EFFECTS OF ADDENDUM MODIFICATION ON ROOT STRESS IN INVOLUTE SPUR GEARS

A THESIS SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

Bachelor of Technology
in
Mechanical Engineering

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and
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CERTIFICATE

This is to certify that the thesis entitled, **"Effects of Addendum modification on root stress in involute spur gears"** submitted by Ms Shweta Nayak and Ms Swetaleena Mishra, Roll No-10303027 and 10303075, in partial fulfillment of the requirements for the award of Bachelor of technology Degree in Mechanical Engineering at the National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by them under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University /Institute for the award of any Degree or Diploma.

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ACKNOWLEDGEMENT

We express our sincere gratitude to Prof. S. K. Acharya, Professor, Department of Mechanical Engineering, N. I. T. Rourkela. who has been a constant source of inspiration, encouragement and motivation and also for the constant advice through discussion for the successful progress of this project work. We thank him for the help he extended in doing our project work.

April, 2007

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ABSTRACT

A study into the effects of addendum modification on root stress in involute spur gears with various pressure angles are presented in this project work. The range of addendum modification co-efficient is taken from negative value to positive value through zero by considering both the upper limit(peaking limit) and the lower limit (undercutting limit). The root stress factor is found out for various loading positions. The variation of root stress factor with addendum modification co-efficient is shown when only the driving gear is modified.

A study into the effects of addendum modification on root stress using mathematical formulation as well as finite element analysis when both the driver and follower are modified at the same time also for different gear ratios.

The value of root stress factor decreases with an increasing addendum modification coefficient when only the driver is modified. The root stress factor also decreases when pressure angle is increased. The root stress factor is further decreased when both the driver and follower are modified at the same time. The root stress factor decreases further as the gear ratio increases.

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NOMENCLATURE

Sl. No.	Description	Symbol	Unit
1	Pitch circle radius of gear	R	mm
2	Base circle radius of gear	R	mm
3	Base circle radius of driver	R_{b1}	mm
4	Base circle radius of follower	R_{b2}	mm
5	Tip circle radius of driver	R_{a1}	mm
6	Tip circle radius of follower	R_{a2}	mm
7	Tip circle radius at pointed tip	R_2	mm
8	Radius of curvature of fillet curve	R_c	mm
9	Edge radius of generating rack	R_o	mm
10	Module of gear	m	mm
11	Circular pitch of gear	P	mm
12	Base pitch of gear	P_b	mm
13	Normal pressure angle at pitch circle	α_1	mm
14	Operating pressure angle	α_b	mm
15	Pressure angle at the tip	α_k	mm
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18	Addendum modification coefficient of follower	X_2	
19	Addendum modification coefficient of driver	X_3	

20	No. of teeth on a gear	z	
21	No. of teeth on a driver	Z_1	
22	No. of teeth on follower	Z_2	
23	Distance from loading point to the critical section	L	mm
24	Distance from loading point to the pitch circle	L_a	mm
25	Distance from loading point to the critical section	L_d	mm
26	Distance from loading point to the tip along the tooth profile	L_p	mm
27	Distance from center line of gear tooth to foot of perpendicular from center of rounded corner of rack tooth on pitch circle	L_o	mm
28	Distance from pitch line to center of rounded corner of rack tooth	H_o	mm
29	Clearance at bottom of tooth space	C_k	mm
30	Face width of the gear	b	mm
31	Distance from loading point to the center line of the gear tooth	y	mm
32	Angle around center of gear	β	radian
33	Rolling angle of rack	γ	radian
34	Angle between the line connecting loading point with center of gear and center line of gear tooth	K	radian

35	Angle between direction of loading and center line of tooth	P	radian
36	Angle between center line of gear tooth and tangent to the fillet	θ	degree
37	Tooth thickness at pitch circle of uncorrected gear	t	mm
38	Tooth thickness at pitch circle of corrected gear	t_1	mm
39	Tooth thickness at critical section of corrected gear	t_2	mm
40	Root stress factor	A	mm^{-1}
41	Root stress	σ_R	Kg/mm^2

Chapter 1

INTRODUCTION

1. INTRODUCTION

The increasing demand for high tooth strength and high load carrying capacity of gears leads to various methods of improvements. One of the major method available till now is "Profile Shift". In gear technology it is known as "Addendum Modification". The amount by which the addendum is increased or decreased is known as "Addendum Modification". The aim of addendum modification is to avoid interference. Previously various methods used to avoid interference were:

- Undercutting at the root.
- Making the mating gear tooth stub.
- Using a minimum number of teeth in a gear for a certain pressure angle.

But undercutting weakens the tooth strength severely and there may be the situation where a smaller number of teeth in a gear is to be adopted. But addendum modification avoids all these difficulties. It is also known that profile-shifted gears as compared to standard gears, offer a lot of advantages. The load carrying capacity of the gears can be greatly improved without any appreciable change in gear dimensions by adopting addendum modification.

Now-a-days profile shifted gears are more often due to its reduced vibration and reduced noise property. All these facilities can be achieved without using any special cutters.
Spur gears with a pressure angle of 20 degrees are usually used as power transmitting gears. But the need for higher load carrying capacity gives way for the selection of higher pressure angles. Gears with higher pressure angles are often used in aircraft applications.

Root stress measures the strength of the gear tooth. So if root stress is more, the gear tooth is weaken and when root stress is less, the gear tooth becomes stronger. In this project work the effects of addendum modification on root stress are investigated. The effects on root stress are investigated by both increasing and decreasing the height of addendum. In order to calculate the root stress at the root fillet of the gear tooth, a formula is adopted as proposed by Japanese author “Aida and Terauchi”. The dimensions required to evaluate the root stress are derived theoretically, assuming that gear tooth is being cut by a basic rack cutter.

In this project report, influence of modification of addendum of involute spur gear tooth on its root stress has been studied for different gear ratios and pressure angles. This work has been carried out in different stages. In the first stage only the driver is modified. In the second stage both the driver and the follower are modified and thirdly both the driver and the follower are modified with different gear ratios. Gear ratio has been varied from 0.33 to 3.0 while pressure angles have been taken as 14.5,20,23, and 27. The value of module is taken as 4 and number of teeth on driver is taken as 26.

In the following modules the detailed literature survey, formulation and numerical of the problem, finite element analysis of the problem and discussion of the results have been dealt with.

1.1 Addendum modification*:*

Interference is a main problem in gears having a small number of teeth. So in order to avoid interference in gears “undercutting” is done. But undercutting weakens the tooth strength. But there are situation at which gears will have to work with a small number of teeth which is less than the stipulated minimum number of teeth required to avoid undercutting. In gear technology it is known as Addendum Modification”.

Addendum modification is done by increasing or decreasing the height of the addendum. The amount by which the addendum is increased or decreased is known as “Addendum Modification”. It is customary to express the amount of modification in terms of module. Hence the modified amount now can be defined as a product of module (M) and a non-dimensional factor(X). This non-dimensional factor (X) is known as “Addendum Modification Coefficient” or “Correction Factor’.

In order to avoid interference while generating the pinion or gear, the cutting arrangement is done in such a way that the cutting tip of the rack-type cutter just touches the point of tangency when the cutting action begins. To affect this, cutter is withdrawn by a specified distance so that the addendum line of the rack just passes through the interference that is the point of tangency. Such a corrected tooth is shown in fig.

When a standard unconnected gear is cut by a rack, the pitch line of the rack is tangent to the pitch circle of the gear at the pitch point. But when addendum modification is done, the reference line of the rack is shifted away by that certain distance that is to be modified.

Addendum modification can be done in both ways, i.e. both positive and negative correction the reference can be done. For positive correction the reference line of the rack is shifted away by a certain distance from the gear centre and this type of gear is known as "S-plus Gear". For negative correction, the reference line of the rack moves towards the gear centre by a certain amount and this type of gear is called "S-Minus" gear. When both the pinion and gear are modified then that type of gearing is known as "S-gearing". The net amount of correction may either be positive or negative. But usually it is made positive to take the advantage of the beneficial effects of positive correction. Now when the net amount of correction becomes zero, then that type of gearing is known as "So-Gearing". In this type of gearing both pinion and gear get the same amount of modification but opposite in sign. Usually the pinion gets the positive value where as the gear gets the negative value. So-Gearing is generally meant where the reduction ratio is large. So-Gearing is not recommended for small reduction ratios as it tends to weaken the teeth of the gear. The So-Gearing also sometimes recommended where for certain specific reasons, the normal tooth-thickness of the gear pair on the specific sliding velocities between the meshing teeth flanks are to be changed.

1.2 ROOT STRESS:

The stress acting at the root acting at the root of the tooth is known as "Root stress". The gear tooth in general is assumed to be a cantilever beam subjected to an end load which is equal to the tangential tooth load. Hence the dangerous section on the critical section becomes the root area where the first sign of damage will take place. Hence he root stress is directly proportional to the root area. is directly proportional to the root area. Now the root area is product of the face-width of the gear and the tooth thickness at the critical section. Now for a particular gear blank the face width is constant. Hence it all depends upon the tooth thickness at the root. So as the tooth thickness at the root increases, the root stress decreases and as the tooth thickness decreases the root stress increases. In a standard gear when number of teeth required to avoid interference, then undercutting is provided. Now due to undercutting, the tooth thickness at the root of the gear decreases. So the root stress becomes higher. The root stress defines the strength of the gear, hence undercutting decreases the strength of the gear tooth.

1.3 INTERFERENCE:

It is well known that the involute curve begins at the base circle and extend outwards to form the gear tooth profile. So the portion of the tooth profile between the base circle and the root circle does not have involute curve. We know that the line of action of the two inter-meshing gears is tangent to the two base circles. The two points of tangency represents the two extreme limiting points of the length of action. These two points are called "**Interference Points**".

It is known to have and maintain conjugate action, the mating teeth profiles of the gear pair must consist of involute curves when of course involute curves are used as teeth profiles. Any meshing outside of the involute portion will result in non-conjugate action. Now it may also happen that the mating teeth are of such proportion that the beginning or the end of contact or both occur outside of the interference points on the path of contact. Then the involute portion of one gear will mate with the non-involute curve of the other gear. In this case the flank of the tooth of the driver is forced into contact with the tip of the tooth of the driven gear. It can be seen that the tip of the driver comes in contact below the base circle of the driver. Hence no conjugate action takes place. This phenomenon is called as "**Interference**" in gear technology.

Interference in gear technology is undesirable because of several reasons. Due to interference the tip of one tooth of the gear pair will tend to dig into portions of the flank of the tooth of the other member of the pair. Moreover, removal of the portions of the involute profile adjacent to the base circle may result in serious reduction in the length of action. All theses factors weaken the teeth and are detrimental to proper tooth action. Interference can of course be eliminated by using more teeth on gear. But this remedy is usually not taken up because this leads to larger gears with their ensuing problems such as increased pitch line velocity, noise, reduced power transmission etc.

Interference can also be avoided by undercutting. Undercutting can be defined as the removal of material from the interference zones. But we know, the gear tooth is the weakest at the root. So undercutting makes the tooth weaker, because undercutting is done in the root region. Hence, it cannot be accepted as a final solution to this problem.

Interference can be avoided by using a higher pressure angle. As higher pressure angle results in smaller base circle and in turn allows more of the tooth profile to be made of involute curve. Another practical way of avoiding interference is by making the tooth of the driven gear as stub.

In order to avoid interference, there should be a minimum stipulated number of teeth on gear. In general it is given by an equation.

$$Z = \frac{2}{\sin^2 \alpha}$$

Where Z=minimum number of teeth on gear.

α =pressure angle of gear.

For 20 degree full depth system, the minimum number of teeth is 17 where as for 14.5 full depth systems it is 32.

1.4 INVOLUTE TOOTH PROFILE:

The condition to be satisfied by the tooth profile so that the teeth of two mating gears will transmit uniform angular velocity is that the common normal to the mating tooth surfaces at their point of contact must pass through the same pitch point i.e. the point where the line of centers intersects the pitch circles. Two tooth profiles satisfying this condition are;

1] CYCLOIDAL

2] INVOLUTE.

Cycloidal curve consists of two curves and is much complicated to generate. Hence its use is limited.

An involute is the curve traced by a point on a line as the line rolls on another curve. It can also be defined as the path traced by a point on a taut string when it is unwrapped from a reel. The circle on which the line rolls is called the “Base circle”. Hence the involute curve starts from the base circle and ends at the tip circle. The universal employment of involute profile is due to

1) Simplicity of manufacture

2) Inter-changeability of gears incase of changes in the speed ratio.

3) Possibility of certain increase in the centre distance without affecting the velocity ratio on the accuracy of engagement.

PRESSURE ANGLE:

It is defined as the angle between the line of action and the perpendicular to the line of centers. It is also known as the “Angle of obliquity”. This angle defines the strength s wear of the gear teeth. This angle is also important as this angle is related to the forces acting on the gear shaft and the bearings. Higher the pressure angle, stronger the tooth as tooth thickness becomes more. Local carrying capacity increases with higher pressure angle and a small no. of teeth can be adopted without undercutting. But with a higher pressure angle, the separating force which is undesirable becomes greater. Two pressure angles 14.5 ° and 20° are commonly used as power transmitting gears .But according to I.S.I. the standard pressure angle is 20°.Hence a gear system is defined by its pressure angle.

LINE OF ACTION:

This is the line along which the point of contact of the two mating tooth profile moves. This is also known as the path of contact. It is common tangent to the two base circles of the mating gears.

LENGTH OF ACTION:

The portion of the line of action on which the point of contact moves during the course of action is known as the length of action. It starts from the point of tangency of base circle of driven gear to the point of tangency of base circle of the driven gear. This can also be defined as the summation of the length of approach and the length of recess.

PITCH CIRCLE:

This is the circumference of an imaginary cylinder which rolls without slipping when in contact with another such cylinder as in friction drive. The two rolling cylinders are called pitch cylinders. According to the law of gearing the angular velocity ratio must remain unchanged. Since this is not practicable in friction drive, the cylinders are replaced by toothed wheels .The pitch circles of the two mating gears are same as the circumferences of the two rolling pitch cylinders having the same angular velocity ratio. In any gear, the relevant pitch circle is the reference circle of that gear and though imaginary, it is the basis of measurement of other parameters of the gear. The diameter of the pitch circle is called the"pitch circle diameter" (P.C.D)

TIP CIRCLE:

This is the outer most circle of the gear. This circle bounds the edges of the teeth of a gear. This is also known as the" Addendum Circle".

ROOT CIRCLE:

It is defined as the circle where the tooth joins the body of the gear. It is also known as "Deddendum Circle". This circle limits the depth of the tooth.

BASE CIRCLE:

This is the circle from which the involute tooth profile is developed. Usually this circle lies in between pitch circle and root circle. But in some cases when pressure angle is very high, then the base circle lies below the root circle. From this circle also the length of action starts and ends at the corresponding base circle.

