VIBRATIONAL ANALYSIS OF SANDWICH BEAMS

A PROJECT REPORT SUBMITTING IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE IN

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Mechanical Engineering

By

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CERTIFICATE

This is to certify that the thesis entitled, "VIBRATIONAL ANALYSIS OF SANDWICH BEAMS" submitted by Sri Alankar Sharan and Sri. Atul Kumar in partial fulfillment of the requirements for the award of Bachelor of Technology Degree in Mechanical Engineering at the NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA (Deemed University) is an authentic work carried out by them under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma .

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CONTENTS

ABSTRACT

The project report deals first with the brief introduction on composites and their types and uses. Then it explains about the vibration of sandwich beams. The consistent higher-order dynamic formulation for foam-type (soft) core sandwich beams was extended to the case of composite sandwich plates. Eight dynamic governing equations and the corresponding boundary conditions were derived through the application of Hamilton's principle. The extended formulation was applied to the free vibration analysis of soft-core and honeycomb-core sandwich plates with anti-symmetric and symmetric lay-ups.

The vibration results for the thin and thick composite sandwich plates obtained using the extended formulation were consistent with the predictions of the higher order mixed layer wise theory for laminated and sandwich plates. To simplify the formulation for the case of symmetric sandwich plates, the general dynamic formulation was decoupled into two systems of equations representing symmetric and anti-symmetric vibrations. The numerical study demonstrates the importance of the present formulation for the prediction of higher mode vibration response of composite sandwich plates.

INTRODUCTION

1.1 History

Composite Materials, which made a new revolution in materials industry, are known to mankind since ancient time. Composite have been used by humans in past. The most primitive composite material that was made by man was a brick for building construction. It comprised straw and mud, wherein straw acted like fibers and mud as matrix. Mankind reinvented the benefits of composite materials in 1960s. Since then, Composite materials have created an impact on the development of the aerospace structures because of their superior fatigue characteristics, damage tolerance and strength to weight ratio compared to that of metals. The use of composite is not only restricted to Aerospace domain and now it is being used extensively in Automotive, Industrial applications and Civil structures domain. Another important reason for the popularity of composite materials is due to the significant operating cost reduction due to its use. Considerable weight reduction is achieved using composite materials.

The composite plates are used in various applications, ranging from Aerospace to marine industry. As composites are having better properties than metals, they can be exposed severe loading conditions and environment.

1.2 What are composites?

In their broadest form, composites are a material which consists of two or more constituents. The constituents are combined in such a way that they keep their individual physical phases and they are neither are not soluble in each other nor to form a new chemical compound. One constituent is called reinforcing phase and the one in which the reinforcing phase is embedded is called matrix. Historical or natural examples of composites are abundant: brick made of clay reinforced with straw, mud wall with bamboo shoots, concrete, concrete reinforced with steel rebar, granite consisting of quartz, mica and feldspar, wood (cellulose fibers in lignin matrix), etc.

1.3 What are hybrid composites?

They are composites hybridized with other materials (either other composites or unreinforced materials) or composites using multiple reinforcements and matrices. For example, carbon fibers and glass fibers are both included in the epoxy matrix; a compressed natural gas tank consists of aluminum liner and carbon fiber/epoxy composite over-wrapping.

1.4 Advantages of the Composites

- High specific stiffness and high specific strength .weight reduction for the same strength applications.
- High corrosion resistance. Acid, alkali resistance of polymers, chemical and marine applications, infrastructure applications.
- High impact resistance .High internal damping of Kevla fiber/epoxy composites, ballistics protection.
- High wear resistance .Ceramic particle reinforced metal matrix composites, ceramic matrix composites.
- Resistance to high temperature and extreme mechanical, environmental conditions:
- Tailor-able properties. Design both materials and structures.

  Choose appropriate combination of reinforcements and matrices.

  Choose optional fiber orientation and lay-up sequences.

1.5 Disadvantages of composite materials

- Higher cost
- Complexity in mechanical characterization and difficulty in analysis
- Weak in transverse direction and low toughness
- Difficulty in attaching (Joining)
- Environmental degradation (Polymer matrix absorb moisture)

1.6 Classification of the composite

1. **According to the types of fiber reinforcement**
    a. Fiber-reinforcement composite (Fibrous composite)
    b. Particle-reinforcement composite (Particulate composite)
2. **According to the matrix material**
    a. Metal matrix composite (MMC)
    b. Polymer matrix composite (PMC)
3. **According to the particulate orientation**
    a. Random orientation composite
    b. Preferred orientation composite
4. **According to the number of fiber constituent**
    a. Simple composite
    b. Hybrid composite
5. **According to the fiber type**
    a. Continuous fiber composite
    b. Discontinuous fiber composite
6. **According to the direction of the fiber**
    a. Unidirectional reinforcement composite
    b. Bi-directional reinforcement composite

1.7 Application of the Composite materials

1. Aerospace industries
2. Civil engineering
3. Space industries
4. Marine applications
5. Defense industries
6. Automobile industries
7. Electrical applications

1.8 Terminology used in composite materials

1. Isotropic materials-Properties of the materials are same in all the direction. (Short, discontinuous fiber reinforcement composite)
2. Anisotropic materials-Properties are different in different direction (Long continuous fiber reinforcement composite)
3. Reinforcement-composite consists of one or more continuous/discontinuous phase embedded in the continuous phase. The discontinuous phase, generally stronger than the continuous phase is called reinforcement or reinforcing material whereas continuous phase is called as matrix.
4. Controlled anisotropy-means the desired ratio of the property values in different direction can easily obtained. Ex. Unidirectional composite in which ration of longitudinal to transverse modulus ratio can easily altered by changing the fiber volume fraction.
5. Volume fraction-Fractional volume of the any constituent (fiber/matrix material) in a composite is referred as a volume fraction.
6. Weight fraction- Fractional weight of the any constituent (fiber/matrix material) in a composite is referred as weight fraction of the individual constituent.
7. Aspect ratio-The ratio of diameter of the fiber to length of the fiber
8. Lamina-A single layer of the composite material is generally referred as a ply or lamina. (Thickness=013mm)
9. Laminate- Several laminas are bonded together to form the laminates. Here each lamina should have different properties.
10. Homogeneous anisotropy- Material properties are the function of the direction but do not change from point to point

2. Higher Order Sandwich Panel Theory

The free vibrations of composite sandwich plates have been extensively studied using classical analytical formulations, finite element analyses based on first- and higher-order shear deformation theories. In the majority of these studies, the sandwich core is assumed to be incompressible in the vertical direction. This assumption is practically accurate only for the vibration analysis of sandwich plates with a honeycomb core. However, in the case of a flexible sandwich core (for example, a foam core), this assumption will preclude modeling of the symmetric vibration modes where the two face sheets move out-of-phase. Furthermore, modeling of a flexible sandwich core with the aid of general-purpose commercial finite element software requires the use of 3-D solid elements, which consumes significant computational resources (memory and cpu time).

