Agility Evaluation in Fuzzy Environment

Thesis submitted in partial fulfillment of the requirements for the Degree of

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In

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By

SURAJ KANT SAHU
Roll No. 109ME0415

Under the Guidance of

Prof. SAURAV DATTA

NATIONAL INSTITUTE OF TECHNOLOGY
ROURKELA 769008, INDIA

NATIONAL INSTITUTE OF TECHNOLOGY
ROURKELA 769008, INDIA

Certificate of Approval

This is to certify that the thesis entitled **Agility Evaluation in Fuzzy Environment** submitted by ***Sri Suraj Kant Sahu*** has been carried out under my supervision in partial fulfillment of the requirements for the Degree of ***Bachelor of Technology*** in ***Mechanical Engineering*** at **National Institute of Technology**, **NIT Rourkela**, and this work has not been submitted elsewhere before for any other academic degree/diploma.

Dr. Saurav Datta

Assistant Professor
Department of Mechanical Engineering
National Institute of Technology, Rourkela

Rourkela-769008

Date:

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Suraj Kant Sahu

Abstract

Agility metrics are difficult to define, mainly due to the multidimensionality and vagueness of the concept of agility (Nikos et al., 2002). In this work, a fuzzy logic, knowledge-based framework is intended to be developed for the assessment of an enterprise's agility; as a case study. The necessary expertise explored to quantitatively determine and evaluate overall agility degree is to be represented via fuzzy logic analyses. Apart from estimating overall agility appraisement index; the study is aimed to be extended to identify agile barriers (obstacles towards achieving agility). The proposed appraisement module would be implemented in an Indian enterprise as a case study. Data obtained thereof, would be critically analyzed to reveal the current scenario of existing agile practices of the said enterprise and to seek for ill-performing areas which need future improvement.

Keywords: Business Agility, Fuzzy Performance Index (FPI), Fuzzy Performance Importance Index (FPII)

Contents

1. State of Art

Business agility is the ability of a business to adapt rapidly and cost efficiently in response to changes in the business environment. Business agility can be maintained by maintaining and adapting goods and services to meet customer demands, adjusting to the changes in a business environment and taking advantage of human resources.

Agility is a concept that incorporates the ideas of flexibility, balance, adaptability, and coordination under one umbrella. In a business context, agility typically refers to the ability of an organization to rapidly adapt to market and environmental changes in productive and cost-effective ways. The agile enterprise is an extension of this concept, referring to an organization that utilizes key principles of complex adaptive systems and complexity science to achieve success.

Tsourveloudis and Valavanis (2002) proposed a knowledge-based framework and presented as a candidate solution for the measurement and assessment of manufacturing agility. Given an enterprise, in order to calculate its overall agility, a set of quantitatively defined agility parameters was proposed and grouped into production, market, people and information infrastructures. The combined, resulting, measure incorporated the individual and grouped infrastructure agility parameters and their variations into one calculated value of the overall agility. The necessary expertise used to quantitatively determine and measure individual agility parameters was represented via fuzzy logic terminology that allows for human-like knowledge representation and reasoning. An example demonstrated the feasibility and applicability of the proposed approach.

Lin and Chu (2006) developed a fuzzy agility index (FAI) based on agility providers using fuzzy logic. The FAI comprises attribute' ratings and corresponding weights, and is aggregated by a fuzzy weighted average. To illustrate the efficacy of the method, this study also evaluated the supply chain agility of a Taiwanese company. This evaluation demonstrated that the method can provide analysts with more informative and reliable information for decision.

Chandna (Kharbanda) (2008) presented a fuzzy logic, knowledge-based framework for the assessment of manufacturing agility. The combined measure incorporated certain operational parameters, their variations, and their effect on the value of agility. The necessary expertise used to quantitatively determine and measure agility was represented via fuzzy logic terminology, which allows for human-like knowledge representation and reasoning. Emerging standards for distributed simulation and virtual reality were utilized to implement a distributed simulation test bed. The test bed was used to simulate, measure, and evaluate agility and its parameters. The simulation test bed integrated the modeling of agility infrastructures, simulation of an enterprise through its infrastructures, real-life data, and a virtual reality based interface. High Level Architecture (HLA) and Virtual Reality Modeling Language (VRML) were standards selected for the implementation of the test bed.

Charles et al. (2010) clearly defined the concept of supply chain agility, and second, built a model for assessing the level of agility of a supply chain. The paper developed first, a framework for defining supply chain agility and second, a model for assessing and improving the capabilities of humanitarian and commercial supply chains in terms of agility, based on an analysis of humanitarian approaches. The model was developed thanks to inputs from humanitarian practitioners and feedbacks from academics. This paper contributed significantly to clarifying the notion of supply chain agility. It also provided a consistent, robust and reproducible method of assessing supply chain agility, which seems appropriate for both humanitarian and business sectors. Finally, it was complementary to existent research on humanitarian logistics. It showed that though humanitarian professionals have a lot to learn from the private sector, the reverse is also true.

Vinodh et al. (2010) reported a research carried out to assess the agility level of an organization using a multi-grade fuzzy approach. Agility refers to the capability of an organization to respond quickly in accordance with the dynamic demands of the customers. During this research, an agility index measurement model containing 20 criteria incorporated with the multi-grade fuzzy approach was designed. Subsequently, the data gathered from a manufacturing company was substituted in this model and the proposals for enhancing the agility level of this company were derived. The usage of the model contributed in this paper would indicate the actions required to enhance an organization's agility level. This process might accelerate the absorption of agility characteristics in modern organizations.

Yaghoubi et al. (2010) studied the effective factors on organizational agility. Many researchers had classified these factors under three sections including drivers, capabilities and enablers of the agility. With reference to this approach, the paper presented some conceptions of agility at the beginning and a brief history of it. Then, drivers, capabilities and 26enablers were introduced with imparting different theories and models. It was expected that this research would be able to accelerate the organizations getting success and helping the future researchers.

