# Decentralized Control Design Approaches for Formation Control of Unmanned Aerial Vehicles

*Thesis submitted in partial fulfilment*

*of the requirements for the award of the degree of*

**Master of Technology**

*in*

**Control & Automation**

*by*

**Pradosh Ranjan Sahoo**

**(210EE3307)**

*Under the supervision*

*of*

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**NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA**

**राष्ट्रीय प्रौद्योगिकी संस्थान, राउरकेला**

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# Abstract

The leader follower type formation of Unmanned Aerial Vehicles usually demands decentralized yet co-operative control among the vehicles. The decentralized control approach is superior to centralized control in view of lesser involvement of delay, minimal information sharing requirement, reduced computational effort for controller design etc. The dynamic model of leader follower formation with an information structure constraint, in which each vehicle except the leader have the information of all the states of vehicle in front of it. The formation is treated as an interconnected system with overlapping control gains in the sense an UAV share information only with its neighbouring ones.

In this thesis, two approaches are used: (i) Inclusion principle (ii) Graph theory based approach for designing control gains. In the inclusion principle approach, control gain is designed separately for each disjoint subsystem in the expanded space. The static state feedback control law and linear matrix inequalities tool boxes are used for designing the controllers for each subsystem. Finally decentralized controllers are contracted back so as to be applied to the original system. In the graph theory approach, an overlapping information flow structure is constructed that determines the outputs of the system available in constructing any input signal of the system. The Graph theory is used to transform the overlapping interconnected system to decentralized one. The static state feedback type controller is used and a DK iterative algorithm is used to find out control gain. Then, a comparison between these two decentralized approaches is reported in the thesis so as to obtain the relative merits and demerits. There is delay in information flow form leader to follower in the formation so frequency domain stability analysis is done for time delay system. Frequency sweeping test is conducted for getting maximum tolerable communication delay between any two UAVs.

Electrical Engineering Department

National Institute of Technology Rourkela

# Certificate

*This is to certify that the Thesis entitled, "**Decentralized Control Design Approaches for Formation Control of Unmanned Aerial Vehicles**" submitted by "**Pradosh Ranjan Sahoo** " to the National Institute of Technology Rourkela is bona fide research work carried out by him under our guidance and is worthy for the award of the degree of "**Master of Technology**" in Electrical Engineering specializing in "**Control & Automation**" from this institute. The embodiment of this thesis is not submitted in any other university and/or institute for the award of any degree or diploma to the best of our knowledge and belief.*

Prof. Sanjib Ganguly

Prof. Sandip Ghosh

Date:

Place : Rourkela

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Pradosh Ranjan Sahoo

Rourkela, May 2012

# Contents

# List of Figures

# Notations and Abbreviations

| | |
|---|---|
| UAV | : Unmanned Aerial vehicle |
| DOF | : Degree of Freedom |
| $R, R^n, R^{n \times n}$ | : Set of real numbers, n components real vector, n by n real matrix |
| $\lVert . \rVert$ | : Vector matrix norm |
| $A \succ 0, A \geq 0$ respectively | : Matrix A is positive definite and positive semi-definite |
| $\lambda_i(A)$ | : $i^{th}$ eigen value of matrix A |
| $\rho(A)$ | : Spectral radius of a matrix A, $\max_i \lvert \lambda_i(A) \rvert$ |
| $\sigma(A)$ | : Spectrum of matrix A |

# *Chapter-1*

# Introduction

## 1.1 Introduction

Formation of Unmanned Aerial Vehicle (UAV) is used in both military and civilian works such as: target selection, vertical damage assessment, surveillance and exploration work, vegetation growth analysis, rapid assessment of topographical changes such as flooding or earthquakes. The formation can be of different shapes such as triangular shape, rectangular and circular. The Formation is better than single UAV due to its better sensitivity and the ability of rapid reconfiguration in case of single point failure [1]. To control the formation centralized or decentralized approaches may be used. For decentralized control, large scale systems or control problems are divided into no of manageable sub-problems which are weakly related with each other and can be solved independently [2-4] .

### 1.1.1 The Formation Control

Formation is defined as maintaining optimal geometric of the agents relative to each other or subject to form a fixed well defined control/sensing and communication architecture for a particular mission. In order to maintain the shape of a formation, it is required to maintain the distance between all pairs of agents being constant. Control of a formation requires the mixing of several tasks. One is the whole formation task of moving from one point to another point (or moving the centre of mass of the formation and adopting a certain orientation). Another is to maintain the relative positions of the agents during formation motion so that the shape is preserved. A third is to avoid obstacles, a fourth may be to handle maximum tolerable delay between the agents in formation etc. Five vehicles are in leader follower type formation in a triangular shape [1] shown below.

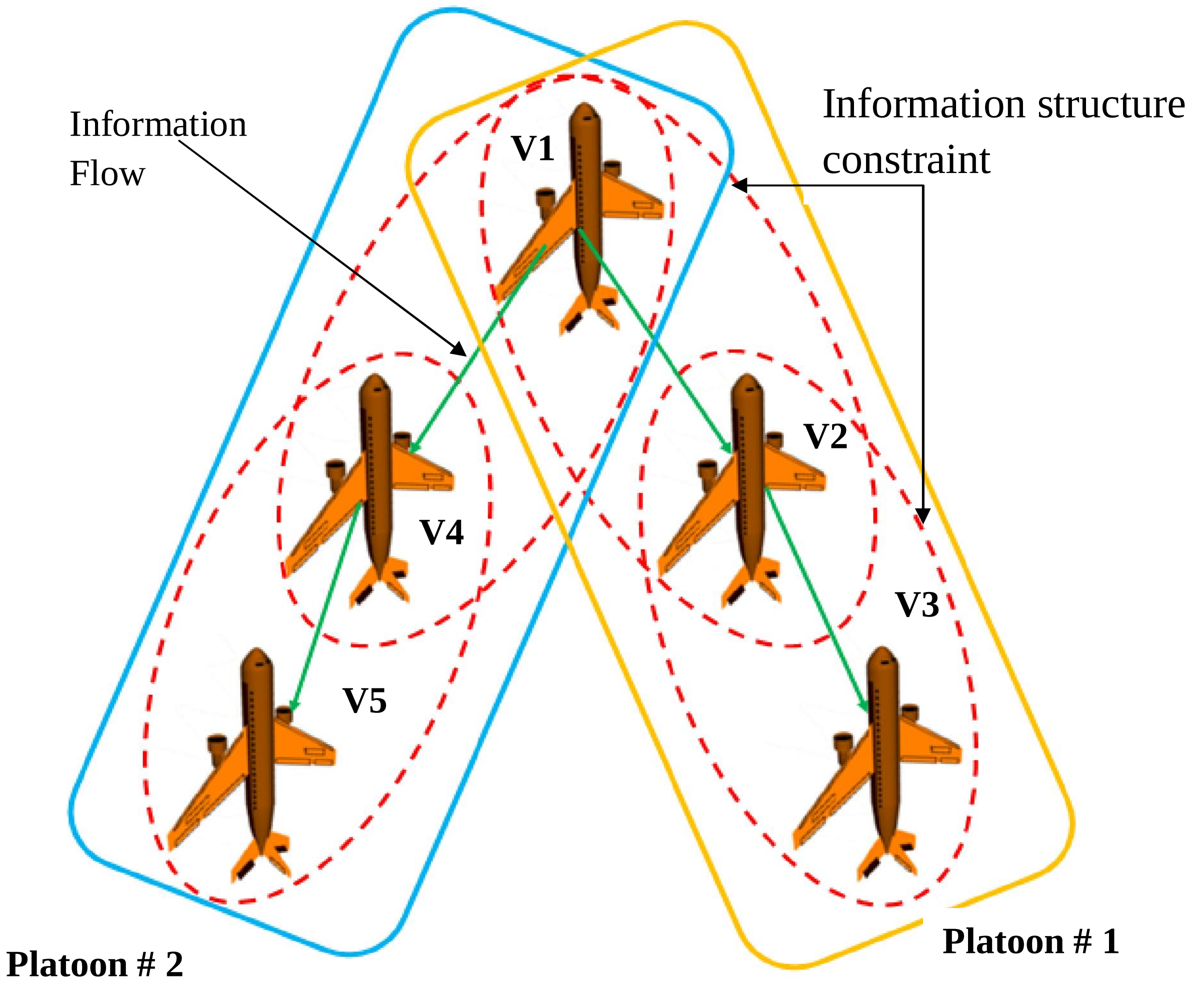


Figure 1.1: Formation of five Unmanned Aerial Vehicles

As an example consider a formation problem presented in Fig.1.1 five vehicles form a triangular formation where dotted lines shows information structure constraint and the arrow line shows the information flow from leader to follower .Formation of unmanned aerial vehicle used mostly in surveillance or exploration work. The whole formation is able to synthesize antenna size which is more than individual agent that results improves sensitivity , the different agents carry different sensors which enhances the multiple functionality of whole formation and also it increases robustness and efficiency. It also decreases system cost.

### 1.1.2 Decentralized Control for System with Overlapping Structure

A large scale system or a control problem is portioned into number of independently manageable sub problems so that the plant is no longer controlled by a single controller but by several independent controller which all together known as a decentralized controller [4,6]. The formation of UAV is a system of interconnected overlapping subsystems. Interconnected overlapping system is that who shares a common state between them. Decomposition is a prerequisite for decentralized control. Generally we represent a large-scale system as a collection of weakly interconnected subsystems of lower dimension. Decomposition of systems with the overlapping structure is important to solve problems in many fields such as, economic systems, automated highway systems, electric power systems, and formation of UAVs. There are different type of approaches for formation like behavioural formation, virtual leader type formation and leader follower type formation. Leader follower type formation is presented here with information structure constraint where each vehicle except leader has state information about the vehicles in front of it [1].Two types of control strategy generally used in formation 1. Centralized control 2. Decentralized control.

With the help of inclusion principle, we can expand the state space [2] (input and output spaces), so that the overlapping subsystems appear as disjoint .Applying standard methods for decentralized control fully decentralized control laws can be designed in this expanded space, and contracted back to the original state space of formation for implementation. The inclusion principles is used to ensure that this expansion/contraction procedure is correctly carried over, that is that solutions of the original system are included in the solutions of the expanded system. Satisfaction of the inclusion conditions is important for transferring properties of the expanded system to the original one

In Graph theory approach [10-11] it is assumed that an overlapping information flow structure is given by a matrix which determines which outputs of the system are available for constructing any input signal of the system. Graph theory is used to transform the overlapping interconnected system to decentralized one by dividing main graphs to bipartite sub graphs having separate edges.

### 1.1.3 Advantages of Decentralized Formation Control

Decentralized formation control posses many advantages than centralized control

- Minimal information structure constraint.
- Reduced computational time due to parallel processing.
- Delay free due to local information sharing.
- Reliable for structure reconfiguration.

## 1.2 Review of some existing Works

This section is devoted in reviewing decentralization techniques used in this formation control problem and stability analysis of time delay system. The review will start with the definition of decentralized control, approaches of decentralization and application of decentralization to different control problems like formation control of UAVs and vehicle platooning problem. Then to find a maximum tolerable delay in the formation using frequency sweeping test.

### 1.2.1 Review of Decentralized Controller Approaches

A large control problem can be partitioned into manageable sub-problems for analysis and synthesis so that the overall plant is controlled by several in depended controllers instead of a single controller together called decentralized controller [4]. The subsystems under consideration for decentralization divided into two types (1) Strongly coupled subsystem and (2) weakly coupled subsystem [4]. In strongly coupled subsystem at least one approximation model of all other sub system is considered where as in weakly coupled subsystem coupling is neglected during the design of individual controller. Overlapping decomposition means to expand the original system with strongly coupled subsystem with weakly coupled subsystem [1]. The Solution of larger dimensional system must include the solution of lower dimension original system. In this thesis two decentralization approaches is considered

(1) Inclusion principle [1,2,13] is one of the method for expanding state space, input & output spaces so that the overlapping sub systems appears as disjoint. Satisfaction of inclusion principle the properties of original system can be transferred to expanded one. In this method both system as well as controller is expanded and generally static state feedback used. Controller structure is designed by knowledge of information structure constraint. Finally the designed controllers contracted back [1-2] to form original control .

(2) Graph theory based approach [8,11] is one of the well known method for decentralization. The control constraint can be represented by binary information flow matrix. Output feedback controller is used for construction of controller gain matrix. The structurally constrained controller can be determined by which output is available to construct any specific input of the system. Some procedures are followed to divide the bipartite graph into no of sub graphs [8]. From the sub graphs the block diagonal expanded controller gain matrix is derived.

The decentralized overlapping control designed approach using inclusion principle is presented in IVHS [9]. In control of platoon of vehicles the original system is decomposed by input/ output expansion. The subsystems are defined in such a way that the state vectors include measurements available from each vehicle. Local control laws for the extracted subsystems are obtained by optimizing local quadratic performance indices. The dynamics of vehicle is considered for problem formulation [9]. Graph theory approach based generally is used when there is a limitation of available of states [10]. Decentralization using graph theory is applicable for overlapping interconnected pants where graph theory approach is adopted to find constrained control gain. The problem of optimal LTI structurally constraint control with respect to quadratic performance is presented in some papers [8]

### 1.2.2 Review of Delay Tolerability in Time Delay System

Delay has significant impact on the stability and performance of the system. Uncertain transmission delay is considered in communication links among different subsystems as referred in [12,15]. The controller gain is decomposed into diagonal and off diagonal components. Graph theory based approach is used to transform the controller gain matrix into diagonal form. LMI based design algorithm is implemented for solving the disturbance attenuation [12] & to achieve stability. Many authors discussed about stability properties of LTI time delay systems. In paper [15,16] stability properties of linear time invariant delay systems in state space form is presented. The sufficient and necessary condition for stability independent of delay is discussed with the help of frequency sweeping test [16]. Delay margin i.e. maximal tolerable delay over which the system under consideration maintains stability is calculated. The necessary and sufficient condition for stability independent of delay can be checked by computing the spectral radii of certain frequency dependant matrix.

## 1.3 Motivation

The motivation behind the thesis is that the formation control problem is a mixture of graph theory, nonlinear system theory and linear algebra. Leader follower type formation is a wide research area. The control issues associated with formation is very challenging using decentralized approach. The decentralization is very useful when the subsystem has contradictory goal and subsystems are handled by different authorities. Here different decentralization approaches have been discussed and comparison has been made. The decentralization using graph theory has some advantages over inclusion principle approach such as expanded system has inherently uncontrollable, contraction of the designed controller is very difficult task and static state feedback controller is not used for practical use.