The higher-order sandwich panel theory was derived to model the behavior of sandwich plates with a flexible core. This model is based on the nonlinear through-the-thickness displacement field in the core in both longitudinal and vertical directions. However, the corresponding acceleration field in the core is assumed to vary linearly with height, which introduces inconsistency in the formulation. For sandwich beams, this inconsistency has been overcome in the recently developed formulation that accounts for a nonlinear acceleration field in the core.

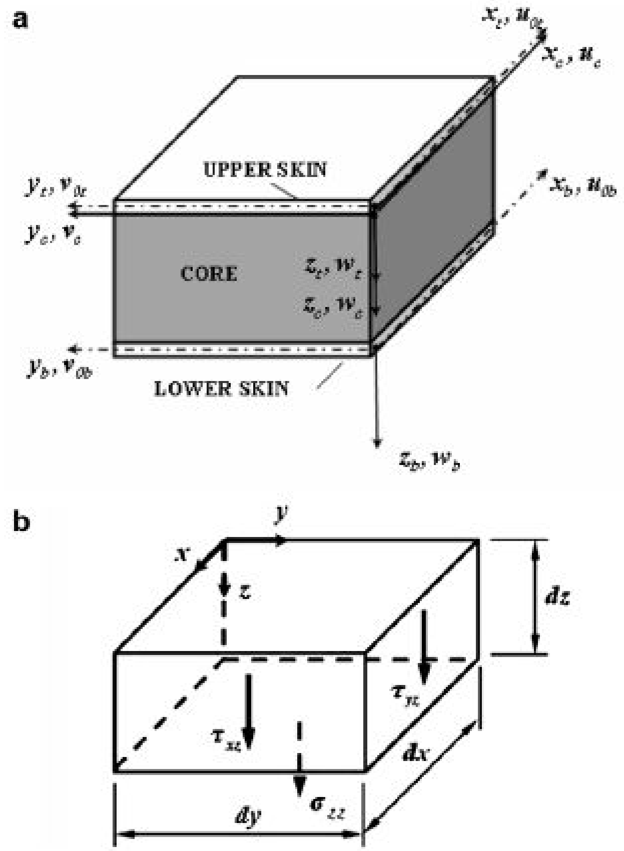
In the present work, the one-dimensional beam formulation described is generalized to the two-dimensional plate analysis. The main difference between the present approach and the higher order mixed formulation in Ref. [3] is that in the latter case, the assumed through-the-thickness displacement field is used, whereas in the former, non-linear displacement field in the core is derived based on well-defined physical assumptions. In what follows, a consistent free vibration formulation for composite sandwich plates is developed in terms of the system of governing partial differential equations and the corresponding boundary conditions.

The derived formulation is applied to the vibration analysis of soft-core and honeycomb-core sandwich plates with anti-symmetric and symmetric lay-ups. Excellent agreement between the calculated vibration response for the thin and thick composite sandwich plates and the results of the higher order mixed layer wise theory is demonstrated.

For the case of symmetric sandwich plates, simplification is achieved through the decoupling of the general formulation into two independent systems of equations representing symmetric and anti-symmetric motions. The numerical study of the free vibration response of composite sandwich plates based on the derived formulation follows next. Differences between the vibration responses of the soft-core sandwich plates with anti-symmetric and symmetric composite layups are studied and discussed. The effect of honeycomb core modulus on the symmetric vibration response of sandwich plates with isotropic face sheets is investigated. Finally, the importance of the present formulation for the prediction of higher-frequency vibration response is discussed and conclusions are drawn.

3. Mathematical Assumptions

3.1. Assumptions



Sandwich Plate conventions: (a) Geometry, co-ordinate systems and displacement functions of the sandwich plate. (b) Stress field in the core

The present formulation is concerned with the linear vibration analysis of sandwich plates (see Fig. 1). The face sheets of a sandwich plate are assumed to behave as Kirchhoff thin plates with negligible shear deformations. The vertically compressible core layer is considered as an antiplane, three-dimensional elastic medium with orthotropic out-of-plane shear properties. Here, an antiplane assumption implies that the stresses in the core in planes parallel to the face sheets are neglected, and only the out of plane shear and normal stresses are accounted for the analysis.

This assumption is nearly exact for a honeycomb core, and is an appropriate approximation for an isotropic foam core, where the core material modulus is significantly lower than the modulus of the face sheets (see Section 1). Note that as a result of the compressibility of the core, the core height may change under loading and the cross section may not remain planar. The interface layers between the face sheets and the core are assumed to provide perfect continuity of the deformations at the interfaces. The acceleration fields of the face sheets are assumed to vary linearly with height, whereas the acceleration field of the compressible core varies nonlinearly with height in the plane and out-of-plane directions. Thus, the dynamic fields in the face sheets and core are consistent with the corresponding static patterns as normally assumed in dynamic analyses

