

BUCKLING ANALYSIS OF SWCNT REINFORCED COMPOSITE PLATE

A Report Submitted
In Partial Fulfilment of the Requirements for the degree of
Bachelor of Technology in Mechanical Engineering

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ROLL NO: 110ME0316
(Department Of Mechanical Engineering)

Under the Supervision of
Prof. Subrata Kumar Panda



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CERTIFICATE

This is to certify that the thesis entitled, “**BUCKLING ANALYSIS OF SWCNTREINFORCED COMPOSITE PLATES**” submitted by Mr **Sourava Jyoti Nayak** in partial fulfilment of the requirement for the award of **Bachelor of Technology** Degree in **Mechanical Engineering** with specialization in **Mechanical Engineering** at the National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/ Institute for the award of any degree or diploma.

Dr. S.K. Panda
Department Of Mechanical Engineering
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Abstract

This work presents the buckling analysis of functionally graded single walled carbon nanotubes reinforced composite plates under in-plane mechanical loads. The effective material properties of uniformly distributed carbon nanotubes are obtained using Mori–Tanaka approach and extended rule of mixtures. The buckling load has been obtained numerically with the help of commercial finite element package ANSYS using ANSYS parametric design language code. The convergence behaviour of the developed model has been checked and validated by comparing the responses with those available literature. Effect of different geometrical parameters such as aspect ratio and side to thickness ratio have been studied and discussed in details.

Key words: SWCNT, FEM, ANSYS, Laminated composite, Bucklin

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Introduction and literature review

Carbon nanotubes (CNTs) are carbon allotropes whose structure resembles that of a cylinder. The length to diameter ratio of CNT's can be in excess of thousands. It is well known from various experiments conducted by the researcher and claimed that the aspect ratio (l/d) exceed 1.32×10^8 to 1. These carbon atoms which are cylindrical in shape have very unusual properties which find application in electronics, optics, nanotechnology and other areas of science and engineering. These tubes possess extremely high thermal conductivity and electrical and mechanical properties and find application in material science and engineering. Nanotubes are used in tiny quantities and in microscopic proportions for common items like golf balls and in military application like gun barrels.

The basic structure of a nanotube resembles that of a fullerene, it is like long hollow walls which are formed by layers of carbon one atom thick which is called graphene. These tubes vary in properties depending upon the angle of rolling and the rolling radius. The angle of rolling assumes prime importance in determining the properties of the CNT and is called as chiral angle. Nanotubes are mainly classified as single walled and multi walled nanotubes (SWCNT/MWCNT). All nanotube molecules are held together by covalent forces which are also known as Vander Walls forces. Quantum chemistry and orbital hybridization can be used to describe the chemical structure of CNT's and it is composed of completely sp^2 bonds, like graphite.

There are numerous applications of CNT's which vary from use in miniscule applications involving nanotechnology to providing strength to gigantic structures. Breakthroughs carried out by Ray H. Baughman at the Nano Tech Institute has proposed that MWCNT and SWCNT can yield substances with great toughness which has been previously unheard of. Many researchers have carried out experiments to reinforce composites of CNT's. Generally mechanical, thermal and electrical properties of composite structures greatly depend on the added reinforcement, and the elasticity and tension strength of CNTs may give strong and stiff composite structures, such as shells, beams and plates.

Buckling of structural parts and equipment is a common failure mode. Buckling is a geometrical instability, which leads to a catastrophic failures. Buckling is characterized by a sudden failure of

a structural member subjected to high compressive stress, where the actual compressive stress at the point of failure is less than the ultimate compressive stresses that the material is capable of withstanding. Numerous studies have been conducted to study and describe the buckling behaviour of composites.

Many studies have been reported in literature to predict the buckling strength of functionally graded carbon nanotube (FGCNT) under mechanical loading.