ADDENDUM:

It is defined as the radial distance between the pitch circle and the tip circle .The standard value of addendum is one module. The addendum can be increased or decreased according to the need.

DEDENDUM:

It is defined as the radial distance between the pitch circle and the root circle .The standard value of dedendum is 1.25 times the module. Dedendum can also be increased or decreased .But usually it is not increased, rather it is decreased.

CLEARENCE:

This is defined as the radial distance between the top land of a tooth and the bottom land of the mating tooth space. The standard value of clearance is taken as 0.157 times the module.

FACE WIDTH:

It is defined as the width of the gear and is the distance from one end of a tooth to the other end of the same tooth. It can also be defined as the thickness of the gear blank from which the gear is cut.

MODULE:

It is defined as the ratio of pitch circle diameter to the number of teeth on the gear. The value of the module is expressed in millimeters. It is one of the major and determining parameters of the gear. Mathematically

Module (M) = pitch circle diameter (D)/no. of teeth in gear (Z)

FILLET CURVE:

It is defined as the curve which is the prolongation of the flank down to the root. It is of complex form and can only be defined by the form of the cutting on finishing tool used. It gives rise to stress concentration. So stress concentration depends on the fillet radius .Now when the fillet radius increases, the fillet curve becomes more flat and hence decreases the stress concentration. The reverse is also true.

ROOT:

This is a term which sometimes means the combined fillet curves that outlines the bottom of a tooth space. But when discussing the strength of the gear teeth, it is the material of a tooth where it joins the body of the gear.

CIRCULAR PITCH:

It is defined as the length of the arc of the pitch circle between two adjacent teeth .It is one of the most important criteria of specification of a gear. It is expressed in terms of module. The standard value of circular pitch is

Circular pitch (P) = $\pi \times M$

BASE PITCH:

It is defined as the length of the arc of the base circle between two adjacent teeth. It can also be defined as the distance along the line of action between two successive and corresponding involute tooth profiles. Mathematically it is given as base pitch **(P_b) =$p \cos\alpha = \pi M \cos\alpha$**

TIP RADIUS:

It is the radius of the rounded corner of the basic rack. If the corner of the basic rack is straight and pointed instead of being rounded, then fillet curve traced by it is different than that of the rounded tip. For rounded tip, the radius of curvature of the fillet curve includes the tip radius whereas for straight and pointed tip, the tip radius is not included.

BASIC RACK:

It is defined as a rack whose teeth are proportioned to a standard tooth form. The proportions and form of the teeth of gears when they are cut, is determined by the basic rack. The profile and proportions of the basic rack in terms of module have been standardized. The reference line of the rack is situated at a distance of one module from the tip of the teeth. Usually gears are cut by basic rack cutters.

ROLLING ANGLE OF RACK:

When a gear is cut by a basic rack cutter, it rolls on the pitch line. The angle through which it rolls to give the shape of the involute curve is known as “Rolling angle of Rack”. In actual case, the basic rack does not roll. It actually reciprocates and at the same time, the gear rotates to give the shape of a gear.

OPERATING PRESSURE ANGLE:

In two standard, uncorrected mating gears, the pitch circles touch at the pitch point. Now when gears are corrected, then there centre distance increases. So the pitch circles do not touch each other any further. Now the two mating gears touch each other at the pitch point with two new circles. This new circle is known as operating circle or working circle. The pressure line, which is tangent to both the base circles as before and which passes through the pitch point, now making a new angle, ’α_b’ instead of the standard pressure angle, ’α’. This new angle, ’α_b’ is known as the “operating pressure angle”.

HIGHEST POINT OF SINGLE TOOTH MESHING:

The choice of load point is an important criteria in designing the gear. Usually the load is assumed to have acting at the tip of the gear tooth. But it may not be the sole case. The load can act lower down the tip along the tooth profile. But for maximum nominal bending stress, assuming perfection in gear tooth, the load is assumed to be acting at the highest point of single tooth meshing. This point now can be defined as the point along the tooth profile which is distant one base pitch from the tip circle when measured along the line of action.

1.5 FINITE ELEMENT ANALYSIS:

In this finite element analysis the continuum is divided into a finite numbers of elements, having finite dimensions and reducing the continuum having infinite degrees of freedom to 'finite' degrees of unknowns. It is assumed that the elements are connected only at the nodal points.

The accuracy of solution increases with the number of elements taken. However, more number of elements will result in increased computer cost. Hence optimum number of divisions should be taken.

In the element method the problem is formulated in two stages

:

The element formulation:

It involves the derivation of the element stiffness matrix which yields a relationship between nodal point forces and nodal point displacements.

The system formulation:

It is the formulation of the stiffness and loads of the entire structure.

BASIC STEPS IN THE FINITE ELEMENT METHOD:

1. **Discretisation of the domain**

   The continuum is divided into a no. of finite elements by imaginary lines or surfaces. The interconnected elements may have different sizes and shapes .The success of this idealization lies in how closely this discretised continuum represents the actual continuum. The choice of the simple elements or higher order elements, straight or curved, its shape, refinement are to be decided before the mathematical formulation starts.

2. **Identification of variables**

   The elements are assumed to be connected at their intersecting points referred to as nodal points. At each node, unknown displacements are to be prescribed. They are dependent on the problem at hand. The problem may be identified in such a way that in addition to the displacement which occurs at the nodes depending on the physical nature of the problem, certain other quantities such as strain may need to be specified as nodal unknowns for the element, which however, may not have a corresponding physical quantity in the generalized forces. The value of these quantities can however be obtained from variation principles.

3. **Choice of approximating functions.**

   After the variables and local coordinates have been chosen, the next step is the choice of displacement function, which is the starting point of mathematical analysis. The function represents the variation of the displacement within the element. The function can be approximated in many ways. A convenient way of expressing it is by polynomial expressions.

   The shape of the element or the geometry may also approximate. The coordinates of corner nodes define the element shape accurately if the element is actually made of straight lines or planes. The weightage to be given to the geometry and displacements also needs to be decided for a particular problem.

4. **Formation of element stiffness matrix**

   After the continuum is discretised with desired element shapes, the element stiffness matrix is formulated. Basically it is a minimization procedure. The element stiffness matrix for majority of elements is not available in explicit form. They require numerical integration for this evaluation. The geometry of the element is defined in reference to the global frame.

5. **Formation of the overall stiffness matrix**

   After the element stiffness matrix in global coordinates is formed, they are assembled to form the overall stiffness matrix. This is done through the nodes which are common to adjacent elements. At the nodes the continuity of the displacement functions and their derivatives are established. The overall stiffness matrix is symmetric and banded.

6. **Incorporation of boundary conditions**

   The boundary restraint conditions are to be imposed in the stiffness matrix. There are various techniques available to satisfy the boundary conditions.

7. **Formation of the element loading matrix.**

   The loading inside an element is transferred at the nodal points and consistent element loading matrix is formed.

8. **Formation of the overall loading matrix**

   The element loading matrix is combined to form the overall loading matrix. This matrix has one column per loading case and it is either a column vector or a rectangular matrix depending on the no. of loading conditions.

9. **Solution of simultaneous equations**

   All the equations required for the solution of the problem is now developed. In the displacement method, the unknowns are the nodal displacement. The Gauss elimination and Choleky's factorization are most commonly used methods.

10. **Calculation of stresses or stress resultants**

    The nodal displacement values are utilized for calculation of stresses. This may be done for all elements of the continuum or may be limited only to some predetermined elements.

1.6 LIMITATIONS OF THE FEM

Due to the requirement of large computer memory and time, computer program based on FEM can be run only in high speed digital computers.

For some problems, there may be considerable amount of input data. Errors may creep up in their preparation and the results thus obtained may also appear to be acceptable which indicates deceptive state of affairs.

In the FEM, the size of problem is relatively large. Many problems lead to round off errors.

CHAPTER 2

LITERATURE REVIEW

2.LITERATURE REVIEW

In the year 2001, Jesper Braucer published a paper named "Analytical geometry of straight conical involute gears". In this paper he derived the parametric equations for a straight conical involute gear tooth and its offset surface. These formulas were then used to create a finite element model with a specific surface layer. Such a surface layer enables meshing control or modeling of surface properties such as case hardening and surface roughness. In addition, he derived an expression for the minimum value of the inner transverse addendum modification coefficient that avoids undercutting of the whole gear tooth.

In the year 1995,R. Maiti and A. K. Roy published a paper named "Minimum tooth difference in internal-external involute gear pair". In the paper the possibility of lowering the difference between the gear teeth and the pinion teeth numbers as much as possible in the internal-external involute gear pair with the help of simple gear corrections has been examined. It is found that by addendum modification the tooth difference can be reduced, though not to unity, from their value with full depth gears. By introducing center distance modification, although this number cannot be reduced further for practical purposes the contact ratio improves. A mathematical relation has been established from which it can be concluded that tip interference can be avoided when the pinion rim is deflected as in the case of harmonic drives. This becomes possible due to the flexion of the pinion rim on the elliptical cam.

In the year 1995, J. I. Pedrero and M. Artes published a paper named "Approximate equation for the addendum modification factors for tooth gears with balanced specific sliding". In this paper an approximate equation for the addendum modification factors for gears with balanced specific sliding is presented. The equation gives the relation between the addendum modification factors of both gears, and other conditions, like minimizing the specific sliding or ensuring pre-established values for the contact ratio or the center distance, could be imposed.

In the year 1995, J. I. Pedrero, M. Artes and J. C. Garcia-Prada published a paper named "Determination of the addendum modification factors for gears with pre-established contact ratio". In this paper an approximate equation for the addendum modification factor gears to have specific values for the contact ratio. The equation gives the relation between the addendum modification factors of both gears, and other conditions, like equalizing specific sliding or ensuring pre-established values for the center distance, could be imposed. The accuracy obtained is high enough for design calculations.

In the year 1986, Satoshi Oda, Takao Koide, Tshiniko Ikeda and Kiyohiko Umezawa published a paper named "Effects of pressure angle on tooth deflection and root stress". In this paper the analysis of tooth deflection and bending moment at root fillet due to a concentrated load on gear tooth with various pressure angles were carried out by the "Finite difference Method" and the results were compared with those obtained from the experiment. The approximate equations for the deflection and bending moment due to a concentrated load on a gear tooth with various pressure angles were derived on the basis of the calculated and measured results. They conclude that tooth deflection and root stress distributions due to a concentrated load on the tip become more localized with an increasing pressure angle and their maximum value increases.

In the year 1985, Satoshi Oda and Takao Koide published a paper named "Study on load bearing capacity of gears with smaller number of teeth". In this paper they presented a study on the surface durability and pitting failure of normalized steel spur gears with small number of teeth from view point of Hertz stress, specific sliding and position of pitch point in the range of engagement. The aim of this study was to obtain more precise data and information to establish the design standards for gears with a smaller number of teeth by examining the load bearing capacity of these gears of various materials and heat treatment conditions. In this paper the surface durability and pitting failure of normalized stell spur gears with a smaller number of teeth with various amounts of addendum modification were investigated by carrying out a running test.

In the year 1986, Satoshi Oda, Takatsure Yamatari and Takao Koide published a paper named "Study on load bearing capacity of gears with smaller number of teeth". In this paper they used a tufftrided gear with smaller number of teeth. They carried out the test on this gear with various amounts of addendum modifications and compared with the results of normalized steel spur gears. The range of applications of these experimental results was examined on the basis of the calculated results of Hertz stress and specific sliding of spur with various number of teeth and addendum modification co-efficient. They found the surface durability is 50% higher in this case than the normalized steel gear.

In the year 1981, Satoshi Oda and Koji Tsubokura published a paper named "Effects of addendum modification on bending fatigue strength of spur gears". In this paper they calculated the root stress factor for different values of addendum modification co-efficient at the worst loading point by both experimentally and theoretically. The gears were made of cast iron and cast steel. In the paper they also presented a study into the effects of the addendum modification on the contact ratio factor.

After going through the literature we inferred that, no work was done on root stress of spur gears. An FEM analysis was done to achieve the targets.

CHAPTER 3

MATHEMATICAL FORMULATION

3. MATHEMATICAL FORMULATION

3.1 RANGE OF ADDENDUM MODIFICATION CO-EFFICIENT:

The range of addendum modification coefficient(x) is defined as "the minimum value of modification co-efficient required to avoid undercut and the maximum value of modification co-efficient required to avoid peaking i.e." the pointed tooth".

3.2 MINIMUM VALUE OF ADDENDUM MODIFICATION COEFFICIENT TO AVOID UNDERCUT

Let α_1 = pressure angle of gear

m = module of gear

z = no. of teeth on gear

R = pitch circle radius of gear

Now referring to the fig. point 'T' is the point of tangency and from this point the involute curve begins.

From the fig. we get;

$\sin \alpha_1 = PQ/PT = (m-x\, m)/PT$………….. (3.1)

Also $\sin \alpha_1 = PT/R$

$\Rightarrow PT = R.\sin\alpha_1$

$\Rightarrow PT = (Mz/2) \sin\alpha_1$………. (3.2)

Now putting the value of PT from equation (3.2) in equation (3.1), we get

$\sin \alpha_1 = (m-x\, m)/ (mZ.\sin\alpha_1/2)$

$\Rightarrow Z/2.\sin^2\alpha_1 = (1-x)$

$\Rightarrow x = 1-(Z.\sin^2\alpha_1)/2$

Hence eq (3.3) gives the minimum value of addendum modification co-efficient to avoid undercutting.

In a standard gear, X=0, so the minimum no. of teeth on a gear to avoid undercutting can be found out by putting X=0 in eq. (3.3)

$0=1-(Z/2) \sin^2\alpha_1$

$Z\ min = Z= 2/\sin^2\alpha_1$

3.3 MAXIMUM VALUE OF ADDENDUM MODIFACTION CO-EFFICIENT:

The maximum value of addendum modification co-efficient goes up to point where the tooth becomes pointed as shown in fig.3.1.2

Tooth thickness at pitch circle= $t = \pi m/2$

Pitch circle radius $= R = mZ/2$

Now the pressure angle at the pointed tip can be found out by the relation

$Inv.\alpha_2 = t/2R + inv.\alpha_1$

Now the radius of the tip circle where the tooth becomes pointed can be found out by the relation

$R_2 = R \, Cos\alpha_1 / Cos\, \alpha_2$

Maximum addendum = $(R_2 - R)$

We know that normal addendum = 1 Module = m

Maximum addendum modification = $(R_2 - R - m)$

We know addendum modification = x m

$x\, m = R_2 - R - m$

$X = (R_2 - R - m)/m$

Hence eq. (3.5) gives the maximum value of addendum modification co-efficient.