Yaghoubiet al, (2011) proposed the following subjects: the concept, importance and necessity of accessing agility and fuzz plus its reasons. Then, they assessed agility with the Goldman methodology based on fuzzy approach. In this respect, several questionnaires were distributed among the top managers of Saipa Yadak car co., Iran. Finally, after precise and through analyses, the sub- criteria were recognized based on the fuzzy approach and the possible obstacles for reaching the agility level and different recommendations were suggested.

Dahmardehand Pourshahabi (2011) proposed a knowledge-based framework for the measurement and assessment of public sector agility using the A.T. Kearney model. Fuzzy logic provided a useful tool for dealing with decisions in which the phenomena are imprecise and vague. In the paper, the authors used the absolute agility index together with fuzzy logic to address the ambiguity in agility evaluation in public sector in a case study.

Literature review depicts some extensive work has been shown in organizational supply chain agility domain and few work has been undertaken in the business or marketing agility perspectives. Therefore, an attempt has been made in this paper to develop a fuzzy based appraisement module in order to assess the business agility as well as to identify the agile barriers which may require for the improvement of business agility. A case study has been performed to identify the important agile barriers of an Indian automotive industry in order to improve the business agility, on the basis of questionnaire survey.

2. Fuzzy Preliminaries

To deal with vagueness in human thought, Zadeh (1965) first introduced the fuzzy set theory, which has the capability to represent/manipulate data and information possessing based on nonstatistical uncertainties. Moreover fuzzy set theory has been designed to mathematically represent uncertainty and vagueness and to provide formalized tools for dealing with the imprecision inherent to decision making problems. Some basic definitions of fuzzy sets, fuzzy numbers and linguistic variables are reviewed from Zadeh (1975), Buckley (1985), Negi (1989), Kaufmann and Gupta (1991).The basic definitions and notations below will be used throughout this paper until otherwise stated.

2.1 Definitions of fuzzy sets:

Definition 1. A fuzzy set $\tilde{A}$ in a universe of discourse X is characterized by a membership function $\mu_{\tilde{A}}(x)$ which associates with each element x in X a real number in the interval $[0,1]$. The function value $\mu_{\tilde{A}}(x)$ is termed the grade of membership of x in $\tilde{A}$ **(Kaufmann and Gupta, 1991)**.

Definition 2. A fuzzy set $\tilde{A}$ in a universe of discourse X is convex if and only if

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\left(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\right) \tag{1}$$

For all x_1, x_2 in X and all $\lambda \in [0,1]$, where $\min$ denotes the minimum operator (Klir and Yuan, 1995).

Definition 3. The height of a fuzzy set is the largest membership grade attained by any element in that set. A fuzzy set $\tilde{A}$ in the universe of discourse X is called normalized when the height of $\tilde{A}$ is equal to 1 (Klir and Yuan, 1995).

2.2 Definitions of fuzzy numbers:

Definition 1. A fuzzy number is a fuzzy subset in the universe of discourse X that is both convex and normal. Fig. 1 shows a fuzzy number $\tilde{n}$ in the universe of discourse X that conforms to this definition (Kaufmann and Gupta, 1991).

Definition 2. The α -cut of fuzzy number $\tilde{n}$ is defined as:

$$\tilde{n}^{\alpha} = \{x_i : \mu_{\tilde{n}}(x_i) \geq \alpha, x_i \in X\}, \tag{2}$$

Here, $\alpha \in [0,1]$

The symbol $\tilde{n}^{\alpha}$ represents a non-empty bounded interval contained in X, which can be denoted by $\tilde{n}^{\alpha} = [n_l^{\alpha}, n_u^{\alpha}]$, n_l^{α} and n_u^{α} are the lower and upper bounds of the closed interval, respectively (Kaufmann and Gupta, 1991; Zimmermann, 1991). For a fuzzy number $\tilde{n}$, if

$n_l^\alpha > 0$ and $n_u^\alpha \le 1$ for all $\alpha \in [0,1]$, then $\tilde{n}$ is called a standardized (normalized) positive fuzzy number (Negi, 1989).

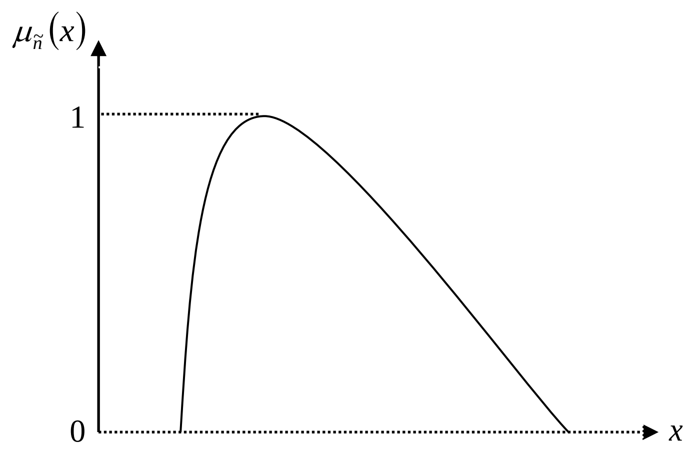

Fig. 1. A fuzzy number $\tilde{n}$

Definition 3. Suppose, a positive triangular fuzzy number (PTFN) is $\tilde{A}$ and that can be defined as (a,b,c) shown in Fig. 2. The membership function $\mu_{\tilde{n}}(x)$ is defined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-a)/(b-a), & if\ a \le x \le b, \\ (c-x)/(c-b), & if\ b \le x \le c, \\ 0, & otherwise, \end{cases} \quad (3)$$

