SIMULATION OF WIRELESS COMMUNICATION SYSTEM USING OFDM PRINCIPLE

A THESIS SUBMITTED IN PARTIAL FULFILMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

Master of Technology

In

VLSI Design and Embedded System

By

D.Jagadeesh

Roll No: 20507003

Department of Electronics and Communication Engineering

National Institute of Technology

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Under the guidance of

Prof. G. Panda

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CERTIFICATE

This is to certify that the thesis entitled, "**Simulation of Wireless Communication System Using OFDM Principle**" submitted by Mr**. Jagadeesh Darapureddy** in partial fulfillment of the requirements for the award of Master of Technology Degree in Electronics and communication Engineering with specialization in "**VLSI Design and Embedded System**" at the National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/ Institute for the award of any degree or diploma.

Date:

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D. Jagadeesh

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ABSTRACT

FDMA, TDMA and CDMA are the well known multiplexing techniques used in wireless communication systems. While working with the wireless systems using these techniques various problems encountered are (1) multi-path fading (2) time dispersion which lead to intersymbol interference (ISI) (3) lower bit rate capacity (4) requirement of larger transmit power for high bit rate and (5) less spectral efficiency. In a typical terrestrial broadcasting, the transmitted signal arrives at the receiver using various paths of different lengths. Since multiple versions of the signal interfere with each other, it becomes difficult to extract the original information. The use of orthogonal frequency division multiplexing (OFDM) technique provides better solution for the above mentioned problems. OFDM technique distributes the data over a large number of carriers that are spaced apart at precise frequencies. This spacing provides the "orthogonality", which prevents the demodulator from seeing frequencies other than their own. The benefits of OFDM are high spectral efficiency, resiliency of RF interference, and lower multi-path distortion. OFDM is a powerful modulation technique that is capable of high data rate and is able to eliminate ISI. The use of FFT technique to implement modulation and demodulation functions makes it computationally more efficient. The OFDM based wireless communication system design includes the design of OFDM transmitter, and OFDM receiver. Using MATLAB, simulation of OFDM was done with different modulation techniques using different transform techniques. The digital modulation schemes such as BPSK and QPSK were selected to assess the performance of the designed OFDM system.

Different transform techniques such as Discrete Fourier transform (DFT) and its inverse (IDFT), discrete Hartley transform (DHT) and its inverse (IDHT), Discrete Cosine Transform and its inverse (IDCT) are used to perform the modulation and demodulation operations and compared the performance of the designed OFDM system.

For OFDM-based transceivers, the modulator needs to compute a long-length inverse discrete Fourier transform (IDFT), and the demodulator needs to compute a long length DFT, where the transform length is up to 512 or more. For such long-length DFT/IDFT computations, a great number of complex multiplications are required and each of them basically involves four real multiplications and two real additions. Clearly, the complexity of an OFDM-based transceiver would be reduced if the corresponding modulator/demodulator could be implemented using fast algorithms like Fast Fourier transform (FFT) algorithm.

The complexity of an OFDM-based transceiver would also be reduced if the corresponding modulator/demodulator could be implemented using purely real transforms like discrete Hartley transform (DHT) and discrete cosine transform (DCT). Since the DHT and IDHT definitions are identical, we can use the same hardware or program to implement the modulator and demodulator of the OFDM system. Like the DFT, there have been a number of fast algorithms and hardware architectures available for the DHT computation.

A discrete cosine transform (DCT) is a Fourier-related transform similar to the discrete Fourier transform (DFT), but using only real numbers. The DCT is often used in signal and image processing, especially for lossy data compression, because it has a strong "energy compaction" property: most of the signal information tends to be concentrated in a few low-frequency components of the DCT. The Discrete cosine transform (DCT) is also a real valued transform, used to find where the power is concentrated in a signal. So it will transmit those parts only where the power was concentrated.

From the simulation results, it is observed that the BPSK allows the BER to be improved in a noisy channel at the cost of maximum data transmission capacity. Use of QPSK allows higher transmission capacity, but at the cost of slight increase in the probability of error. From the results, use of OFDM with QPSK is beneficial for short distance transmission link, whereas for long distance transmission link OFDM with BPSK will be preferable.

From the results it is also observed that the DHT-based OFDM achieves the same transmission performance as the DFT-based OFDM from the BER vs SNR point of view, but requires less computational complexity with reduced implementation cost, because of it is purely based on the real valued DHT. And the DCT based OFDM has better BER performance compared to the DFT, DHT based OFDM methods, because of its strong "energy compaction" property and real valued nature. So, from the BER vs SNR, implementation and performance points of view, DHT, DCT transform techniques are may be alternatives to DFT for the designing of OFDM-based communication systems.

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Chapter 1

INTRODUCTION

1. INTRODUCTION

1.1 INTRODUCTION:

In a basic communication system, the data are modulated onto a single carrier frequency. The available bandwidth is then totally occupied by each symbol. This kind of system can lead to inter-symbol-interference (ISI) in case of frequency selective channel. The basic idea of OFDM is to divide the available spectrum into several orthogonal subchannels so that each narrowband subchannel experiences almost flat fading. Orthogonal frequency division multiplexing (OFDM) is becoming the chosen modulation technique for wireless communications. OFDM can provide large data rates with sufficient robustness to radio channel impairments. Many research centers in the world have specialized teams working in the optimization of OFDM systems. In an OFDM scheme, a large number of orthogonal, overlapping, narrow band sub-carriers are transmitted in parallel. These carriers divide the available transmission bandwidth. The separation of the sub-carriers is such that there is a very compact spectral utilization. With OFDM, it is possible to have overlapping subchannels in the frequency domain, thus increasing the transmission rate. The attraction of OFDM is mainly because of its way of handling the multipath interference at the receiver. Multipath phenomenon generates two effects (a) Frequency selective fading and (b) Intersymbol interference (ISI).

The "flatness" perceived by a narrowband channel overcomes the frequency selective fading. On the other hand, modulating symbols at a very low rate makes the symbols much longer than channel impulse response and hence reduces the ISI. Use of suitable error correcting codes provides more robustness against frequency selective fading. The insertion of an extra guard interval between consecutive OFDM symbols can reduce the effects of ISI even more. The use of FFT technique to implement modulation and demodulation functions makes it computationally more efficient. OFDM systems have gained an increased interest during the last years. It is used in the European digital broadcast radio system, as well as in wired environment such as asymmetric digital subscriber lines (ADSL). This technique is used in digital subscriber lines (DSL) to provides high bit rate over a twisted-pair of wires.

1.2 MOTIVATION:

Multimedia is effectively an infrastructure technology with widely different origins in computing, telecommunications, entertainment and publishing. New applications are emerging, not just in the wired environment, but also in the mobile one. At present, only low bit-rate data services are available to the mobile users. The radio environment is harsh, due to the many reflected waves and other effects. Using adaptive equalization techniques at the receiver could be the solution, but there are practical difficulties in operating this equalization in real-time at several Mb/s with compact, low-cost hardware. A promising candidate that eliminates a need for the complex equalizers is the Orthogonal Frequency Division Multiplexing (OFDM), a multiple carrier modulation technique. OFDM is robust in adverse channel conditions and allows a high level of spectral efficiency. It effectively mitigates performance degradations due to multipath and is capable of combating deep fades in part of the spectrum. The OFDM waveform can be easily modified to adjust to the delay spread of the channel. OFDM can handle large delay spreads easier to due the independence of the carriers and the flexibility of varying the cyclic prefix length. OFDM allows efficient operation in both FDD and TDD mode as very short or no pre-ambles are needed. Multiple access techniques which are quite developed for the single carrier modulations (e.g. TDMA, FDMA) had made possible of sharing one communication medium by multiple number of users. Multiple techniques schemes are used to allow many mobile users to share simultaneously a finite amount of radio spectrum. The sharing is required to achieve high capacity by simultaneously allocating the available bandwidth (or the available amount of channels) to multiple users.

For the quality communications, this must be done without severe degradation in the performance of the system. FDMA, TDMA and CDMA are the well known multiplexing techniques used in wireless communication systems. While working with the wireless systems using these techniques various problems encountered are (1) multi-path fading (2) time dispersion which lead to intersymbol interference (ISI) (3) lower bit rate capacity (4) requirement of larger transmit power for high bit rate and (5) less spectral efficiency. Disadvantage of FDMA technique is its Bad Spectrum Usage. Disadvantages of TDMA technique is Multipath Delay spread problem. In a typical terrestrial broadcasting, the transmitted signal arrives at the receiver using various paths of different lengths. Since multiple versions of the signal interfere with each other, it becomes difficult to extract the original information. The use of orthogonal frequency division multiplexing (OFDM) technique provides better solution for the above mentioned problems.

1.3 LITERATURE SURVEY:

The concept of using parallel data transmission by means of frequency division multiplexing (FDM) was published in mid 60's [23, 24]. Some early development with this can be traced back to the 50s. A U.S. patent was filled and issued in January 1970. The idea was to use parallel data streams and FDM with overlapping sub channels to avoid the use of high-speed equalization and to combat impulsive noise, and multipath distortion as well as to fully use the available bandwidth. The initial applications were in the military communications. In the telecommunications field, the terms of discrete multi-tone (DMT), multichannel modulation and multicarrier modulation (MCM) are widely used and sometimes they are interchangeable with OFDM. In OFDM, each carrier is orthogonal to all other carriers. However, this condition is not always maintained in MCM. OFDM is an optimal version of multicarrier transmission schemes. Weinstein and Ebert [25] applied the discrete Fourier transform (DFT) [12] to parallel data transmission system as part of the modulation and demodulation process. In the 1980s, OFDM has been studied for high speed modems, digital mobile communications [10] and high-density recording.

Various fast modems were developed for telephone networks. In 1990s, OFDM has been exploited for wideband data communications over mobile radio FM channels [14], wireless LAN [13] wireless multimedia communication, high-bit-rate digital subscriber lines (HDSL) [16], asymmetric digital subscriber lines (ADSL) [20], very high speed digital subscriber lines (VHDSL), digital audio broadcasting (DAB) [18] and HDTV terrestrial broadcasting.

In a classical parallel data system, the total signal frequency band is divided into N nonoverlapping frequency subchannels. Each subchannel is modulated with a separate symbol and then the N subchannels are frequency-multiplexed. It seems good to avoid spectral overlap of channels to eliminate interchannel interference. However, this leads to inefficient use of the available spectrum. To cope with the inefficiency, the ideas proposed from the mid-1960s were to use parallel data and FDM with overlapping subchannels, in which, each carrying a signaling rate b is spaced b apart in frequency to avoid the use of high-speed equalization and to combat impulsive noise and multipath distortion, as well as to fully use the available bandwidth.

1.4 CONTRIBUTION:

Using MATLAB, simulation of OFDM was done with different modulation techniques using different transform techniques. The digital modulation schemes such as BPSK and QPSK were selected to assess the performance of the designed OFDM system by finding their Bit Error rate for different values of SNR.

Different transform techniques such as Discrete Fourier transform (DFT) and its inverse (IDFT), discrete Hartley transform (DHT) and its inverse (IDHT), Discrete Cosine Transform and its inverse (IDCT) are used to perform the modulation and demodulation operations in the implementation of OFDM and compared their performance by finding their Bit Error rate for different values of SNR.

1.5 THESIS OUTLINE:

Following the introduction, the rest of the thesis is organized as follows. Chapter 2 gives a review on existing multiple access techniques used in wireless communication systems. Chapter 3 describes different digital modulation techniques that are used in OFDM.
Chapter 4 discribes about different transform techniques that are available like FFT, DHT, and DCT. Chapter 5 gives an over view of OFDM. It discribes OFDM basic principle, its working model, properties, parameters, and applications. In Chapter 6 discribes OFDM simulations and results. Then I made a conclusion to my work and the points to possible directions for future work in Chapter 7.

Chapter 2

MULTIPLE ACCESS TECHNIQUES

2. MULTIPLE ACCESS TECHNIQUES

2.1 INTRODUCTION

Multiple access techniques are employed side by side in cellular systems. The need for multiple access techniques arises from the necessity to share a limited resource of radio spectrum amongst many users. Multiple access schemes are used to allow many mobile users to share simultaneously a finite amount of radio spectrum. The sharing of spectrum is required to achieve high capacity by simultaneously allocating the available bandwidth (or the available amount of channels) to multiple users.

2.2 DUPLEXING

In wireless communications systems, it is often desirable to allow the subscriber to send simultaneously while receiving information to the base station while receiving information from the base station. This effect is called *duplexing* and a device called duplexer is used inside each subscriber unit and base station. Duplexing may be done using frequency or time domain techniques. Figure 2.1 illustrates FDD (Frequency Division Duplexing) and TDD (Time Division Duplexing).

Frequency division duplexing (FDD) provides two distinct bands of frequencies for every user. The *forward band* provides traffic form the base station to mobile, and the reverse provides traffic from the mobile to base station.

Time division duplexing (TDD) uses time instead of frequency to provide both a forward and reverse link. In TDD, multiple users share a single radio channel by taking turns in the time domain. Individual users allowed to access the channel in assigned time slots, and each duplex channel has both forward time slot and a reverse time slot to facilitate bidirectional communication.

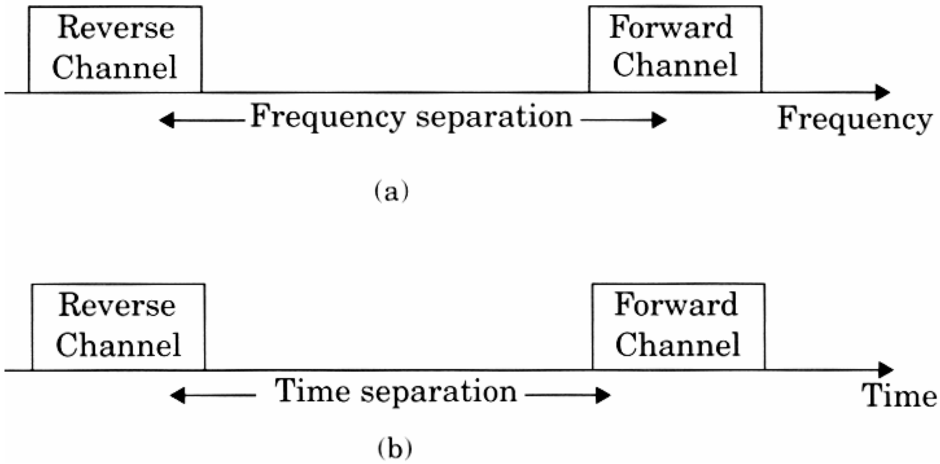


Figure 2.1: (a) FDD provides two simplex channels at the same time; (b) TDD provides two simplex time slots at the same frequency.

2.3 INTRODUCTION TO MULTIPLE ACCESS:

Frequency division multiple access (FDMA), *time division multiple access* (TDMA), *and code division multiple access* (CDMA) are the three major access techniques used to share the available bandwidth in a wireless communication system

2.3.1 FDMA (*Frequency Division Multiple Access*)**:** Frequency Division Multiple Access was the initial multiple-access technique for cellular systems. In this technique a user is assigned a pair of frequencies when placing or receiving a call. One frequency is used for downlink (base station to mobile) and one pair for uplink (mobile to base). This is called frequency division duplexing. That frequency pair is not used in the same cell or adjacent cells during the call. During the period of the call, no other user can share the same channel. If an FDMA channel is not in use, then it sits idle and cannot be used by other users to increase or share capacity. It is essentially a wasted resource. Figure 2.2 illustrates a FDMA system. Even though the user may not be talking, the spectrum cannot be reassigned as long as a call is in place. Frequency Division Multiple Access (FDMA) is the most common analog system. It is a technique whereby spectrum is divided up into frequencies and then assigned to users. With FDMA, only one subscriber at any given time is assigned to a channel. The channel therefore is closed to other conversations until the initial call is finished, or until it is handed-off to a different channel. A "full-duplex" FDMA transmission requires two channels, one for transmitting and the other for receiving. FDMA has been used for first generation analog systems.

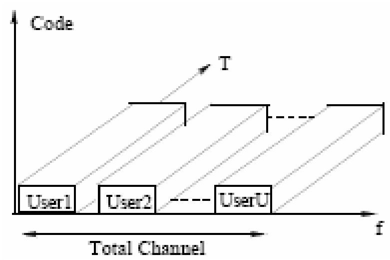


Figure2.2: Frequency division multiple access (FDMA).

2.3.2 TDMA (*Time Division Multiple Access*)**:** Time Division Multiple Access (TDMA) improves spectrum capacity by splitting each frequency into time slots. TDMA allows each user to access the entire radio frequency channel for the short period of a call. Other users share this same frequency channel at different time slots. The base station continually switches from user to user on the channel. TDMA is the dominant technology for the second generation mobile cellular networks. TDMA system divide the radio spectrum into time slots, and in each slot only one user is allowed to transmit and receive. It can be seen from Figure 2.3 that each user occupies a cyclically repeating time slot, so a channel may be thought of as a particular time slot that reoccurs every frame, where N time slots comprise a frame.

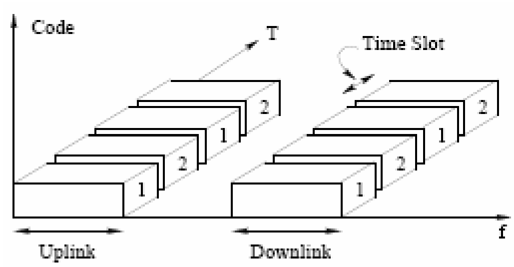


Figure 2.3: Time division multiple access (TDMA).

2.3.3 CDMA (*Code Division Multiple Access*)**:** Code Division Multiple Access is based on "spread" spectrum technology. Since it is suitable for encrypted transmissions, it has long been used for military purposes. CDMA increases spectrum capacity by allowing all users to occupy all channels at the same time. Transmissions are spread over the whole radio band, and each voice or data call are assigned a unique code to differentiate from the other calls carried over the same spectrum. CDMA allows for a "soft hand-off", which means that terminals can communicate with several base stations at the same time. In CDMA systems, the narrowband message signal is multiplied by a very large bandwidth signal called the *spreading signal*. The spreading signal is a pseudo-noise code sequence that has a chip rate which is orders of magnitudes greater than the data rate of the message. All users in a CDMA system, as seen from figure 2.4, use the same carrier frequency and may transmit simultaneously. Each user has its own pseudorandom codeword which is approximately orthogonal to all other code words. The receiver performs a time correlation operation to detect only the specific desired codeword. All other code words appear as noise due to decorrelation. For detection of the message signal, the receiver needs to know the codeword used by the transmitter. Each user operates independently with no knowledge of the other users.

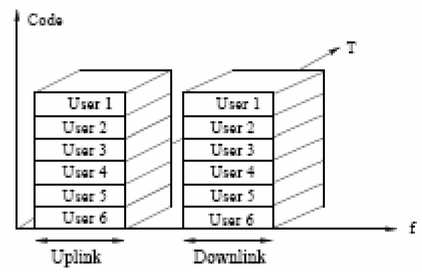


Figure 2.4: Code division multiple access (CDMA).

Chapter 3

DIGITAL MODULATION TECHNIQUES

3. DIGITAL MODULATION TECHNIQUES

3.1 BASIC CONCEPTS OF MODULATION

3.1.1 Three kinds of modulations

Modulation is the process of facilitating the transfer of information over a medium. Sound transmission in air has limited range for the amount of power your lungs can generate. To extend the range your voice can reach, we need to transmit it through a medium other than air, such as a phone line or radio. The process of converting information (voice in this case) so that it can be successfully sent through a medium (wire or radio waves) is called modulation.

There are three basic types of digital modulation techniques. These are

1. Amplitude-Shift Keying (ASK)
2. Frequency-Shift Keying (FSK)
3. Phase-Shift Keying (PSK)

All of these techniques vary a parameter of a sinusoid to represent the information which we wish to send. A general carrier wave may be written:

$$C(t) = A\sin(2\pi ft + \phi)$$

A sinusoid has three different parameters than can be varied. These are its amplitude, phase and frequency. Modulation is a process of mapping such that it takes your voice (as an example of a signal) converts it into some aspect of a sine wave and then transmits the sine wave, leaving the actual voice behind. The sine wave on the other side is remapped back to a near copy of your sound.

The medium is the thing through which the sine wave travels. So wire is a medium and so are air, water and space. The sine wave is called the carrier. The information to be sent, which can be voice or data is called the information signal. Once the carrier is mapped with the information to be sent, it is no longer a sine wave and we call it the signal.

3.2 ASK

In ASK, the amplitude of the carrier is changed in response to information and all else is kept fixed. In Binary ASK Bit 1 is transmitted by a carrier of one particular amplitude. To transmit 0, we change the amplitude keeping the frequency constant. On-Off Keying (OOK) is a special form of ASK, where one of the amplitudes is zero as shown in fig 3.1 and fig 3.2.

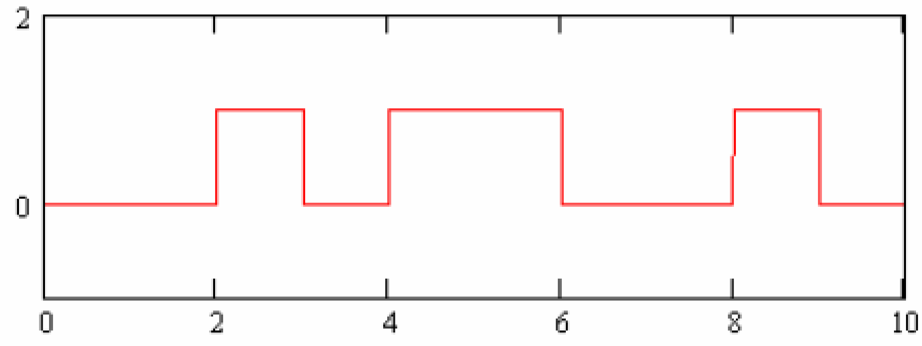


Figure 3.1 - Baseband information sequence – 0010110010

Binary $ASK(t) = s(t)\sin(2\pi ft)$

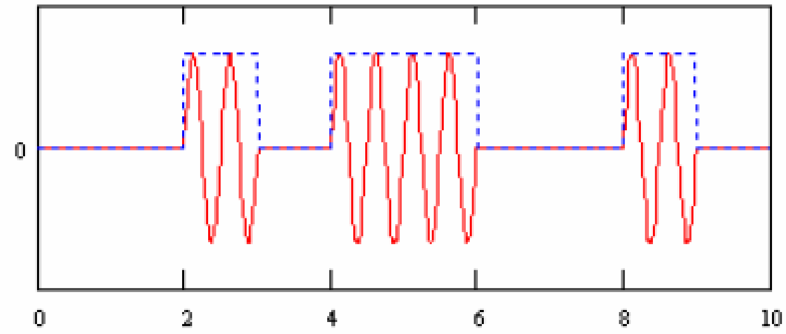


Figure 3.2 - Binary ASK (OOK) signal

3.3 FSK

In FSK, we change the frequency in response to information, In Binary FSK one particular frequency for a 1 and another frequency for a 0 is used as shown in fig 3.3 for the same bit sequence as above. In the example below, frequency f1 for bit 1 is higher than f2 used for the 0 bit.

$$\text{Binary FSK(t)} = \begin{cases} \sin(2\pi f_1 t) & \textbf{\textit{for bit 1}} \\ \sin(2\pi f_2 t) & \textbf{\textit{for bit 0}} \end{cases}$$

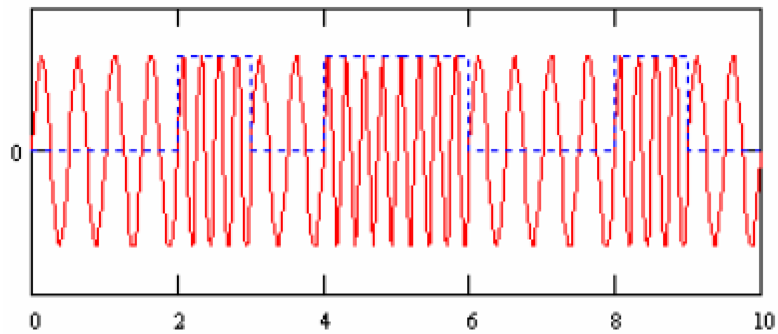


Figure 3.3 - Binary FSK signal

3.4 PSK

In PSK, we change the phase of the sinusoidal carrier to indicate information. Phase in this context is the starting angle at which the sinusoid starts. The transmitted signal is a sinusoid of fixed amplitude. Phase shift keying is a modulation process whereby the input signal, a binary PCM waveform, shifts the output waveform to one of a fixed number of states. The general analytic expression for PSK is

$$S_i(t) = (2E/T)^{\wedge}\, 1/2^{*} \cos\left[\omega_o + \varphi_i(t)\right]$$

$0 \le t \le T$ i=1,…,M Where the phase term $\varphi_i(t)$ will have M discrete values, typically given by $\varphi_i(t) = 2\pi i/M$ i=1,.....,M E is the symbol energy, T is symbol time duration.