Most examinations of carbon nanotubes-fortified composites (CNTRC) have concentrated on material properties and specialists have uncovered that mechanical, electrical and warm properties of polymer composites could be respectably enhanced by including little measures of Cnts. Odegard et al. [1] displayed a constitutive displaying of nanotubes-fortified polymer composites with nanotubes/ polymer interface displayed as a powerful continuum fiber by utilizing a proportional-continuum model. Gary et al. [2] acquired the powerful flexible properties of CNTRCs through an assortment of micromechanics systems with the powerful properties of CNTs ascertained using a composite barrels micromechanics method as a first venture in a two-stage process. Fidelus et al. [3] analyzed the thermo-mechanical properties of epoxy-based nanocomposites focused around low weight parts of arbitrarily arranged single- and multi-walled carbon nanotubes. Han and Elliot [4] displayed established sub-atomic motion (MD) enactments' of model polymer/CNT composites developed by implanting a solitary divider (10, 10) CNT into two separate indistinct polymer networks. By utilizing MD technique, the stress–strain conduct of carbon nanotube-fortified Epon862 composites was additionally considered by Zhu et al [5]. Despite the fact that these studies are truly helpful, a definitive reason for improvement of this propelled material is to investigate potential provisions of CNTs in genuine structures, for example, CNT-fortified pillars, plates or shells. Wuite and Adali [6] examined diversion and stress practices of nanocomposite fortified shafts utilizing a multiscale investigation. Their effects indicated that fortification by including a little extent of nanotube prompts noteworthy change in shaft solidness. Vodenitcharova and Zhang [7] displayed investigates of immaculate bowing and twisting-prompted neighbourhood clasping of a nanocomposite shaft focused around a continuum mechanical model and found that solitary-walled carbon nanotube (SWCNT) clasps at littler curving edges and more excellent straightening degrees in thicker framework layers. Formica et al. [8] explored vibration practices of CNTRC plates by utilizing an identical continuum model focused around the Eshelby–Mori–Tanaka approach.

Based on the above review, it is clear that the buckling behaviour of FGCNT composite have been reported in open literature but the number is very few. Hence, here we aim to analyse the mechanical behaviour of FGCNT composite plates under mechanical loading. As a first step, FGCNT model has been developed in ANSYS using ANSYS parametric design language code (APDL). The effective material properties of the FGCNT has been obtained using Mori-Tanaka extended rule of mixture. The model has been discretised using the SHELL281 element from ANSYS library. The convergence behaviour of the developed model has been checked and compared with those available literature. Some new results are also computed based on the different (thickness ratio and aspect ratio) parameters.

Mathematical Formulation

For the modelling purpose, a SHELL 281 element as shown in Fig. 1 is selected from the ANSYS 13.0 element library. This is an element having eight nodes and contains six degrees of freedom (three of the six degrees belongs to translations along x , y and z axes and the remaining three degrees belongs to rotations along x , y and z axes) per node. It works on shear deformation theory of the first order and is most appropriate for linear, large rotation, and/or large strain nonlinear applications. The formulation of is based on logarithmic strain and true stress measures.

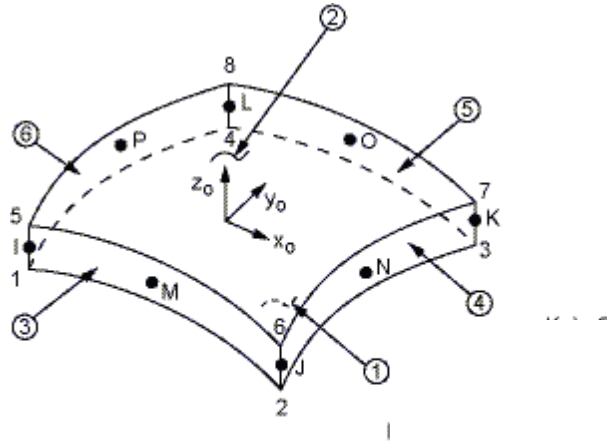


Fig. 1 Geometrical representation of SHELL281 [10]

x_0 = Element x-axis if element orientation is not provided.

x_1 = Element x-axis if element orientation is provided.