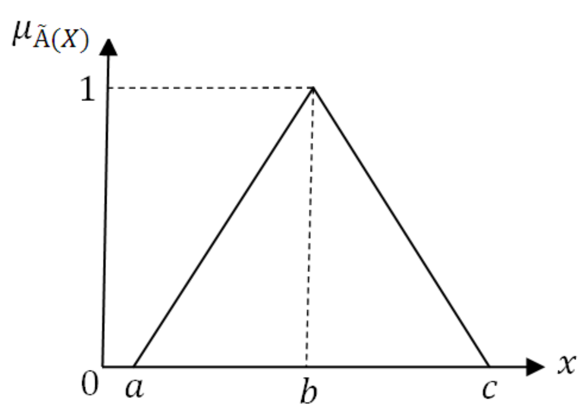

Fig. 2. A triangular fuzzy number $\tilde{A}$

Based on extension principle, the fuzzy sum $\oplus$ and fuzzy subtraction Θ of any two triangular fuzzy numbers are also triangular fuzzy numbers; but the multiplication $\otimes$ of any two triangular fuzzy numbers is only approximate triangular fuzzy number (Zadeh, 1975). Let's have a two positive triangular fuzzy numbers, such as $\tilde{A}_1 = (a_{1,} b_1, c_1)$, and $\tilde{A}_2 = (a_2, b_2, c_2)$, and a positive real number $r = (r, r, r)$, some algebraic operations can be expressed as follows:

$$\tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2) \tag{4}$$

$$\tilde{A}_1 \Theta \tilde{A}_2 = (a_1 - a_2, b_1 - b_2, c_1 - c_2), (5)\ \tilde{A}_1 \otimes \tilde{A}_2 = (a_1 a_2, b_1 b_2, c_1 c_2), \tag{6}$$

$$r \otimes \tilde{A}_1 = (ra_1, rb_1, rc_1), \tag{7}$$

$$\tilde{A}_1 \varnothing \tilde{A}_2 = (a_1 / c_2, b_1 / b_2, c_1 / a_2), \tag{8}$$

The operations of $\vee$ (max) and $\wedge$ (min) are defined as:

$$\tilde{A}_1 (\vee) \tilde{A}_2 = (a_1 \vee a_2, b_1 \vee b_2, c_1 \vee c_2), \tag{9}$$

$$\tilde{A}_1 (\wedge) \tilde{A}_2 = (a_1 \wedge a_2, b_1 \wedge b_2, c_1 \wedge c_2), \tag{10}$$

Here, $r > 0$, and $a_1, b_1, c_1 > 0$,

Also the crisp value of triangular fuzzy number set $\tilde{A}_1$ can be determined by defuzzification which locates the Best Non-fuzzy Performance (BNP) value. Thus, the BNP values of fuzzy number are calculated by using the center of area (COA) method as follows: (Moeinzadeh and Hajfathaliha, 2010)

$$\text{BNP}_i = \frac{[(c-a)+(b-a)]}{3} + a, \qquad \forall_i, \tag{11}$$

Definition 4. A matrix $\tilde{\mathbf{D}}$ is called a fuzzy matrix if at least one element is a fuzzy number (Buckley, 1985).

2.3 Linguistic variable:

Definition 1. A linguistic variable is the variable whose values are not expressed in numbers but words or sentences in a natural or artificial language (Zadeh, 1975). The concept of a linguistic variable is very useful in dealing with situations, which are too complex or not well-defined to be reasonably described in conventional quantitative expressions (Zimmermann, 1991). For example, 'weight' is a linguistic variable whose values are 'very low', 'low', 'medium', 'high', 'very high', etc. Fuzzy numbers can also represent these linguistic values.

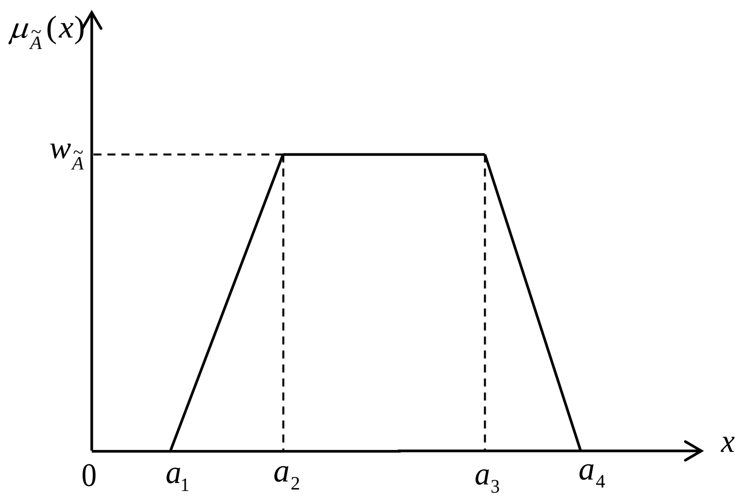

Fig. 3 Trapezoidal fuzzy number $\tilde{A}$

2.4 The concept of generalized trapezoidal fuzzy numbers

By the definition given by (Chen, 1985), a generalized trapezoidal fuzzy number can be defined as $\tilde{A} = \left(a_1, a_2, a_3, a_4; w_{\tilde{A}}\right)$, as shown in Fig. 3.

and the membership function $\mu_{\tilde{A}}(x): R \rightarrow [0,1]$ is defined as follows:

$$\mu_{\tilde{A}}(x)=\begin{cases}\frac{x-a_1}{a_2-a_1}\times w_{\tilde{A}}, & x\in(a_1,a_2)\\ w_{\tilde{A}}, & x\in(a_2,a_3)\\ \frac{x-a_4}{a_3-a_4}\times w_{\tilde{A}}, & x\in(a_3,a_4)\\ 0, & x\in(-\infty,a_1)\cup(a_4,\infty)\end{cases} \quad (12)$$

Here, $a_1 \le a_2 \le a_3 \le a_4$ and $w_{\tilde{A}} \in [0,1]$

The elements of the generalized trapezoidal fuzzy numbers $x \in R$ are real numbers, and its membership function $\mu_{\tilde{A}}(x)$ is the regularly and continuous convex function, it shows that the membership degree to the fuzzy sets. If $-1 \le a_1 \le a_2 \le a_3 \le a_4 \le 1$, then $\tilde{A}$ is called the normalized trapezoidal fuzzy number. Especially, if $w_{\tilde{A}} = 1$, then $\tilde{A}$ is called trapezoidal fuzzy number (a_1,a_2,a_3,a_4); if $a_1 < a_2 = a_3 < a_4$, then $\tilde{A}$ is reduced to a triangular fuzzy number. If $a_1 = a_2 = a_3 = a_4$, then $\tilde{A}$ is reduced to a real number.