3.2. Equations of motion

The governing differential equations and the corresponding boundary conditions are derived here through the use of Hamilton's variational principle, namely,

$$\int_{t_1}^{t_2} (\delta T - \delta V)\,\mathrm{d}t = 0$$

where T is the kinetic energy and V is the strain energy of a sandwich plate; t is the time coordinate; and t1 and t2 are the values of the time coordinate at the beginning and end of the motion, respectively. The strain energy of the composite face sheets is given by

$$V_F = \int_0^a \int_0^b \sum_{i=t,b} [1/2A_{i11}u_{0i,x}^2 + 1/2A_{i22}v_{0i,y}^2 + 1/2A_{i66}(u_{0i,y}+v_{0i,x})^2 + A_{i\,12}u_{0i,x}v_{0i,y} + A_{i16}(u_{0i,y}+v_{0i,x})u_{0i,x} + A_{i26}(u_{0i,y}+v_{0i,x})v_{0i,y} - B_{i11}u_{0i,x}w_{i,xx} - B_{i22}v_{0i,y}w_{i,yy} - 2B_{i66}(u_{0i,y}+v_{0i,x})w_{t,xy} - B_{i12}(v_{0i,y}w_{i,xx} + u_{0i,y}w_{i,yy}) - B_{i16}(u_{0i,y}+v_{0i,x})w_{i,xx} - B_{i26}(u_{0i,y}+v_{0i,x})w_{i,yy} - 2B_{i16}u_{0i,x}w_{i,xy} - 2B_{i26}v_{0i,y}w_{i,xy} + ½\,D_{i11}w_{i,xx}^2 + 1/2D_{i22}w_{i,yy}^2 + 2D_{i66}w_{i,xy}^2 + D_{i12}w_{i,xx} + 2D_{i16}w_{i,xx}w_{i,xy} + 2D_{i26}w_{i,yy}w_{i,xy}]\,dx_i dy_i \qquad (2)$$

where a and b are the in-plane dimensions of the sandwich plate; $u_{0i}(x_i, y_i, t)$, $v_{0i}(x_i, y_i, t)$, and $w_i(x_i, y_i, t)$ are the longitudinal, transverse and vertical unknown displacement functions of the centroid line of the face sheets, respectively $(i = t, b)$; t and b refer to quantities affiliated to top and bottom face sheets, respectively; A_{ijk}, B_{ijk}, and D_{ijk} ($i = t, b$; $j, k = 1, 2, 6$) are the coefficients of the stiffness matrices **A**, **B**, **D** in the force–strain relations for the face sheets; and the comma stands for differentiation with respect to the spatial coordinate. The kinetic energy of the face sheets reads

$$T_F = \tfrac{1}{2} \int_0^a \int_0^b [\rho_t d_t(u_{0t}^2 + v_{0t}^2 + w_t^2) + 1/12\rho_t d_t^3(w_{t,y}^2 + w_{t,x}^2)]dx_t dy_t +$$

$$\tfrac{1}{2} \int_0^a \int_0^b [\rho_b d_b(u_{0b}^2 + v_{0b}^2 + w_b^2) + 1/12\rho_b d_b^3(w_{b,y}^2 + w_{b,x}^2)]dx_b dy_b$$

where d_t and d_b are the thicknesses of the upper and lower face sheets, respectively; ρ_t and ρ_b are the densities of the upper and lower face sheets, respectively; and the dot represents time differentiation.

As described in Section 3.1, the transversely flexible core is treated as a three-dimensional elastic medium with small deformations, where the core height may change under loading, and the core cross-section does not remain planar (nonlinear displacement field in the core). The in-plane stresses in the core are neglected because of the negligible in-plane strength compared to the face sheets, which implies that

$$\sigma_{xxc} = \sigma_{xxy} = \tau_{xyc}$$

where σ_{xxc}; σ_{xxy} and τ_{xyc} are the in-plane normal and shear stresses in the core. For this case, the partial differential equations describing the equilibrium in the core can be uncoupled and solved analytically for the longitudinal, transverse and vertical displacements, and the resultant expressions are generalized here to include the time parameter, as shown below

$$u_c(x_c y_c, z_{c,t}) = [\{z_c^2(2z_c - c)\}/12](\tau_{xx,xx}(x_c,y_c,t) + \tau_{yx,xy}(x_c,y_c,t)) + [\{z_c^2\}/2c](w_{t,x}(x_c,y_c,t) - w_{bx,x}(x_c,y_c,t) + z_c\tau_{xzc}(x_c,y_c,t) - z_c w_{t,x}(x_c,y_c,t) + u_{0t}(x_c,y_c,t) - 1\backslash 2d_t w_{t,x}(x_c,y_c,t) \quad (5)$$

$$v_c(x_c,y_c,z_c,t) = [\{z_c^2(2z_c\text{-}3c)\}/12E_c](\tau_{xx,xy}(x_c,y_c,t)+\tau_{yzc,yy}(x_c,y_c,t))+ [\{z_c^2\}/2c](w_{t,y}(x_c,y_c,t) - w_{b,y}(x_c,y_c,t)) + [z_c/G_{cy}]\ \tau_{yx}(x_c,y_c,t) - z_c w_{t,y}(x_c,y_c,t) + v_{0t}(x_c,y_c,t) - ½\ d_t w_{t,y}(x_c,y_c,t) \quad (6)$$

$$w_c(x_c,y_c,z_c,t) = [\{z_c(z_c - c)\}/2E_c]\ (\tau_{xzc,x}(x_c,y_c,t) + \tau_{yzc,y}(x_c,y_c,t)) - \{w_t(x_c,y_c,t) + w_b(x_c,y_c,t)\}/c]z_c + w_t(x_c,y_c,t) \quad (7)$$

Here, E_c is the vertical Young's modulus of the core, G_{cx} and G_{cy} are the vertical shear moduli of the core, τ_{xzc} and τ_{yzc} are the shear stresses in the core, and c is the thickness of the core. The strain energy and kinetic energy of the transversely flexible core can be calculated from the displacements in Eqs. (5)–(7). The strain energy of the core reads

$$Vc = [c^3/24Ec(\tau_{xzc,x} + \tau_{yzc,y})^2 + E_c/2c(w_t - w_b)^2 + c/2(\tau_{xzc}^2/G_{cx} + \tau_{yzc}^2/G_{cy})]dx_c dy_c \quad (8)$$

The expression for the kinetic energy of the core reads

$$T_c = 1\backslash 2\ \rho_c(u_c^2+v_c^2+w_c^2)dx_c dy z_c dz_c \quad (9)$$

where ρ_C is the density of the core.

Eqs. (5)–(7) account only for the compatibility between the top face sheet and the core. The compatibility conditions at the lower interface are accounted for by

$$u_c(x_c,y_c,c,t) = u_{0b}(x_c,y_c,t)+1\backslash 2d_b w_{b,x}(x_c,y_c,t) \quad (10)$$

$$v_c(x_c,y_c,c,t) = v_{0b}(x_c,y_c,t)+1\backslash 2d_b w_{b,y}(x_c,y_c,t) \quad (11)$$

The governing equations are then obtained from Eq. (1) using the Lagrange multiplier method with the auxiliary equations given in Eqs. (10) and (11). The Lagrange multipliers can be shown to be the shear stresses in the core, τ_{xzc} and τ_{yzc} [15].