It is well known that the mathematical model have been used the FSDT to model in ANSYS and the displacement field is conceded as [10]:

$$u(x, y, z) = u_0(x,y) + z\theta_x(x,y)$$

$$v(x, y, z) = v_0(x,y) + z\theta_y(x,y) \dots\dots\dots(1)$$

$$w(x, y, z) = w_0(x,y) + z\theta_z(x,y)$$

The displacements u, v and w can be represented in terms of shape functions (N_i) as [10]:

$$\delta = \sum_{i=1}^j N_i \delta_i \dots\dots\dots(2)$$

where $\delta_i = [u_{0i} \ v_{0i} \ w_{0i} \ \theta_{xi} \ \theta_{yi} \ \theta_{zi}]^T$

The shape functions for the 8 noded (j=8) shell element (SHELL 281) are given by [10] :

$$\begin{aligned} N_1 &= 1/4 \times (1 - \zeta) (1 - \eta) (-\zeta - \eta - 1); & N_2 &= 1/4 \times (1 + \zeta) (1 - \eta) (\zeta - \eta - 1) \\ N_3 &= 1/4 \times (1 + \zeta) (1 + \eta) (\zeta + \eta - 1); & N_4 &= 1/4 \times (1 - \zeta) (1 + \eta) (-\zeta + \eta - 1) \dots\dots\dots(3) \\ N_5 &= 1/2 \times (1 - \zeta^2) (1 - \eta); & N_6 &= 1/2 \times (1 + \zeta) (1 - \eta^2) \\ N_7 &= 1/2 \times (1 - \zeta^2) (1 + \eta); & N_8 &= 1/2 \times (1 - \zeta) (1 - \eta^2) \end{aligned}$$

Strains can be got by derivation of displacements which can be shown as:

$$\{\varepsilon\} = \{u_{,x} \quad v_{,y} \quad w_{,z} \quad u_{,y} + v_{,x} \quad v_{,z} + w_{,y} \quad w_{,x} + u_{,z}\}$$

where $\{\varepsilon\} = \{\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx}\}^T$ is the strain matrix containing normal and shear strain components of the mid-plane in in-plane and out of plane direction.

The strain components are now rearranged in the following steps.

The in-plane strain vector is given by [10]:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{x0} \\ \varepsilon_{y0} \\ \gamma_{xy0} \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \dots\dots\dots(4)$$

The transverse strain vector is given by [10] :

$$\begin{Bmatrix} \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{z0} \\ \gamma_{yz0} \\ \gamma_{xz0} \end{Bmatrix} + z \begin{Bmatrix} \kappa_z \\ \kappa_{yz} \\ \kappa_{xz} \end{Bmatrix} \dots\dots\dots(5)$$

The deformation components are explained as [10] :

$$\begin{Bmatrix} \varepsilon_{x0} \\ \varepsilon_{y0} \\ \gamma_{xy0} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix}; \begin{Bmatrix} \kappa_z \\ \kappa_{yz} \\ \kappa_{xz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial \theta_x}{\partial x} \\ \frac{\partial \theta_y}{\partial y} \\ \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} \end{Bmatrix} \dots\dots\dots(6)$$

$$\begin{Bmatrix} \varepsilon_{zo} \\ \gamma_{yz0} \\ \gamma_{xz0} \end{Bmatrix} = \begin{Bmatrix} \theta_z \\ \frac{\partial w_0}{\partial y} + \theta_y \\ \frac{\partial w_0}{\partial x} + \theta_x \end{Bmatrix}; \begin{Bmatrix} \kappa_z \\ \kappa_{yz} \\ \kappa_{xz} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \frac{\partial \theta_z}{\partial y} \\ \frac{\partial \theta_z}{\partial x} \end{Bmatrix} \dots\dots\dots(7)$$

The strain vector are represented in terms of nodal displacement vector

$$\{\varepsilon\} = [B] \{\delta\} \dots\dots\dots(8)$$

[B] is the strain displacement matrix having shape functions and their derivatives and { δ } is the nodal displacement vector.

For a laminate, the stress-strain relationship with respect to its reference plane can be expressed as:

$$\{\sigma\} = [D]\{\varepsilon\} \dots\dots\dots(9)$$

where, { σ } and { ε } is the stress and strain vector respectively and [D] is the rigidity matrix containing the effective material properties of the orthotropic matrix of the composite plate.

