TRANSIENT RADIATIVE TRANSFER IN HOMOGENEOUS/NONHOMOGENEOUS PARTCIPATING MEDIUM

A THESIS SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

Bachelor of Technology
In
Mechanical Engineering

By

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National Institute of Technology
Rourkela

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CERTIFICATE

This is to certify that the thesis entitled, "**Transient Radiative Transport in Homogeneous/ Non-Homogeneous Participating Medium**" submitted by **Sri Amit Bikram Kar** and **Sri Abhisek Saran** in partial fulfillment of the requirements for the award of Bachelor of Technology Degree in Mechanical Engineering at the National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by them under our supervision and guidance.

To the best of our knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

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NIT Rourkela

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Abstract

Thermal radiation is important in many applications, and its analysis is difficult in the presence of a participating medium. In traditional engineering studies, the transient term of the radiative transfer equation (RTE) can be neglected. The assumption does not lead to important errors since the temporal variations of the observables e.g. temperature are slow as compared to the time of light of a photon. However in many new applications such as pulsed LASER interaction with materials, the transient effect must be considered in the RTE. In the transient phase, the reflected and the transmitted signals have temporal signatures that persist for a time period greater than the duration of the source pulse. This could be a source of information about the properties field inside the medium. Hence sufficiently accurate solution methods are required. Predicted signals are dependent on the considered models. The results vary significantly from approximate models.

In the last few years, the finite volume method (FVM) and discrete transfer method have emerged as one of the most attractive methods for modeling steady state radiative transfer.

The present research work deals with the analysis of transient radiative transfer in one dimensional scattering medium using FVM. One boundary is subjected to short pulse irradiation and the other boundary is assumed to be diffused. The effects of short pulse LASER on variation of transmittance and reflectance with time have been observed for a given value of optical thickness and scattering albedo. The results obtained were in accordance with that obtained from the discrete transfer method (DTM).

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Chapter 1

INTRODUCTION

Background
Present Objectives
Review of existing methods

Chapter 1

INTRODUCTION

Thermal radiation is important in many applications and its analysis is difficult in the presence of participating medium. Thermal radiation being electromagnetic waves, it propagates at the speed of light. In most of the traditional engineering application, such as in the thermal analysis of boilers, furnaces, IC engines etc. as temporal variations in thermal quantities of interests are much slower than the time scale associated with the propagation of thermal radiation, transient term from the radiative transfer equation is neglected, i.e. radiation is assumed an instantaneous(steady state) process. However there are certain situations in which temporal variations are required at time scales as low as 10^{-12} to 10^{-15} seconds. Therefore such situations necessitate inclusion of transient term in the radiative transfer equation, thereby making the problems further complicated.

Some examples where transient nature of thermal radiation has to be considered are the micro scale systems, pulsed laser interaction with materials, laser induced shock waves, laser therapy, optical tomography, remote sensing of turbid area of ocean and atmosphere, probing the characteristics of the particulate medium by examining the transmitted or back scattered intensities, particle detection and sizing. A detail review dealing in various aspects of transient radiative transport has been done. The details of transport phenomenon involving thermal radiation can be found in Chandrasekhar [30].

In the literature, applications of various existing methods used for solving steady state radiative transport problems have been extended to the analysis of transient radiative transfer transport in the participating medium. Rackmill and Buckius [15] used the finite difference method with adding double scheme to solve transient equation of transfer for a plane parallel slab. Diffusion approximation has been used by Yamada [8] and Flock et al [16]. As is true with the solution of steady state radiative transport problems, diffusion approximation has been found to give correct predictions only for optically thick conditions. For thin optical thickness condition it has not been found suitable. Kumar et al.[18] and mitra et al.[19] have extended the use of P_1 approximation to the solution of transient radiative transport for 1D and 2D rectangular geometries. Mitra and Churnside [11] have applied discrete ordinate method for the solution of transient radiative equation applied to oceanographic lidar. Monte carlo method has been used by Guo et al. [20] and Schwieger et al.[21]. Tan and Hsu [22] have used a time dependent integral formulation for modeling transient radiative transfer, and the application of the radiation element method for this class of problems has been extended by Guo and Kumar [23].

A comparative study of two-flux method, P_1 approximation and the discrete ordinate method has been presented by Mitra and Kumar [24]. They compared both the parabolic as well as the hyperbolic nature of the short pulse radiative transport, and found that for higher optical thickness, both hyperbolic ad parabolic solutions become identical. Also it has been observed that the wave propagation speed depends upon the method used. With two-flux method and P_1 approximation, propagation speed of wave front has been found to be far away from the speed of pulse. This lower propagation speed could have the potential drawback of predicting results that have a significant temporal mismatch with observed data. But such discrepancy is low with the DOM. However DOM, is prone to ray and fall scattering effects and it is severe in multidimensional geometries hence scope for further study remains open with other numerical methods.

Discrete transfer method (DTM) [25] is one of the popular methods for solving radiative transfer problems. For steady state problems, this method has been extensively used for pure radiation as well as radiation, conduction and/or convection heat transfer problems. However for the transient radiative heat transfer problems, applicability of this method has not been explored so far.

1.1 Present objectives

Following are the objectives in the present work.

- ➔ Analysis of transient radiative transfer equation subjected to short pulse laser at one boundary of a homogeneous 1-D planar medium. The medium is assumed absorbing, emitting/non-emitting and scattering.

- ➔ Applicability of finite volume method to transient radiative transfer problems.
- ➔ Variation of heat flux with respect to optical thickness.
- ➔ Variation if transmittance with respect to time.
- ➔ Validation of finite volume method with discrete transfer method(DTM)

1.2 REVIEW OF EXISTING METHODS

1.2.1 EXACT ANALYTICAL METHOD

Analytical solution of transient radiative heat transfer problem is a very difficult task and can only be obtained for a highly simplified and idealized situations such as a 1D plane parallel medium. Transient radiative transfer has got its wide application in the areas of atmospheric sciences, neutron transport, laser applications etc. Tan and Hsu have given an integral formulation for the case of an 1D plane parallel absorbing, non-emitting and scattering medium. The non-emission assumption is valid in many situations where short pulse laser or light source is directed into a cold medium. The non-emission assumption is also valid in the situation where temperature disturbance caused by the incident pulse in the material cannot be immediately revealed within the time scale of the radiative transport.