## 1.4 The Scope of the Present Work

The salient objective of this thesis is

1. To realize the decentralized control strategy of formation of UAV in a planner motion.

2. To study the static feedback problems.

3. To make a comparison study of two approaches of decentralization.

   I. Inclusion principle approach.
   II. Graph theory approach.

4. To find a maximum tolerable time delay in communication channel in the formation.

## 1.5 Organisation of this Thesis

The work done in this thesis is organised as follows

- Chapter-1 provides a brief background of formation control, decentralization approach, motivation and objective.
- In Chapter-2 the kinematics model and dynamics of formation of 3 UAV is presented.
- Chapter-3 Decentralized overlapping controller is designed using inclusion principle method , robust state feedback control law used to find the control gains and simulation results are presented
- In Chapter-4 Procedure for designing decentralization using Graph theory based approaches is presented. The problem is formulated as a convex optimization problem

in terms of linear matrix inequalities .DK iteration procedure is used to find the control gains using output feedback.

- Chapter-5 provides the brief idea about time delay systems and frequency domain analysis. Frequency sweeping test is carried out to known about delay independent stability and to get delay margin for stability of system
- Chapter-6 provides conclusion and scope of present work

# *Chapter-2*

# Model Description

## 2.1 Kinematics of a single UAV

Kinematics describes the motion of points or objects without considering the forces that cause it. Unmanned Aerial Vehicle has 6 degree of freedom that shown below in fig. 2.1. For formation of flight the UAVs move in planner motion along XY plane at that time we only consider two degrees of freedom i.e. yaw and surg.

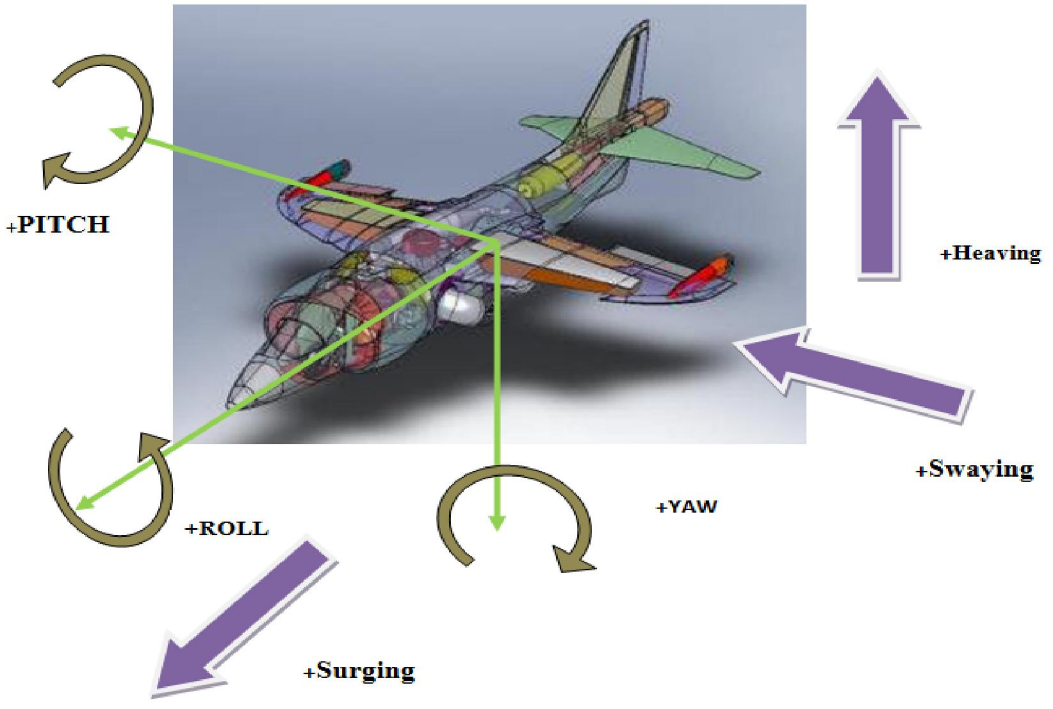


Figure 2.1: A single UAV with six Degrees of Freedom

Courtesy: unmanned.co.uk

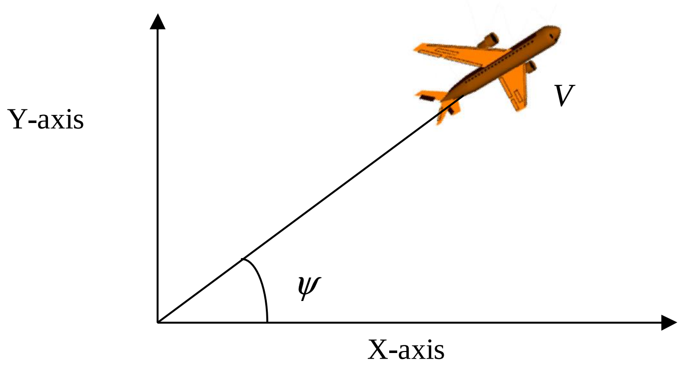


Figure 2.2: Representation of a single UAV in XY-plane

The planar kinematics model for a single UAV as shown in above figure is

$$\begin{aligned} \dot{X} &= V\cos\psi \\ \dot{Y} &= V\sin\psi \\ \dot{\psi} &= \omega \end{aligned} \tag{2.1}$$

where $X$ and $Y$ are rectangular coordinates of the UAV ,

$\psi$ is the heading angle in the plane,

The speed $V$ and angular turn rate $\omega$ are reference input. As vector relative degree of of the above model is singular then to solve this type of problem we have to add some states and input variables. So considered $V$ as a new state variable, and acceleration $a$ as a new input variable.

The state and input variables for nonlinear model is declared as

$$\begin{bmatrix}\xi_1\\ \xi_2\\ \xi_3\\ \xi_4\end{bmatrix} = \begin{bmatrix}X\\ Y\\ \psi\\ V\end{bmatrix}, \qquad \eta = \begin{bmatrix}\eta_1\\ \eta_2\end{bmatrix} = \begin{bmatrix}a\\ \omega\end{bmatrix} \tag{2.2}$$

The nonlinear kinematic model can be written as

$$\dot{\xi} = f(\xi) + g(\xi)\eta$$

where $$f(\xi)=\begin{bmatrix}\xi_4\cos\xi_3\\ \xi_4\sin\xi_3\\ 0\\ 0\end{bmatrix}=\begin{bmatrix}V\cos\psi\\ V\sin\psi\\ 0\\ 0\end{bmatrix}\quad,\quad g(\xi)=\begin{bmatrix}0&0\\0&0\\0&1\\1&0\end{bmatrix} \tag{2.3}$$

$$\dot{\xi}=\begin{bmatrix}\dot{X}\\ \dot{Y}\\ \dot{\psi}\\ \dot{V}\end{bmatrix}=\begin{bmatrix}\xi_4\cos\xi_3\\ \xi_4\sin\xi_3\\ \omega\\ a\end{bmatrix}=\begin{bmatrix}V\cos\psi\\ V\sin\psi\\ \omega\\ a\end{bmatrix} \tag{2.4}$$

Applying input to state feedback linearization and mapping the change of state variables

$$Z=T(\xi)\quad,\quad\begin{bmatrix}Z_1\\ Z_2\\ Z_3\\ Z_4\end{bmatrix}=\begin{bmatrix}\xi_1\\ \xi_2\\ \xi_4\cos\xi_3\\ \xi_4\sin\xi_3\end{bmatrix}=\begin{bmatrix}X\\ Y\\ V\cos\psi\\ V\sin\psi\end{bmatrix} \tag{2.5}$$

The input variable is defined as $\eta=M(\xi)u$ where $u$ is new input variable

with $$M(\xi)=\begin{bmatrix}\cos\xi_3 & \sin\xi_3\\ -\sin(\xi_3)/\xi_4 & \cos(\xi_3)/\xi_4\end{bmatrix} \tag{2.6}$$

The linearization of the nonlinear model

$$Z=T(\xi)\qquad\Rightarrow\dot{Z}=\frac{\partial T}{\partial\xi}\dot{\xi}$$

The kinematics of single UAV can be written in linear form as

$$\dot{Z}=\begin{bmatrix}0&0&1&0\\0&0&0&1\\0&0&0&0\\0&0&0&0\end{bmatrix}Z+\begin{bmatrix}0&0\\0&0\\1&0\\0&1\end{bmatrix}u \tag{2.7}$$

$$\Rightarrow\dot{Z}=EZ+Fu$$

we can rewrite equation- as $\dot{Z}=EZ+Fu$

$$\dot{Z} = \begin{bmatrix} 0_2 & I_2 \\ 0_2 & 0_2 \end{bmatrix} Z + \begin{bmatrix} 0_2 \\ I_2 \end{bmatrix} u \quad (2.8)$$

with $Z \in R^4$ and $u \in R^2$ are the state and input to the system, respectively. $0_2$ denotes the 2 × 2 zero matrix and $I_2$ denotes the 2×2 identity matrix. In order to simplify the notation these two matrices will be simply denoted as $0$ and $I$ .

## 2.2 The UAV Formation Problem under consideration

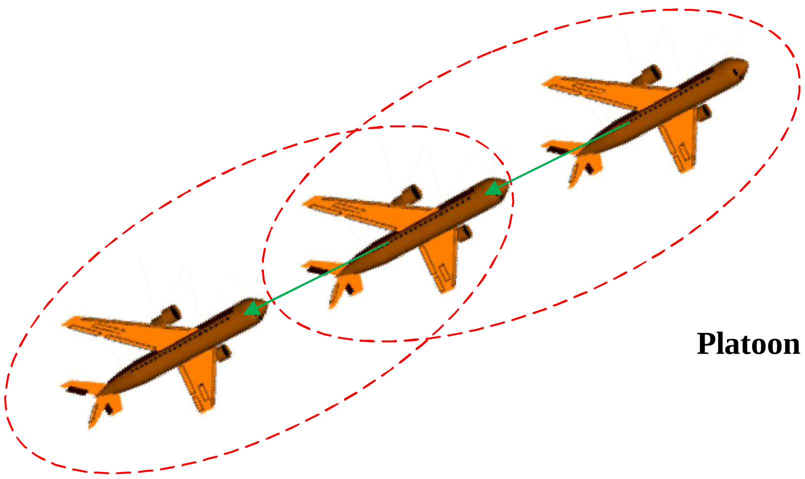


Figure 2.3: Vehicles having interconnected overlapping structure

Here in the Fig.2.3 we have taken into account one platoon where 3 UAVs are present. The dotted line shows information structure constraint and the arrow is showing the information flow from leader to follower. As the whole formation is symmetric so one platoon is considered for calculation.

For $i^{th}$ vehicle out of q vehicles of a formation

$$Z_i = \begin{bmatrix} Z_{i1} \\ Z_{i2} \\ Z_{i3} \\ Z_{i4} \end{bmatrix} = \begin{bmatrix} Z^{I}{}_i \\ Z_i^{II} \end{bmatrix} \in R^4$$

. with

$$Z_i^{p} = \begin{bmatrix} Z_{i1} \\ Z_{i2} \end{bmatrix} = \begin{bmatrix} X_i \\ Y_i \end{bmatrix} \in R^2 \quad , \; Z_i^{v} = \begin{bmatrix} Z_{i3} \\ Z_{i4} \end{bmatrix} = \begin{bmatrix} V_i \cos\psi_i \\ V_i \sin\psi_i \end{bmatrix} \in R^2 \quad (2.9)$$

The vector $Z_i$ is spitted into two sub vectors, where the first sub vector $Z_i^p$ includes position coordinates and the second sub vector $Z_i^v$ includes speed coordinates of the $i^{th}$ vehicle [1]. This type of decomposition is chosen due to different treatment of the state variables. The goal is to control the vehicles in a formation by controlling variables that represent distances between vehicles (i.e., not positions of the vehicles), and variables that represent speed coordinates for each independent vehicle. The control input for the $i^{th}$ vehicle as defined in (2.6) will be denoted as $u_i$ , where $u_i \in R^2$ .By imposing the information structure constraint that each vehicle, except the leading one, has state information about the vehicle in front of it, it is natural to decompose the formation into two platoons that share the leading vehicle. In Fig.1.1, the number of vehicles in the formation is equal to five and each platoon has three vehicles.

For simplicity and without loss of generality, let us consider a platoon of 'r' vehicles and introduce change of variables

$$\begin{aligned} e_1^v &= Z_1^v - v_{d1} \\ e_i^p &= Z^p{}_{i-1} - Z^p{}_i - d_{i-1} \\ e_i^v &= Z_i^v - v_{di} \end{aligned} \qquad (2.10)$$

where $e_1^v$ is the velocity error for the leader, $e_i^p$ is the position error for the $i^{th}$ vehicle and $e_i^v$ is the velocity error for $i^{th}$ the vehicle and $i \in (2,3,....r)$. $d_{i-1} \in R^2$ is a constant desired Euclidean distance between the $(i-1)st$ and $i^{th}$ vehicles, $i \in (2,3,....r)$ and $v_{di} \in R^2$ , represents the desired speed for the $i^{th}$ vehicle $i \in (2,3,....r)$. Let's take a assumption that $v_{di} = v_d$ for all the vehicles as Euclidian distance between vehicles are assumed to be constant.

The error dynamics can be formulated as

$$\begin{aligned} \dot{e}_1^v &= u_1 \\ \dot{e}_i^p &= e^v{}_{i-1} - e_i^v \\ \dot{e}_i^v &= u_i \end{aligned} \qquad (2.11)$$

where $e_1^v$ is error dynamics for leader and $i \in (2,3,....r)$

The goal is for the whole platoon i.e. formation to fly at constant desired speed $v_d$ with desired spacing between vehicles, uniquely determined by desired Euclidean distances between successive vehicles equal to $d_i$, Let us take 3 vehicle as shown in Fig 2.2 or take platoon-1. The position of the leading vehicle is not needed because the leader is not following some desired path.