3.3. Symmetric sandwich plate

The majority of sandwich plates used in practical applications have symmetric cross-sections. Therefore, it is worthwhile to simplify the general formulation derived in Section 3.2 for the symmetric case. For this purpose, the general horizontal and vertical motions of the face sheets are represented as the superposition of the symmetric and anti-symmetric displacements (see Fig. 2).

$$u_{oi} = u' \pm u' \quad (12)$$

$$v_{oi} = v' \pm v' \quad (13)$$

$$w_i = w' \pm w' \quad (14)$$

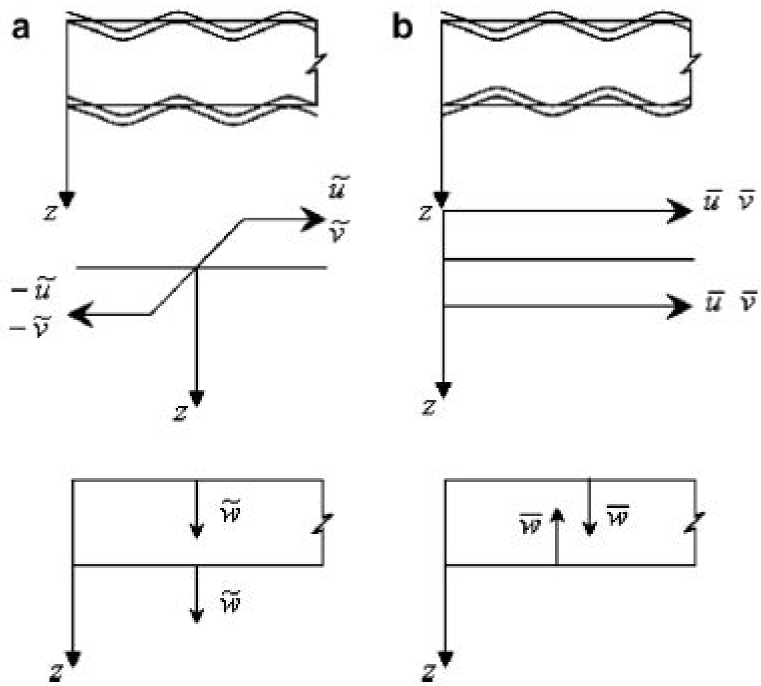
where i = t, b stands for the top and bottom face sheets, respectively, the plus and minus signs are used for the upper and lower face sheets, respectively, and u', v', w' denote the symmetric and anti-symmetric displacements of the face sheets, respectively.

In the following, the displacement field in the core flexible in the vertical direction, corresponding to symmetric and anti-symmetric vibrations, is derived. Substitution of the relations $u_{0t} = u_{0b} = u'$ and $v_{0t} = v_{0b} = v'$, which characterize the symmetric response, into Eqs. (5)–(7) yields

$$-c^3 \backslash 12E_c(\tau_{xzc,xx}+\tau_{yzc,xy})+c \backslash G_c x\tau_{xyc} = 0 \quad (15)$$

$$-c^3 \backslash 12E_c(\tau_{xzc,xy}+\tau_{yzc,yy})+c \backslash G_{cy}\tau_{yzc} = 0 \quad (16)$$

These equations are satisfied for $\tau_{xzc} = \tau_{yzc} = 0$. This is a mathematical manifestation of the fact that the symmetric motion of the sandwich plate is characterized only by the expansion and compression of the core material, which are caused by the out-of-phase vertical displacements of the face sheets, in the absence of shear deformation. Substitution of the relations $u_{0t} = u_{0b} = u'$, $v_{0t} = v_{0b} = v'$ and $\tau_{xzc} = \tau_{yzc} = 0$ into Eqs. (5)–(7) produces the following displacement field in the core for the case of symmetric motion



$$u'_c \quad = \quad z_c^2 w'_{,x}\text{-}(z_c+d\backslash 2)w'_{,x}+\bar{u} \qquad (17)$$

$$v_c' \quad = \quad z_c^2\backslash 2w'_{,y}\text{-}(z_c+d\backslash 2)w'_{,y}+v' \qquad (18)$$

$$w'_c \quad = \quad (1\text{-}2z_c\backslash c)w' \qquad (19)$$

Eqs. (17)–(19) show that the tangential displacements in the core for symmetric motion are given by polynomials of the second order in the vertical coordinate z_c, whereas the vertical displacements depend linearly on z_c. Note particularly that neglecting the non-linear terms in Eqs. (17) and (18) will result in a tangential displacement field for ordinary plates consistent with the Kirchhoff assumptions. From Eq. (19), the vertical displacements of the core are seen to be the linear interpolation of the vertical displacements of the face sheets. Similarly, for the anti-symmetric motion, substitution of the relations $u_{0t} = u_{0b} = u'$, $v_{0t} = v_{0b} = v'$ and $w_{0t} = w_{0b} = w'$ into Eqs. (5)–(7) yields the following displacement field in the core:

$$u'_c = [\{2z_c^3\text{-}3cz_c^2\backslash 12E_c\}](\tau_{xzc,xx}+\tau_{yzc,xy})+\{[z_c\backslash G_{cx}\}]\tau_{xzc}(z_c+d\backslash 2)w'_{,x}+\bar{u} \quad (20)$$

$$v'_c = [\{2z_c^3\text{-}3cz_c^2\}]12E_c(\tau_{xzc,xy}+\tau_{yzc,yy})+[\{z_c\backslash G_{cy}\}]\tau_{yc}\_(z_c+d\backslash 2)w'_{,y}+v' \quad (21)$$

$$w'_c = -[\{z_c^2+cz_c\}]\backslash 2E_c(\tau_{xzc,}x+\tau_{yzc,y})+w' \quad (22)$$