When a radiation beam travels through a participating medium, it is attenuated by absorption and by out scattering and at the same gains energy by emission as well as by in-scattering from other direction into the direction of travel. The resulting equation is shown in Eq.(1.2 .1)The equation for transient radiative transfer of the intensity in the direction *s* is given by

$$\frac{dI(z,\theta,t)}{ds} = \frac{DI(z,\theta,t)}{cDt} = -kI(z,\theta,t) + aI_b(z,\theta,t) + \frac{\sigma}{4\pi}\int_{\Omega'=4\pi} I(z,\theta,t)\phi(\theta',\theta)d\Omega' \tag{1.2.1}$$

or

$$\frac{\partial I(z,\theta,t)}{c\partial t} + \frac{\partial I(z,\theta,t)}{\partial s} = -kI(z,\theta,t) + aI_b(z,\theta,t) + \frac{\sigma}{4\pi}\int_{\Omega'=4\pi} I(z,\theta,t)\phi(\theta',\theta)d\Omega' \tag{1.2.2}$$

where D, represents the substantial derivative, a - the absorption coefficient, k and σ are scattering coefficients and I is the intensity inside the medium. Equation (1.2.1) is the Lagrangian viewpoint and Eq. (1.2.2) the Eulerian. It is found that the Lagrangian view point can simplify the analysis of the time-of-f1ight of a photon and allows for deduction of the domain of influence. Since in the Lagrangian viewpoint the observer is moving with the wave, the dependence upon sand t could not be separated from each other. The dependence is evident from the relationship of $dt = \frac{ds}{c}$. Therefore, only one independent variable (either t or s) is needed in the transient analysis. Once the distance ds through which the radiation travels along a certain path is given, the relative location s + ds and the time interval dt = ds/c (hence the instant t' = t + ds/ c) can be determined definitely.

In the figure (1.1 the layer width is Z_0 in the z direction. The transient radiative transfer equation can be written as (Lagrangian viewpoint)

$$\frac{dI(z,\theta,t)}{ds} = -kI(z,\theta,t) + \frac{\sigma}{4\pi}\int_{\Omega=4\pi} I(z,\theta',t)\phi(\theta',\theta)d\Omega \tag{1.2.3}$$

where ¢(θ', θ) is the scattering phase function. For isotropic scattering, ¢(θ', θ) = 1.
This leads to

$$\frac{dI(z,\theta,t)}{ds} = -kI(z,\theta,t) + \frac{\sigma}{4\pi}G(z,t) \tag{1.2.4}$$

where G(z, t) is the incident(or integrated) radiation defined as

$$G(z,t) = \int_{4\pi} I(z,\theta,t)d\Omega. \tag{1.2.5}$$

Using an integrating factor, Eq. (1.2.4) can be written as

$$d\left[I(z,\theta,t)e^{ks}\right] = \frac{\sigma}{4\pi}G(z,t)e^{ks}ds. \tag{1.2.6}$$

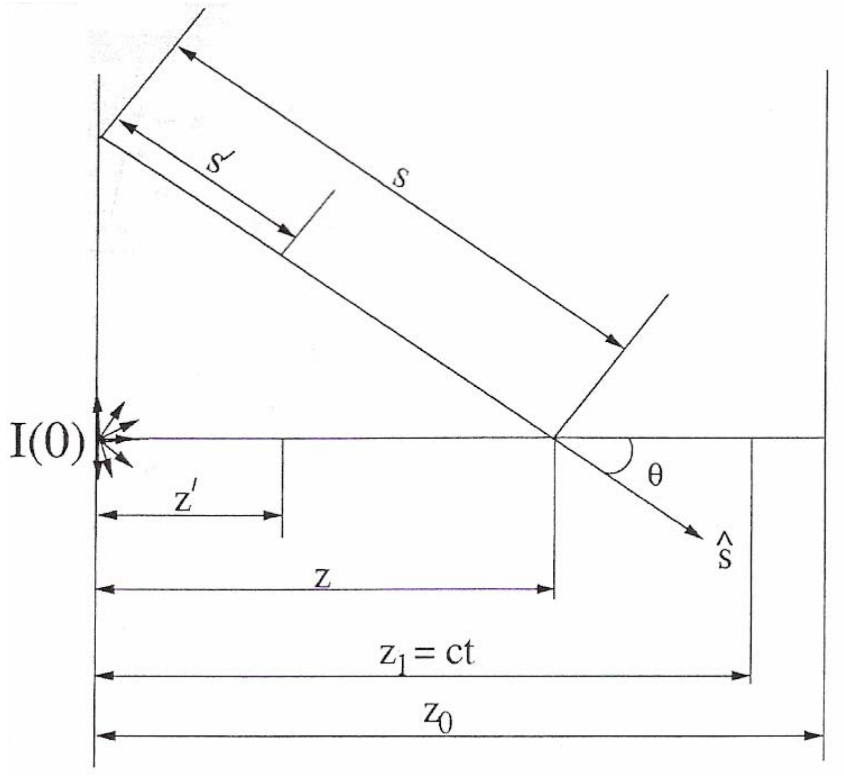


Figure 1.1: Geometry for analysis of exact analytical method.

Integrating Eq. (1.2.6) on both sides, one obtains

$$\int_{s'=0}^{s'=s} d\left[I(z',\theta,t')e^{ks'}\right] = \int_{s'=0}^{s'=s} \frac{\sigma}{4\pi} G(z',t')e^{ks'}\,ds' \qquad (1.2.7)$$

$$\left[I(z',\theta,t')e^{ks'}\right]_{s'=s} - \left[I(z',\theta,t')e^{ks'}\right]_{s'=0} = \int_{s'=0}^{s'=s} \frac{\sigma}{4\pi} G(z',t')e^{ks'}\,ds' \qquad (1.2.8)$$

When $s' = s$, the corresponding coordinate is $z' = z$ and $t' = t$, which are the location and the instant under consideration (Fig. 1.1). Since the time interval between $s' = 0$ and $s' = s$ is $\Delta t = s/c$, therefore, at $s' = 0$ the same photon can be described by $z' = 0$ and $t' = t - s/c$. Substituting the above relationships into the left hand side of Eq. (1.2.8) gives

$$I(z,\theta,t) = I(0,\theta,t - s/c)e^{-ks} + e^{-ks}\int_{s'=0}^{s'=s} \frac{\sigma}{4\pi} G(z',t')e^{ks'}\,ds'. \qquad (1.2.9)$$

In order to carryout the integration on the right side of Eq. (1.2.9), the dummy variables z' and t' of the integrated intensity G should be expressed in terms of the integrating variables s', which can easily be found. Therefore Eq. (1.2.9) becomes

$$I(z,\theta,t) = I(0,\theta,t - s/c)e^{-ks}$$

$$+\frac{1}{4\pi}\int_0^s \sigma(z')G(z-(s-s')\cos\theta, t-(s-s')/c)e^{-k(s-s')}ds' \qquad (1.2.10)$$

Physically, one can readily appreciate that the first term on the right-hand side of Eq. (1.2.10) is the contribution to the local intensity by the intensity entering the medium from the boundary at $s' = 0$, which decays exponentially due to extinction over the optical distance ks (Fig. 1.1). Since it takes the photon an amount of time s/c to travel from the boundary (at $s' =$ s), the boundary contribution can only come from the radiation entering the medium at the instant of $t' = t - s / c$. The integrand of the second term, $\sigma G(z',t')ds'/4\pi$, is the contribution from the local scattering at s', attenuated exponentially by extinction over the distance between the scattering point and the point under consideration, $s - s'$. Since the time interval between these two points is $\Delta t = (s-s')/c$, only the photons at and after the instant of $t' = t - (s - s')/c$, and in the position $z' >= z - (s - s') \cos \theta$ contribute to the radiative intensity at the location z at the instant t, $I(z, \theta, t)$. The integral sums all of the contributions over the entire scattering path from $s' = 0$ to $s' = s$. Eq.(1.10) shows that radiative intensity in a certain direction depends on the time history along the path in that direction. Furthermore, radiative intensity at location z, at time t and in direction e depends on the radiation emitted from the boundary at an earlier instant $(t - s / c)$ in the same direction and the entire time history of incident intensity along the path of travel, rather than the values at the same instant t. Thus, transient radiative transfer presents a strong time dependence, which in turn implies a path length dependence.