Suppose that $\tilde{a} = (a_1,a_2,a_3,a_4;w_{\tilde{a}})$ and $\tilde{b} = (b_1,b_2,b_3,b_4;w_{\tilde{b}})$ are two generalized trapezoidal fuzzy numbers, then the operational rules of the generalized trapezoidal fuzzy numbers $\tilde{a}$ and $\tilde{b}$ are shown as follows (Chen and Chen, 2009):

$$\tilde{a}\oplus\tilde{b} = (a_1,a_2,a_3,a_4;w_{\tilde{a}})\oplus(b_1,b_2,b_3,b_4;w_{\tilde{b}}) =$$

$$(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4;\min(w_{\tilde{a}},w_{\tilde{b}})) \quad (13)$$

$$\tilde{a}-\tilde{b} = (a_1,a_2,a_3,a_4;w_{\tilde{a}})-(b_1,b_2,b_3,b_4;w_{\tilde{b}}) =$$

$$(a_1-b_4,a_2-b_3,a_3-b_2,a_4-b_1;\min(w_{\tilde{a}},w_{\tilde{b}})) \quad (14)$$

$$\tilde{a}\otimes\tilde{b} = (a_1,a_2,a_3,a_4;w_{\tilde{a}})\otimes(b_1,b_2,b_3,b_4;w_{\tilde{b}}) =$$

$$(a,b,c,d;\min(w_{\tilde{a}},w_{\tilde{b}})) \quad (15)$$

Here,

$$a = \min\left(a_1 \times b_1, a_1 \times b_4, a_4 \times b_1, a_4 \times b_4\right)$$

$$b = \min\left(a_2 \times b_2, a_2 \times b_3, a_3 \times b_2, a_3 \times b_3\right)$$

$$c = \max\left(a_2 \times b_2, a_2 \times b_3, a_3 \times b_2, a_3 \times b_3\right)$$

$$d = \max\left(a_1 \times b_1, a_1 \times b_4, a_4 \times b_1, a_4 \times b_4\right)$$

If $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4$ are real numbers, then

$$\tilde{a} \otimes \tilde{b} = \left(a1 \times b1, a2 \times b2, a3 \times b3, a4 \times b4; \min\left(w_{\tilde{a}}, w_{\tilde{b}}\right)\right)$$

$$\tilde{a} / \tilde{b} = \left(a_1, a_2, a_3, a_4; w_{\tilde{a}}\right) \Big/ \left(b_1, b_2, b_3, b_4; w_{\tilde{b}}\right)$$

$$= \left(a_1 / b_4, a_2 / b_3, a_3 / b_2, a_4 / b_1; \min\left(w_{\tilde{a}}, w_{\tilde{b}}\right)\right) \tag{16}$$

Chen and Chen (2003) proposed the concept of COG point of generalized trapezoidal fuzzy numbers, and suppose that the COG point of the generalized trapezoidal fuzzy number $\tilde{a} = \left(a_1, a_2, a_3, a_4; w_{\tilde{a}}\right)$ is $\left(x_{\tilde{a}}, y_{\tilde{a}}\right)$, then:

$$y_{\tilde{a}} = \begin{cases} \dfrac{w_{\tilde{a}} \times \left(\dfrac{a_3 - a_2}{a_4 - a_1} + 2\right)}{6}, & if\ a_1 \neq a_4 \\ \dfrac{w_{\tilde{a}}}{2}, & if\ a_1 = a_4 \end{cases} \tag{17}$$

$$x_{\tilde{a}} = \frac{y_{\tilde{a}} \times \left(a_2 + a_3\right) + \left(a_1 + a_4\right) \times \left(w_{\tilde{a}} - y_{\tilde{a}}\right)}{2 \times w_{\tilde{a}}} \tag{18}$$

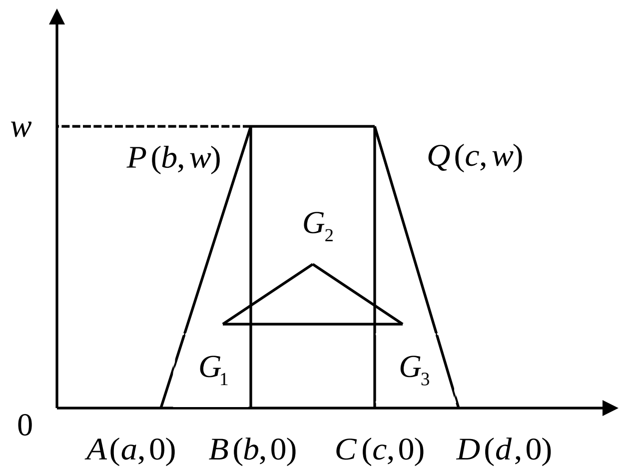

Fig. 4. Trapezoidal Fuzzy Number [Thorani et al. (2012)]

2.5 Ranking of Generalized Trapezoidal Fuzzy Numbers **[**Thorani et al. (2012)]

The centroid of a trapezoid is considered as the balancing point of the trapezoid (Fig. 4). Divide the trapezoid into three plane figures. These three plane figures are a triangle (APB), a rectangle (BPQC), and a triangle (CQD), respectively. Let the centroids of the three plane figures be G_1, G_2, and G_3 respectively. The Incenter of these Centroids G_1, G_2 and G_3 is taken as the point of reference to define the ranking of generalized trapezoidal fuzzy numbers. The reason for selecting this point as a point of reference is that each centroid point are balancing points of each individual plane figure, and the Incentre of these Centroid points is a much more balancing point for a generalized trapezoidal fuzzy number. Therefore, this point would be a better reference point than the Centroid point of the trapezoid.

Consider a generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$, (Fig. 4). The Centroids of the three plane figures are $G_1 = \left(\frac{a+2b}{3}, \frac{w}{3}\right)$, $G_2 = \left(\frac{b+c}{2}, \frac{w}{2}\right)$ and $G_3 = \left(\frac{2c+d}{3}, \frac{w}{3}\right)$ respectively.

Equation of the line $\overline{G_1G_3}$ is $y=\dfrac{w}{3}$ and G_2 does not lie on the line $\overline{G_1G_3}$. Therefore, G_1G_2 and G_3 are non-collinear and they form a triangle.

