LINEARIZATION AND ANALYSIS OF LEVEL AS WELL AS THERMAL PROCESS USING LABVIEW

A Thesis Submitted in Partial Fulfilment
of the Requirements for the Award of the Degree of

Master of Technology

in

Electronics and Instrumentation Engineering

by

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National Institute of Technology, Rourkela

Odisha- 769008, India

May 2012

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CERTIFICATE

This is to certify that the Thesis Report entitled "**LINEARIZATION AND ANALYSIS OF LEVEL AS WELL AS THERMAL PROCESS USING LABVIEW"** submitted by **Mr. SANKATA BHANJAN PRUSTY** bearing roll no. **210EC3216** in partial fulfilment of the requirements for the award of Master of Technology in Electronics and Communication Engineering with specialization in **"Electronics and Instrumentation Engineering"** during session 2010-2012 at National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

Dr. Umesh Chandra Pati

Place:

Date:

Associate Professor

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Rourkela-769008

Dedicated

to

My Family

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ABSTRACT

The processes encountered in the real world are usually multiple input multiple output (MIMO) systems. Systems with more than one input and/or more than one output are called MIMO system. The MIMO system can either be interacting or non-interacting. If one output is affected by only one input, then it is called non-interacting system, otherwise it is called interacting system. The control of interacting system is more complex than the control of non-interacting system. The output of MIMO system can either be linear or non-linear. In process industries, the control of level, temperature, pressure and flow are important in many process applications.

In this work, the interacting non-linear MIMO systems (i.e. level process and thermal process) are discussed. The process industries require liquids to be pumped as well as stored in tanks and then pumped to another tank. Most of the time the liquid will be processed by chemical or mixing treatment in the tanks, but the level and temperature of the liquid in tank to be controlled at some desired value and the flow between tanks must be regulated. The interactions existing between loops make the process more difficult to design PI/PID controllers for MIMO processes than that for single input single output (SISO) ones and have attracted attention of many researcher in recent years.

In case of level process, the level of liquid in the tank is controlled according to the input flow into the tank. Two input two output (TITO) process and four input four outputs (FIFO) process are described in the thesis work. The aim of the process is to keep the liquid levels in the tanks at the desired values. The output of the level process is non-linear and it is converted into the linear form by using Taylor series method. By using Taylor series method in the non-linear equation, the converted linear equation for the MIMO process is obtained.

The objective of the thermal process is to cool a hot process liquid. The dynamic behaviour of a thermal process is understood by analysing the features of the solutions of the mathematical models. The mathematical model of the thermal process is obtained from the energy balance equation. The nonlinear equation is linearized by using Taylor series. The responses of the higher-order thermal process (3 x 2 and 3 x 3) are obtained and analysed.

Laboratory Virtual Instrumentation Engineering Workbench (LabVIEW) is used to communicate with hardware such as data acquisition, instrument control and industrial automation. Hence LabVIEW is used to simulate the MIMO system.

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LIST OF FIGURES

LIST OF ABBREVIATIONS

MIMO	Multiple Input Multiple Output
SISO	Single Input Single Output
TITO	Two Input Two Output
FIFO	Four Input Four Output
GMC	Generic Model Control
PI	Proportional Integral
PID	Proportional Integral and Derivative
VI	Virtual Instrument
LabVIEW	Laboratory Virtual Instrumentation Engineering Workbench

CHAPTER 1

Introduction

Overview

Literature Review

Motivation

Objectives

Organisation of the Thesis

INTRODUCTION

This chapter gives the general overview of the work. This comprises of a brief description of level process and thermal process followed by literature survey. The objectives and organisation of the thesis are mentioned in this chapter.

1.1 OVERVIEW

The control of level and temperature of liquid in tanks and flow between tanks are the basic problem in the process industries. The process control industries require liquids are to be pumped as well as stored in tanks and then pumped to another tank. Most of the times, the liquids will be processed in the tanks, but always the level and temperature of liquid in the tanks have to be controlled and the flow between tanks have to be regulated. As the tanks are so coupled together that the levels in tanks interact with each other and this must also be controlled. Systems which have more than one input and/or more than one output are called multiple input multiple output (MIMO) systems [1].

Tank level control systems are used frequently in different processes. All of the process industries, the human body and fluid handling system depend upon tank level control systems. The control system engineers have to understand how tank control systems work and how the level control problem is achieved. However, interactions existing between loops make it more difficult to design PI/PID controllers [2] for MIMO processes than that for single input single output (SISO) ones and have attracted attention of many researcher in recent years.

Thermal control systems are also used frequently in different processes. The thermal control system is nonlinear, time varying and consists of multivariable. Hence, the control of such system is complex and challenging. It is a challenging task for control system engineers to understand how thermal control systems work and how the control of temperature of liquid in the tank is achieved. The description of a system using mathematical concepts and language is called a mathematical model. Mathematical modeling is the process of developing a mathematical model. A mathematical model helps to explain a system and to study the effects of different components, and to make predictions about their behavior. Mathematical models can take many forms like dynamical systems, statistical models or differential equations. Hence, Mathematical model of the components of the thermal control system is very important for design and analysis of the control system.

The output of the MIMO process can either be interacting or non-interacting. If one output is affected by only one input, then it is called non-interacting, otherwise it is called interacting system. In MIMO system, the input variable affects the output variable which causes interaction between the input/output loops. So, the control of multivariable systems is much more difficult compared to the SISO system. Therefore, the degree of interaction plays an important role to quantify the proper input/output pairings that minimize the impact of the interaction. The interaction between inputs and outputs can be eliminated by using decouplers, cross controllers or many other methods.

Laboratory Virtual Instrumentation Engineering Workbench (LabVIEW) [4 - 5] is used to communicate with hardware such as data acquisition, instrument control and industrial automation. LabVIEW is a system design platform and development environment for a visual programming language and also allows to create programs with graphics instead of text code. LabVIEW can do the following operations.

- Monitoring and controlling water levels
- Monitoring and controlling temperature limits in thermal processes
- Analysing and processing of electric signals
- Vibration analysis

Hence, LabVIEW is used to simulate the MIMO system. The level process and thermal process are designed and simulated using the mathematical models of the corresponding processes in LabVIEW.

1.2 LITERATURE REVIEW

The literature study for this work begins with the MIMO systems [1-2]. The processes which deal with the real world are usually MIMO systems. The modelling of the system is necessary whenever the analysis of control systems is required [3]. The modelling is necessary for the analysis of the processes like level process [6-10] and thermal process [11]. In the paper titled "The Level Control of Three Water Tanks Based on Self-Tuning Fuzzy-PID Controller", H. Y. Sun, D. W. Yan and B. Li have controlled the level in three water tanks using Fuzzy-PID controller [12]. D. Subbulekshmi and J. Kanakaraj have discussed about the decoupling and linearization feedback and Generic Model Control (GMC) algorithms for an approximated model of interacting thermal process [13]. Decentralized Proportional, Integral and Derivative (PID) controller synthesis methods for closed loop stabilization of linear timeinvariant plants subject to I/O delays have been presented by A.N. Mete, A. N. Gundes and H. Ozbay [14]. The MIMO system can either be interacting or non-interacting. The controls of interacting systems [15-16] are much more complicated than the non-interacting systems. The designing of multi-loop controllers for the multivariable interacting systems [17-22] are complicated. The designing of multivariable decoupling and multiloop PI/PID controllers in a sequential fashion have been presented by S. J. Shiu and S. Hwang [23]. They have proposed the tuning technique which is appropriate for a wide range of process dynamics in a multivariable environment.

E. Cornieles, M. Saad, G. Gauthier and H. Saliah-Hassane [24] included five different PID controllers (Ziegler-Nichols, Integral of Time-Weighted Absolute Error (ITAE), Internal Model Control (IMC), poles placement and dual loop). They have presented the modelling of the physical system and real time simulations using these PID structures and applied for the regulation of level and the temperature of a water reservoir control process. The responses of the systems have discussed by C. C. Hang [25], Un-Chul Moon and Kwang. Y. Lee [26]. The synthesis of a decentralized fuzzy controller for level control of four stage cascaded tank system has presented by Q. J. Bart [27]. J. P. Su, C. Y. Ling, and H. M. Chen have controlled a class of nonlinear system and discussed its application [28].

1.3 MOTIVATION

All of the process industries, the human body and the fluid handling system depend upon tank level control systems. The processes in which tank control systems work and the level control problem is solved are the main goal of this work. Control systems are applied to the nonlinear processes which give high performances. We got the idea to control the level and temperature in tank. The nonlinearities of the processes are very difficult to model. So, we got the scope to work with the nonlinearities of the process. Different linearization techniques are present to linearize the nonlinear equations. For linearization purpose, Taylor series method is used. Then, a control system is developed and designed based on the linear model. The control system developed for the linear model gives limited performance for the closed loop nonlinear system. Hence, a controller is used to improve the performance of the closed loop nonlinear system. Thermal control systems are also used frequently in different processes. The outlet process liquid temperature changes with the change of inlet process liquid temperature, the inlet cooling water temperature and the inlet cooling water flow rate. Hence, we got the scope to work on the level process and thermal process.

1.4 OBJECTIVES

The objectives of the thesis are as follows.

- To keep the liquid levels in the tanks at the desired values.
- Linearize the multivariable nonlinear processes
- To eliminate the interaction present in the system
- To control the temperature of process liquid present in the tank
- To cool a hot process liquid in thermal process
- Analyze the step responses of the thermal process

1.4 ORGANISATION OF THE THESIS

Including the introductory chapter, the thesis is divided into 5 chapters. The organisation of the thesis is presented below.

Chapter – 2 Level Process

The SISO system and TITO system are discussed in this chapter. The general formulation of the MIMO process is described. The interaction of input/output and also the linearization of the nonlinear level process are explained in detail. The interaction between the input/output are eliminated using cross controllers. The LabVIEW implementations of level process using the mathematical models have been discussed.

Chapter – 3 Thermal Process

In this chapter, the control of temperature of process liquid in tank is discussed and the step responses of the (3 x 2) and (3 x 3) processes are analyzed. The nonlinear equations are linearized using Taylor series method. The mathematical models of thermal process are implemented in LabVIEW. This chapter also explains the effect of inlet temperature, inlet cooling water temperature and inlet flow rates to outlet process temperature and cooling water temperature.

Chapter – 4 Four Input Four Output Tank System

In this chapter, the basics of FIFO tank system are discussed. The nonlinear model of FIFO tank system is converted into linear form and a control system is developed for this linear model. The step responses of (4 x 4) process are analyzed thoroughly. The LabVIEW implementations of the mathematical models of FIFO tank system are explained.

Chapter – 5 Conclusion

The overall conclusion of the thesis is presented in this chapter. It also contains some future research topics which need attention and further investigation.

CHAPTER 2

Level Process

General Formulation of MIMO System

Control of Interacting Systems

Mathematical Modelling of Tank Level Process

Linearization

Response of TITO Tank System

LabVIEW Implementation of Level Process

Simulation Results and Discussion

LEVEL PROCESS

This chapter describes about the single input single output (SISO) and two input two output (TITO) tank systems. The modelling of the SISO and TITO tank systems are very important for the analysis of the control system. The mathematical models of SISO and TITO tank systems are presented here. The interaction of input/output and also the linearization of the nonlinear level process are explained.

2.1 GENERAL FORMULATION OF MIMO SYSTEM

Consider a closed loop stable multivariable system with n-inputs and n-outputs as shown in Fig.1.1. In the figure *Ri, i*=1, 2 ….n are the reference inputs; *Ui, i*=1, 2 ….n are manipulated variables; *Yi, i*=1, 2 ….n are the system outputs [2; 13; 14].

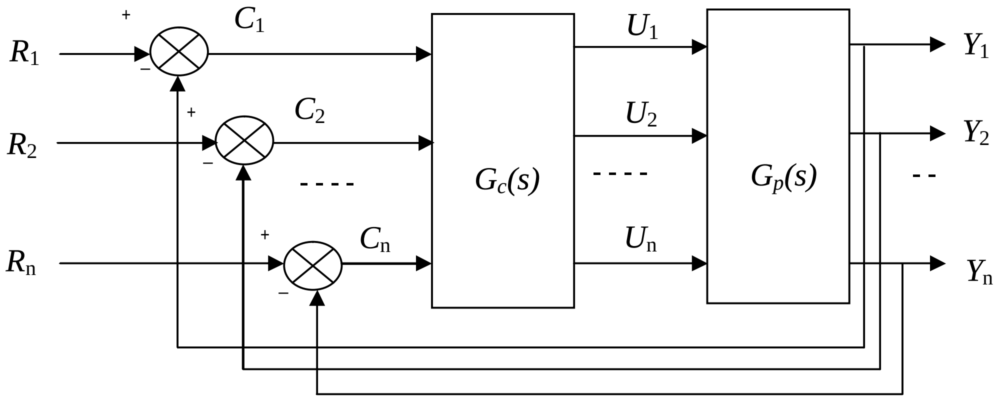


Fig. 2.1: Closed loop multivariable control system

In Fig. 2.1, $G_p(s)$ is the process transfer matrix and $G_c(s)$ is the controller matrix with compatible dimensions, expressed by the Eqs. (2.1) and (2.2), respectively.

$$G_p(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) & -- & G_{1n}(s) \\ G_{21}(s) & G_{22}(s) & -- & G_{2n}(s) \\ -- & -- & -- & -- \\ G_{n1}(s) & G_{n2}(s) & -- & G_{nn}(s) \end{bmatrix} \quad (2.1)$$

and

$$G_c(s) = \begin{bmatrix} G_{c11}(s) & G_{c12}(s) & -- & G_{c1n}(s) \\ G_{c21}(s) & G_{c22}(s) & -- & G_{c2n}(s) \\ -- & -- & -- & -- \\ G_{cn1}(s) & G_{cn2}(s) & -- & G_{cnn}(s) \end{bmatrix} \quad (2.2)$$

2.2 CONTROL OF INTERACTING SYSTEM

The interacting two tank liquid level system is shown in Fig. 2.2. In the figure, there is one input, the flow to tank 1 (q_{i1}) and one output, the level in tank 2 (h_2). In this figure, q_{i1} and h_2 are related by a second order transfer function. So it is a SISO system and control of this system is not much difficult compared to MIMO system.

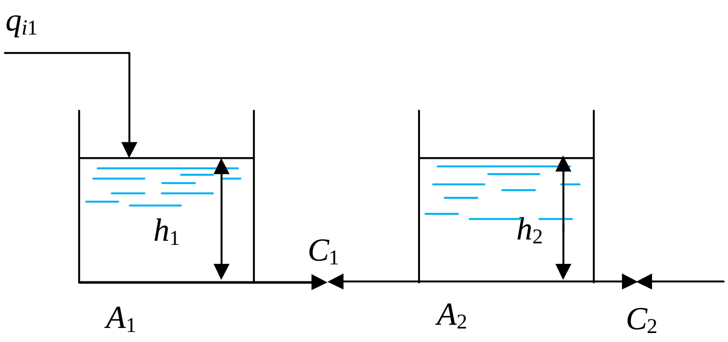


Fig. 2.2: Single-input single-output (SISO) interacting tank system

The interacting two tank liquid level system is shown in Fig. 2.3. In the figure, there are two inputs, the flow to tank 1 and tank 2 (q_{i1} and q_{i2}) and two outputs, the levels in tank 1 and tank 2 (h_1 and h_2), respectively. A change in q_{i1} alone will affect both the outputs (h_1 and h_2). A change in q_{i2} alone will also affect both the outputs. This is an interacting process for which the level in tank 1 is affected by the level in tank 2. So it is called the MIMO system, more specifically called two input two outputs (TITO) system.