For Binary PSK It has one fixed phase usually 0° when the data is 1. To transmit 0, we shift the phase of the sinusoid by 180°. Phase shift represents the change in the state of the information in this case. ASK techniques are most susceptible to the effects of non-linear devices which compress and distort signal amplitude. To avoid such distortion, the system must be operated in the linear range, away from the point of maximum power where most of the non-linear behavior occurs. The use of phase shift keying produces a constant amplitude signal and was chosen for its simplicity and to reduce problems with amplitude fluctuations due to fading.

$$\text{Binary PSK(t)} = \begin{cases} \sin(2\pi f t) & \textbf{\textit{for bit}}\ \mathbf{1} \\ \sin(2\pi f t + \pi) & \textbf{\textit{for bit}}\ \mathbf{0} \end{cases}$$

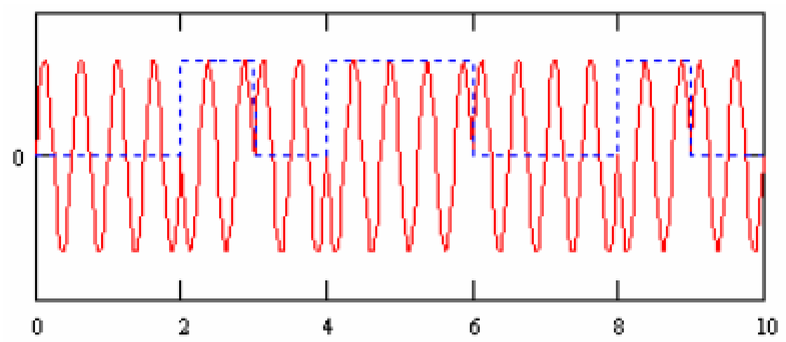


Figure 3.4 - Binary PSK Carrier (Note the 180° phase shifts at bit edges)

3.4.1 BPSK

In binary phase shift keying (BPSK) the transmitted signal is a sinusoid of fixed amplitude. BPSK is the simplest form of PSK. It uses two phases which are separated by 180° and so can also be termed 2-PSK. It has one fixed phase when the data is at one level and when the data is at another level the phase is different by 180°. It does not particularly matter exactly where the constellation points are positioned, and in this figure they are shown on the real axis, at 0° and 180°. This modulation is the most robust of all the PSK's, since it takes serious distortion to make the demodulator reach an incorrect decision. It is, however, only able to modulate at 1 bit/symbol (as seen in the figure 3.4) and so is unsuitable for high data-rate applications when bandwidth is limited.

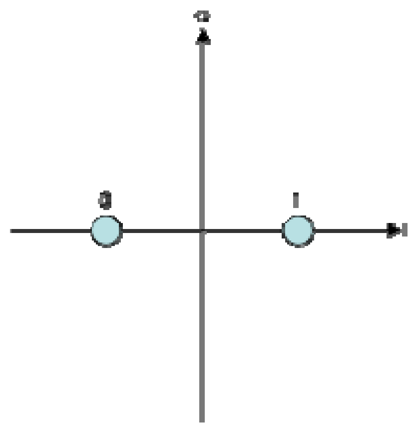


Fig 3.5 Constellation diagram for BPSK.

The bit error rate (BER) of BPSK in AWGN can be calculated as:

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$$

Where $No/2$ is noise power spectral density (W/Hz)

Where T_b is bit interval

Where $E_b = P_s T_b$ is the energy contained in a bit duration

Where P_s is power of sinusoid of amplitude A

So, $P_s = \frac{1}{2}A^2$

In binary phase-shift keying (BPSK) the transmitted signal is a sinusoid of fixed amplitude.

If the sinusoid is of amplitude A it has a power $P_s = \frac{1}{2}A^2$ so that $A = \sqrt{2P_s}$.

Thus the transmitted signal is either

$$V_{BPSK}(t) = \sqrt{2P_s}\cos(\omega_0 t)$$

$$V_{BPSK}(t) = \sqrt{2P_s}\cos(\omega_0 t + \pi)$$

$$= -\sqrt{2P_s}\cos(\omega_0 t)$$

In BPSK the data b(t) is a stream of binary digits with voltage levels which, as a matter of convenience, we take to be at +1v and -1v. When b(t) = 1v we say it is at logic level 1 and when b(t) = -1v we say it is at logic level 0. Hence $V_{BPSK}(t)$ can be written as

$$V_{BPSK}(t) = b(t)\sqrt{2P_s}\cos(\omega_0 t)$$

So, the BPSK signal is generated as shown in the figure 3.6

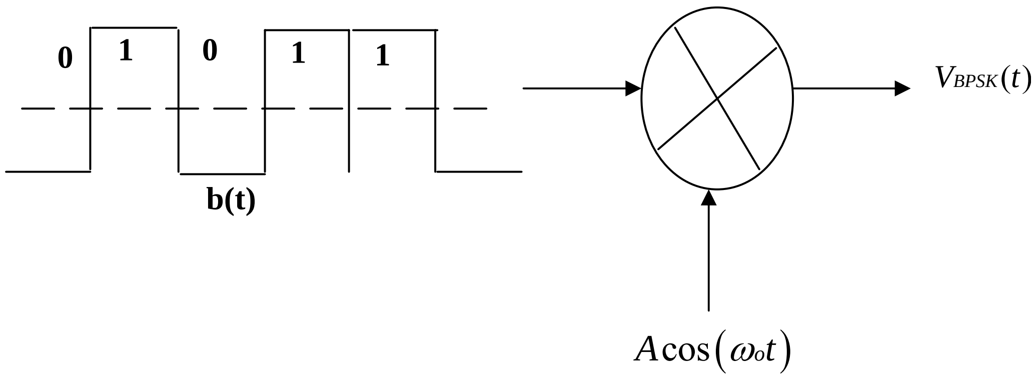


Figure 3.6: Illustrating of the modulation of a message signal to yield a BPSK signal.

In practice, a BPSK signal is generated by applying the waveform $\cos(\omega_0 t)$, as a carrier, to a balanced modulator and applying the baseband signal b(t) as the modulating waveform. In this sense BPSK can be thought of as an AM signal.

If f_c is the frequency of the sinusoid and T is the bit interval then the spectrum of the resulting BPSK signal is shown in the figure 3.7

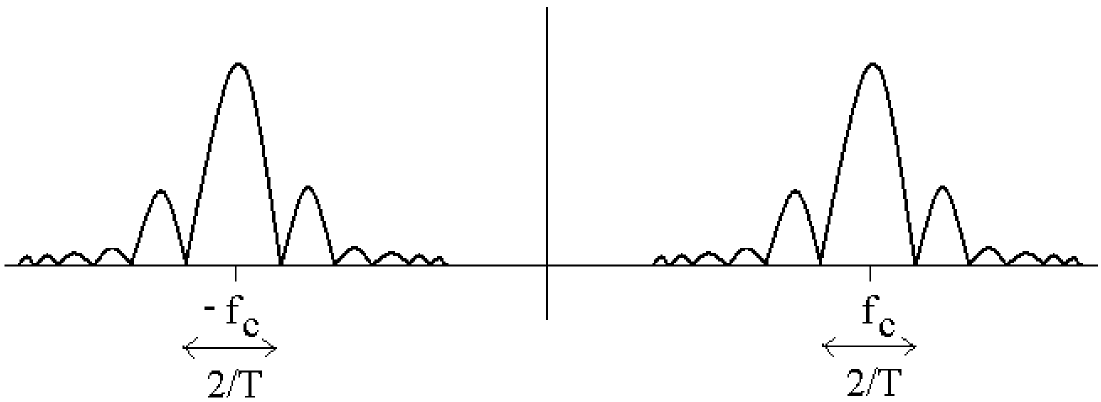
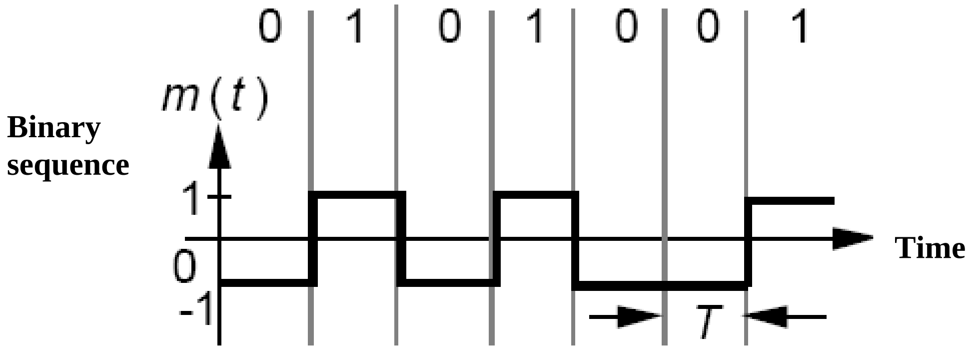
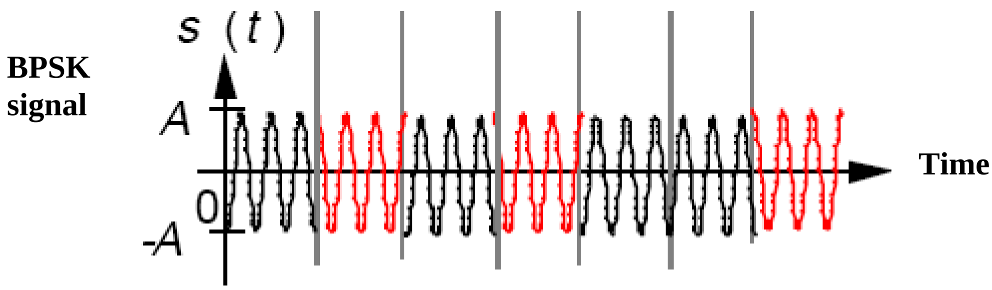


Figure 3.7: Amplitude spectrum of BPSK.

For binary sequence m(t) = 0 1 0 1 0 0 1, and if the sinusoid s(t) is of amplitude of A, then the resulting BPSK signal will be as shown in the figure 3.8



(a)



(b)

Figure 3.8 (a) Binary modulating signal, and (b) BPSK signal.

3.4.2 QPSK

QPSK (4-ary PSK) involves changing the phase of the transmitted waveform. Each finite phase change represents unique digital data. A phase-modulated waveform can be generated by using the digital data to change the phase of a signal while its frequency and amplitude stay constant. A QPSK modulated carrier undergoes four distinct changes in phase that are represented as symbols and can take on the values of π/4, 3π/4, 5π/4, and 7π/4. Each symbol represents two binary bits of data. The constellation diagram of a QPSK modulated carrier is shown in Figure 3.9

$$S(t) = x\cos(wt) + y\sin(wt)$$

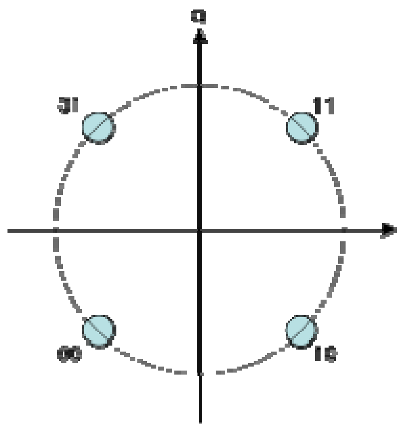


Figure 3.9: Constellation diagram for QPSK with Gray coding.
Each adjacent symbol only differs by one bit.

Note that

$$I_{amplitude} = (symbol\ expression)^{*}\cos(phase)$$

$$Q_{amplitude} = (symbol\ expression)^{*}\sin(phase)$$

Table 3.1: Four symbols mapping definitions for QPSK

Symbol	Bits	Expression	Phase	I	Q
S1	00	$(2E/T)^{\wedge}1/2^{*}\cos(wt+\frac{\pi}{4})$	π/4	$2^{\wedge}(-1/2)$	$2^{\wedge}(-1/2)$
S2	10	$(2E/T)^{\wedge}1/2^{*}\cos(wt+3\frac{\pi}{4})$	3π/4	$-2^{\wedge}(-1/2)$	$2^{\wedge}(-1/2)$
S3	11	$(2E/T)^{\wedge}1/2^{*}\cos(wt+5\frac{\pi}{4})$	5π/4	$-2^{\wedge}(-1/2)$	$-2^{\wedge}(-1/2)$
S4	01	$(2E/T)^{\wedge}1/2^{*}\cos(wt+7\frac{\pi}{4})$	7π/4	$2^{\wedge}(-1/2)$	$-2^{\wedge}(-1/2)$

3.4.2.1 Gray Code

The system performance of a digital communication network can be enhanced by incorporating a coding technique, within the system, known as Gray coding. The gray encoder is used to map the data in such a way as to help reduce bit errors. A QPSK system takes the input data bits, two at a time, and creates a symbol that represents on of four phase states. The gray encoder therefore is used to map every two input data bits to one of four unique symbol values so that the bit pairs that are used to generate the symbols are only one bit different from each adjacent symbol. This technique proves to help with error performance because if a symbol is received in error, it will contain only one error bit if it was received in error to an adjacent symbol. This can be more easily observed by viewing the QPSK constellation diagram shown in Fig 3.9 This QPSK constellation diagram shows symbols, each represented by two data bits that were first gray encoded. One can see that each adjacent symbol is represented by two data bits that vary by one bit. The performance of digital communication networks can further be enhances by the use of error correcting codes.

3.4.2.2 QPSK Modulation

Figure 3.10 represents the process of a QPSK modulator. First, the input binary bit stream is split into two bit streams which are the even and odd bit streams (quadrature and in-phase streams) by the serial to parallel converter. Then, send alternating bits to I, Q channels: even bits to Q channel, odd bits to I channel.

Second, using the method of NRZ, the even and odd bits are converted from a unipolar sequence to a bipolar sequence (0 to -1). Next, multiply Q channel with a sine of fc and multiply I channel with a sine but shifted by 90 degree which is –cosine. Notice that the

90 degrees block in the figure transmits the upper sine sequence to the lower -cosine sequence.

Finally, combining or adding the upper (I) and lower (Q) parts and passing through a harmonic or channel filter will get the QPSK modulated output.

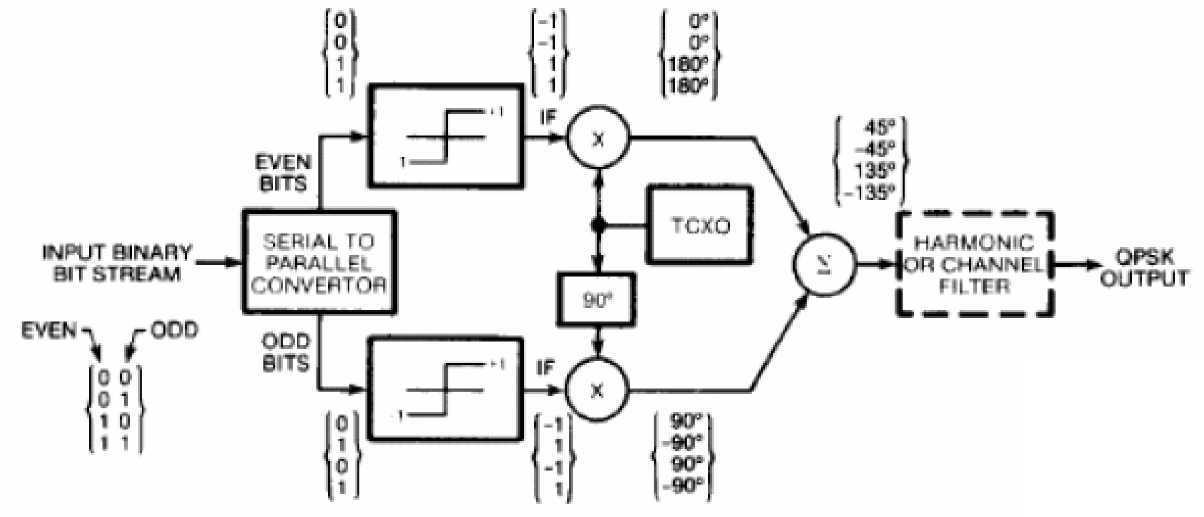


Figure 3.10: QPSK Modulator

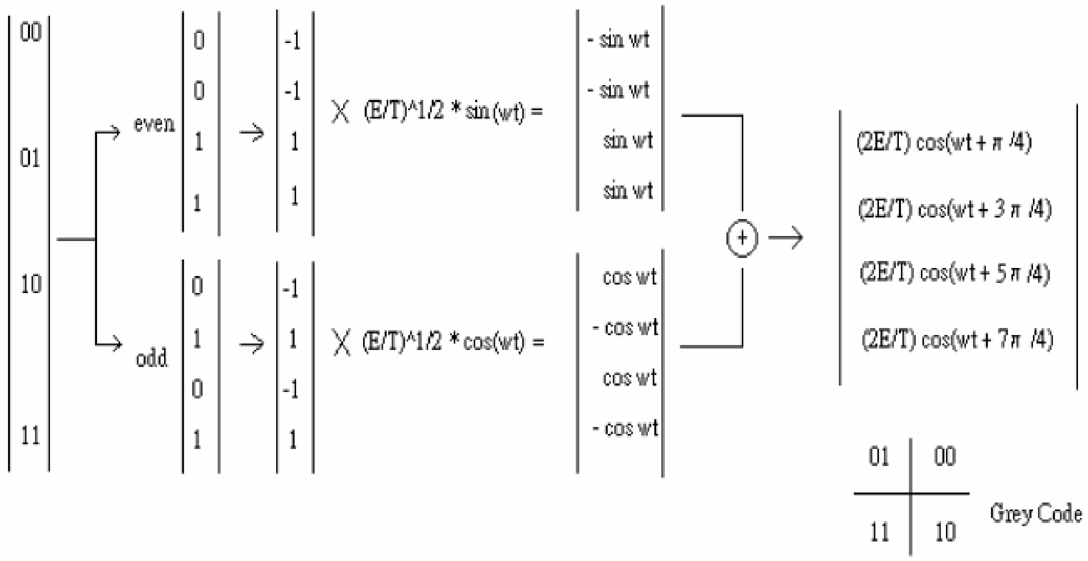
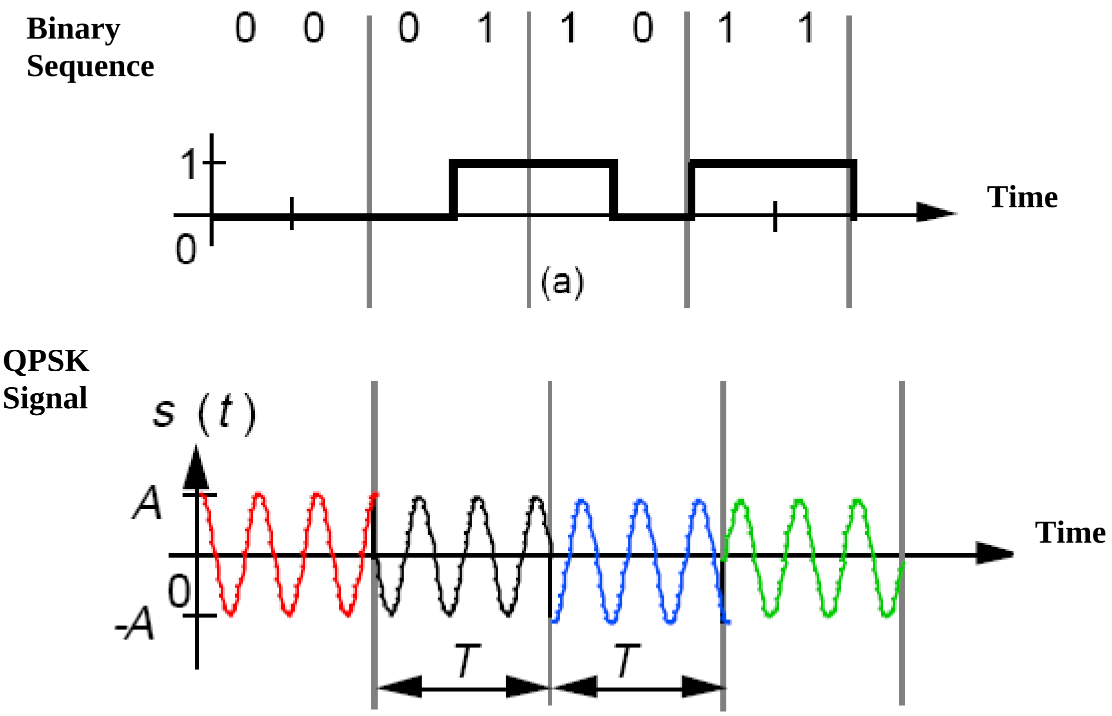


Figure 3.11: The Mathematic representation of the process of the QPSK modulator

For a binary sequence m(t) = 0 0 0 1 1 0 1 1, if the sinusoid s(t) is of amplitude of A, then the resulting QPSK signal will be as shown in the figure 3.12. Phase of the sinusoid is shifted by 90°, 180°, 270°, 360° for data 00, 01, 10, 11 respectively



(b)

Figure 3.12 QPSK modulation: (a) binary sequence and (b) QPSK signal.

3.4.3 MPSK

In BPSK we transmit each bit individually. Depending on whether b(t) is logic 0 or logic 1, we transmit one or another of sinusoid for the bit time T_b, the sinusoids differing in phase by $2\pi/2 = 180°$. In QPSK we lump together two bits. Depending on which of the four two bits words develops, we transmit one or another of four sinusoids of duration $2T_b$, the sinusoids differing in phase by amount $2\pi/4 = 90°$. The scheme can be extended. Let us lump together N bits so that in this N-bit symbol, extending over the time NT_b, there are $2^N = M$ possible symbols. Now let us represent the symbols by sinusoids of duration $NT_b = T_s$ which differ from on another by the phase of $2\pi/M$.