The element stiffness matrix [K] may be easily obtained by virtual work method which can be expressed as [10] :

$$[K] = \int_{-1}^{+1} \int_{-1}^{+1} [B]^T [D] [B] |J| d\zeta d\eta \dots\dots\dots(10)$$

where, |J| : Determinant of the Jacobian matrix, [N]: Shape function matrix and [m] : the inertia matrix.

The integration has been done using the Gaussian quadrature method.

The nodal displacements can be written in the form of their shape functions and corresponding nodal values as follows [10]:

$$u^0 = \sum_{i=1}^8 N_i u_i^0, v^0 = \sum_{i=1}^8 N_i v_i^0, w^0 = \sum_{i=1}^8 N_i w_i^0, \theta_x = \sum_{i=1}^8 N_i \theta_{xi}^0, \theta_y = \sum_{i=1}^8 N_i \theta_{yi}^0, \theta_z = \sum_{i=1}^8 N_i \theta_{zi}^0$$

The above expression can be reshown in i th nodal displacement as follows :

$$\{\delta_i^*\} = \{u_i^0 \quad v_i^0 \quad w_i^0 \quad \theta_{xi}^0 \quad \theta_{yi}^0 \quad \theta_{zi}^0\} \dots\dots\dots(11)$$

$$\{\delta_i^*\} = [N_i] \{\delta_i\} \dots\dots\dots(12)$$

$$\text{where } [N_i] = \begin{bmatrix} N_i & 0 & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & 0 & N_i \end{bmatrix} \dots\dots\dots(13)$$

Substituting the value of nodal displacement in strain, strain energy we will get

$$\{\varepsilon_i\} = [B_i] \{\delta_i\}$$

$$U = \frac{1}{2} \int [B_i]^T \{\delta_i\}^T [D] [B_i] \{\delta_i\} dA - \{F\}_m \dots\dots\dots(14)$$

where $[B_i]$: strain displacement relation matrix,
 $\{F\}_m$: mechanical force, respectively.

The final form of the equation can be obtained by minimizing the total potential energy (TPE) as follows:

$$\delta \Pi = 0$$

Where, Π is the total potential energy.

$$[K]\{\delta\} = \{F\}_m$$

where, $[M]$ and $[K]$ are the global mass and stiffness matrix.

The buckling equation can be obtained for the laminated shell/plate and conceded as [10] :

$$[K]\{\delta\} = \{F\}_m \dots \dots \dots (15)$$

In this study eigenvalue type of buckling has studied and the eigenvalue equation will be obtained by dropping the force vectors and considering their effects in geometry the new form of the Eq. (21) will be expressed as [10] :

$$\{[K] + \lambda_{cr}[K_G]\}\{\delta\} = 0 \dots \dots \dots (16)$$

where

$[K_G]$ =the geometric stiffness matrix

λ_{cr} = critical mechanical load at which the structure buckles.

Carbon nanotube reinforced composite plates

An ANSYS model of UD -CNTRC plate has been developed by defining the length a , width b , and thickness h . The compelling material properties of the two-stage nano composites, mixture of CNTs and an orthotropic polymer as per the Mori–Tanaka rule of mixture

$$E_{11} = \eta V_{CNT} E_{11}^{CNT} + V_m E^m \quad (1a)$$

$$\frac{\eta_2}{E_{22}} = \frac{V_{CNT}}{E_{22}^{CNT}} + \frac{V_m}{E_{12}^{CNT}} + \frac{V_m}{E^m} \quad (1b)$$

(1c)

$$\frac{\eta_3}{G_{12}} = \frac{V_{CNT}}{G_{12}^{CNT}} + \frac{V_m}{G^m}$$

where, E_{11}^{CNT} , E_{22}^{CNT} and G_{12}^{CNT} indicate the young's modulus and shear modulus of SWCNTS, respecting and E^m and G^m represent the properties of the orthotropic matrix. Similarly, V_{CNT}^* is the volume part of the carbon nanotube and V_{CNT} is the volume portion of the matrix.

$$V_{CNT} = V_{CNT}^* \text{ (UD CNTRC)} \quad 2(a)$$

$$\text{where, } V_{CNT}^* = \frac{W_{CNT}}{W_{CNT} + (\rho^{CNT} / \rho^m) - (\rho^{CNT} / \rho^m)W_{CNT}} \quad 2(b)$$

where, W_{CNT} = weight fraction of the CNT in the composite plate

ρ^{CNT} = Densities of the carbon nanotubes

ρ^m = Densities of the matrix.