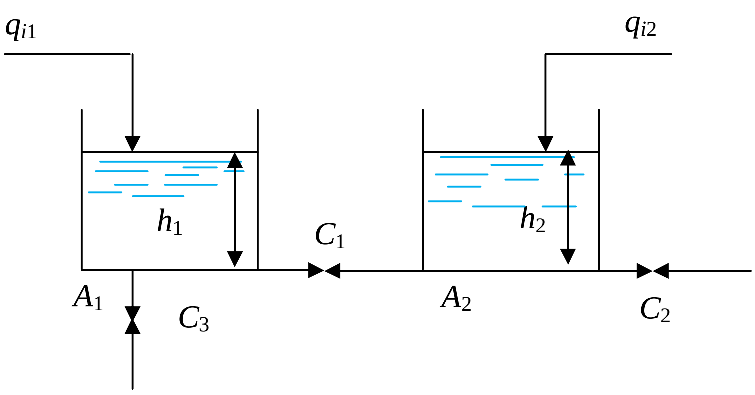


Fig. 2.3: Two-input two-output (TITO) tank System

The interaction between the inputs and outputs can be shown by the block diagram of Fig. 2.4. The change in one of the inputs affects both of the outputs is shown in Fig. 2.4. If Q_{i1} alone will change, it will affect both the outputs H_1 and H_2 simultaneously, because a disturbance enters the lower loop through the transfer function G_{21}. Similarly, if Q_{i2} alone will change, it will also affect both the outputs H_1 and H_2 simultaneously.

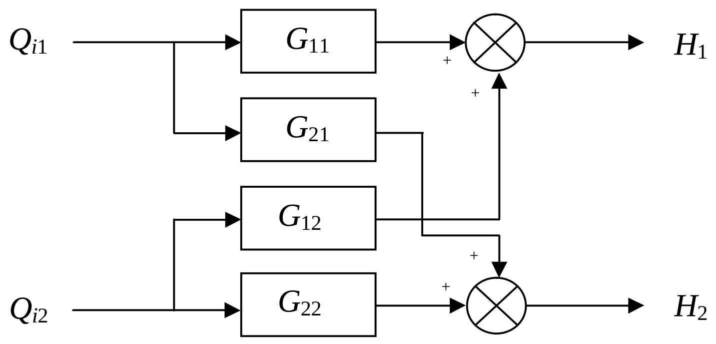


Fig. 2.4: Block diagram of two tank interacting level process

2.2.1 Control objective

The control objective is to control H_1 and H_2 independently, in spite of changes in Q_{i1} and Q_{i2} or other load variables [1]. So two control loops are added to the process shown in Fig. 2.4. Each loop contains a block for the controller, the valve and the measuring element as shown in Fig. 2.5.

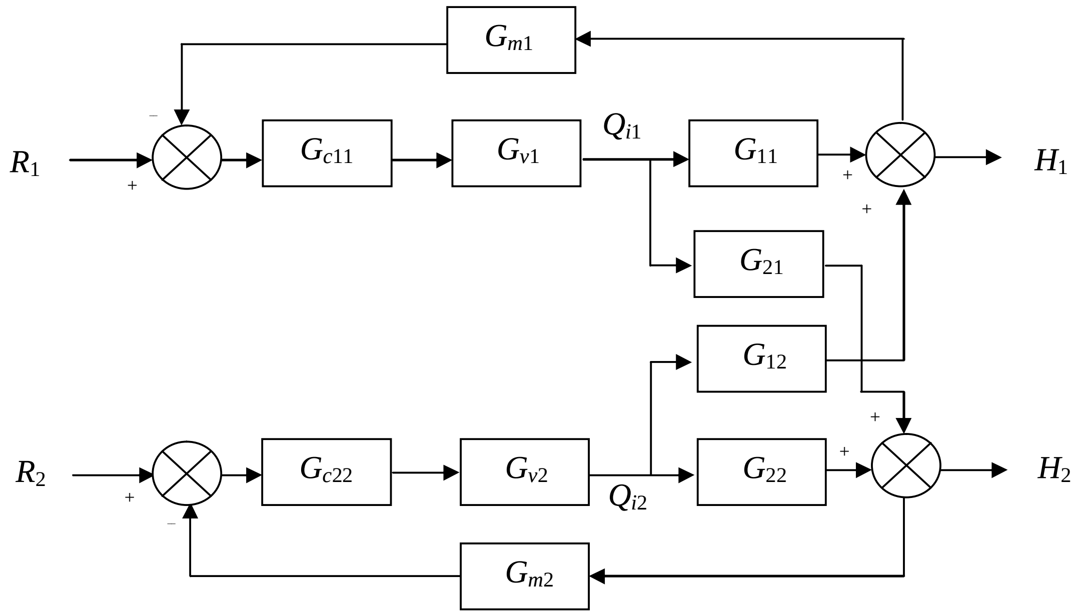


Fig. 2.5: Block diagram of two tank interacting level process with two controllers

G_{c11}, G_{c22} are the controller transfer function, (G_{v1}, G_{v2}) *are* the transfer function of the valve and (G_{m1}, G_{m2}) are the transfer function of measuring element as shown in Fig. 2.5. Due to the interaction in the system, a change in R_1 will also cause H_2 to vary because a disturbance enters the lower loop through the transfer function G_{21}. Similarly, a change in R_2 will also cause H_1 to vary because a disturbance enters the upper loop through the transfer function

G_{12}. Both outputs (H_1 and H_2) will change if a change is made in either input alone due to the interaction present in the system. As the transfer functions G_{21} and G_{12} provide weak interaction, the two controllers of Fig.2.5 will give the satisfactory control. If $G_{12} = G_{21} = 0$, there is no interaction and the two control loops are isolated from each other.

The interaction between outputs and set points are completely eliminated by using two more controllers called cross controllers are added to the process shown in Fig. 2.5. The two tank interacting level process with two primary controllers and two cross controllers is shown in Fig. 2.6. G_{c12} and G_{c21} are the cross controllers transfer function. When the cross controllers are added into the process, there is no interaction occurs between R and H as shown in Fig. 2.6, i.e. R_1 affects only H_1 and R_2 affects only H_2 respectively. If only R_1 is changed, then the output value H_1 only changes and no change occur in H_2.

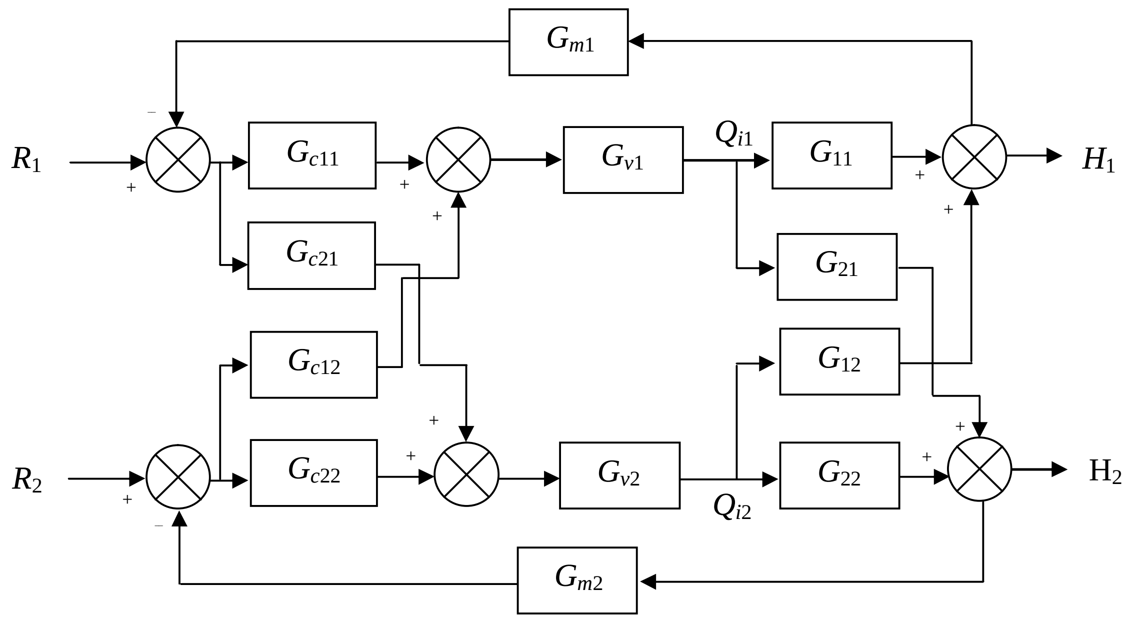


Fig. 2.6: Block diagram of two tank interacting level process with two primary controllers and two cross controllers

2.3 MATHEMATICAL MODELLING OF TANK LEVEL PROCESS

Mathematical modelling is the process of developing a mathematical model. A mathematical model helps to explain a system and to study the effects of different components, and to make predictions about their behaviours. Mathematical models can take many forms like dynamical systems, statistical models, or differential equations as discussed previously.

2.3.1 Single-Input Single-Output (SISO) Tank System

The tank level process to be simulated is single-input single-output (SISO) tank system as shown in Fig. 2.7. The user can adjust the inlet flow by adjusting the control signal, *w*. During the simulation, the level '*h*' will be calculated and displayed in the front panel of the tank system at any instant of time. In the SISO tank system, the liquid will flow into the tank through the valve K_1 and the liquid will come out from the tank through valve K_2. Here, we want to maintain the level of the liquid in the tank at desired value; so the measured output variable is the liquid level *h*.

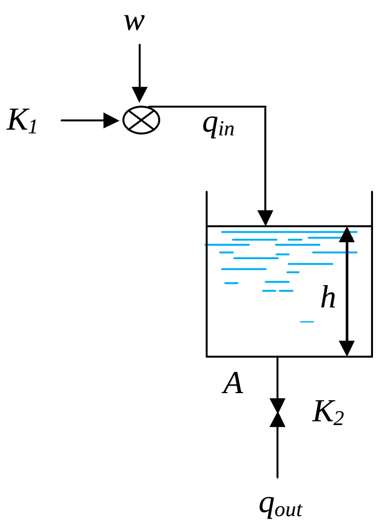


Fig. 2.7: Single-input single-output (SISO) tank system

For simulating the SISO tank system, its mathematical model [3] can be developed. The system is designed according to the mathematical model. For developing the mathematical model for SISO tank system, the density of liquid in the inlet, in the outlet and in the tank is assumed to be same and also the tank has straight vertical walls.

The notations used in modeling the SISO tank system are

q_{in} = Inlet volumetric flow rate [m^3/sec]

q_{out} = Outlet volumetric flow rate [m^3/sec]

V = Volume of liquid in the tank [m^3]

h = Height of liquid in the tank [m]

ρ = Liquid density [Kg/m^3]

A = Cross sectional area of the tank [m^2]

The mass of the liquid in the tank can be expressed as

$$m(t) = \rho A h(t) \tag{2.3}$$

The inlet volumetric flow into the tank is given by

$$q_{in}(t) = K_1 w(t) \tag{2.4}$$

The outlet volumetric flow through the valve is expressed as the square root of the pressure drop over the valve.

$$q_{out}(t) = K_2 \sqrt{\rho\, g\, h(t)} \tag{2.5}$$

According to the Mass balance equation

$$\left| \begin{array}{c} \text{Rate of change of} \\ \text{total mass of fluid} \\ \text{inside the tank} \end{array} \right| = \left| \begin{array}{c} \text{Mass flow rate} \\ \text{of fluid} \\ \text{into the tank} \end{array} \right| - \left| \begin{array}{c} \text{Mass flow rate} \\ \text{of fluid} \\ \text{out of the Tank} \end{array} \right| \tag{2.6}$$

$$\frac{dm(t)]}{dt} = \rho\, q_{in}(t) - \rho\, q_{out}(t) \tag{2.7}$$

$$\Rightarrow \frac{d[\rho A h(t)]}{dt} = \rho\, K_1 w(t) - \rho\, K_2 \sqrt{\rho\, g\, h(t)}$$

$$\Rightarrow \frac{dh(t)}{dt} = \left(\frac{1}{A}\right) \times \left[K_1 w(t) - K_2 \sqrt{\rho\, g\, h(t)}\right] \tag{2.8}$$

The SISO tank system is designed according to the model in Eq. (2.8). The mathematical block diagram for the model in Eq. (2.8) is shown in the Fig. 2.8.

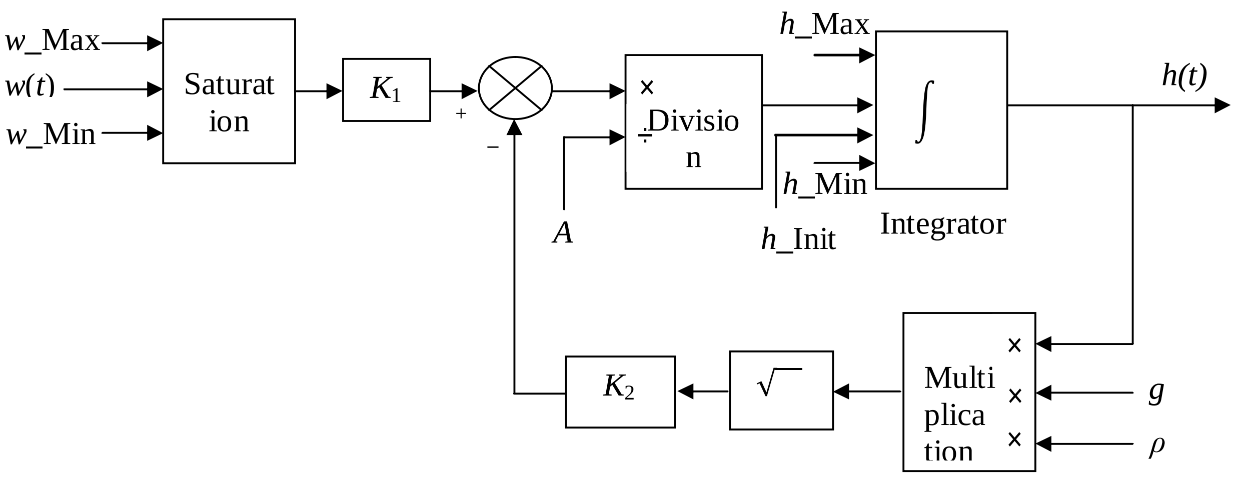


Fig. 2.8: Mathematical block diagram of SISO tank system

If the process is initially at steady state, the inlet and outlet flow rates are equal. If the inlet volumetric flow rate is suddenly increased while the outlet volumetric flow rate remains constant, the liquid level in the tank will increase until the tank overflows. Similarly, if the outlet volumetric flow rate is increased while the inlet volumetric flow rate remains constant, the tank level will decrease until the tank is empty.

2.3.2 Two-Input Two-Output (TITO) Tank System

The level process to be modelled, is the two-input two-output (TITO) tank system as shown in Fig. 2.3. In the figure, q_{i1} and q_{i2} are the two inputs to the tank where as h_1 and h_2 are the output levels for the two tank systems. The user can adjust the input by adjusting the input volumetric flow rates (q_{i1} and q_{i2}) and simultaneously the output levels 'h_1 and h_2' are calculated and displayed at any instant of time in the simulation.

An unsteady-state mass balance Eq. (2.6) around the first tank can be written as

$$\rho A_1 \frac{dh_1(t)}{dt} = \rho q_{i1}(t) - \rho q_{12}(t) - \rho q_1(t)$$

$$\Rightarrow A_1 \frac{dh_1(t)}{dt} = q_{i1}(t) - q_{12}(t) - q_1(t) \qquad (2.9)$$

An unsteady-state mass balance Eq. (2.6) around the second tank can be given by

$$\rho A_2 \frac{dh_2(t)}{dt} = \rho q_{i2}(t) + \rho q_{12}(t) - \rho q_2(t)$$

$$\Rightarrow A_2 \frac{dh_2(t)}{dt} = q_{i2}(t) + q_{12}(t) - q_2(t) \qquad (2.10)$$

The flow of liquid through a valve is given by the valve equation [3] as

$$q(t) = \frac{C_v}{7.48}\sqrt{\frac{\Delta P(t)}{G}} = \frac{C_v}{7.48}\sqrt{\frac{\rho\, g\, h(t)}{144\, g_c G}}$$

$$\Rightarrow q(t) = C_v' \sqrt{h(t)} \qquad (2.11)$$

where

C_v = Valve coefficient

$h(t)$ = Level in the tank

g_c = Conversion factor

$\Delta P(t)$ = Pressure drop across the valve

G = Specific gravity of liquid flowing through the valve

$$C'_v = \frac{C_v}{7.48}\sqrt{\frac{\rho g}{144 g_c G}} = C_v Z$$

The interaction between the tanks is shown from the valve Eq. (2.11) for the flow, q_{12} is

$$q_{12}(t) = C'_{v12}\sqrt{h_1(t) - h_2(t)} \tag{2.12}$$

where

$C'_{v12} = C_{v12} Z$

C_{v12} = Coefficient of the valve connected between the two tanks.