The error dynamics for interconnected system is written as

$$\begin{bmatrix} \dot{e}_1^{\,v} \\ \dot{e}_2^{\,p} \\ \dot{e}_2^{\,v} \\ \dot{e}_3^{\,p} \\ \dot{e}_3^{\,v} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ I & 0 & -I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & -I \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1^{\,v} \\ e_2^{\,p} \\ e_2^{\,v} \\ e_3^{\,p} \\ e_3^{\,v} \end{bmatrix} + \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \qquad (2.12)$$

$$\Rightarrow \dot{e} = Ae + Bu$$

The system described by (2.11) can be considered as an interconnected system with subsystems having state variables that are defined as $e_1 = e_1^{\,v}, e_i = \begin{bmatrix} e_i^{\,p} \\ e_i^{\,v} \end{bmatrix}$ for all $i \in (2, 3, \dots. r)$

# *Chapter-3*

# Inclusion Principle Based Decentralized Overlapping Controller Design

## 3.1 Introduction

Decomposition is a pre-requirement of decentralized control. A large scale system can be breakdown to number of lower dimension subsystem. There are different decomposition methods such as epsilon decomposition, BBD decomposition and overlapping decomposition. Within the mathematical frame work of inclusion principle the overlapping system is expanded into disjoint subsystems. Satisfaction of the inclusion principle is necessary for transferring all the properties of original system to expanded system. Consider a continues LTI system

$$S_i : \dot{x}_i = A_{ii}x_i + B_{ii}u_i \quad and \quad y_i = C_{ii}x_i \tag{3.1}$$

where $x_i \in R^{ni}, u_i \in R^{m_i}, y_i \in R^{l_i}$ are the state, input and output vectors respectively.

while $A = \begin{bmatrix} A_{11} & A_{12} & . & . & A_{1n} \\ A_{21} & A_{22} & . & . & A_{2n} \\ . & . & . & . & . \\ . & . & . & . & . \\ A_{n1} & A_{n2} & . & . & A_{nv} \end{bmatrix}$ and

$B = blockdiag[B_{11}B_{22}....B_{NN}]$

$C = blockdiag[C_{11}C_{22}......C_{NN}]$

In which $A_{ij}, B_{ii}$ *and* $C_{ii}$ are sub matrices of order $(n_i \times n_j)$, $(n_i \times m_i)$ *and* $(l_i \times n_i)$ where $i = 1, 2, ......N$ *and* $j = 1, 2, 3......N$

Pair wise subsystem can be defined as

$$S_{ij} : \begin{bmatrix} \dot{x}_i \\ \dot{x}_j \end{bmatrix} = \begin{bmatrix} A_{ii} & A_{ij} \\ A_{ji} & A_{jj} \end{bmatrix} \begin{bmatrix} x_i \\ x_j \end{bmatrix} + \begin{bmatrix} B_{ii} & 0 \\ 0 & B_{jj} \end{bmatrix} \begin{bmatrix} u_i \\ u_j \end{bmatrix}, \qquad \begin{bmatrix} y_i \\ y_j \end{bmatrix} = \begin{bmatrix} C_{ii} & 0 \\ 0 & C_{jj} \end{bmatrix} \begin{bmatrix} x_i \\ x_j \end{bmatrix} \tag{3.2}$$

In this case, each subsystem $S_i$ $(i = 1, 2, ...N)$ is shared with N−1 different "pair-wise" subsystems $S_{ij}$, $j = 1, 2, 3....N$ and $j \neq i$ so that it represents their over lapping part. Out of three structures viz : longitudinal ,radial and loop we considered here longitudinal structure where each subsystem $S_i$ is shared by only adjacent subsystems $S_{i-1,i}$ *and* $S_{i,i+1}$

Let's consider a state matrix A having interconnected overlapping elements represents below

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 & . & 0 \\ A_{21} & A_{22} & A_{23} & . & 0 \\ 0 & A_{32} & A_{33} & . & 0 \\ . & . & . & . & 0 \\ 0 & 0 & 0 & 0 & A_{NN} \end{bmatrix}$$

After applying inclusion principle [2] the matrix $A$ converted to expanded form $\tilde{A}$ and the interconnected blocks are now decoupled into disjoint diagonal blocks.

$$\tilde{A} = \begin{bmatrix} A_{11} & A_{12} & 0 & . & . & 0 & 0 \\ A_{21} & A_{22} & 0 & . & 0 & 0 & 0 \\ A_{21} & 0 & A_{22} & . & 0 & 0 & . \\ . & . & . & . & . & . & . \\ . & . & . & . & A_{N-1,N-1} & 0 & A_{N-1,N} \\ 0 & 0 & 0 & . & . & A_{N-1,N-1} & A_{N-1,N} \\ 0 & 0 & 0 & . & . & A_{N,N-1} & A_{NN} \end{bmatrix}$$

## Overlapping Controller Design

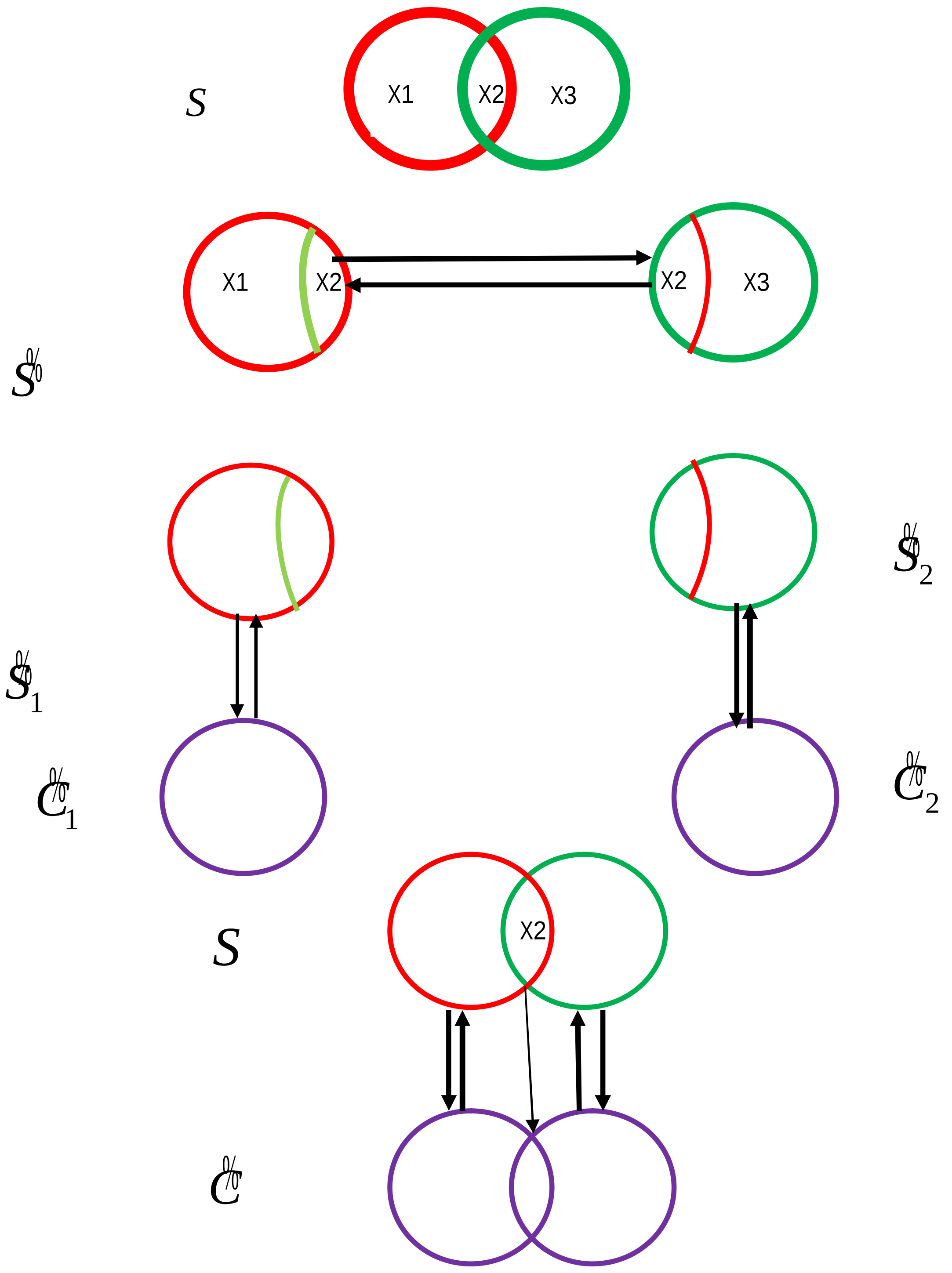


Figure3.1: Overlapping controller design

(a) Overlapping subsystems; (b) expanded system (c) decentralized controller design

(d) Contracted closed-loop system

In the Fig.3.1 overlapping controller design is presented. X1 and X3 is overlapping interconnected system where X2 is overlapping part. First apply some transformation so that the overlapping part is converted to disjoint subsystems $\tilde{S}1$ & $\tilde{S}2$ .Then for each subsystem controller $\tilde{C}1$ *and* $\tilde{C}2$ is designed independently and again applying some transformation the controllers are contracted back .

## 3.2 Decentralization using Inclusion Principle

Our objective here is to expand the interconnected system represented (2.12) into a space in which the subsystems will be decoupled. In order to do this we have to use the inclusion principle [1-2] for linear systems.

Consider the system

$$S : \dot{x} = Ax + Bu \ \ , \ \ x(t_0) = x_0 \tag{3.3}$$

$x \in R^n$ is the state and $u \in R^m$ is the control input

The expanded system

$$\tilde{S} : \dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}\tilde{u} \ \ , \ \tilde{x}(t_0) = \tilde{x}_0 \tag{3.4}$$

$\tilde{x} \in R^{\tilde{n}}$ is the state , $\tilde{u} \in R^{\tilde{m}}$ is the control input with $\tilde{n} > n$ and $\tilde{m} > m$

Trajectories of system $S$ and $\tilde{S}$ is denoted as $x(t; x_{0,} u)$ and $\tilde{x}(t; \tilde{x}_{0,} \tilde{u})$ respectively.

The system $S$ and $\tilde{S}$ are related to each other by a transformation

$$\begin{aligned} \tilde{x} &= Vx \quad , \ x = U\tilde{x} \\ \tilde{u} &= Ru \quad , \ u = Q\tilde{u} \\ \tilde{y} &= Ty \quad , \ y = S\tilde{y} \end{aligned}$$

The state expansion and contraction matrices are given below

$$V \in R^{\tilde{n}\times n}, U \in R^{n\times\tilde{n}}, UV = I \in R^{n\times n} \tag{3.5}$$

Similarly input expansion and contraction matrices are

$$R \in R^{\tilde{m}\times m}, Q \in R^{m\times\tilde{m}}, QR = I \in R^{m\times m} \tag{3.6}$$

### 3.2.1 Inclusion Principle

**Definition 1[1,2] :**

System $\tilde{S}$ includes system $S$ if for any initial state $x_0$ and any input $u(t)$, If the following is valid: $x(t;x_{0,}u) = U\tilde{x}(t;V\tilde{x}_{0,}R\tilde{u})$

Theorems presented here are referred in [1, 2]

**Theorem 1**:

System $\tilde{S}$ includes system $S$ if and only if $A^i = U\tilde{A}^iV, A^iB = U\tilde{A}^i\tilde{B}R$ for $i \in \{0,1,...\tilde{n}-1\}$ .In other words, the inclusion principle formulates conditions under which the trajectories of the original system $S$ are included in the set of trajectories of the expanded system $\tilde{S}$.

**Theorem -2**:

$S$ is a restriction of $\tilde{S}$ if one of the following is true

(a) $\tilde{A}V = VA$ and $\tilde{B}R = VB$ (restriction type(a))

(b) $\tilde{A}V = VA$ and $\tilde{B} = VBQ$ ( restriction type(b))

If static feedback control laws for both systems are assumed to be in the form

$$\begin{aligned} u &= Kx, K \in R^{m\times n} \\ \tilde{u} &= \tilde{K}\tilde{x}, \tilde{K} \in R^{\tilde{m}\times\tilde{n}} \end{aligned} \tag{3.7}$$

The closed loop system in the original space

$$\overline{S} : \dot{x} = (A + BK)x \tag{3.8}$$

is included in the closed loop system in the expanded space

$$\overline{\tilde{S}} : \dot{\tilde{x}} = (\tilde{A} + \tilde{B}\tilde{K})\tilde{x} \tag{3.9}$$

if it satisfies Theorem-3

**<u>Theorem 3</u>**:

$\bar{S}$ is a restriction of $\tilde{S}$ if one of the following is true:

(a) $\tilde{A}V = VA$, $\tilde{B}R = VB$, and $\tilde{K}V = RK$ (restriction type (a)).

(b) $\tilde{A}V = VA$, $\tilde{B} = VBQ$, and $K = Q\tilde{K}V$ (restriction type (b)).

The interconnected system with subsystems that overlap can be expanded simply repeating overlapping parts such that in the expanded space subsystems appear disjoint.

By applying the inclusion principle to the error dynamics that is expanding both the states and inputs by repeating the second vehicles state and input it can be written as

$$\tilde{e}_1 = e_1, \qquad \tilde{u}_1 = u_1$$

$$\tilde{e}_i = \begin{bmatrix} e^{ll}{}_{i-1} \\ e^{l}{}_{i} \\ e^{ll}{}_{i} \end{bmatrix} \text{ and } u_i = \begin{bmatrix} u_{i-1} \\ u_i \end{bmatrix} \quad , i \in \{2,3\}$$

The error dynamics for one platoon is interconnected overlapping subsystems & can be written as where dotted lines denote interconnected systems.

$$\begin{bmatrix} \dot{e}_1^{\,v} \\ \dot{e}_2^{\,p} \\ \dot{e}_2^{\,v} \\ \dot{e}_3^{\,p} \\ \dot{e}_3^{\,v} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ I & 0 & -I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & -I \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1^{\,v} \\ e_2^{\,p} \\ e_2^{\,v} \\ e_3^{\,p} \\ e_3^{\,v} \end{bmatrix} + \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{3.10}$$

$$\Rightarrow \dot{e} = Ae + Bu$$

The expansion and contraction matrices [2] for the state given as

$$V = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}, U = \begin{bmatrix} \frac{1}{2}I & \frac{1}{2}I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}I & \frac{1}{2}I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix} \tag{3.11}$$

and similarly the input expansion and contraction matrices are

$$R=\begin{bmatrix} I & 0 & 0 \\ I & 0 & 0 \\ 0 & I & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, Q=\begin{bmatrix} \frac{1}{2}I & \frac{1}{2}I & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}I & \frac{1}{2}I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix} \tag{3.12}$$

Using (3.10) and (3.7 -3.12) it is verified that $A_D V = VA$ and $B_D R = VB$ . Then, from Theorem-2 it follows that this expansion/contraction procedure satisfies the conditions definition 2(a) restriction type (a)).

$$\begin{bmatrix} \dot{e}_1^{\,v} \\ \dot{e}_1^{\,v} \\ \dot{e}_2^{\,p} \\ \dot{e}_2^{\,v} \\ \dot{e}_2^{\,v} \\ \dot{e}_3^{\,p} \\ \dot{e}_3^{\,v} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & -I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & -I \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1^{\,v} \\ e_1^{\,v} \\ e_2^{\,p} \\ e_2^{\,v} \\ e_2^{\,v} \\ e_3^{\,p} \\ e_3^{\,v} \end{bmatrix} + \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} u_1 \\ u_1 \\ u_2 \\ u_2 \\ u_3 \end{bmatrix} \tag{3.13}$$

$$\Rightarrow \dot{\tilde{e}} = \tilde{A}_D \tilde{e} + \tilde{B}_D \tilde{u}$$

Static state feedback control law for expanded system is

$$\tilde{u} = \tilde{K}_D \tilde{e} \tag{3.14}$$

$$u_1 = \tilde{K}_{11} e_1^{\,v} \qquad Subsystem-1$$

$$\begin{aligned} u_1 &= \tilde{K}_{22} e_1^{\,v} \\ u_2 &= \tilde{K}_{32} e_1^{\,v} + \tilde{K}_{33} e_2^{\,p} + \tilde{K}_{34} e_2^{\,v} \qquad Subsystem-2 \end{aligned} \tag{3.15}$$

$$\begin{aligned} u_2 &= \tilde{K}_{45} e_2^{\,v} \\ u_3 &= \tilde{K}_{55} e_2^{\,v} + \tilde{K}_{56} e_3^{\,p} + \tilde{K}_{57} e_3^{\,v} \qquad Subsystem-3 \end{aligned}$$

It is clear from the information structure constraint that the control action for each vehicle except leader depends on previous vehicles velocity, its own velocity and the distance between them.