Thus in M-ary PSK the waveforms used to identify the symbols are

$$v_m(t) = \sqrt{2P_s}\cos(\omega_o + \phi_m) \quad (m = 0, 1, \ldots\ldots, M\text{-}1)$$

With the symbol phase angel given by

$$\phi_m = (2m+1)\frac{\pi}{M}$$

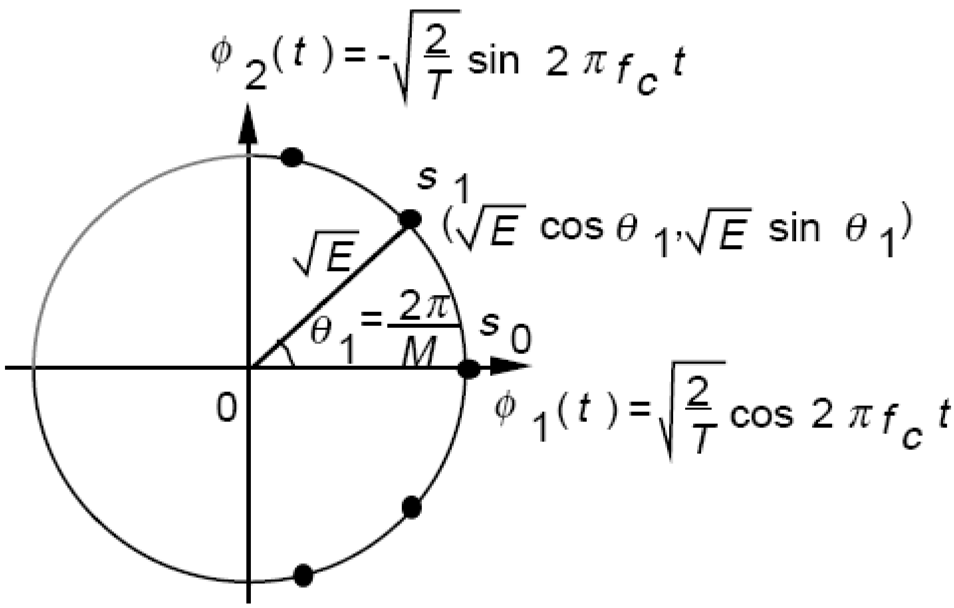


Figure 3.13: Constellation diagram for M-ary PSK

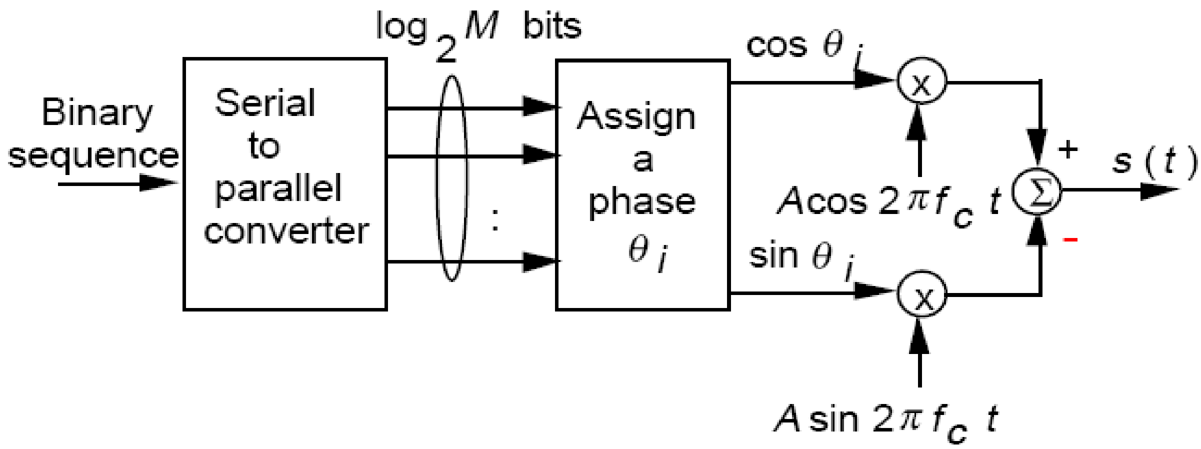


Figure 3.14 M-ary PSK modulator

3.5 QAM

In BPSK, QPSK, and M-ary PSK we transmit, in any symbol interval, one signal or another which are distinguished from one another in phase but are all of the same amplitude. In each of these individual systems the end points of the signal vectors in signal space falls on the circumference of a circle. Now we have note that our ability to distinguished one signal vector from another in the presence of noise will depend on the distance between the vector end points. It is hence rather apparent that we shall be able to improve the noise immunity of a system by allowing signal vectors to differ, not only in phase but also in amplitude. We call this as *amplitude and phase shift keying* or Quadrature amplitude modulation (QAM).

ASK is also combined with PSK to create hybrid systems such as Quadrature Amplitude Modulation (QAM) where both the amplitude and the phase are changed at the same time. QAM is a modulation scheme which conveys data by changing (*modulating*) the amplitude of two carrier waves. These two waves, usually sinusoids, are out of phase with each other by 90° and are thus called quadrature carriers—hence the name of the scheme.

As for many digital modulation schemes, the constellation diagram is a useful representation. In QAM, the constellation points are usually arranged in a square grid with equal vertical and horizontal spacing, although other configurations are possible. Since in digital telecommunications the data is usually binary, the number of points in the grid is usually a power of 2 (2, 4, 8...). Since QAM is usually square, some of these are rare—the most common forms are 16-QAM, 64-QAM, 128-QAM and 256-QAM. By moving to a higher-order constellation, it is possible to transmit more bits per symbol. However, if the mean energy of the constellation is to remain the same (by way of making a fair comparison), the points must be closer together and are thus more susceptible to noise and other corruption; this results in a higher bit error rate and so higher-order QAM can deliver more data less reliably than lower-order QAM.

If data-rates beyond those offered by 8-PSK are required, it is more usual to move to QAM since it achieves a greater distance between adjacent points in the I-Q plane by distributing the points more evenly. The complicating factor is that the points are no longer all the same amplitude and so the demodulator must now correctly detect both phase and amplitude, rather than just phase.

64-QAM and 256-QAM are often used in digital cable television and cable modem applications. In the US, 64-QAM and 256-QAM are the mandated modulation schemes for digital cable. In the UK, 16-QAM and 64-QAM are currently used for digital terrestrial television (Freeview and Top Up TV).

3.5.1 Rectangular QAM

Rectangular QAM constellations are, in general, sub-optimal in the sense that they do not maximally space the constellation points for a given energy. However, they have the considerable advantage that they may be easily transmitted as two pulse amplitude modulation (PAM) signals on quadrature carriers, and can be easily demodulated. The non-square constellations achieve marginally better bit-error rate (BER) but are harder to modulate and demodulate.

The first rectangular QAM constellation usually encountered is 16-QAM, its constellation diagram is shown in fig. 3.15. A Gray coded bit-assignment is also given. The reason that 16-QAM is usually the first is that a brief consideration reveals that 2-QAM and 4-QAM are in fact binary phase-shift keying (BPSK) and quadrature phase-shift keying (QPSK), respectively. Also, the error-rate performance of 8-QAM is close to that of 16-QAM (only about 0.5dB better), but its data rate is only three-quarters that of 16-QAM.

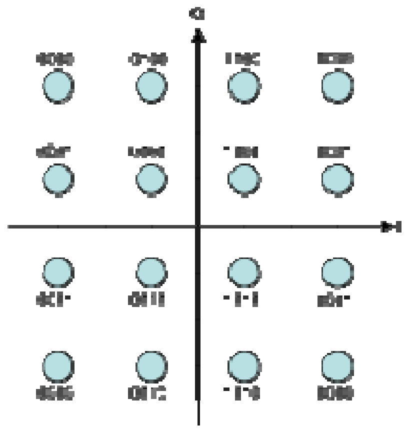


Figure 3.15: Constellation diagram for rectangular 16-QAM.

Expressions for the symbol error-rate of rectangular QAM are not hard to derive but yield rather unpleasant expressions. For an even number of bits per symbol, *k*, exact expressions are available. They are most easily expressed in a *per carrier* sense:

$$P_{sc} = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1}\frac{E_s}{N_o}}\right)$$

So, $$P_s = 1 - \left(1 - P_{sc}\right)^2$$

The bit-error rate will depend on the exact assignment of bits to symbols, but for a Gray-coded assignment with equal bits per carrier:

$$P_{bc} = \frac{4}{k}\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3k}{M-1}\frac{E_b}{N_o}}\right)$$

So, $$P_b = 1 - \left(1 - P_{bc}\right)^2$$

M = Number of symbols in modulation constellation

E_b = Energy-per-bit

E_s = Energy-per-symbol = kE_b with k bits per symbol

N_o = Noise power spectral density (W/Hz)

P_b = Probability of bit-error

P_{bc} = Probability of bit-error per carrier

P_s = Probability of symbol-error

P_{sc} = Probability of symbol-error per carrier

Chapter 4

DIFFERENT TRANSFORMS

4. DIFFERNET TRAMSFORMS

4.1 DISCRETE FOURIER TRANSFORM (DFT)

In mathematics, the discrete Fourier transform (DFT), occasionally called the finite Fourier transform, is a transform for Fourier analysis of finite-domain discrete-time signals. It is widely employed in signal processing and related fields to analyze the frequencies contained in a sampled signal, to solve partial differential equations, and to perform other operations such as convolutions. The DFT can be computed efficiently in practice using a fast Fourier transform (FFT) algorithm.

Since FFT algorithms are so commonly employed to compute the DFT, the two terms are often used interchangeably in colloquial settings, although there is a clear distinction: "DFT" refers to a mathematical transformation, regardless of how it is computed, while "FFT" refers to any one of several efficient algorithms for the DFT. This distinction is further blurred, however, by the synonym "finite Fourier transform" for the DFT.

4.1.1 Definition

The sequence of N complex numbers $x_0, ..., x_{N-1}$ is transformed into the sequence of N complex numbers $X_0, ..., X_{N-1}$ by the DFT according to the formula:

$$X_k = \sum_{n=0}^{N-1} x_n e^{-\frac{2\pi i}{N}kn} \qquad k = 0, \dots, N-1$$

Where e is the base of the natural logarithm, i is the imaginary unit $(i^2 = -1)$, and π is pi. The inverse discrete Fourier transform (IDFT) is given by

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{\frac{2\pi i}{N}kn} \qquad n = 0, \dots, N-1$$

Note that the normalization factor multiplying the DFT and IDFT (here 1 and $1/N$) and the signs of the exponents are merely conventions, and differ in some treatments. The only requirements of these conventions are that the DFT and IDFT have opposite-sign exponents and that the product of their normalization factors be $1/N$.

A normalization of $1/\sqrt{N}$ for both the DFT and IDFT makes the transforms unitary, which has some theoretical advantages, but it is often more practical in numerical computation to perform the scaling all at once as above (and a unit scaling can be convenient in other ways). (The convention of a negative sign in the exponent is often convenient because it means that X_k is the amplitude of a "positive frequency" $2\pi k / N$. Equivalently, the DFT is often thought of as a matched filter: when looking for a frequency of +1, one correlates the incoming signal with a frequency of −1.)

4.1.2 Properties

Completeness

The discrete Fourier transform is an invertible, linear transformation $f : C^N \to C^N$ with C denoting the set of complex numbers. In other words, for any $N > 0$, an N-dimensional complex vector has a DFT and an IDFT which are in turn N-dimensional complex vectors.

Orthogonality

The vectors $e^{\frac{2\pi i}{N}kn}$ form an orthogonal basis over the set of N-dimensional complex vectors:

$$\sum_{n=0}^{N-1} e^{\frac{2\pi i}{N}kn} \left(e^{-\frac{2\pi i}{N}k^1 n} \right) = N\delta_{kk^1}$$

Where δ_{kk^1} is the Kronecker delta. This orthogonality condition can be used to derive the formula for the IDFT from the definition of the DFT.

Periodicity

If the expression that defines the DFT is evaluated for all integers k instead of just for $k = 0,....., N-1$ then the resulting infinite sequence is a periodic extension of the DFT, periodic with period N.

The periodicity can be shown directly from the definition:

$$X_{k+N} = \sum_{n=0}^{N-1} x_n e^{-\frac{2\pi i}{N}(k+N)n} = \sum_{n=0}^{N-1} x_n e^{-\frac{2\pi i}{N}kn} e^{-2\pi in} = \sum_{n=0}^{N-1} x_n e^{-\frac{2\pi i}{N}kn} = X_k$$

Where we have used the fact that $e^{-2\pi i} = 1$. In the same way it can be shown that the IDFT formula leads to a periodic extension.

Circular convolution theorem and cross-correlation theorem

The cyclic or circular convolution **x*y** of the two vectors **x** = x_k and **y** = y_n is the vector **x*y** with components

$$(x*y)_n = \sum_{m=0}^{N-1} x_m y_{n-m} \quad n = 0,\ldots\ldots, N-1$$

Where we continue **y** cyclically so that

$$y_{-m} = y_{N-m} \qquad m = 0,\ldots\ldots, N-1$$

The discrete Fourier transform turns cyclic convolutions into component-wise multiplication. That is, if

$$z_n = (x * y)_n$$

Then

$$Z_k = X_k Y_k \qquad k = 0,\ldots\ldots\ldots, N-1$$

Where capital letters (X, Y, Z) represent the DFTs of sequences represented by small letters (x, y, z). The direct evaluation of the convolution summation, above, would require $O(N^2)$ operations, but the DFT (via an FFT) provides an $O(N \log N)$ method to compute the same thing.

4.1.3 Applications

The DFT has seen wide usage across a large number of fields; we only sketch a few examples below (see also the references at the end). All applications of the DFT depend crucially on the availability of a fast algorithm to compute discrete Fourier transforms and their inverses, a Fast Fourier Transform.

Partial differential equations

Discrete Fourier transforms are often used to solve partial differential equations, where again the DFT is used as an approximation for the Fourier series (which is recovered in the limit of infinite N). The advantage of this approach is that it expands the signal in complex exponentials e^{jnx} which are eigen functions of differentiation: $d/dx\, e^{jnx} = jn\, e^{jnx}$

Thus, in the Fourier representation, differentiation is simple—we just multiply by $i\ n$. A linear differential equation with constant coefficients is transformed into an easily solvable algebraic equation. One then uses the inverse DFT to transform the result back into the ordinary spatial representation. Such an approach is called a spectral method.

Spectral analysis

When the DFT is used for spectral analysis, the $\{x_n\}$ sequence usually represents a finite set of uniformly-spaced time-samples of some signal $x(t)$ where *t* represents time. The conversion from continuous time to samples (discrete-time) changes the underlying Fourier transform of x(t) into a discrete-time Fourier transform (DTFT), which generally entails a type of distortion called aliasing. Choice of an appropriate sample-rate (see Nyquist frequency) is the key to minimizing that distortion. Similarly, the conversion from a very long (or infinite) sequence to a manageable size entails a type of distortion called *leakage*, which is manifested as a loss of detail (aka resolution) in the DTFT. DFT is just a discrete sampling of the DTFT, which is a function of a continuous frequency domain. The inefficiency of performing multiplications and additions is more than offset by the inherent efficiency of the FFT. As already noted, leakage imposes a limit on the inherent resolution of the DTFT. So there is a practical limit to the benefit that can be obtained from a fine-grained DFT.

Data compression

The field of digital signal processing relies heavily on operations in the frequency domain (i.e. on the Fourier transform). For example, several lossy image and sound compression methods employ the discrete Fourier transform: the signal is cut into short segments, each is transformed, and then the Fourier coefficients of high frequencies, which are assumed to be unnoticeable, are discarded. The decompressor computes the inverse transform based on this reduced number of Fourier coefficients. (Compression applications often use a specialized form of the DFT, the discrete cosine transform or sometimes the modified discrete cosine transform).

Multiplication of large integers

The fastest known algorithms for the multiplication of large integers or polynomials are based on the discrete Fourier transform: the sequences of digits or coefficients are interpreted as vectors whose convolution needs to be computed; in order to do this, they are first Fourier-transformed, then multiplied component-wise, then transformed back.

4.2 FAST FOURIER TRANSFORM (FFT)

A Fast Fourier Transform (FFT) is an efficient algorithm to compute the discrete Fourier transform (DFT) and its inverse. The Discrete *Fourier Transform* (DFT) is used to produce frequency analysis of discrete non-periodic signals. The FFT is a faster version of the Discrete Fourier Transform (DFT).

The FFT utilizes some clever algorithms to do the same thing as the DTF, but in much less time. FFTs are of great importance to a wide variety of applications, from digital signal processing to solving partial differential equations to algorithms for quickly multiplying large integers. Let $x_0, ..., x_{N-1}$ be complex numbers. The DFT is defined by the formula

$$X_k = \sum_{n=0}^{N-1} x_n e^{-\frac{2\pi i}{N} nk} \qquad k = 0, ..., N-1$$

Evaluating these sums directly would take O(N^2) arithmetical operations An FFT is an algorithm to compute the same result in only O(N log N) operations. In general, such algorithms depend upon the factorization of N, but (contrary to popular misconception) there are FFTs with O(N log N) complexity for all N, even for prime N.

Many FFT algorithms only depend on the fact that $e^{-\frac{2\pi i}{N}}$ is a primitive root of unity. Since the inverse DFT is the same as the DFT, but with the opposite sign in the exponent and a 1/N factor, any FFT algorithm can easily be adapted for it as well.

4.3 DISCRETE HARTLEY TRANSFORM (DHT)

A discrete Hartley transform (DHT) is a Fourier-related transform of discrete, periodic data similar to the discrete Fourier transform (DFT), with analogous applications in signal processing and related fields. Its main distinction from the DFT is that it transforms real inputs to real outputs, with no intrinsic involvement of complex numbers. Just as the DFT is the discrete analogue of the continuous Fourier transform, the DHT is the discrete analogue of the continuous Hartley transform, introduced by R. V. L. Hartley in 1942.

Because there are fast algorithms for the DHT analogous to the fast Fourier transform (FFT), the DHT was originally proposed by R. N. Bracewell in 1983 as a more efficient computational tool in the common case where the data are purely real. It was subsequently argued, however, that specialized FFT algorithms for real inputs or outputs can ordinarily be found with slightly fewer operations than any corresponding algorithm for the DHT.

4.3.1 Definition:

Formally, the discrete Hartley transform is a linear, invertible function H: $R^n \rightarrow R^n$

(Where R denotes the set of real numbers)**.** The N real numbers $x_0, ..., x_{N-1}$ are transformed into the N real numbers $H_0, ..., H_{N-1}$ according to the formula

$$H_k = \sum_{n=0}^{N-1} x_n \left[\cos\left(\frac{2\pi}{N} nk\right) + \sin\left(\frac{2\pi}{N} nk\right) \right] \qquad k = 0, ….., N-1$$

Where π is Pi.

The IDHT is given by

$$x_n = (1/N)\sum_{k=0}^{N-1} H_k \left[\cos\left(\frac{2\pi}{N}nk\right) + \sin\left(\frac{2\pi}{N}nk\right)\right] \qquad n = 0,....,N-1$$

The combination $\cos(z)+\sin(z)=\sqrt{2}\cos(z-\frac{\pi}{4})$ is sometimes denoted $cas(z)$, and should be contrasted with the $e^{-iz} = \cos(z) - i\sin(z)$ that appears in the DFT definition (where i is the imaginary unit). Like for the DFT, the overall scale factor in front of the transform and the sign of the sine term are a matter of convention, and differ in some treatments, but do not affect the essential properties.

4.3.2 Properties:

The transform can be interpreted as the multiplication of the vector (x_0, ..., x_{N-1}) by an N-by-N matrix; therefore, the discrete Hartley transform is a linear operator. The matrix is invertible; the inverse transformation, which allows one to recover the x_n from the H_k, is simply the DHT of H_k multiplied by $1/N$. That is, the DHT is its own inverse (involutary), up to an overall scale factor.

The DHT can be used to compute the DFT, and vice versa. For real inputs x_n, the DFT output X_k has a real part $(H_k + H_{N-k})/2$ and an imaginary part $(H_{N-k} - H_k)/2$. Conversely, the DHT is equivalent to computing the DFT of x_n multiplied by $1+i$, then taking the real part of the result.

As with the DFT, a cyclic convolution $\mathbf{z} = \mathbf{x}*\mathbf{y}$ of two vectors $\mathbf{x} = (x_n)$ and $\mathbf{y} = (y_n)$ to produce a vector $\mathbf{z} = (z_n)$, all of length N, becomes a simple operation after the DHT. In particular, suppose that the vectors $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Z}$ denote the DHT of $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ respectively. Then the elements of $\mathbf{Z}$ are given by:

$$Z_k = \left[X_k\left(Y_k + Y_{N-k}\right) + X_{N-k}\left(Y_k - Y_{N-k}\right)\right]/2$$

$$Z_{N-k} = \left[X_{N-k}\left(Y_k + Y_{N-k}\right) - X_k\left(Y_k - Y_{N-k}\right)\right]/2$$

Where we take all of the vectors to be periodic in N ($X_N = X_0$, etcetera). Thus, just as the DFT transforms a convolution into a point wise multiplication of complex numbers (*pairs* of real and imaginary parts), the DHT transforms a convolution into a simple combination of *pairs* of real frequency components. The inverse DHT then yields the desired vector z**.** In this way, a fast algorithm for the DHT (see below) yields a fast algorithm for convolution. (Note that this is slightly more expensive than the corresponding procedure for the DFT, not including the costs of the transforms below, because the pair wise operation above requires 8 real-arithmetic operations compared to the 6 of a complex multiplication. This count doesn't include the division by 2, which can be absorbed e.g. into the $1/N$ normalization of the inverse DHT.)

Fast algorithms:

Just as for the DFT, evaluating the DHT definition directly would require $O(N^2)$ arithmetical operations. There are fast algorithms similar to the FFT, however, that compute the same result in only $O(N \log N)$ operations. Nearly every FFT algorithm, from Cooley-Tukey to Prime-Factor to Winograd, has a direct analogue for the discrete Hartley transform. In particular, the DHT analogue of the Cooley-Tukey algorithm is commonly known as the fast Hartley transform (FHT) algorithm, and was first described by Bracewell in 1984. As mentioned above, DHT algorithms are typically slightly less efficient (in terms of the number of floating-point operations) than the corresponding DFT algorithm (FFT) specialized for real inputs (or outputs). On present-day computers, performance is determined more by cache and CPU pipeline considerations than by strict operation counts, and a slight difference in arithmetic cost is unlikely to be significant. Since FHT and real-input FFT algorithms have similar computational structures, neither appears to have a substantial *a priori* speed advantage. As a practical matter, highly optimized real-input FFT libraries are available from many sources (e.g. from CPU vendors such as Intel), whereas highly optimized DHT libraries are less common.

On the other hand, the redundant computations in FFTs due to real inputs are much more difficult to eliminate for large prime N, despite the existence of $O(N \log N)$ complex-data algorithms for such cases, because the redundancies are hidden behind intricate permutations and/or phase rotations in those algorithms. In contrast, a standard prime-size FFT algorithm, Rader's algorithm, can be directly applied to the DHT of real data for roughly a factor of two less computation than that of the equivalent complex FFT. This DHT approach currently appears to be the only known way to obtain such factor-of-two savings for large prime-size FFTs of real data.

4.3.3 Advantages

The Discrete Hartley Transform is closely related to the Discrete Fourier Transform, but unlike the DFT, the DHT has the advantage of producing real numbers. Furthermore, it is (quasi-)symmetrical. The Hartley transform produces real output for a real input, and is its own inverse. It therefore can have computational advantages over the discrete Fourier transform.

4.4 DISCRETE COSINE TRANSFORM (DCT)

A discrete cosine transform (DCT) is a Fourier-related transform similar to the discrete Fourier transform (DFT), but using only real numbers. DCTs are equivalent to DFTs of roughly twice the length, operating on real data with even symmetry (since the Fourier transform of a real and even function is real and even), where in some variants the input and/or output data are shifted by half a sample.

Like other transforms, the Discrete Cosine Transform (DCT) attempts to decorrelate the image data. After decorrelation each transform coefficient can be encoded independently without losing compression efficiency. Like any Fourier-related transform, discrete cosine transforms (DCTs) express a function or a signal in terms of a sum of sinusoids with different frequencies and amplitudes. Like the discrete Fourier transform (DFT), a DCT operates on a function at a finite number of discrete data points. The obvious distinction between a DCT and a DFT is that the former uses only cosine functions, while the latter uses both cosines and sines (in the form of complex exponentials). However, this visible difference is merely a consequence of a deeper distinction: a DCT implies different boundary conditions than the DFT or other related transforms.

The Fourier-related transforms that operate on a function over a finite domain, such as the DFT or DCT or a Fourier series, can be thought of as implicitly defining an *extension* of that function outside the domain. That is, once you write a function $f(x)$ as a sum of sinusoids, you can evaluate that sum at any x, even for x where the original $f(x)$ was not specified. The DFT, like the Fourier series, implies a periodic extension of the original function. A DCT, like a cosine transform, implies an even extension of the original function.

However, because DCTs operate on *finite*, *discrete* sequences, two issues arise that do not for the continuous cosine transform. First, one has to specify whether the function is even or odd at *both* the left and right boundaries of the domain (i.e. the min-n and max-n boundaries in the definitions below, respectively). Second, one has to specify around *what point* the function is even or odd. In particular, consider a sequence *abcd* of four equally spaced data points, and say that we specify an even *left* boundary. There are two sensible possibilities: either the data is even about the sample *a*, in which case the even extension is *dcbabcd*, or the data is even about the point *halfway* between *a* and the previous point, in which case the even extension is *dcbaabcd* (*a* is repeated).