1.2.2. Collapsed Dimension Method

This method was proposed by [28] and its detailed information and development is given by Mishra [29]. This method seems to be promising in terms of accuracy and applicability to conjugate and complex geometries problem. In CDM, 3-D radiative information mapped into 2-D plane in terms of effective intensity and optical thickness coefficient (OTC). At a point in the system, all the actual intensities contained in the discrete plane are represented by an effective intensity. Since effective intensities are obtained only in 2-D plane, it is unlike actual intensities contained in dictate plane are represented by an effective intensity. Since effective intensities are obtained only in 2-D plane, it is unlike actual intensities where we need two angles one is polar and another is azimuthally angles, here in CDM, they are identified by only one angle, i.e., planar angle. Due to application of a single angle in effective intensity, mathematical expressions for all radiation parameters (radiative heat flux, incident radiation, divergence of radiative heat flux, boundary intensity) are different from other methods. Thus, absence of one angular dimension from the analysis makes formulation and solution procedure simpler. Collapsed Dimension Method (CDM) developed by Mishra [29] is employed.

1.2.3. Discreate Ordinate Method

This method was developed by Chandrasekhar [30] to astrophysical problems and later applied to neutron transport by Carlson and Lathrop. This method was initially used in radiative heat transfer by Hyde and Truelove. The DOM is based on a discrete representation of directional variation of radiative intensity. In this method angular intensity at any point in space can be represented by a discrete set intensities which can span over entire solid angle of 4π. Any quantity that involves the integral of intensity is estimated by an angular quadrature scheme. For instance, the irradiance is expressed as

$$\int_{4\pi} \mathrm{I}(\mathrm{r},\mathrm{w})\mathrm{dw} = \sum_{i=1}^{M} w_i I_i(r) \qquad (1.2.15)$$

Where w_i are the quadrature weights, M is the total number of discrete directions. The evaluation of radiant het amounts to the solution of the RTE in these specific directions as $M = N(N + 2)$.

This method was implemented by Khail and Truelove for cylindrical enclosures and by Fiveland for two and three dimensional rectangular enclosures with anisotropic scattering medium. The S_4 method is generally found slightly more accurate than the spherical harmonics P_3 method with better computational economy. The major drawback of this method is the presence of ray effects, i.e., inability to discretize intensity distribution to fully represent the actual continuous distribution especially when radiant energy conservation is not insisted upon. Consequently, radiation can not be ‘lost’ if it does not fall into one of the discrete ordinate directions where ray effects can not be totally eliminated. Another drawback of S_N method is to extend its application beyond Cartesian geometry . This is evidenced even in the simple cylindrical system where formulation is made complicated by additional restrictions entailed by the presence of curved surface.

1.2.4 Simple Harmonics (*P_N* - Approximation)

This method is a vehicle to obtain an approximate arbitrary high order (i.e., accuracy), by transforming the equation of transfer into a set of simultaneous partial differential equations. In this method radiative intensity field *I(r,s)* at location r within the medium as the value of scalar function on the surface of a sphere of unit radius, surrounding the point r. Any such function may be expressed in terms of a two-dimensional generalized series.

$$I(r,s) = \sum_{l=0}^{N} \quad \sum_{m=-l}^{l} I_l^m(r) y_l^m(s) \qquad (1.2.16)$$

Where $I_l^m(r)$, are space functions and the spherical harmonics $Y_l^m(s)$, are defined in terms of the associated Legendre polynomials $P_l^m(s)$. The factor N specifies the truncation polynomial series and the order of approximations. The general procedure to determine $I_l^m(r)$ is to substitute in Eq. (1.2.16) into RTE and integrate over 4π the product of the RTE and Y_l^m. This step essentially forces the assumed functional form of intensity to satisfy the RTE over the entire solid angle of 4π. The result is $(N + 1)^2$ coupled PDE for the $(N + 1)^2$ unknowns of I_l^m. All the coefficient I_l^m are replaced by the moments of intensity in these equations which are then solved to determine the radiant
heat fluxes. Because of strong mathematical basis, determination of radiative heat
transfer by this method is better than the simple flux method. The P_N method has been applied to one, two and three dimensional problems with good accuracy . Incorporation of antisotropic scattering has also been achieved in combustion system by Menğuc and by Viskanta and Verma and Menğuc . The main weakness of this method is its inflexibility. The order of approximation N, cannot be changed without a substantial derivation of the

governing equations. Actually, the P_3 approximation is considered the accurate one, because changing N from 3 to 5 requires complex derivations and high computational cost with marginal improvement in accuracy as quoted by Mengüç and Viskanta. Accurate results below 0.5 optical thickness and/or sharp intensity distributions have sharp angular variations. And, the result are sensitive to boundary condition, the table (1.1) shown above shows summary of various methods used in solving transient radiative transfer problem.

Method	*Summary and Remarks*
Exact	These solutions are obtained by solving RTE analytically. These are accurate byt can only obtained in highly idealized situations [22].
P_N	RTE is transformed into set of PDE's. Here angularly dependent intensity field is expressed in a series of approximations. It provide accurate results for simple as well as for complex geometries [37 – 42]
DOM	In this method, angular distribution of intensity, at any point in the enclosure is represented by a discrete set of intensities spanned over the entire solid angle of 4π. This is widely used in the field of astrophysics [33,34]. The major drawback is presence of ray effect and false scattering.
DTM	Method was proposed by Lockwood [27] and Shah [25]. This is a combination of zone method, MCM and flux method. This is numerically exact and geometrically flexible. It can easily be implemented on conjugate problems. The weakness is large number of rays needed for complex geometry.
CDM	This is a ray tracing method proposed by Mishra [29], actual intensities are mapped into 2-D plane and instead of two angles, i.e., polar and azimuthal only one angle, i.e., planar angle is used. This method is computationally less expensive.

Table 1.1: Summary of Various Radiative Heat Transfer Method

Chapter 2

FORMULATION

TRT Equation
FVM formulation
Dimensionless TRTE
Solution Procedure

Chapter 2

FORMULATION

2.1 TRANSIENT RADIATIVE TRANSPORT EQUATION

Radiation Transfer Phenomenon can take place in two ways.