Thus the controller in the expanded space is designed in the following way

$$\tilde{K}_D = \begin{bmatrix} \tilde{K}_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_{32} & \tilde{K}_{33} & \tilde{K}_{34} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{K}_{45} & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{K}_{55} & \tilde{K}_{56} & \tilde{K}_{57} \end{bmatrix} \tag{3.16}$$

In order to satisfy the Theorem-3 the matrix and for proper contraction $\tilde{K}_D$ is modified as

$$\tilde{K}_{DM} = \begin{bmatrix} \frac{(\tilde{K}_{11}+\tilde{K}_{22})}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{(\tilde{K}_{11}+\tilde{K}_{22})}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_{32} & \tilde{K}_{33} & \frac{(\tilde{K}_{34}+\tilde{K}_{45})}{2} & 0 & 0 & 0 \\ 0 & \tilde{K}_{32} & \tilde{K}_{33} & 0 & \frac{(\tilde{K}_{34}+\tilde{K}_{45})}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{K}_{55} & \tilde{K}_{56} & \tilde{K}_{57} \end{bmatrix} \tag{3.17}$$

Here all subsystems are equal and so if $\tilde{K}_{11} = \tilde{K}_{22} = \tilde{K}_1$, and $\tilde{K}_{34} = \tilde{K}_{45} = \tilde{K}_{57} = \tilde{K}_1$, from (3.16) and (3.17), it follows that the stability of the expanded closed-loop system will be preserved after modification. Let's take $\tilde{K}_{32} = \tilde{K}_{55} = \tilde{K}_2$ and $K_{33} = K_{56} = K_3$ for simplicity and $K_{DM}$ is computed in the overlapping form as

$$\tilde{K}_{DM} = \begin{bmatrix} \tilde{K}_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 & 0 & 0 & 0 \\ 0 & \tilde{K}_2 & \tilde{K}_3 & 0 & \tilde{K}_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 \end{bmatrix} \tag{3.18}$$

Then the controller gain in original system is contracted to

$$K_M = \begin{bmatrix} \tilde{K}_1 & 0 & 0 & 0 & 0 \\ \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 & 0 & 0 \\ 0 & 0 & \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 \end{bmatrix} \tag{3.19}$$

So that the relation $K_{DM} V = R K_M$ is valid.

## 3.3 Robust Feedback Control law

It is a new approach for robust stabilization of nonlinear system within LMI. The main goal is to formulate linear constant feedback laws that stabilize the system and maximize the bounds on the nonlinearity that the system can tolerate without going unstable. Here a method is discussed to compute a gain matrix in (3.18) that will robustly stabilize the expanded system, so that its contraction will stabilize the original system as well.

Let the perturbed kinematics model is

$$\dot{\xi} = f(\xi) + g(\xi) + w \tag{3.20}$$

where $w = [w_1, w_2, w_3, w_4]^T \in R^4$ is a perturbation in the system which represents wind gust disturbances or uncertainties in the model description. Here only sector bounded perturbations will be considered i.e. perturbations that reside in some conical sector.

$$\dot{Z} = \frac{\partial T}{\partial \xi}\dot{\xi} = \frac{\partial T}{\partial \xi} f(\xi) + \frac{\partial T}{\partial \xi} g(\xi)u + \frac{\partial T}{\partial \xi} w$$

$$\Rightarrow \dot{Z} = EZ + Fu + \overline{w}$$

where $$\overline{w} = \frac{\partial T}{\partial \xi} w$$

$$\overline{w} = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & -\xi_4 \sin\xi_3 & \cos\xi_3 \\ 0 & 0 & \xi_4 \cos\xi_3 & \sin\xi_3 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ 0 \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ -Z_4 w_3 \\ Z_3 w_3 \end{bmatrix} \tag{3.21}$$

It can be decomposed in another form

where

$$\overline{w} = \begin{bmatrix} \overline{w}^p \\ \overline{w}^v \end{bmatrix} \in R^4 \quad , \overline{w}^p = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}, \overline{w}^v = \begin{bmatrix} -Z4 \\ Z3 \end{bmatrix} w_3 \in R^2 \tag{3.22}$$

After introducing perturbations, the coupled (2.12) become

$$\begin{aligned} \dot{e}_1 &= u_1 + \bar{w}_1^{\,v} \\ \dot{e}_i^{\,p} &= \dot{e}_{i-1}^{\,v} - \dot{e}_i^{\,v} + \bar{w}_{i-1}^{\,p} - \bar{w}_i^{\,p} \\ \dot{e}_i^{v} &= u_i + \bar{w}_i^{\,v} \end{aligned} \tag{3.23}$$

where $i \in (2,3,....r)$

The perturbation for the leading vehicle $\hat{w}_1 = \bar{w}_1^v$

for leading vehicle

$$\hat{w}_i = \begin{bmatrix} \hat{w}_i^{\,p} \\ \hat{w}_i^{\,v} \end{bmatrix} = \begin{bmatrix} \bar{w}_{i-1}^p - \bar{w}_i^p \\ \bar{w}_i^v \end{bmatrix}, \qquad i \in (2,3,....r)$$

In the case of three vehicles in the platoon, in the expanded space we got

$$\tilde{w}_1 = \hat{w}_1,$$

$$\tilde{w}_i = \begin{bmatrix} \hat{w}_{i-1}^v \\ \hat{w}_i^p \\ \hat{w}_i^v \end{bmatrix} = \begin{bmatrix} \hat{w}_1 \\ \hat{w}_1^v \\ \hat{w}_2^p \\ \hat{w}_2^v \\ \hat{w}_2^{\,v} \\ \hat{w}_3^p \\ \hat{w}_3^v \end{bmatrix} \qquad \text{Where} \quad \hat{w}_1 = \bar{w}_1^v$$

By introducing perturbation into the kinematic model in (3.13) becomes

$$\Rightarrow \dot{\tilde{e}} = \tilde{A}_D \tilde{e} + \tilde{B}_D \tilde{u} + \tilde{w} \tag{3.24}$$

In order to compute stabilizing feedback gains in the expanded space that only the $i^{th}$ subsystem from (3.24) is considered, because all the subsystems are identical and the subsystems are completely decoupled.

$$\dot{\tilde{e}}_i = \begin{bmatrix} 0 & 0 & 0 \\ I & 0 & -I \\ 0 & 0 & 0 \end{bmatrix} \tilde{e}_i + \begin{bmatrix} I & 0 \\ 0 & 0 \\ 0 & I \end{bmatrix} \tilde{u}_i + \tilde{w}_i \tag{3.25}$$

where $\tilde{x}_i \in R^6$

$$\dot{\tilde{x}}_i = \tilde{A}\tilde{x}_i + \tilde{B}\tilde{u}_i + \tilde{w}_i$$

$$\tilde{u}_i = \begin{bmatrix} \tilde{K}_1 & 0 & 0 \\ \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 \end{bmatrix} \tilde{x}_i \in R^4 \qquad (3.26)$$

and $\tilde{w}_i \in R^6$

$$\tilde{u}_i = \tilde{K}\tilde{x}_i$$

With $\tilde{w}_i$ residing in the sector, that is

$$\tilde{w}_i^T \tilde{w}_i \le \alpha^2 \tilde{x}_i^T W^T W \tilde{x}_i \qquad (3.27)$$

where $\alpha$ is a positive number to be maximized and $W \in R^{P\times 6}$ (P being an arbitrary positive integer) is a constant matrix (usually set to be identity). The matrix $W$ is chosen according to a predetermined knowledge about the perturbations, and if no particular knowledge about the perturbations is available it is set to be an identity matrix, meaning that the norm of perturbations is bounded by the scaled norm of the state variables

As the subsystems are identical, the subsystems' parameters $\tilde{A}, \tilde{B}, \tilde{K}, \alpha$ and $W$ are independent of $i$.

To stabilize each subsystem, a quadratic Lyapunov function is taken

$$V(\tilde{x}_i) = \tilde{x}_i^T P \tilde{x}_i \quad ,$$

where $P \in R^{6\times 6}$ is a positive definite matrix is considered.

## S-Procedure

Let F and G be symmetric matrices of dimension $n \times n$.

Then $y^T F y < 0$ holds whenever $y^T G y \le 0$

If there exists a number $\tau > 0$ such that $F - \tau G < 0$

## Schur Complement

Schur complement is used to convert nonlinear inequalities to LMI.

$$\text{Let}\begin{bmatrix} Q(x) & S(x) \\ S(x)^T & R(x) \end{bmatrix} > 0$$

Where $Q(x) = Q(x)^T$ and $R(x) = R(x)^T$, $S(x)$ depends affinely on $x$ is equivalent to

$R(x) \succ 0,\ Q(x) - S(x)R(x)^{-1}S(x)^T \succ 0$

From Lyapunov function we can write

$$\begin{bmatrix} \tilde{e}_i \\ w_i \end{bmatrix}^T \begin{bmatrix} A_K{}^T P + PA_K & P \\ P & 0 \end{bmatrix} \begin{bmatrix} \tilde{e}_i \\ w_i \end{bmatrix} \prec 0 \quad \text{where } A_K = \tilde{A} + \tilde{B}\tilde{K}$$

From (3.27) we can write inequality form as

$$\begin{bmatrix} \tilde{e}_i \\ w_i \end{bmatrix}^T \begin{bmatrix} -\alpha^2 W^T W & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \tilde{e}_i \\ w_i \end{bmatrix} \le 0$$

By putting S-procedure and Schur complement method finally we got LMI as

Minimize $\gamma$

Subjected to $Y \succ 0$

$$\begin{bmatrix} \tilde{A}Y + Y\tilde{A}^T + \tilde{B}L + L^T\tilde{B}^T & I & YW^T \\ I & -I & 0 \\ WY & 0 & -\gamma I \end{bmatrix} \prec 0 \tag{3.28}$$

which is an LMI optimization problem in the scalar variable $\gamma = 1/\alpha^2$ and the matrix variables $L$ and $Y$ ($Y$ is a scaled inverse of $P$)where $\tilde{K} = LY^{-1}$ and $Y = \tau P^{-1}$ .the matrix $\tilde{K}$ is same structure as $L$ with an imposed structure as follows:

$$L = \begin{bmatrix} L_1 & 0 & 0 \\ L_2 & L_3 & L_1 \end{bmatrix}, Y = \begin{bmatrix} Y_1 & 0 & 0 \\ 0 & Y_2 & 0 \\ 0 & 0 & Y_1 \end{bmatrix} \tag{3.29}$$

Contraction of the feedback gains is carried over to the original space according to the inclusion principle.

## 3.4 Simulation results

The formation of UAVs is designed by taking the nonlinear model of plant. The linear plant model is used to design control gains of error dynamics. As input to linear model and error model is same we use the K value the control gain in nonlinear plant. The control gain value K is calculated by the help of YALMIP toolbox [17] and sedumi solver. From K value the K1, K2 and K3 value for 3 vehicles are extracted. Then by taking help of ODE-45 solver the nonlinear plant dynamics was solved for 3 vehicles. Different plots are drawn for different initial values of 3 vehicles taking some fixed desired velocity $V_d = [300 \ \ 0] \ ft/s$ or $\|V_d\| = 300 \ ft/s$ [1] and desired distance between $d = [400 \quad 400]^T \ ft$ [1]. The parameter $\alpha$ which determines the size of the sector is maximized at value 0.93. The matrix $W$ describing the shape of the sector was set to be identity matrix.

### Results of Formation of five vehicles using Inclusion principle

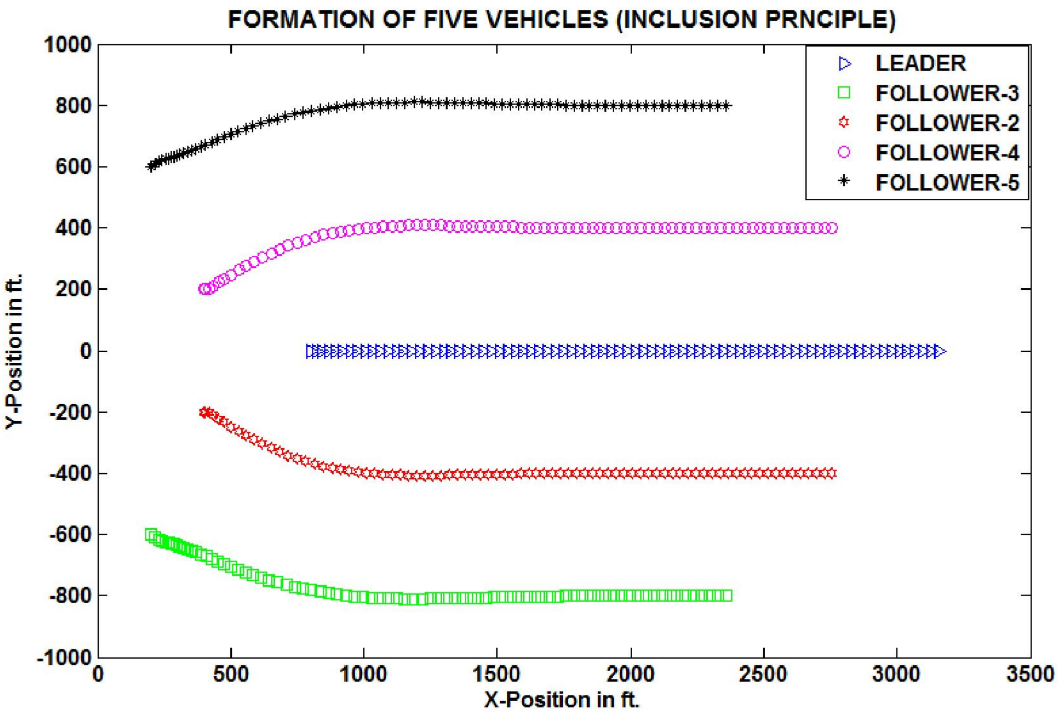


Figure 3.2: Snapshots of the formation for one set of initial condition using inclusion principle