So within the maximum and minimum value of 'x' any design can be a feasible design. Hence by considering both the upper limit and the lower limit, in this project the modification co-efficient are

-0.5, -0.3,-0.15, 0, 0.15, 0.30, 0.50.

The zone of feasible design by considering both the upper limit and lower limit of addendum modification co-efficient is shown in fig.3.1.3 for various no. of tooth.

3.4 TOOTH THICKNESS OF CORRECTED GEARS AT PITCH CIRCLE:

The tooth thickness of a corrected gear is different from that of the standard gear when measured along the pitch circle. The generation of a positively corrected gear is shown in fig.3.1.2.The amount of correction is x m millimeter. The profile reference line is shifted by an amount of x m from the generating line which contacts the pitch circle of the gear at the point 'p'.

Let p = circular pitch= π m

We know the tooth thickness along the pitch circle before correction is given by

T = p/2 = π m/2

Now after correction, it is clear from the fig.3.1.2.the tooth thickness increases by an amount 2xm tanα_1.Hence for an s-plus gear, the tooth thickness on pitch circle becomes

$t_1 = p/2 + 2xm \tan \alpha_1$

For s-minus gear

$t_1 = p/2 - 2xm \tan \alpha_1$

3.5 DIMENSIONS REQUIRED FOR CALCULATION OF ROOT STRESS:

When rack tooth represents the form of the generating tool, then trochoid gives the form of the fillet of the gear tooth. Taking the co-ordinate system for the trochoid and the symbols as shown in fig.3.1 and 3.2,the co-ordinates (x_0,y_0) of the centre of the rounding of rack tooth and the co-ordinates($x_{11}y_1$) on the fillet curve at the root of gear tooth are derived as follows;

Let

R = pitch radius of gear,

m = module,

z = no. of teeth on gear,

x = addendum modification co-efficient,

α_1 = pressure angle of rack cutter,

R_0 = edge radius of generating rack,

H = distance from centre line to centre of rounded corner of rack tooth,

L_0 = distance from centre line of gear tooth to foot of perpendicular from centre of rounded corner of rack tooth on pitch line.

C_K = clearance at bottom of tooth space,

P = angle around centre of gear given by L_0/R,

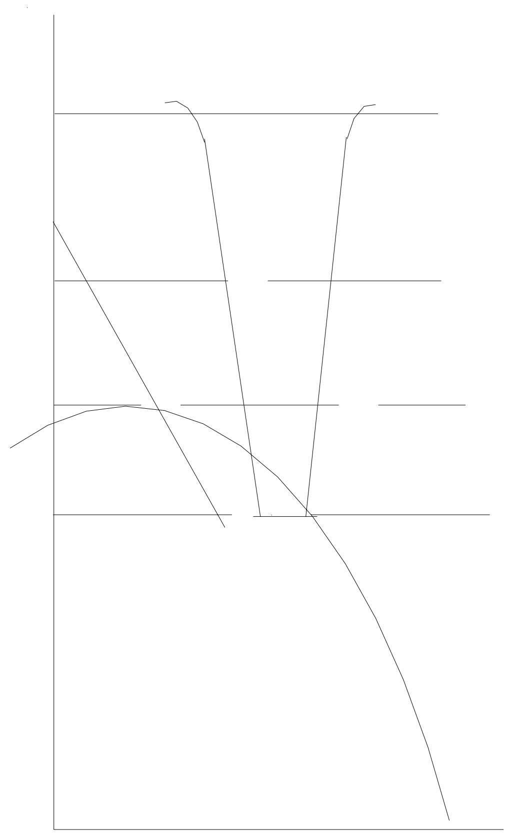


Fig 3.1

Construction of root fillet curve and coordinate system for determination of tooth thickness ‘T2’ at critical section and the radius of curvature (Rc) at the root fillet

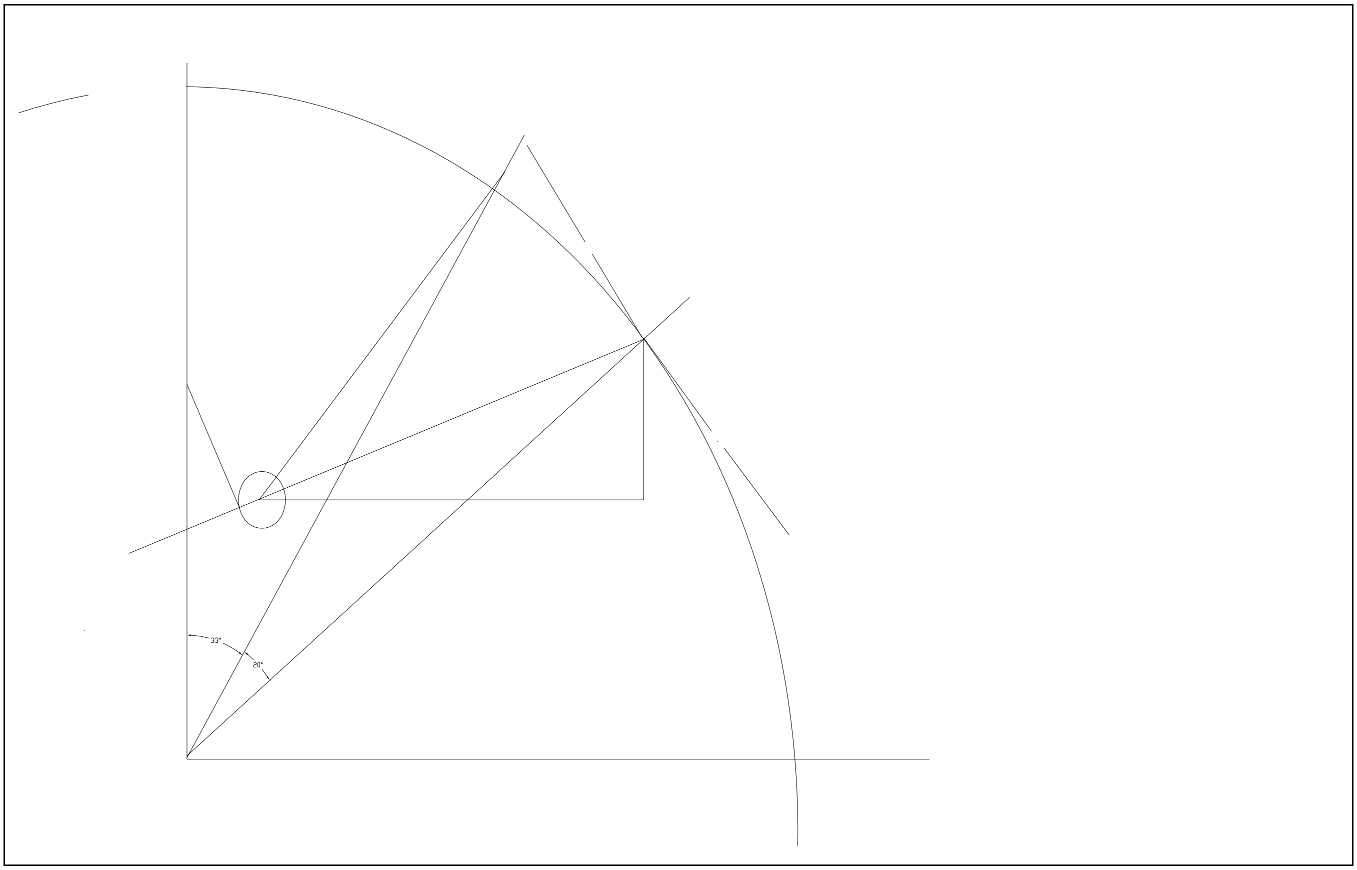


Fig 3.2

Construction of root fillet curve and coordinate system for determination of tooth thickness 'T2' at critical section and the radius of curvature (Rc) at the root fillet

γ= rolling angle of rack,

Φ = centre of gear (origin of co-ordinate system)

Now let the point (x_0, y_0) is shown in figures as point A. The point (x_1, y_1) is shown as point 'G'. Our objective is to find the co-ordinate (x_1, y_1) of point 'G'.

3.6 CONSTRUCTION:

1) Drop a perpendicular from point 'B' to the line 'OX'. Let it touch at point 'C'.

2) From point 'A', draw a line parallel to OX. Let this line touches the line BC at point 'D'.

3) Draw a perpendicular to line AD from point 'a' and it touches at E.E.

4) From point 'B', draw a line parallel to AD and it touches at point F.F.

Analysis:

Now from the geometry of figure 3.3.1 and 3.3.2, we get

$\angle BOH = \angle Aae = \angle OBC = \beta+\gamma$

$\angle Fab = 90^0 - (\beta+\gamma)$

$\angle Abf = 90^0 - [90^0 - (\beta+\gamma)] = (\beta+\gamma)$

Now $OC = OB \times \sin\angle OBC = R \sin(\beta+\gamma)$

$AE = Aa \times \sin\angle Aae = H_0 \sin(\beta+\gamma)$

From Δ aoB, we get a B = R γ (for γ being small)

Now $BF = a B \times \cos\angle Abf = R \cos(\beta+\gamma)$

$BF = ED = R \cos(\beta+\gamma)$

$AD = AE+ED = H_0 \sin(\beta+\gamma) + R \cos(\beta+\gamma)$

(Where R_γ = length of the arc on the pitch circle of gear)

Now $X_0 = OC - AD$

$\Rightarrow X_0 = OC-(AE+ED)$

$\Rightarrow X_0 = R \sin(\beta+\gamma) - H_0 \sin(\beta+\gamma) - R \cos(\beta+\gamma)$

$\Rightarrow X_0 = (R-H_0) \sin(\beta+\gamma) - R \cos(\beta+\gamma$

Similarly $BC = OB \cos\angle OBC = R \cos(\beta+\gamma)$

$a E = Aa \cos\angle AaE = H_0 \cos(\beta+\gamma)$

$BD = FE = a\,E - a\,F = H_0 \, Cos\,(\beta+\gamma) - R_\gamma \, Sin\,(\beta+\gamma)$

$\gamma_0 = BC - BD$

$=> \gamma_0 = BC - FE$

$=> \gamma_0 = BC - (a\,E - a\,F)$

$=> \gamma_0 = R\,Cos\,(\beta+\gamma) - H_0\,Cos\,(\beta+\gamma) + R\,Sin\,(\beta+\gamma)$

$=> \gamma_0 = (R - H_0)\,Cos\,(\beta+\gamma) + R\,Sin\,(\beta+\gamma)$

Now redrawing the part of fig 2 as shown in fig 3, a parallel line GM is drawn to AD and a perpendicular AN is dropped to GM.

From fig 2, we get

$$BD = H_0 \cos(\beta+\gamma) - R_\gamma * \sin(\beta+\gamma)$$

$$AD = MN = H_0 * \sin(\beta+\gamma) + R_\gamma \cos(\beta+\gamma)$$

$$AB = \sqrt{h_0^2 + R^2\gamma^2}$$

$$AG = R_0$$

To find the co-ordinates(X_1, Y_1)

Now from the similar triangles ABD and AGM, we get

GN/AD=AG/AB

=>GN=(AD/AB) AG

$$=> GN = \frac{H_0 * \sin(\beta+\gamma) + R\gamma * \cos(\beta+\gamma)}{\sqrt{h_0^2 + R^2\gamma^2}}$$

Again AN/BD=AG/AB

=>AN=(BD/AB) AG

$$=> AN = \frac{H_0 * \sin(\beta+\gamma) + R\gamma * \cos(\beta+\gamma)}{\sqrt{h_0^2 + R^2\gamma^2}} * R_o$$

$X_1 = X_0 - GN$

$$=> X_1 = (R - H_0)\sin(\beta+\gamma) - R_\gamma \cos(\beta+\gamma) - \frac{H_0 * \sin(\beta+\gamma) + R\gamma * \cos(\beta+\gamma)}{\sqrt{h_0^2 + R^2\gamma^2}} * R_o$$

Y_1=Y_0-AN

$$Y_1 = (R - H_0)\sin(\beta+\gamma) + R_\gamma \cos(\beta+\gamma) - \frac{H_0 * \sin(\beta+\gamma) + R\gamma * \cos(\beta+\gamma)}{\sqrt{h_0^2 + R^2\gamma^2}} * R_o$$

Again from fig3, we get

$$H_o = (1-x)m - R_0 \sin\alpha_1$$

$$L_0 = \pi * m/4 + H_0 \tan\alpha_1 + R_0 \sec\alpha_1 + xm\tan\alpha_1$$

$$R_0 = Ck/1 - \sin\alpha_1$$

$\beta = L_0 / R$ (for β being small)

Let θ = Angle between the centre line of gear tooth and tangent to the fillet curve.(as shown in fig1)

(Usually its value is taken as 30°)

Now referring fig3, we get

$$\tan\theta = -dx/dy = -\frac{dx/d\gamma}{dy/d\gamma}$$

$$= \frac{H_0 - R_\gamma \tan(\beta+\gamma)}{H_0 \tan(\beta+\gamma) + R_\gamma}$$

Now the radius of curvature of the fillet curve at the critical section is given by

$$R_c = \frac{(H_0^2 + R^2\gamma^2)^{3/2}}{R^2\gamma^2 + RH_0 + H_0^2} + R_0$$

After getting the co-ordinates of critical section at the root fillet and referring fig1, we get,

The tooth thickness 'T_2' at the critical section

$T_2=2*X_2$

The distance of the critical section from the pitch circle is given by the relation

$L_d=R-Y_1$

The pressure angle at the tip of gear tooth is found by the relation

$$\alpha_k = \cos^{-1}\left[z\cos\alpha_1 / \{z+2(1+x)\}\right]$$

Now pressure angle at the loading point is found by the relation

$$\alpha_p = \tan^{-1}\tan^2\alpha_k - 2L_p / R_b$$

Where R_b= base circle radius of gear.

L_p=Distance from the loading point to tip along the tooth profile

Angle between the line connecting loading point with center of gear and center line of gear tooth is given by

$$K = \frac{\pi + 4x\tan\alpha_1}{2z} + inv.\alpha_1 - inv.\alpha_p$$

Now angle between direction of loading and center line of tooth is given by the relation.

$$P = \pi/2 + K - \alpha_p$$

The distance from loading point to the pitch circle is given by

$$L_a = mz/2(\cos\alpha_1/\cos\alpha_p * \cos K - 1)$$

The distance from loading point to the critical section is given by the relation

$L=L_a+L_d$

Distance from loading point to the center line of gear tooth is given by the relation.

$$Y = mz/2 * \cos\alpha_1/\cos\alpha_p \sin K$$

All these dimensions which are required to evaluate the root stress at the fillet of gear tooth is shown in fig4

3.7 FORMULA FOR CALCULATING ROOT STRESS

The root stress in the gear is given by the relation

$\sigma_r = A * P_1 / b$

Where σ_r=root stress

P1=load applied on gear tooth

b=face width of the gear tooth

A=root stress factor

Now as load applied and face width remains constant, hence root stress is directly related to the root stress factor.