We define the Incentre $I_{\tilde{A}}(\bar{x}_0,\bar{y}_0)$ of the triangle with vertices G_1, G_2 and G_3 of the generalized trapezoidal fuzzy number $\tilde{A}=(a,b,c,d;w)$ as

$$I_{\tilde{A}}(\bar{x}_0,\bar{y}_0)=\left(\frac{\alpha\left(\frac{a+2b}{3}\right)+\beta\left(\frac{b+c}{2}\right)+\gamma\left(\frac{2c+d}{3}\right)}{\alpha+\beta+\gamma},\frac{\alpha\left(\frac{w}{3}\right)+\beta\left(\frac{w}{2}\right)+\gamma\left(\frac{w}{3}\right)}{\alpha+\beta+\gamma}\right) \tag{19}$$

Here

$$\alpha=\frac{\sqrt{(c-3b+2d)^2+w^2}}{6}$$

$$\beta=\frac{\sqrt{(2c+d-a-2b)^2}}{3}$$

$$\gamma=\frac{\sqrt{(3c-2a-b)^2+w^2}}{6}$$

As a special case, for triangular fuzzy number $\tilde{A}=(a,b,c,d;w)$, i.e. $c=b$ the incentre of Centroids is given by

$$I_{\tilde{A}}(\bar{x}_0,\bar{y}_0)=\left(\frac{x\left(\frac{a+2b}{3}\right)+yb+z\left(\frac{2b+d}{3}\right)}{x+y+z},\frac{x\left(\frac{w}{3}\right)+y\left(\frac{w}{2}\right)+z\left(\frac{w}{3}\right)}{x+y+z}\right) \tag{20}$$

Here

$$x=\frac{\sqrt{(2d-2b)^2+w^2}}{6}$$

$$y = \frac{\sqrt{(d-a)^2}}{3}$$

$$z = \frac{\sqrt{(2b-2a)^2 + w^2}}{6}$$

The ranking function of the generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$, which maps the set of all fuzzy numbers to a set of real numbers is defined as,

$$R(\tilde{A}) = x_0 \times y_0 = \left(\frac{x\left(\frac{a+2b}{3}\right) + yb + z\left(\frac{2b+d}{3}\right)}{x+y+z} \times \frac{x\left(\frac{w}{3}\right) + y\left(\frac{w}{2}\right) + z\left(\frac{w}{3}\right)}{x+y+z} \right) \tag{21}$$

This is the Area between the incenter of the centroids $I_{\tilde{A}}(\bar{x}_0, \bar{y}_0)$ as defined in Eq. (19) and the original point.

The Mode (m) of the generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$, is defined as:

$$m = \frac{1}{2}\int_0^w (b+c)dx = \frac{w}{2}(b+c) \tag{22}$$

The Spread(s) of the generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$, is defined as:

$$s = \int_0^w (d-a)dx = w(d-a) \tag{23}$$

The left spread (ls) of thegeneralized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$, is defined as:

$$ls = \int_0^w (b-a)dx = w(b-a) \tag{24}$$

The right spread (rs) of thegeneralized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$, is defined as:

$$rs = \int_0^w (d-c)dx = w(d-c) \tag{25}$$

Using the above definitions we now define the ranking procedure of two generalized trapezoidal fuzzy numbers.

Let $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ be two generalized trapezoidal fuzzy numbers.

The working procedure to compare $\tilde{A}$ and $\tilde{B}$ is as follows:

Step 1: Find $R(\tilde{A})$ and $R(\tilde{B})$

Case (i) If $R(\tilde{A}) > R(\tilde{B})$ then $\tilde{A} > \tilde{B}$

Case (ii) If $R(\tilde{A}) < R(\tilde{B})$ then $\tilde{A} < \tilde{B}$

Case (iii) If $R(\tilde{A}) = R(\tilde{B})$ comparison is not possible, then go to ***step 2***.

Step 2: Find $m(\tilde{A})$ and $m(\tilde{B})$

Case (i) If $m(\tilde{A}) > m(\tilde{B})$ then $\tilde{A} > \tilde{B}$

Case (ii) If $m(\tilde{A}) < m(\tilde{B})$ then $\tilde{A} < \tilde{B}$

Case (iii) If $m(\tilde{A}) = m(\tilde{B})$ comparison is not possible, then go to ***step 3***.

Step 3: Find $s(\tilde{A})$ and $s(\tilde{B})$

Case (i) If $s(\tilde{A}) > s(\tilde{B})$ then $\tilde{A} < \tilde{B}$

Case (ii) If $s(\tilde{A}) < s(\tilde{B})$ then $\tilde{A} > \tilde{B}$

Case (iii) If $s(\tilde{A}) = s(\tilde{B})$ comparison is not possible, then go to ***step 4***.

Step 4: Find $ls(\tilde{A})$ and $ls(\tilde{B})$

Case (i) If $ls(\tilde{A}) > ls(\tilde{B})$ then $\tilde{A} > \tilde{B}$

Case (ii) If $ls(\tilde{A}) < ls(\tilde{B})$ then $\tilde{A} < \tilde{B}$

Case (iii) If $ls(\tilde{A}) = ls(\tilde{B})$ comparison is not possible, then go to ***step 5***.

Step 5: Examine w_1 and w_2

Case (i) If $w_1 > w_2$ then $\tilde{A} > \tilde{B}$

Case (ii) If $w_1 < w_2$ then $\tilde{A} < \tilde{B}$

Case (iii) If $w_1 = w_2$ then $\tilde{A} \approx \tilde{B}$

3. Proposed Appraisement Module

A fuzzy based performance appraisement module in agile manufacturing proposed in this paper has been present below. General hierarchy criteria (GHC) for evaluating overall organizational agility degree, adapted in this paper has been shown in Table 1 [Dahmardeh and Pourshahabi, 2011]. It consists of two-level index system; which aims at achieving the target to evaluate overall appraisement index. 1st level lists out a number of agile capabilities/ enablers; 2nd level comprises of various agile attributes. Procedural steps for agility evaluation have been presented as follows:

1. Selection of linguistic variables towards assigning priority weights (of individual agile capabilities as well as attributes) and appropriateness rating (performance extent) corresponding to each 2ndlevel agile attributes.

