 + \sigma_y^2 + (K_2L)^2)^2} = \mathcal{C}_2' \frac{K_2}{(1 + K_2^2(\frac{L^2}{\sigma_x^2+\sigma_y^2}))^2} \tag{3.6}$$

where $\mathcal{C}_2' = \mathcal{C}_1' \frac{2L^2(\sigma_x-\sigma_y)^2}{(\sigma_x^2+\sigma_y^2)^2}$.

From the derived equations 3.4 and 3.6, we can see that the sensitivity equations are of the form $f(x) = ax/(1 + bx^2)$ where a, b are constants and x in our case are K_1 and K_2. As the range in which K_1 and K_2 are varied are same then the sensitivity parameters depend upon the constants 'a' and 'b'. These constants in case of equation 3.4 are the μ terms of image X and Y, and in case of 3.6 are the σ terms of image X and Y. To understand the effect of μ and σ, in case of an image, we have to consider the range of values in which the μ & σ vary.

Let us consider the minimum and maximum for μ. The minimum can be considered to be zero which is not possible in an image, as μ=0 implies that either all the pixel values of image will be zero (no information content) or that there are some pixel having a negative intensity values which is naturally not possible to occur in an image. The max value that μ can reach is 255, which is again highly unlikely as it would imply that all the pixel intensity have value as 255 which would have no information content at all. For simplicity let us consider the min value as 0 and max value as 255. So the range in which the μ varies is 0 to 255.

Considering σ now for its estimation of max and min. The equation $\sigma_x = \sqrt{\frac{1}{N}\sum(x_i - \overline{x})^2}$ suggests that there cannot be any negative value of σ. So the minimum value which the σ can achieve is zero, which implies that there is no variation in the image or no information content in the image. This case is highly unlikely in a natural image. To consider the case of max value we have to consider that in an image there are pixel values of 0 and 255 intensity levels only. Let the number of 0 in an image be denoted as N_0 and number of 255 be N_{255}. For maximum value of σ, $0 < N_0 \ll N_{255}$ or $0 < N_{255} \ll N_0$, if this occurs then the maximum value of σ is around 20. So the range of value of σ is 0 to 20.

A plot of sensitivity curve for both μ and σ variation, keeping K_1 and K_2 constant, were plotted and shown in figure 3.5. From the plot we can see that the variation in σ are more dominating to the sensitivity curve than the variation in the values of μ. As σ is associated to K_2, SSIM variation is dependent on K_2 and not on K_1, because variation due to K_2 dominates K_1. This can be again proved in the figure 3.6.

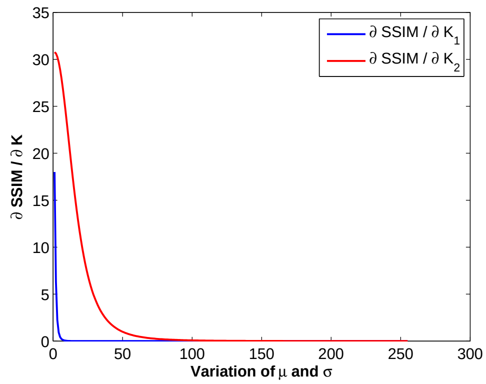

Figure 3.5: Sensitivity plot for variation of μ and σ

To show the effect of the sensitivity equations 3.4 & 3.6, an image of 'cameraman.tif' was taken in which the sensitivity curves were plotted. While plotting equation 3.4 K_2 was considered as 0.01, and while plotting equation 3.6 K_1 was considered as 0.01. The sensitivity plot shows the plot of the equation on a single patch of the image. The plots are shown in figure 3.6.

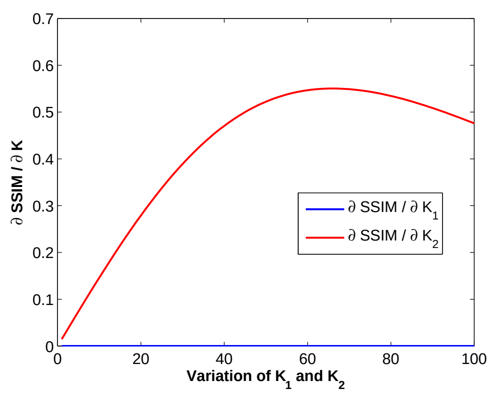

Figure 3.6: Sensitivity plots of K_1 & K_2

From figure 3.6 we can see that the sensitivity of SSIM or the change in

SSIM for corresponding change in K_2 is linearly increasing and is dominating to the changes due to K_2.