$$\nu_{12} = (V_{CNT}^* \nu_{12}^{CNT} + V_m \nu^m) \dots \dots \dots 3(a)$$

$$\rho = (V_{CNT} \rho^{CNT} + V_m \rho^m) \dots \dots \dots 3(b)$$

$$\alpha_{11} = V_{CNT} \alpha_{11}^{CNT} + V_m \alpha^m \dots \dots \dots 3(c)$$

$$\alpha_{11} \alpha_{22} = (1 + \nu_{12}^{CNT}) V_{CNT} \alpha_{22}^{CNT} + (1 + \nu^m) V_m \alpha^m - \nu_{12} \alpha_{11} \dots \dots \dots 3(d)$$

where, ν_{12}^{CNT} and ν^m are Poisson's ratio of CNT and matrix and α_{11}^{CNT} , α_{22}^{CNT} , α^m are the thermal expansion coefficients in 1 and 2-directions of the CNT and matrix, respectively. The ν_{12} remains invariable along the transverse direction of the CNTRC plate. The details of UD FGCNT is presented in Fig. 2

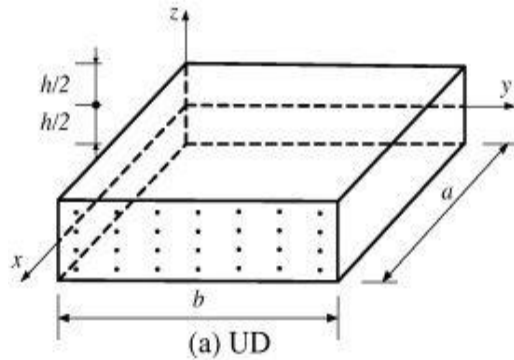


Fig.2. FGCNT plate model with uniformly distributed CNT [9]

Results and Discussion

In this section, the FGCNT model of uniformly distributed (UD) composite plate have been developed using APDL code in ANSYS. As discussed a plate having dimensions $a = b = 20$ mm and $h = 2$ mm and the effective modulus of elasticity of matrix (E_m) = 2.1×10^9 N/mm² have been developed and presented in Fig. 3.

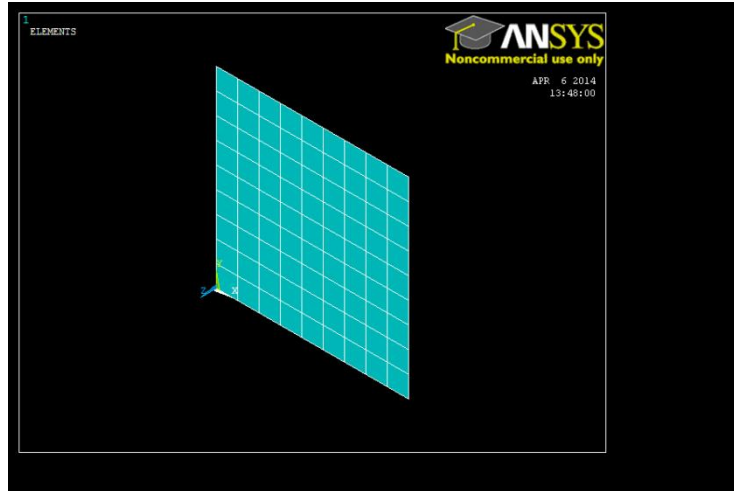


Fig. 3. FGCNT plate model

The following simply supported boundary conditions are used for the analysis [9]:

$$v_0 = w_0 = \phi_y = 0 \text{ at } x = 0 \text{ and } x = a$$

$$u_0 = w_0 = \phi_x = 0 \text{ at } y = 0 \text{ and } y = b$$

Convergence and Comparison Study

In order to check the convergence behaviour of the present model, the present model has been developed in ANSYS and presented in Fig.4. It is clearly observed that the present results are converging well with mesh refinement. In continuation to that, the nondimensional critical buckling load parameters are computed using the present APDL code and the comparison study has been presented in Table 1. The material and geometrical parameters are same as the reference. The ANSYS model is showing very good agreement with that of the reference.