Eq. (2.12) shows that the flow between the two tanks depends on the levels in both the tanks, each affecting the other. Hence, the system is called as an interacting system.

The flow through the valve connected to the first tank is given as

$$q_1(t) = C'_{v1}\sqrt{h_1(t)} \tag{2.13}$$

where

$C'_{v1} = C_{v1} Z$

C_{v1} = Coefficient of the valve connected to the first tank.

The flow through the valve connected to the second tank is given as

$$q_2(t) = C'_{v2}\sqrt{h_2(t)} \tag{2.14}$$

where

$C'_{v2} = C_{v2} Z$

C_{v2} = Coefficient of the valve connected to the second tank.

Substituting Eqs. (2.12), (2.13) and (2.14) into Eqs. (2.9) and (2.10), we get

$$A_1 \frac{dh_1(t)}{dt} = q_{i1}(t) - C'_{v12}\sqrt{h_1(t) - h_2(t)} - C'_{v1}\sqrt{h_1(t)} \tag{2.15}$$

$$A_2 \frac{dh_2(t)}{dt} = q_{i2}(t) + C'_{v12}\sqrt{h_1(t) - h_2(t)} - C'_{v2}\sqrt{h_2(t)} \tag{2.16}$$

Eqs. (2.15) and (2.16) are the nonlinear equations due to the square root terms present in the relations, i.e. the outputs will vary nonlinearly with the inputs.

2.4 LINEARIZATION

Two nonlinear equations (2.15) and (2.16) are derived for modelling the TITO tank system. The nonlinearities of the processes are very difficult to model. Hence, the nonlinearities of the process are converted into linear form using Taylor series method. Then a control system is developed and designed based on the linear model.

The Taylor series is given as

$$f(x) = f(x_0) + \left.\frac{df}{dx}\right|_{ss} \frac{(x - x_0)}{1!} + \left.\frac{d^2 f}{dx^2}\right|_{ss} \frac{(x - x_0)^2}{2!} + \ldots\ldots \tag{2.17}$$

All the terms except the first two terms can be neglected . The nonlinear terms can be linearized with respect to h_1 and h_2, the linearization will be done about the steady-state values $\bar{h}_1$ and $\bar{h}_2$ respectively.

The nominal steady-state value of $q_{12}(t)$ is

$$q_{12}(t) = \bar{q}_{12} + \left.\frac{\partial q_{12}(t)}{\partial h_1}\right|_{ss} (h_1(t) - \bar{h}_1) + \left.\frac{\partial q_{12}(t)}{\partial h_2}\right|_{ss} (h_2(t) - \bar{h}_2)$$

$$\Rightarrow q_{12}(t) = \bar{q}_{12} + C_1(h_1(t) - \bar{h}_1) - C_1(h_2(t) - \bar{h}_2) \tag{2.18}$$

where

$$C_1 = \frac{C'_{v12}}{2} (\bar{h}_1 - \bar{h}_2)^{-1/2} \tag{2.19}$$

The nominal steady-state value of $q_2(t)$ is

$$q_2(t) = \bar{q}_2 + \left.\frac{\partial q_2(t)}{\partial h_2}\right|_{ss} (h_2(t) - \bar{h}_2)$$

$$\Rightarrow q_2(t) = \bar{q}_2 + C_2(h_2(t) - \bar{h}_2) \tag{2.20}$$

where

$$C_2 = \frac{C'_{v2}}{2} (\bar{h}_2)^{-1/2} \tag{2.21}$$

The nominal steady-state value of $q_1(t)$ is

$$q_1(t) = \bar{q}_1 + \left.\frac{\partial q_1(t)}{\partial h_1}\right|_{ss} (h_1(t) - \bar{h}_1)$$

$$\Rightarrow\ q_1(t) = \bar{q}_1 + C_3(h_1(t) - \bar{h}_1) \tag{2.22}$$

where

$$C_3 = \frac{C'_{v1}}{2}(\bar{h}_1)^{-1/2} \tag{2.23}$$

Substituting equations (2.18) and (2.22) into the equation (2.9), we get

$$A_1\frac{dh_1(t)}{dt} = q_{i1}(t) - \bar{q}_{12} - C_1(h_1(t) - \bar{h}_1) + C_1(h_2(t) - \bar{h}_2) - \bar{q}_1 - C_3(h_1(t) - \bar{h}_1)$$

$$\Rightarrow A_1\frac{dh_1(t)}{dt} = q_{i1}(t) - \bar{q}_{12} - \bar{q}_1 - C_1(h_1(t) - \bar{h}_1) + C_1(h_2(t) - \bar{h}_2) - C_3(h_1(t) - \bar{h}_1) \tag{2.24}$$

A steady-state mass balance around the tank 1 can be written as

$$\rho\,\bar{q}_{i1} - \rho\,\bar{q}_{12} - \rho\,\bar{q}_1 = 0$$

$$\Rightarrow\ \bar{q}_{i1} - \bar{q}_{12} - \bar{q}_1 = 0 \tag{2.25}$$

Subtracting Eq. (2.25) from Eq. (2.24) and simplifying, we get

$$A_1\frac{dH_1(t)}{dt} = Q_{i1}(t) - (C_1 + C_3)H_1(t) + C_1H_2(t) \tag{2.26}$$

where $Q_{i1}(t)$, $H_1(t)$ and $H_2(t)$ are the deviation variables of $q_{i1}(t)$, $h_1(t)$ and $h_2(t)$, respectively.

$$\left.\begin{aligned} Q_{i1}(t) &= q_{i1}(t) - \bar{q}_{i1} \\ H_1(t) &= h_1(t) - \bar{h}_1 \\ H_2(t) &= h_2(t) - \bar{h}_2 \end{aligned}\right\} \tag{2.27}$$

Substituting Eqs. (2.18) and (2.20) into the Eq. (2.10), we get

$$A_2\frac{dh_2(t)}{dt} = q_{i2}(t) + \bar{q}_{12} + C_1(h_1(t) - \bar{h}_1) - C_1(h_2(t) - \bar{h}_2) - \bar{q}_2 - C_2(h_2(t) - \bar{h}_2)$$

$$\Rightarrow A_2\frac{dh_2(t)}{dt} = q_{i2}(t) + \bar{q}_{12} - \bar{q}_2 + C_1(h_1(t) - \bar{h}_1) - C_1(h_2(t) - \bar{h}_2) - C_2(h_2(t) - \bar{h}_2) \tag{2.28}$$

A steady-state mass balance around the tank 2 can be written as

$$\rho\,\bar{q}_{i2} + \rho\,\bar{q}_{12} - \rho\,\bar{q}_2 = 0$$

$$\Rightarrow\ \bar{q}_{i2} + \bar{q}_{12} - \bar{q}_2 = 0 \tag{2.29}$$

Subtracting Eq. (2.29) from Eq. (2.28) and simplifying, we get

$$A_2 \frac{dH_2(t)}{dt} = Q_{i2}(t) + C_1 H_1(t) - (C_1 + C_2) H_2(t) \tag{2.30}$$

where $Q_{i2}(t)$ is the deviation variable of $q_{i2}(t)$.

$$Q_{i2}(t) = q_{i2}(t) - \bar{q}_{i2} \tag{2.31}$$

The definition and use of deviation variables in the analysis and design of process control systems is most important.

2.4.1 Advantages of deviation variable

The advantages of deviation variable are presented below.

- The value of the deviation variable indicates that the degree of deviation from some operating steady-state value. The steady-state value may be the desired value of the variable.
- Their initial value is zero, it helps for simplifying the solution of differential equations.

From Eqs. (2.26) and (2.30), we can easily observe that the output levels vary linearly with the input flows. The control system for these models can be easily designed. The expressions of C_1, C_2 and C_3 are shown in the Eqs. (2.19), (2.21) and (2.23), respectively. Eqs. (2.26) and (2.30) are in terms of deviation flow rates and deviation level outputs. The solution of Eq. (2.26) yields $H_1(t)$, the deviation level in tank 1 versus time for a certain inflow rate $Q_{i1}(t)$. If the actual output level $h_1(t)$ is desired, the steady-state value $\bar{h}_1$ must be added to $H_1(t)$. Similarly, the solution of Eq. (2.30) yields $H_2(t)$, the deviation level in tank 2 versus time for a certain inflow rate $Q_{i2}(t)$. The steady-state value $\bar{h}_2$ is added with $H_2(t)$ to get the actual output level $h_2(t)$.

2.5 RESPONSES OF TITO TANK SYSTEM

The time behavior of the outputs of a system when its inputs change from zero to one in a very short time is called step response. The step response of a system gives the information on the stability characteristics of that system, and it has the ability to reach the steady state. The step response describes the reaction of the system as a function of time. The overall system cannot perform until the component’s output settles down to its final state. The overall system depends on the parameters of the system and delays the overall system response.

Taking the Laplace Transform of Eq. (2.26) ,we get

$$A_1\, s\, H_1(s) \;=\; Q_{i1}(s) \;-\; (C_1 + C_3)\; H_1(s) \;+\; C_1\, H_2(s)$$

$$\Rightarrow\; H_1(s) \;=\; \frac{K_1}{\tau_1 s + 1}\, Q_{i1}(s) \;+\; \frac{K_2}{\tau_1 s + 1} H_2(s) \qquad (2.32)$$

where

$$K_1 \;=\; \frac{1}{C_1 + C_3} \qquad , \qquad K_2 \;=\; \frac{C_1}{C_1 + C_3}$$

$$\tau_1 \;=\; \frac{A_1}{C_1 + C_3}$$

Eq. (2.32) relates the level in tank 1 with the input flow into the tank 1 and the level in tank 2. The parameter τ_1 is the time constant and K_1 is the gain or sensitivity gives the amount of change of levels in both the tanks per unit change of flow into the tank 1. This change takes place while a constant opening is kept in the outlet valves of both the tanks. The parameter K_2 gives the amount of change of level in tank 1 per change of level in tank 2.

Similarly, taking the Laplace Transform of Eq. (2.30) ,we get

$$A_2\, s\, H_2(s) \;=\; Q_{i2}(s) \;+\; C_1\, H_1(s) - (C_1 + C_2)\; H_2(s)$$

$$\Rightarrow\; H_2(s) \;=\; \frac{K_3}{\tau_2 s + 1}\, Q_{i2}(s) \;+\; \frac{K_4}{\tau_2 s + 1} H_1(s) \qquad (2.33)$$

where

$$K_3 \;=\; \frac{1}{C_1 + C_2} \qquad , \qquad K_4 \;=\; \frac{C_1}{C_1 + C_2}$$

$$\tau_2 \;=\; \frac{A_2}{C_1 + C_2}$$

Eq. (2.33) relates the level in tank 2 with the input flow into the tank 2 and the level in tank 1. The parameter τ_2 is the time constant and K_3 is the gain or sensitivity gives the amount of change of levels in both the tanks per unit change of flow into the tank 2. This change takes place while a constant opening is kept in the outlet valves of both the tanks. The parameter K_4 gives the amount of change of level in tank 2 per change of level in tank 1.

Eqs. (2.32) and (2.33) can be represented by the block diagram as shown in Fig. 2.9. This is the interacting two tank system. In this diagram, the change of one of the inputs affects both the outputs. Suppose, if a change occurs in only $Q_{i1}(s)$, the responses of both $H_1(s)$ and $H_2(s)$

are affected. The transfer functions in Fig. 2.9 will be worked out for a specific set of process parameters. The larger time constant for the interacting case is greater than for the noninteracting case, resulting in a slower responding system.

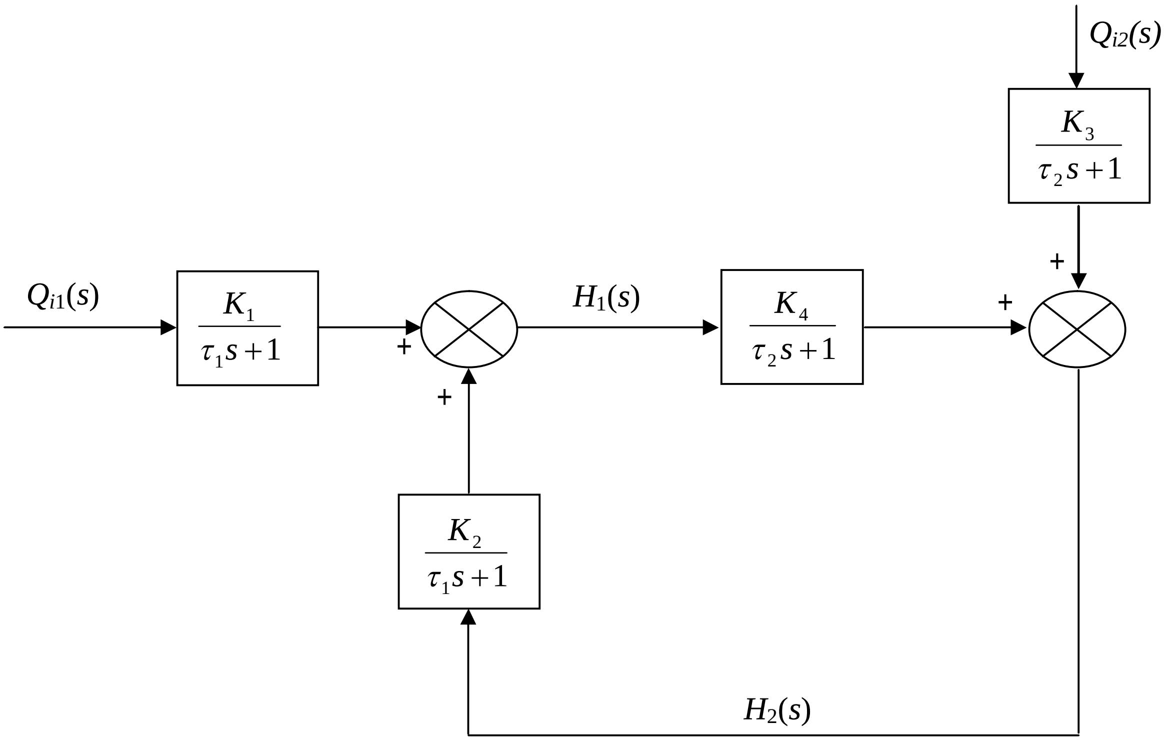


Fig. 2.9: Block diagram of TITO interacting tank system

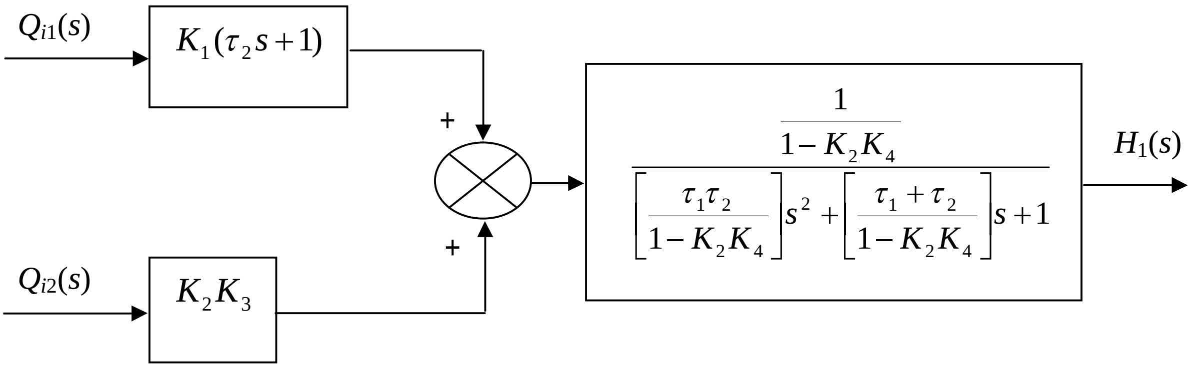
Now, substituting Eq. (2.33) into Eq. (2.32) and simplifying, we have

$$H_1(s)=\frac{K_1\left[\dfrac{\tau_2 s+1}{1-K_2K_4}\right]}{\left[\dfrac{\tau_1\tau_2}{1-K_2K_4}\right]s^2+\left[\dfrac{\tau_1+\tau_2}{1-K_2K_4}\right]s+1}Q_{i1}(s)+\frac{\dfrac{K_2K_3}{1-K_2K_4}}{\left[\dfrac{\tau_1\tau_2}{1-K_2K_4}\right]s^2+\left[\dfrac{\tau_1+\tau_2}{1-K_2K_4}\right]s+1}Q_{i2}(s) \quad (2.34)$$