3.4 Down-Sampling of Image

If one observes the MATLAB code being in the link [15], one can find that the image of certain size (size of images above 200x200 pixels) are low pass filtered and down sampled for the calculation of the SSIM index. This process is done to reduce the time complexity of the code. This method of calculation of quality score index can lead to error in calculation of the SSIM index as there is high probability that certain aspect of image which is highly affected by noise be ignored.

For analysis purpose the SSIM algorithm was altered, where the step where down-sampling occurs has been removed and the SSIM is estimated. This algorithm is termed as 'SSIM ALTERED'. This algorithm will be used in chapter 5, which would help in justification of use of the developed algorithm explained in chapter 4 called CM-SSIM.

This brings us to the concluding part of this chapter from which we have understood the effect of the constant on the value of SSIM and also the down sampling of image done in the SSIM algorithm.

3.5 Chapter Summary

In the results shown in figures 3.3, 3.4, 3.5 and 3.6 we can infer that the value of SSIM is more dependent on K_2 rather than K_1. The sensitivity of SSIM were shown with both variation of σ, μ, K_1 & K_2 variations. The reasons were explained with both mathematical and experimental analysis.

For this reason there should be a fixed value of the constants taken for the calculation of SSIM. As seen from the graph from figure 3.6 the SSIM is less sensitivity to K_2 for lower values, so it would be better if we taken the values in range of $0 < K_2 < 0.4$. For these reasons the values that have been considered for analysis purpose are, K_1=0.01 and K_2=0.03.

The last part just gave a rough idea of what has been discussed in chapter 5. The down sampling method can lead to erroneous estimation of the quality of the image, which can lead to failure of any system which is going to use this index parameter for its operation. So a revised method has to be adopted to encounter this problem. This solution is being discussed in the next chapters.

Chapter 4

CM-SSIM

To find an alternative method to down-sampling of images for calculation of quality score, or to calculate the quality score in the spectral domain (to avoid the inverse transform of the transform coefficient) we have suggested another algorithm called as Compressive Measurement-SSIM (CM-SSIM). This metric will have the advantage of reduction in number of computation required, as well as we can help in designing a real time embedded system for estimation of huge stream of data like the HD video streaming.

The development of CM-SSIM required help of the Compressive Measurement (CM) Theory [3] and the developed SSIM [2][14] algorithm. There was also requirement of certain mathematical tools which have been mentioned in this chapter. To understand the CM theory one has to understand the Compressive Sampling (CS) Theory, explained in section 4.1.

4.1 Compressive Measurement

This section is about compressive sampling/sensing [4][5]. The following are the general discussion on the theory with the mathematical formulation involved in the theory.

Let f be the signal of concern, let ϕ be the sampling operator and y is the sampled output.

So we have,

$$y_k = \langle f, \varphi_k \rangle \tag{4.1}$$

where k indicates the indices.
Now generally in the normal sampling system the sampling matrix is a square matrix, meaning the number of samples y will be equal to number of equation formed by the inner product $\langle *, * \rangle$ of signal(f) and the sampling matrix(ϕ). At the receivers end(or output) we will be receiving the samples(y). We will be knowing the sampling scheme(ϕ), by which information we will be able to get the signal(f) by solving simultaneously the set of equations.

Now consider a case that we take lesser amount of samples and we have to reconstruct the signal. Logically the set of equations are under-determined and there is no unique solution to it. To solve this problem we can take help of CS theory which says that such under determined system can be solved under the condition that the signal is sparse in nature. To make the signal sparse we can make use of a transform domain where the signal can be sparse in nature. Let's say that the signal is sparse in Ψ domain and the transform coefficients of signal be x. Then solution of the problem can be explained as follows.

If $y_k = \langle f, \varphi_k \rangle$ where the samples(y) will be lesser than required $(m < n)$.

$$\min_{\tilde{x} \in R^n} \|\tilde{x}\|_{l_1} \tag{4.2}$$

subject to $y_k = \langle \varphi_k, \psi \tilde{x} \rangle$.

There is one more condition that the system developed has to satisfy which is that the sampling matrix and the transform matrix that gives sparse solution of the signal are to be incoherent with each other. This condition ensure that we get maximum information from the lesser amount of samples available with us, which help in determining error in the reconstruction of the system.

The next consideration that we have to go through is the sampling rate suggested by the CS theory. The factor that affect the sampling rate for CS are as follows.

- Firstly we have to consider the saparsity (S) of the signal. It is the count of the least amount of non-zero coefficient required to represent the signal. In general let Ψ be the domain in which the signal of concern is sparse.

- Second we consider the incoherence measure between the sampling matrix and the transform matrix. Let the equation denote the incoherence,

  $$\mu(\phi, \psi) = \sqrt{n} \cdot \max_{1 \le k,j \le n} |\langle \varphi_k, \psi_j \rangle|$$

  where n is the Nyquist sampling rate.