Now root stress factor (A) is given by the relation

$$A = (1 + 0.08\frac{T_2}{R_c})(0.66A_b + 0.04\sqrt{(A_b^2 + 36A_c^2)} + 1.15A_c)$$

Where $A_b = 6L\sin P / T_2^2$

$A_t = \sin P / T_2$

$A_c = -\cos P / T_2 - 6Y\cos P / T_2^2$

Now as T_2, R_c, P, L, Y etc depends upon the addendum modification coefficient (X), so the root stress also depends on addendum modification coefficient (X). Hence as 'X' varies 'A' also varies. So our aim is to study the effect of 'X' on root stress factor (A) as 'x' varies. In this project work 'X' is varied from -0.5 to 0.5 through -0.3, -0.15, 0, 0.15, 0.3

The standard values adopted to evaluate the root stress factor (A) are given below.

Module, m=4

No. of teeth on gear, z=26

Pressure angles used are 14.5, 20, 23, 27

3.8 Effect of Addendum modification on root stress when both driver and follower are modified at the same time and with different gear ratios:

Previously addendum modification was applied only to the driver wheel. In this section a study is presented when both the driver wheel and the follower are modified at the same time and the effect of this modification on root stress in driver wheel only is presented for different gear ratios.

In this section, the load is assumed to be acting at the highest point of single tooth meshing.

Now after correction the gear pairs do not touch each other at the previous pitch point. More over they touch each other at some other point. The circles passing through this point is known as working pitch circle. Now as pitch circle changes, so the pressure angle also changes. Hence the new pressure angle is known as "operating pressure angle (α_b)".

Now this operating pressure angle can be found out by using the relation

$$Inv\alpha_b = Inv\alpha_1 + 2\tan\alpha_1(x_2 + x_3 / z_1 + z_2)$$

Where α_b=operating pressure angle

α_1=normal pressure angle

x_2, x_3 = addendum modification coefficient of driver and follower respectively

z_1, z_2=no. of teeth on driver and follower respectively.

In case of So-gearing, $x_2 + x_3 = 0$, so $\alpha_b = \alpha_1$

So even though the tooth proportion changes, the pressure angle does not change and they contact each other at the same pitch point 'P'.

Determination of pressure angle at the load point (highest point of meshing)

From geometry of the fig, we get

GA=R_b=Base circle radius of driver gear

=R_1.cos α_1

Where R_1=pitch circle radius of driver gear.

Rb_2=base circle radius of driven gear

=R2 cos α_1

Where R2=pitch circle radius of driven gear.

$$AP = Rb_1 \tan\alpha_b = R_1 \cos\alpha_1 \tan\alpha_b$$

$$PC = Rb_2 \tan\alpha_b = R_2 \cos\alpha_1 \tan\alpha_b$$

$$AC = AP + PC = (R_1 + R_2)\cos\alpha_1 \tan\alpha_b = m(Z_1 + Z_2)/2\cos\alpha_1 \tan\alpha_b$$

Now $BC = R^2a_2 - R^2b_2$

Where Ra_2 =tip circle radius of follower

$=R_2+m(1+x_3)$

BD $=P_b$=base pitch $=p \cos \alpha_1 = \pi m \cos \alpha_1$

AB=AC-BC

$$= m(Z_1 + Z_2)/2\cos\alpha_1 \tan\alpha_b - \sqrt{(R^2a_2 - R^2b_2)}$$

AD=AC+BD

$$= m(Z_1 + Z_2)/2\cos\alpha_1 \tan\alpha_b - \sqrt{(R^2a_2 - R^2b_2)} + \pi m\cos\alpha_1$$

Pressure angle at the load point is found by the relation

$\tan \alpha_p = AD/O_1A$

After getting the value of α_p from equation,it is put into the other equations to find out the different dimensions required to calculate the root stress and hence the root stress is calculated.

The standard values used to evaluate the root stress factor are

Module, m=4

Pressure angle=20^o

No. of teeth on driver=26

Gear ratio used=0.33, 0.5, 1, 2 and 3.

3.9 GEAR MODELING IN CATIA AND ANALYSIS USING ANSYS:

Pitch circle radius R=52mm

Tooth thickness=6.283mm

Minimum value of addendum modification coefficient to avoid undercutting

$$x_{\min} = 1 - \left(\sin^2 \alpha\right) z / 2$$

$$x_{\min} = -1.6794$$

Maximum value of addendum modification coefficient to avoid pointed tooth

$$\frac{1}{\alpha_2} = \frac{t}{2 * R} + \frac{1}{\alpha_1}$$

$\alpha_2 = 26.2525$

$$R_2 = R\frac{\cos\alpha_1}{\cos\alpha_2} = 51.661$$

$x_{\max} = (R_2 - R - m)/m = -1.08475$

Base radius= $R\cos\alpha = 46.332$mm

Points for drawing the involute tooth profile

$$x = r_b\sin(t\pi) - r_b t\pi\cos(t\pi)$$

$$y = r_b\cos(t\pi) - r_b t\pi\sin(t\pi)$$

Where t is a parameter that is varied from 0 to 0.5

Values of outer circle radii for different values of addendum modification coefficients:

(49.6, 50, 50.4, 50.8, 51.2, 51.6)

Fillet radius=3mm

Material used:

Structural steel with E=2e6 and μ=0.3

Constraints applied:

Displacements at the inner surface=0

Pressure on the tooth surface=100kPa

CHAPTER 4

RESULTS AND DISCUSSION

4. RESULTS AND DISCUSSION

4.1 Analytical method:

In the **fig 4.1,** the effects of addendum modification coefficient on tooth thickness at critical section is presented with various pressure angles. It is observed from the fig that as ‘x’ increases from negative to positive value of tooth thickness also increases and is maximum at x=0.5

.

In **fig 4.2** the effects of addendum modification coefficient on root stress factor is presented for various pressure angles. It is observed that as ‘x’ increases, the root stress factor decreases and obtains the lowest value at ‘x’=0.5

When different pressure angles are used, it is seen that, the root stress factor decreases with increase in pressure angle. At maximum value that is 27 degrees the root stress becomes minimum irrespective of addendum modification coefficient. Hence it is clear that positive value of modification decreases root stress where as negative value increases it.

Hence as positive correction leads to higher load carrying capacity as tooth thickness at critical section becomes larger and the root stress is lower. So for greater load carrying capacity usually addendum modification is done or a higher value of pressure angle is used.

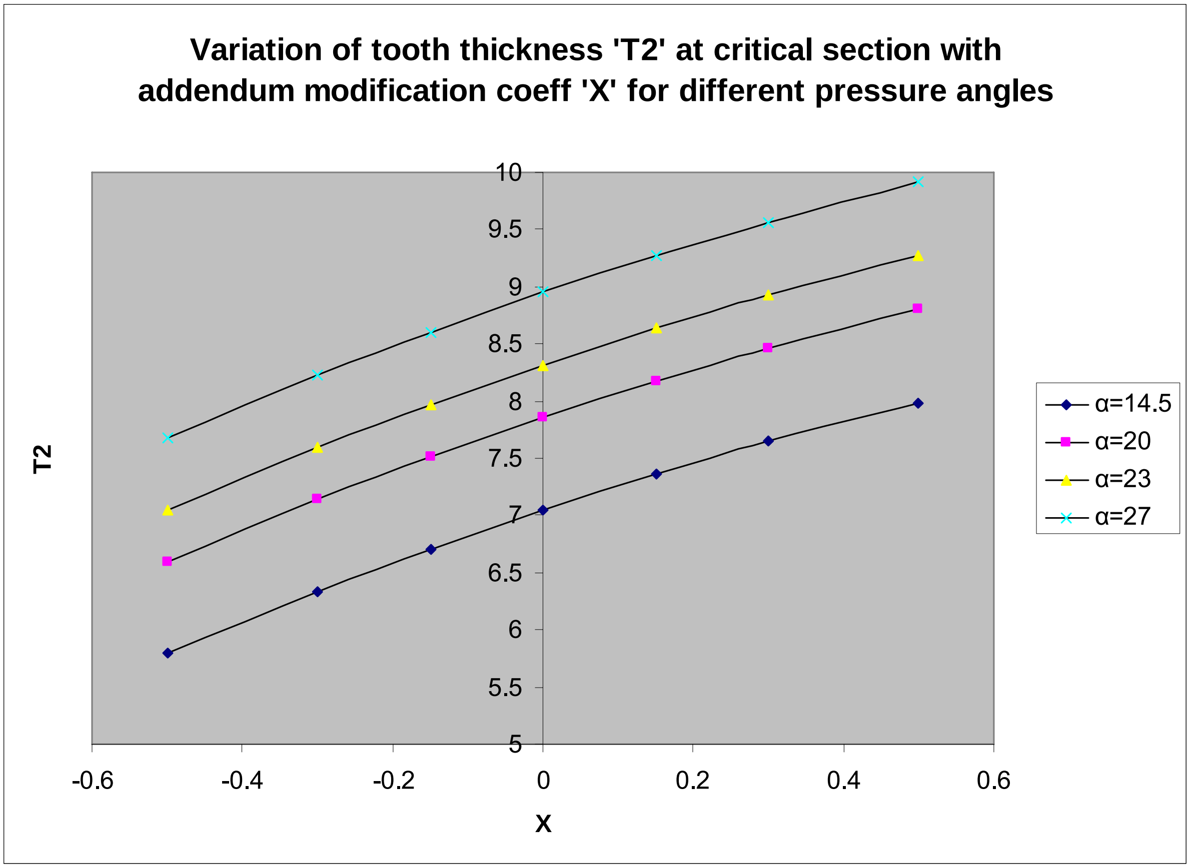

Variation of tooth thickness 'T2' at critical section with addendum modification coeff 'X' for different pressure angles
T2
X
10
9.5
9
8.5
8
7.5
7
6.5
6
5.5
5
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
α=14.5
α=20
α=23
α=27

Fig 4.1

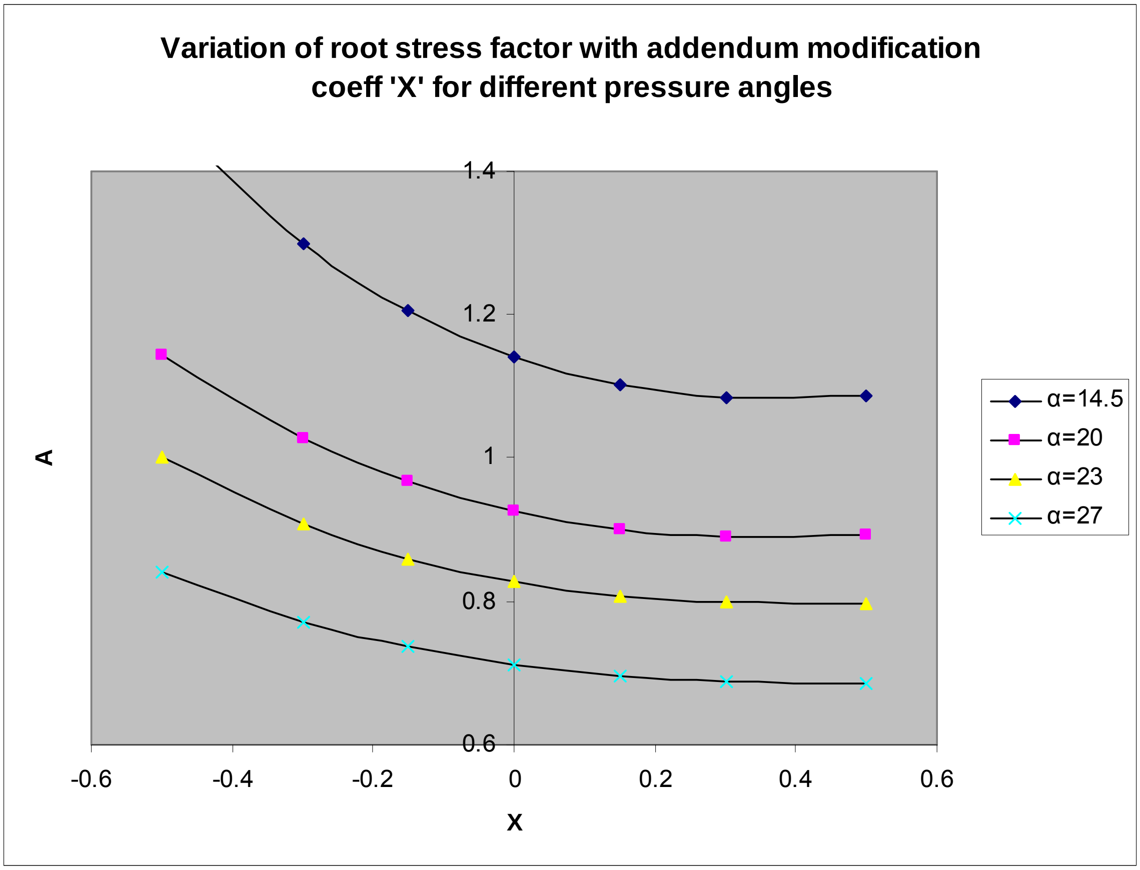

Variation of root stress factor with addendum modification coeff 'X' for different pressure angles
A
X
1.4
1.2
1
0.8
0.6
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
α=14.5
α=20
α=23
α=27

Fig 4.2

From the **fig 4.3** to **fig 4.9** it is found that when both driver and follower wheel are corrected, then the root stress factor is decreased further as compared to the case of modification done only to the driver wheel.

The root stress factor decreases further with an increase in gear ratio 'Q'. Root stress factor for Q=3 is less than that for Q=1. But when Q<1, the root stress factor is increased. 'A' is more for Q=0.5 than Q=0.33, but they lie very close to each other and they become equal at x_2=-0.5 and x_3=0.5. but as x_3 value decreases from 0.5 to -0.5, the root stress factor for Q=0.33 becomes more than that for Q=0.5.

From **fig 4.3** and **fig 4.4** we find that for increasing gear ratio, the effect of addendum modification coefficient of follower becomes smaller as all the curves lie very close to each other.

From **fig 4.5** it is found that for decreasing gear ratio, the effect of x_3 on root stress factor becomes higher.

Optimized value of x_2 and x_3 for different gear ratios:

In order to have minimum root stress in the fillet of gear tooth such that the tooth strength becomes more, the following optimized value of addendum modification coefficient of driver (x_2) and follower (x_3) are suggested.