2. Collection of expert opinion from a selected decision-making group (subjective judgment) in order to express the priority weight as well as appropriate rating against each of the evaluation indices.

3. Representing decision-makers' linguistic judgments using appropriate fuzzy numbers set.

4. Use of fuzzy operational rules towards estimating aggregated weight as well as aggregated rating (pulled opinion of the decision-makers) for each of the selection criterion.

5. Calculation of computed performance rating of 1st level agile capabilities and also overall agility performance index called Fuzzy Performance Index (FPI) at last.

Appropriateness rating for each of the 1st level capability U_i (rating of i_{th} agile capability) has been computed as follows:

$$U_i = \frac{\sum U_{ij} \otimes w_{ij}}{\sum w_{ij}} \tag{26}$$

In this expression (Eq. 26) U_{ij} is denoted as the aggregated fuzzy appropriateness rating against j_{th} agile attribute (at 2nd level) which is under i_{th} main criterion in the 1st level. w_{ij} is the aggregated fuzzy weight against j_{th} agile attribute (at 2nd level) which is under i_{th} main criterion in 1st level.

The *Fuzzy Performance Index (FPI)* has been computed as:

$$U(FPI) = \frac{\sum U_i \otimes w_i}{\sum w_i} \tag{27}$$

In this expression (Eq. 27) U_i is denoted as the computed fuzzy appropriateness rating (obtained using Eq. 26) against i_{th} agile capability at 1st level. w_i is the aggregated fuzzy priority weight against i_{th} agile capability in 1st level.

6. Investigation for identifying ill-performing areas those seek for future improvement.

4. Numerical Illustrations

The proposed appraisement module has been implemented in a famous automobile sector at eastern part of India. The module encompasses of various agile capabilities as well as agile attributes. An evaluation team has been deployed to assign priority weights (importance extent) against different agile capabilities/ attributes considered in the proposed appraisement model. A questionnaire has been formed and circulated among the decision-makers (experts) to provide the required detail. Collected data has been explored to investigate application feasibility of the proposed appraisement platform. After critical investigation and scrutiny each decision-maker has been instructed to explore the linguistic scale (Table 2) towards assignment of priority weight and appropriateness rating against each evaluation indices. Appropriateness rating for 2nd level agile attributes has been furnished in Table 3. Tables 4-5 provide subjective judgment of the evaluation team members expressed through linguistic terms in relation to weight assignment against various agile capabilities as well as attributes, respectively. These linguistic expressions (human judgment) have been converted into appropriate generalized trapezoidal fuzzy numbers as presented in Table 2. The method of *simple average* has been used to obtain aggregated priority weights and aggregated ratings of 2nd level agile attributes (Tables 6). Computed fuzzy performance ratings (obtained by using Eqs. 27) and aggregated fuzzy priority weight for 1st level agile capabilities and tabulated in Table 7. Finally, Eq. 28 has been used to obtain overall FPI.

The concept of '*Ranking of fuzzy numbers*' [Thorani et al. (2012)] has been adapted here to indentify ill-performing areas of agile performance. 2nd level agile attributes have been ranked based on their individual *Fuzzy Performance Importance Index* (FPII) [Lin et al., 2006]. It has been computed as follows:

$$FPII_j = \left[1 - w_{ij}\right] \otimes U_{ij} \tag{28}$$

Here $FPII_j$ is denoted as the *Fuzzy Performance Importance Index* of j_{th} agile attribute; whose aggregated performance rating is U_{ij} and aggregated priority weight w_{ij}. The equivalent crisp measure corresponding to $R(FPII_{Individual})$ has been computed; thus, agile criterions have been ranked accordingly (Table 8).

5. Managerial Implications and Conclusions

Agile paradigm has become an important avenue in recent times. Many organizations around the world have been attempting to implement agile concepts in their supply chain. The agility metric is an important indicator in agile performance measure. Aforesaid study aimed to develop a quantitative analysis framework and a simulation methodology to evaluate the efficacy of an agile organization by exploring the concept of Generalized Trapezoidal Fuzzy Numbers (GTFNs). Exploration of fuzzy logic helps in dealing with decision-makers' linguistic evaluation information efficiently, thereby eliminating ambiguity, imprecision and vagueness arising from subjective human judgment. The procedural hierarchy presented here could help the industries to assess their existing agile performance extent, to compare and to identify week-performing areas towards implementing agility successfully. The specific contributions of this research have been summarized below.

1. Development of fuzzy-based integrated agility appraisement module. Industries/ enterprises can utilize this appraisement module as a test kit to assess and improve agility degree.
2. Estimation of overall agility index; identification of agile barriers.
3. Based on estimated overall agility index; different agile industries can be ranked accordingly.

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Table 1: Agility Appraisement Index hierarchy System [Dahmardeh and Pourshahabi, 2011]

1st Level Agile Capabilities	Index	2nd Level Agile Attributes	Index
Leadership	C_1	Establishment of a clear vision for the organization	C_{11}
		Focusing on new trends and strategic goals	C_{12}
		Using resources for strategic goals	C_{13}
		Assuring implementation of organizational change plans	C_{14}
Culture and Values	C_2	Organizational Flexibility for restructuring	C_{21}
		Decision-making based on consensus	C_{22}
		Readiness for change in organization	C_{23}
		Employee access to needful knowledge	C_{24}
		Characterizing the goals and premiums of team working	C_{25}
		Extent of centralization in organization	C_{26}
		Ability of decision-making by employees	C_{27}
Customer Service	C_3	Existing strategies for management in relation with customers	C_{31}
		Access to managers by the customers	C_{32}
		Instruction of employees about relationship with customers	C_{33}
		Work evaluation about customer	C_{34}
		Extent of management involvement with customers	C_{35}
E-Government	C_4	Extent of acceptance of new technologies	C_{41}
		Setting needful information in web site	C_{42}
		Possibility of E-Consultation for customers	C_{43}
		Emphasis on inputs of citizens for decision-making	C_{44}
		Incentives for shifting customers to low cost channels	C_{45}
Performance Management	C_5	Existence of continuum work evaluation system	C_{51}
		Adjustment and centralization on priorities	C_{52}
		Producing adequate and on time services to customers	C_{53}
		Instruction people for future works	C_{54}
Organizational Change	C_6	Existence of comprehensive method for realization of customer's prospect	C_{61}
		Identifying opportunities and needs for improvement of processes	C_{62}
		Existence of comprehensive system for transforming customer needs to services	C_{63}
		Renovation in organization	C_{64}
		Implementation of new technologies in producing services	C_{65}