4.4.1 Definition:

Let x (n) be a sequence of length N

Its DCT is given by

$$X(k) = \sum_{n=0}^{N-1} x(n)\cos(2\pi nk / N) \qquad k = 0, 1, 2, \ldots, N-1$$

And the IDCT is given by

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\cos(2\pi nk / N) \qquad n = 0, 1, 2, \ldots, N-1$$

4.4.2 Properties of DCT

Decorrelation

The principle advantage of image transformation is the removal of redundancy between neighboring pixels. This leads to uncorrelated transform coefficients which can be encoded independently. The amplitude of the autocorrelation after the DCT operation is very small at all lags. Hence, it can be inferred that DCT exhibits excellent decorrelation properties.

Energy Compaction

Efficacy of a transformation scheme can be directly gauged by its ability to pack input data into as few coefficients as possible. This allows the quantizer to discard coefficients with relatively small amplitudes without introducing visual distortion in the reconstructed image. DCT exhibits excellent energy compaction for highly correlated images.

Orthogonality

DCT basis functions are orthogonal. Thus, the inverse transformation matrix of A is equal to its transpose i.e. $A^{-1} = A^{T}$. Therefore, and in addition to its decorrelation characteristics, this property renders some reduction in the pre-computation complexity.

4.4.3 Applications

The DCT, is often used in signal and image processing, especially for lossy data compression, because it has a strong "energy compaction" property: most of the signal information tends to be concentrated in a few low-frequency components of the DCT, see figure 4.1, approaching the Karhunen-Loève transform (which is optimal in the decorrelation sense) for signals based on certain limits of Markov processes.

For example, a DCT is used in JPEG image compression, MJPEG, MPEG, and DV video compression. There, the two-dimensional DCT of $N \times N$ blocks are computed and the results are quantized and entropy coded. In this case, N is typically 8 and the DCT-II formula is applied to each row and column of the block. The result is an 8×8 transform coefficient array in which the (0,0) element is the DC (zero-frequency) component and entries with increasing vertical and horizontal index values represent higher vertical and horizontal spatial frequencies. A related transform, the *modified* discrete cosine transform, or MDCT (based on the DCT-IV), is used in AAC, Vorbis, and MP3 audio compression.

DCTs are also widely employed in solving partial differential equations by spectral methods, where the different variants of the DCT correspond to slightly different even/odd boundary conditions at the two ends of the array.

DCTs are also closely related to Chebyshev polynomials, and fast DCT algorithms are used in Chebyshev approximation of arbitrary functions by series of Chebyshev polynomials, for example in Clenshaw-Curtis quadrature.

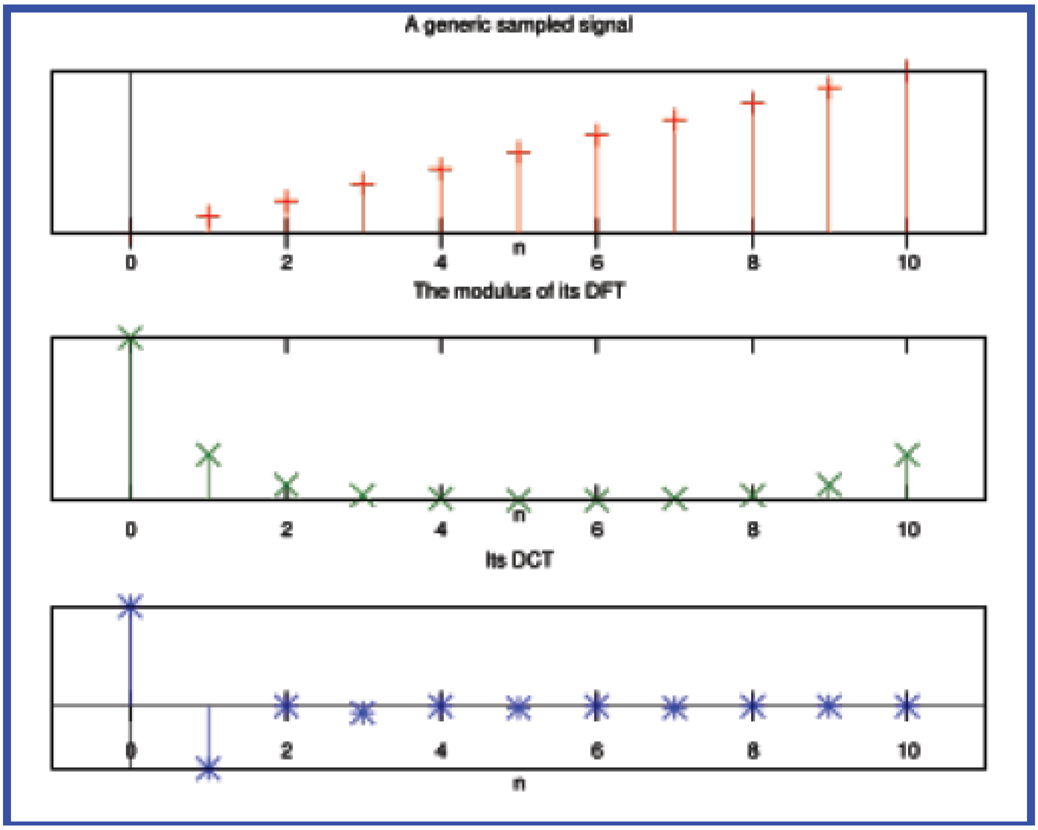


Fig 4.1: DCT (bottom) compared to the DFT (middle) of an input signal (top)

Chapter 5

ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING

5. ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING

5.1 EVOLUTION OF OFDM

Frequency Division Multiplexing (FDM)

Frequency Division Multiplexing (FDM) has been used for a long time to carry more than one signal over a telephone line. FDM divides the channel bandwidth into subchannels and transmits multiple relatively low rate signals by carrying each signal on a separate carrier frequency. To ensure that the signal of one subchannel did not overlap with the signal from an adjacent one, some guard-band was left between the different subchannels. Obviously, this guard-band led to inefficiencies.

Orthogonal Frequency Division Multiplexing (OFDM)

In order to solve the bandwidth efficiency problem, orthogonal frequency division multiplexing was proposed, where the different carriers are orthogonal to each other. With OFDM, it is possible to have overlapping subchannels in the frequency domain, thus increasing the transmission rate. This carrier spacing provides optimal spectral efficiency. Today, OFDM has grown to be the most popular communication system in high-speed communications. OFDM is becoming the chosen modulation technique for wireless communications. OFDM can provide large data rates with sufficient robustness to radio channel impairments.

5.2 INTRODUCTION TO OFDM

Orthogonal Frequency Division Multiplexing (OFDM)

***Modulation** - a mapping of the information on changes in the carrier phase, frequency or amplitude or combination.*

***Multiplexing** - method of sharing a bandwidth with other independent data channels.*

OFDM is a combination of modulation and multiplexing. Multiplexing generally refers to independent signals, those produced by different sources. In OFDM the question of multiplexing is applied to independent signals but these independent signals are a sub-set of the one main signal. In OFDM the signal itself is first split into independent channels, modulated by data and then re-multiplexed to create the OFDM carrier.

OFDM is a special case of Frequency Division Multiplex (FDM). In an OFDM scheme, a large number of orthogonal, overlapping, narrow band sub-carriers are transmitted in parallel. These carriers divide the available transmission bandwidth. The separation of the sub-carriers is such that there is a very compact spectral utilization.

As an analogy, a FDM channel is like water flow out of a faucet, a whole bunch of water coming all in one stream; In contrast the OFDM signal is like a shower from which same amount of water will come as a lot of small streams. In a faucet all water comes in one big stream and cannot be sub-divided. OFDM shower is made up of a lot of little streams.

Fig 5.1 (a) A Regular-FDM single carrier (b) Orthogonal-FDM

The advantage one over the other is that if I put my thumb over the faucet hole, I can stop the water flow but I cannot do the same for the shower. So although both do the same thing, they respond differently to interference. Both methods carry the exact same amount of data. But in case of any interfere to some of these small streams, only some part of data in the OFDM method will suffer.

These small streams when seen as signals are called the sub-carriers in an OFDM system and they must be orthogonal for this idea to work. The independent sub-channels can be multiplexed by frequency division multiplexing (FDM), called multi-carrier transmission or it can be based on a code division multiplex (CDM), in this case it is called multi-code transmission.

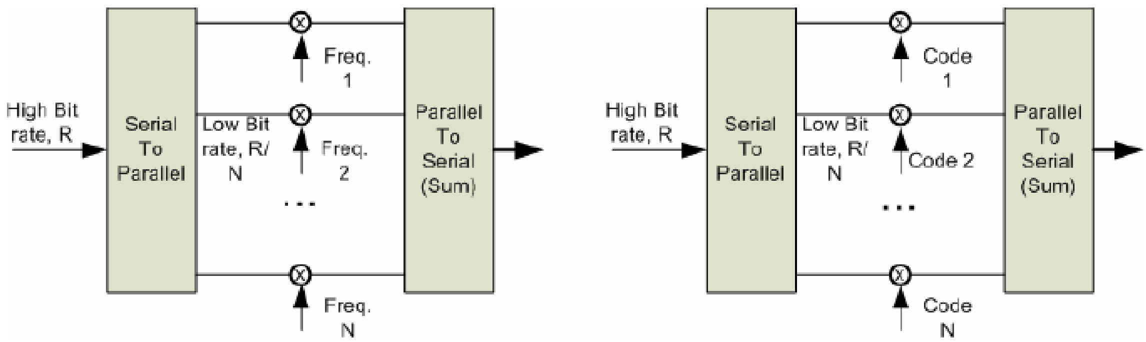


Fig 5.2 Multi-carrier FDM and Multi-code division multiplex

5.3 IMPORTANCE OF ORTHOGONALITY

The main concept in OFDM is orthogonality of the sub-carriers. The "orthogonal" part of the OFDM name indicates that there is a precise mathematical relationship between the frequencies of the carriers in the system. It is possible to arrange the carriers in an OFDM Signal so that the sidebands of the individual carriers overlap and the signals can still be received without adjacent carriers interference. In order to do this the carriers must be mathematically orthogonal. The Carriers are linearly independent (i.e. orthogonal) if the carrier spacing is a multiple of $1/T_s$. Where, T_s is the symbol duration. The orthogonality among the carriers can be maintained if the OFDM signal is defined by using Fourier transform procedures. The OFDM system transmits a large number of narrowband carriers, which are closely spaced. Note that at the central frequency of the each sub channel there is no crosstalk from other sub channels.

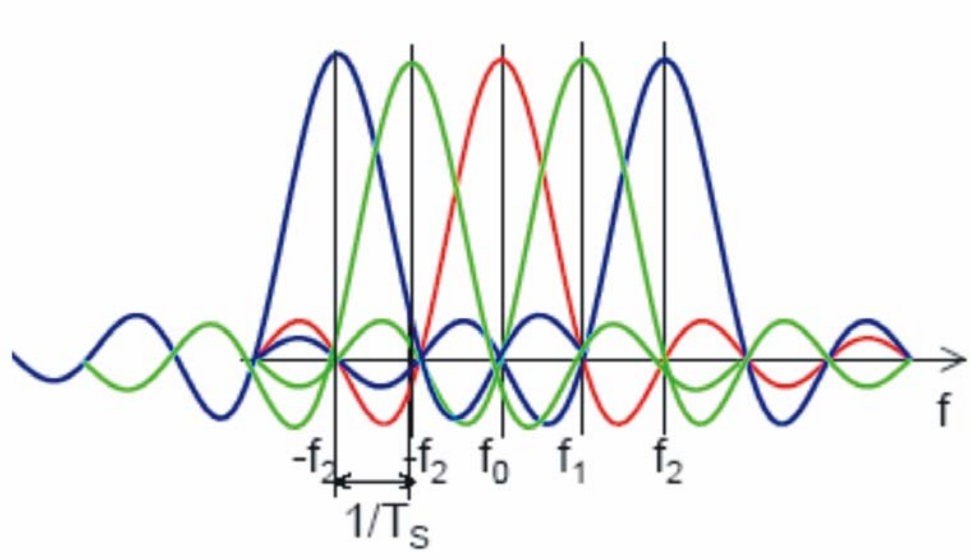


Fig 5.3Example of OFDM spectrum for 5 orthogonal carriers

Since the carriers are all sine/cosine wave, we know that area under one period of a sine or a cosine wave is zero. This is easily shown.

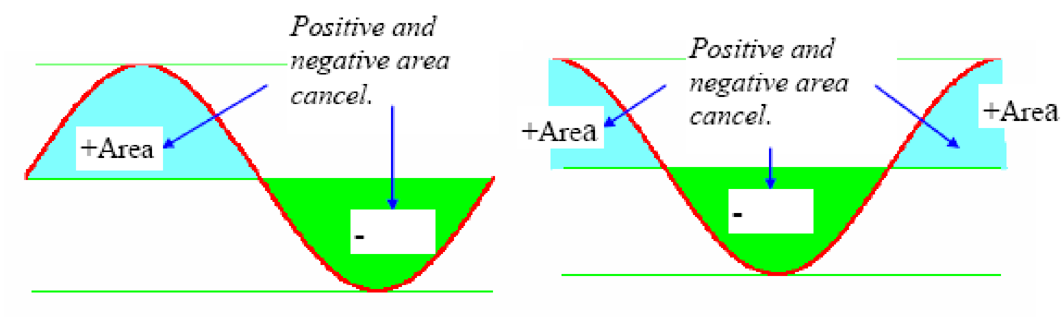


Fig. 5.4 - The area under a sine and a cosine wave over one period is always zero.

If a sine wave of frequency m multiplied by a sinusoid (sine or cosine) of a frequency n,

$$f(t) = \sin mwt \times \sin nwt$$

Where both m and n are integers, since these two components are each a sinusoid, the integral is equal to zero over one period. The integral or area under this product is given by

$$= \int_0^{2\pi} \frac{1}{2}\cos(m-n)wt - \int_0^{2\pi} \frac{1}{2}\cos(m+n)wt$$

$$= 0 - 0$$

So when a sinusoid of frequency *n* multiplied by a sinusoid of frequency *m/n*, the area under the product is zero. In general for all integers *n* and *m*, sin *mx*, cos *mx*, cos *nx*, sin *nx* are all orthogonal to each other. These frequencies are called harmonics.

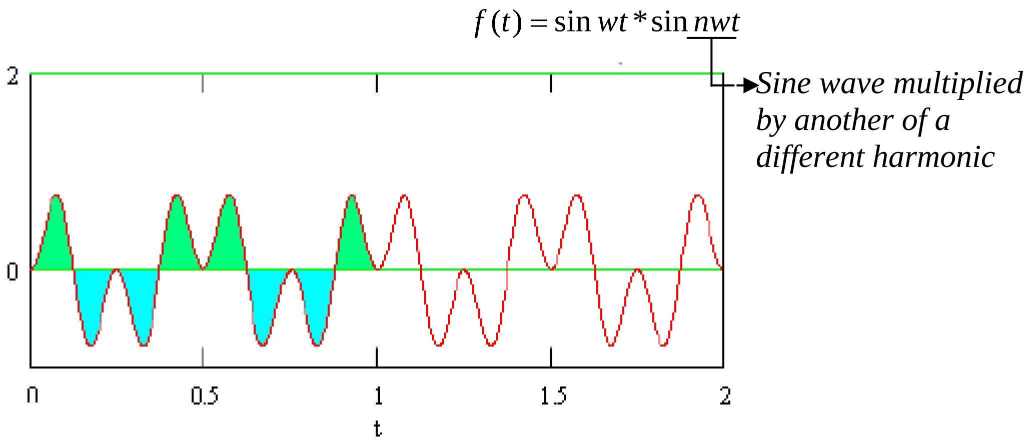


Fig 5.5 the area under a sine wave multiplied by its own harmonic is always zero

The orthogonality allows simultaneous transmission on a lot of sub-carriers in a tight frequency space without interference from each other. In essence this is similar to CDMA, where codes are used to make data sequences independent (also orthogonal) which allows many independent users to transmit in same space successfully.

5.4 OFDM is a special case of FDM

Frequency Division Multiplexing FDM is a special case of FDM. If I have a bandwidth that goes from frequency say *a* to *b*, I can subdivide this into a frequency of equal spaces. In frequency space the modulated carriers would look like this.

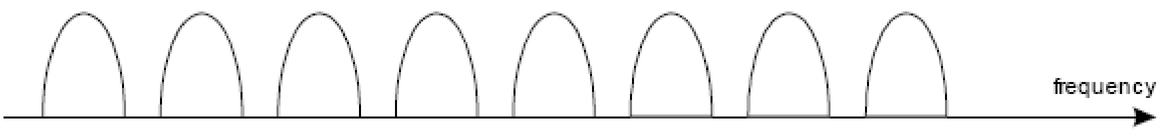


Fig. 5.6 – bandwidth utilization of FDM

The frequencies *a* and *b* can be anything, integer or non-integer since no relationship is implied between *a* and *b*. same is true of the carrier center frequencies which are based on frequencies that do not have any special relationship to each other. If frequency C_1 and C_n were such that for any *n*, an integer, the following holds.

$$c_n = n \times c_1$$

So that

$C_2 = 2C_1$

$C_3 = 3C_1$

$C_4 = 4C_1$

All three of these frequencies are harmonic to c1.

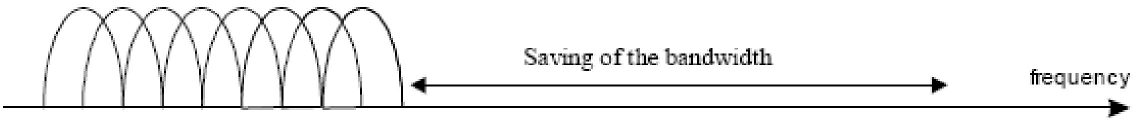


Fig 5.7 bandwidth utilization of OFDM

In this case, since these carriers are orthogonal to each other, when added together, they do not interfere with each other. In FDM, since we do not generally have frequencies that follow the above relationship, we get interference from neighbor carriers. To provide adjacent channel interference protection, signals are moved further apart. Each carrier may be placed apart allowing for a 10% guard band. The frequencies would not be orthogonal but in FDM we don't care about this. It's the guard band that helps keep interference under control.

5.5 An example of OFDM using 4 sub-carriers

In OFDM we have N carriers, N can be anywhere from 16 to 1024 in present technology and depends on the environment in which the system will be used. Let's examine the following bit sequence we wish to transmit and show the development of the ODFM signal using 4 sub-carriers. The signal has a symbol rate of 1 and sampling frequency is 1 sample per symbol, so each transition is a bit.

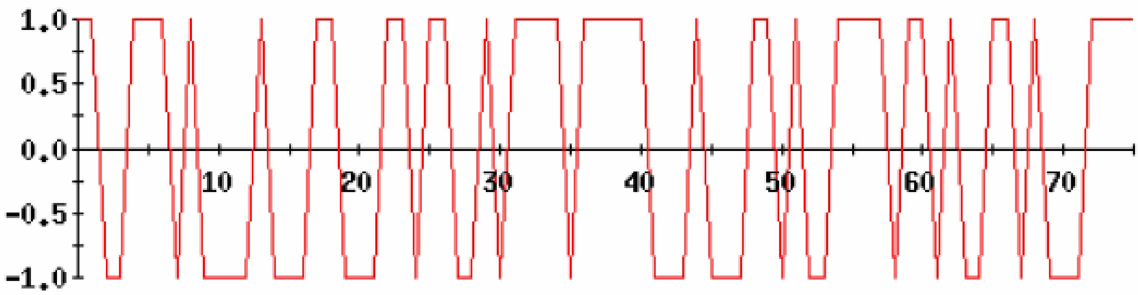


Fig. 5.8 – A bit stream that will be modulated using a 4 carrier OFDM.

First few bits are 1, 1, -1, -1, 1, 1, 1, -1, 1, -1, -1, -1, -1, 1, -1, -1, -1, 1,…

Let's now write these bits in rows of fours see Table. 5.1, since we are using only four sub carriers. We have effectively done a serial to parallel conversion.

Each column represents the bits that will be carried by one sub-carrier. Let's start with the first carrier, c1. From the Nyquist sampling Theorem, we know that smallest frequency that can convey information has to be twice the information rate. In this case, the information rate per carrier will be 1/4 or 1 symbol per second total for all 4 carriers. So the smallest frequency that can carry a bit rate of 1/4 is 1/2 Hz. But we picked 1 Hz for convenience. Had I picked 1/2 Hz as my starting frequency, then my harmonics would have been 1, 3/2 and 2 Hz. I could have chosen 7/8 Hz to start with and in which the harmonics would be 7/4, 7/2, 21/2 Hz.

Table 5.1 – Serial to parallel conversion of data bits.

C_1	C_2	C_3	C_4
1	1	-1	-1
1	1	1	-1
1	-1	-1	-1
-1	1	-1	-1
-1	1	1	-1
-1	-1	1	1

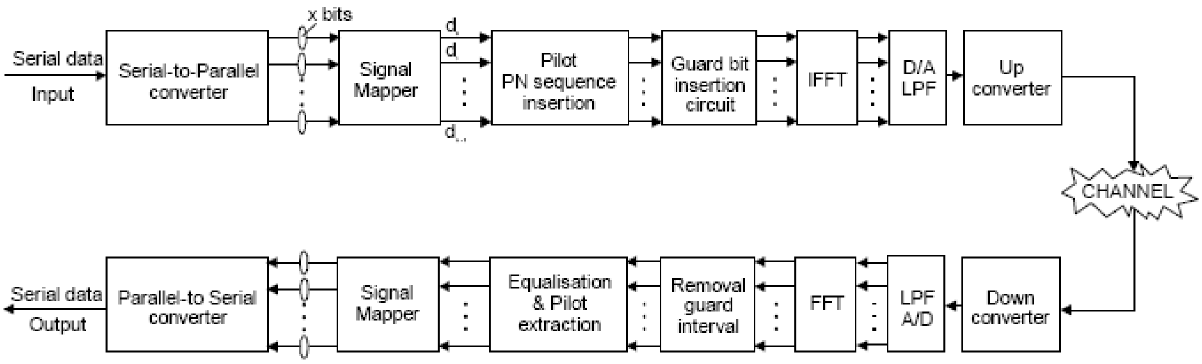


Fig. 5.9 General Block diagram of an OFDM system

We pick BPSK as our modulation scheme for this example. (For QPSK, just imagine the same thing going on in the Q channel, and then double the bit rate while keeping the symbol rate the same.) Note that I can pick any other modulation method, QPSK, 8PSK 32-QAM or whatever.

Carrier 1 - We need to transmit 1, 1, 1 -1, -1, -1 which are superimposed on the BPSK carrier of frequency 1 Hz. First three bits are 1 and last three -1.

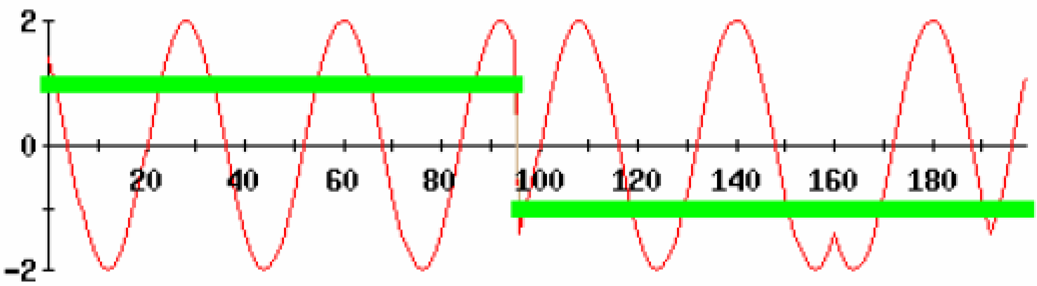


Fig. 5.10 – Sub-carrier 1 and the bits it is modulating (the first column of Table 5.1)

Carrier 2 - The next carrier is of frequency 2 Hz. It is the next orthogonal/harmonic to frequency of the first carrier of 1 Hz. Now take the bits in the second column, marked c2, 1, 1, -1, 1, 1, -1 and modulate this carrier with these bits as shown in Fig. 5.11.