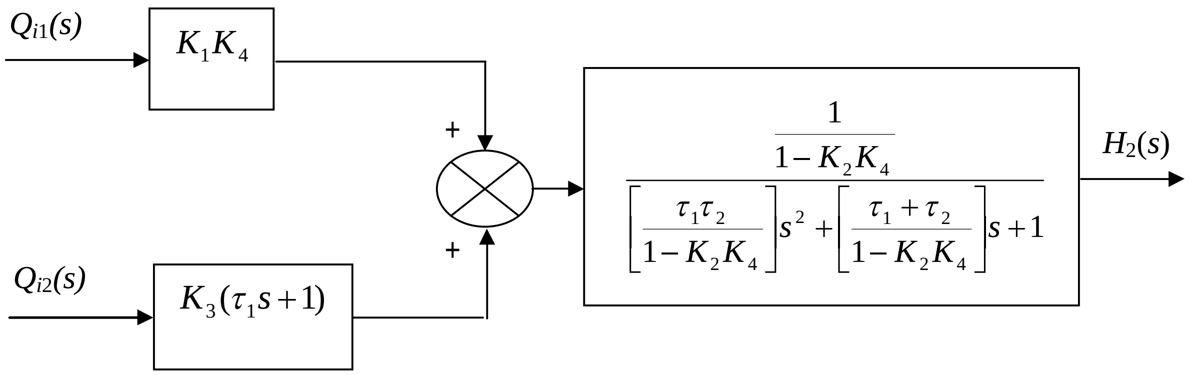
Similarly, substituting Eq. (2.32) into Eq. (2.33) and simplifying, we get

$$H_2(s)=\frac{\dfrac{K_1K_4}{1-K_2K_4}}{\left[\dfrac{\tau_1\tau_2}{1-K_2K_4}\right]s^2+\left[\dfrac{\tau_1+\tau_2}{1-K_2K_4}\right]s+1}Q_{i1}(s)+\frac{K_3\left[\dfrac{\tau_1 s+1}{1-K_2K_4}\right]}{\left[\dfrac{\tau_1\tau_2}{1-K_2K_4}\right]s^2+\left[\dfrac{\tau_1+\tau_2}{1-K_2K_4}\right]s+1}Q_{i2}(s) \quad (2.35)$$

The outputs $H_1(s)$ and $H_2(s)$ can be diagrammatically shown by the block diagram of Fig. 2.10 (*a*) and (*b*).



(a)



(b)

Fig. 2.10: Block diagram of TITO interacting tank system: (a) $H_1(s)$, (b) $H_2(s)$

2.6 LABVIEW IMPLEMENTATION OF LEVEL PROCESS

LabVIEW is used to communicate with hardware such as data acquisition, instrument control and industrial control. Hence, LabVIEW is used to simulate the level process.

2.6.1 Front Panel of SISO Tank System

The front panel of SISO tank system is shown in Fig. 2.11. In the figure, there is only one input into the tank i.e. q_{in} and the liquid can come out from the tank at q_{out}.

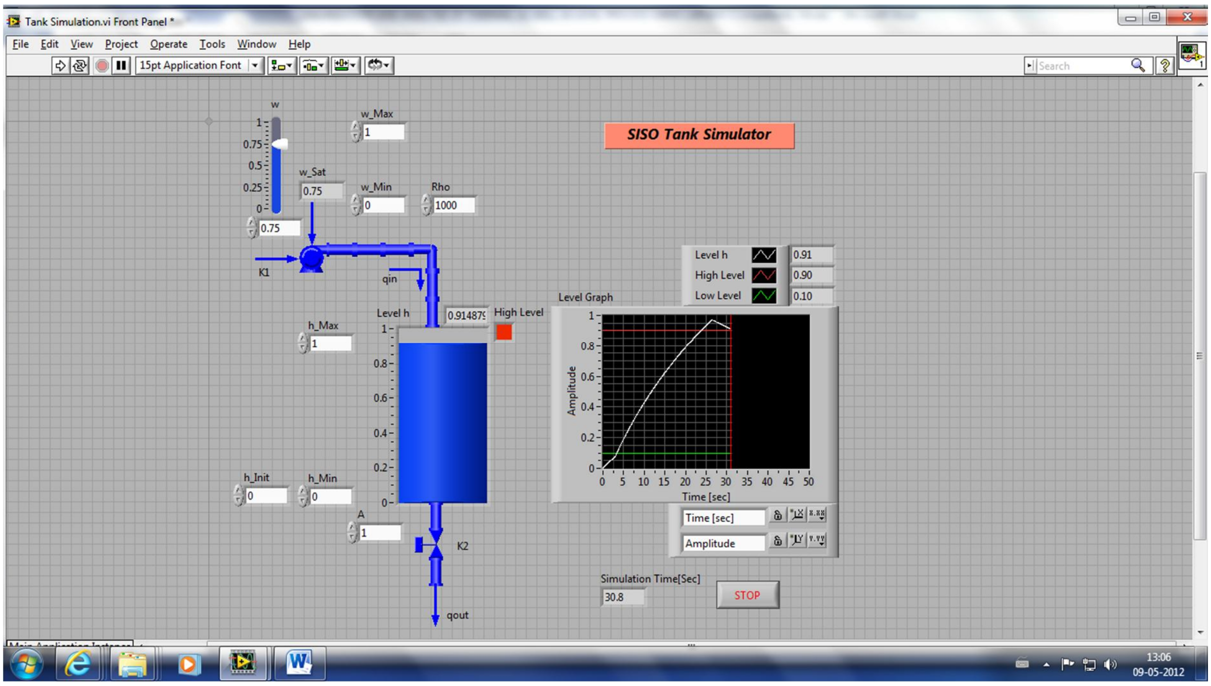


Fig. 2.11: Front panel of SISO tank system.

The level in the tank 'h' is changed and controlled according to the inlet volumetric flow rate q_{in}. When the level in the tank is greater than the higher level or below the lower level, then the alarm will start and a red light will blink.

2.6.2 Front Panel and Block Diagram of TITO Tank System

The front panels are very user friendly because the user can interact with the programs directly by entering the data and viewing the results. The front panel looks like a real system. For simulating the TITO tank system, the following assumptions are made.

- Coefficients of the valves used in the models are kept constant
- Area of cross-sections of both the tanks A_1 and A_2 are constant

2.6.2.1 Front Panel of Nonlinear TITO Tank System

The front panel of nonlinear TITO tank system is shown in Fig. 2.12. The user can adjust the input volumetric flow rates q_{i1} and q_{i2} to get the desired output levels h_1 and h_2, respectively. The output levels change nonlinearly with the input flow rates. The user can either change one input volumetric flow keeping the other input constant or change both the input volumetric flows simultaneously.

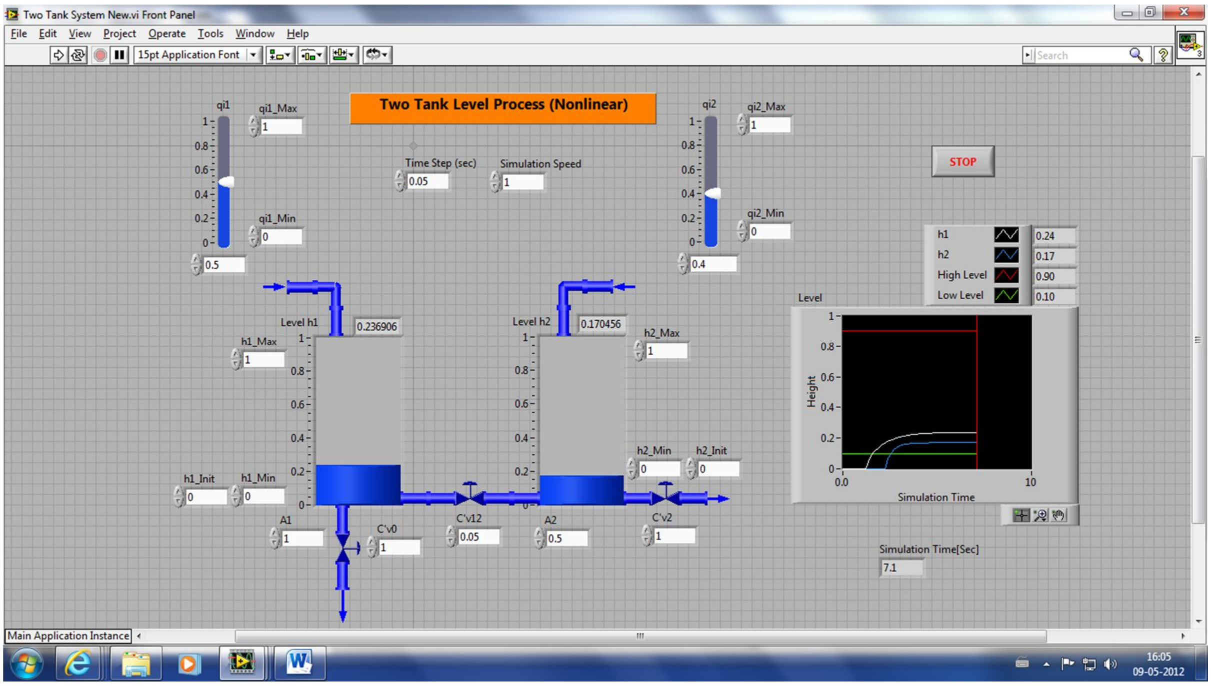


Fig. 2.12: Front panel of nonlinear TITO tank system

2.6.2.2 Block Diagram of Nonlinear TITO Tank System

The block diagram for nonlinear TITO tank system is designed based on the models in Eqs. (2.15) and (2.16) is shown in Fig. 2.13. In the block diagram, the programming can be written with graphics instead of text codes.

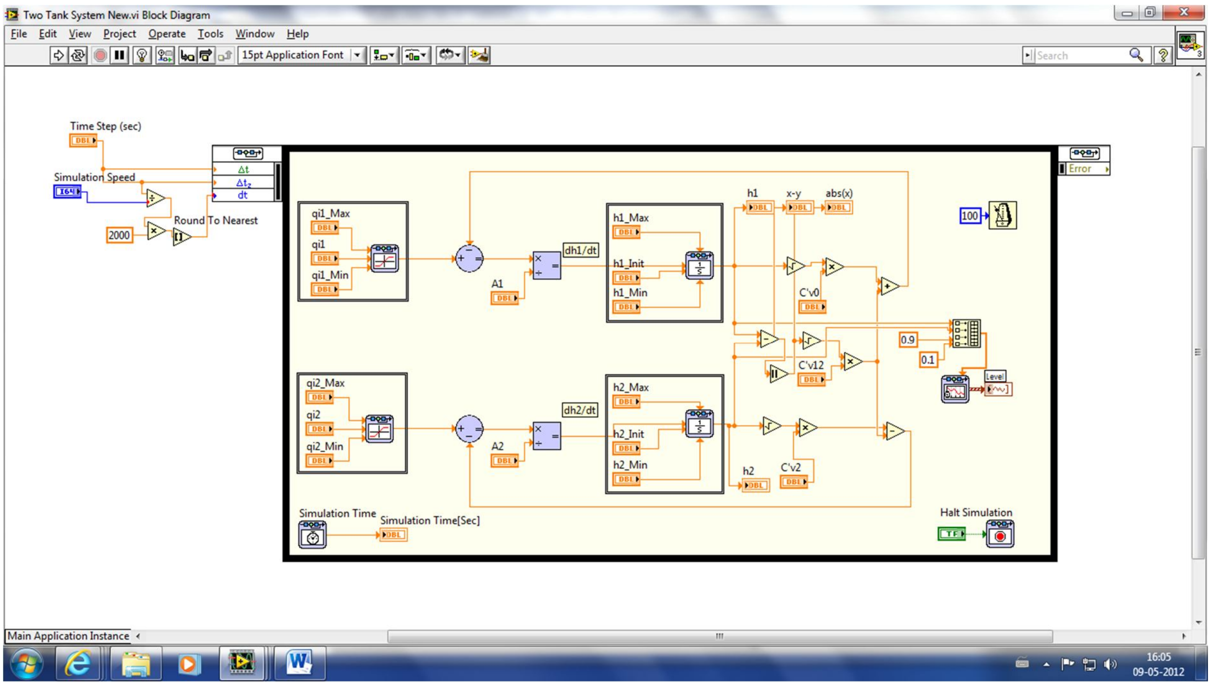


Fig. 2.13: Block diagram of nonlinear TITO tank system

2.6.2.3 Front Panel of Linear TITO Tank System

The front panel of linear TITO tank system is shown in Fig. 2.14. The linear TITO tank system is designed based on the deviation variables. The deviation variables of input volumetric flow rates Q_{i1} and Q_{i2} are adjusted to get the desired deviation variables of output levels H_1 and H_2, respectively. The output levels change linearly with the input flow rates as observed from Eqs. (2.26) and (2.30), respectively. We can either change one input flow while keeping the other input constant or change both the input flows simultaneously. The steady-state value $\bar{h}_1$ must be added to $H_1(t)$ to get the actual output level $h_1(t)$. Similarly, if the actual output level $h_2(t)$ is required, the steady state value $\bar{h}_2$ is added to the deviation level $H_2(t)$.

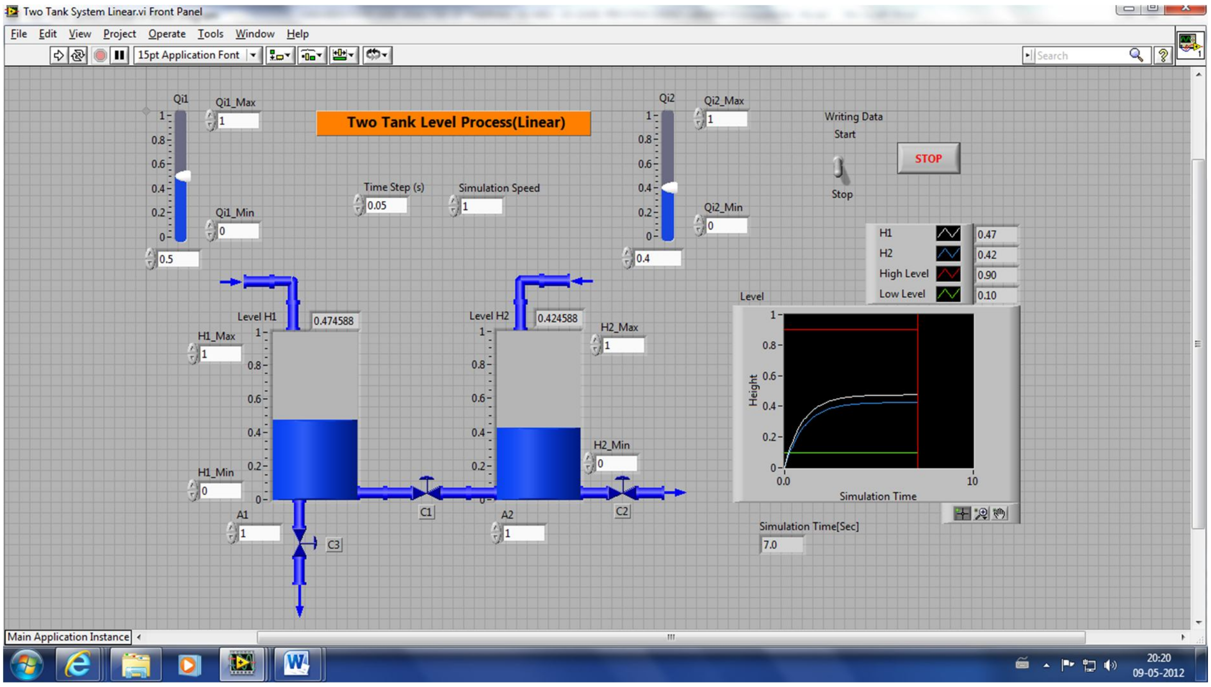


Fig. 2.14: Front panel of linear TITO tank system

2.6.2.4 Block Diagram of Linear TITO Tank System

The block diagram for linear TITO tank system is designed based on the models in Eqs. (2.26) and (2.30) as shown in Fig. 2.15.