The sampling rate in CS is defined as

$$m \geq C \cdot \mu^2(\phi, \psi) \cdot S \cdot \log n \tag{4.3}$$

where C is some positive constant.

In many of the literature [11][12] the use of random sampling has been suggested. These random sampling are not exactly always incoherent with the transform domain but they are independent of the transform used. The incoherence measure is also significant and these noiselet transform[12] can be a good sampling matrix for most of the cases. In our thesis we have made use of random sampling to calculate the Compressive Samples which are to be used to calculate the quality index.

As we have got a glimpse of the Compressive Sampling theory we can now move towards compressive measurement[3]. The theory states that is we have obtained the compressive samples then with the use of these samples itself we can estimate the parameter of the signal. These samples can help in other signal processing tools also but we are concerned about the estimation of parameter of the signal.

4.2 Mathematical Tools

We have understood the basics of CM and SSIM. To assimilate these theories certain basic algorithm were being used. The principles and how are they were being applied in the algorithm have been shown below. The principles being used are as follows:

- Parseval's Theorem.
- Image in frequency domain.
- Convolution in Frequency Domain

4.2.1 Parseval's Theorem

Parseval's theorem[16] is the basic building block in the estimation of quality of the image in the spectral domain. The theorem help in conversion of the parameters, mentioned in the chapter 2, where the parameters where in the spatial domain to be formulated in the spectral domain.

The theorem gives the energy conversion equality between two different domains. The constraint on the domains are that they should be represented by basis. Basis are those set of equations which are either orthogonal or ortho-normal to each other[17]. As Parseval's theorem hold only for bases we are going to consider one such type of bases called as the fourier transform matrix. The set of equation below shows the Parseval's equality:

$$\sum_{n\in N} f(n)\,g(n) = \frac{1}{N}\sum_{k\in N} F(k)G^*(k) \tag{4.4}$$

$$\sum_{n\in N} f^2(n) = \frac{1}{N}\sum_{k\in N} |F(k)|^2 \tag{4.5}$$

The theorem states that sum of square of function is equal to sum of square of function's transform coefficients. In other words the energy content of function in spatial domain is same as that in its spectral domain.

The formula for calculation of mean, variance and covariance is given below

- Mean: The DC value of signal is the present in the zero frequency at the spectral domain i.e. at the zeroth position. In case of 2D signal the DC of the image is center pixel in spectral domain.

$$\sum_{i\in N}\sum_{j\in M} f(x_i, y_j) = F_{u,v}(0,0) \tag{4.6}$$

$$\mu = \frac{\sum\limits_{i\in N}\sum\limits_{j\in M} f(x_i, y_j)}{NM} = \frac{F_{u,v}(0,0)}{NM} \tag{4.7}$$

- Variance: The use of Parseval identity can be used in calculation of variance. The definition of variance is given by $\sigma_x^2 = E\left[(X-\overline{x})^2\right]$.

$$E\left[(X-\bar{x})^2\right] = E\left[X^2 - 2\overline{x}X + (\overline{x})^2\right]$$

$$= E\left[X^2\right] - 2\bar{x}E[X] + \bar{x}^2$$

$$= E\left[X^2\right] - \bar{x}^2$$

These equation tells that variance of signal is equal to difference between mean square of signal and square of mean of signal.
The mean square of signal can be calculated using the parseval theorem as shown in equation(4.5).

$$MS = \frac{\sum_{i \in N} \sum_{j \in M} f^2\left(x_i, y_j\right)}{NM} = \frac{\sum_{i \in N} \sum_{j \in M} \left|F\left(u_i, v_j\right)\right|^2}{\left(NM\right)^2} \tag{4.8}$$

$$\sigma_x^2 = MS - \mu_x^2 \tag{4.9}$$

- Covariance: The definition of covariance between two signal is given by $\sigma_{xy} = E\left[(X - \overline{x})(Y - \overline{y})\right]$.
  $E\left[(X - \bar{x})(Y - \overline{y})\right] = E\left[XY - X\overline{y} - Y\overline{y} + (\overline{x})(\overline{y})\right]$

  $$= E\left[XY\right] - \bar{y}E\left[X\right] - \bar{x}E\left[Y\right] + \bar{x}\bar{y}$$

  $$= E\left[XY\right] - \bar{x}\bar{y}$$

The mean square of signal can be calculated using the parseval theorem as shown in equation(4.4).