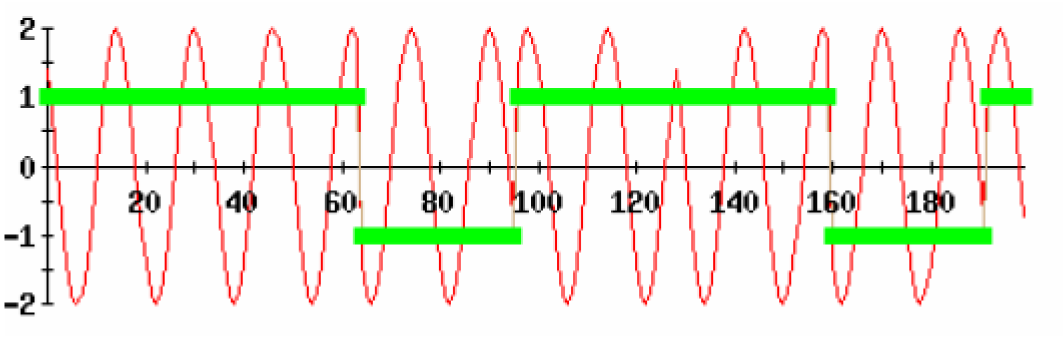


Fig. 5.11 Sub-carrier 2 and the bits that it is modulating (the 2nd column of Table 5.1)

Carrier 3 – Carrier 3 frequency is equal to 3 Hz and fourth carrier has a frequency of 4 Hz. The third carrier is modulated with -1, 1, 1, -1, -1, 1 and the fourth with -1, -1, -1, -1, -1, -1, 1 from Table 5.1.

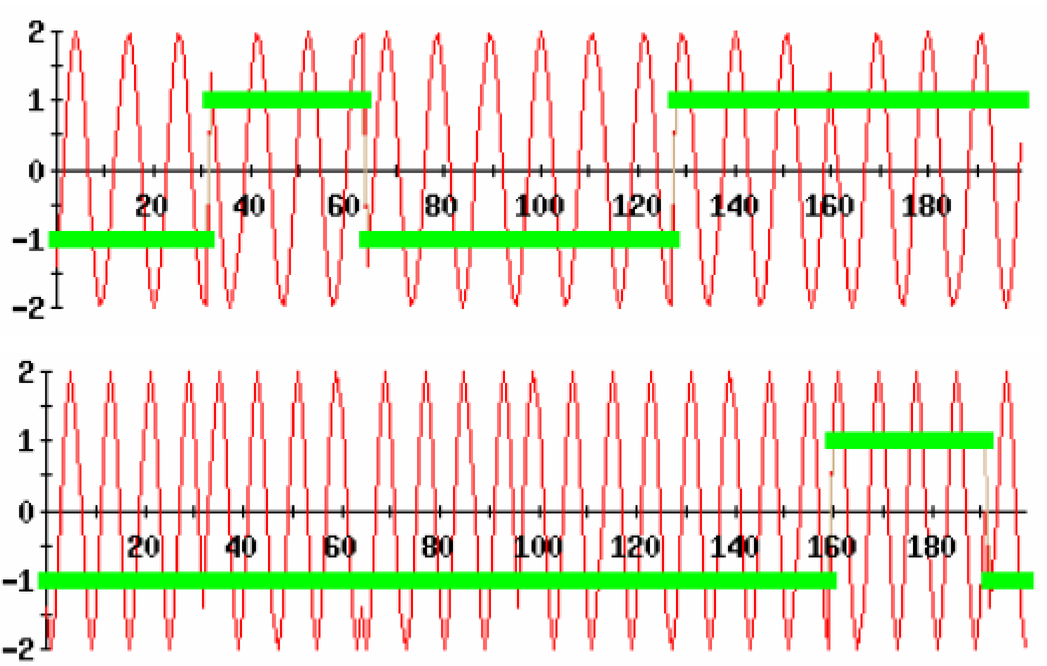


Fig. 5.12 – Sub-carrier 3 and 4 and the bits that they modulating (the 3rd and 4th columns of Table 5.1)

Now we have modulated all the bits using four independent carriers of orthogonal frequencies 1 to 4 Hz. We have taken the bit stream, distributed the bits, one bit at a time to the four sub-carriers as shown in figure below.

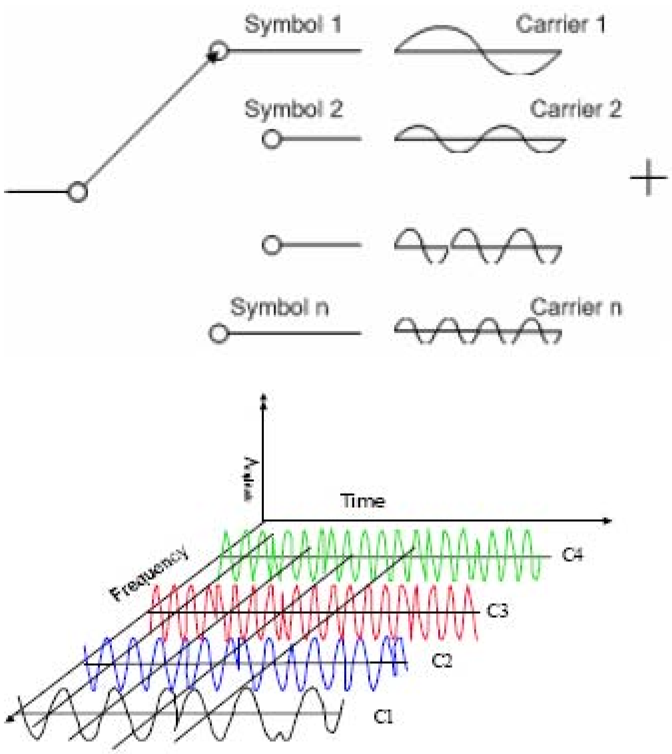


Fig. 5.13 – OFDM signal in time and frequency domain.

Now add all four of these modulated carriers to create the OFDM signal, often produced by a block called the IFFT.

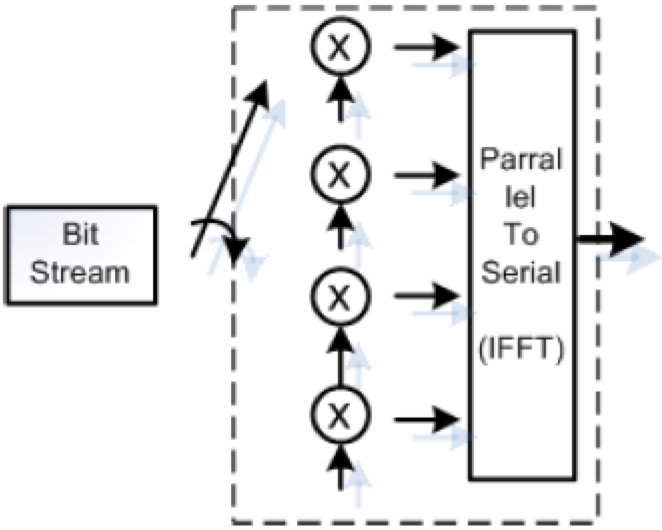


Fig. 5.14 – Functional diagram of an OFDM signal creation. The outlined part is often called an IFFT block.

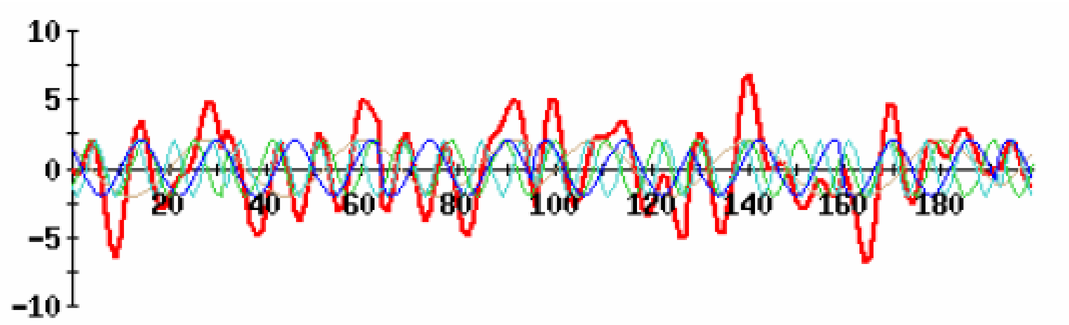


Fig. 5.15 the generated OFDM signal, Note how much it varies compared to the Underlying constant amplitude sub-carriers.

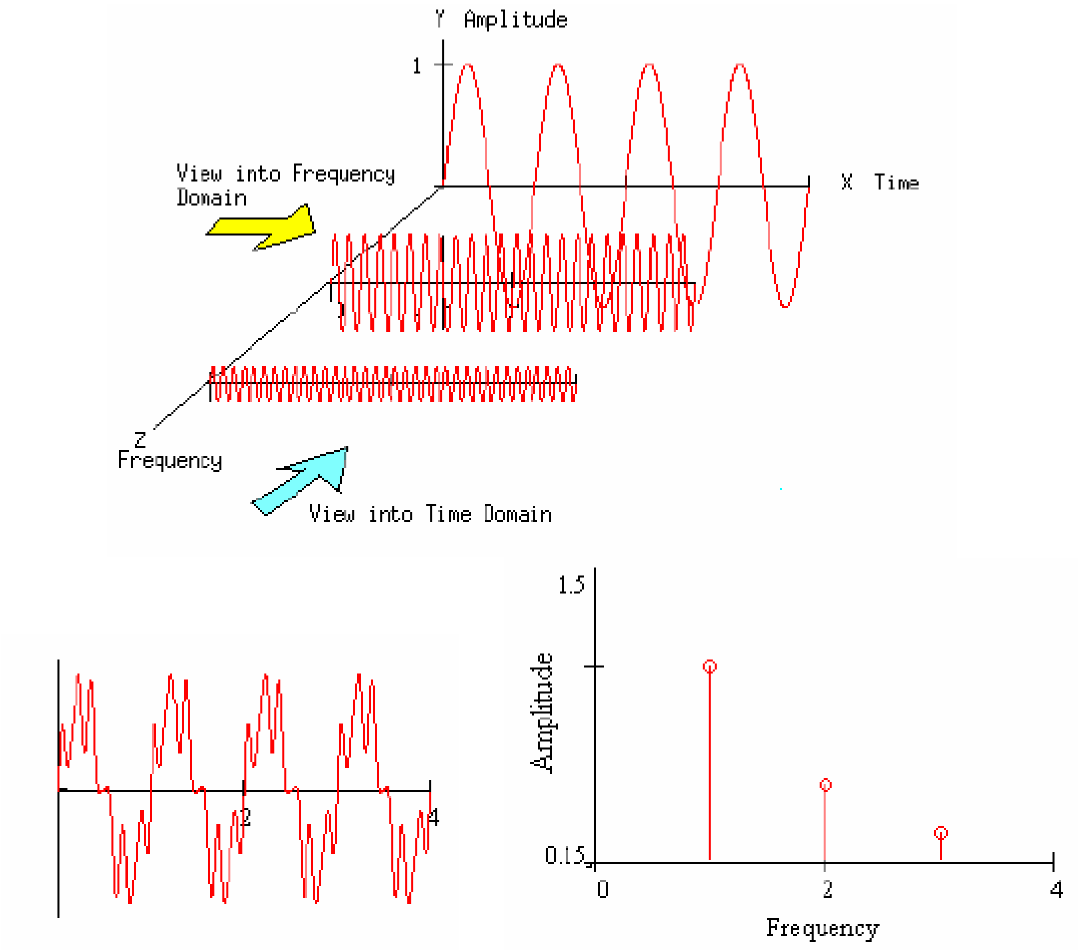
In short-hand, we can write the process above as

$$c(t) = \sum_{n=1}^{N} m_n(t)\sin(2\pi nt)$$

It is basically an equation of an Inverse FFT.

5.5.1 Using Inverse FFT to create the OFDM symbol

The above equation is essentially an inverse FFT.



(a) Time domain view (b) Frequency domain view
Fig. 5.16 - The two views of a signal

Forward FFT takes a random signal, multiplies it successively by complex exponentials over the range of frequencies, sums each product and plots the results as a coefficient of that frequency. The coefficients are called a spectrum and represent “how much” of that frequency is present in the input signal. The results of the FFT in common understanding is a frequency domain signal.

We can write FFT in sinusoids as

$$x(k) = \sum_{n=0}^{N-1} x(n)\sin\left(\frac{2\pi kn}{N}\right) + j\sum_{n=0}^{N-1} x(n)\cos\left(\frac{2\pi kn}{N}\right)$$

Here x(n) are the coeffcients of the sines and cosines of frequency $2\pi\, k/N$, where k is the index of the frequencies over the N frequencies, and n is the time index. x(k) is the value of the spectrum for the kth frequeny and x(n) is the value of the signal at time n. In Fig. 5.16(b), the x(k =1) = 1.0 is one such value.

The inverse FFT takes this spectrum and converts the whole thing back to time domain signal by again successively multiplying it by a range of sinusoids.

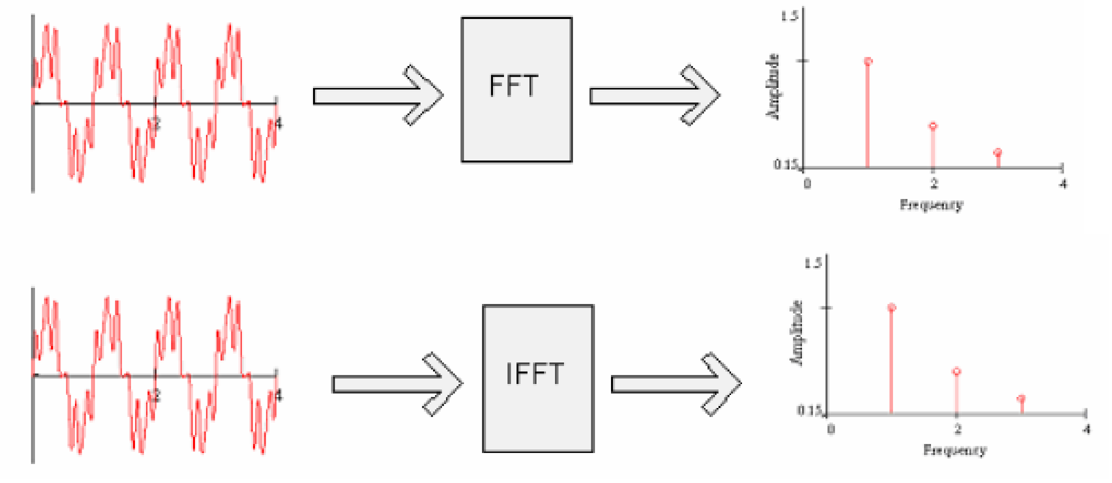
The equation for IFFT is

$$X(n) = \sum_{n=0}^{N-1} x(k)\sin\left(\frac{2\pi kn}{N}\right) + j\sum_{n=0}^{N-1} x(n)\cos\left(\frac{2\pi kn}{N}\right)$$

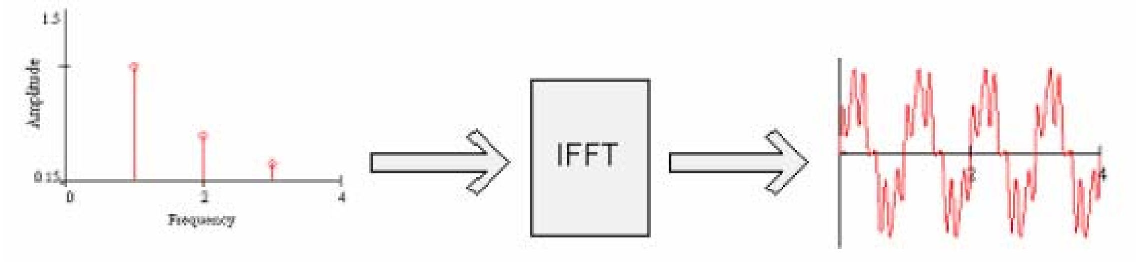
The difference between the last two equations is the type of coefficients the sinusoids are taking, and the minus sign, and that’s all. The coefficients by convention are defined as time domain samples x(k) for the FFT and X(n) frequency bin values for the IFFT. The two processes are a linear pair. Using both in sequence will give the original result back.

Column 1 of Table 5.1 the signal bits, can be considered the amplitudes of a certain range of sinusoids. So we can use the IFFT to produce a time domain signal. Here we pretend that the input bits are not time domain representations but are frequency amplitudes which if you are thinking clearly, will see that that is what they are. In this way, we can take these bits and by using the IFFT, we can create an output signal which is actually a time-domain OFDM signal.

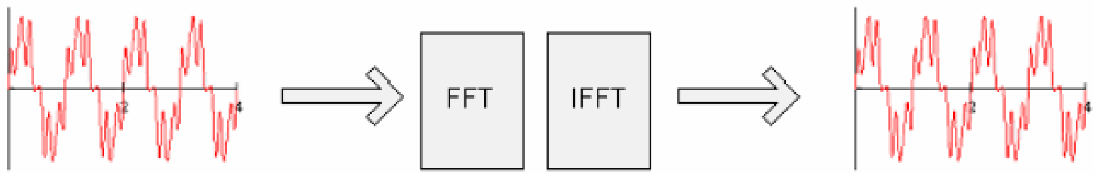
The IFFT is a mathematical concept and does not really care what goes in and what goes out. As long as what goes in is amplitudes of some sinusoids, the IFFT will crunch these numbers to produce a correct time domain result. Both FFT and IFFT will produce identical results on the same input. But most of us are not used to thinking of FFT/IFFT this way. We insist that only spectrums go inside the IFFT.



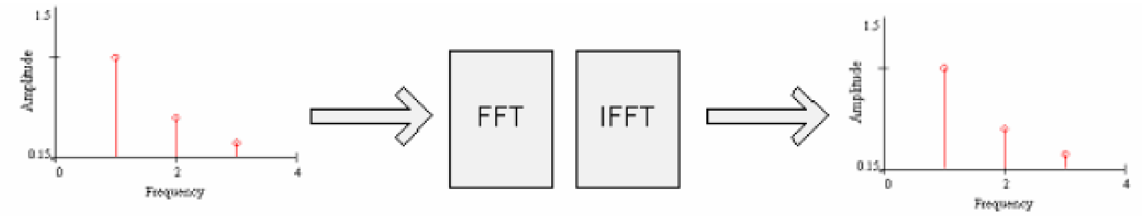
(a) a time domain signal comes out as a spectrum out of a FFT and IFFT.
They both do the same thing.



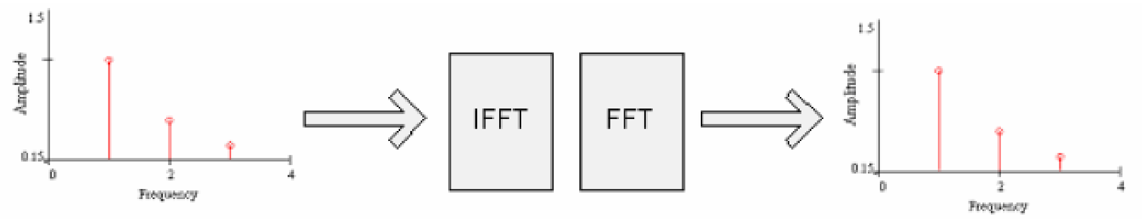
(b) A frequency domain signal comes out as a time domain signal out of a IFFT.



c) The pair return back the original input.



(d) The pair return back the input no matter what it is.



(e) the pair is commutable so they can be reversed and they will still return the original input.

Fig 5.17 – FFT and IFFT are a matched linear pair.

Keeping with that mindset, each row of Table 5.1 can be considered a spectrum as plotted below. These rows aren't actually spectrums, but that does not matter. Each row spectrum has only 4 frequencies which are 1, 2, 3 and 4 Hz. Each of these spectrums can be converted to produce a time-domain signal which is exactly what an IFFT does. Only in this case, the input is really a time domain signal disguising as a spectrum.

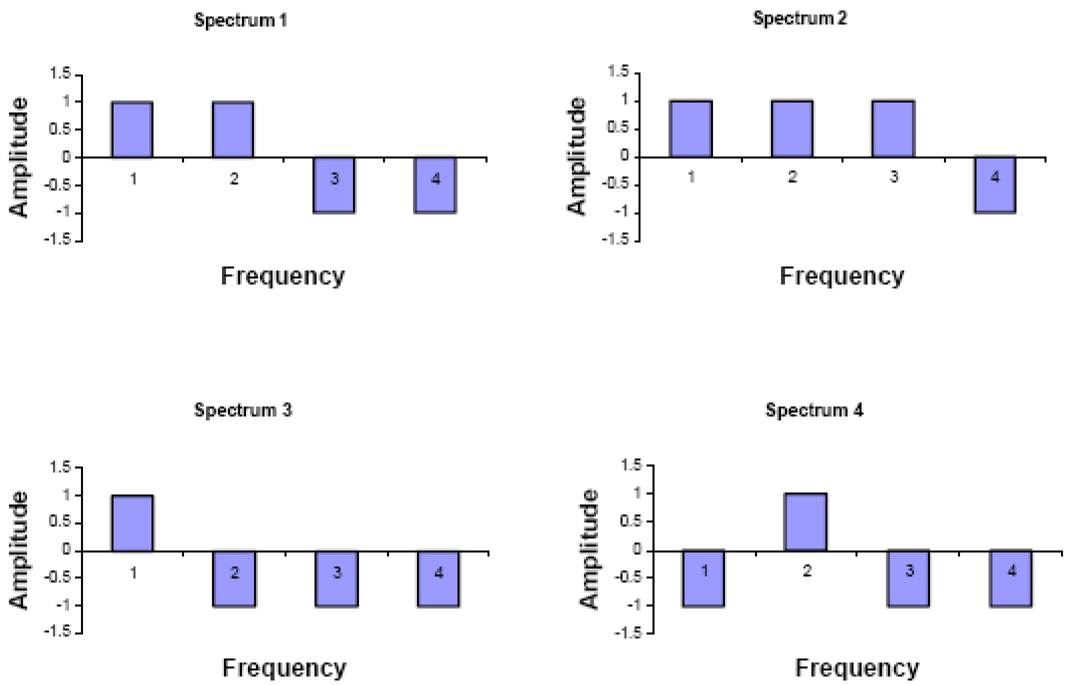


Fig. 5.18 – The incoming block of bits can be seen as a four bin spectrum, The IFFT converts this "spectrum" to a time domain OFDM signal for one symbol, which actually has four bits in it.

The key components of an OFDM system are the inverse FFT at the transmitter and FFT at the receiver. IFFT quickly computes the time-domain signal instead of having to do it one carrier at time and then adding. An N-point FFT only requires Nlog(N) multiplications, which is much more computationally efficient than an equivalent system with equalizer in time domain. Calling this functionality IFFT may be more satisfying because we are producing a time domain signal, but it is also very confusing. Because FFT and IFFT are linear processes and completely reversible, it should be called a FFT instead of a IFFT. The results are the same whether you do FFT or IFFT. In literature you will see it listed as IFFT everywhere. The functional block diagram of how the signal is modulated/demodulated is shown below in fig. 5.19.

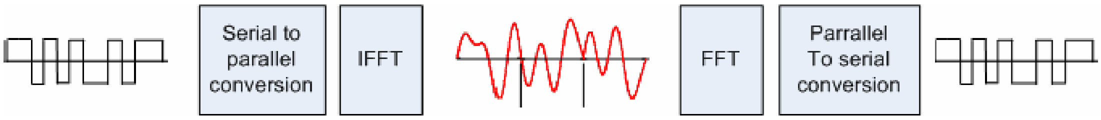


Fig 5.19 – The OFDM link functions

5.6 FADING

The attraction of OFDM is mainly because of its way of handling the multipath interference at the receiver. Multipath phenomenon generates two effects (a) Frequency selective fading and (b) Intersymbol interference (ISI). The "flatness" perceived by a narrow-band channel overcomes the frequency selective fading. On the other hand, modulating symbols at a very low rate makes the symbols much longer than channel impulse response and hence reduces the ISI. Use of suitable error correcting codes provides more robustness against frequency selective fading. The insertion of an extra guard interval between consecutive OFDM symbols can reduce the effects of ISI even more.