1.Non Participating medium e.g. (Vacuum)

2.Participating medium e.g. (Glass, Wood, tissue)

Radiation energy passing through dA_1 and falling

(at a later time) on dA_2

$$= I_\lambda d\lambda \; dA_1 d\Omega_{1-2} = I_\lambda (S_1, t_1) d\lambda \frac{dA_1 dA_2}{(S_2 - S_1)^2}$$

$$t_2 = t_1 + \frac{S_1 - S_2}{C} \approx t_1$$

Radiation energy passing through dA_2 from dA_1

$$= I_\lambda (S_2, t_2) d\lambda dA_2 \frac{dA_1}{(S_2 - S_1)^2}$$

$$\therefore \; I_\lambda (S_1, t_1) d\lambda dA_1 \frac{dA_2}{(S_2 - S_1)^2} = I_\lambda (S_2, t_1) d\lambda dA_2 \frac{dA_1}{(S_2 - S_1)^2}$$

$$\Rightarrow I_\lambda (S_1) = I_\lambda (S_2)$$

$\Rightarrow I_\lambda (S) =$ Constant for surface radiation.

Radiation in Participating medium Absorption:-

$$dI_\lambda = - K_\lambda I_\lambda dS$$

$K_\lambda =$ absorption coefficient

$$\int \frac{dI_\lambda}{I_\lambda} = \int - K ds$$

$$I_\lambda = I_\lambda (0) e^{-\int_O^S kds}$$

$\int_O^S Kds =$ optical thickness of the medium

Dimensionless absorption coefficient

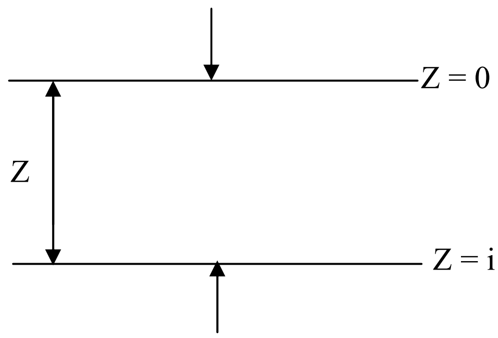
$$=Z_\lambda = \int_O^S K_\lambda ds = K\int_O^S ds = Ks$$

Emission-: Augmentation of intensity inside the medium

$$dI_\lambda = K_\lambda I_b d_s$$

I_b = Intensity of radiation of black body

$$= aI^4/\pi$$



Absorbing Emitting medium

$$dI_\lambda = -K_\lambda I_\lambda ds + K_\lambda I_b ds$$

$$\Rightarrow \frac{dI_\lambda}{ds} = -K_\lambda I_\lambda + K_\lambda I_b$$

$$\Rightarrow I_\lambda(Z) = I_\lambda(0)e^{-Z} + I_b\left(1-e^{-Z}\right)$$

Scattering-: it may be of two types

i) OUTSCATTERING: - involves/results in the attenuation of radiation energy.
$dI_\lambda = -a_{s\lambda}I_\lambda ds$ where $a_{s\lambda}$ is scattering coefficient.

ii) INSCATTERING: - augmentation of radiation energy.

Radiation energy falling onto dA within the solid angle $d\Omega_i$. $= I_\lambda d\lambda dA\mathrm{Cos}\theta d\Omega_i$

Radiation energy after traveling through distance ds

$= (as_\lambda I_\lambda\, d\lambda dA\mathrm{Cos}\theta d\Omega_i)ds/\mathrm{Cos}\theta.$

Fraction. of radiation energy traveling along $\hat{s}$

$$= (a_i I_\lambda dx dA d\Omega_i ds\ )\int_0^{4\pi}\phi\frac{d\Omega}{4\pi}$$

$$\phi = Phase\quad function\ (isotropic)$$

.

INSCATTERING

$$\frac{dI_x}{4\pi} = \frac{as\lambda}{4\pi}\int_{4\pi} I_\lambda{}'\phi\left(\hat{S}_l \rightarrow \hat{S}\right)d\Omega_i$$

Radiative Energy Balance:-

$$I(s+ds, s,\ t+dt) - I\ s, \hat{s}, t)$$

$$kI_b ds - kIds - a_s Ids + \left[\frac{a_s}{4\pi}\int_{4\pi} I^{'}\left(s^{'}, t+dt\right)\phi_n d\Omega_i\right]ds$$

$$I\left(s+ds, \hat{s}, t+dt\right) - I\left(s, \hat{s}, t\right) + ds\frac{dI}{ds} + dt\frac{dI}{dt} + \ldots\ldots\ldots\ \ Taylors\ Equation$$

$$\Rightarrow I\left(s, +ds, s, \hat{t}+dt\right) - I\left(s, \hat{s}, t\right) = ds\frac{\partial I}{\partial s} + dt\frac{\partial I}{\partial t}$$

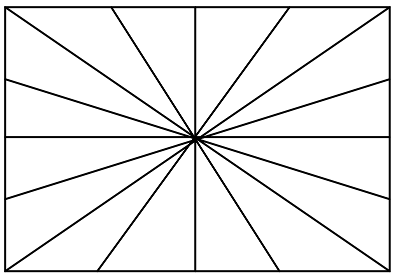
$$\Rightarrow \frac{\partial I}{\partial s} + \frac{dt}{dt}\frac{\partial I}{\partial t} = kI_b - kI - a_s I + \left[\frac{as}{4\pi}\int_{4\pi} I^{'}\left(s^{'},\ t+dt\right), \phi_n d\Omega_i\right]$$

$$\Rightarrow \frac{1}{C}\frac{\partial I}{\partial t} + \frac{\partial I}{\partial s} = kI_b - kI - a_s I + \left[\frac{as}{4\pi}\int_{4\pi} I^{'}\left(s^{'},\ t+dt\right), \phi_n d\Omega_i\right]$$

This is the basic TRTE or Transient Radiadtion Transfer Equation.

2.2 Formation of TRTE using Finite Volume method

$$\frac{1}{C}\frac{dI}{dt} + \frac{dI}{ds} = -\beta I + kI_b + \frac{as}{4\pi}\int_{4\pi} I^{'}\left(r_1\ \hat{s}, t\right)\phi\left(\hat{s} - s\right)d\Omega \qquad 1$$



Control Angle

$$\frac{1}{C}\frac{dI^l}{dt} + \frac{dI^l}{ds} = -\beta I^l + kI_b + \frac{as}{4\pi}I^l\phi^{ll}\Delta\Omega^l + \frac{as}{4\pi}\sum_{\substack{l^{'}=1 \\ l\ \neq 1}}^{m} I^{l^{'}}\left(r, \hat{s}, t\right)\phi^{l^{'}l}\Delta\Omega^{l^{'}}$$

Φ^{ll} = Phase function from the direction l to the direction l.

$\Phi^{l'l}$ = Phase function from direction $l^{'}$ to the direction l where l' = 1 to M and $l^{'} \neq$ M

$$\Rightarrow \frac{1}{C}\frac{dI^{l}}{dt}+\frac{dI^{l}}{ds}=\left(-\beta+\frac{as}{4\pi}\quad \phi^{ll}\Delta\Omega^{l}\quad\right)I^{l}+kI_{b}+\frac{as}{4\pi}\sum_{l'=1}^{m}I^{l}\phi^{l'l}\Delta\Omega^{\quad l'}$$

$$\Rightarrow \frac{1}{C}\frac{\partial I^{'}}{\partial t}+\frac{\partial I^{l}}{\partial s}=-\beta_{m}I^{l}+S_{m} \qquad (2)$$

β_m = modified extinction Coefficient.