Eqs. (20)–(22) reveal that the non-linear behavior of the core depends on the second-order derivatives of the shear stress. Therefore, for harmonic motion, the influence of the non-linear terms on the response of the core will increase with increasing frequency of the vibration modes (see Section 4). Finally, using the auxiliary Eqs. (10), (11) and Eqs. (5), (6), the tangential displacements in the core can be expressed in the form

$$u_c'=[\{z^2(2z\text{-}3c)\backslash c^3\}](2u'+c\backslash G_{cx}\tau_{xyc}\text{-}(c+d)w'_{,x})+z\backslash G_{cx}\tau_{xzc}+u'\text{-}(z+d\backslash 2)w'_{,x} \quad (23)$$

$$v_c'=[\{z^2(2z\text{-}3c)\backslash c^3\}](2v'+c\backslash G_{cx}\tau_{yzc}\text{-}(c+d)w'_{,y}+z\backslash G_{cy}\tau_{yzc}+v'\text{-}(z+d\backslash 2)w'_{,y} \quad (24)$$

where the second-order derivatives of the shear stress in the core have been eliminated. Thus, the general formulation for the free vibrations of symmetric soft-core sandwich plates with composite laminated face sheets has been decoupled into two formulations representing symmetric and anti-symmetric motions. The symmetric motion is represented by the three governing equations, nine boundary conditions, and three equations describing the displacements in the core, Eqs. (17)–(19), in terms of u', v' and w'. The anti-symmetric vibrations are represented by five governing equations, 17 boundary conditions and three equations for the displacements of the core, Eqs. (20)–(22), in terms of u' , v', w', τ_{xzc} and τ_{yzc}.

4. Validation

For the free vibrations of a simply supported sandwich plate, the boundary conditions are identically satisfied by the following harmonic functions

$$u_{0t}=U_t cos(m\pi\backslash a\ x)sin(n\pi\backslash b\ y)e^{i\omega t} \quad (25)$$

$$u_{0b}=U_b cos(m\pi\backslash a\ x)sin(n\pi\backslash b\ y)\ e^{i\omega t} \quad (26)$$

$$v_{0t}=V_t sin(m\pi\backslash a\ x)cos(n\pi\backslash b\ y)\ e^{i\omega t} \quad (27)$$

$$v_{0b}=V_b sin(m\pi\backslash a\ x)cos(n\pi\backslash b\ y)\ e^{i\omega t} \quad (28)$$

$$w_t=W_t sin(m\pi\backslash a\ x)sin(n\pi\backslash b\ y)\ e^{i\omega t} \quad (29)$$

$$w_b=W_b sin(m\pi\backslash a\ x)sin(n\pi\backslash b\ y)\ e^{i\omega t} \quad (30)$$

$$\tau_{xzc}=\Gamma_x cos(m\pi\backslash a\ x)sin(n\pi\backslash b\ y)\ e^{i\omega t} \quad (31)$$

$$\tau_{yzc}=\Gamma_y cos(m\pi\backslash a\ x)sin(n\pi\backslash b\ y)\ e^{i\omega t} \quad (32)$$

Substitution of Eqs. (25)–(32) into the general governing differential equations, produces the generalized algebraic Eigen value problem from which the natural frequencies and corresponding vibration modes of a sandwich plate can be calculated

4.1. Sandwich plate with composite face sheets and a soft core

The free vibration response of a five-layer (0/90/core/0/90), simply supported sandwich plate with planar dimensions a x b is analyzed here. The computed results are subsequently compared with the predictions of the higher order mixed layer wise theory for laminated and sandwich plates. The material constants for graphite-epoxy T300/934 composite face sheets and the isotropic core used in the calculations are given in Table 1. Note that the elastic moduli of the face sheets are given with respect to the material coordinates.

A comparison between the normalized natural frequencies, Ω, calculated using the present formulation and the results of Rao and Desai is presented in Table 2. The normalized frequencies are given by the equation $\Omega = \omega b^2 (\rho_t/E_2)^{1/2}/H$, where Ω is the natural frequency and H is the total thickness of the sandwich plate. Table 2 demonstrates excellent agreement between the predictions based on the present formulation and those in Ref. [3] Note particularly that the close agreement between the two theories holds in a broad range of length-to-thickness ratios. This means that the present formulation can be successfully used for the vibration analysis of both thin and thick soft core sandwich plates.

Note also that for each pair (m, n) (see Eqs. (25)–(32)), there are eight natural frequencies with eight corresponding vibration modes. Physically, this represents various combinations of the displacement patterns of the face sheets and the core corresponding to the same pair of wave numbers. Variations of the in-plane and vertical displacements through the thickness of the sandwich plate under consideration vibrating at the fundamental frequency (m = 1, n = 1), as obtained by the present analysis and that Ref. [3] , appear in Fig. 3.

The variations of the longitudinal displacements (along the x-axis) are given at the middle of the left edge of the sandwich plate (x = 0, y = b=2), Fig. 3a. Similarly, the through-the-thickness variation of the transverse displacements (along the y-axis) are given at the middle of the front edge of the plate (x = a/2, y = 0), Fig. 3b.

Table 1 : Material parameters of the antisymmetric sandwich plates

Component	Elastic modulus*(GPa)*	*Poisson's Ratio*	*Shear modulus(GPa)*	*Mass density(kg/m3)*
Face Sheets	$E_1 = 131$	$\nu_{12} = 0.22$	$G_{12} = 6.985$	$\rho_t = 1627$
	$E_2 = 10.34$	$\nu_{13} = 0.22$	$G_{13} = 6.985$	$\rho_b = 1627$
	$E_3 = 10.34$	$\nu_{23} = 0.49$	$G_{23} = 6.205$	
Core	$E_c = 6.89 \times 10^{-3}$	$\nu_c \approx 0$	$G_c = 3.45 \times 10^{-3}$	$\rho_c = 97$

Table 2 : Comparison of non dimensional frequencies Ω of (0/90/core/0/90) sandwich plate with a/b = 1 and c/d_t = 10

M	*n*	Rao and Desai [3]	Present Analysis
a/H = 100			
1	1	11.9401	11.8593
1	2	23.4017	23.3419
1	3	36.1434	36.1150
2	2	30.9432	30.8647
2	3	41.4475	41.3906
3	3	49.7622	49.7091
a/H = 10			
1	1	1.8480	1.8470
1	2	3.2196	3.2182
1	3	5.2234	5.2286
2	2	4.2894	4.2882
2	3	6.0942	6.0901
3	3	7.6762	7.6721