For Q=0.33

x_2=0.5, x_3=0.5

For Q=0.5

x_2=0.5, x_3=0.5

For Q=1

x_2=0.5, x_3=-0.5

For Q=3

x_2=0.5, x_3=-0.5

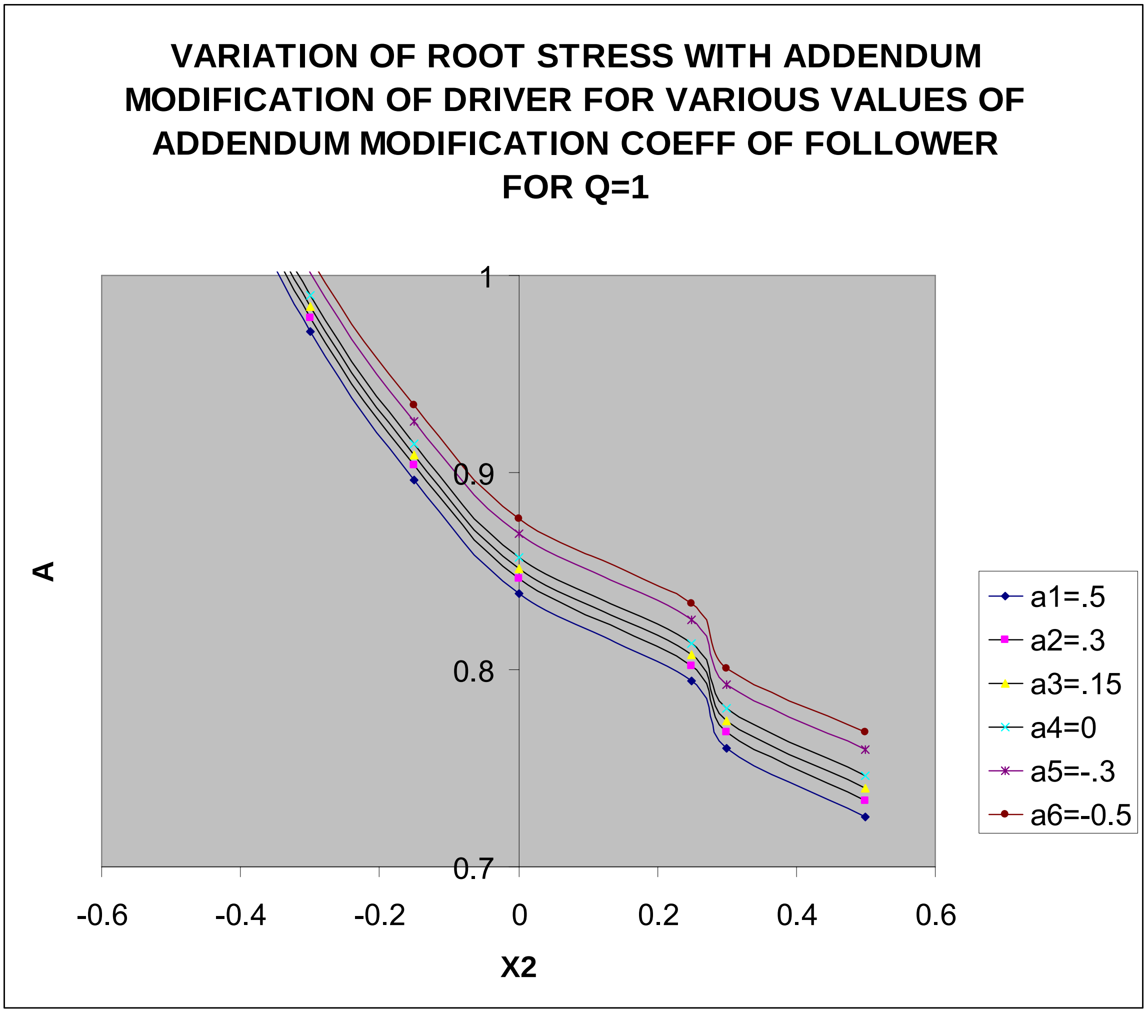
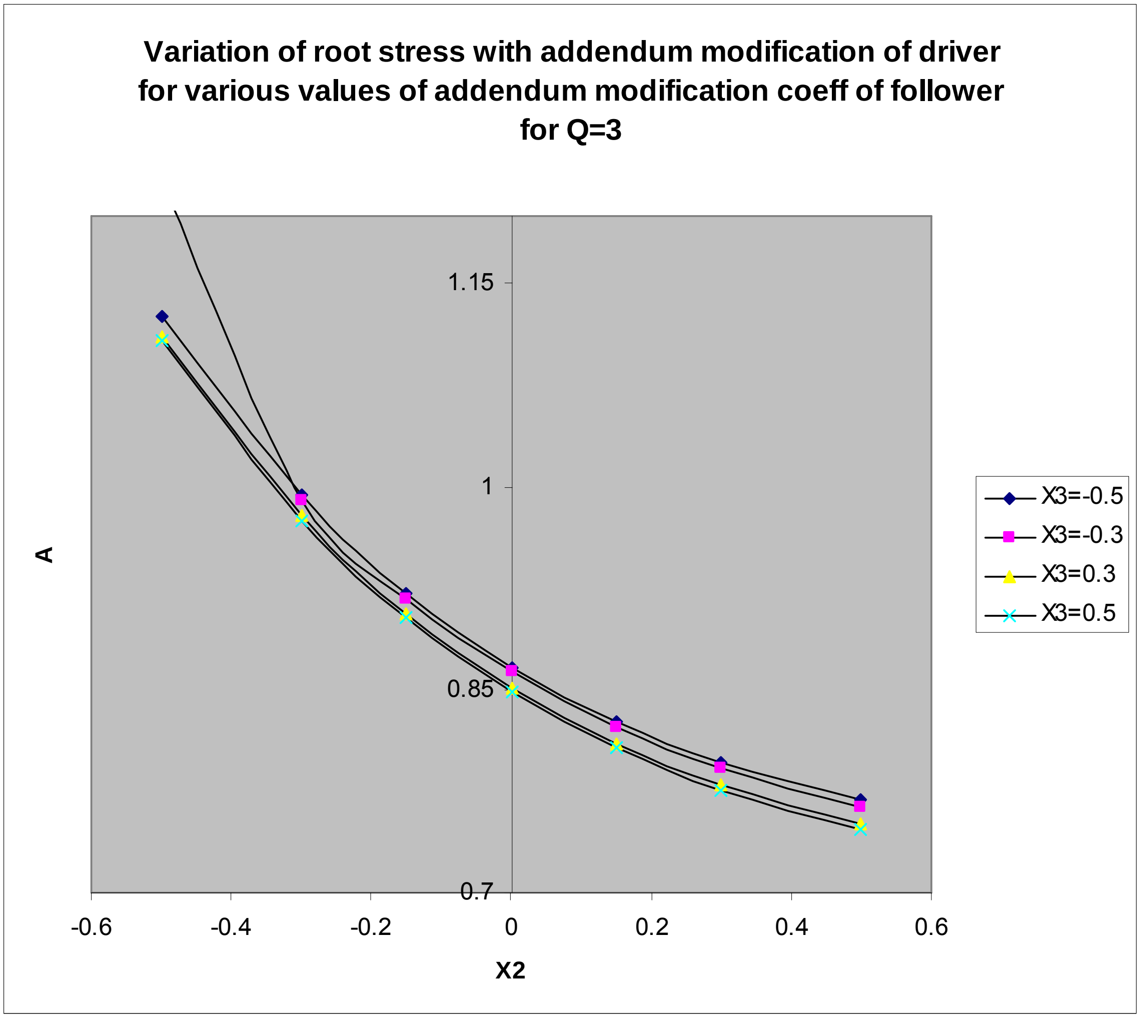

VARIATION OF ROOT STRESS WITH ADDENDUM MODIFICATION OF DRIVER FOR VARIOUS VALUES OF ADDENDUM MODIFICATION COEFF OF FOLLOWER FOR Q=1
A
1
0.9
0.8
0.7
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
X2
a1=.5
a2=.3
a3=.15
a4=0
a5=-.3
a6=-0.5

Fig 4.3



Variation of root stress with addendum modification of driver for various values of addendum modification coeff of follower for Q=3
1.15
1
0.85
0.7
A
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
X2
X3=-0.5
X3=-0.3
X3=0.3
X3=0.5

Fig 4.4

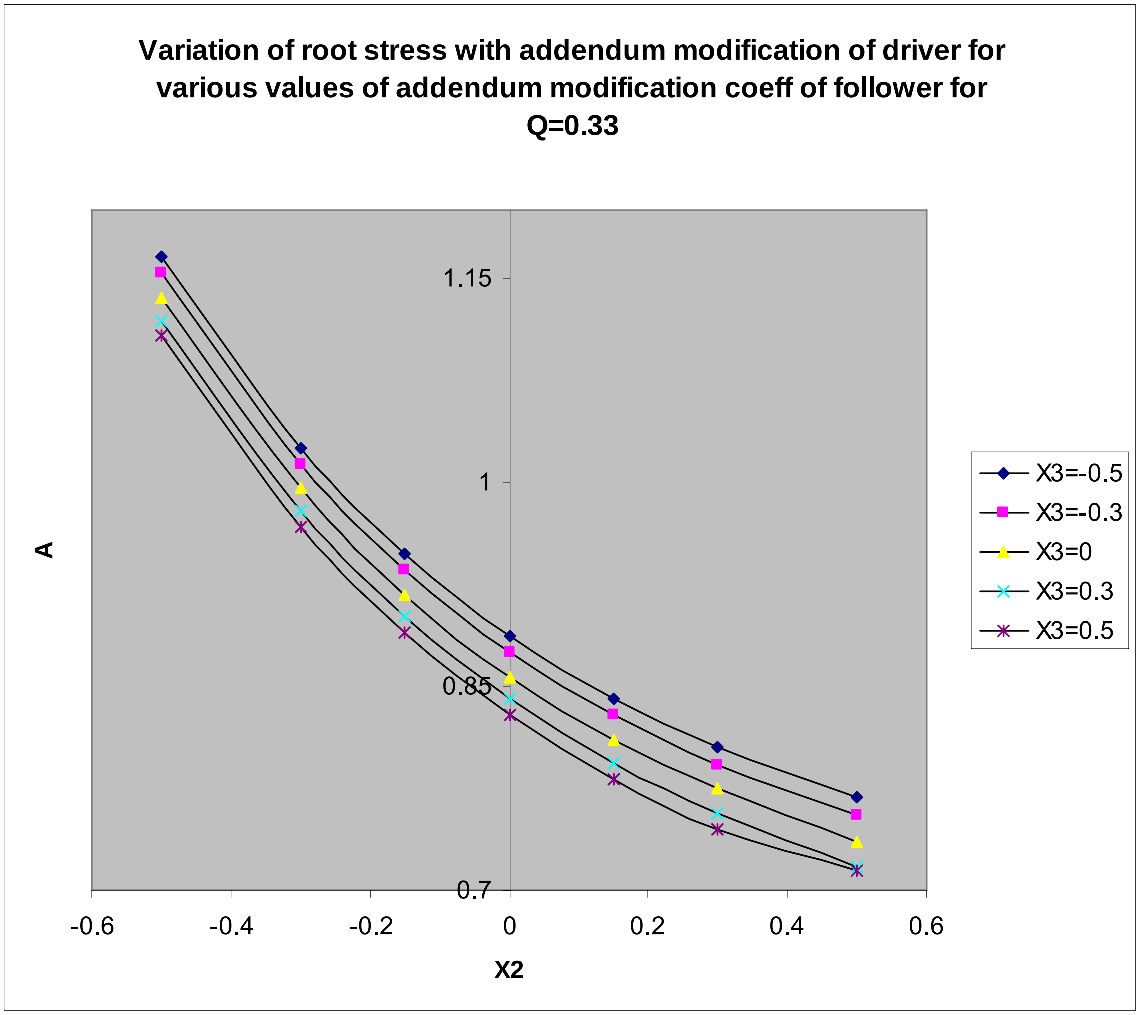
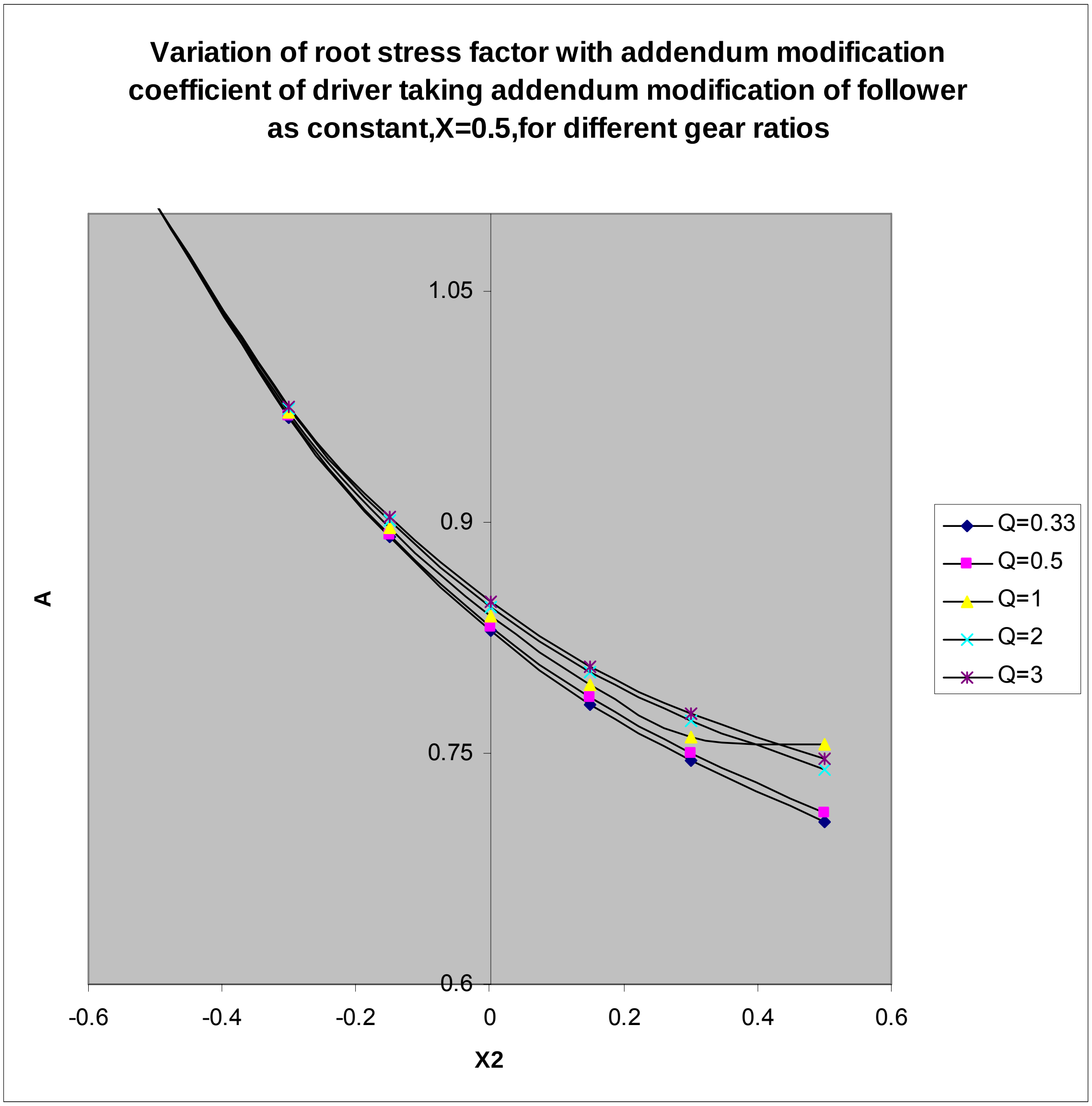