S_m = Modified Source function.

$$\beta_m-\beta+\frac{as}{4\pi}\phi^{ll}\ +\Delta\Omega^{l}$$

$$S_{m}=kI_{b}+\frac{as}{4\pi}\sum_{\substack{l'=1\\ l'\neq l}}^{m}I^{l}\phi^{l'l}\Delta\Omega^{l'}$$

Integrating equation (2) over a Control Volume, over a Control angle and over a time interval t to t+Δt

$$\int_{t}^{t+\Delta t}\int_{\Delta\Omega^{'}}\int_{\Delta v}\frac{1}{C}\frac{\partial I^{l}}{\partial t}dvd\,\Omega\ \ dt+\int_{t}^{t+\Delta t}\int_{\Delta\Omega^{l}}\int_{\Delta v}\frac{\partial I^{l}}{\partial t}dvd\,\Omega\ \ dt\,.$$

$$=\int_{t}^{t+\Delta t}\int_{\Delta\Omega^{l}}\int_{\Delta v}-\beta_{m}I_{l}+S_{m}dvd\,\Omega\ \ dt$$

LHS

$$\int_{t}^{t+\Delta t}\int_{\Delta\Omega^{l}}\int_{\Delta v}\frac{1}{C}\frac{dI^{l}}{dt}dvd\Omega dt=\frac{1}{C}\Delta v\ \ \Delta\Omega^{l}\left[I_{p}{}^{l}(t+\Delta t)-I_{p}{}^{l}(t)\right]$$

$$\int_{t}^{t+\Delta t}\int_{\Delta v'}\int_{\Delta\Omega'}\frac{\partial I^l}{\partial t}dvd\Omega dt=\int_{t}^{t+dt}\left[\sum_{i=nb}I_i^l A_i\int_{\Delta\Omega'}\left(\hat{s}\,\hat{n}_i\right)d\Omega^l\right]dt \qquad GaussTheorem$$

$$=\left[\sum_{i=nb}I_i^l(t+\Delta t)A_i\int_{\Delta\Omega_i}\left(\hat{s}^l,\hat{n},\right)d\Omega^l\right]\Delta t$$

For a 2D Control volume.

$$\left[\sum_{i=nb}I_i^l(t+\Delta t)A_i\int_{\Delta\Omega_i}\left(\hat{s}^{nl},\hat{n}_i\right)d\Omega'\right]d\Omega^l$$

$$=A_eD^l{}_{ce}I_p^l+A_wD_{cw}I_w^l+A_nD_{cn}^lI_p^l+A_sD_{cs}^lI_s^l$$

$$Where \qquad D_{ce}{}^l=\int_{\Delta_i}\left(\hat{s}^l\,\hat{l}_x\right)d\Omega$$

$$D_{ce}{}^l=\int_{\Delta_i}\left(\hat{s}^l\,\hat{l}_y\right)d\Omega$$

$$But \qquad D_{cw}^l=-D_{ce}^l \quad and \quad D_{cs}^l=-D_{cl}^l$$

$$Here\ \hat{l}_x\ and\ \hat{l}_y\ are\ unit\ vectors\ along\ x\ and\ y\ directions$$

$A_e = A_w = \Delta y$, $A_n = A_s = \Delta x$.

For the given orientation in figure (2) LHS (2) can be written by.

$$\left[\sum_{i=nb}I_i^l(t+\Delta t)A_i\int_{\Delta\Omega_i}\left(\hat{s}^{nl},\hat{n}_i\right)d\Omega^l\right]$$

$$=A_eD^l{}_{ce}I_p^l(t+\Delta t)$$
$$+A_wD_{cw}I_w^l(t+\Delta t)$$
$$+A_nD_{cn}^lI_p^l(t+\Delta t)$$
$$+A_sD_{cs}^lI_s^l(t+\Delta t)$$

R.H.S:-

$$= \int_{t}^{t+\Delta t} \int_{\Delta v} \int_{\Delta\Omega^l} \left(-\beta_m^l I_p^l + S_m^l \right) dv d\Omega_l^o dt$$

$$= \left[-\beta_m^l I_p^l (t+\Delta t) + S_m^l (t+\Delta t)\right] \Delta v \Delta\Omega^l \Delta t$$

Hence equation -3 for a typical control volume in a given orientation as shown in figure is given by

$$\frac{1}{C}\Delta v \Delta\Omega^l \left[I_p^l(t+\Delta t) - I_p^l(t)\right] + \begin{bmatrix} A_e D_{ce}^l I_p^l(t+\Delta t) + A_w D_{cw}^l I_w^l(t+\Delta t) \\ + A_n D_{cn}^l I_p^l(t+\Delta t) + A_s D_{cs}^l I_s^l(t+\Delta t) \end{bmatrix} \Delta t$$

$$= \left[-\beta_m^l I_p^l(t+\Delta t) + S_m^l(t+\Delta t)\right] \Delta v \Delta\Omega^l \Delta t$$

$$\Rightarrow \left[A_e D_{ce}^l + A_n D_{cn}^l + \beta_m^l \Delta v \Delta\Omega^l + \frac{\Delta v \Delta\Omega^l}{c\Delta t} \right] I_p^l(t+\Delta t)$$

$$= A_w D_{cw}^l I_w^l(t+\Delta t) + A_n D_{cn}^l I_p^l(t+\Delta t) + S_m^l(t+\Delta t) \Delta v \Delta\Omega^l + \frac{\Delta v \Delta\Omega^l}{c\Delta t} I_p^l(t).$$

$$\Rightarrow a_p^l I_p^l(t+\Delta t) = a_w^l I_w^l(t+\Delta t) + a_s^l I_s^l(t+\Delta t) + b^l$$

$$a_p^l = A_e D_{ce}^l + A_n D_{cn}^l + \beta_m^l \Delta\Omega^l \Delta v + \frac{\Delta v \Delta\Omega^l}{c\Delta t}$$

$$a_w^l = A_w D_{cw}^l$$

$$a_s^l = A_s D_{cs}^l$$

$$b^l = S_m^l(t+\Delta t) \Delta v \Delta\Omega^l + \frac{\Delta v \Delta\Omega^l}{c\Delta t} I_p^l(t)$$

$$RNUM = a_w^l I_w^l(t+\Delta t) + \frac{\Delta v \Delta\Omega^l}{c\Delta t} I_p^l(t) + a_s^l I_s^l(t+\Delta t) + b^l$$

$$DENO = a_p^l$$

In a 2-dimensional control volume

$I_s^1 \neq I_w^1$ and $I_n = I_e = I_p$

2.3 Non dimensional TRTE equation

The TRTE equation using finite volume formulation is

$$\frac{1}{c}\frac{dI^{l}}{dt} + \frac{dI^{l}}{ds} = -\beta_m I^{l} + S_m$$

Where β_m = modified extinction coefficient

S_m = modified source function

To make the equation non-dimensional, we divide the equation by β

$$\frac{1}{\beta c}\frac{dI^{l}}{dt} + \frac{1}{\beta}\frac{dI^{l}}{ds} = -\frac{\beta_m}{\beta} I^{l} + \frac{S_m}{\beta}$$