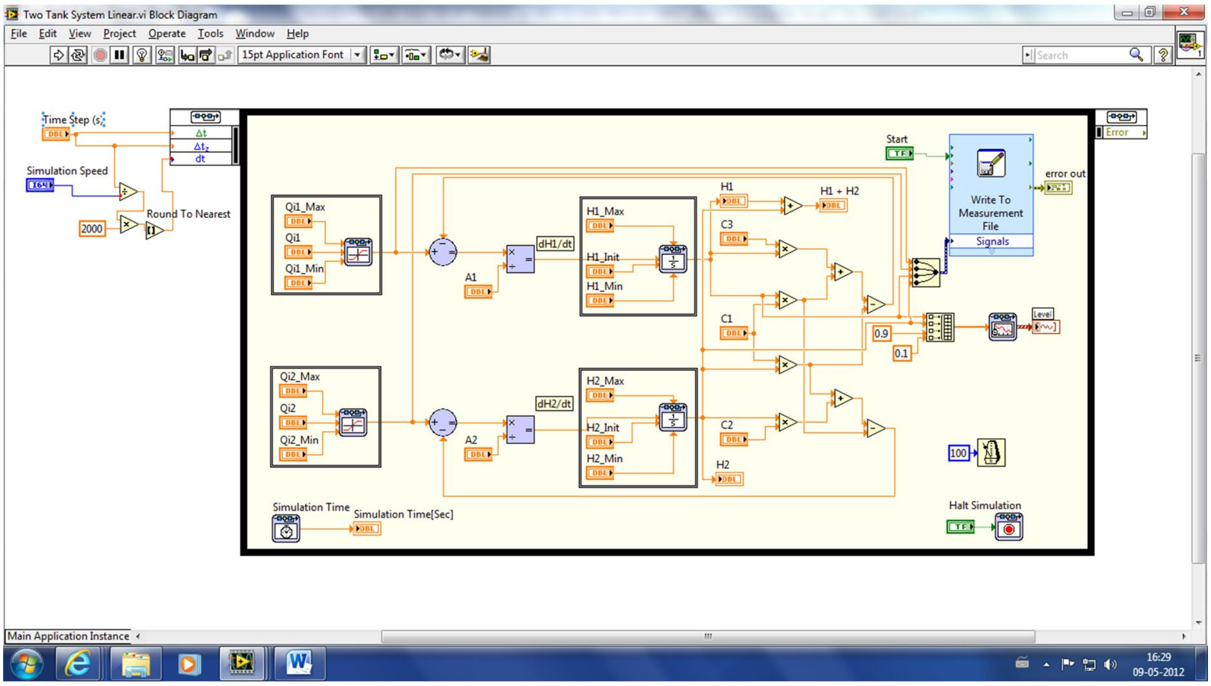


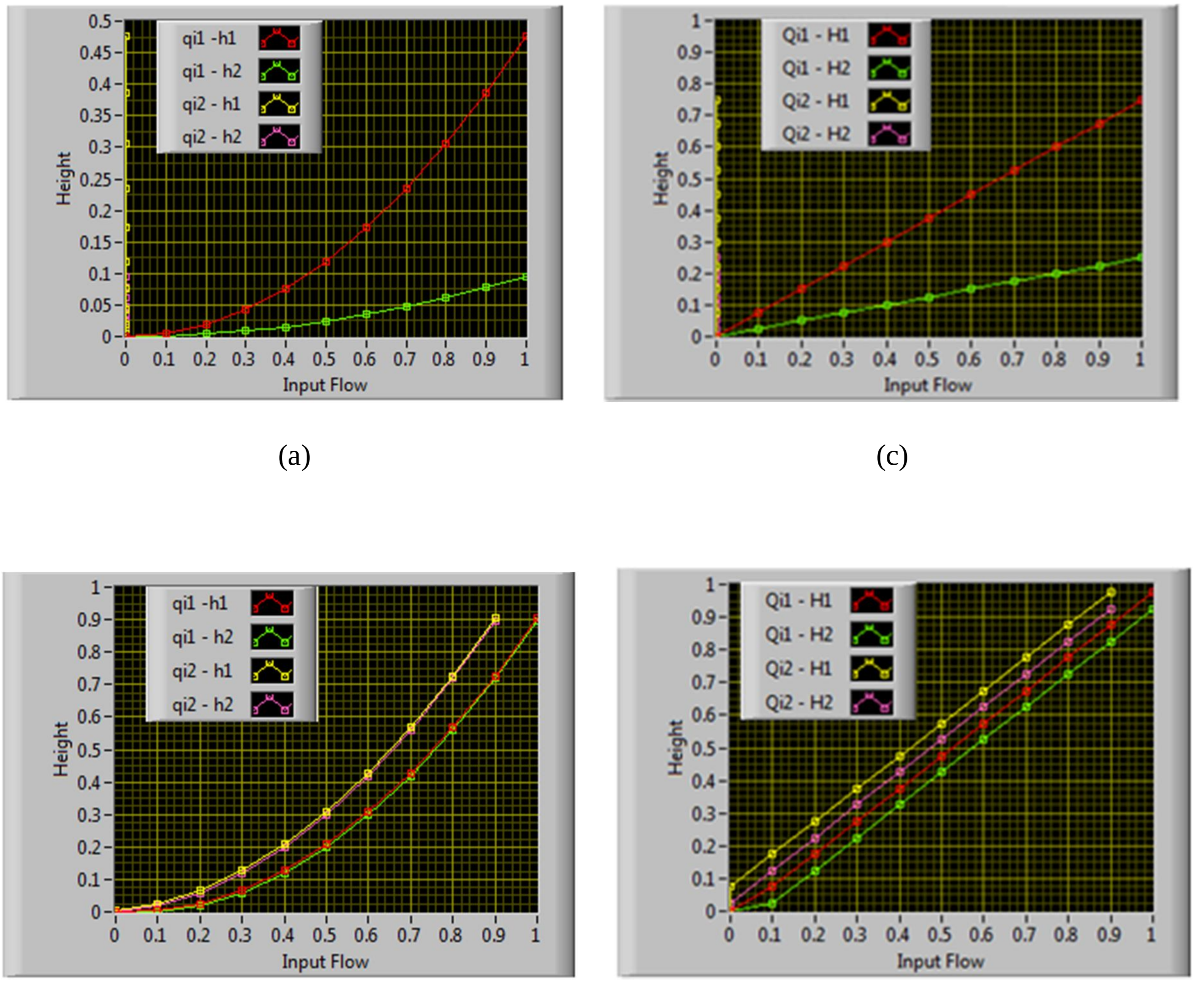
Fig. 2.15: Block diagram of linear TITO tank system

2.7 SIMULATION RESULTS AND DISCUSSIONS

The control system and process model are simulated with LabVIEW. The linearization of the nonlinear equations and its response graphs are analysed and discussed.

2.7.1 Nonlinear and Linear Level Graphs

The level graph of TITO tank system for both nonlinear and linear case are shown in Fig. 2.16. When the input flow rates q_{i1} and q_{i2} change, the output levels h_1 and h_2 change. Here either one input can be varied while keeping other one constant or both the inputs can be varied simultaneously. The variation of input flows as well as output levels in case of nonlinear system is shown in Fig. 2.16 (a) and (b) where as in case of linear system is shown in Fig. 2.16 (c) and (d). From the figures, there are two things clearly observed. First one is the nonlinearities of the system are eliminated and are converted into linear form using Taylor's series method. The second one is that the variation of one input while keeping other one constant, both the output levels is changed. Hence, it shows the interacting nature of the system. The interactions occurred in the system are eliminated by using the cross controllers.



(a) (c)

(b) (d)

Fig. 2.16: Graph between input flow versus level (height) of TITO tank system: (a) nonlinear, (b) nonlinear, (c) linear and (d) linear

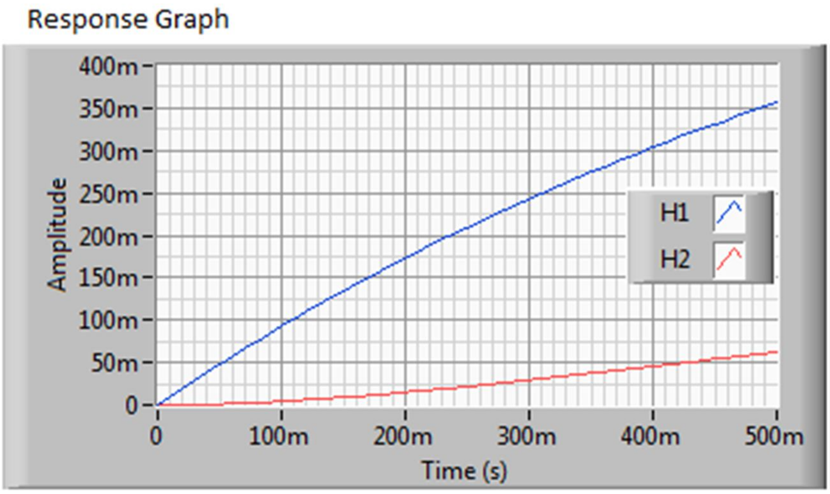
2.7.2 Responses of TITO Tank Level System

The process parameters for TITO tank level system can be calculated from Eqs. (2.32) and (2.33) as

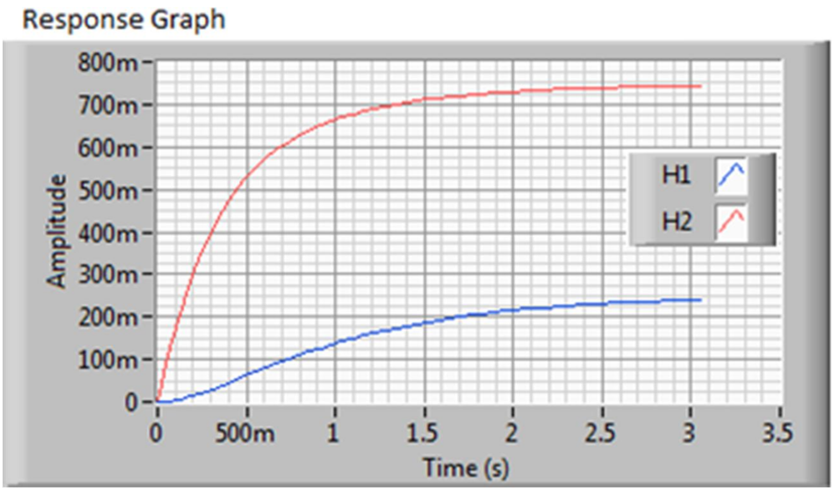
$$\left.\begin{aligned} G_{11}(s) &= \frac{0.25s+0.75}{0.25s^2+1.125s+1} \\ G_{12}(s) &= \frac{0.25}{0.25s^2+1.125s+1} \\ G_{21}(s) &= \frac{0.25}{0.25s^2+1.125s+1} \\ G_{22}(s) &= \frac{0.5s+0.75}{0.25s^2+1.125s+1} \end{aligned}\right\} \tag{2.36}$$

2.7.2.1 Open Loop Responses of TITO Tank System

The open loop response for TITO system in Fig. 2.4 has been shown in Fig. 2.17 (a) and (b). From the figures, it can be observed that the change of only one input variable will affect both the outputs. From the response graph, it is clearly analysed that a change in Q_{i1} alone affects both the output levels H_1 and H_2 as shown in Fig. 2.16 (*a*). Similarly, a change in Q_{i2} alone affects both the output levels H_1 and H_2 as shown in Fig. 2.16 (*b*).



(a)

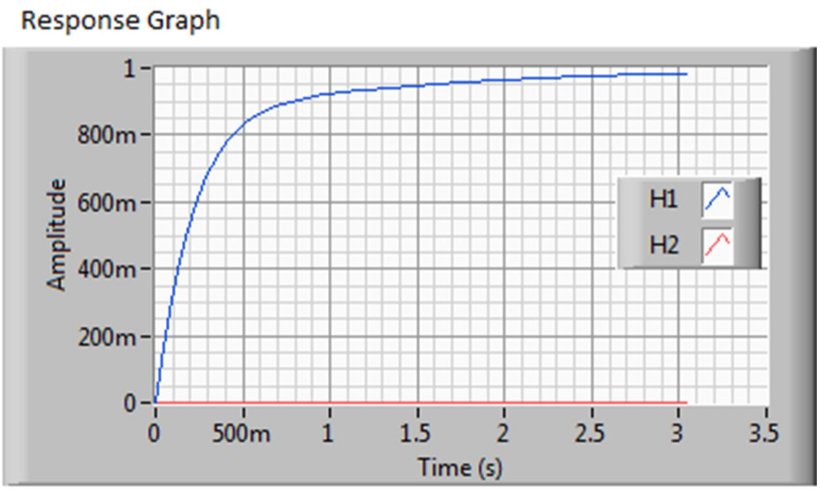


(b)

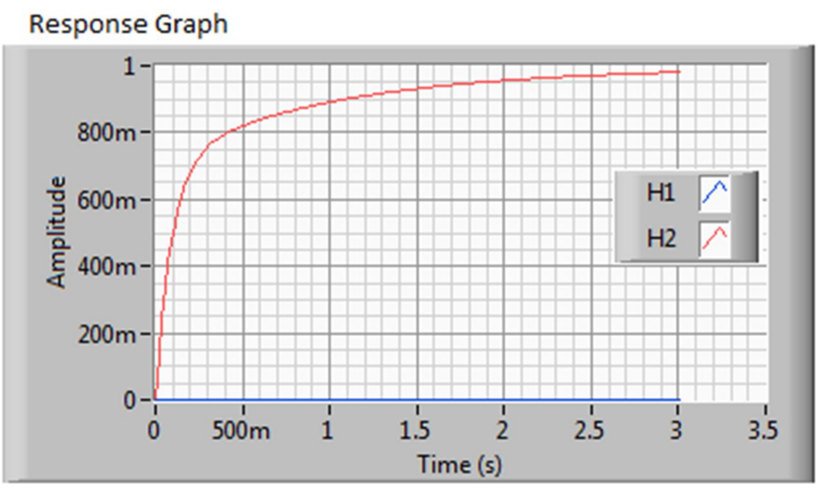
Fig. 2.17: Open loop response for: (a) $Q_{i1} = 1/s$, $Q_{i2} = 0$, (b) $Q_{i1} = 0$, $Q_{i2} = 1/s$.

2.7.2.2 Closed Loop Responses of TITO Tank System

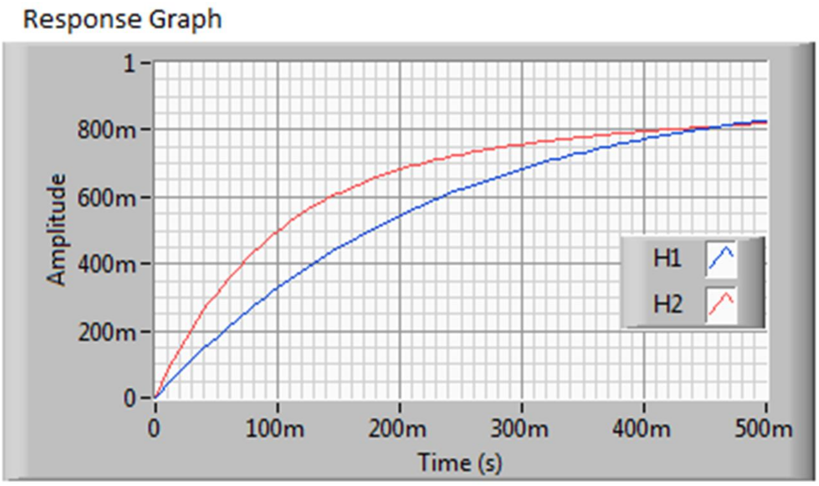
The closed loop responses for TITO tank interacting level process are shown in Fig. 2.18 (a), (b) and (c). It is clearly observed from the response graph that the output responses have no interaction with the set points, i.e. R_1 affects only H_1 and R_2 affects only H_2. By using the cross controllers (G_{c12} and G_{c21}), the interactions present in the system are eliminated. If only R_1 is changed, it only affects H_1 and there is no change in H_2 because two cross controllers are used to make the outputs non-interacting.