5.6.1 Defining fading

If the path from the transmitter to the receiver either has reflections or obstructions, we can get fading effects. In this case, the signal reaches the receiver from many different routes, each a copy of the original. Each of these rays has a slightly different delay and slightly different gain. The time delays result in phase shifts which added to main signal component (assuming there is one.) causes the signal to be degraded. See fig.5.20.

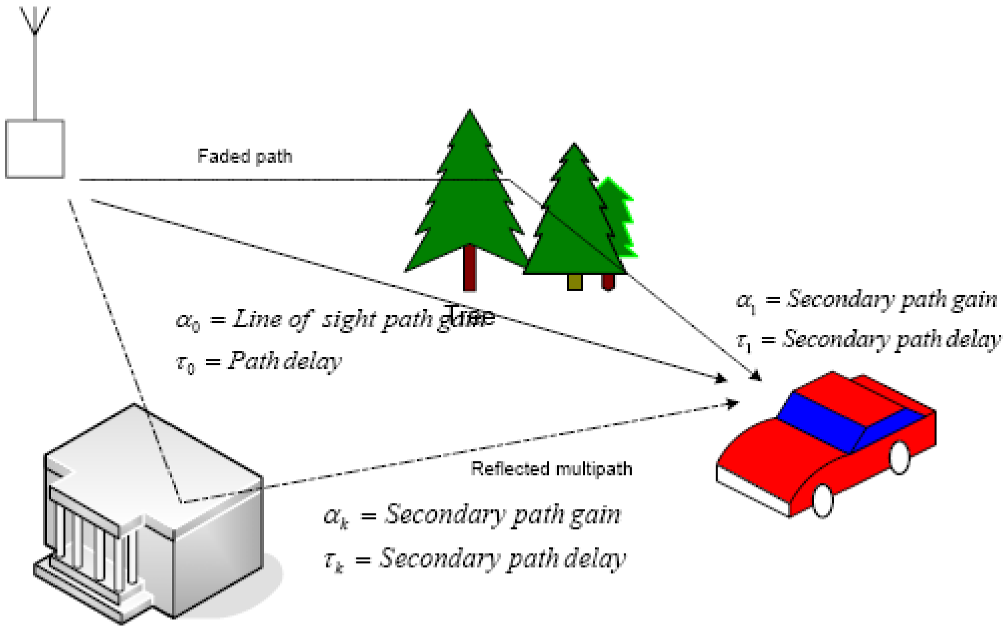
Fading is big problem for signals. The signal is lost and demodulation must have a way of dealing with it. In fading, the reflected signals that are delayed add to the main signal and cause either gains in the signal strength or deep fades. And by deep fades, we mean that the signal is nearly wiped out. The signal level is so small that the receiver can not decide what was there.

The maximum time delay that occurs is called the delay spread of the signal in that environment. This delay spread can be short so that it is less than symbol time or larger. Both cases, cause different types of degradations to the signal. The delay spread of a signal changes as the environment is changing.

Fig. 5.22 shows the spectrum of the signal, the dark line shows the response we wish the channel to have. It is large enough that it allows the signal to go through without bending or distortion. A fading response of the channels is something like shown in fig.8b, we note that at some frequencies in the band, the channel does not allow any information to go through, so called deep fades frequencies. This form of channel frequency response is called frequency selective fading.

Because it does not occur uniformly across the band. It occurs at selected frequencies. If the environment is changing, then this response is also changing. In general when the delay Spread is less than one symbol, we get what is called flat fading. When delay spread is much larger than one symbol that it is called frequency-selective fading.

Rayleigh fading is a term used when there is no direct component and all signals reaching the receiver are reflected. This type of environment is called Rayleigh fading.



$$h_c(t) = \sum_{k=0}^{K-1} \alpha_k \delta(t - \tau_k)$$

α_k = Complex path gain

τ_0 = Normalized path delay relative to LOS

$\Delta_k = \tau_k - \tau_0$ difference in path time

Fig. 5.20 –Fading is particular problem when the link path is changing, such as for a moving car or inside a building or in a populated urban area with tall building.

If we draw the interferences as impulses, they look like this

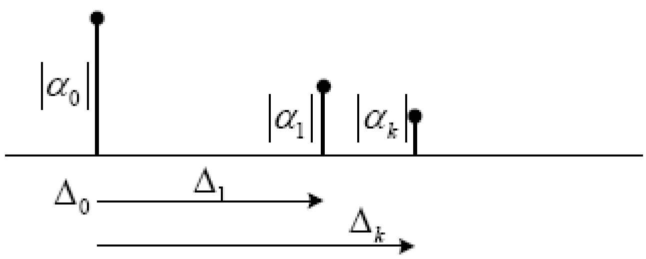


Fig. 5.21– Reflected signals arrive at a delayed time period and interfere with the main line of sight signal, if there is one. In pure Raleigh fading, we have no main signal, all components are reflected.

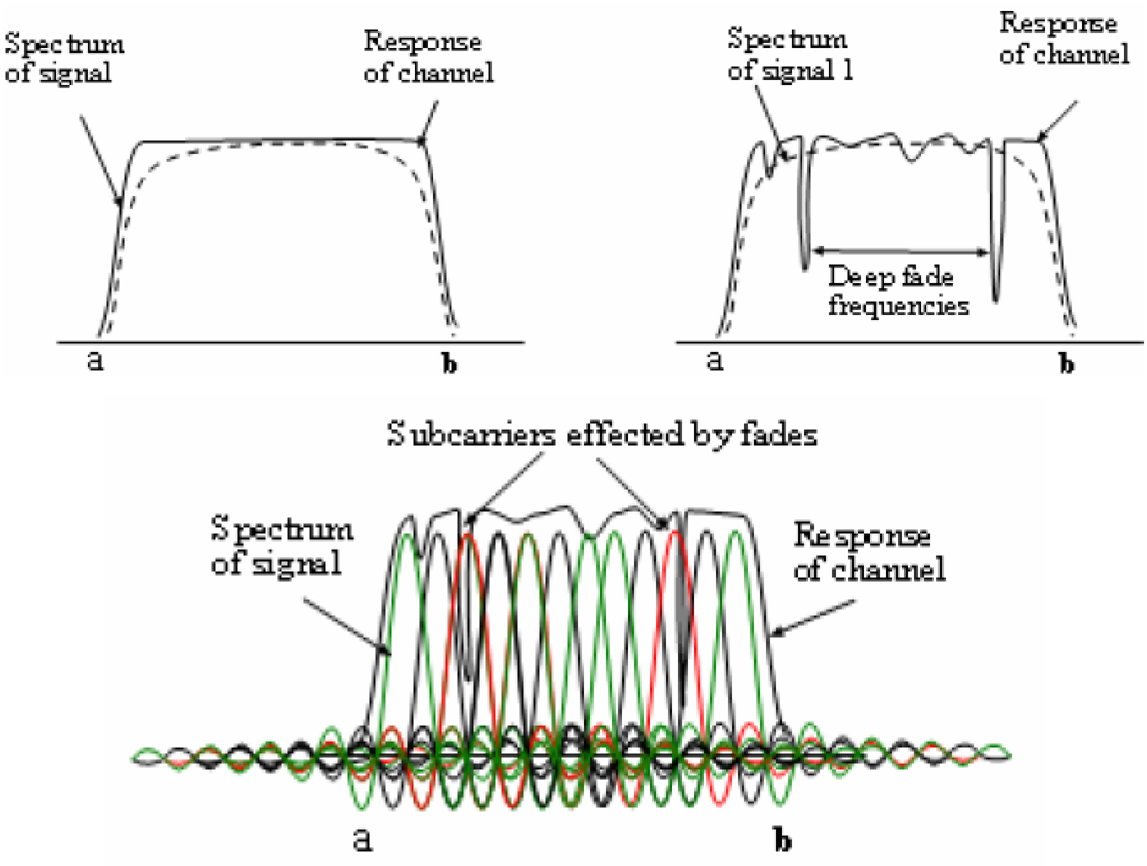


Fig. 5.22 – (a) the signal we want to send and the channel frequency response are well matched. (b) A fading channel has frequencies that do not allow anything to pass. Data is lost sporadically. (c) With OFDM, where we have many little sub-carriers, only a small sub-set of the data is lost due to fading.

An OFDM signal offers an advantage in a channel that has a frequency selective fading response. As we can see, because of the fading response of the channel, only two sub-carriers are affected, all the others arc perfectly OK. Instead Of the whole symbol being knocked out, we lose just a small subset of the (l/N) bits. With proper coding, this can be recovered.

The BER performance of an OFDM signal in a fading channel is much better than the performance of QPSK/FDM which is a single carrier wideband signal. The BER of the OFDM signal is same as the BER of 8PSK signal in Gaussian channel. But in channels that are fading, the OFDM offers far better BER than a wide band signal of exactly the same modulation. The advantage here is coming from the diversity of the multi-carrier such that the fading applies only to a small subset. In FDM carriers, often the signal is shaped with a Root Raised Cosine shape to reduce its bandwidth, in OFDM since the spacing of the carriers is optimal, there is a natural bandwidth advantage and use of RRC does not buy us as much.

5.6.2 Use of cyclic prefix to mitigate delay spread:

You are driving in rain, and the car in front splashes a bunch of water on you. What do you do? You move further back, you put a little distance between you and the front car, far enough so that the splash won't reach you. If we equate the reach of splash to delay spread of a splashed signal then we have a better picture of the phenomena and how to avoid it.

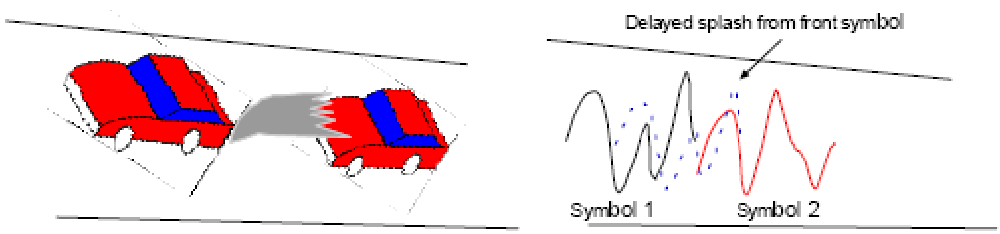


Fig. 5.23 – Delay spread is like the undesired splash you might get from the car ahead of you. In fading, the front symbol similarly throws a splash backwards which we wish to avoid.

Increase distance from car in front to avoid splash. The reach of splash is same as the delay spread of a signal. Fig. 5.23 shows the symbol and its splash. In composite, these splashes become noise and affect the beginning of the next symbol as shown in fig. 5.24

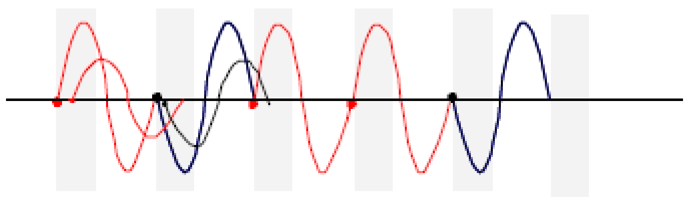


Fig. 5.24 (Composite interference) The PSK symbol and its delayed version

To mitigate this noise at the front of the symbol, we will move our symbol further away from the region of delay spread as shown below. A little bit of blank space has been added between symbols to catch the delay spread.

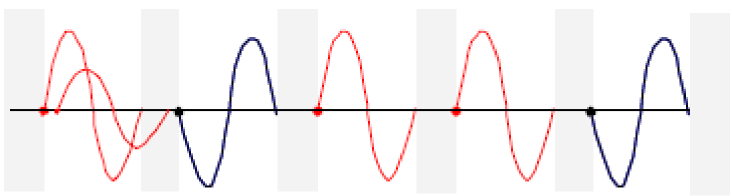


Fig 5.25 – Move the symbol back so the arriving delayed signal peters out in the gray region. No interference to the next symbol

But we can not have blank spaces in signals. This is won't work for the hardware which likes to crank out signals continuously. So it's clear it need to have something there. So, just let the symbol run longer as a first choice.

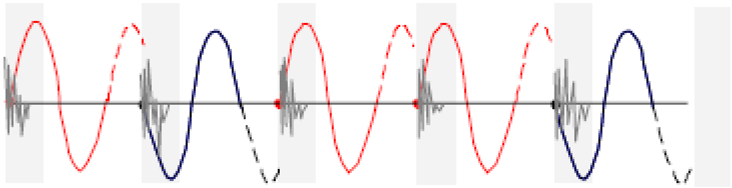


Fig. 5.26 – If we just extend the symbol, then the front of the symbol which is important to us since it allows figuring out what the phase of this symbol is, is now corrupted by the "splash".

We extend the symbol into the empty space, so the actual symbol is more than one cycle. But now the start of the symbol is still in the danger zone, and this start is the most important thing about our symbol since the slicer needs it in order to make a decision about the bit. We do not want the start of the symbol to fall in this region, so lets just slide the symbol backwards, so that the start of the original symbol lands at the outside of this zone. And then fill this area with something.

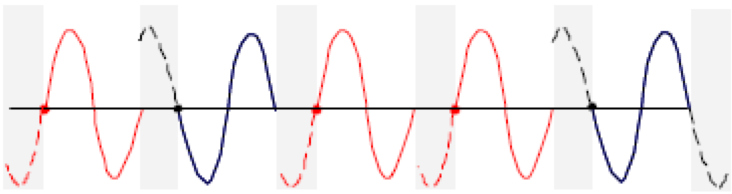


Fig. 5.27 – If we move the symbol back and just put in convenient filler in this area, then not only we have a continuous signal but one that can get corrupted and we don't care since we will just cut it out anyway before demodulating.

1. We want the start of the symbol to be out of the delay spread zone so it is not corrupted.
2. We start the signal at the new boundary such that the actual symbol edge falls out side this zone.

So, Slide the symbol to start at the edge of the delay spread time and then fill the guard space with a copy of what turns out to be tail end of the symbol. We will be extending the symbol so it is 1.25 times as long, to do this, copy the back of the symbol and glue it in the front. In reality, the symbol source is continuous, so all we are doing is adjusting the starting phase and making the symbol period longer.

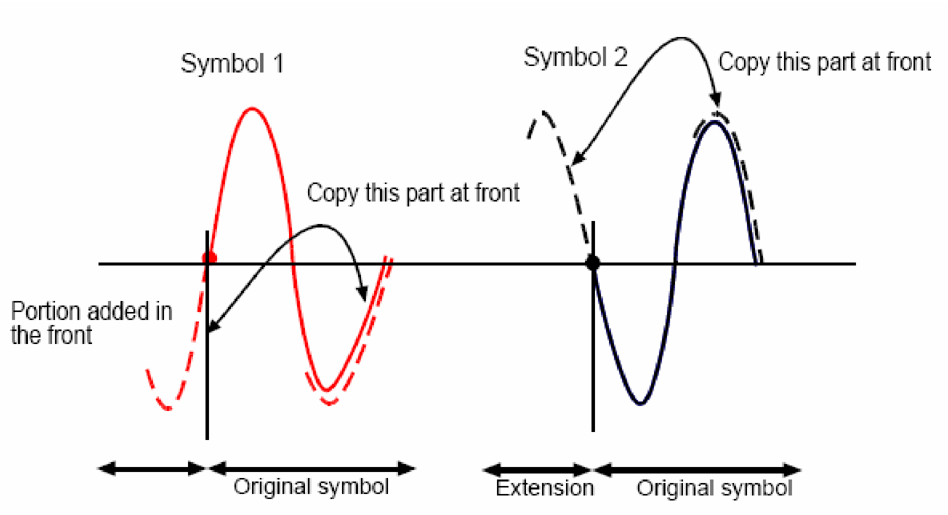


Fig. 5.28-Cyclic prefix is copy of back of the symbol we add to the front of the symbol.

This procedure is called adding a cyclic prefix. Since OFDM, has a lot of carriers, we would do this to each and every carrier. But that's only in theory. Since the OFDM signal is a linear combination, the whole process can be done only once to the OFDM signal, rather than doing it to each and every sub-carrier. We can add cyclic prefix just once to the composite OFDM signal. The prefix is any where from 10% to 25% of the symbol time. We add the prefix after doing the IFFT just once to the composite signal. After the signal has arrived at the receiver, first remove this prefix, to get back the perfectly periodic signal so it can be FFT'd to get back the symbols on each carrier.

However, the addition of cyclic prefix which mitigates the effects of link fading and inter symbol interference, increases the bandwidth.

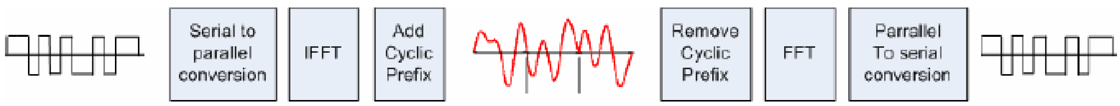


Fig 5.29 – Addition of Cyclic prefix to the OFDM signal further improves its ability to deal with fading and interference.

Here is an OFDM signal with period equal to 32 samples. We want to add a 25% cyclic shift to this signal.

1. First we cut pieces that are 32 samples long.
2. Then we take the last .25 (32) = 8 samples, copy and append them to the front as shown.

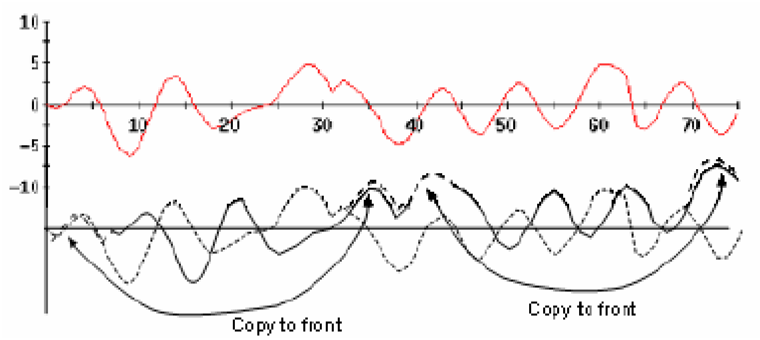


Fig. 5.30 – The whole process can be done only once to the OFDM signal, rather than doing it to each and every sub-carrier.

5.7 Properties of OFDM

5.7.1 Spectrum and performance

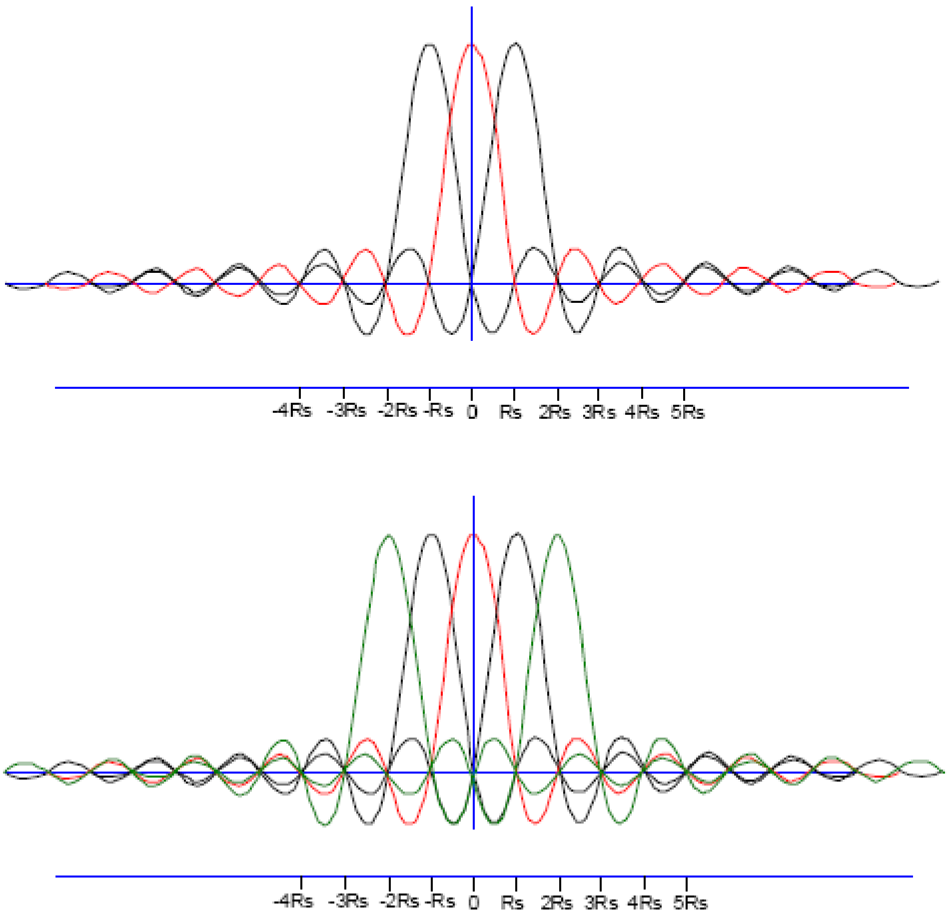


Fig. 5.31 – The spectrum of an OFDM signal (without addition of cyclic prefix) is much more bandwidth efficient than QPSK.

Unshaped QPSK signal produces a spectrum such that its bandwidth is equal to (1+ α)Rs. In OFDM, the adjacent carriers can overlap in the manner shown here. The addition of two carriers, now allows transmitting 3Rs over a bandwidth of -2Rs to 2Rs or total of 4Ts. This gives a bandwidth efficiency of 4/3 Hz per symbol for 3 carriers and 6/5 for 5 carriers.
As more and more carriers are added, the bandwidth approaches,

$$\frac{N+1}{N} \text{ Bits per Hz.}$$

So the larger the number of carriers, the better.
Here is a spectrum of an OFDM signal. Note that the out of band signal is down by 50 dB without any pulse shaping.

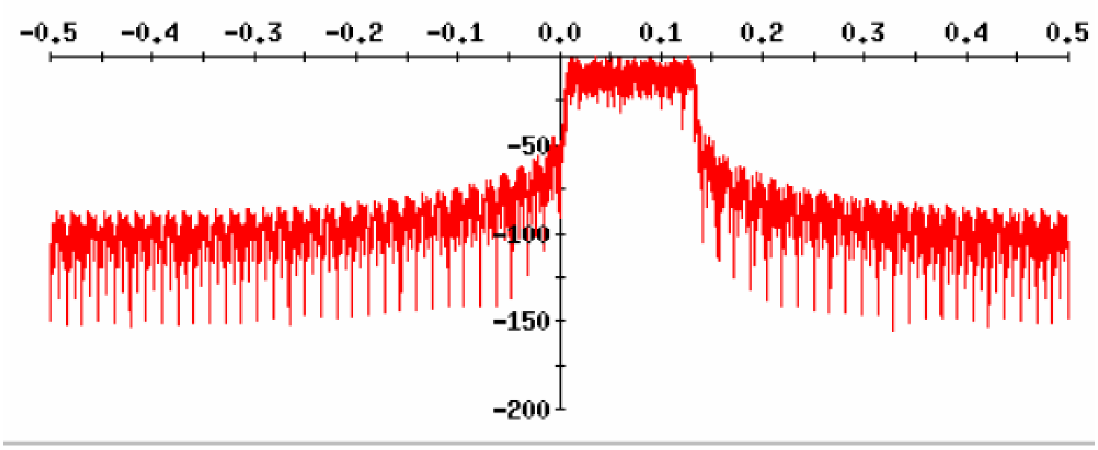


Fig. 5.32 – the spectrum of an OFDM signal with 1024 sub-carriers

Compare this to the spectrum of a QPSK signal, note how much lower the sidebands are for OFDM and how much less is the variance.