β cdt = dimensionless time =dt*

β ds=dτ=change in optical thickness

$$\beta_m \frac{\beta_m}{\beta} = -\frac{\beta}{\beta} + \frac{as}{4\pi\beta}\phi^{ll} + \Delta\Omega^{l}$$

$$\beta^* = -1 + \frac{\omega}{4\pi}\phi^{ll'} + \Delta\Omega^{l}$$

Where ω=scattering albedo

$$\frac{S_m}{\beta} = \frac{kI_b}{\beta} + \frac{as}{4\pi\beta}\sum_{\substack{l'=1\\ i\neq 1}}^{m} I^{l}\phi^{l'l}\Delta\Omega^{\ l'}$$

$$\Rightarrow \frac{S_m}{\beta} = \frac{\beta - a_s}{\beta} + \frac{as}{4\pi}\sum_{\substack{l'=1\\ i\neq 1}}^{m} I^{l}\phi^{l'l}\Delta\Omega^{\ l'}$$

$$\Rightarrow S_m^{\ *} = (1-\omega)I_b + \frac{\omega}{4\pi}\sum_{\substack{l'=1\\ i\neq 1}}^{m} I^{l}\phi^{l'l}\Delta\Omega^{\ l'}$$

So the dimensionless equation is

$$\frac{dI^{\ l}}{dt^*} + \frac{dI^{\ l}}{d\tau} = -\beta_m^{\ *} I^{l} + S_m^{\ *}$$

Integrating equation (2) over a Control Volume, over a Control angle and over a time interval t *to t*+Δt*

$$\int_{t^*}^{t^*+\Delta t^*}\int_{\Delta\Omega'}\int_{\Delta v}\frac{1}{C}\frac{\partial I^l}{\partial t^*}dvd\,\Omega\,dt^* + \int_{t^*}^{t^*+\Delta t^*}\int_{\Delta\Omega'}\int_{\Delta v}\frac{\partial I^l}{\partial t}dvd\,\Omega\,dt^*$$
$$= \int_{t^*}^{t^*+\Delta t^*}\int_{\Delta\Omega^l}\int_{\Delta v}(-\beta_m{}^*I_l + S_m{}^*)dvd\,\Omega\;\;dt^*$$

LHS

1st term

$$\int_{t^*}^{t^*+\Delta t^*}\int_{\Delta\Omega'}\int_{\Delta v}\frac{dI^l}{dt}dvd\Omega dt = \frac{1}{\beta C}\Delta v\;\;\Delta\Omega^l\left[I_p{}^l(t^*+\Delta t^*) - I_p{}^l(t^*)\right]$$

2nd term

$$\int_{t^*}^{t^*+\Delta t^*}\int_{\Delta v'}\int_{\Delta\Omega'}\frac{\partial I^l}{\partial\tau}dvd\Omega dt = \int_{t^*}^{t^*+dt^*}\left[\sum_{i=nb}I_i^l\Delta_i\int_{\Delta\Omega'}\left(\hat{s}\,\hat{n}_i\right)d\Omega^l\right]dt$$
$$= \left[\sum_{i=nb}I_i^l(t^*+\Delta t^*)A_i\int_{\Delta\Omega_i}\left(\hat{s}^l,\hat{n},\right)d\Omega'\right]\Delta t^*\,(Green's Theorem)$$

So LHS should be written as

$$\left[\sum_{i=nb}I_i^l(t^*+\Delta t^*)A_i\int_{\Delta\Omega_i}\left(\hat{s}^{nl},\hat{n}_i\right)d\Omega^l\right]$$
$$= A_e D^l{}_{ce}I_p^l(t^*+\Delta t^*) + A_w D_{cw}I_w^l(t^*+\Delta t^*) + A_n D_{cn}^l I_p^l(t^*+\Delta t^*) + A_s D_{cs}^l I_s(t^*+\Delta t^*)$$

R.H.S:-

$$= \int_{t^*}^{t^*+\Delta t^*} \int_{\Delta v} \int_{\Delta\Omega^l} \left(-\beta_m^{l^*} I_p^l + S_m^{l^*}\right) dv d\Omega_l^o dt^*$$

$$= \left[-\beta^{*l}_m I_p^l \left(t^* + \Delta t^*\right) + S_m^{l*}\left(t^* + \Delta t^*\right)\right] \Delta v \Delta\Omega^l \Delta t^*$$

Hence equation -3 for a typical control volume in a given orientation as shown in figure is given by

$$\frac{1}{C\beta} \Delta v \Delta\Omega^l \left[I_p^l\left(t^* + \Delta t^*\right) - I_p^l\left(t^*\right)\right] + \begin{bmatrix} A_e D_{ce}^l I_p^l \left(t^* + \Delta t^*\right) + A_w D_{cw}^l I_w^l \left(t^* + \Delta t^*\right) \\ + A_n D_{cn}^l I_p^l\left(t^* + \Delta t^*\right) + A_s D_{cs}^l I_s^l \left(t^* + \Delta t^*\right) \end{bmatrix} \Delta t^*$$

$$= \left[-\beta^{l*}_m I_p^l\left(t^* + \Delta t^*\right) + S_m^{*l}\left(t^* + \Delta t^*\right)\right] \Delta v \Delta\Omega^l \Delta t^*$$

$$\Rightarrow \left[A_e D_{ce}^l + A_n D_{cn}^l + \beta^{l*}_m \Delta v \Delta\Omega^l + \frac{\Delta v \Delta\Omega^l}{c\Delta t}\right] I_p^l\left(t^* + \Delta t^*\right)$$

$$= A_w D_{cw}^l I_w^l\left(t^* + \Delta t^*\right) + A_n D_{cn}^l I_p^l\left(t^* + \Delta t^*\right) + S_m^{l*}\left(t^* + \Delta t^*\right) \Delta v \Delta\Omega^l + \frac{\Delta v \Delta\Omega^l}{c\Delta t} I_p^l\left(t^*\right).$$

$$\Rightarrow a_p^l I_p^l\left(t^* + \Delta t^*\right) = a_w^l I_w^l\left(t^* + \Delta t^*\right) + a_s^l I_s^l\left(t^* + \Delta t^*\right) + b^l$$

where $a_p^{l,}$ $a_w^{l,}$ $a_s^{l,}$ and b^l can be found out by equating the coefficients.

2.4 SOLUTION PROCEDURE

1. Advance the time step ***t*** **= t +** $\boldsymbol{\Delta}$**t**
2. Guess the intensity field in all the interior grids in the computational domain.
3. Start the calculation from one of the boundaries in the current time step.
4. Calculate the modified source function and modified extinction coefficient.
5. Compute Ip based on the guessed values.
6. Approximate the values of 5p at the boundaries using immediate neighbor values.
7. Calculate the maximum error from all the interior grid points.