$(V_d = [300 \quad 0] \, ft/s, \; d = [400 \quad 400]^T \; ft)$

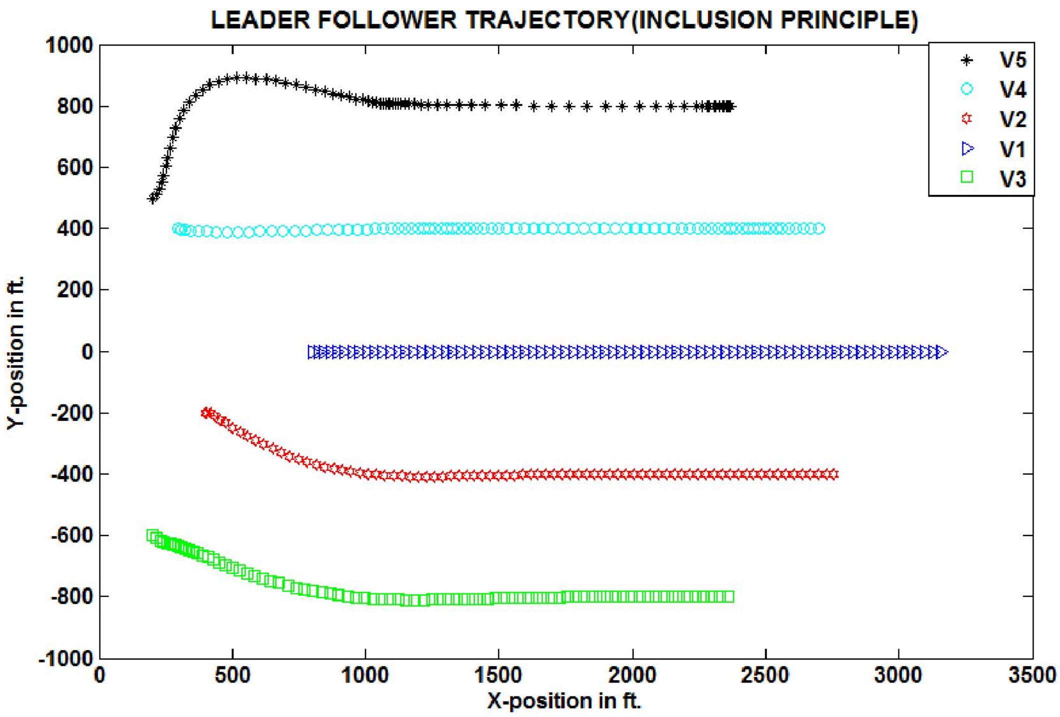


Figure 3.3: Snapshots of the formation for second set of initial condition using inclusion principle

$(V_d = [300 \quad 0]\, ft/s, d = [400 \quad 400]^T \; ft)$

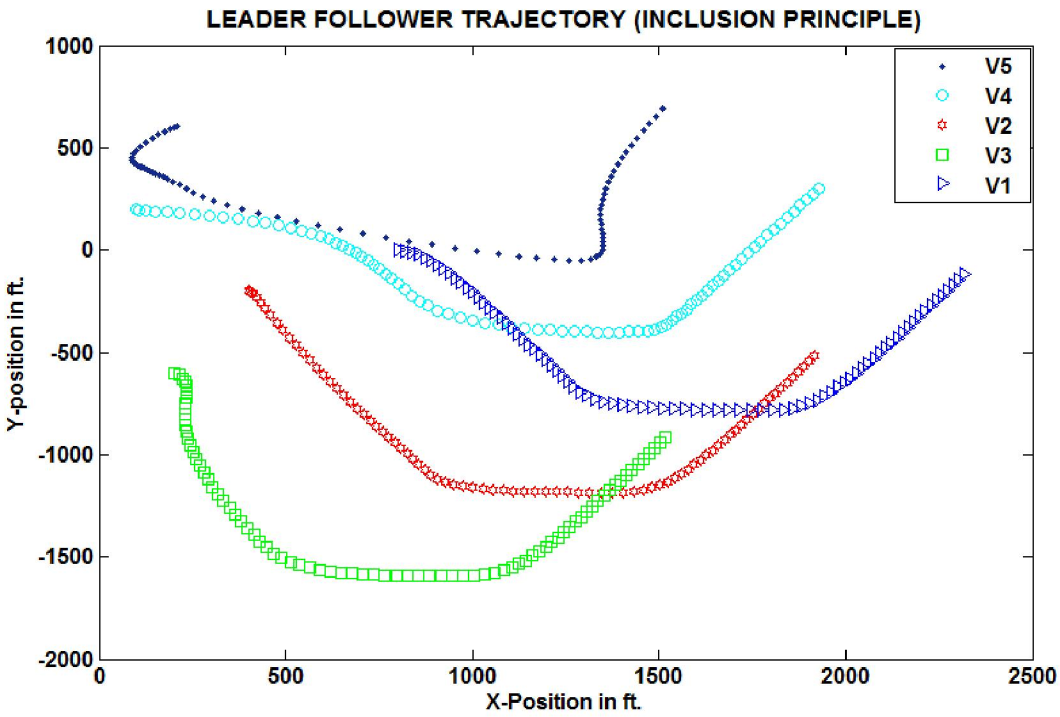


Figure 3.4 Snapshots of the formation for the piecewise defined trajectory using inclusion principle

$(\|V_d\| = 300\, ft/s \;\; ; \; d = [400 \quad 400]^T \; ft)$

In Fig.3.2 and Fig.3.3 superimposed snapshots of formations for different sets of initial condition is presented taking time interval between 0 to 8 sec. In Fig.3.4 the snapshots of the formation with a desired trajectory which is piecewise continues is presented. The nominal speed $V_d$ is $[300\cos(-\pi/3), 300\sin(\pi/3)], [300, 0], [300\cos(\pi/3), 300\sin(\pi/3)]$ $ft/s$ where $\|V_d\| = 300\, ft/s$ .

Horizontal distance between vehicle-1 and vehicle-2 and vehicle-2 and vehicle-3 is presented in Fig.3.5 and Fig.3.6 for one set of initial condition.

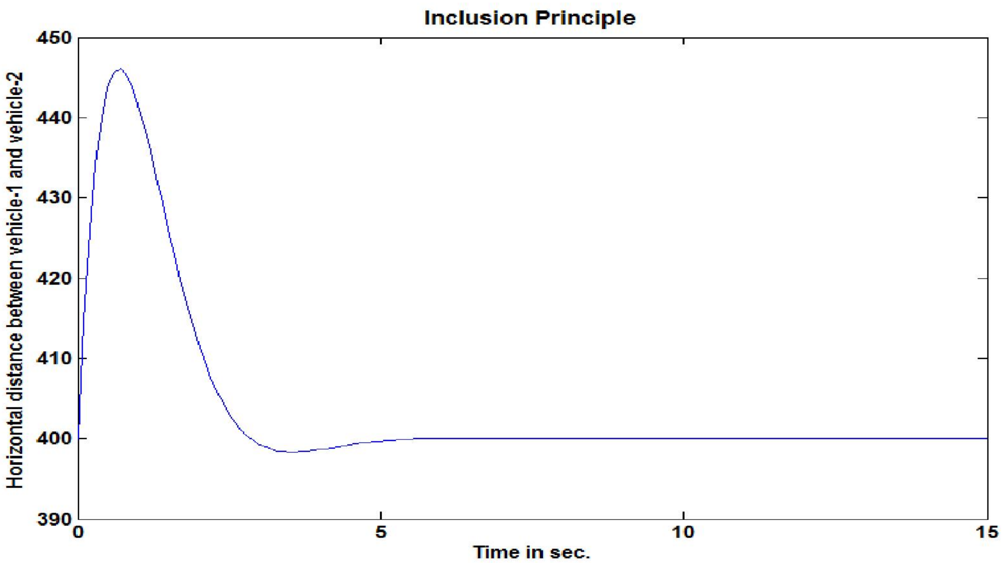


Figure 3.5: Horizontal distance between V1 and V2; $(V_d = [300 \quad 0]\, ft/s, d = [400 \quad 400]^T\ ft)$

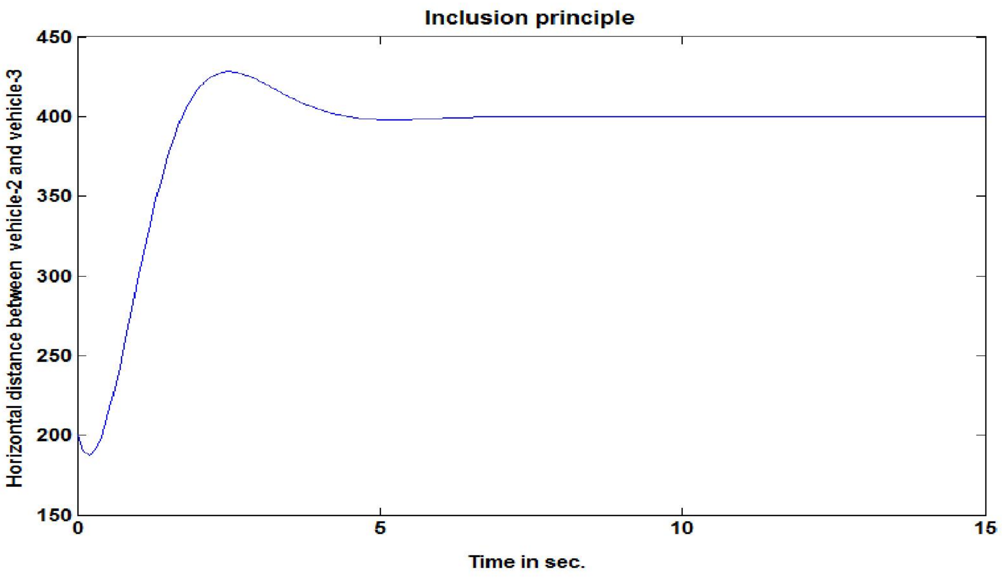


Figure 3.6: Horizontal distance between V2 and V3; $(V_d = [300 \quad 0]\, ft/s, d = [400 \quad 400]^T\ ft)$

## 3.5 Chapter Summary

In this chapter the interconnected overlapping system is decoupled into disjoint sub system by inclusion principle method. In section-3.2 inclusion principle is stated and explained .In section-3.3 robust static state feedback laws are designed and the stability of system is checked by Lyapunov stability criterion. S-procedure & Schur complement is explained. Finally the simulation results for the formation is presented taking different initial condition and the horizontal distance between vehicles are plotted.

# *Chapter-4*

# Graph Theory Based Approach for Decentralized Overlapping Controller Design

## 4.1 Introduction

Graph Theory based approach for decentralization is used when there is a limitation on the availability of states i.e. only few numbers of outputs is available for constructing the control output. The control constraint is represented by binary information flow matrix [8]. When the matrix is block diagonal having block elements one, then the control structure is decentralized. In this approach only controller is decentralized into diagonal form.

## 4.2 Decentralization using Graph Theory Approach

Consider a LTI interconnected system $S$ consisting of $v$ subsystems. Assume that the state-space model for the $i^{th}$ subsystem is described by

$$\dot{x}_i(t) = A_{ii}x_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{v} A_{ij}x_j(t) + B_i u_i(t) \tag{4.1}$$

where $i \in \overline{v} := \{1, 2, 3, ....v\}$

Where $x_i \in R^{n_i}$ $and\, u_i \in R^{m_i}$ is the state and input for the $i^{th}$ subsystem, respectively. In (4.1) the term $A_{ij}x_j,\ j \in \overline{v}$ represents the effect of the $j^{th}$ subsystem on the dynamics of subsystem $i$. The system matrices $A_i, B_i$ $and$ $A_{ij}, i, j \in \overline{v}$ are constant and have appropriate dimensions. The overall dynamics of the interconnected system $S$ can be expressed as

$$\dot{x}(t) = Ax(t) + Bu(t) + Ew(t)$$
$$where$$
$$x(t) = [x_1(t)^T \; x_2(t)^T .........x_v(t)^T]^T$$
$$u(t) = [u_1(t)^T \; u_2(t)^T .........u_v(t)^T]^T$$
$$w(t) = [w_1(t)^T \; w_2(t)^T .........w_v(t)^T]^T$$

The local measurement output signal for $i^{th}$ local controller is represented by

$$y_i(t) = C_i x_i(t)$$

*where* $y_i \in R^{r_i}$ *and* $C_i$ is given a constant matrix of appropriate dimension

and $m = \sum_{i=1}^{v} m_i$, $r = \sum_{i=1}^{v} r_i$

It is necessary to stabilize the system $S$ by using a structurally constrained controller. These constraints determine which outputs $y_i(i \in v)$ are available to construct any specific input $u_j(j \in v)$ of the system. In order to simplify the formulation of the control constraint, a block matrix $K$ with binary entries is defined, where its $(i, j)$ block entry, $i, j \in v$ is a $m_i \times r_j$ matrix with all entries equal to 1 if the output of the $j^{th}$ subsystem can contribute to the construction of the input of the $i^{th}$ subsystem, and is a $m_i \times r_j$ zero matrix otherwise. The matrix $K$ represents the control constraint, and will be denoted to as the information flow matrix. To control the system $S$ a local static output feedback controller be considered for $i^{th}$ sub system

$$u_i(t) = K_i y_i(t)$$

Overall we can say that

$$u(t) = Ky(t) = KCx(t) \tag{4.2}$$

Construct the matrix $K(s) \in R^{m \times r}$ from K as follows.

Replace the $(i, j)$ block entry of $K$, i, j$\in v$, with $K_{ij}(s) \in R^{m_i \times r_j}$ if it is not a zero matrix, where $K_{ij}(s)$ is the rational transfer function matrix of a controller whose input and output are the output of the $j^{th}$ subsystem and the input of the $i^{th}$ subsystem, respectively. It is to be noted that $K(s) \in R^{m \times r}$ represents the transfer function matrix of any structurally constrained controller.

### 4.2.1 Graph Theory Procedures for Decentralization

Consider the system S given by (4.1) Some procedures [8] are followed to construct a bipartite graph $G$ for any given information flow matrix $K$, associated with $S$.

### Procedure-1[8]

Construct the graph $G$ as follows

1. Define two sets of $v$ vertices. Label the sets as set 1 and set 2, and the vertices in each set as vertex 1 to vertex $v$.

2. For any $i, j \in v$, connect the $i^{th}$ vertex of the first set to the $j^{th}$ vertex of the second set with an edge, if the $(i, j)$ block entry of $K$ is not a zero matrix, i.e., if the output of the $j^{th}$ subsystem can contribute to the construction of the input of the $i^{th}$ subsystem. Denote the gain of this edge with $K_{ij}(s)$.

### Procedure-2 (construction of bipartite graph $\bar{G}$ with a decentralized structure from the graph G.

Partition the graph $G$ into a set of complete bipartite sub-graphs such that each edge of the graph $G$ appears in only one of the sub-graphs. It may happen that this partition may require some of the vertices of the graph $G$ to appear in multiple sub-graphs. Denote the resultant graph with $\bar{G}$.