$$\sigma_{xy} = \frac{\sum_{i \in N} \sum_{j \in M} f\left(x_i, y_j\right) g\left(x_i, y_j\right)}{NM} - \mu_x \mu_y \tag{4.10}$$

$$\sigma_{xy} = \frac{\left|\sum_{i \in N} \sum_{j \in M} F\left(u_i, v_j\right) G^*\left(u_i, v_j\right)\right|}{\left(NM\right)^2} - \mu_x \mu_y \tag{4.11}$$

4.2.2 Image in Frequency Domain

When fourier transform of an image, which is a 2D signal, is taken we get 2D fourier transform coefficient[18]. When observed the transform coefficient that contribute the most to the image are concentrated around the center of 2D fourier image. The required information of the image can be obtained from this region. The parameter that where discussed earlier namely mean, covariance and variance can be obtained with good accuracy if we concentrate around the central region of the 2D fourier image. This will reduce the computation requirement for estimation of such parameters.

The SSIM calculates the index with use of a Gaussian window(default) which is convoluted over the image in spatial domain. The fourier equivalent of convolution is multiplication. So while we are developing to incorporate the SSIM in frequency domain to get the required effect as in spatial domain we have to multiply the fourier transform of image with the fourier transform of the gaussian window[19].

$$f(x) = X, f(y) = Y \tag{4.12}$$

$$f(x * y) = XY \tag{4.13}$$

In Compressive Measurement as explained earlier the estimation of the signal is done in its compressed domain with only few samples, so if we were to concentrate our samples in the center of the 2D fourier domain then the accuracy in estimation of SSIM will increase. This will help in achieving two of our goals which are less computation time requirement with good accuracy.

4.3 Compressive Measurement SSIM

Understanding the basic concept explained in the above section a MATLAB function was developed.
The function takes 2D fourier coefficients of the two images for estimation of the SSIM. This function takes in two images one being the reference image and other being the image for which the index is to be calculated.
The samples are taken in random format. After the samples are chosen then the calculation of the statistical parameters required for CM-SSIM estimation are calculated. For each randomly selected sample the index is calculated and then the mean of these index gives us the required CM-SSIM value.

4.4 Chapter Summary

The developed algorithm has been explained with all mathematical explanations. The development required use of many signal processing tools. There formulation and definition have been incorporated in the chapter.

The next chapter will include all the result that have been helpful in validating the developed algorithm. The results conclusion have been also provided in the same.

Chapter 5

RESULTS AND DISCUSSIONS

This chapter will be dealing with the results of the developed CM-SSIM. The results are acquired with the help of MATLAB platform. Use of some of the toolbox in MATLAB like, image processing toolbox, statistical toolbox, signal processing toolbox etc where used. The description of the statistical comparison parameters have been discussed.

5.1 Image Database

For the verification of authenticity of the developed algorithm there are image database developed which have been made open source by there developer. These database consists of certain number of images of who's distorted images have been provided. The distortion of the original image vary in type and distortion level. The database provides both the original images and there distorted images in separate folder according to the type of distortion and with different levels.

From these database one can make use of original image to find the quality score of the distorted images for a FRIQ algorithms. For each distorted image there has been a quality score being provided namely called as DMOS(Differential Mean Opinion Score). This score is a subjective score being provided by group of user who had observed these images. Each image had been provided to the group of people to rate the distorted image and provide a quantitative score to the image. From the various score available for an image the DMOS was calculated. These score have been provided for each images in the database these score and the score available from the testing algorithm will help in verification of the algorithms authenticity.

The set of database used for the authenticity of the developed algorithm are given as follows with the description of the database along with it.

LIVE Database[20] There are 29 original images in this data-base[21]. These images have five different set of distortion types. The distortion types are:

- JPEG2000 compression
- JPEG compression
- Gaussian Blur
- White Noise
- Fast Fading

5.2 Comparison Parameters

The DMOS score obtained is used for assessment of the developed quality score indexing algorithm. The DMOS values are compared with the score made available from the algorithm under test. The score and the DMOS values are compared using various statistical parameters. These parameter have been explained below:

5.2.1 Pearson Correlation

Equation 5.1 defines the Pearson correlation[22] of two set of variables X and Y.

$$\rho = \frac{cov(X,Y)}{\sigma_x \sigma y} \tag{5.1}$$

where -1 $\leq \rho \leq$ 1.

$\rho > 0$ There is a positive association between two variables.

$\rho < 0$ There is a negative association between two variables.

$\rho = 0$ There is no association between two variable.

For values of $|\rho| = 1$ the two functions are in perfect relationship with each other. This geometrically means that a linear equation describes the values of X and Y, for perfect value of 1 all the points of (X,Y) lie on this line. These are the ideal values we require for testing of the developed algorithm.