(a)



(b)



(c)

Fig. 2.18: Closed loop response for: (a) R_1 = 1/*s*, R_2 = 0, (b) R_1 = 0, R_2 = 1/*s*, (c) R_1= 1/*s*, R_2 = 1/*s*

From the transient response, it can be observed that H_1 moves towards the set point of 1.0 and that H_2 does not change as shown in Fig. 2.18 (*a*) and also in Fig. 2.18 (b), H_2 moves towards the set point of 1.0 and that H_1 does not change. When both R_1 and R_2 change, then both H_1 and H_2 will affect and move towards the set point of 1.0 as shown in Fig. 2.18 (c).

2.8 SUMMARY

This Chapter describes about the MIMO level process i.e. SISO and TITO tank system. The models of MIMO system are nonlinear and these nonlinear systems are linearized using Taylor series method. The interactions between the input/output present in a MIMO system are eliminated using cross controllers. The LabVIEW implementations of level process using the mathematical models are explained.

CHAPTER 3

Thermal Process

Thermal Control System

Mathematical Modelling of Thermal Process

Response of Thermal Process

LabVIEW Implementation of Thermal Process

Simulation Results and Discussion

THERMAL PROCESS

This chapter has two objectives. The first one is to describe the method for developing the thermal process models and this modeling plays an important role for the analysis of the control system. The second one describes the personality of the processes. The personality of the process is the physical meaning of some process parameters. The modeling of industrial processes is based on the balance equation.

3.1 THERMAL CONTROL SYSTEM

The control of temperature of liquid in the tank is a basic problem in the process industries. The process control industries require liquids are to be pumped as well as stored in tanks and then the temperature of the liquid in tank is to be controlled. Thermal control systems are used frequently in different processes. The thermal control system is nonlinear, time varying and consists of multivariable.

The well-stirred tank [3] is shown in Fig. 3.1. In the figure, the inlet and outlet volumetric flow is same and denoted by q in (m^3/s). The inlet temperature is $T_i(t)$ and the outlet temperature is $T(t)$. The densities and heat capacities of liquid are assumed to be constant. The thermal process is an adiabatic process. An adiabatic process is also called an isocaloric process. An adiabatic process is a thermodynamic process in which the net heat transfers to or from the working liquid is zero. The process in which no heat is gained or lost by the system is called an adiabatic process. In such a process the tank has thermally insulated walls. When the inlet temperature $T_i(t)$ changes by keeping the inlet and outlet flow of the tank constant, then the outlet temperature $T(t)$ changes.

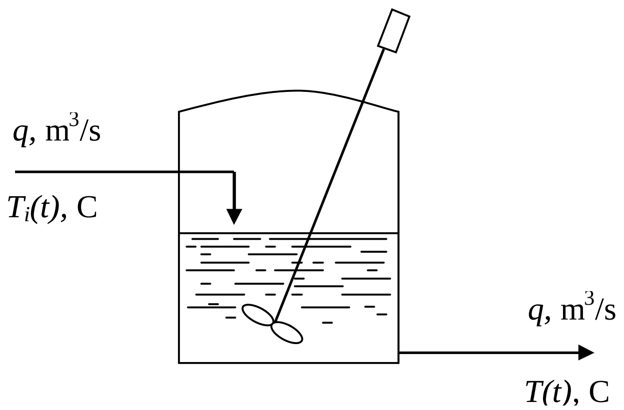


Fig. 3.1: Well-stirred tank

3.2 MATHEMATICAL MODELLING OF THERMAL PROCESS

The well-stirred tank with metal wall is shown in Fig. 3.2. The inlet and outlet volumetric flows are same and are denoted by q (m^3/s). The objective of the thermal process is to cool a hot process liquid. For performing this operation, the following assumptions are made.

- Constant inlet and outlet volumetric flow rates
- Constant densities and heat capacities of the process liquid and the cooling water
- Water in the cooling jacket and the liquid in the tank are well mixed
- The tank is well insulated

The process liquid is cooled by passing water as cooling medium through a jacket. The outlet process liquid temperature, $T(t)$ and the outlet cooling water temperature, $T_C(t)$ can be changed by the variation of the inlet process liquid temperature, $T_i(t)$, the inlet cooling water temperature, $T_{Ci}(t)$ and the cooling water flow rate, $q_C(t)$, respectively.

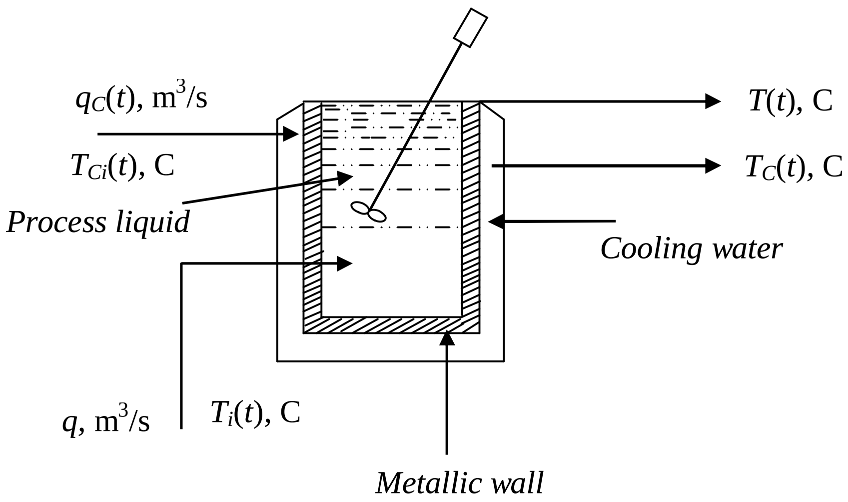


Fig. 3.2: Well-stirred tank with metal wall

The notations used in modeling for the thermal control system are

ρ = Liquid density, kg/m^3

ρ_C = Density of cooling water, kg/m^3

C_p = Liquid heat capacity at constant pressure, J/Kg-C

C_v = Liquid heat capacity at constant volume, J/Kg-C

C_{pC} = Heat capacity of cooling water at constant pressure, J/Kg-C

C_{vC} = Heat capacity of cooling water at constant volume, J/Kg-C

V = Volume of liquid in the tank, m^3

V_C = Volume of cooling jacket, m^3

U = Overall heat transfer coefficient, J/m^2-C-s

A = Heat transfer area, m^2

The overall heat transfer coefficient (U) is used to calculate the total heat transfer through a wall and it depends on the liquids and their properties on both sides of the wall, the properties of the wall and the transmitting surface. Also it is a measure of the overall ability of a series of conductive and convective barriers to transfer heat and it is assumed to be constant. The height of liquid in the tank is constant as the mass of liquid in the tank and its density are assumed to be constant and the heat transfer area (A) is also constant.

According to the Energy balance equation

$$\left| \begin{array}{c} \text{Rate of change of} \\ \text{energy of liquid} \\ \text{in the process} \end{array} \right| = \left| \begin{array}{c} \text{Energy rate of} \\ \text{liquid} \\ \text{into the process} \end{array} \right| - \left| \begin{array}{c} \text{Energy rate of} \\ \text{liquid} \\ \text{out of the process} \end{array} \right| \quad (3.1)$$

An unsteady-state energy balance (3.1) on the process liquid is

$$V\rho C_v \frac{dT(t)}{dt} = q\rho C_p T_i(t) - UA\left[T(t) - T_C(t)\right] - q\rho C_p T(t) \quad (3.2)$$

A steady-state energy balance (3.1) on the process liquid is given by

$$q\rho C_p \overline{T}_i - UA\left[\overline{T} - \overline{T}_C\right] - q\rho C_p \overline{T} = 0 \quad (3.3)$$

Subtracting Eq. (3.3) from Eq. (3.2) and simplifying, we get

$$V\rho C_v \frac{d\,\mathbf{T}(t)}{dt} = q\rho C_p \mathbf{T}_i(t) - UA\left[\mathbf{T}(t) - \mathbf{T}_C(t)\right] - q\rho C_p \mathbf{T}(t) \quad (3.4)$$

where

$$\mathbf{T}_i(t) = T_i(t) - \overline{T}_i$$

$$\mathbf{T}(t) = T(t) - \overline{T}$$

$$\mathbf{T}_C(t) = T_C(t) - \overline{T}_C$$

$\overline{T}_i$, $\overline{T}$, $\overline{T}_C$ = Steady-state values of inlet, outlet and cooling water temperatures, respectively, C

$\mathbf{T}_i(t)$, $\mathbf{T}(t)$, $\mathbf{T}_C(t)$ = Deviation variables of inlet, outlet and cooling water temperatures, respectively, C

An unsteady-state energy balance (3.1) around the cooling jacket is given as

$$V_C \rho_C C_{vC} \frac{dT_C(t)}{dt} = q_C(t)\, \rho_C C_{pC} T_{Ci}(t) \; + \; UA\left[\, T(t) - T_C(t) \,\right] - q_C(t) \rho_C C_{pC} T_C(t) \tag{3.5}$$

The nonlinear terms are the first and last terms of the right hand side of Eq. (3.5). So, Eq. (3.5) is nonlinear and it can be linearized using Taylor series method in Eq. (2.17).

Eq. (3.5) after using Taylor series method can be written as

$$\begin{aligned} V_C \rho_C C_{vC} \frac{dT_C(t)}{dt} = {} & \bar{q}_C\, \rho_C C_{pC} T_{Ci}(t) + q_C(t)\, \rho_C C_{pC} \bar{T}_{Ci} \; + \; UA\left[\, T(t) - T_C(t) \,\right] \\ & - q_C(t)\, \rho_C C_{pC} \bar{T}_C - \bar{q}_C\, \rho_C C_{pC} T_C(t) \end{aligned} \tag{3.6}$$

Subtracting the steady-state value around the cooling jacket from Eq. (3.6), we get

$$V_C \rho_C\, C_{vC} \frac{d\,\mathbf{T}_C(t)}{dt} = \left[X_1 - X_3\right] \mathbf{Q}_C(t) + X_2\, \mathbf{T}_{Ci}(t) - X_2\, \mathbf{T}_C(t) + UA\left[\mathbf{T}(t) - \mathbf{T}_C(t)\right] \tag{3.7}$$

where

$$\left.\begin{aligned} X_1 &= \rho_C C_{pC} \bar{T}_{Ci} \\ X_2 &= \bar{q}_C\, \rho_C C_{pC} \\ X_3 &= \rho_C C_{pC} \bar{T}_C \end{aligned}\right\} \tag{3.8}$$

and the deviation variables are

$$\left.\begin{aligned} \mathbf{Q}_C(t) &= q_C(t) - \bar{q}_C \\ \mathbf{T}_{Ci}(t) &= T_{Ci}(t) - \bar{T}_{Ci} \end{aligned}\right\} \tag{3.9}$$

When the temperature of the cooling water changes, the heat transfer to the wall of the tank changes and consequently, the temperature of the wall starts to change. Then, the heat transfer rate from the wall to the process liquid changes. Hence, the dynamics of the wall of the tank has been taken into consideration. Hence, the wall of the tank acts as capacitance in the system and it’s magnitude depends on thickness, density, heat capacity and other physical properties of the material of the wall. The surface next to the process liquid and the surface next to the cooling water are assumed to be at the same temperature.

The notations used in modeling for the thermal control system are

ρ_m = Density of the metal wall, kg/m^3

C_{vm} = Liquid heat capacity at constant volume, J/Kg-C

V_m = Volume of the metal wall, m^3

h_i = Inside film heat transfer coefficient, J/m^2-C-s

h_o = Outside film heat transfer coefficient, J/m^2-C-s

A_i = Inside heat transfer area, m^2

A_o = Outside heat transfer area, m^2

$T_m(t)$ = Temperature of the metal wall, C

An unsteady- state energy balance on the process liquid is then written as

$$V\rho C_v \frac{dT(t)}{dt} = q\rho C_p T_i(t) - h_i A_i \left[T(t) - T_m(t)\right] - q\rho C_p T(t) \tag{3.10}$$

A steady-state energy balance on the process liquid is written as

$$q\rho C_p \overline{T}_i - h_i A_i \left[\overline{T} - \overline{T}_m\right] - q\rho C_p \overline{T} = 0 \tag{3.11}$$

Subtracting Eq. (3.11) from Eq. (3.10), we get

$$V\rho C_v \frac{d\,\mathbf{T}(t)}{dt} = q\rho C_p \mathbf{T}_i(t) - h_i A_i \left[\mathbf{T}(t) - \mathbf{T}_m(t)\right] - q\rho C_p \mathbf{T}(t) \tag{3.12}$$

where the deviation variable is

$$\mathbf{T}_m(t) = T_m(t) - \overline{T}_m \tag{3.13}$$

An unsteady-state energy balance on the wall of the tank is given as

$$V_m \rho_m C_{vm} \frac{dT_m(t)}{dt} = h_i A_i \left[T(t) - T_m(t)\right] - h_o A_o \left[T_m(t) - T_C(t)\right] \tag{3.14}$$

A steady- state energy balance on the wall of the tank is written as

$$h_i A_i \left[\overline{T} - \overline{T}_m\right] - h_o A_o \left[\overline{T}_m - \overline{T}_C\right] = 0 \tag{3.15}$$

Subtracting Eq. (3.15) from Eq. (3.14), we get

$$V_m \rho_m C_{vm} \frac{d\,\mathbf{T}_m(t)}{dt} = h_i A_i \left[\mathbf{T}(t) - \mathbf{T}_m(t)\right] - h_o A_o \left[\mathbf{T}_m(t) - \mathbf{T}_C(t)\right] \tag{3.16}$$

An unsteady-state energy balance Eq. (3.1) on the cooling water is given as

$$V_C \rho_C C_{vC} \frac{dT_C(t)}{dt} = q_C(t)\rho_C C_{pC} T_{Ci}(t) + h_o A_o \left[T_m(t) - T_C(t)\right] - q_C(t)\rho_C C_{pC} T_C(t) \tag{3.17}$$

The nonlinear terms are the first and last terms of the right hand side of Eq. (3.17). So, Eq. (3.17) is nonlinear and it can be linearized using Taylor series method in Eq. (2.17).

Eq. (3.17) after using Taylor series method can be written as

$$V_C \rho_C C_{vC} \frac{dT_C(t)}{dt} = q_C(t)\rho_C C_{pC} \bar{T}_{Ci} + \bar{q}_C \rho_C C_{pC} T_{Ci}(t) - q_C(t)\rho_C C_{pC} \bar{T}_C - \bar{q}_C \rho_C C_{pC} T_C(t) + h_o A_o \left[\bar{T}_m - \bar{T}_C\right] \quad (3.18)$$

Subtracting the steady-state value around the cooling jacket from Eq. (3.18), we get

$$V_C \rho_C C_{vC} \frac{d\,\mathbf{T}_C(t)}{dt} = \left[X_1 - X_3\right]\mathbf{Q}_C(t) + X_2\,\mathbf{T}_{Ci}(t) - X_2\,\mathbf{T}_C(t) + h_o A_o \left[\mathbf{T}_m(t) - \mathbf{T}_C(t)\right] \quad (3.19)$$

Eq. (3.19) is the linearized form of the nonlinear model of thermal process in Eq. (3.17). The value of X_1, X_2 and X_3 are derived and explained in Eq. (3.8). As discussed earlier, $\mathbf{T}_C(t)$, $\mathbf{T}_m(t)$, $\mathbf{T}_{Ci}(t)$ and $\mathbf{Q}_C(t)$ are the deviation variables.

3.3 RESPONSES OF THERMAL PROCESS

The outlet process liquid temperature , the cooling water temperature and the metal wall temperature depend on parameter called process gain or steady-state gain.