   Guess Ip_n - Ip
8. Check (max error < tolerance limit)

   If no

   guess Ip = Ip

Go to step (3)

If yes

Check (time required = time step reached)

If yes stop.

If no

repeat step 1-7

Chapter 3

RESULTS AND DISCUSSIONS

Chapter 3

RESULTS AND DISCUSSIONS

First the grid independent tests were performed to check the values of number of control volumes, number of control angles and the incremental time at which the result did not change from their neighboring values. In the following sections, the results of the time-varying non-dimensional transmissivity and reflectivity obtained from the FVM are presented for a particular value of optical thickness T for a scattering medium (scattering albedo ω = 0.998). For pulse LASER irradiation normal to the top boundary (μ_0=1), all these results are presented for the non-dimensional pulse width βct=1.

In figure-3.1, the variation of dimensionless heat flux (ψ) with optical thickness (T) for normal collimated irradiation using different control volume sizes is plotted. It is found that the results did not change for number of control volumes greater than 40.

In figure-3.2, the variation of dimensionless heat flux (ψ) with optical thickness (T) for normal collimated irradiation using different control angle sizes is plotted taking number of control volumes as 40. It is found that the results did not change for number of control angles greater than 30.

In figure-3.3, the variation of dimensionless heat flux (ψ) with optical thickness (T) for normal collimated irradiation using different time step increments is plotted with number of control volumes as 40 and number of control angles as 30. it is found that the results did not vary for time step increment less than 0.01.

In figure-3.4 the variation of dimensionless heat flux (ψ) with optical depth (Y) at different times are plotted. In this case albedo=1, the number of control volumes CV=40, number of control angles CA=30. dimensionless time increment DT=0.01. from this plot we find that at time t*=8, the non-dimensional heat flux becomes constant throughout the optical thickness and thus the steady state is achieved. The divergence of flux is zero.

In figure-3.5 the transient characteristics is studied. It shows that in transient radiation analysis, radiation takes some finite time to travel the optical depth of the medium. The greater the optical depth of the medium, the more time is taken by the radiation to travel from one boundary to the other. So the transmittance signals are found available only after the time radiation has reached the other boundary. However as found from figure-3.6, the reflected signal is available as soon as the boundary is subjected to the pulsed-laser source.