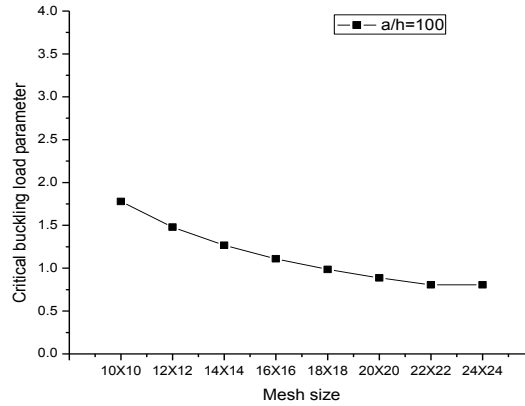


Fig.4 Convergence behaviour of nondimensional buckling load parameter

Table 1. Comparison of critical buckling load.

| Mode No. | Present \bar{N}_{cr} | Reference [9] | Percentage of difference |
|----------|------------------------|---------------|--------------------------|
| 1 | 27.44 | 28.4768 | 3.78 |
| 2 | 27.46 | 28.8410 | 5.03 |
| 3 | 27.64 | 29.5768 | 7.01 |
| 4 | 28.94 | 30.1219 | 4.08 |

Numerical Illustration

Some new results are computed for FGCNT composite plate for thickness ratio and aspect ratio and presented in Fig. 5 and 6, respectively. It is observed that the buckling load parameters increases as the thickness ratio increases and the decreases with aspect ratio. In addition to that four different buckling mode shapes are presented in Fig. 7 (a), (b), (c) and (d), respectively.