Variation of root stress with addendum modification of driver for various values of addendum modification coeff of follower for Q=0.33
A
1.15
1
0.85
0.7
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
X2
X3=-0.5
X3=-0.3
X3=0
X3=0.3
X3=0.5

Fig 4.5



Variation of root stress factor with addendum modification coefficient of driver taking addendum modification of follower as constant,X=0.5,for different gear ratios
1.05
0.9
0.75
0.6
A
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
X2
Q=0.33
Q=0.5
Q=1
Q=2
Q=3

Fig 4.6

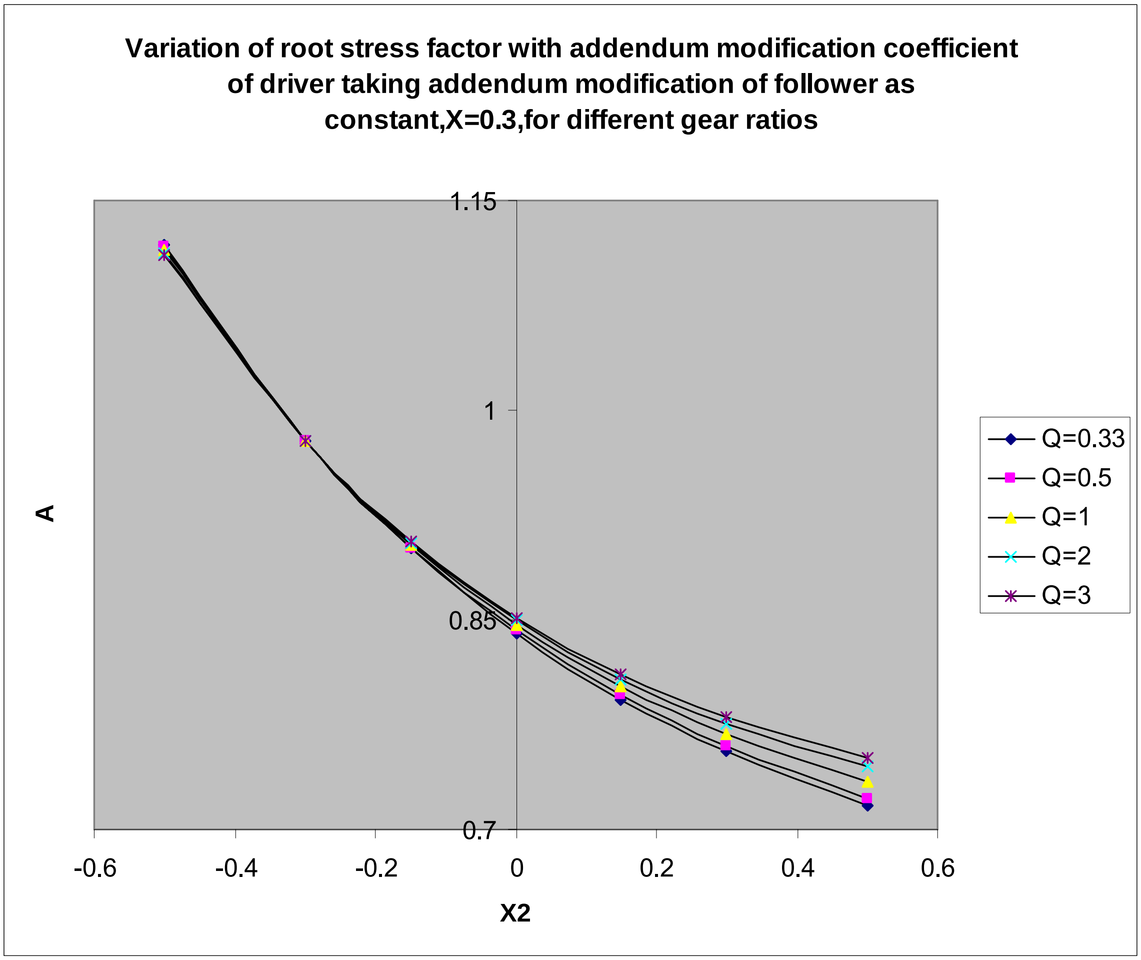

Variation of root stress factor with addendum modification coefficient of driver taking addendum modification of follower as constant,X=0.3,for different gear ratios
1.15
1
0.85
0.7
A
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
X2
Q=0.33
Q=0.5
Q=1
Q=2
Q=3

Fig 4.7

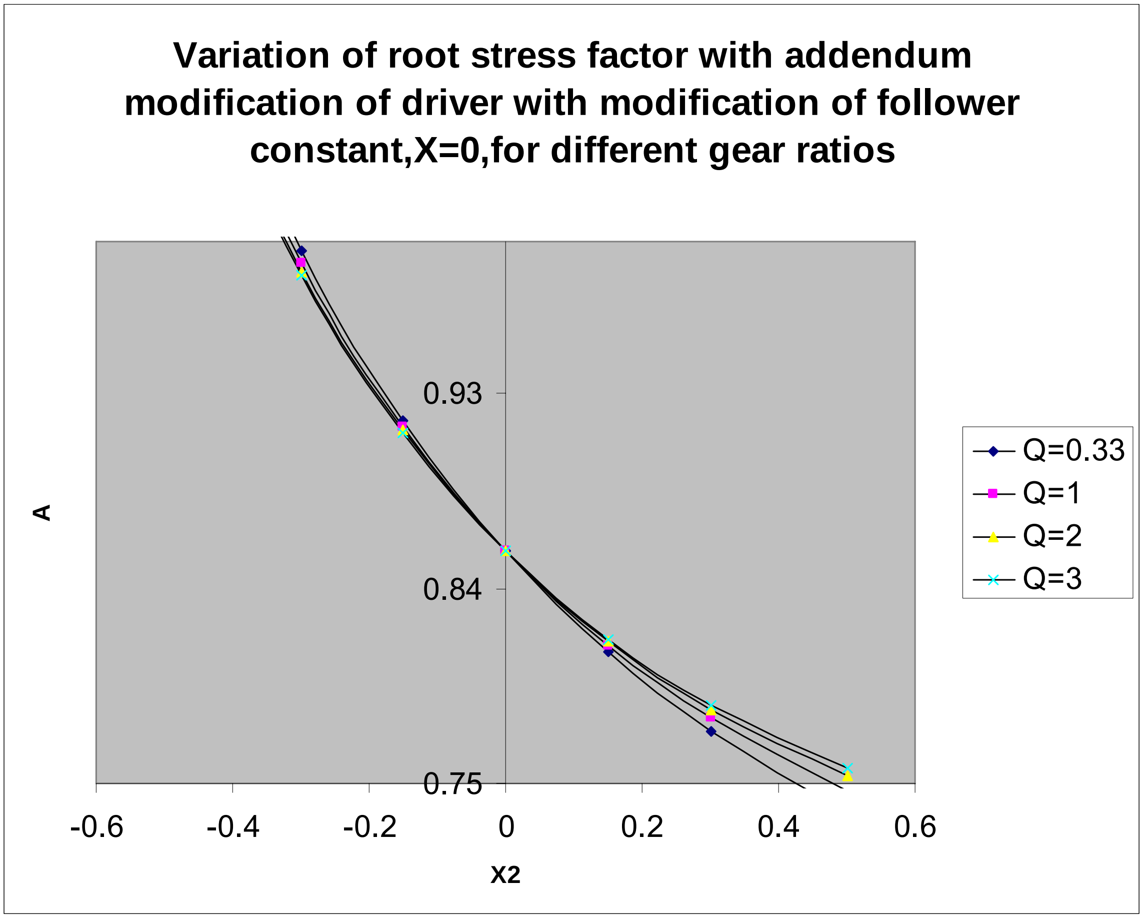

Variation of root stress factor with addendum modification of driver with modification of follower constant,X=0,for different gear ratios
A
0.93
0.84
0.75
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
X2
Q=0.33
Q=1
Q=2
Q=3

Fig 4.8

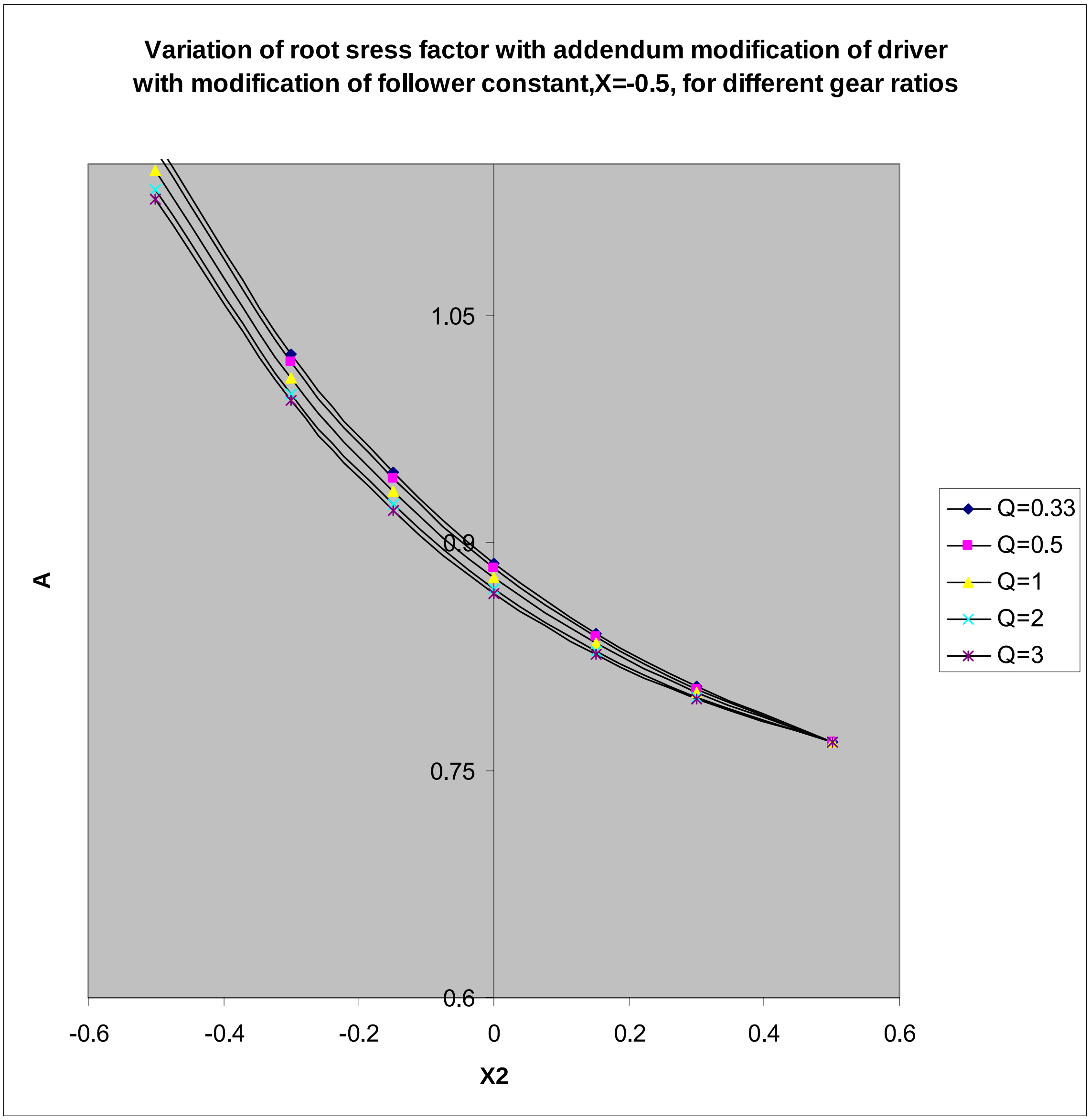

Variation of root sress factor with addendum modification of driver with modification of follower constant,X=-0.5, for different gear ratios
A
1.05
0.9
0.75
0.6
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
X2
Q=0.33
Q=0.5
Q=1
Q=2
Q=3

Fig 4.9

4.2 Finite element analysis:

Fig 4.10, fig 4.11 and **fig 4.12** show the stress distribution along the tooth section of gears with different addendum modification coefficients. It can be observed that maximum stress is obtained at the nodal elements at the root of the gear tooth.

Fig 4.13, fig 4.14 and **fig 4.15** show the deformations at the different nodal points of the gear due to stress.