It is not necessarily result in a unique decentralized graph $\bar{G}$ for a given graph $G$

### Procedure-3(Construction of matrix function $\bar{K}(s)$ correspond to graph $\bar{G}$ )

Form a $\bar{m} \times \bar{r}$ block diagonal matrix $\bar{K}(s)$, where $\bar{m}$ and $\bar{r}$ are the number of vertices in sets 1 and 2of $\bar{G}$, respectively, and the number of blocks on its main diagonal is equal to the number of partitioned sub-graphs in $\bar{G}$. Label the complete bipartite sub graphs of $\bar{G}$ as sub graphs 1 to $\bar{v}$. Furthermore, label the vertices of sub graph $l$, $l$ =1, 2, ..., $\bar{v}$ , as vertex $l$, ..., η in subset 1 (corresponding to set 1) and vertex 1, ..., $\overline{\eta}_l$ in subset 2 (corresponding to set 2). The $(l,l)$ block entry of $\overline{K}(s)$, $l$ =1, ..., $\overline{v}$, is a matrix whose $(i,j)$ block entry is equal to the gain of the edge connecting vertex $i$ of subset 1 to vertex $j$ of subset 2 in sub graph $l$ of $\overline{G}$, for any $i$ {1, ..., $\eta_l$ }, $j$ {1, ..., $\overline{\eta}_l$ }. Denote the $(l,l)$ block entry of $\overline{K}(s)$ with $\overline{K}_l(s) \in R^{\overline{m}_l \times \overline{r}_l}$, for $l$ =1, 2, ..., $\overline{v}$.

Suppose that $\overline{K}(s)$ is derived from $K(s)$ using procedures 1, 2 and 3.There exit constant matrices $\phi_1$ and $\phi_2$ which satisfy

$$K(s) = \phi_1 \overline{K}(s) \phi_2 \tag{4.3}$$

$\phi_1$ and $\phi_2$ are denoted as transformation matrix and can be calculated as follows

Consider a matrix $\overline{K}(s)$ with $\overline{v} \times \overline{v}$ block structure. Choose at least one nonzero block entry from each block column and each block row of $\overline{K}(s)$ and let them be denoted by $K_{i_1 j_1}, K_{i_2 j_2} ......K_{i_p j_p}$ Suppose that $K_{i_l j_l}(s)$ $l$ =1, 2, ..., p, is the $(i_l', j_l')$ block entry of the matrix $\overline{K}(s)$ For any 1 ≤ $g$ ≤ ν, 1 ≤ $l$ ≤ p, $g \neq i_l$, the (g, $i_l'$) block entry of the matrix $\phi_1$ is $0_{m_g \times m_{i_l}}$ and also, for any 1 ≤ $g$ ≤ ν, 1 ≤ $l$ ≤ p, $g \neq j_l$, the ( $j_l'$ ,g) block entry of the matrix $\phi_2$ is $0_{r_{j_r} \times r_g}$ Furthermore, for any 1 ≤ $l$ ≤ p, the ( $(i_l, i_l')$ *and* $(j_l', j_l)$ block entries of the matrices

and $\phi_2$ are $I_{m_{i_l}}$ and $I_{r_{j_l}}$, respectively.

The system $\overline{S}$ is an interconnected system an be represented as

$$\begin{aligned} \dot{\overline{x}}(t) &= A\overline{x}(t) + \overline{B}\overline{u}(t) \\ \overline{y}(t) &= \overline{C}\overline{x}(t) \end{aligned} \tag{4.4}$$

$$where \;\; \overline{B} = B\phi_1, \quad \overline{C} = \phi_2 C$$

Where the set of closed-loop modes of the system $S$ under the controller $K(s)$ is equivalent to the set of closed loop modes of the system $\overline{S}$ under the controller $\overline{K}(s)$. Since the system S for the controller $K(s)$ behaves identically to the system under $\overline{S}$ the corresponding controller $\bar{K}(s)$ the problem of structurally constrained control design K(s) for the system $S$ can be redesigned as a decentralized controller $\bar{K}(s)$ for the system $\bar{S}$ .

Let the system $S$ is stabilizable by a information flow matrix $K$ . It is necessary to find a structurally constrained LTI controller with the zero initial state and the transfer function $K(s)$ corresponding to $K$ , such that it minimizes the following LQR performance index:

$$J = \int_0^{\infty} ((x(t)^T Qx(t) + u(t)^T Ru(t))dt \tag{4.5}$$

where $R \in R^{m \times n}$ and $Q \in R^{n \times n}$ are positive definite and positive semi-definite matrices.

The $\bar{K}(s)$ can be calculated through Procedures 1, 2 and 3 so that $\phi_1$ obtained is equal to $I_m$

So finally we got $\phi_1 = I_m$ , $K(s) = \bar{K}(s)\phi_2$ *and* $\bar{B} = B\phi_1, \quad \bar{C} = \phi_2 C$ *for* $\forall t \geq 0$ .

If the controller $K(s)$ is the optimal controller for the system S then the corresponding decentralized controller $\bar{K}(s)$ obtained by using Procedures 1, 2 and 3 is the optimal decentralized controller for the system $\bar{S}$ with respect to the performance index $\bar{J}$

## 4.3 Controller Gain restructuring using Graph Theory

According to information structure constraint each vehicle except leader has information about the states of the vehicle in front of it and its own state .We can say that the output from vehicle-1 is fed to its own and vehicle-2 and similarly for vehicle-2 its information is available for vehicle-2 and vehicle-3.If we use output feedback approach then the control gain matrix for system $S$

$$K(s) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} (K_{11})_{2\times2} & 0_{2\times4} & 0_{2\times4} \\ (K_{21})_{2\times2} & (K_{22})_{2\times4} & 0_{2\times4} \\ 0_{2\times2} & (K_{32})_{2\times4} & (K_{33})_{2\times4} \end{bmatrix}_{6\times10} \end{array} \tag{4.6}$$

If we will draw the graph for controller gain matrix it looks like

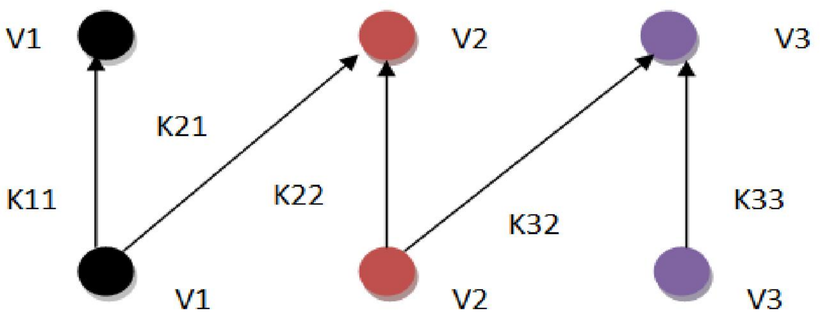


Figure 4.1: Graph $G$ corresponding to gain matrix $K(s)$

Then applying procedure 1, 2 and 3 and we got optimal LTI controller gain $\bar{K}(s)$ .the arrow marks shows the flow of information from one vertex to other [10].

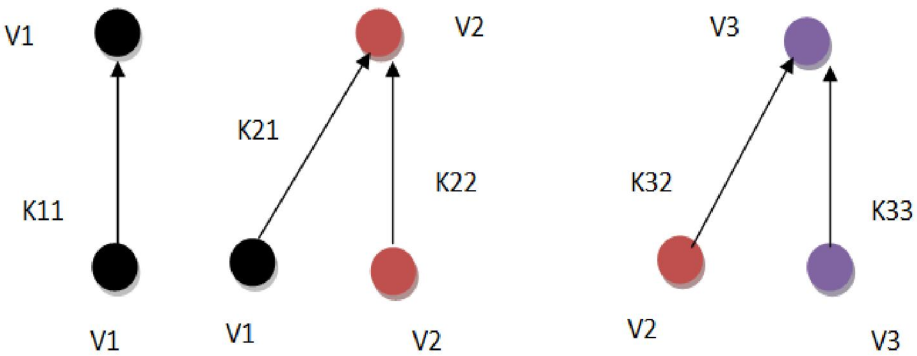


Figure 4.2: Decentralized graph $\bar{G}$ obtained from $G$

From the Fig.4.2 the decentralized gain matrix is formulated as

$$\bar{K}(s) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 1 & 2 & 2 & 3 \end{array} \\ \begin{bmatrix} (K_{11})_{2\times2} & 0_{2\times2} & 0_{2\times4} & 0_{2\times4} & 0_{2\times4} \\ 0_{2\times2} & (K_{21})_{2\times2} & (K_{22})_{2\times4} & 0_{2\times4} & 0_{2\times4} \\ 0_{2\times2} & 0_{2\times2} & 0_{2\times4} & (K_{32})_{2\times4} & (K_{33})_{2\times4} \end{bmatrix}_{6\times16} \end{array} \quad (4.7)$$

We got $\phi_1 = I_6$ and 

$$\phi_2 = \begin{bmatrix} I_{2\times2} & 0_{2\times4} & 0_{2\times4} \\ I_{2\times2} & 0_{2\times4} & 0_{2\times4} \\ 0_{4\times2} & I_{4\times4} & 0_{4\times4} \\ 0_{4\times2} & I_{4\times4} & 0_{4\times4} \\ 0_{4\times2} & 0_{4\times4} & I_{4\times4} \end{bmatrix}_{16\times10}$$

The transformed C matrix is $\bar{C} = \phi_2 C$

So closed loop dynamics of decentralized system $\bar{S}$ is

$$\dot{\bar{x}}(t) = (A + B\bar{K}\bar{C})\bar{x}(t) \quad where\ B = \bar{B}$$

## 4.4 A D-K type iteration algorithm

Consider a linear time invariant continuous time system

$$\dot{x} = Ax + Bu$$
$$y = cx$$

The static output feedback control law is $u = Ky = Kcx$

The closed loop system is defined as $\dot{x} = (A + BKC)x$

### Algorithm

1. Initialize gain $K = 0$ , $i > 0$ $(any\ positive\ value)$
2. Solve for $P$ using the following optimization problem:

$$\text{Minimize} \quad \alpha$$
$$PA + A'P + PBKC + C'K'B'P + \alpha I \prec 0$$
$$P \succ 0$$

3. Consider a small perturbation in $P$ as $P = P + \varepsilon \hat{I}$ ,where $\hat{I}$ is an appropriate dimensional matrix with all its elements equal to 1.This is to avoid convergence of $P$ to a block-diagonal matrix, which is important considering co-operative nature of the decentralized control problem. In present case, we have used $\varepsilon = 0.1$.

4. With the above value of $P$ , obtain $K$ from the following:

$$\text{Minimize} \quad \alpha$$

$$PA + A^T P + PBKC + C^T K^T B^T P + \alpha I < 0$$
$$P > 0$$
$$\begin{bmatrix} \bar{k} & K \\ K^T & I \end{bmatrix} > 0$$
$$\bar{k} - 0.1 > 0$$
$$\bar{k} - 100 < 0$$

5. If $\alpha$ is negative then stop and the obtained value of $K$ is the desired solution. Else go to Step 2 with the updated $K$.

## 4.5 Simulation results

The formation of UAVs is designed by taking the nonlinear model of plant. The linear plant model is used to design control gains of error dynamics. As input to linear model and error model is same we use the K value the control gain in nonlinear plant. The control gain value K is calculated by the help of LMI toolbox. From K value the K11, K21, K22, K32, K33 value for 3 vehicles are extracted. Then by taking help of ODE-45 the nonlinear plant dynamics was solved for 3 vehicles. Different plots are drawn for different initial values of 3 vehicles taking some fixed desired velocity $V_d = [300 \;\; 0] \; ft/s$ or $\|V_d\| = 300 \; ft/s$ and desired distance between $d = [400 \;\;\; 400]^T \; ft$ [1]. The formation is symmetric triangular structure so after getting the result for one platoon the second platoon is solved. It is a closed loop system so by putting only initial condition i.e. by giving only initial position and velocity of vehicles we will get result and from the result it is observed that how the followers are following the leader.

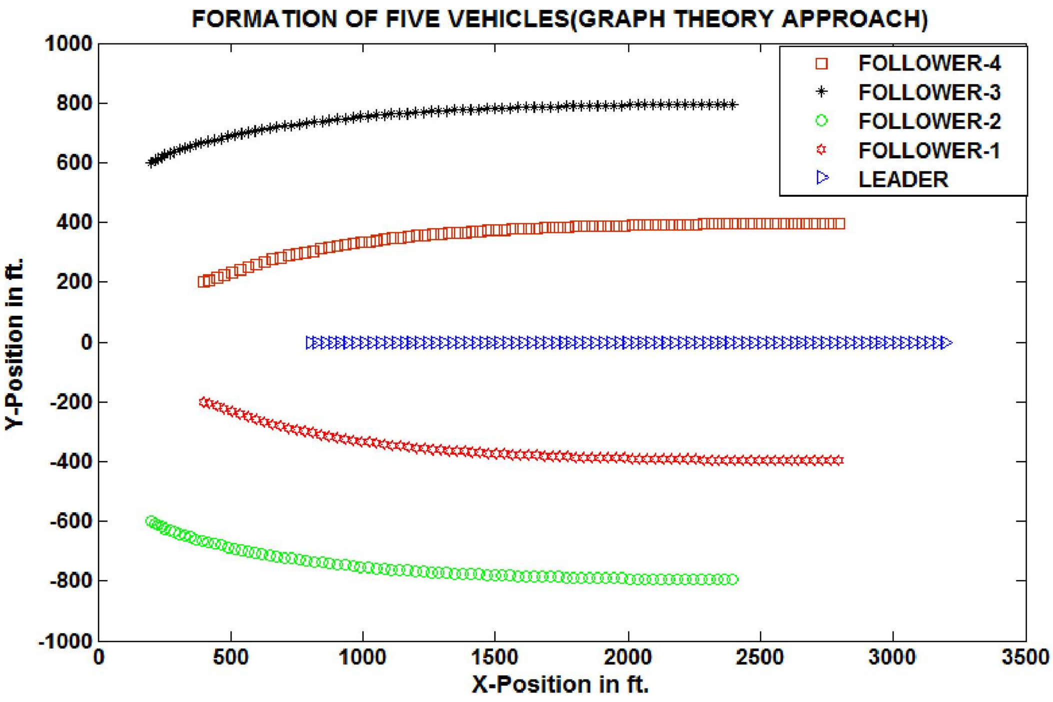


Figure 4.3: Snapshots of the formation for the piecewise defined trajectory using Graph Theory approach
$(V_d = [300 \;\; 0]\, ft/s, d = [400 \;\; 400]^T \;\; ft)$