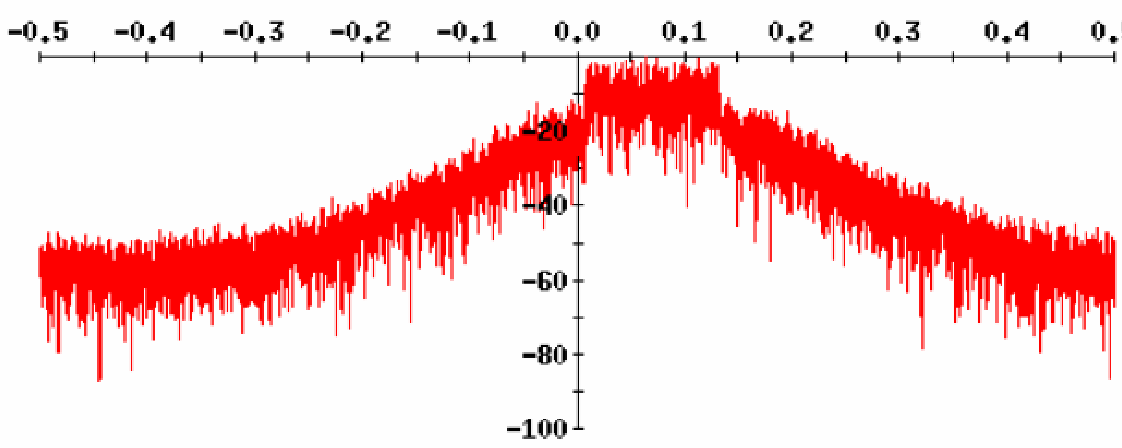


Fig. 5.33 – the spectrum of a QPSK signal

5.7.2 OFDM is a special case of FDM:

Frequency division multiplexing:

Each signal will occupy separate frequency band. To provide adjacent channel interference protection, signals are moved further apart. So here the spectrum is not used completely.

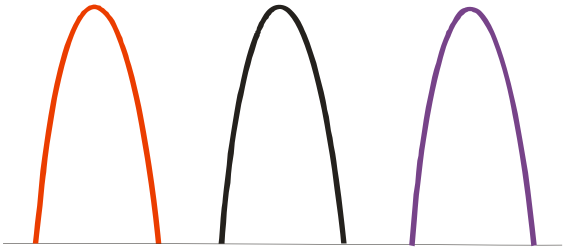


Fig. 5.34 Spectrum of FDM

OFDM frequency dividing:

Large number of orthogonal, overlapping, narrow band sub carriers is transmitted. Since these carriers are orthogonal to each other, when added together, they do not interfere with each other. In FDM, we do not generally have frequencies that follow the above relationship. Here spectrum is used almost twice. So there is a earn in spectral efficiency

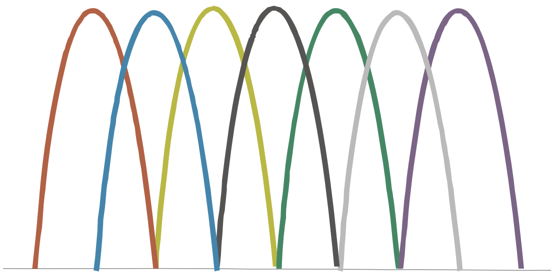


Fig. 5.35 Spectrum of OFDM

5.7.3 Peak to average power ratio (PAPR)

If a signal is a sum of N signals each of max amplitude equal to 1 v, then it is conceivable that we could get a max amplitude of N that is all N signals add at a moment at their max points. The PAPR is defined as

$$R = \frac{|x(t)|^2}{P_{avg}}$$

For an OFDM signal that has 128 carriers, each with normalized power of 1 w, then the max PAPR can be as large as log (128) or 21 dB. This is at the instant when all 128 carriers combine at their maximum point, unlikely but possible. The RMS PAPR will be around half this number or 10-12 dB. The large amplitude variation increases in-band noise and increases the BER when the signal has to go through amplifier non-linearities.

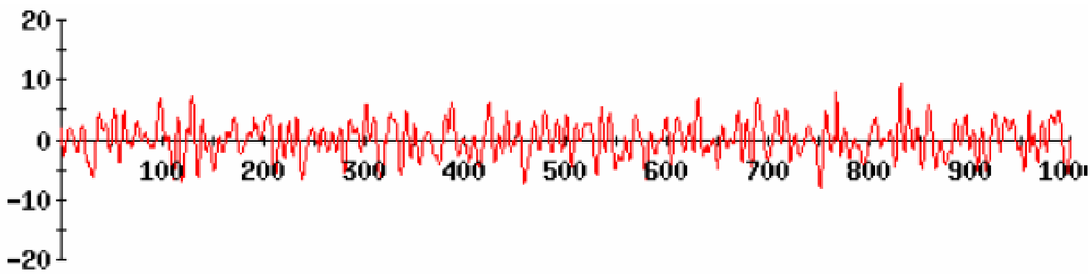


Fig.5.36 AN OFDM signal is very noise like. It looks just like a composite multi-FDM signal.

The OFDM time-domain signal has a relatively large peak-to-average ratio. This tends to reduce the power efficiency of the RF amplifier. Non-linear amplification destroys the orthogonality of the OFDM signal and introduced out-of-band radiation. Several ideas are used to mitigate it.

1.Clipping

We can just clip the signal at a desired power level. This reduces the PAPR but introduces other distortions and ICI.

2. Selective Mapping

Multiply the data signal by a set of codes, do the IFFT on each and then pick the one with the least PAPR. This is essentially doing the process many times using a CDMA like code.

3. Partial IFFT

Divide the signal in clusters, do IFFT on each and then combine these. So that if we subdivide 128 carrier in to a group of four 32 carriers, each, the max PAPR of each will be 12 dB instead of 21 for the full. Combine these four sequences to create the transmit signal.

5.7.4 Synchronization:

The other problem is that tight synchronization is needed. OFDM is sensitive to frequency, clock and phase offset. Often pilot tones are served in the sub carrier space. These are used to lock on phase and to equalize the channel.

5.7.5 Coding:

The sub-carriers are typically coded with Convolutional coding prior to going through IFFT. The coded version of OFDM is called COFDM or Coded OFDM.

5.8 OFDM APPLICATIONS:

OFDM is digital transmission technique developed into a popular scheme for wideband digital communication systems. It is well suited for wideband, high data rate transmissions. The main advantage is that less equalization is necessary. The OFDM use has increased greatly in the last 10 years. Nowadays, OFDM is mainly used for one to many (broadcast) communications like radio or television broadcasting. Examples are digital broadcasting systems such as DAB and DVB.

It is now proposed for Digital audio broadcasting such as in Eureka 147 standard and Digital Radio Mondiale (DRM). Digital Audio Broadcasting (DAB) is an international, standardized digital broadcasting system developed by the European EUREKA-147 Project.

OFDM is used for modem/ADSL application where it coexists with phone line. For ADSL use, the channel, the phone line, is filtered to provide a high SNR. OFDM here is called Discrete Multi Tone (DMT.) HDSL: High bit rate Digital Subscriber Line is another implementation for symmetric speeds (uplink rate = downlink rate).

HiperLAN2 is the all new high performance radio technology, specifically suited for operating in LAN environments. HiperLAN2 is a technology being developed within the European Telecommunications Standardisation Institute (ETSI). OFDM is the modulation used in the physical layer of HiperLAN2.

OFDM is also in use in wireless internet modem and this usage is called 802.1la.

Examples of applications are:

ADSL and VDSL broadband access via telephone network copper wires.

IEEE 802.11a and 802.11g Wireless LANs.

Terrestrial digital TV systems DVB-T, DVB-H, T-DMB and ISDB-T.

IEEE 802.16 or WiMax Wireless MAN.

IEEE 802.20 or Mobile Broadband Wireless Access (MBWA).

Flash-OFDM cellular system.

Some Ultra wideband (UWB) systems.

Power line communication (PLC).

Digital audio, video broadcasting.

Wireless ATM transmission system.

Proposed scheme for UMTS air interface for bit rates higher than 384 kbps

5.9 PARAMETERS OF REAL OFDM

OFDM is used in wireless internet modem and this usage is called 802.11a. Let's take a look at some Parameters of this application of OFDM. The summary of these are given below.

Data rates

6 Mbps to 48 Mbps

Modulation

BPSK, QPSK, 16 QAM and 64 QAM

Coding

Convolutional concatenated with Reed Solomon

FFT size

64 with 52 sub-carriers uses, 48 for data and 4 for pilots.

Sub carrier frequency spacing

20 MHz divided by 64 carriers or .3125 MHz

FFT period

Also called symbol period, 3.2 μ sec = 1/Δf

Guard duration

One quarter of symbol time, 0.8 μ sec

Symbol time

4 μ sec

Chapter 6

OFDM SIMULATION AND RESULTS

6. OFDM SIMULATION AND RESULTS

6.1 WORKING OF THE OFDM MODEL:

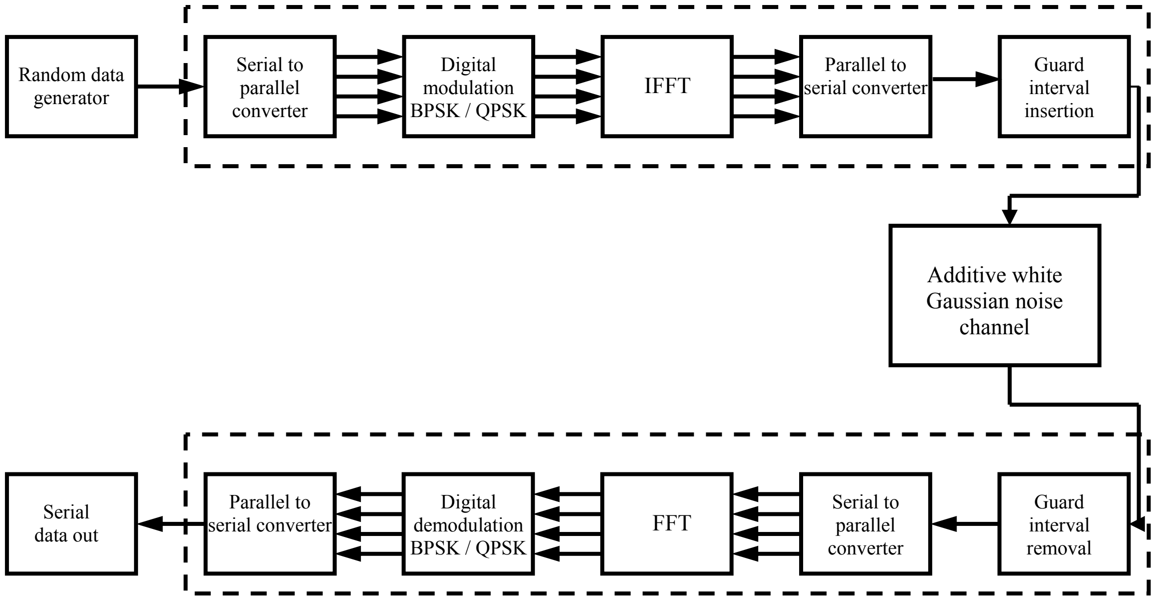


Fig 6.1 OFDM model used for simulation

Figure 6.1 shows the basic block diagram of OFDM transmitter and receiver used for simulation. OFDM is generated by choosing the spectrum required, based on the input data, and modulation scheme used. Each carrier to be produced is assigned data to be transmitted. The required amplitude and phase of the carrier is then calculated based on the modulation scheme (typically BPSK, QPSK, or QAM). For example, if we have to transmit incoming 8 bit digital data, we have to choose 8 different carrier signals, which are orthogonal to each other. Each carrier is assigned to a different bit and its amplitude and phase are chosen according to modulation scheme used. The required spectrum is then converted back to its time domain signal using an Inverse Fourier Transform.

In most applications, an Inverse Fast Fourier Transform (IFFT) is used [6, 7]. They are already in time domain, but here we pretend that the input bits are not time domain representations but are frequency amplitudes which if you are thinking clearly, will see that that is what they are. In this way, we can take these bits and by using the IFFT, we can create an output signal which is actually a time-domain OFDM signal. The IFFT is a mathematical concept and does not really care what goes in and what goes out. As long as what goes in is amplitudes of some sinusoids, the IFFT will crunch these numbers to produce a correct time domain result. Both FFT and IFFT will produce identical results on the same input. We insist that only spectrums go inside the IFFT. IFFT quickly computes the time-domain signal instead of having to do it one carrier at time and then adding. Calling this functionality IFFT may be more satisfying because we are producing a time domain signal, but it is also very confusing. Because FFT and IFFT are linear processes and completely reversible, it should be called a FFT instead of a IFFT. The results are the same whether you do FFT or IFFT. In literature you will see it listed as IFFT everywhere. This block can also be a FFT as long as on the receive side, you do the reverse. The IFFT performs the transformation very efficiently, and provides a simple way of ensuring the carrier signals proceed are orthogonal. The reverse process guarantees that the carriers generated are orthogonal.

Consider the model shown in Fig 6.1. The random data generator generates the data system. This input serial data stream is formatted into the word size required for transmission. For example, 1 bit/word for BPSK & 2 bits/word for QPSK and then shifted into a parallel format. The data is then transmitted in parallel by assigning each data word to one carrier in the transmission. The data to be transmitted on each carrier is then mapped into a Phase Shift Keying (PSK) format. The data on each symbol is mapped to a phase angle based on the modulation method. For example, in QPSK the phase angles used are 0°, 90°, 180°, and 270°. The use of phase shift keying produces a constant amplitude signal and was chosen for its simplicity and to reduce problems with amplitude fluctuations due to fading.

After the required spectrum is worked out, an Inverse Fourier Transform is used to find the corresponding time domain waveform. The guard period is then added to the start of each symbol as shown in the fig 6.2.

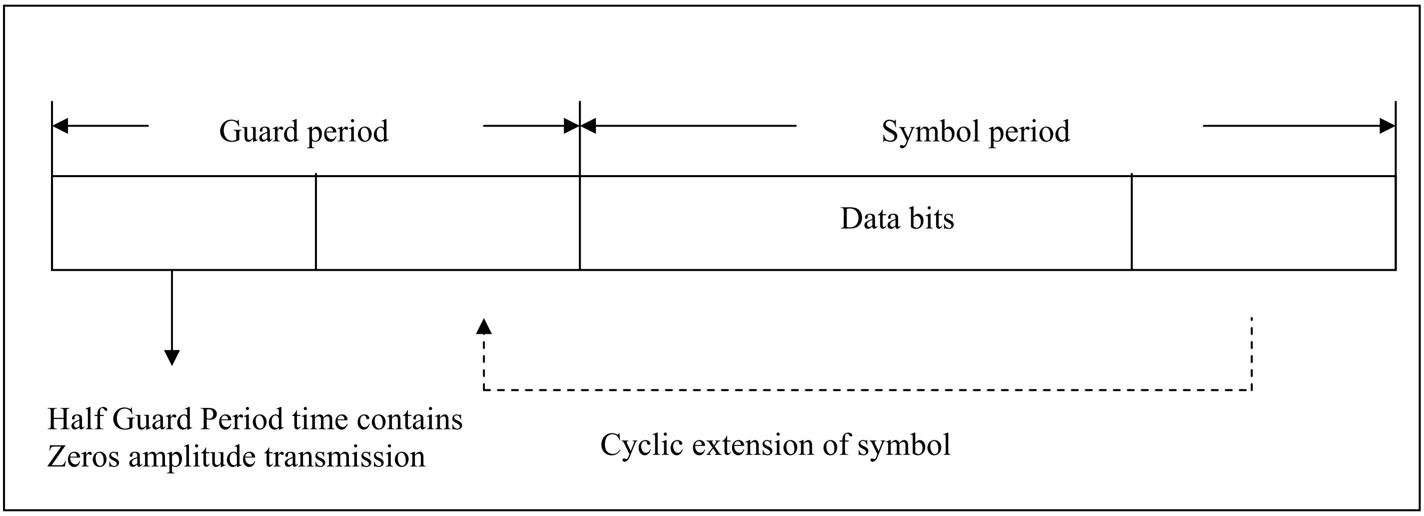


Fig 6.2 addition of guard band interval with symbol

One of the most important properties of OFDM transmissions is its high level of robustness against multipath delay spread. This is a result of the long symbol period used, which minimizes the inter-symbol interference. The level of multipath robustness can be further increased by the addition of a guard period between transmitted symbols. The guard period allows time for multipath signals from the previous symbol to die away before the information from the current symbol is gathered. The guard period used, can be made up of two sections. Half of the guard period time is a zero amplitude transmission called Zero padding and the other half of the guard period is a cyclic extension of the symbol to be transmitted. After the guard has been added, the symbols are then converted back to a serial time waveform. This is then the base band signal for the OFDM transmission. A channel model is then applied to the transmitted signal. The model allows for the signal to noise ratio. It is set by adding a known amount of white noise to the transmitted signal. The channel output is given to the receiver. The receiver basically does the reverse operation to the transmitter. The guard period is removed from the received signal. The FFT of each symbol is then taken to find the original transmitted spectrum. The phase angle of each transmission carrier is then evaluated and converted back to the data word by demodulating the received phase. The data words are then combined, which gives the same word size as that of original data.

6.2 OFDM SYSTEM SIMULATION WITH BPSK:

The random binary generator block generates random binary numbers that are applied to BPSK modulator. This modulates using the binary phase shift keying method. The output is a baseband representation of the modulated signal. The input must be a discrete-time binary-valued signal. If the input bit is 0 or 1, then the modulated symbol is exp(θ) *or* -exp(θ) respectively, where θ is the Phase offset parameter. This frequency domain data is then applied to IFFT block as shown in the Fig 6.1

The IFFT block computes the Inverse Fast Fourier Transform (IFFT) of length-M input, where M must be a power of two. While working with other input sizes, the Zero Pad blocks can be used to pad or to truncate the length so that it will be of size M. The output is always frame-based, and each output frame contains the M-point Inverse Discrete Fourier Transform (IDFT) of the corresponding input. Thus IFFT converts the frequency domain data in to time domain signal and at the same time maintains the orthogonality among the carriers. The AWGN channel block adds white Gaussian noise to a real or complex input signal. When the input signal is real, this block adds real Gaussian noise and produces a real output signal. When the input signal is complex, this block adds complex Gaussian noise and produces a complex output signal.

At the receiver basically the reverse operation to the transmitter will be done. After the removal of guard band the FFT block computes the Fast Fourier Transform (FFT) of length-M input, where M must be a power of two. To work with other input sizes, use the zero pad block to pad or truncate the length-M dimension to a power-of-two length. The data is converted back into frequency domain so that it can be processed by the BPSK Demodulator block. This block demodulates a signal that was modulated using the binary phase shift keying method. The input is a baseband representation of the modulated signal. The input must be a discrete-time complex signal. The block maps the point $exp(\theta)$ and - $exp(\theta)$ to 0 and 1, respectively, where θ is the Phase offset parameters.

The bit error rate calculated by comparing the input data from a transmitter with input data from a receiver. It calculates the bit error rate by dividing the total number of unequal pairs of data elements by the total number of input data elements from source.

6.2.1 Simulation Results for OFDM with BPSK:

No. of bits transmitted = 12000

No. carriers used = 6

Bits per each carrier = 2000

TABLE 6.1: BER results for OFDM model using BPSK modulation

SNR(dB)	BER(using BPSK)
0	0.0757
1	0.0564
2	0.0388
3	0.0215
4	0.0118
5	0.0057
6	0.0022
7	$8.3x\ 10^{-4}$
8	$2.5x\ 10^{-4}$
9	0

These are categorized as tabular results, and graphical results. Signal to Noise ratio (SNR) also called as E_b /N_o, where E_b is bit energy and No is noise energy. SNR values in dB are adjusted every time by adding noise in the AWGN channel. For particular SNR value system is simulated and corresponding probability of error (Bit Error Rate**,** BER) is calculated. These results are noted in Table 6.1. Figure 6.3 shows the nature of the BER versus SNR curve. As we go on increasing the SNR value, bit error rate reduces.

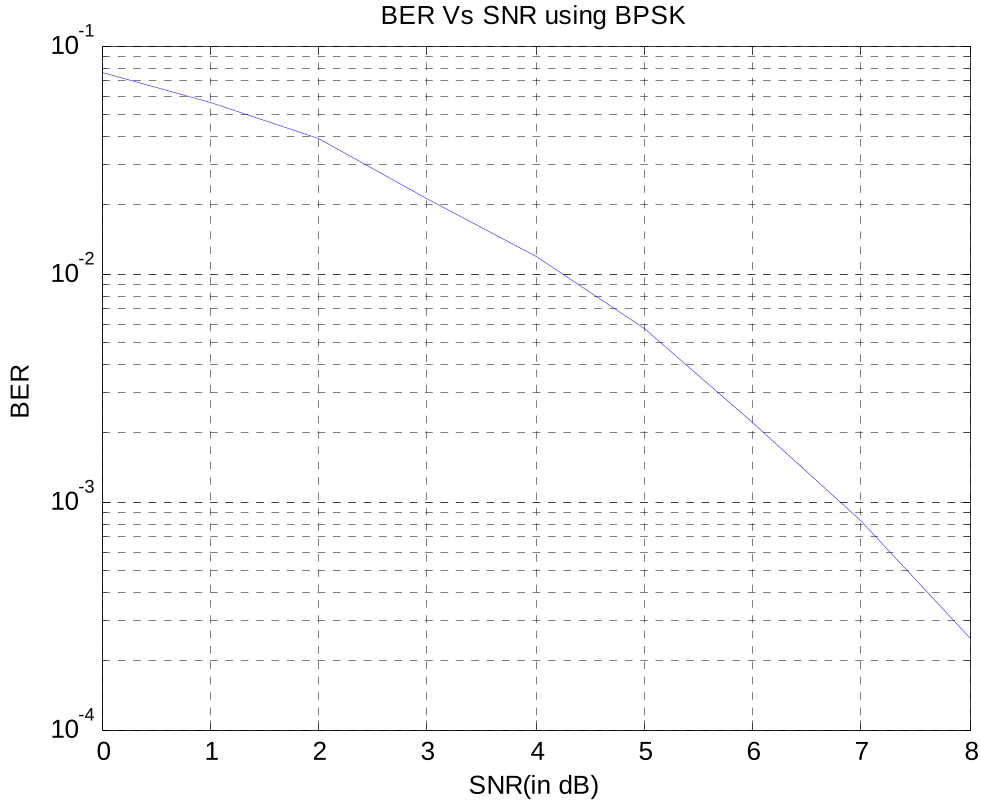


Fig 6.3 BER versus SNR curve for OFDM with BPSK

6.3 OFDM SYSTEM SIMULATION WITH QPSK:

Simulation of OFDM with QPSK modulation technique is similar to the simulation of OFDM with BPSK modulation. The difference is that the BPSK modulator/demodulators are replaced by the QPSK modulator/demodulators. The QPSK modulator modulates using the quaternary phase shift keying method. The output is a baseband representation of the modulated signal. The QPSK demodulator demodulates a signal that was modulated using the quaternary phase shift keying method.

6.3.1 Simulation Results for OFDM with QPSK:

The entire simulation process to be carried out is similar to that of OFDM with BPSK scheme. For particular SNR value system is simulated and corresponding probability of error (Bit Error Rate, BER) is calculated. These results are noted in Table 6.2. Figure 6.4 shows the nature of the BER versus SNR curve. As we go on increasing the SNR value, bit error rate reduces.