Table 2: Nine-member linguistic terms and their corresponding fuzzy numbers

Linguistic terms for weight assignment	Linguistic terms for ratings	fuzzy numbers
Absolutely low, AL	Absolutely poor, AP	(0.0, 0.0, 0.0, 0.0; 1.0)
Very low, VL	Very poor, VP	(0.0, 0.0, 0.02, 0.07; 1.0)
Low, L	Poor, P	(0.04, 0.10, 0.18, 0.23; 1.0)
Fairly low, FL	Fairly poor, FP	(0.17, 0.22, 0.36, 0.42; 1.0)
Medium, M	Medium, M	(0.32, 0.41, 0.58, 0.65; 1.0)
Fairly High, FH	Fairly satisfactory, FS	(0.58, 0.63, 0.80, 0.86; 1.0)
High, H	Satisfactory, S	(0.72, 0.78, 0.92, 0.97; 1.0)
Very High, VH	Very Impressive, VI	(0.93, 0.98, 1.0, 1.0; 1.0)
Absolutely high, AH	Absolutely impressive, AI	(1.0, 1.0, 1.0, 1.0; 1.0)

Table 3: Appropriateness rating (linguistic) of 2nd level indices assigned by DMs

2nd level indices	Appropriateness rating (linguistic) of 2nd level indices assigned by DMs				
	DM1	DM2	DM3	DM4	DM5
C_{11}	S	VI	VI	S	S
C_{12}	S	S	VI	S	S
C_{13}	FS	S	S	S	FS
C_{14}	M	FS	S	S	S
C_{21}	VI	VI	AI	VI	S
C_{22}	M	FS	M	M	M
C_{23}	S	S	S	S	VI
C_{24}	AI	VI	VI	VI	VI
C_{25}	S	VI	S	S	S
C_{26}	S	S	VI	S	S
C_{27}	FS	S	VI	S	FS
C_{31}	M	FS	S	S	S
C_{32}	VI	VI	VI	VI	S
C_{33}	M	FS	FS	M	M
C_{34}	S	S	S	S	VI
C_{35}	AI	VI	S	VI	VI
C_{41}	S	VI	VI	S	S
C_{42}	S	S	S	S	S
C_{43}	FS	S	S	S	FS
C_{44}	M	FS	S	S	S
C_{45}	VI	VI	VI	VI	S
C_{51}	M	FS	M	M	M
C_{52}	S	S	S	S	VI
C_{53}	AI	VI	AI	VI	VI
C_{54}	S	VI	VI	S	S
C_{61}	S	S	VI	S	S
C_{62}	FS	S	VI	S	FS
C_{63}	M	FS	S	S	S
C_{64}	VI	VI	VI	VI	S
C_{65}	M	FS	M	FS	M

Table 4: Priority Weight (linguistic) of 2nd level indices assigned by DMs

2nd level indices	Priority Weight (linguistic) of 2nd level indices assigned by DMs				
	DM1	DM2	DM3	DM4	DM5
C_{11}	VH	H	H	H	VH
C_{12}	AH	VH	VH	VH	VH
C_{13}	H	H	H	H	H
C_{14}	FH	H	VH	H	H
C_{21}	H	VH	H	VH	VH
C_{22}	AH	H	H	H	H
C_{23}	H	VH	VH	VH	VH
C_{24}	VH	VH	H	H	VH
C_{25}	AH	H	VH	VH	VH
C_{26}	H	H	H	H	H
C_{27}	FH	H	VH	H	H
C_{31}	H	VH	H	VH	VH
C_{32}	AH	AH	H	H	H
C_{33}	H	VH	VH	VH	VH
C_{34}	VH	H	H	H	VH
C_{35}	AH	VH	H	H	VH
C_{41}	H	H	H	H	H
C_{42}	FH	H	H	H	H
C_{43}	H	VH	H	VH	VH
C_{44}	AH	H	H	H	H
C_{45}	H	VH	H	VH	VH
C_{51}	VH	H	H	H	VH
C_{52}	AH	VH	VH	VH	VH
C_{53}	H	H	H	H	H
C_{54}	FH	H	H	H	H
C_{61}	H	VH	H	VH	VH
C_{62}	AH	H	H	H	H
C_{63}	H	H	H	VH	VH
C_{64}	VH	H	H	H	VH
C_{65}	AH	VH	H	VH	VH

Table 5: Priority Weight (linguistic) of 1st level indices assigned by DMs

2^{nd}level indices	Priority Weight (linguistic) of 2^{nd}level indices assigned by DMs				
	DM1	DM2	DM3	DM4	DM5
C_1	VH	VH	H	H	VH
C_2	AH	VH	VH	VH	VH
C_3	H	VH	H	H	H
C_4	FH	H	VH	H	H
C_5	H	H	H	VH	VH
C_6	H	H	H	H	H

Table 6: Aggregated fuzzy weight and aggregated fuzzy rating of 2nd level indices