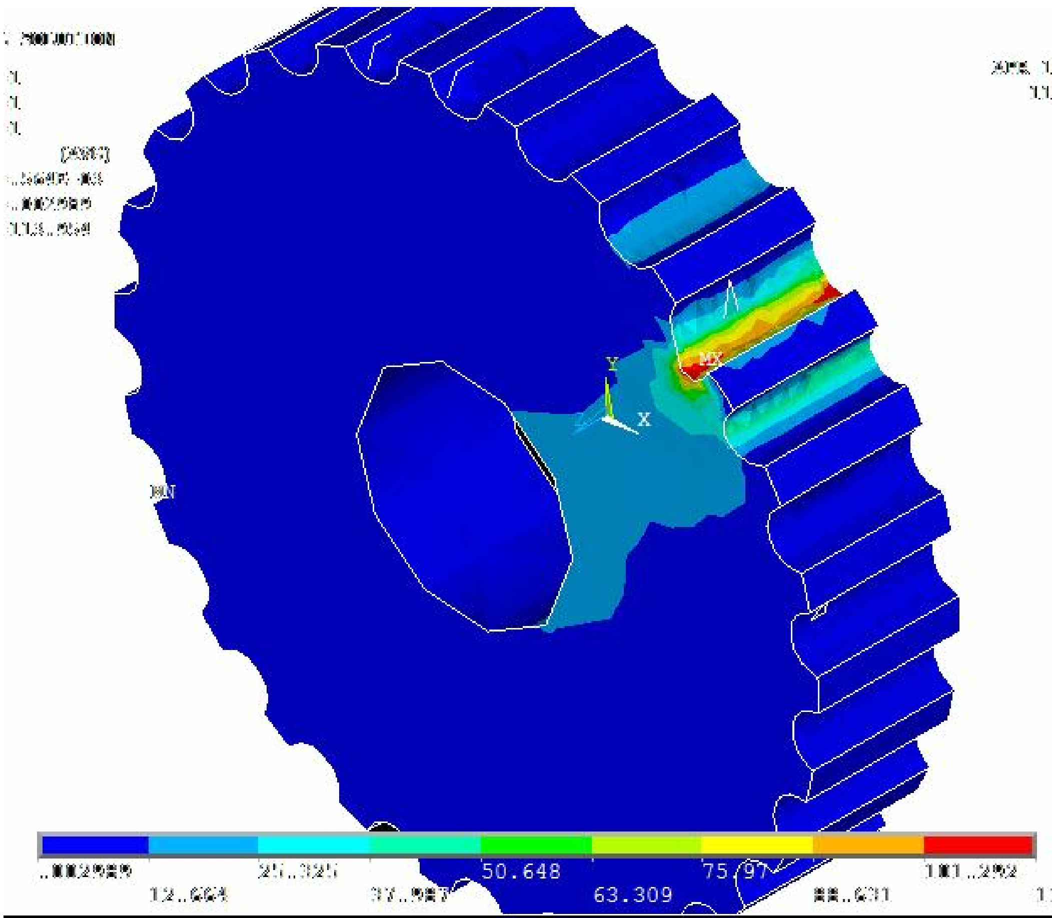


Fig 4.10: **stress concentration in tooth section**

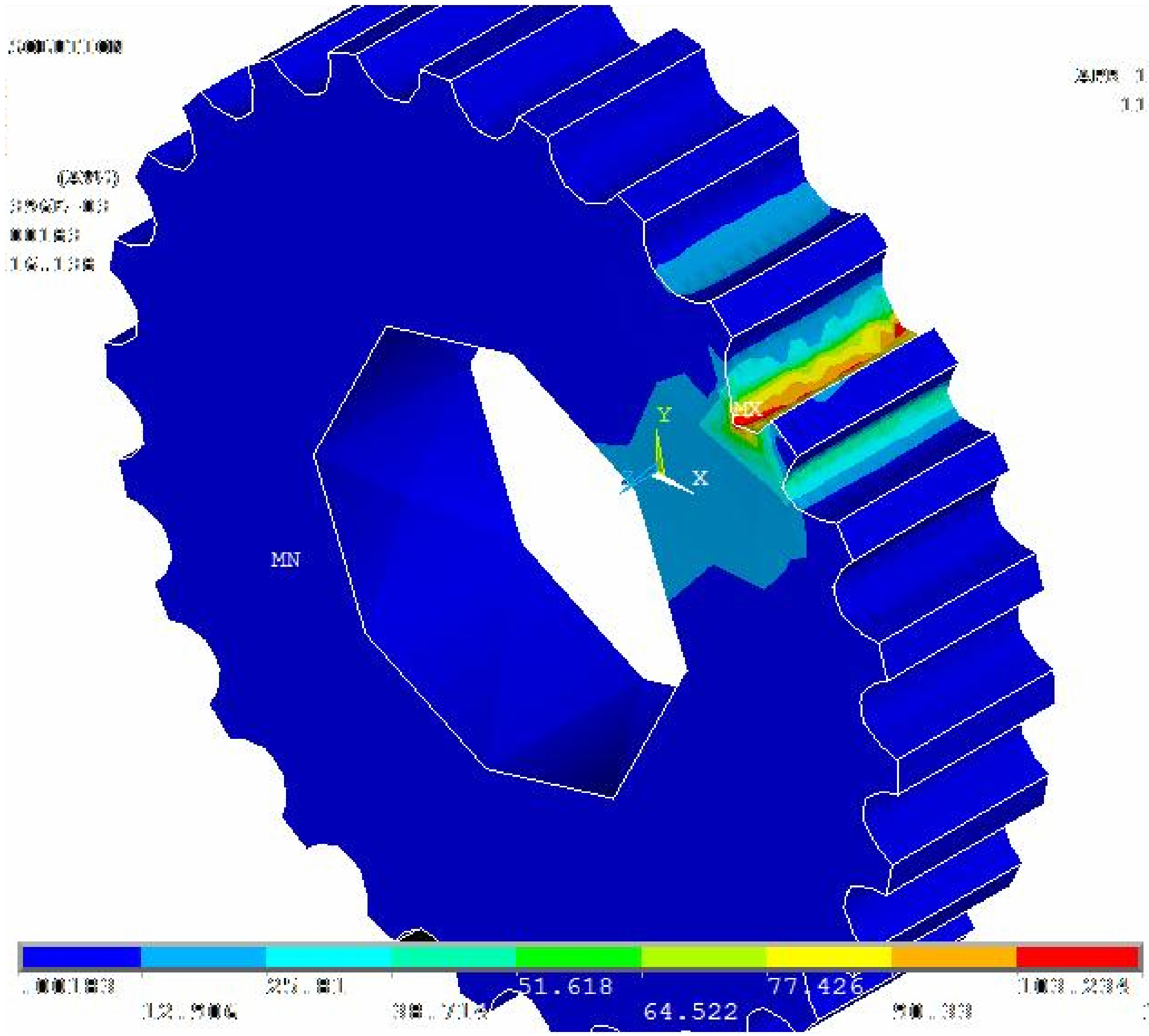


Fig 4.11: **stress concentration at tooth section**

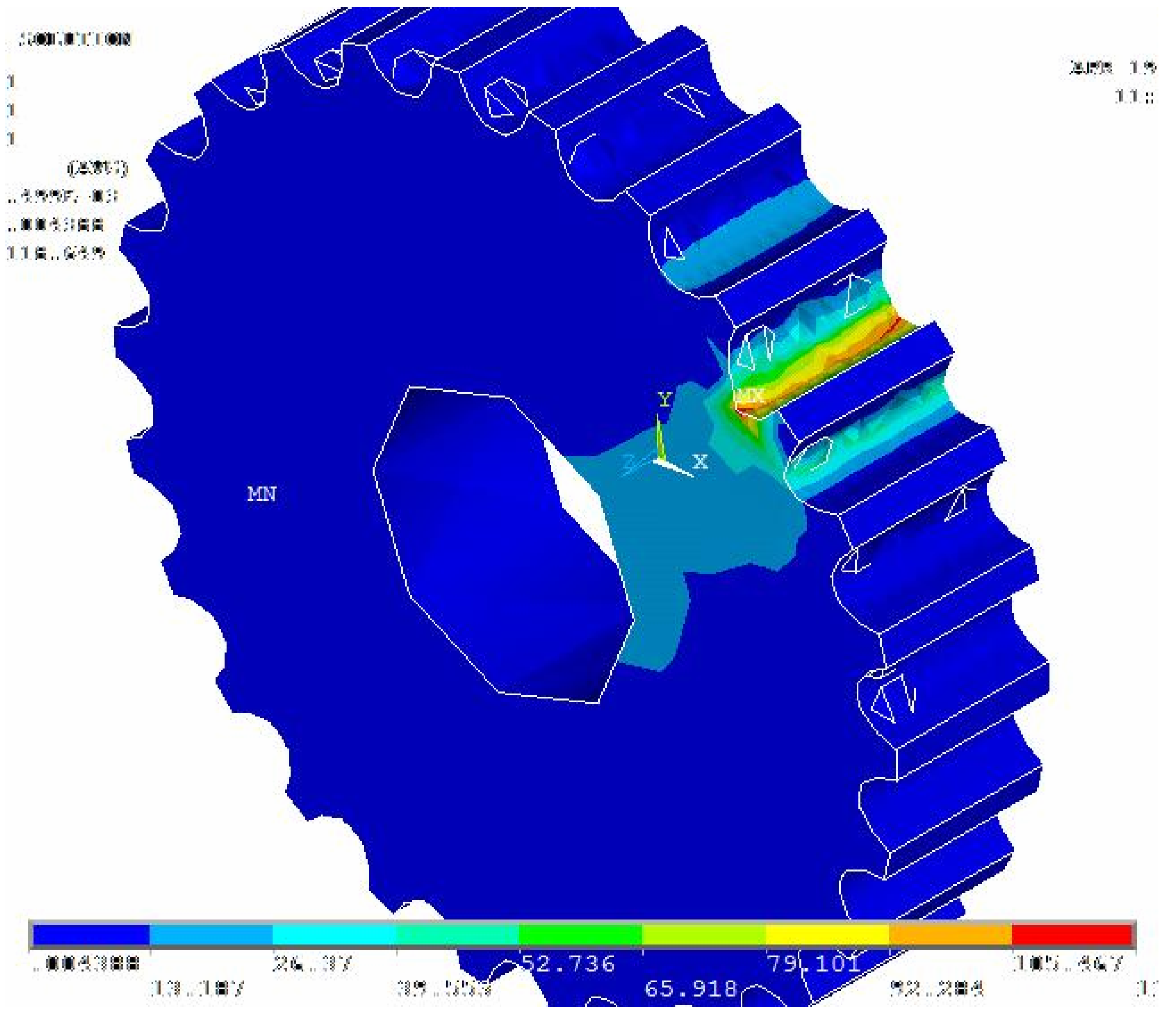


Fig 4.12: stress concentration at tooth section

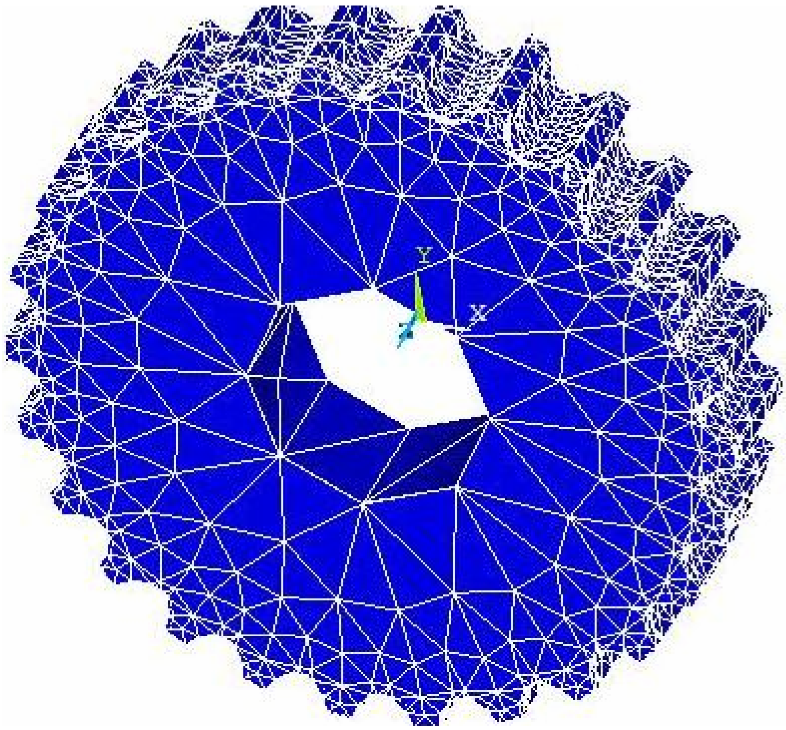


Fig 4.13: **tooth deformation**

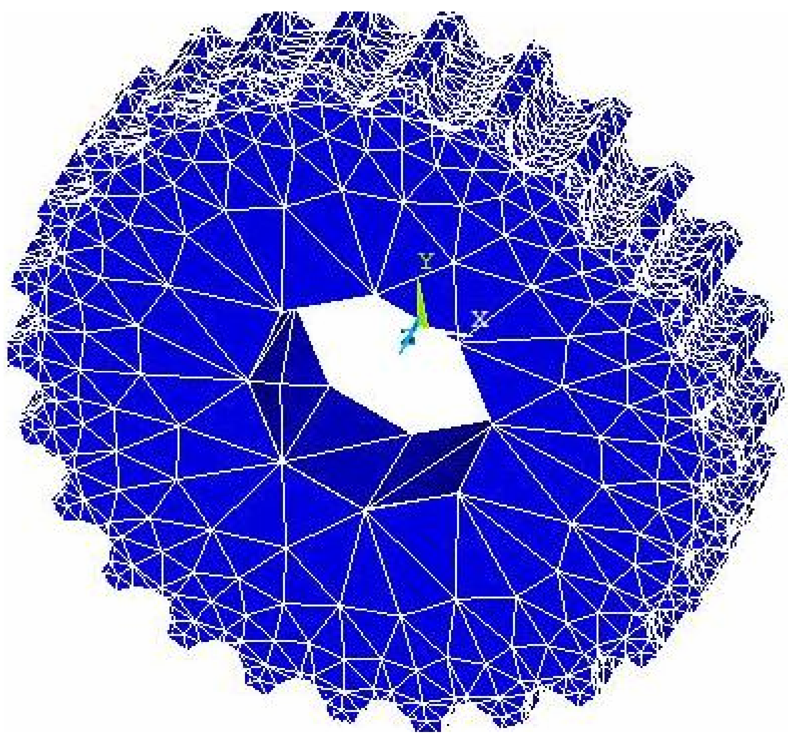


Fig 4.14: tooth deformation

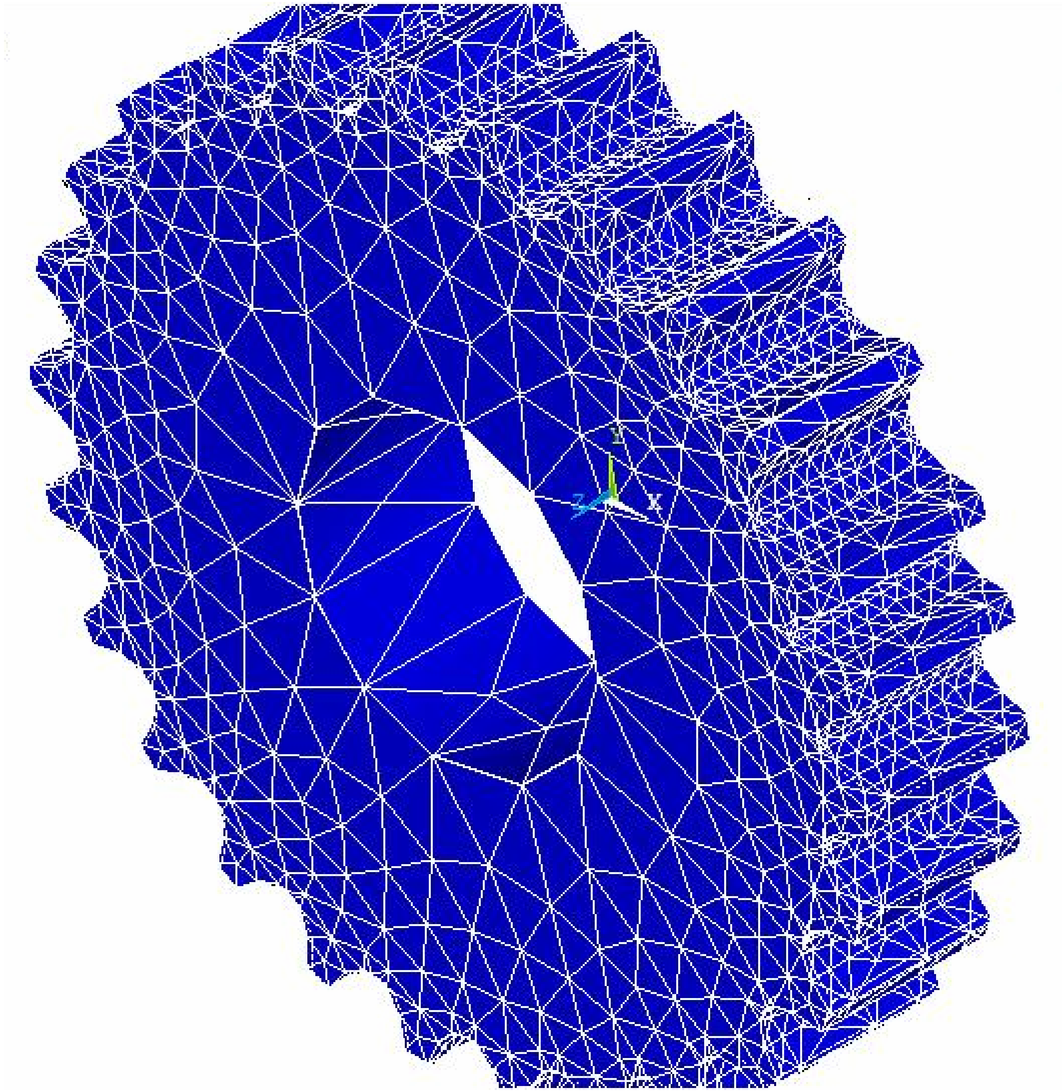


Fig 4.15: **tooth deformation**

CHAPTER 5

CONCLUSION

5.CONCLUSION:

Root stress Factor depends upon the Addendum modification coefficient as shown in the graphs. The effect of correction factor on root stress when only the driven gear is corrected is found out for various pressure angles. Then Root stress factor is also found when both driver gear and the follower gear are modified at the same time for different gear ratios. The results are discussed in detail in the previous chapters.

The results obtained from the investigation are summarized as follows:

1. The root stress factor for calculating root stress decreases significantly with an increasing addendum modification coefficient. It also decreases with an increase in pressure angle.
2. The tooth thickness at critical section becomes higher with positive addendum modification coefficient. Tooth thickness also increases with an increase in pressure angle. So as the tooth thickness becomes greater at critical section the load carrying capacity of gear increases considerably.
3. The root stress factor decreases further when both the driver and the follower wheel are modified at the same time. Root stress factor attains the lowest value when driver wheel gets positive maximum correction where as the follower gets the negative correction.
4. The root stress factor is also decreased as gear ratio increases but the root stress factor increases when gear ratio decreases.
5. The values obtained from finite element analysis of the gears also confirm with the above results.

From the above results, it is clear that by doing proper addendum modification, the root stress on gear tooth can be decreased considerably and hence the strength of gear tooth is increased. It is also clear that by suitable addendum modification, the load carrying capacity of the gear can be increased to a great extent. In addition to all these, a smaller number of teeth on gear can be adopted and the most useful advantage is that interference can be avoided. Hence the choice of addendum modification is of great use.

CHAPTER-6

FUTURE WORK

6.SCOPE FOR FURTHER WORK

1. We know, when the profiles of two meshing teeth contact at the pitch point, the motion is one of pure rolling without slippage. As the contact point moves up or down the line of contact, the motion is a combination of rolling and sliding, so when we carry out addendum modification the motion becomes both rolling and sliding. Now the sliding velocity has got a direct impact on the amount of abrasion wear. Moreover sliding velocity is an important design criterion for high speed gears. Hence an investigation should be carried out to study the effect of addendum modification on sliding phenomena in gear teeth.
2. Due to addendum modification, the pure rolling contact changes to a combination of rolling and sliding contact. Hence vibration in gear teeth is likely to change and due to vibration noise will also vary. So the effects of addendum modification on noise due to vibration must be studied to see whether addendum modification will increase noise or reduce it.

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APPENDIX

This is a programme on root stress and calculates the root stress factor for various pressure angles.

```
#include<stdio.h>
#include<conio.h>
#include<math.h>
FILE *IN,*OUT;
int main()
{

   float Lo,L,La,Ld,Lp,K,m,z,R,Rb,Ck,Ro,Ho,B,C,E,Rc,x,x1,y1,T2,P,Y,Ab,At,Ac,A;
   float Gamma,Gamma0,Gamma1,alpha,alpha0,alpha1,beta,alpha2,alphap,alphak;
   int i;
   OUT=fopen("Output2.txt","w");
   if(OUT==NULL)
   {
         printf("Cant write data in output file");
         exit(0);
   }

   fprintf(OUT,"hello");
   IN=fopen("Input2.txt","r");
   if(IN==NULL)
   {
         printf("Cant read data from the input file");
         exit(0);
   }
   fscanf(IN,"%f%f%f%f",&m,&z,&alpha1,&Lp);
   for(i=0;i<7;i++)
   {
              fscanf(IN,"%f",&x);
              alpha=(3.143/180.0)*alpha1;
              R=(m*z)/2.0;
              Rb=R*cos(alpha);
              Ck=.157*m;
              Ro=Ck/(1-sin(alpha));
              Ho=(1-x)*m-Ro*sin(alpha);
              Lo=3.143*m/4.0+(Ho*tan(alpha))+(Ro/cos(alpha))+(x*m*tan(alpha));
              beta=Lo/R;
              Gamma0=0.01;
              Gamma=(Ho-
.5773*Ho*tan(beta+Gamma0))*(1/(R*(.5773+tan(beta+Gamma0))));
              Gamma1=abs(Gamma-Gamma0);
```

```
		if(Gamma1>10e-5)
		{
			Gamma0=Gamma;
			Gamma=(Ho-
.5773*Ho*tan(beta+Gamma0))*(1/(R*(.5773+tan(beta+Gamma0))));
			Gamma1=abs(Gamma-Gamma0);
		}

printf("\nGamma%f\nR%f\nRo%f\nRb%f\nHo%f\nbeta%f",Gamma,R,Ro,Rb,Ho,beta);

Rc=(pow((Ho*Ho+R*R*Gamma*Gamma),1.5))/(R*R*Gamma*Gamma+R*Ho+Ho*Ho
)+Ro;
		B=sin(beta+Gamma);
		C=cos(beta+Gamma);
		E=R*Gamma;
		x1=(R-Ho)*B-E*C-Ro*(E*C+Ho*B)/sqrt(Ho*Ho+E*E);
		y1=(R-Ho)*C+E*B-Ro*(E*B+Ho*C)/sqrt(Ho*Ho+E*E);
		T2=2*x1;
		Ld=R-y1;
		alphak=acos(z*cos(alpha)/(z+2*(1+x)));
		alphap=atan(sqrt(((tan(alphak))*(tan(alphak)))-(2*Lp/Rb)));
		alpha0=tan(alpha)-alpha;
		alpha2=tan(alphap)-alphap;
		K=(3.143+(4*x*tan(alpha)))/(2*z)+alpha0-alpha2;
		P=(3.143/2.0)+K-alphap;
		La=(m*z/2.0)*(cos(alpha)*cos(K)/cos(alphap)-1);
		L=La+Ld;
		Y=m*z*cos(alpha)*sin(K)/(2*cos(alphap));
		Ab=6*L*sin(P)/(T2*T2);
		At=sin(P)/T2;
		Ac=-cos(P)/T2-6*Y*cos(P)/(T2*T2);
		A=(1+.08*T2/Rc)*((.66*Ab+0.4*(sqrt(Ab*Ab+36*At*At)))+1.15*Ac);
		fprintf(OUT,"\n X = %f\tAlpha1 = %f\tLp = %f",x,alpha1,Lp);
		fprintf(OUT,"\n Rc = %f\tT2 = %f\tLd = %f",Rc,T2,Ld);
		fprintf(OUT,"\n L = %f\tLa = %f\tK = %f",L,La,K);
		fprintf(OUT,"\n P = %f\tY = %f\tA = %f",P,Y,A);
	}
	return 0;
}
```

This is a programme on root stress and calculates the root stress factor, tooth thickness at critical section for different gear ratios when both the driver and follower are modified.

```
 #include<stdio.h>
#include<conio.h>
#include<math.h>
FILE *IN,*OUT;
int main()
{
   float
Lo,La,L,Ld,K,m,z1,Q,z2,R,Rb,Ck,Ro,Ho,B,C,E,Rc,x,x1,x3,y1,T1,T2,P,Y,Ab,At,Ac,A;
   float
Gamma,Gamma0,Gamma1,alpha,alpha0,alpha1,beta,alpha2,alpha3,alphap,alphak;
   int i;
   OUT=fopen("Output3.txt","w");
   if(OUT==NULL)
   {
        printf("Cant write data in output file");
        exit(0);
   }
   IN=fopen("Input3.txt","r");
   if(IN==NULL)
   {
        printf("Cant read data from the input file");
        exit(0);
   }

   fscanf(IN,"%f%f%f%f",&m,&z1,&alpha1,&x3,&Q);
   for(i=0;i<7;i++)
   {
             fscanf(IN,"%f",&x);
             alpha=(3.143/180.0)*alpha1;
             R=(m*z1)/2.0;
             Rb=R*cos(alpha);
             Ro=0.375*m;
             Ho=(1-x)*m-Ro*sin(alpha);
             Lo=3.143*m/4.0+(Ho*tan(alpha))+(Ro/cos(alpha))+(x*m*tan(alpha));
             beta=Lo/R;
             Gamma0=0.01;
             Gamma=(Ho-
.5773*Ho*tan(beta+Gamma0))*(1/(R*(.5773+tan(beta+Gamma0))));
             Gamma1=abs(Gamma-Gamma0);
             if(Gamma1>10e-5)
             {
```

```
                    Gamma0=Gamma;
                    Gamma=(Ho-
.5773*Ho*tan(beta+Gamma0))*(1/(R*(.5773+tan(beta+Gamma0))));
                    Gamma1=abs(Gamma-Gamma0);
            }

fprintf(OUT,"\nGamma%f\nR%f\nRo%f\nRb%f\nHo%f\nbeta%f",Gamma,R,Ro,Rb,Ho,
beta);

Rc=(pow((Ho*Ho+R*R*Gamma*Gamma),1.5))/(R*R*Gamma*Gamma+R*Ho+Ho*Ho
)+Ro;
            B=sin(beta+Gamma);
            C=cos(beta+Gamma);
            E=R*Gamma;
            x1=(R-Ho)*B-E*C-Ro*(E*C+Ho*B)/sqrt(Ho*Ho+E*E);
            y1=(R-Ho)*C+E*B-Ro*(E*B+Ho*C)/sqrt(Ho*Ho+E*E);
            T2=2*x1;
            Ld=R-y1;
            alphak=acos(z1*cos(alpha)/(z1+2*(1+x)));
            T1=((3.143/2.0)+2*x*tan(alpha))*m;
            z2=Q*z1;
            alpha0=tan(alpha)-alpha;
            alpha3=alpha0+2*tan(alpha)*((x+x3)/(z1+z2));
            alphap=tan(alphak)-(T1/(2*R))-alpha3;
            alpha2=tan(alphap)-alphap;
            K=(3.143+(4*x*tan(alpha)))/(2*z1)+alpha0-alpha2;
            P=(3.143/2.0)+K-alphap;
            La=(m*z1/2.0)*(cos(alpha)*cos(K)/cos(alphap)-1);
            L=La+Ld;
            Y=m*z1*cos(alpha)*sin(K)/(2*cos(alphap));
            Ab=6*L*sin(P)/(T2*T2);
            At=sin(P)/T2;
            Ac=-cos(P)/T2-6*Y*cos(P)/(T2*T2);
            A=(1+.08*T2/Rc)*((.66*Ab+0.4*(sqrt(Ab*Ab+36*At*At)))+1.15*Ac);
            fprintf(OUT,"\n X = %f\tAlpha1 = %f\tQ = %f",x,alpha1,Q);
            fprintf(OUT,"\n Rc = %f\tT2 = %f\tLd = %f",Rc,T2,Ld);
            fprintf(OUT,"\n L = %f\tLa = %f\tK = %f",L,La,K);
            fprintf(OUT,"\n P = %f\tY = %f\tA = %f",P,Y,A);
            fprintf(OUT,"\n X3 = %f",x3);
   }
   return 0;
}
```

This is a programme on addendum modification coefficient.

```
#include<stdio.h>
#include<conio.h>
#include<math.h>
FILE *IN,*OUT;
int main()
{
   float m,Zmin,T1,Z,R,R2,X1,Xu;
   float alpha,alpha0,alpha1,alpha2,alpha3,alpha4,alpha5;
   int i;
   OUT=fopen("Output1.txt","w");
   if(OUT==NULL)
   {
        printf("Cant write data in output file");
        exit(0);
   }
   IN=fopen("Input1.txt","r");
   if(IN==NULL)
   {
        printf("Cant read data from the input file");
        exit(0);
   }

   fscanf(IN,"%f%f",&m,&alpha1);
   alpha=(3.143/180.0)*alpha1;
   Zmin=2.0/((sin(alpha))*(sin(alpha)));
   T1=(3.143*m)/2.0;
   for(i=0;i<6;i++)
   {
             fscanf(IN,"%f",&Z);
             R=(Z*m)/2.0;
             alpha0=tan(alpha)-alpha;
             alpha2=T1/(2*R)+alpha0;
             alpha4=.01;
             alpha3=tan(alpha4)-alpha2;
             alpha5=abs(alpha3-alpha4);
             if(alpha5>10e-5)
             {
                     alpha4=alpha3;
                     alpha3=tan(alpha4)-alpha2;
                     alpha5=abs(alpha3-alpha4);
             }
             fprintf(OUT,"\nAlpha3 = %f",alpha3);
```

```
            R2=(R*cos(alpha))/cos(alpha3);
            X1=(Zmin-Z)/Zmin;
            Xu=(R2-R-m)/m;
            fprintf(OUT,"\nX1 = %f\tXu = %f",X1,Xu);
            fprintf(OUT,"\nZmin = %f\tAlpha1 = %f",Zmin,alpha1);
            fprintf(OUT,"\nZ =  %f",Z);
   }
   return 0;
}
```