No. of bits transmitted = 12000

No. carriers used = 6

Bits per each carrier = 2000

TABLE 6.2: BER results for OFDM model using QPSK modulation

SNR(dB)	BER(using QPSK)
0	0.2088
1	0.1802
2	0.1475
3	0.1125
4	0.0807
5	0.0554
6	0.0313
7	0.0172
8	0.0090
9	0.0035
10	0.0015
11	5.83×10^{-4}

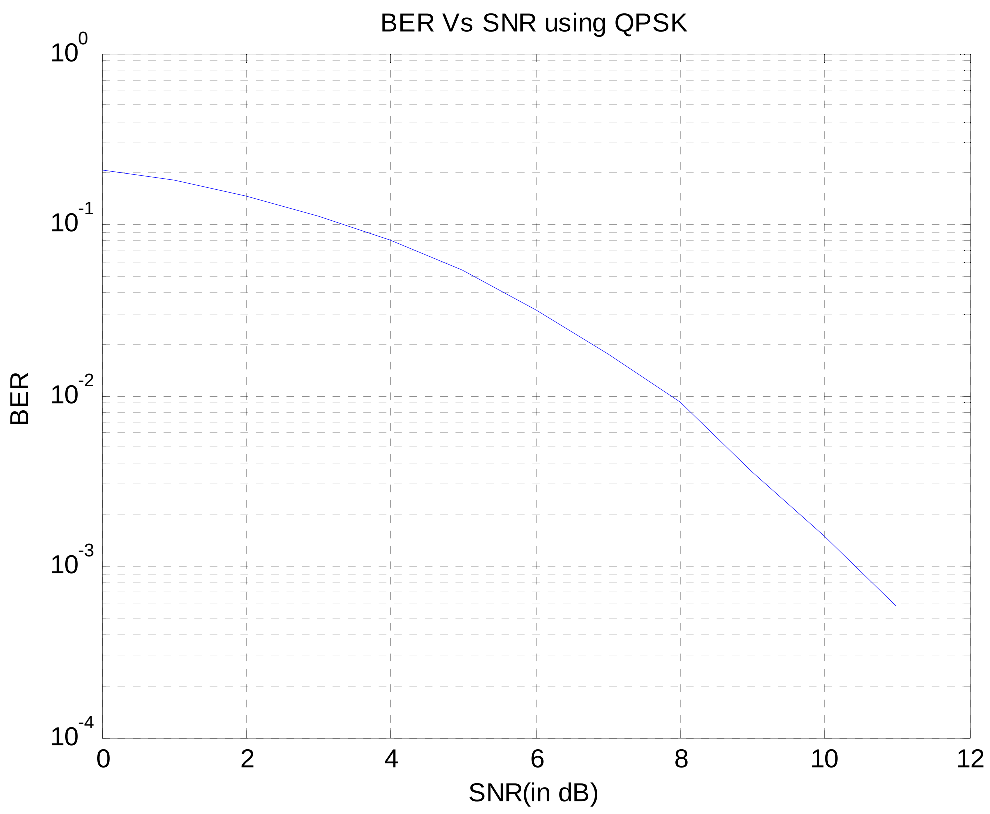


Fig 6.4 BER versus SNR curve for OFDM with BPSK

6.4 COMPARISON OF OFDM SYSTEMS WITH QPSK AND BPSK

Figure 6.5 shows comparison of the BER versus SNR curves obtained in the OFDM systems with BPSK and QPSK modulation schemes. It is clearly observed that the curve in latter system is always above the curve in the earlier system

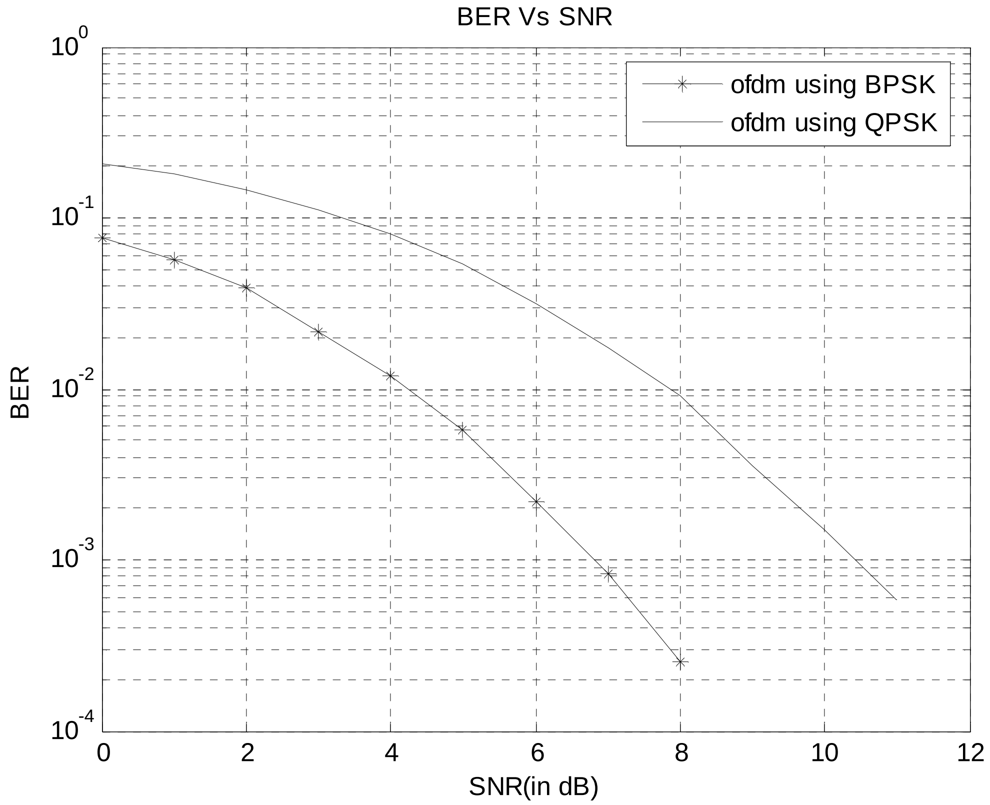


Fig 6.5 Comparison of BER versus SNR of Two Modulation Schemes

6.5 OFDM IMPLEMENTATION USING FFT

In the OFDM system implementation Fast Fourier Transform (FFT) and its inverse (IFFT) were used to perform the modulation and demodulation operations. BPSK is used as digital modulation technique. The Fourier transform allows us to relate events in time domain to events in frequency domain. OFDM system Use discrete Fourier transform (DFT) to modulate and demodulate parallel data. The fast Fourier transform (FFT) is merely a rapid mathematical method for computer applications of DFT. In most applications, an Inverse Fast Fourier Transform (IFFT) is used. Both transmitter and receiver can be implemented using efficient FFT techniques that reduce the number of operations from N^2 in DFT, down to NlogN. It is the availability of this technique, and the technology that allows it to be implemented on integrated circuits at a reasonable price, that has permitted OFDM to be developed as far as it has. The orthogonality of subchannels in OFDM can be maintained and individual subchannels can be completely separated by the FFT at the receiver when there are no intersymbol interference (ISI) and intercarrier interference (ICI) introduced by transmission channel distortion.

IFFT quickly computes the time-domain signal instead of having to do it one carrier at time and then adding. The IFFT performs the transformation very efficiently, and provides a simple way of ensuring the carrier signals proceed are orthogonal. The reverse process guarantees that the carriers generated are orthogonal. FFT based OFDM is computationally efficient due to the use of FFT techniques to implement modulation and demodulation functions.

6.5.1 Simulation Results

Results are noted in Table 6.3. Figure 6.6 shows the nature of the BER versus SNR curve. As we go on increasing the SNR value, bit error rate reduces.

No. of bits transmitted = 12000

No. of carriers used = 6

Bits per each carrier = 2000

Spacing between the each carrier = 6 KHz

Carrier frequencies are 6 KHz, 12 KHz, 18 KHz, 24 KHz, 30 KHz, and 36 KHz

Modulation technique used is BPSK

TABLE 6.3: BER results for FFT based OFDM

SNR(dB)	BER(using FFT)
0	0.09042
1	0.07660
2	0.06800
3	0.05730
4	0.05042
5	0.04420
6	0.03683
7	0.03090
8	0.02375
9	0.02040
10	0.01308
11	0.01190
12	6.75×10^{-3}
13	5.20×10^{-3}
14	2.92×10^{-3}
15	6.70×10^{-4}
16	4.17×10^{-4}

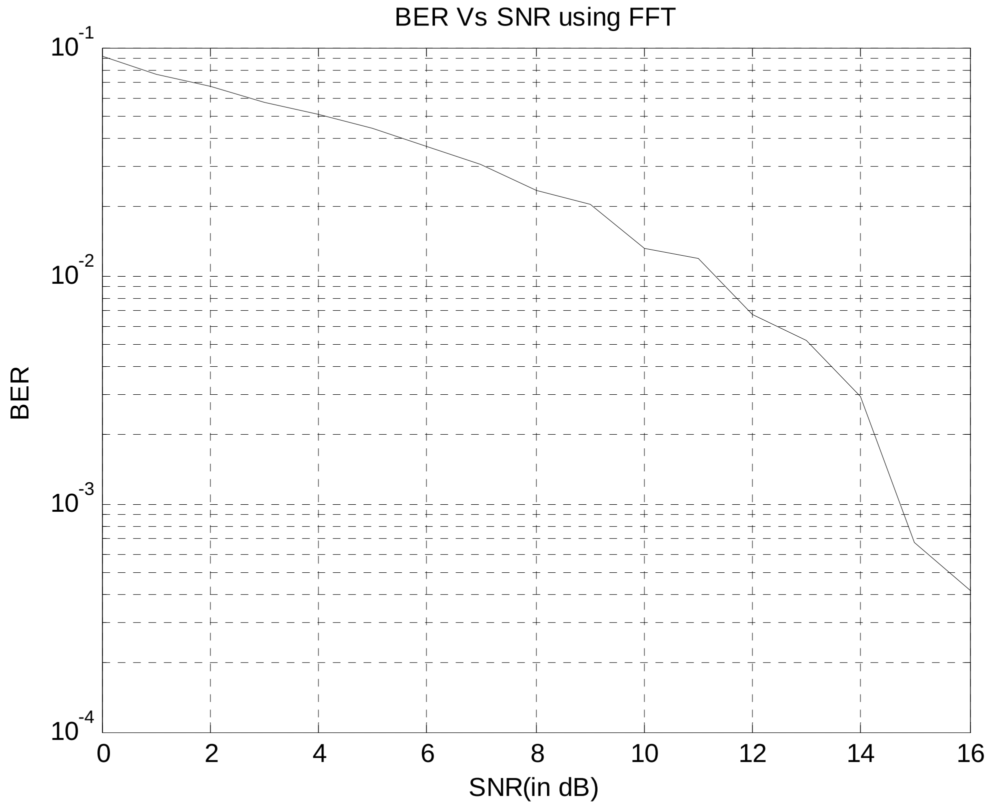


Fig 6.6 BER versus SNR curve for FFT based OFDM

6.6 OFDM IMPLEMENTATION USING DHT

In the OFDM system implementation real-valued discrete Hartley transform (DHT) and its inverse (IDHT) were used to perform the modulation and demodulation operations. BPSK is used as digital modulation technique. Since the DHT and IDHT definitions are identical, we can use the same hardware or program to implement the modulator and demodulator.

For the current OFDM-based transceivers, the modulator needs to compute a long-length inverse discrete Fourier transform (IDFT), and the demodulator needs to compute a long length DFT, where the transform length is up to 512 or more. For such long-length DFT/IDFT computations, a great number of complex multiplications are required and each of them basically involves four real multiplications and two real additions. Clearly, the complexity of a OFDM-based transceiver would be reduced if the corresponding modulator/demodulator could be implemented using purely real transforms while fast algorithms similar to the fast Fourier transform **(**FFT**)** algorithm can still be applied.

The DHT involves only real-valued arithmetic and has an identical inverse. Like the DFT, there have been a number of fast algorithms and hardware architectures available for the DHT computation. From the results it is seen that the DHT-based OFDM method achieves the same BER performance as the DFT-based OFDM method, but requires less computational complexity.

6.6.1 Simulation Results

Results are noted in Table 6.4. Figure 6.7 shows the nature of the BER versus SNR curve. As we go on increasing the SNR value, bit error rate reduces.

No. of bits transmitted = 12000

No. of carriers used = 6

Bits per each carrier = 2000

Spacing between the each carrier = 6 KHz

Carrier frequencies are 6 KHz, 12 KHz, 18 KHz, 24 KHz, 30 KHz, and 36 KHz

Modulation technique used is BPSK

TABLE 6.4: BER results for DHT based OFDM

SNR(dB)	BER(using DHT)
0	0.1072
1	0.0882
2	0.0775
3	0.0649
4	0.0523
5	0.0447
6	0.0378
7	0.0340
8	0.0246
9	0.0189
10	0.0137
11	0.0106
12	0.0080
13	0.0042
14	$2.6\text{x}10^{-3}$
15	$5.0\text{x } 10^{-4}$
16	$3.3\text{x } 10^{-4}$

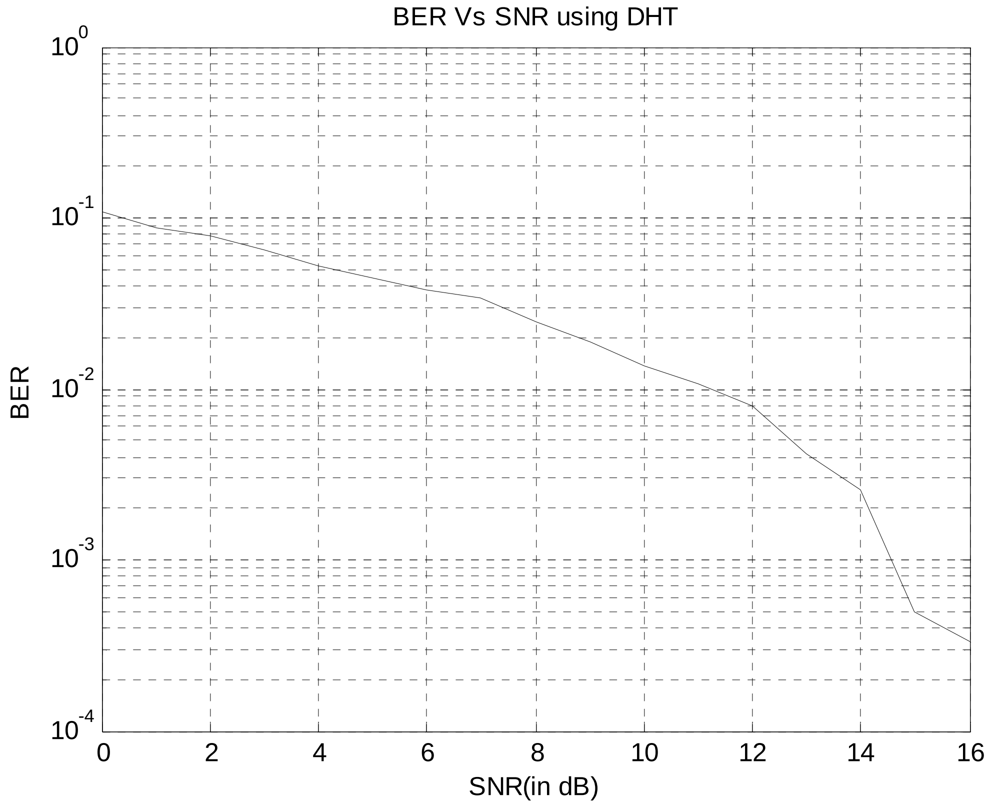


Fig 6.7 BER versus SNR curve for DHT based OFDM

6.7 OFDM IMPLEMENTATION USING DCT

In the OFDM system implementation real-valued Discrete Cosine Transform (DHT) and its inverse (IDCT) were used to perform the modulation and demodulation operations. BPSK is used as digital modulation technique. The DCT uses only real arithmetic, as opposed to the complex-valued DFT. This reduces the signal-processing complexity/power consumption. For the current OFDM-based transceivers, the modulator needs to compute a long-length inverse discrete Fourier transform (IDFT), and the demodulator needs to compute a long length DFT, where the transform length is up to 512 or more. For such long-length DFT/IDFT computations, a great number of complex multiplications are required and each of them basically involves four real multiplications and two real additions. The DCT basis is well known to have excellent spectral compaction and energy concentration properties.

6.7.1 Simulation Results

Results are noted in Table 6.5. Figure 6.8 shows the nature of the BER versus SNR curve. As we go on increasing the SNR value, bit error rate reduces.

No. of bits transmitted = 12000

No. of carriers used = 6

Bits per each carrier = 2000

Spacing between the each carrier = 6 KHz

Carrier frequencies are 6 KHz, 12 KHz, 18 KHz, 24 KHz, 30 KHz, and 36 KHz

Modulation technique used is BPSK

TABLE 6.5: BER results for DCT based OFDM

SNR(dB)	BER(using DCT)
0	0.0237
1	0.0210
2	0.0162
3	0.0109
4	0.0083
5	0.0050
6	0.0028
7	0.0014
8	4.167x 10
9	1.667x 10
10	0

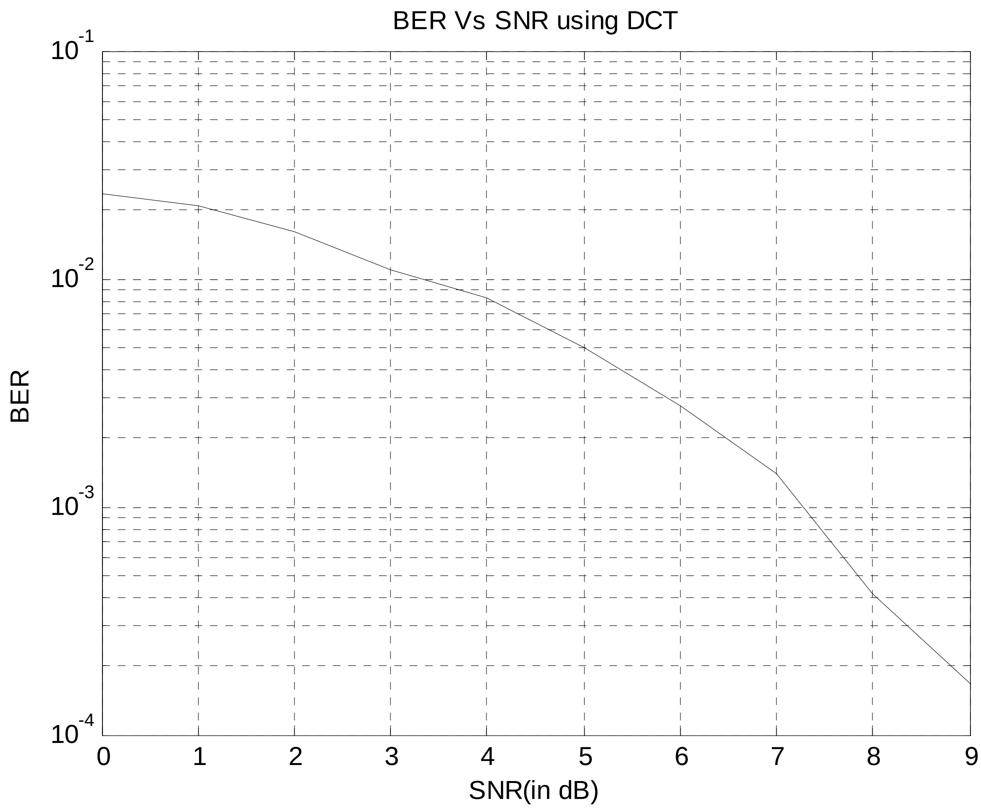


Fig 6.8 BER versus SNR curve for DCT based OFDM

6.8 COMPARISON OF PERFORMANCE OF OFDM SYSTEMS

Figure 6.9 shows comparison of the BER versus SNR curves obtained in the FFT, DHT, and DCT based OFDM systems with BPSK as a modulation technique. It is observed that for all the systems BER is getting reduced as SNR increases. The BER performance of FFT and DHT is almost same. It is also observed that the DCT based BER curve is always below the curves of other two systems.

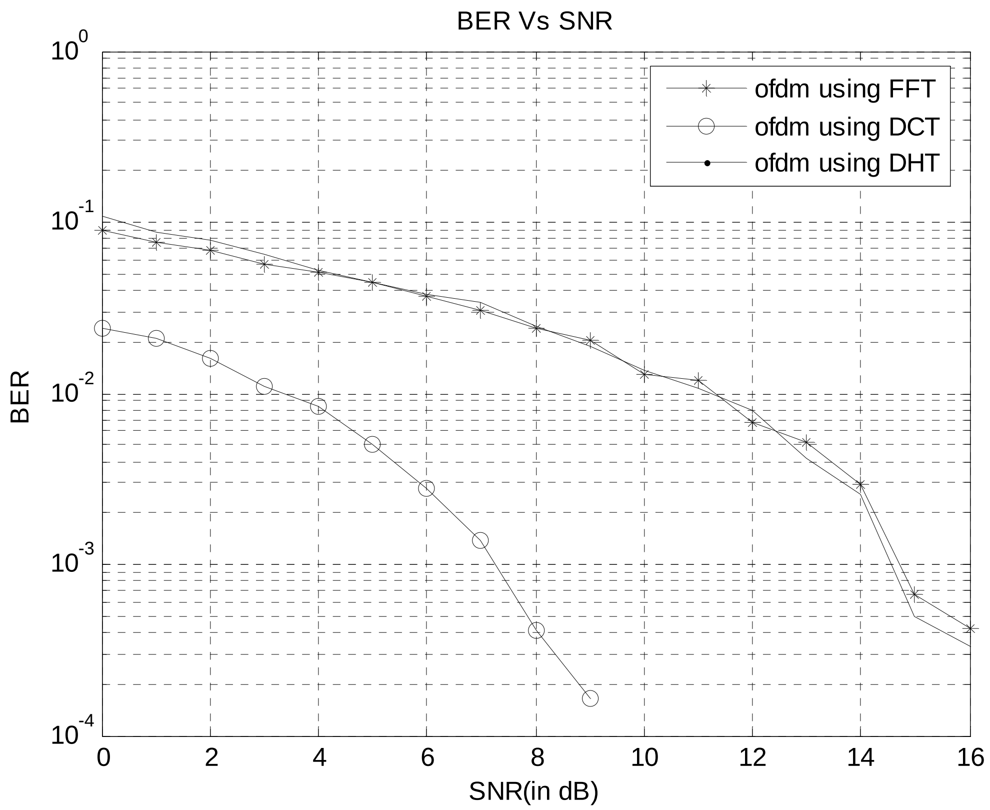


Fig 6.9 Comparison of Bit Error Rates of FFT, DHT, and DCT based OFDM systems

Chapter 7

CONCLUSION

7. CONCLUSION

This chapter gives a summary of the work presented in this thesis. An outline for the future work based on this is also given.

7.1 SUMMARY

The OFDM makes efficient use of available spectrum by allowing overlapping among the carriers. It basically converts the high data rate stream in to several parallel lower data rate streams and thereby eliminating the frequency selective fading. It has been seen that the OFDM is a powerful modulation technique that is capable of high data rate and is able to eliminate ISI. It is computationally efficient due to the use of FFT techniques to implement modulation and demodulation functions.

Using MATLAB software, the performance of OFDM system was tested for two digital modulation techniques namely BPSK and QPSK. The performance of OFDM system was also tested for three different transform techniques namely FFT, DHT, and DCT.

From the simulation results, it is observed that the BPSK allows the BER to be improved in a noisy channel at the cost of maximum data transmission capacity. Use of QPSK allows higher transmission capacity, but at the cost of slight increase in the probability of error. This is because of the fact that QPSK uses two bits per symbol. Hence QPSK is easily affected by the noise. Therefore OFDM with QPSK requires larger transmit power. From the results, use of OFDM with QPSK is beneficial for short distance transmission link, whereas for long distance transmission link OFDM with BPSK will be preferable.

From the results it is also observed that the DHT-based OFDM achieves the same transmission performance as the DFT-based OFDM from the BER vs SNR point of view, but requires less computational complexity with reduced implementation cost, because of it is purely based on the real valued DHT. And the DCT based OFDM has better BER performance compared to the DFT, DHT based OFDM methods, because of its strong "energy compaction" property and real valued nature. So, from the BER vs SNR results we conclude that implementation and performance points of view, DHT, DCT transform techniques are may be alternatives to DFT for the designing of OFDM-based communication systems.

7.2 SCOPE OF FUTURE WORK

The following are the some of the interesting extensions of the present work:

1) An interesting topic for future research is to perform more extensive performance comparisons between FFT based OFDM, DHT based OFDM, and DCT based OFDM systems under additional real-world channel impairments, such as multipath fading, time dispersion whish leads to inter symbol interference (ISI).
2) The main problems with OFDM signal is very sensitive to carrier frequency offset, and its high Peak to Average Power Ratio (PAPR). So, these three transform based OFDM systems can be tested for these problems.

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