2nd level indices	Aggregated fuzzy weight, wij	Aggregated fuzzy rating, Uij
C_{11}	(0.804,0.860,0.952,0.982;1)	(0.804,0.860,0.952,0.982;1)
C_{12}	(0.944,0.984,1.000,1.000;1)	(0.762,0.820,0.936,0.976;1)
C_{13}	(0.320,0.410,0.580,0.650;1)	(0.664,0.720,0.872,0.926;1)
C_{14}	(0.734,0.790,0.912,0.954;1)	(0.612,0.676,0.828,0.884;1)
C_{21}	(0.846,0.900,0.968,0.988;1)	(0.902,0.944,0.984,0.994;1)
C_{22}	(0.776,0.824,0.936,0.952;1)	(0.372,0.454,0.624,0.692;1)
C_{23}	(0.888,0.940,0.984,0.994;1)	(0.762,0.820,0.936,0.976;1)
C_{24}	(0.846,0.900,0.968,0.988;1)	(0.944,0.984,10.00,1.000;1)
C_{25}	(0.902,0.944,0.984,0.994;1)	(0.762,0.820,0.936,0.976;1)
C_{26}	(0.720,0.780,0.920,0.970;1)	(0.762,0.820,0.936,0.976;1)
C_{27}	(0.734,0.790,0.912,0.954;1)	(0.706,0.760,0.888,0.932;1)
C_{31}	(0.846,0.900,0.968,0.988;1)	(0.612,0.676,0.828,0.884;1)
C_{32}	(0.832,0.868,0.952,0.982;1)	(0.888,0.940,0.984,0.994;1)
C_{33}	(0.888,0.940,0.984,0.994;1)	(0.424,0.498,0.668,0.734;1)
C_{34}	(0.804,0.860,0.952,0.982;1)	(0.762,0.820,0.936,0.976;1)
C_{35}	(0.860,0.904,0.968,0.988;1)	(0.902,0.944,0.984,0.994;1)
C_{41}	(0.720,0.780,0.920,0.970;1)	(0.804,0.860,0.952,0.982;1)
C_{42}	(0.692,0.750,0.896,0.948;1)	(0.720,0.780,0.920,0.970;1)
C_{43}	(0.846,0.900,0.968,0.988;1)	(0.664,0.720,0.872,0.926;1)
C_{44}	(0.776,0.824,0.936,0.952;1)	(0.612,0.676,0.828,0.884;1)
C_{45}	(0.846,0.900,0.968,0.988;1)	(0.888,0.940,0.984,0.994;1)
C_{51}	(0.804,0.860,0.952,0.982;1)	(0.372,0.454,0.624,0.692;1)
C_{52}	(0.944,0.984,10.00,1.000;1)	(0.762,0.820,0.936,0.976;1)
C_{53}	(0.720,0.780,0.920,0.970;1)	(0.958,0.988,1.000,1.000;1)
C_{54}	(0.692,0.750,0.896,0.948;1)	(0.804,0.860,0.952,0.982;1)
C_{61}	(0.846,0.900,0.968,0.988;1)	(0.762,0.820,0.936,0.976;1)
C_{62}	(0.776,0.824,0.936,0.952;1)	(0.706,0.760,0.888,0.932;1)
C_{63}	(0.804,0.860,0.952,0.982;1)	(0.612,0.676,0.828,0.884;1)
C_{64}	(0.804,0.860,0.952,0.982;1)	(0.888,0.940,0.984,0.994;1)
C_{65}	(0.902,0.944,0.984,0.994;1)	(0.424,0.498,0.668,0.734;1)

Table 7: Aggregated fuzzy weight and computed fuzzy rating 1st level indices

2ndlevel indices	Aggregated fuzzy weight, wi	Computed fuzzy rating, Ui
C_1	(0.846,0.900,0.968,0.988;1)	(0.565,0.744,1.019,1.208;1)
C_2	(0.944,0.984,1.000,1.000;1)	(0.626,0.734,0.990,1.121;1)
C_3	(0.762,0.820,0.936,0.976;1)	(0.612,0.715,0.948,1.068;1)
C_4	(0.734,0.790,0.912,0.954;1)	(0.591,0.705,1.028,1.188;1)
C_5	(0.804,0.860,0.952,0.982;1)	(0.581,0.693,0.979,1.125;1)
C_6	(0.720,0.780,0.920,0.970;1)	(0.568,0.673,0.939,1.071;1)

Table 8: Ranking order of 2nd level indices

2nd level indices	FPII	Crisp Value	Ranking Order
C_{11}	(0.158,0.120,0.046,0.018;1)	0.025	11
C_{12}	(0.043,0.013,0.000,0.000;1)	0.003	29
C_{13}	(0.452,0.452,0.366,0.324;1)	0.093	1
C_{14}	(0.163,0.142,0.073,0.041;1)	0.029	8
C_{21}	(0.139,0.094,0.031,0.012;1)	0.020	16
C_{22}	(0.083,0.080,0.040,0.033;1)	0.016	21
C_{23}	(0.085,0.049,0.015,0.006;1)	0.011	25
C_{24}	(0.145,0.098,0.032,0.012;1)	0.021	15
C_{25}	(0.075,0.046,0.015,0.006;1)	0.010	26
C_{26}	(0.213,0.180,0.075,0.029;1)	0.037	6
C_{27}	(0.188,0.160,0.078,0.043;1)	0.033	7
C_{31}	(0.094,0.068,0.026,0.011;1)	0.014	23
C_{32}	(0.149,0.124,0.047,0.018;1)	0.025	12
C_{33}	(0.047,0.030,0.011,0.004;1)	0.006	27
C_{34}	(0.149,0.115,0.045,0.018;1)	0.024	14
C_{35}	(0.126,0.091,0.031,0.012;1)	0.019	18
C_{41}	(0.225,0.189,0.076,0.029;1)	0.039	5
C_{42}	(0.222,0.195,0.096,0.050;1)	0.040	4
C_{43}	(0.102,0.072,0.028,0.011;1)	0.015	22
C_{44}	(0.137,0.119,0.053,0.042;1)	0.025	13
C_{45}	(0.137,0.094,0.031,0.012;1)	0.020	17
C_{51}	(0.073,0.064,0.030,0.012;1)	0.013	24
C_{52}	(0.043,0.013,0.000,0.000;1)	0.003	30
C_{53}	(0.268,0.217,0.080,0.030;1)	0.045	2
C_{54}	(0.248,0.215,0.099,0.051;1)	0.045	3
C_{61}	(0.117,0.082,0.030,0.012;1)	0.017	20
C_{62}	(0.158,0.134,0.057,0.045;1)	0.028	9
C_{63}	(0.120,0.095,0.040,0.016;1)	0.019	19
C_{64}	(0.174,0.132,0.047,0.018;1)	0.027	10
C_{65}	(0.042,0.028,0.011,0.004;1)	0.006	28