While testing we calculate the correlation between the DMOS values and the acquired quality index from the developed algorithm. Values close to 1 or -1 suggests that the developed algorithm is closely related to the DMOS values. This values gives the prediction accuracy of the developed algorithm[23][2].

5.2.2 Spearman Correlation

Equation 5.2 defines the Spearman correlation[22] of two set of variables X and Y. This is a rank based correlation i.e. instead of taking the actual value of the variables we consider the rank of the variable in its set to calculate the correlation.

$$r_s = \frac{\sum_{i=1}^{n}((rank(x_i) - \overline{rank(x_i)}))((rank(y_i) - \overline{rank(y_i)}))}{\sqrt{\sum_{i=1}^{n}((rank(x_i) - \overline{rank(x_i)}))^2 \sum_{i=1}^{n}((rank(y_i) - \overline{rank(y_i)}))^2}} \tag{5.2}$$

where -1 $\leq \rho \leq$ 1.

$r_s > 0$ There is a positive monotonic association between two variables.

$r_s < 0$ There is a negative monotonic association between two variables.

$r_s = 0$ There is no association between two variable.

For values of $|r_s| = 1$ the two functions are in perfect monotonic relationship with each other. These are the ideal values we require for testing of the developed algorithm.

While testing we calculate the correlation between the DMOS values and the acquired quality index from the developed algorithm. Values close to 1 or -1 suggests that the developed algorithm is closely related to the DMOS values. This values gives the monotonic accuracy of the developed algorithm[23][2].

5.2.3 Kendall tau Correlation

Equation 5.3 defines the Kendall tau correlation[22] of two set of variables X and Y.

$$\tau = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} sgn(x_i - x_j) sgn(y_i - y_j)}{n(n-1)} \tag{5.3}$$

$$\begin{aligned} sgn(x_i - x_j) &= +1 \qquad (x_i - x_j) > 0 \\ &= \ 0 \qquad (x_i - x_j) = 0 \\ &= -1 \qquad (x_i - x_j) < 0 \end{aligned}$$

where -1 $\leq \rho \leq$ 1.

$\tau > 0$ There is a positive association between two variables.

$\tau < 0$ There is a negative association between two variables.

$\tau = 0$ There is no association between two variable.

This correlation is calculated irrespective of any assumption of the distribution of variables under consideration. For values of $|\tau| = 1$ the two functions are in perfect relationship with each other. These are the ideal values we require for testing of the developed algorithm.

While testing we calculate the correlation between the DMOS values and the acquired quality index from the developed algorithm. Values close to 1 or -1 suggests that the developed algorithm is closely related to the DMOS values[23][2].

5.2.4 Deviation measure

Box-plot is a graphical representation of the data points of a variable, in which we get to visually see the extent of the variation of the data-points about its median. The box plot is in the form of a rectangular figure with a vertical line passing through it. The horizontal edges of this box represent the 25th and 75th percentiles of the data set. The line passing through the box is called as whiskers, the end points of the whiskers represent the extreme points of data-set excluding any outliers if present. The outliers are the values which are about twice the standard deviation from the mean of the data set. The box-plot also indicates the median of the set, which is indicated by a horizontal line which is indicated inside the box.

5.3 Results

This section will display all the results obtained using all the comparison parameters discussed in section 5.2 for the image data-set discussed in section 5.1.

The following set of results will deal with the correlation values obtained. The correlation values, as discussed in 5.2.1, 5.2.2 & 5.2.3, will be calculated between SSIM and DMOS and another set of values between CMSSIM and DMOS. Coming to the discussion done in section 3.4 we are going to calculate the correlation values for SSIM ALTERED also. These values will help in understanding how accuracy of method of down-sampling which is adopted for SSIM. Table 5.1, 5.2 & 5.3 shows the correlation values for the LIVE image data set, for individual degradations type as well as the total LIVE data-set taken together.

Table 5.1: Pearson Correlation of SSIM, CM-SSIM & SSIM ALTERED with DMOS for LIVE data-base

Distortion	SSIM	CM-SSIM	SSIM ALTERED
JP2K	-0.8263	-0.9104	-0.8892
JPEG	-0.7978	-0.8846	-0.8748
White Noise	-0.9662	-0.9656	-0.9471
Gaussian Blur	-0.8632	-0.9402	-0.8553
Fast Fading	-0.8512	-0.9348	-0.9045
Total	-0.6932	-0.8707	-0.7319

Table 5.2: Spearman Correlation of SSIM, CM-SSIM & SSIM ALTERED with DMOS for LIVE data-base