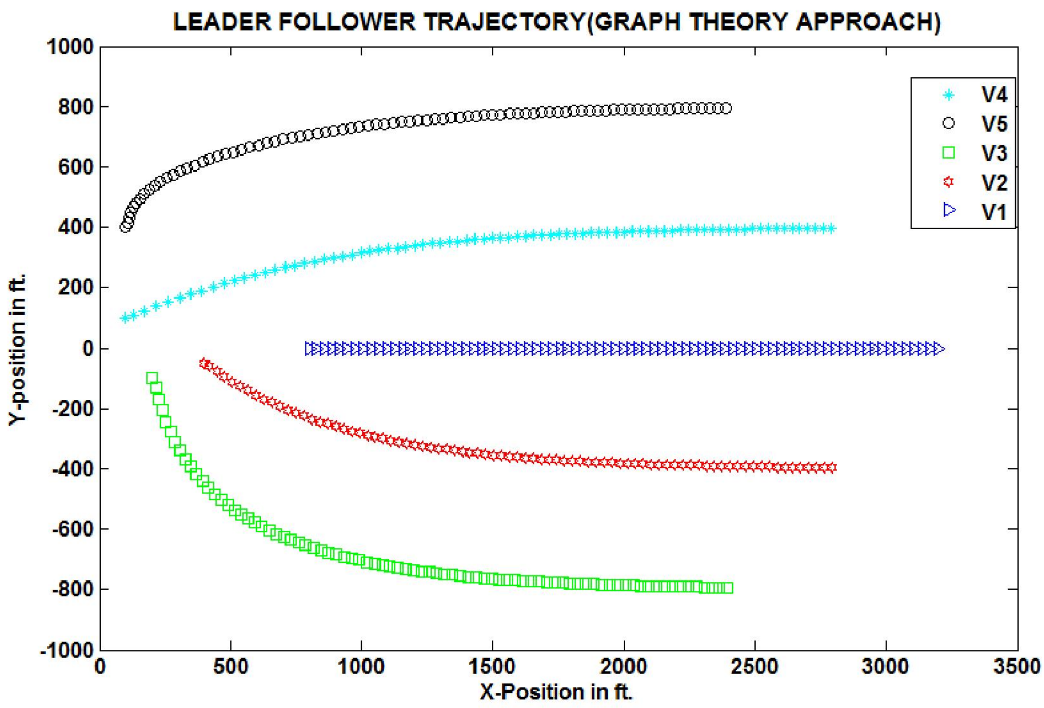


Figure 4.4: Snapshots of the formation for the piecewise defined trajectory using Graph Theory approach
$(V_d = [300 \;\; 0]\, ft/s, d = [400 \;\; 400]^T \;\; ft)$

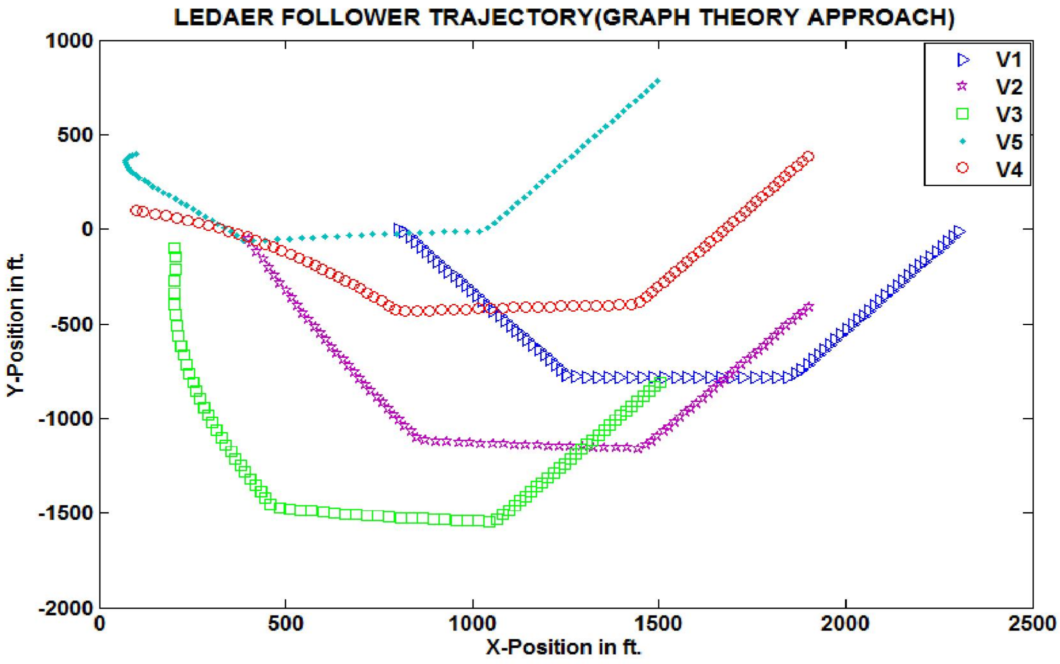


Figure 4.5: Snapshots of the formation for the piecewise defined trajectory using Graph Theory approach ($\|V_d\| = 300\, ft/s$ , $d = [400 \;\; 400]^T \; ft$)

In Fig.4.3 and Fig.4.4 superimposed snapshots of formations for different sets of initial condition is presented taking time interval between 0 to 8 sec. In Fig.4.5 the snapshots of the formation with a desired trajectory which is piecewise continues is presented . The nominal speed $V_d$ is $[300\cos(-\pi/3), 300\sin(\pi/3)], [300, 0], [300\cos(\pi/3), 300\sin(\pi/3)]\; ft/s$ where $\|V_d\| = 300\, ft/s$ . Horizontal distance between vehicle-1 and vehicle-2 and vehicle-2 and vehicle-3 is presented in Fig.4.6 and Fig.4.8 for one set of initial condition

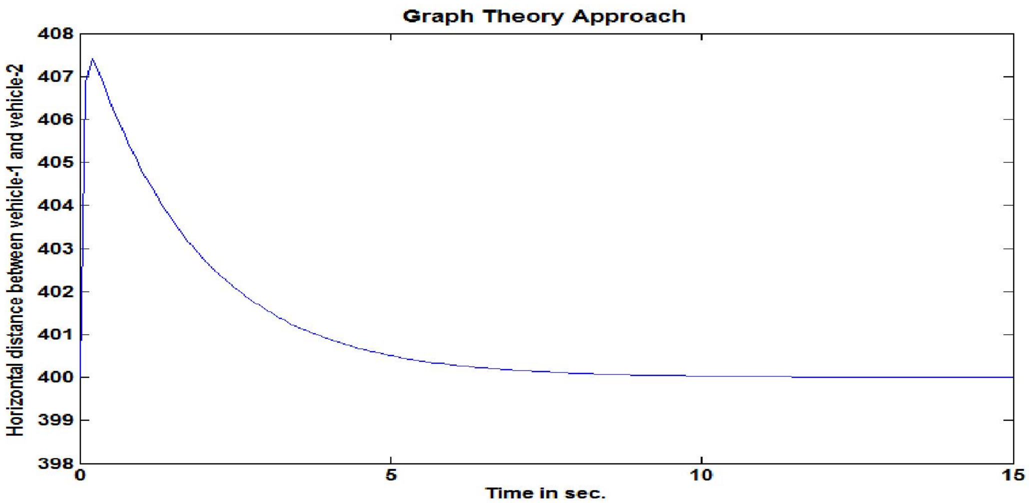


Figure 4.6: Horizontal distance between V1 and V2 ; ($V_d = [300 \;\; 0]\, ft/s, d = [400 \;\; 400]^T \; ft$)

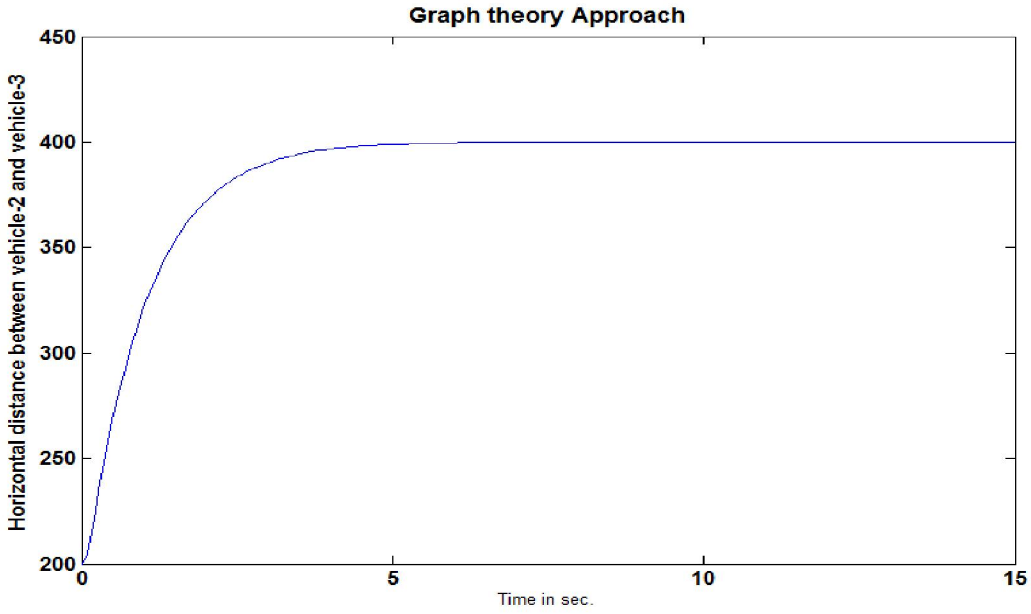


Figure 4.7: Horizontal distance between V2 and V3; $(V_d = [300 \quad 0]\, ft/s, d = [400 \quad 400]^T \; ft)$

## Discussion

From the above simulation results it is clear that all the vehicles formation tracks its desired velocity and desired spacing more quickly in the inclusion principle method than the graph theory approach. Generally it is taking 5 sec more than the inclusion principle to converge to its steady state. In inclusion method the expanded system which is produced is uncontrollable one. The contraction of the designed controller is very difficult task when there is multiple overlapping. In graph theory based approach only controller structure is expanded to decentralized structure instead of decentralization of whole plant structure. So graph theory approach based decentralization is more efficient and computationally less complex.

## 4.6 Chapter Summary

In this chapter graph theory based approach is used for decentralization of control gain. Here the original graph corresponding to gain matrix $K$ is transformed to number of bipartite sub graphs from which decentralized gain matrix $\bar{K}$ is obtained. Then the transformation of all other matrices are obtained by transformation matrix .Finally D-K iteration is used to get the control gain values and simulation results are presented for formation of 5 UAVs.

# *Chapter 5*

# Delay Tolerability in Overlapping Control

## 5.1 Introduction

In a distributed or decentralized control system time delays exists in information exchange between different control agents. Time delays in system dynamics have significant impact on performance and stability of the system. So this problem is generally taken into account at the time of controller design. Basically there are two kinds of delay in multi agents systems like the formation control problem (1) Communication Delay (2) input delay. Delay which occurs in between the communication from one agent to another is called communication delay and another delay is related to processing and connecting time for the packets arriving at each agent. It can also occur between actuator and controllers when they are connected by networks. Here communication delay is considered in our problem. Although decentralized control is used where delay is minimal but our goal is to determine the maximum delay, the formation can tolerate. Delay margin is defined as maximum time delay, the system can tolerate before going to unstable.

## 5.2 Closed loop Dynamics

The overlapping control gain $K$ which shows the information flow between agents is decomposed into diagonal and off-diagonal blocks. From chapter-4 it is clear that the $i^{th}$ local output feedback gain is denoted $K_i$.

The diagonal matrix   of control gain $K$ is known as decentralized gain matrix $\bar{K}$ and the $(i,i)$ block entry of $\bar{K}$ is equal to $K_{ii}$. The off diagonal matrix is known as overlapping gain matrix and denoted by $\tilde{K}$. Its $(i,j)$ block entry $i \neq j$ is $K_{ij}$.

So finally we can write as the control input as

$$u_i(t) = K_{ii} C_i x_i(t) + \sum_{j=1, j\neq i}^{v} K_{ij} C_j x_j(t-h) \tag{5.1}$$

and generalized expression $u(t) = \bar{K}Cx(t) + \tilde{K}Cx(t-h)$

where '$h$' is the communication delay

the closed loop system dynamics is written as

$$\dot{x}(t) = (A + B\bar{K}C)x(t) + B\tilde{K}Cx(t-h) \tag{5.2}$$

Here some assumptions have taken for nontrivial solution. It is assumed that at least one of the local controllers can access at least one of the other subsystem's measurement signals through a communication link. Let the vehicle 2 can access the local measurement of other vehicles and while vehicle -1 and vehicle-3 can access the measurements of vehicle-2.

The control gain matrix $K$ is written as

$$K = \begin{bmatrix} (K_{11})_{2\times2} & (K_{12})_{2\times4} & 0_{2\times4} \\ (K_{21})_{2\times2} & (K_{22})_{2\times4} & (K_{23})_{2\times4} \\ 0_{2\times2} & (K_{32})_{2\times4} & (K_{33})_{2\times4} \end{bmatrix}_{6\times10}$$

The corresponding bipartite graph is

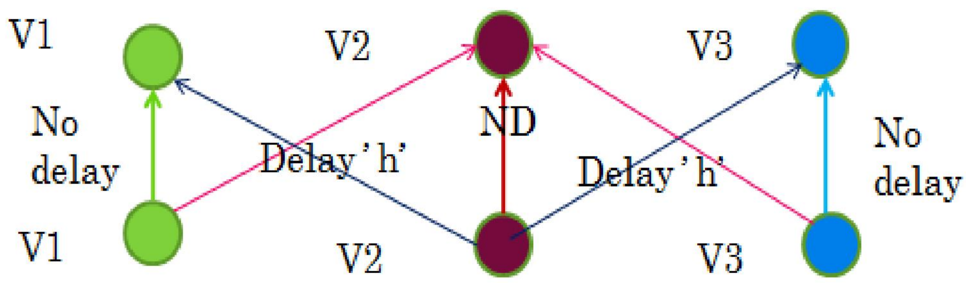


Figure 5.1 Graph $G$ for time delay system

The above Fig.5.1 shows that time delay exists when information is transmitted from one vertex to another vertex (different number of vertex). There is no time delay for the same vehicle or vertex.