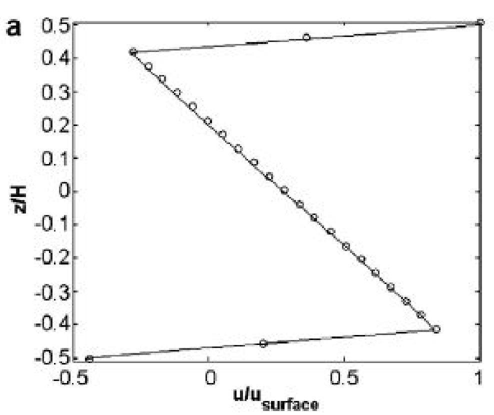


Fig. 3a

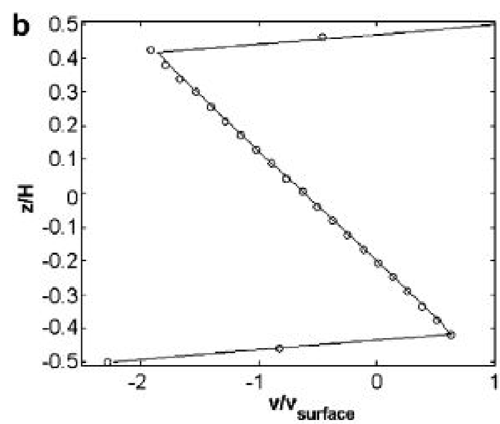


Fig. 3b

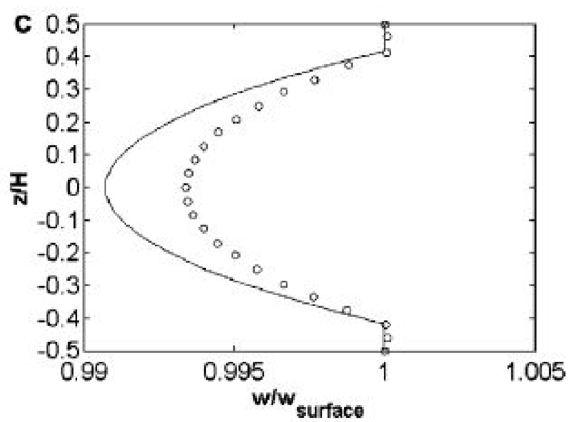


Fig. 3c

Finally, the distribution of the vertical displacements through the height of the plate are calculated in the center of the plate (x = a/2, y = b/2), as shown in Fig. 3c.

Fig. 3 shows that the distributions of the in-plane displacements from the present analysis practically coincide with those of Rao and Desai. The displacements in Fig. 3 were normalized to the longitudinal ($u_{surface}$), transverse ($v_{surface}$), and vertical ($w_{surface}$) displacement components at the outer surface of the lower skin (see Fig. 1). The same normalization is used in the following figures. A negligible discrepancy between the two theories (~0.3%) is detected for the vertical displacements of the core, which is explained by the fact that in the present formulation, the horizontal stresses in the core are neglected.

4.2 Symmetric sandwich plate with isotropic face sheets and a honeycomb core

A simply supported sandwich plate (1.83 m x 1.22 m) with symmetric cross-section, aluminum face sheets, and an aluminum honeycomb core is considered next. The material properties of the plate constituents are given in Table 3. In Table 4, the natural frequencies of the honey comb core sandwich plate computed using the present analysis are compared with the analytical results of Ref. [1]. and the experimental and analytical results of Ref. [4].. As shown in Table 4, the frequencies, which are obtained using the present formulation, are consistent with both previous experimental and numerical predictions. Thus, the present formulation yields accurate results for both soft core and honeycomb-core sandwich plates.

Table 3 : Material Parameters from Ref [3].

Component	*Thickness (mm)*	*Elastic modulus (GPa)*	*Poisson's ratio*	*Shear modulus(GPa)*	*Mass density (kg/m³)*
Face Sheets	0.4064	68.984	0.3	25.924	2768
Core	6.35	0.1379	0	$G_{xy} = 0$ $G_{xz} = 0.13445$ $G_{yz} = 0.05171$	121.8

Table 4: Natural frequencies of sandwich plate with aluminum face sheets and aluminum honeycomb core

Natural Frequency	Ref. [4]		FEM	SFPM	Present Analysis
	Experimental	Analytical			
f_1	-	23	23	23.29	23.04
f_2	45	44	44	44.47	44.16
f_3	69	71	70	71.15	69.76
f_4	78	80	80	78.78	79.17
f_5	92	91	90	91.57	90.24
f_6	125	126	125	125.10	124.27

FEM : Finite Element Method

SFPM : Spline Finite Point Method

5. Numerical study and discussion

5.1. Vibration response of soft-core sandwich plates with symmetric and anti-symmetric lay-ups of composite laminated face sheets

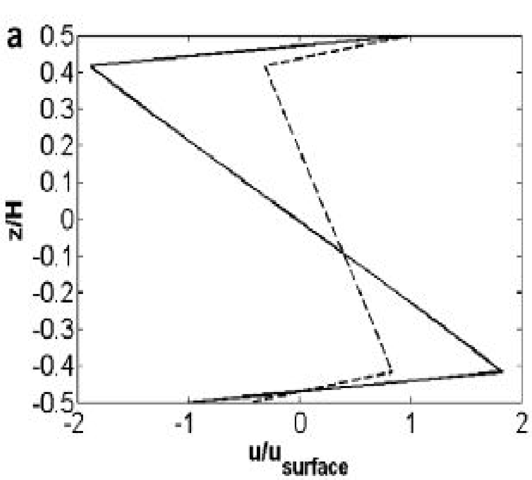
A comparison of the vibration response of soft-core sandwich plates with anti-symmetric (0/90/core/0/90) and symmetric (0/90/core/90/0) lay-ups is considered. The properties of the composite laminates and the core are given in Table 1. The first six normalized natural frequencies Ω for both anti-symmetric and symmetric sandwich plates are presented in Table 5. Note that the vibration frequencies of both thin (a/H = 100) and thick (a/H = 10) soft-core sandwich plates are presented. As evident from Table 5, the differences between the natural frequencies of the anti-symmetric and symmetric sandwich plates are negligible. Note that this equivalence of values for anti-symmetric and symmetric cases was tracked up to the pair of wave numbers m = 101, n = 101.