3.3.1 Responses of (3 x 2) order Thermal Process

Taking the Laplace Transform of Eq. (3.4), we have

$$V\rho C_v s\mathbf{T}(s) = q\rho C_p \mathbf{T}_i(s) - UA\left[\mathbf{T}(s) - \mathbf{T}_C(t)\right] - q\rho C_p \mathbf{T}(s)$$

$$\Rightarrow \quad \mathbf{T}(s) = \frac{Y_1}{\tau_1 s + 1}\mathbf{T}_i(s) + \frac{Y_2}{\tau_1 s + 1}\mathbf{T}_C(s) \quad (3.20)$$

where

$$Y_1 = \frac{q\rho C_p}{UA + q\rho C_p}, \quad Y_2 = \frac{UA}{UA + q\rho C_p}$$

$$\tau_1 = \frac{V\rho C_v}{UA + q\rho C_p}$$

Similarly, taking the Laplace Transform of Eq. (3.7), we get

$$V_C \rho_C C_{vC} s\mathbf{T}_C(s) = \left[X_1 - X_3\right]\mathbf{Q}_C(s) + X_2\,\mathbf{T}_{Ci}(s) - X_2\,\mathbf{T}_C(s) + UA\left[\mathbf{T}(s) - \mathbf{T}_C(s)\right]$$

$$\Rightarrow \mathbf{T}_C(s) = \frac{Y_3}{\tau_2 s+1}\mathbf{T}_{Ci}(s) + \frac{Y_4}{\tau_2 s+1}\mathbf{Q}_C(s) + \frac{Y_3}{\tau_2 s+1}\mathbf{T}_C(s) \tag{3.21}$$

where

$$Y_3 = \frac{X_2}{UA+X_2} \quad , \quad Y_4 = \frac{X_1 - X_3}{UA+X_2}$$

$$\tau_2 = \frac{V_C \rho_C C_{vC}}{UA+X_2} \quad , \quad Y_5 = \frac{UA}{UA+X_2}$$

After solving Eqs. (3.20) and (3.21), we get

$$\begin{bmatrix} \mathbf{T}(s) \\ \mathbf{T}_C(s) \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \end{bmatrix} \begin{bmatrix} \mathbf{T}_i(s) \\ \mathbf{T}_{Ci}(s) \\ \mathbf{Q}_C(s) \end{bmatrix} \tag{3.22}$$

The transfer functions $G_{ij}(s)$ are as follows

$$\left.\begin{aligned}
G_{11}(s) &= \frac{0.487s+0.993}{0.606s^2+1.727s+1} \\
G_{12}(s) &= \frac{0.000335}{0.606s^2+1.727s+1} \\
G_{13}(s) &= \frac{0.000010786}{0.606s^2+1.727s+1} \\
G_{21}(s) &= \frac{0.000266}{0.606s^2+1.727s+1} \\
G_{22}(s) &= \frac{1.236s+0.999}{0.606s^2+1.727s+1} \\
G_{23}(s) &= \frac{0.0398s+0.0322}{0.606s^2+1.727s+1}
\end{aligned}\right\} \tag{3.23}$$

3.3.2 Responses of (3 x 3) order Thermal Process

Taking the Laplace Transform of Eq. (3.12), we have

$$V\rho C_v s\mathbf{T}(s) = q\rho C_p \mathbf{T}_i(s) - h_i A_i \left[\mathbf{T}(s) - \mathbf{T}_m(s)\right] - q\rho C_p \mathbf{T}(s)$$

$$\Rightarrow \ \mathbf{T}(s) = \frac{Y_6}{\tau_3 s+1}\mathbf{T}_i(s) + \frac{Y_7}{\tau_3 s+1}\mathbf{T}_m(s) \tag{3.24}$$

where

$$Y_6 = \frac{q\rho C_p}{h_i A_i + q\rho C_p} \quad , \quad Y_7 = \frac{h_i A_i}{h_i A_i + q\rho C_p}$$

$$\tau_3 = \frac{V\rho C_v}{h_i A_i + q\rho C_p}$$

By using the procedure already discussed, the transfer function derives from Eq. (3.16) as

$$\mathbf{T}_m(s) = \frac{Y_8}{\tau_4 s+1}\mathbf{T}(s) + \frac{Y_9}{\tau_4 s+1}\mathbf{T}_C(s) \tag{3.25}$$

where

$$Y_8 = \frac{h_i A_i}{h_i A_i + h_o A_o} \quad , \quad Y_9 = \frac{h_o A_o}{h_i A_i + h_o A_o}$$

$$\tau_3 = \frac{V_m \rho_m C_{vm}}{h_i A_i + h_o A_o}$$

Similarly, the transfer function derives from Eq. (3.19) as

$$\mathbf{T}_C(s) = \frac{Y_{10}}{\tau_5 s+1}\mathbf{T}_{Ci}(s) + \frac{Y_{11}}{\tau_5 s+1}\mathbf{T}_m(s) + \frac{Y_{12}}{\tau_5 s+1}\mathbf{Q}_C(s) \tag{3.26}$$

where

$$Y_{10} = \frac{X_2}{X_2 + h_o A_o} \quad , \quad Y_{11} = \frac{h_o A_o}{X_2 + h_o A_o}$$

$$Y_{12} = \frac{X_1 - X_3}{X_2 + h_o A_o} \quad , \quad \tau_5 = \frac{V_C \rho_C C_{vC}}{X_2 + h_o A_o}$$

The simplification of Eqs. (3.24), (3.25) and (3.26) gives the solution as

$$\begin{bmatrix} \mathbf{T}(s) \\ \mathbf{T}_C(s) \\ \mathbf{T}_m(s) \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} \mathbf{T}_i(s) \\ \mathbf{T}_{Ci}(s) \\ \mathbf{Q}_C(s) \end{bmatrix} \tag{3.27}$$

The parameters of (3 x 3) order thermal process are

$$\left.\begin{aligned}
G_{11}(s) &= \frac{0.769s^2 + 2.058s + 0.963}{0.986s^3 + 3.356s^2 + 3.311s + 0.981} \\
G_{12}(s) &= \frac{0.000203}{0.986s^3 + 3.356s^2 + 3.311s + 0.981} \\
G_{13}(s) &= -\frac{0.00531}{0.986s^3 + 3.356s^2 + 3.311s + 0.981} \\
G_{21}(s) &= \frac{0.00854}{0.986s^3 + 3.356s^2 + 3.311s + 0.981} \\
G_{22}(s) &= \frac{1.939s^2 + 2.789s + 0.972}{0.986s^3 + 3.356s^2 + 3.311s + 0.981} \\
G_{23}(s) &= -\frac{0.97s^2 + 1.394s + 0.486}{0.986s^3 + 3.356s^2 + 3.311s + 0.981} \\
G_{31}(s) &= \frac{0.359s^2 + 0.96s + 0.454}{1.563s^4 + 6.431s^3 + 8.755s^2 + 4.911s + 0.981} \\
G_{32}(s) &= \frac{1.033s^2 + 1.486s + 0.523}{1.563s^4 + 6.431s^3 + 8.755s^2 + 4.911s + 0.981} \\
G_{33}(s) &= -\frac{0.518s^2 + 0.724s + 0.262}{1.563s^4 + 6.431s^3 + 8.755s^2 + 4.911s + 0.981}
\end{aligned}\right\} \tag{3.28}$$

3.4 LABVIEW IMPLEMENTATION OF THERMAL PROCESS

LabVIEW is used to simulate the thermal process. The thermal process is designed according to the mathematical modeling. LabVIEW is very user friendly because the programs are created with graphics instead of text codes. The user can directly interact with the programs by entering the input data and viewing the output in the Virtual Instruments (VI).

3.4.1 Front Panel and Block Diagram of Thermal Process

The front panel of thermal process to cool a hot process liquid in the tank is shown in Fig. 3.3. As discussed earlier, water is used as cooling medium to cool a hot process liquid and flows through a jacket. For simulating the thermal process, the volume of liquid in the tank, volume of cooling jacket and the value of C_{pC} and C_{vC} are assumed to be constant. As the inlet temperature increases, the outlet temperature increases gradually as shown in the front panel of the thermal process. The outlet cooling water temperature is affected by the inlet cooling water temperature. Both the outputs ($T(t)$ and $T_C(t)$) will change if a change is made in any one input (i.e. $T_i(t)$, $T_{Ci}(t)$ or $q_C(t)$) alone. Hence, the thermal process is an interacting process. The interaction in the thermal process is due to the heat loss in the system and also due to the metal wall.

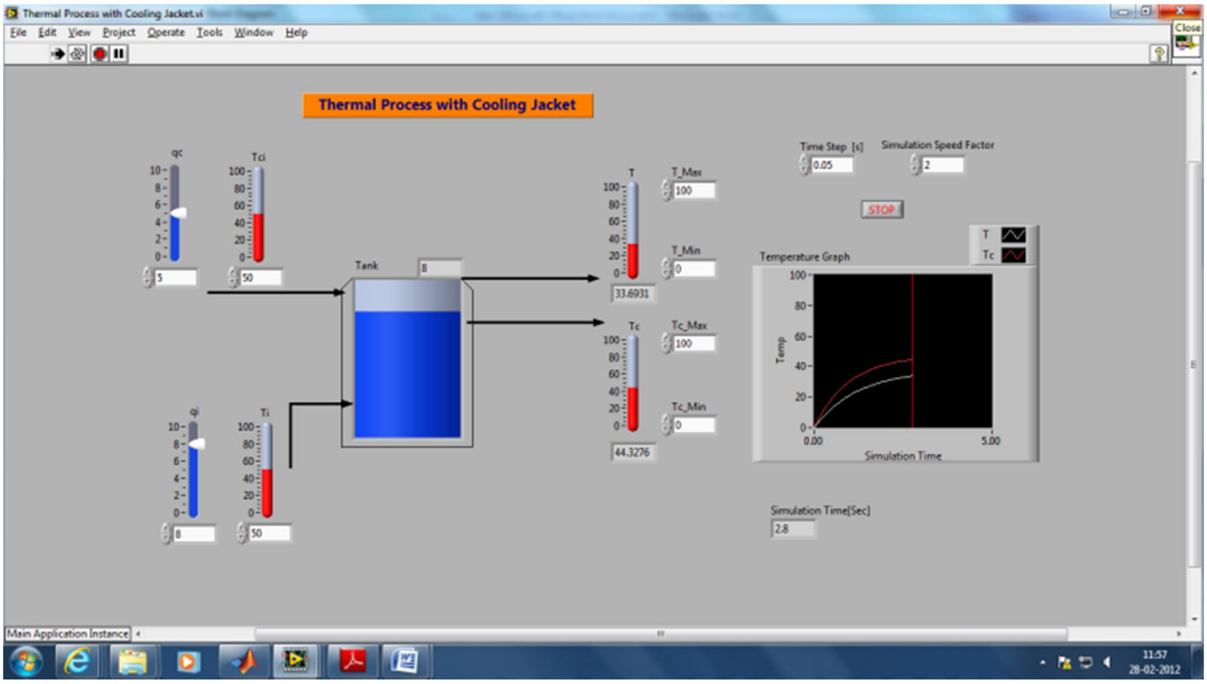


Fig. 3.3: Front panel to cool a hot process liquid in thermal process

Fig. 3.4 shows the block diagram of the thermal process to cool a hot process liquid. The block diagram of the thermal process is designed according to the mathematical models shown in Eqs. (3.4) and (3.7) respectively.

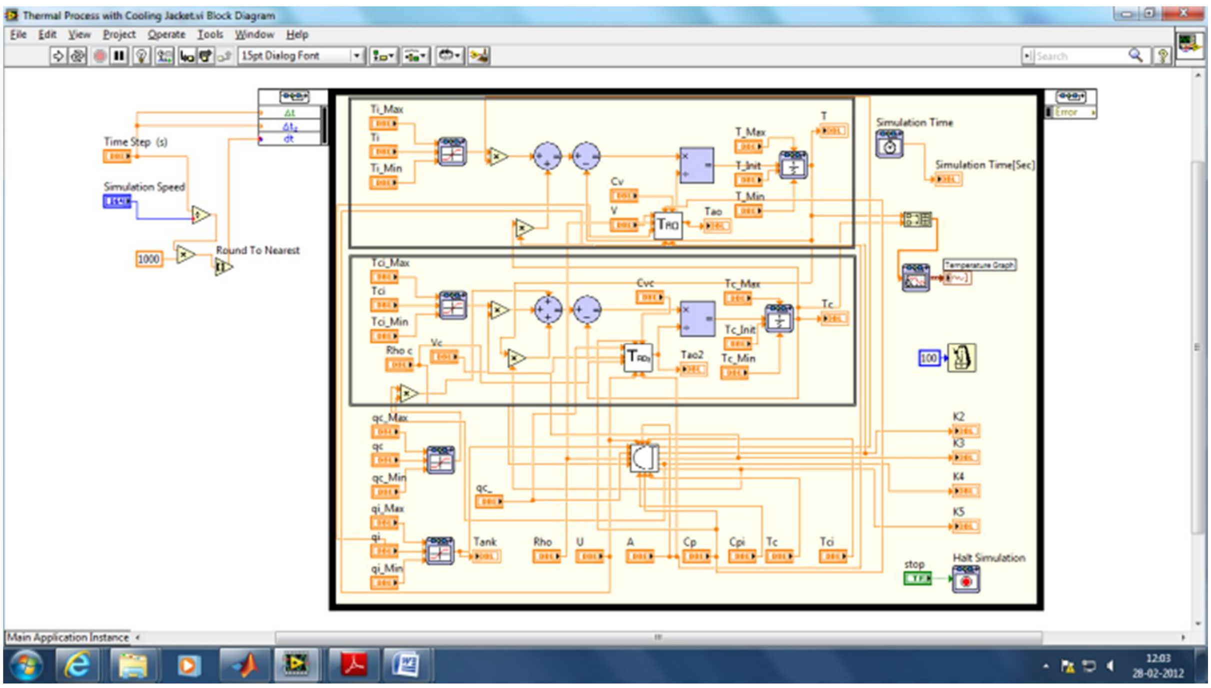


Fig. 3.4: Block diagram of the thermal process to cool a hot process liquid

Fig. 3.5 shows the front panel of the thermal process when the metal wall has been taken into consideration.

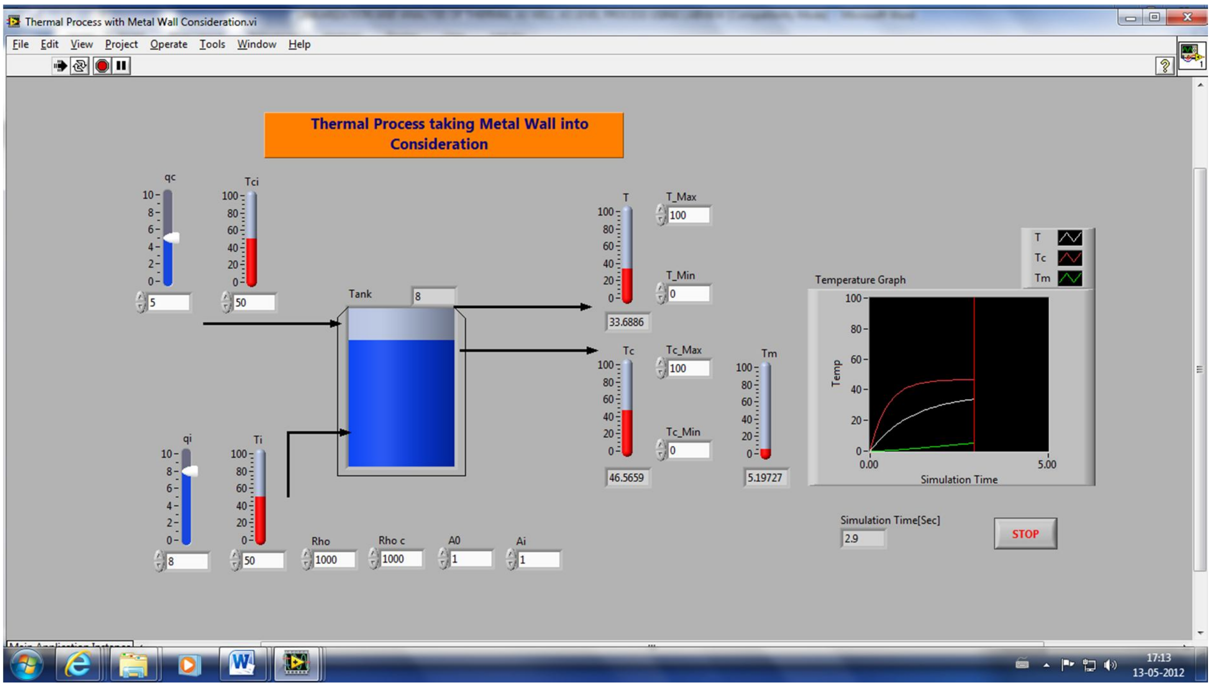


Fig. 3.5: Front panel of the thermal process taking metal wall into consideration

When the temperature of the cooling water changes, the temperature of process liquid changes through the metal wall. For simulating the thermal process, the following assumptions are made.