FIGURES

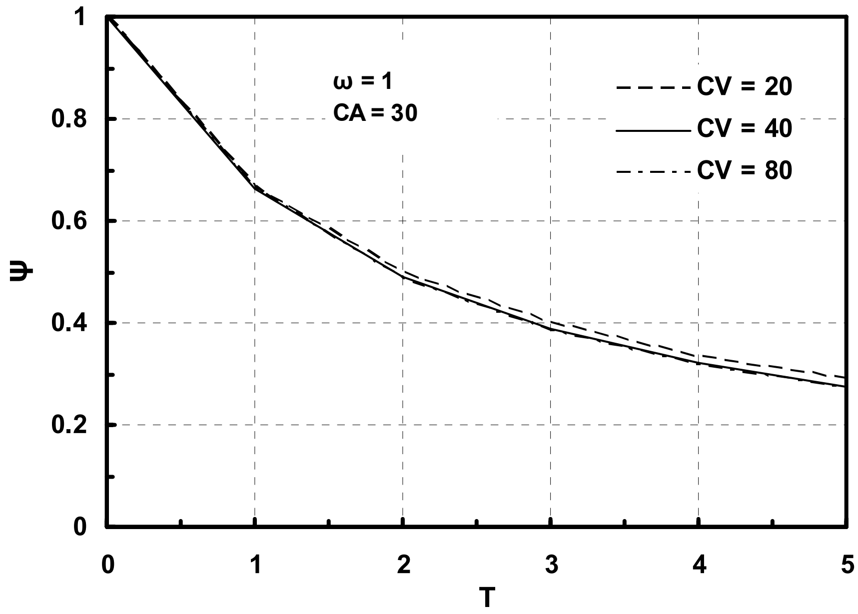


Figure-3.1

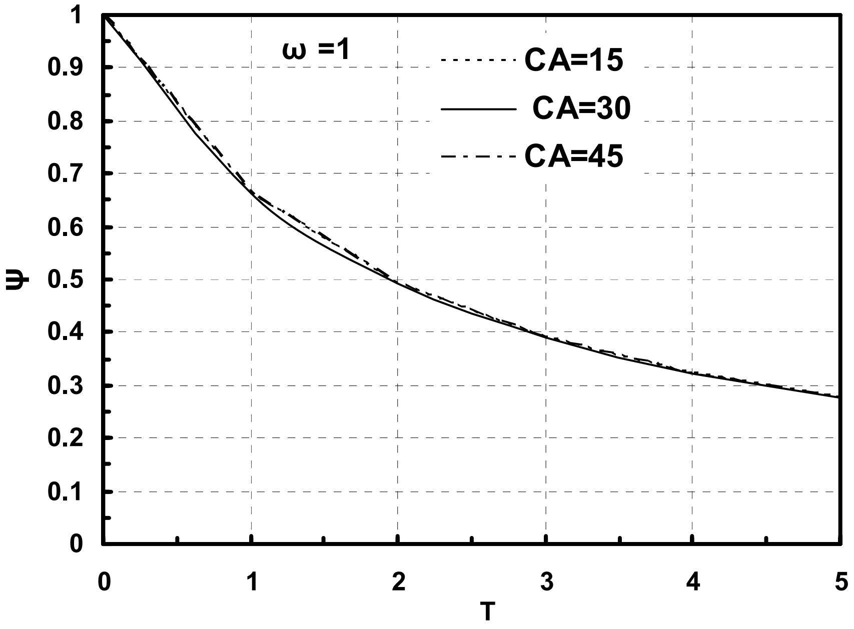


Figure-3.2

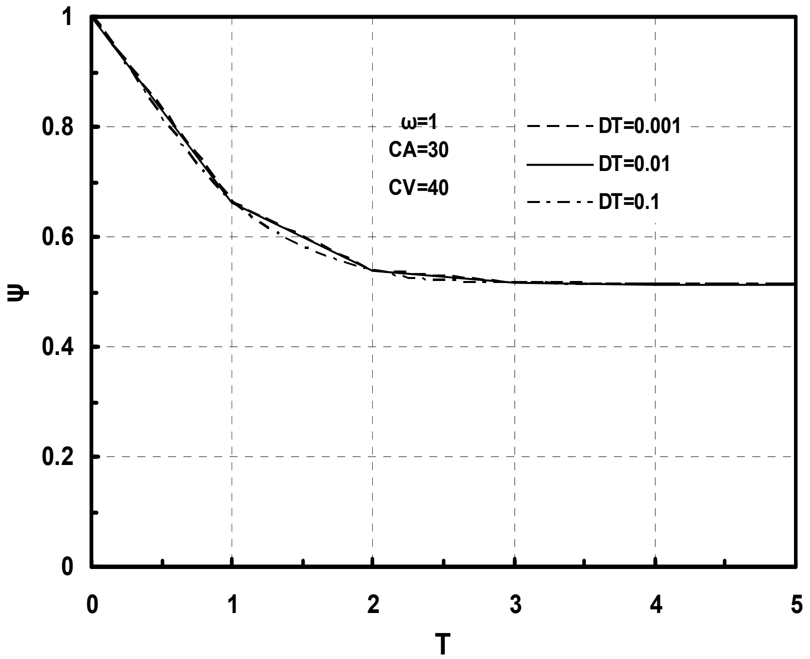


Figure-3.3

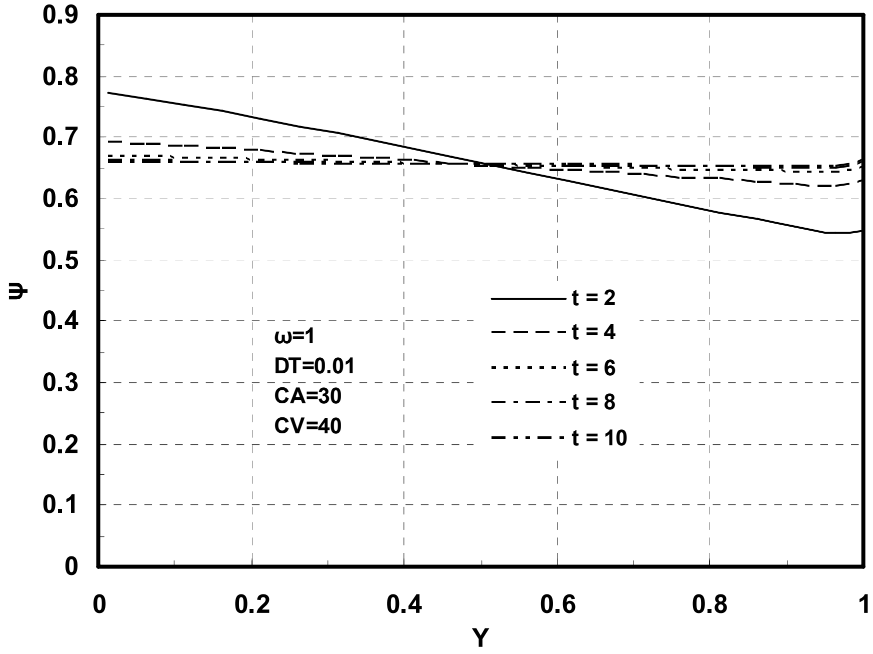


Figure-3.4

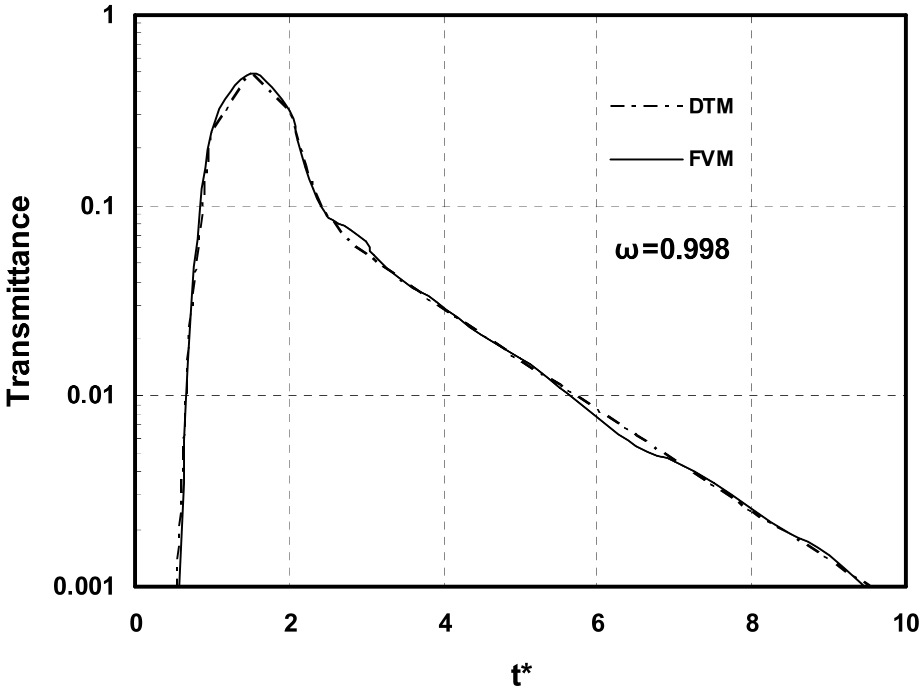


Figure-3.5

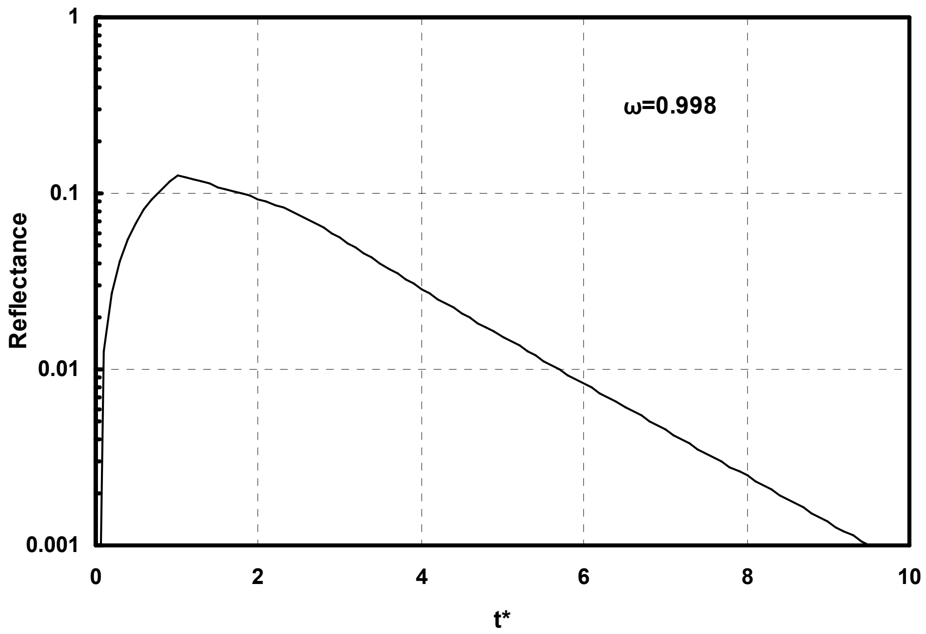


Figure-3.6

Conclusion

Application of the finite volume method has been made, to solve transient radiative transport problems in a participating medium. The formulation presented has been validated by solving transient radiative transfer problems in a one-dimensional planar absorbing and scattering medium, one boundary of which is subjected to a short-pulse laser and the other boundary of which is cold. Effects of optical thickness on transmittance and reflectance have been studied. For some sample cases, results have been compared with those available in the literature. The finite volume method has been found to work well for the transient radiative transport problems.

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