Distortion	SSIM	CM-SSIM	SSIM ALTERED
JP2K	-0.9633	-0.9623	-0.9581
JPEG	-0.9411	-0.9406	-0.9399
White Noise	-0.9641	-0.9615	-0.9609
Gaussian Blur	-0.9246	-0.9595	-0.8995
Fast Fading	-0.9396	-0.9652	-0.9397
Total	-0.9115	-0.9208	-0.8775

Table 5.3: Kendall tau Correlation of SSIM, CM-SSIM & SSIM ALTERED with DMOS for LIVE data-base

Distortion	SSIM	CM-SSIM	SSIM ALTERED
JP2K	-0.8376	-0.8371	-0.8256
JPEG	-0.7963	-0.8039	-0.7918
White Noise	-0.8345	-0.8307	-0.8308
Gaussian Blur	-0.7536	-0.8256	-0.7211
Fast Fading	-0.7856	-0.8356	-0.7812
Total	-0.7457	-0.7646	-0.7060

The above tables 5.1 5.2 5.3 give the correlation values of SSIM, SSIM altered and CM-SSIM for LIVE data-base.

From the correlation values it can be observed that the values of correlation for CM-SSIM with DMOS are almost nearing one and are comparable to those obtained from SSIM. For some of the cases the correlation values are better than that of SSIM. If we observe the values of Pearson correlation for SSIM ALTERED, then we can see that the value for the total LIVE database is more than that of SSIM. This brings use to the point that the prediction accuracy of SSIM is less than that of SSIM ALTERED. This justifies the claim that the SSIM calculated by down-sampling can lead to to an erroneous decision. So CM-SSIM can be used as an alternate method for calculation of SSIM in spectral domain.

The next set of result will show the visual representation of the performance of the algorithms. These were mentioned in 5.2.4. First set of images for the box-plot representation.

The data base as we know has set of original image of which the degraded version of images with both type and level have been stored. A set of degraded images, of same degradation type and related to a single original image, are taken and the SSIM, CM-SSIM & SSIM ALTERED are calculated. After these calculations the values are correlated with the corresponding DMOS values and stored. This procedure is carried out for all the set of original images in the data-set. After this procedure the box-plot is plotted for SSIM, CM-SSIM & SSIM ALTERED. For convenience of observation absolute value of the correlations are taken for plotting purpose. The results are as shown below,