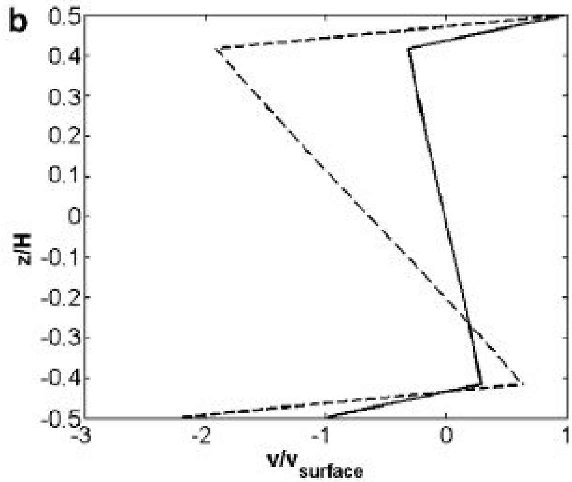
Table 5 : Natural frequencies of anti-symmetric and symmetric sandwich plates

M	*N*	Anti-symmetric (0/90/core/0/90)	Symmetric (0/90/core/0/90)
a/H = 100			
1	1	11.8593	11.8674
1	2	23.3419	22.7200
1	3	36.1150	34.9339
2	2	30.8647	30.8859
2	3	41.3906	40.7379
3	3	49.7091	49.7455
a/H = 10			
1	1	1.8470	1.8483
1	2	3.2182	3.1645
1	3	5.2286	5.1399
2	2	4.2882	4.2845
2	3	6.0901	6.0441
3	3	7.6721	7.6753

However, the corresponding vibration modes exhibit quite different through-the-thickness, in-plane displacement patterns, as illustrated in Fig. 4. As expected, for the symmetric sandwich plate, the in-plane displacements of the face sheets corresponding to the fundamental frequency are identical in magnitude and opposite in sign, as shown in Fig. 4a and b.

Fig(a):longitudinal displacement along x-axis Fig(b):transverse displacement along y-axis





Fig(c) : vertical displacement along z- axis

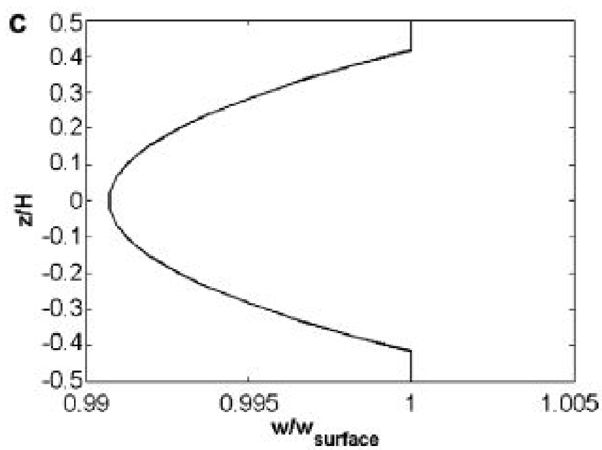


Fig 4 : Comparison of the through-the-thickness variations of the normalized displacements corresponding to the fundamental frequency (m = 1, n = 1) of the anti-symmetric and symmetric sandwich plates with a/H = 10

5.2. Effect of the modulus of a honeycomb core on the symmetric vibration response of sandwich plates with isotropic face sheets

The vibration response of a honeycomb-core sandwich plate is anti-symmetric up to very high frequencies because a honeycomb core is extremely stiff in the vertical direction. Therefore, symmetric vibration modes of a honeycomb core sandwich plate are observed only in the much higher frequency range. For example, in the frequency range below 200 Hz, the vibration response of the sandwich plate with aluminum face sheets and honeycomb core, which was considered in Section 3.2, is characterized by anti-symmetric vibration modes.

On the other hand, the first symmetric vibration mode for this plate is numerically predicted at a frequency above 600 Hz, even for a core with a very low value of elastic modulus in the vertical direction. To illustrate this point, the effect of the vertical stiffness of the honeycomb core Ec on the magnitude of the first symmetric natural frequency f_1^s of the sandwich plate is presented in Fig. 5. As shown in Fig. 5, the magnitude of f_1^s strongly depends on the vertical stiffness of the core up to $Ec \approx 0.3$ MPa.

Beyond this value, the magnitude of the first symmetric frequency is not affected by the increase in E_c. This behavior is explained by the fact that there are three natural frequencies and three corresponding vibration modes for each pair (m, n) in the case of the symmetric vibration response (compare with Section 3.1). For the low value of E_c the magnitude of the vertical displacements W is significantly larger than that of the in-plane displacements (see Fig. 6a).

On the other hand, if E_c is increased by several orders of magnitude, the reverse picture is observed. Namely, the in-plane displacements become significantly larger than the vertical displacements (see Fig. 6b). Therefore, for the low value of E_c, the pronounced vertical motion of the sandwich plate occurs, whereas for the significantly increased value of E_c, the longitudinal motion dominates the symmetric vibration response. This in-plane vibration response happens to be independent of the value of E_c (see the plateau in Fig. 5)

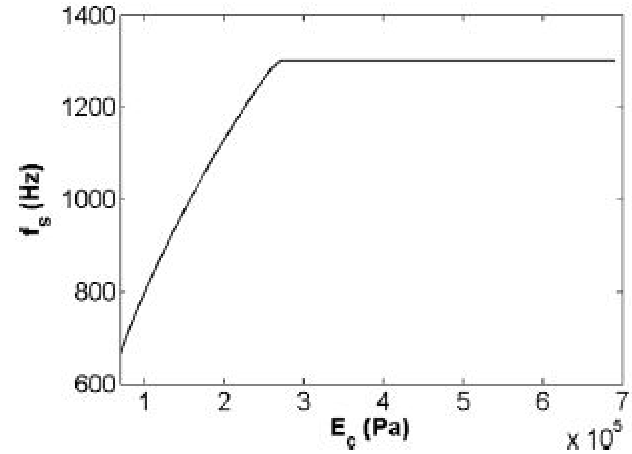


Fig 5 : Dependence of the first symmetric natural frequency on the Young's modulus of the core (m = 1, n = 1)