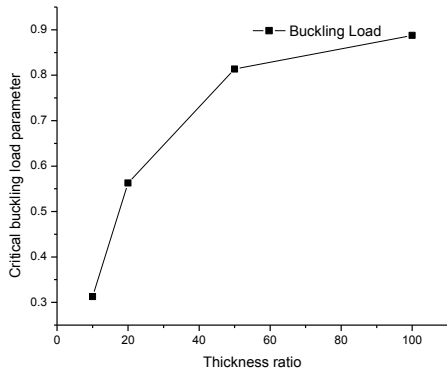


Fig. 5. Effect of thickness ratio on buckling load parameter

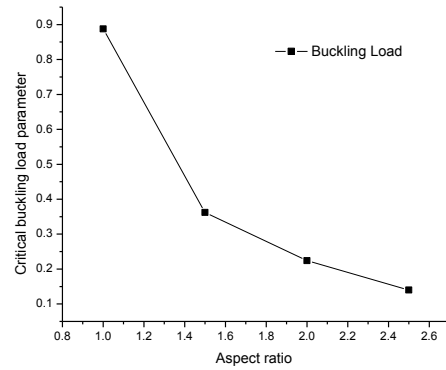


Fig. 6. Effect of aspect ratio on buckling load parameter

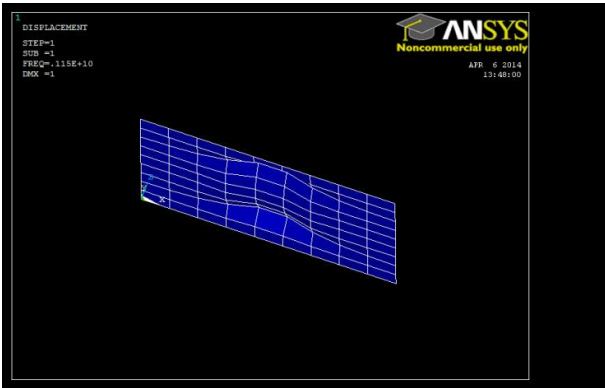


Fig. 7 (a). 1st Mode

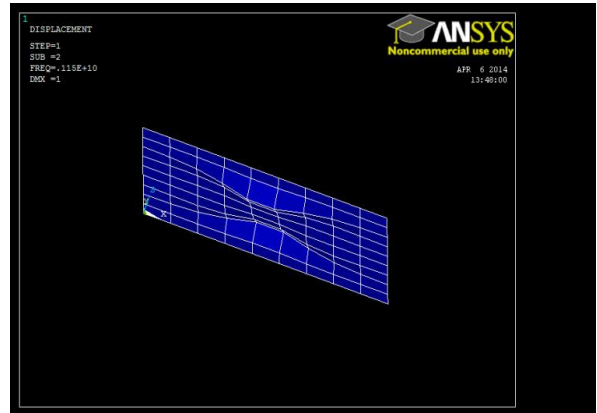


Fig. 7(b). 2nd Mode

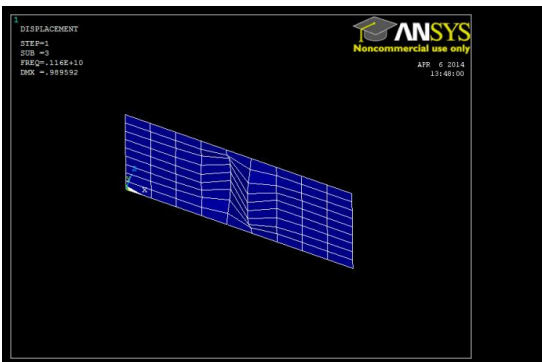


Fig. 7(c). 1st Mode

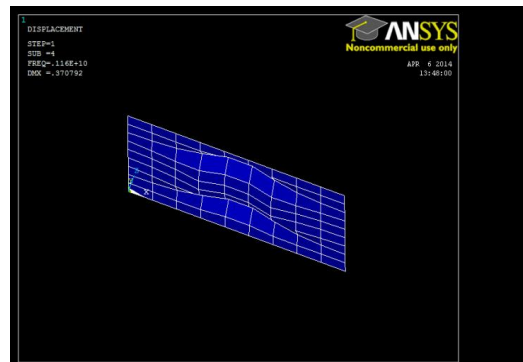


Fig. 7(d). 1st Mode

Conclusions:

In this study, the nondimensional buckling load parameter have been obtained numerically. The model has been developed in ANSYS using APDL code. The convergence and comparison behaviour of the developed model has been checked. Some new results are computed for thickness ratio and aspect ratio. The results are following the expected line.

References

- [1] Odegard GM, Gates TS, Wise KE, Park C, Siochi EJ. Constitutive modeling of nanotube-reinforced polymer composites. *Compos Sci Technol* 2003;63:1671–87.
- [2] Seidel GD, Lagoudas DC. Micromechanical analysis of the effective elastic properties of carbon nanotube reinforced composites. *Mech Mater* 2006;38:884–907.
- [3] Fidelus JD, Wiesel E, Gojny FH, Schulte K, Wagner HD. Thermo-mechanical properties of randomly oriented carbon/epoxy nanocomposites. *Composites:Part A* 2005;36:1555–61.
- [4] Han Y, Elliott J. Molecular dynamics simulations of the elastic properties of polymer/carbon nanotube composites. *Comput Mater Sci* 2007;39:315–23.
- [5] Zhu R, Pan E, Roy AK. Molecular dynamics study of the stress-strain behaviour of carbon-nanotube reinforced Epon 862 composites. *Mater Sci Eng, A* 2007;447:51–7.
- [6] Wuite J, Adali S. Deflection and stress behaviour of nanocomposite reinforced beams using a multiscale analysis. *Compos Struct* 2005;71:388–96.
- [7] Vodenitcharova T, Zhang LC. Bending and local buckling of a nanocomposite beam reinforced by a single-walled carbon nanotube. *Int J Solids Struct* 2006;43:3006–24.
- [8] Formica G, Lacarbonara W, Alessi R. Vibrations of carbon nanotube-reinforced composites. *J Sound Vib* 2010;329:1875–89.
- [9] Z.X. Lei, K.M. Liew, J.L. Yu, Buckling analysis of functionally graded carbon nanotube-reinforced composite plates using the element-free kp-Ritz method.
- [10] Santanu Kumar Sahu, Static and Buckling analysis of Laminated Sandwich Plates with Orthotropic Core using FEM, Department of Mechanical Engg., NIT Rourkela, Odisha, India.