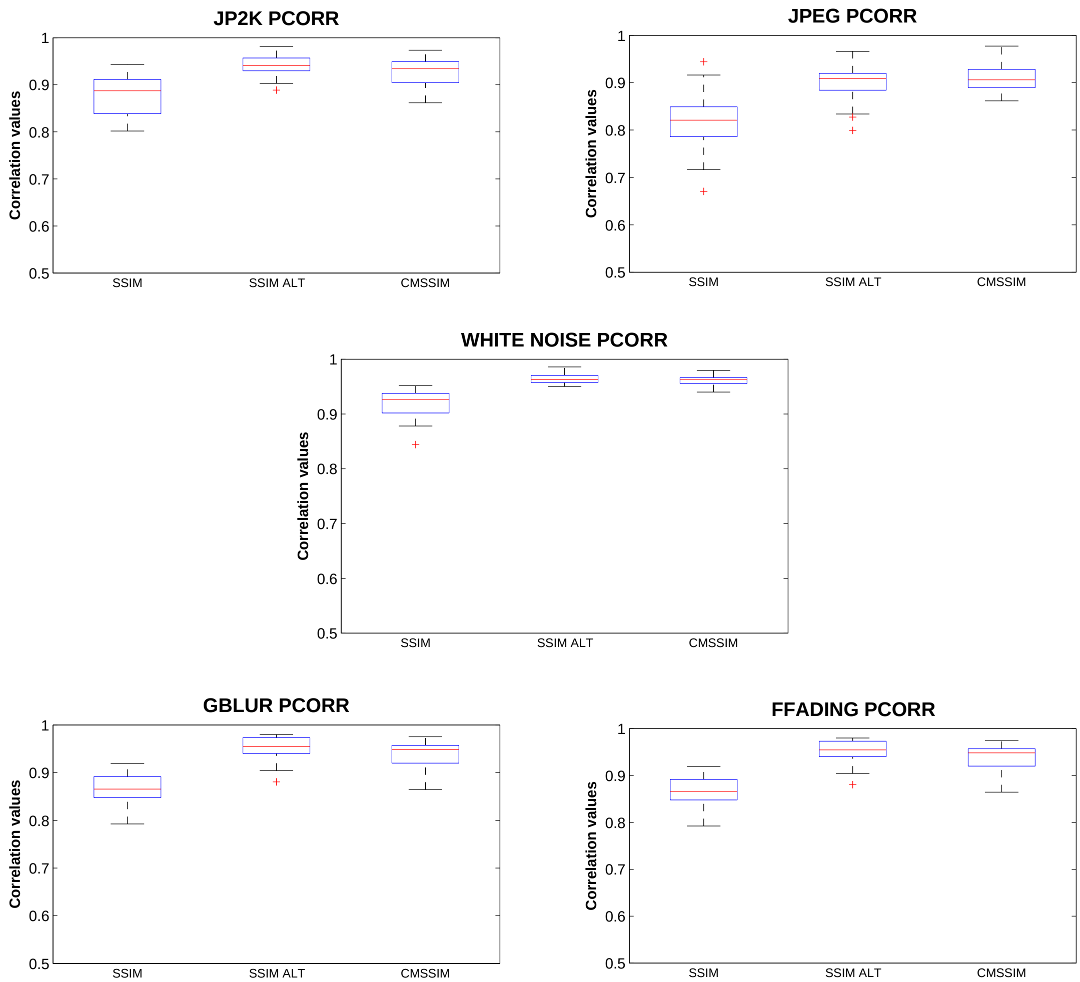

Figure 5.1: Box-Plot for Pearson Correlation Values(LIVE)

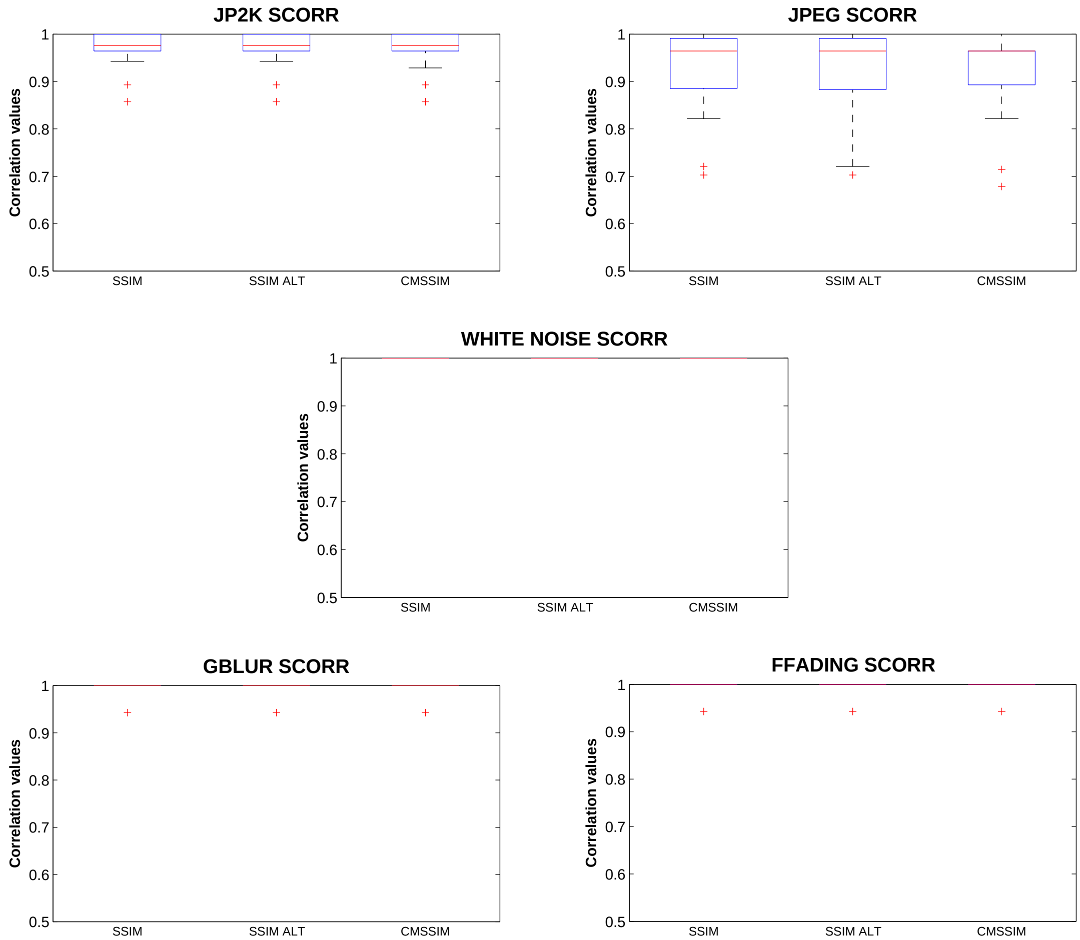

Figure 5.2: Box-Plot for Spearman Correlation Values(LIVE)

From figure 5.1 we can see that the variation in the Pearson correlation values is very much less in comparison to that obtained by SSIM and these values are almost closer to what the SSIM ALTERED shows. This helps in concluding that the CM-SSIM can be an alternative method for calculation of SSIM, rather than going for the down-sampling method used in SSIM. This also helps to conclude in stating that CM-SSIM can be a good alternative method to calculate the quality index in the spectral domain. It can also be observed that the number of outlier for CM-SSIM are lesser than compared to other two algorithm. The median shown for CM-SSIM is much closer to 1 than others.