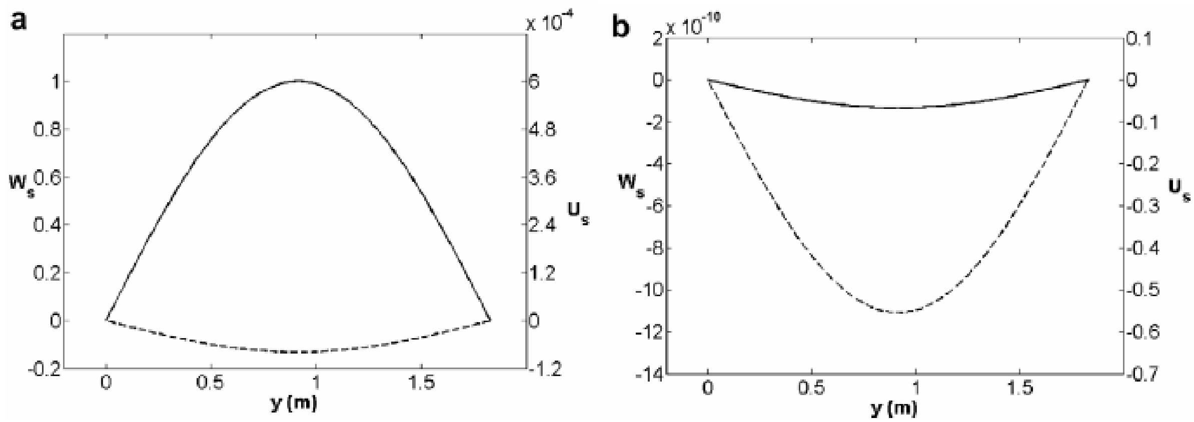


Fig 6 : The horizontal (U_s) and vertical (W_s) displacement amplitudes at x = a/2 corresponding to the first symmetric frequency (f_1^s) of the honeycomb-core sandwich plate with isotropic face sheets for: (a) Ec ≈ 0:07 MPa, {U_s = -0.5271e - 4, V_s = - 0.7908e - 4, W_s = 1.0000}; and (b) Ec ≈ 70:0 MPa, {U_s = -.8320, V_s = 0.5547, W_s = -0.1336e - 9} (- - -, U; —, W).

5.3 Importance of the present formulation for the prediction of the higher-frequency vibration response

As shown in Fig. 7, the variation of the normalized inplane displacements corresponding to the anti-symmetric vibration modes, which are characterized by the dominant vertical displacements, changes significantly with increasing mode pair (m,n). Namely, the through-the-thickness variation of the in-plane displacements is linear for m = 1, n = 1, whereas it becomes increasingly nonlinear for higher values of m and n. This is because the higher order terms in Eqs. (20) and (21) involve the second derivatives of the shear stress in the core, which increase with the mode numbers m and n in the harmonic assumptions of Eqs. (31) and (32). Therefore, the linear assumptions for the acceleration field in the core [9] are suitable only in the case of low vibrations modes. The present analysis, however, can be applied equally well for all frequency ranges of a sandwich plate.

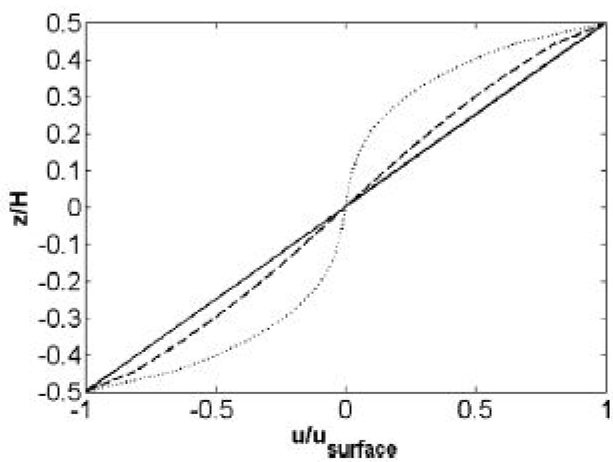


Fig 7 : Distribution of the anti-symmetric longitudinal displacements u' at x = 0; y = b/2 [—, (m = 1, n = 1), - - -, (m = 5, n = 5),· · ·, (m = 7, n = 7)].

6. Conclusions

The consistent higher-order free vibration equations and the corresponding boundary conditions for sandwich plates have been derived, taking into consideration the nonlinear acceleration field in the core. In addition, the general formulation was decoupled into two systems of equations representing symmetric and anti-symmetric vibrations. The present formulation has been validated by comparisons with results of the higher order mixed layer wise theory for laminated and sandwich plates and the results of experimental studies.

A numerical study of the free vibration analysis of soft and honeycomb-core thin and thick sandwich plates with anti-symmetric and symmetric lay-ups was undertaken. The study showed that the vibration modes of the soft-core sandwich plates with anti-symmetric and symmetric layups exhibit quite different through-the-thickness in-plane displacement patterns in spite of the almost identical natural frequencies. In the case of honeycomb sandwich plates, it was shown that the change in the modulus of the core not only brings about the shift in the first symmetric frequency, but also the qualitative change in the corresponding vibration pattern.

The importance of the present formulation for the prediction of the higher-frequency response of sandwich plates was demonstrated. The main difference between the present approach and the higher order mixed formulation is that unlike the latter, which is based on the assumed through-the-thickness displacement field in the core, the former uses the non-linear displacement field that is derived based on the well-defined physical assumptions. The present two-dimensional formulation constitutes an efficient alternative to the application of commercial finite element software, which requires three-dimensional modeling to achieve comparable results.

Furthermore, in the case of symmetric sandwich plates, the decoupled formulations, which represent symmetric and anti-symmetric motions, can be easily coded and applied to the accurate and efficient assessment of the free vibration response of simply supported sandwich plates with either foam or honeycomb cores.

7. References

SL NO.	REFERENCE NO.	NAME OF THE BOOK AND AUTHOR
1.	1.	Monforton GR, Schmit LA Jr. Finite element analysis of sandwich plates and cylindrical shells with laminate faces.
2.	2.	Rao MK, Desai YM. Analytical solutions for vibrations of laminated and sandwich plates using mixed theory
3.	3.	Zhou HB, Li GY. Free vibration analysis of sandwich plates with laminated faces using spline finite point method
4.	4.	Raville ME, Veng ES. Determination of natural frequencies of vibration of a sandwich plate
5.	5.	TonganWang,Vladimir Sokolinsky,Shankar Rajaram,Steven R Nutt.Consistent higher-order free vibration analysis of composite sandwich plates.