From this the diagonal and off-diagonal blocks are extracted as

$$\tilde{K} = \begin{bmatrix} 0_{2\times2} & (K_{12})_{2\times4} & 0_{2\times4} \\ (K_{21})_{2\times2} & 0_{2\times4} & (K_{23})_{2\times4} \\ 0_{2\times2} & (K_{32})_{2\times4} & 0_{2\times4} \end{bmatrix}_{6\times10}$$

$$\bar{K} = \begin{bmatrix} (K_{11})_{2\times2} & 0_{2\times4} & 0_{2\times4} \\ 0_{2\times2} & (K_{22})_{2\times4} & 0_{2\times4} \\ 0_{2\times2} & 0_{2\times4} & (K_{33})_{2\times4} \end{bmatrix}_{6\times10}$$

and we got transformation matrix is $T = \begin{bmatrix} I_{2\times2} & 0_{2\times4} & 0_{2\times4} \\ 0_{4\times2} & I_{4\times4} & 0_{4\times4} \\ I_{2\times2} & 0_{2\times4} & 0_{2\times4} \\ 0_{4\times2} & I_{4\times4} & 0_{4\times4} \\ 0_{4\times2} & 0_{4\times4} & I_{4\times4} \\ 0_{4\times2} & I_{4\times4} & 0_{4\times4} \\ 0_{4\times2} & 0_{4\times4} & I_{4\times4} \end{bmatrix}_{24\times10}$

Frequency domain analysis is widely used for testing the stability criterion both for linear and nonlinear systems with tools such as describing function, Popov criterion, circle criteria and some frequency sweeping tests. Frequency sweeping test are generally more favoured because of its simplicity and computational ease and can be checked graphically.

There are two stability notions

- Delay independent stability
- Delay dependent stability

Consider a LTI delay system described by state space equation

$$\dot{x}(t) = A_0 x(t) + \sum_{k=1}^{m} A_k x(t - r_k) \qquad , \ r_k \geq 0, \tag{5.3}$$

where $A_0$ , $A_k \in R^{n\times n}$ are system matrices and $r_k$ are delay times.

The above system is stable independent of delay if the stability persists with respect to all possible nonnegative delays and if the stability persists only for a subset of nonnegative delays then it is known as delay dependent stability.

### 5.2.1 Frequency Sweeping Test

It is one of the frequency domain stability analysis method of time delay system.

Let’s take a time delay system with a single delay

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-\tau), \qquad \tau \geq 0 \tag{5.4}$$

The quasipolynomial for the system is given by

$$a(s, e^{-\tau s}) = \det(sI - A_0 - e^{-\tau s} A_1) \tag{5.5}$$

The necessary and sufficient condition for stability independent of delay

## Theorem -1[12]

The system is stable independent of delay if and only if

1. $A_0$ is stable
2. $A_0 + A_1$ is stable and
3. $$\rho((j\omega I - A_0)^{-1} A_1) < 1, \qquad \forall \omega > 0 \tag{5.6}$$

## Theorem-2[12]

Let the system in eqn-5.4 is stable at $\tau = 0$

Let $rank(A_1) = q$

Define

$$\bar{\tau}_i := \begin{cases} \min\limits_{1 \leq k \leq n} \dfrac{\theta^i{}_k}{\omega^i{}_k} & if\ \lambda_i(j\omega^i{}_k I - A_0, A_1) = e^{-j\theta^i{}_k}\ for\ some\ \omega^i{}_k \in (0,\infty), \theta^i{}_k \in [0, 2\pi] \\ \infty & \underline{\rho}(j\omega I - A_0, A_1) > 1, \qquad \forall \omega \in (0,\infty) \end{cases} \tag{5.7}$$

Then $\bar{\tau} = \min\limits_{1 \leq i \leq q} \bar{\tau}_i$

The system is stable for all $\tau \in [0, \bar{\tau})$ but becomes unstable at $\tau = \bar{\tau}$

where $\bar{\tau}$ is known as delay margin (maximum tolerable delay)

### 5.2.2 Computation of Delay Margin

1. Compute the generalized eigen values $\lambda_i(j\omega^i{}_k I - A_0, A_1)$ by gridding the frequency axis. At each gridding point calculate the generalized Eigen value

2. If $\underline{\rho}(j\omega I - A_0, A_1)$ f 1, $\forall \omega$ f 0 then we conclude that the system is stable for all $\tau \in [0, \infty)$

3. Otherwise find the critical values $(\omega^i{}_k, \theta^i{}_k)$ which yields delay margin when $\det(j\omega^i{}_k I - A_0 - A_1) \neq 0$

From DK iteration and the graph theory based approach discussed in chapter-4 the controller gain values obtained as follows

$$k_{11} = \begin{bmatrix} -18.0874 & 0.0005 \\ 0.0005 & -18.0874 \end{bmatrix},$$

$$k_{12} = \begin{bmatrix} -17.2478 & -0.0013 & 1.9965 & -0.0013 \\ -0.0013 & -17.2478 & -0.0013 & 1.9965 \end{bmatrix},$$

$$k_{21} = \begin{bmatrix} -6.0567 & 0.0053 \\ 0.0053 & -6.0567 \end{bmatrix},$$

$$k_{22} = \begin{bmatrix} 2.0180 & 0.0015 & -17.0089 & -0.0005 \\ 0.0015 & 2.0180 & -0.0005 & -17.0089 \end{bmatrix},$$

$$k_{23} = \begin{bmatrix} -7.6645 & 0.0002 & -0.5902 & -0.0006 \\ 0.0002 & -7.6645 & -0.0006 & -0.5902 \end{bmatrix},$$

$$k_{32} = \begin{bmatrix} 5.7726 & -0.0002 & -2.6051 & 0.0036 \\ -0.0002 & 5.7726 & 0.0036 & -2.6051 \end{bmatrix},$$

$$k_{33} = \begin{bmatrix} 10.5003 & 0.0008 & -17.0949 & 0.0012 \\ 0.0008 & 0.5003 & 0.0012 & -17.0949 \end{bmatrix}$$

The error model of the formation in the relative co-ordinates is obtained from chapter-2, (2.12)

$$\begin{bmatrix} \dot{e}_1^{II} \\ \dot{e}_2^{I} \\ \dot{e}_2^{II} \\ \dot{e}_3^{I} \\ \dot{e}_3^{II} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ I & 0 & -I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & -I \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1^{II} \\ e_2^{I} \\ e_2^{II} \\ e_3^{I} \\ e_3^{II} \end{bmatrix} + \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{5.8}$$

$$\Rightarrow \dot{e} = Ae + Bu$$

By taking closed loop system model and assuming that the matrix C as identity matrix the time delay equation is obtained. From frequency sweeping test delay margin is obtained as $h = \tau =$ 0.19 sec

The state variables of error dynamics of single UAV are plotted below for delay margin 0.17 sec.

The error states are broadly discussed in chapter-2 now it can be represented by new variable like $e_1^{II} = [Z13\ \ Z14]^T$, $\begin{bmatrix} e_2^{I} \\ e_2^{II} \end{bmatrix} = [Z21\ Z22\ Z23\ Z24]^T$, $\begin{bmatrix} e_3^{I} \\ e_3^{II} \end{bmatrix} = [Z31\ Z32\ Z33\ Z34]^T$

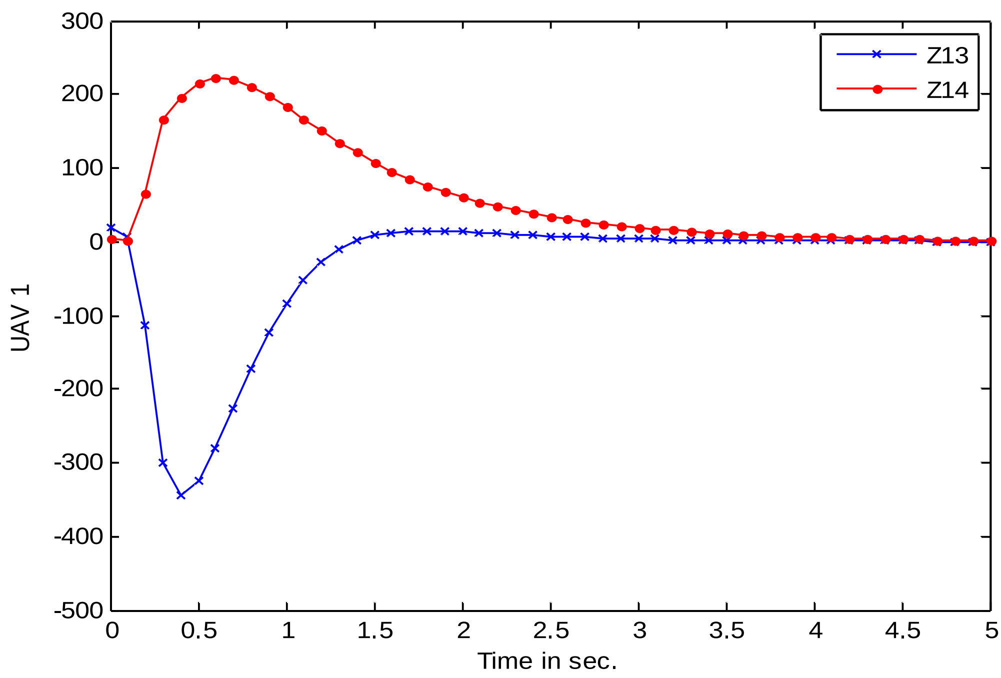


Figure 5.2:State response of vehicle-1 for h=0.17 sec.

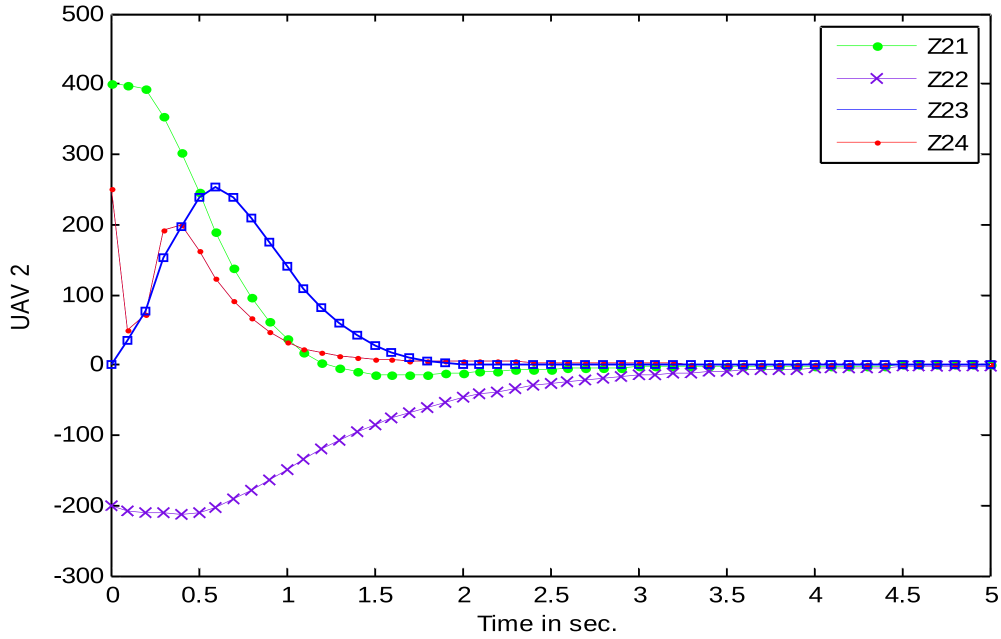


Figure 5.3: State response of vehicle-2 for h=0.17sec

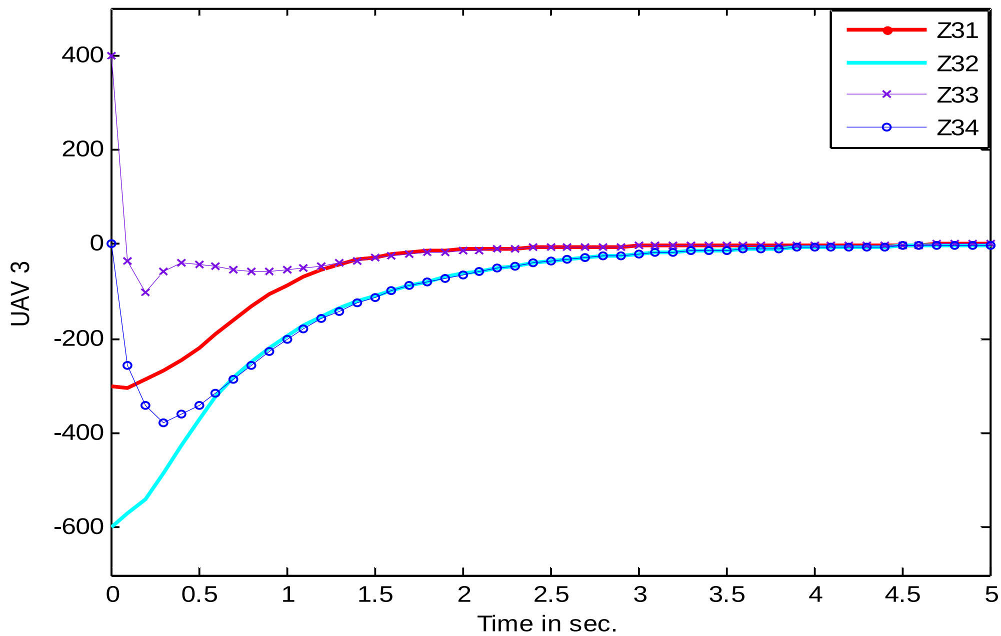


Figure 5.4: state response of vehicle-3 for h=0.17 sec

In the above figures we plotted the states of error equation for 3 UAVS taking delay margin into account. From the graphs it is found that the error goes to zero after 3.5 sec.

## 5.3 Chapter Summary

In this chapter, different time delays of a large scale system have been discussed .The delay tolerability and the system stability of system is calculated by frequency sweeping test in section 5.1.1 and 5.2.2 .Some simulation results are presented for error dynamics by taking communication delay into account.

# *Chapter 6*

# Conclusion and Scope for Future Work

## 6.1 Conclusion

In this thesis leader follower type formation for UAVs has been designed with information structure constraint that the follower has only the information about leader in front of it hence reducing the communication overhead. Two decentralization techniques, i.e. inclusion principle and graph theory approach have been studied and applied to triangular formation of five UAVs. In the inclusion principle, the original interconnected system is expanded to disjoint subsystems and controller has been designed separately, where as in the graph theory approach only the controller is expanded to disjoint ones. The Static state feedback control laws are designed in the expanded space using inclusion principle and the graph theory approaches. D-K iteration, S-procedure techniques and LMI and YALMIP toolboxes are used to obtain the control gains of the formation. Then expanded subsystems are contracted back to original space for implementation. From the simulation result it is clear that inclusion principle approach takes less time for formation but the expanded system is uncontrollable and contraction of the designed controller is very difficult task. The graph theory approach gives better stability to system. Delay tolerability is calculated by adding communication delay in information exchange between two UAVs. The system is converted to time delay system and then frequency sweeping test is carried out to check the stability analysis and to get the maximum tolerable delay i.e. delay margin.

## 6.2 Scope for future work

In future work the discrete time domain can be used for design of formation control problem. Instead of constant time delay, uncertain time delay information flow may be considered. Performance criterion like pole placement, disturbance rejection will be considered in future. Different shape of formation which increases the no of overlapping can be taken into account and the dynamics of UAV may be taken.

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