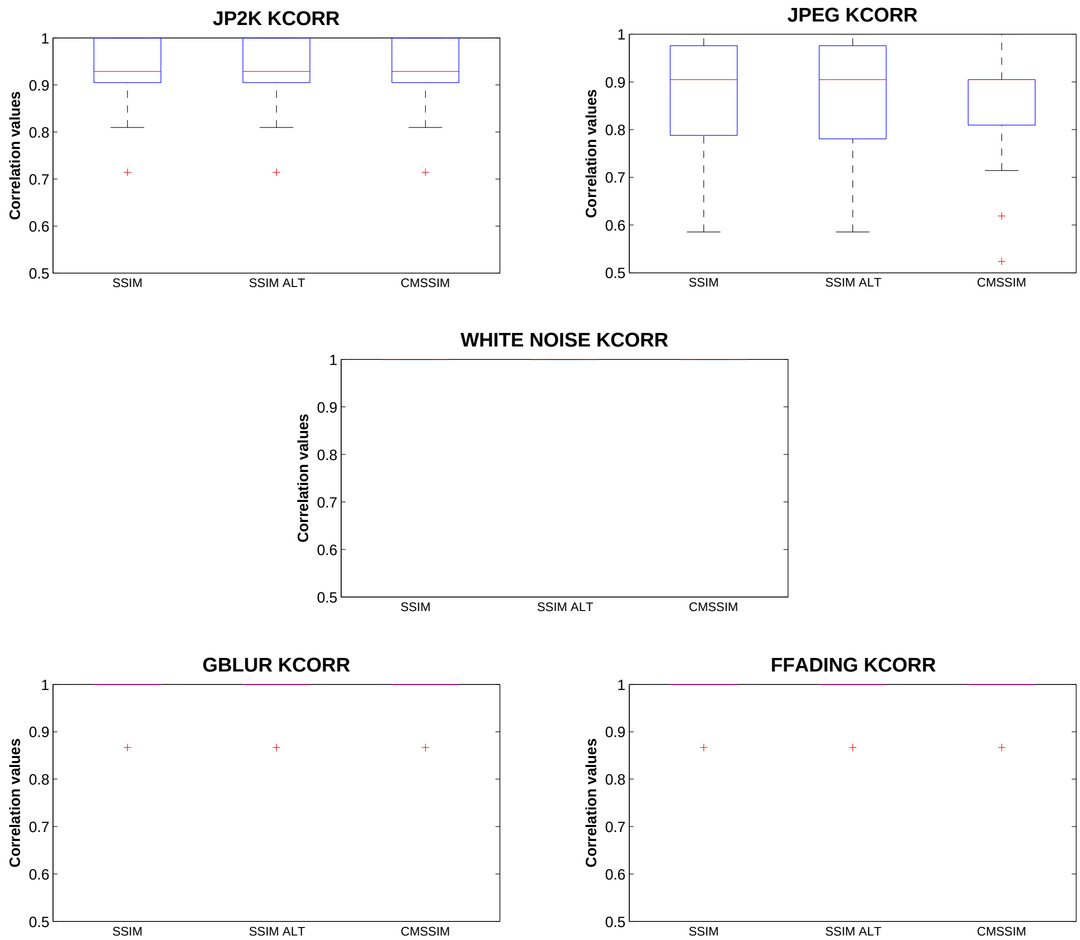

Figure 5.3: Box-Plot for Kendall's tau Correlation Values(LIVE)

5.4 Chapter Summary

The chapter describes the results carried out for performance measurement of the developed algorithm. The results obtained were discussed and studied upon.

Chapter 6

Conclusion

6.1 Conclusion

From the set of result which were presented in section 5.3 we can conclude that the developed algorithm can be helpful in estimation of quality of an image in spectral domain using just the fourier coefficients. This was achieved with help of CM theory. The results have also helped in concluding that the developed algorithm can be replacement to the down-sampling method mentioned in section 3.4.

6.2 Future Scope

This algorithm has helped us to understand the implementation of CM with the quality measurement. The algorithm can be further developed for implementing with more better compressed domain for images like the wavelet or curvelet domain. This can be the future scope of research from this thesis. The real-time implementation of the image quality metric is another reasearch area that can be worked upon.

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