- Constant density and volume of the metal wall
- Constant heat transfer coefficient inside and outside the film
- Inside and outside heat transfer area are kept constant

The block diagram of the thermal process when the metal wall has been taken into consideration is shown in Fig. 3.6. The block diagram of the thermal process is designed by using the mathematical model Eqs. (3.10), (3.13) and (3.16) respectively.

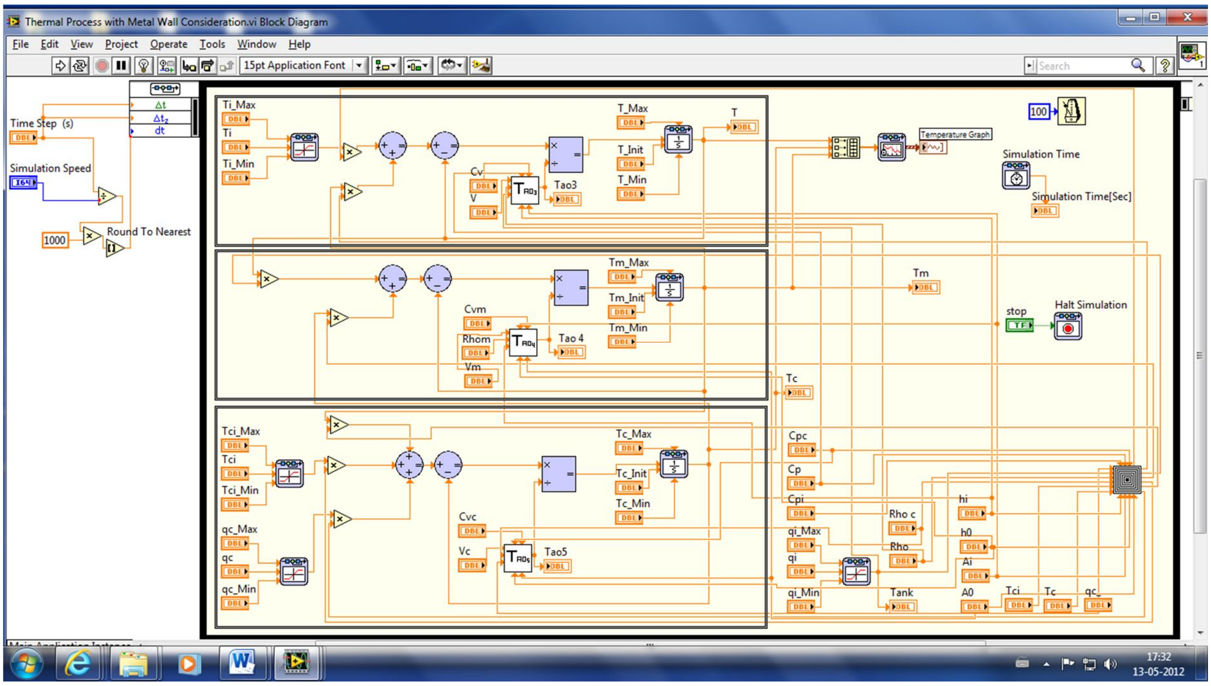


Fig. 3.6: Block diagram of the thermal process taking metal wall into consideration

3.5 SIMULATION RESULTS AND DISCUSSIONS

The mathematical models of the thermal process are simulated using LabVIEW. The step responses of (3 x 2) and (3 x 3) order thermal process are analyzed and discussed.

3.5.2 Responses of (3 x 2) order Thermal Process

The response of outlet process temperature to a step change in $T_i(t)$, $T_{Ci}(t)$ and $q_C(t)$ due to the cooling medium is shown in Fig. 3.7.

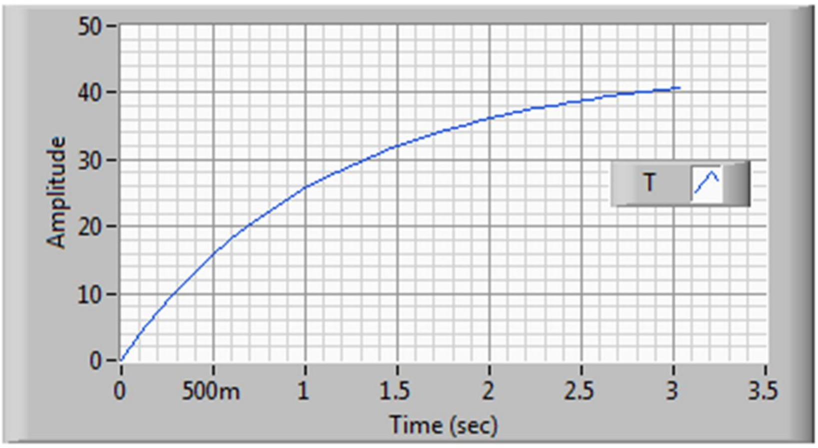


Fig. 3.7: Response of outlet process liquid temperature to a step change in $T_i(t)$, $T_{Ci}(t)$ and $q_C(t)$ due to the cooling medium

If $T_i(t)$ changes, then it directly affects the process liquid temperature, but if $T_{Ci}(t)$ changes, it affects first the jacket temperature and then the process liquid temperature. Hence, in the response of $T(t)$ w.r.t. $T_{Ci}(t)$, two first-order systems are in series. From the response, it is observed that $\mathbf{T}(t)$ converges towards the set point of amplitude 50. The actual outlet process liquid temperature $T(t)$ can be obtained by adding the steady-state value $\bar{T}$ with $\mathbf{T}(t)$.

Fig. 3.8 shows the response of outlet cooling water tempertature to a step change in $T_i(t)$, $T_{Ci}(t)$ and $q_C(t)$. If the inlet cooling water temperature, $T_{Ci}(t)$ as well as the cooling water flow rate, $q_C(t)$ change, it affects the outlet cooling water temperature $\mathbf{T}_C(t)$. The actual outlet cooling water temperature $T_C(t)$ can be obtained by adding the steady-state value $\bar{T}_C$ with $\mathbf{T}_C(t)$. It can be observed that $\mathbf{T}_C(t)$ converges towards the set point of amplitude 50.

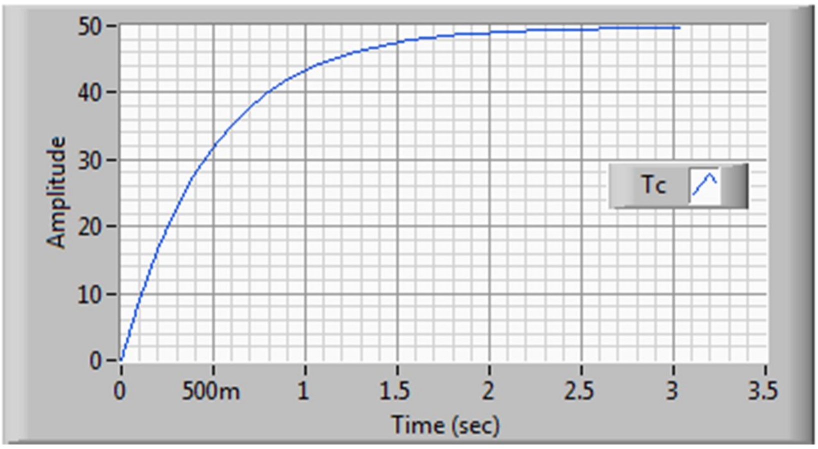


Fig. **3.**8: Response of outlet cooling water temperature to a step change in $T_i(t)$, $T_{Ci}(t)$ and $q_C(t)$ due to the cooling medium

3.5.3 Responses of (3 x 3) order Thermal Process

The response of the outlet process liquid temperature T(t) while taking the dynamics of the wall of the tank into consideration is shown in Fig. 3.9. If inlet liquid temperature $T_i(t)$ changes, it directly affects the process liquid temperature, but if $T_{Ci}(t)$ changes, it first affects the cooling water temperature $\mathbf{T}_C(t)$, then this temperature affects the metal wall temperature $\mathbf{T}_m(t)$ and finally this temperature affects the process liquid temperature $\mathbf{T}(t)$. So, there are three first-order systems in series. The steady-state value $\bar{T}$ is added to $\mathbf{T}(t)$ to get the actual outlet process liquid temperature $T(t)$. From the response, it is observed that $\mathbf{T}(t)$ converges towards the set point of amplitude 50.

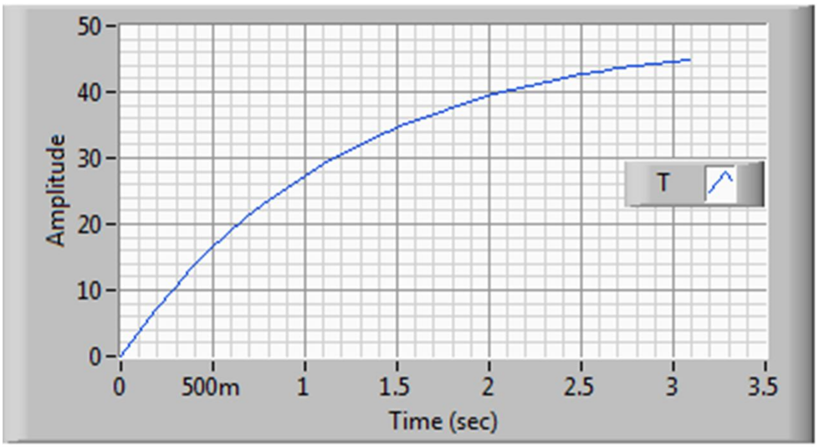


Fig. 3.9: Response of outlet process liquid temperature to a step change in $T_i(t)$, $T_{Ci}(t)$ and $q_C(t)$ taking metal wall into consideration

The response of outlet cooling water temperature and metal wall temperature to a step change in $T_i(t)$, $T_{Ci}(t)$ and $q_C(t)$ taking metal wall into consideration are shown in Fig. 3.10 and Fig. 3.11 respectively.

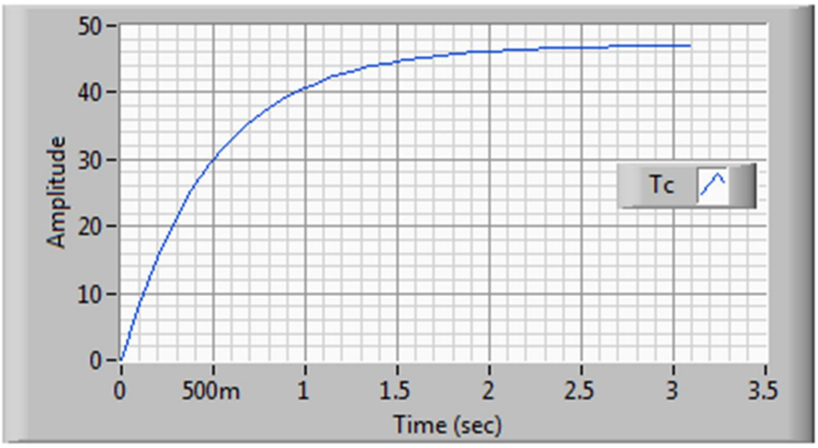


Fig. 3.10: Response of outlet cooling water temperature to a step change in $T_i(t)$, $T_{Ci}(t)$ and $q_C(t)$ taking metal wall into consideration

If the actual outlet process liquid temperature $T_C(t)$ is required, the steady-state value $\overline{T}_C$ is added to the deviation temperature $\mathbf{T}_C(t)$. From the response, it is observed that $\mathbf{T}_C(t)$ converges towards the set point of amplitude 50. Similarly, the steady-state value $\overline{T}_m$ can be added to $\mathbf{T}_m(t)$ to get the actual metal wall temperature $T_m(t)$.

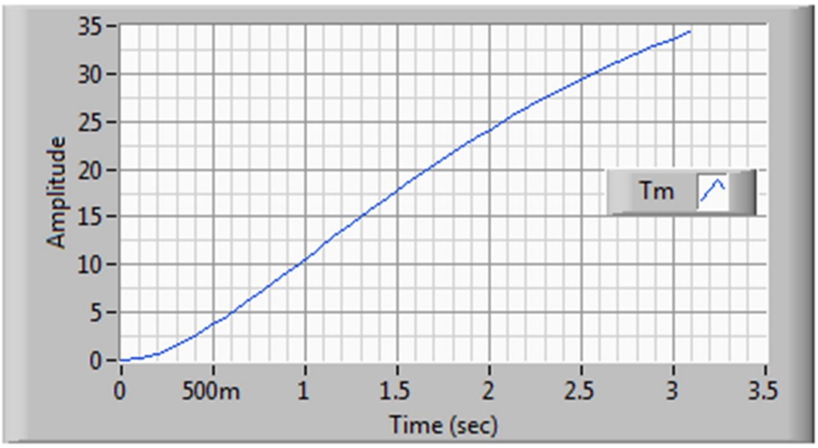


Fig. 3.11: Response of metal wall temperature to a step change in $T_i(t)$, $T_{Ci}(t)$ and $q_C(t)$ taking metal wall into consideration

3.6 SUMMARY

This chapter describes about the control of temperature of process liquid in tank. The nonlinear equations are linearized using Taylor series method. The effect of inlet temperature, inlet cooling water temperature and inlet cooling water flow rates to outlet process temperature and cooling water temperature are explained here. The mathematical models of thermal process are implemented in LabVIEW. The step responses of the (3 x 2) and (3 x 3) processes are analyzed in this chapter. The response curves give the acceptable results.

CHAPTER 4

Four Input Four Output Tank System

Four Input Four Output Tank System

Mathematical Modelling of FIFO Tank System

Linearization

Response of FIFO Tank System

LabVIEW Implementation of FIFO Tank System

Simulation Results and Discussion

FOUR INPUT FOUR OUTPUT TANK SYSTEM

This chapter describes about the four input four outputs (FIFO) tank system. The modelling of the FIFO tank system plays an important role for the analysis of the control system. The mathematical models of FIFO tank system are presented. The physical parameters which describe the personality of the FIFO tank system are explained. The (4 x 4) order process is described by the ordinary differential equations.

4.1 FOUR INPUT FOUR OUTPUT (FIFO) TANK SYSTEM

The interacting four tank liquid level system is shown in Fig. 4.1. In the figure, there are four inputs, the flow to tank 1, tank 2, tank 3 and tank 4 (q_{i1}, q_{i2}, q_{i3} and q_{i4}) and four outputs, the levels in tank 1, tank 2, tank 3 and tank 4 (h_1, h_2, h_3 and h_4), respectively. A change in q_{i1} alone will affect all the four outputs (h_1, h_2, h_3 and h_4). Similarly, a change in any one of the inputs alone will also affect all the four outputs. This is an interacting process for which the level in tank 1 is affected by the level in tank 2 and so on. So, it is called the MIMO system, more specifically called four input four outputs (FIFO) system.

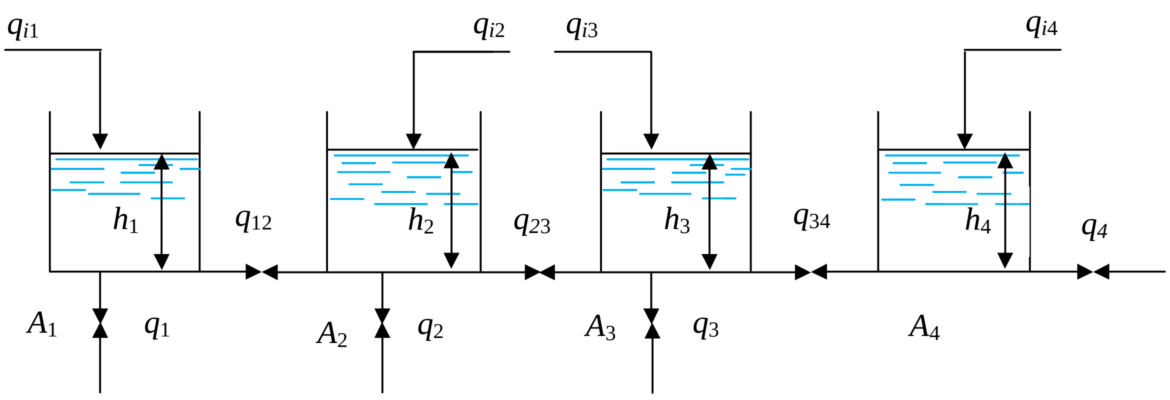


Fig. 4.1: Block diagram of FIFO tank system

The four tanks are interconnected with each other through the valves. The liquid will flow from one tank to other through the valve. Also the liquid will come out from the tank through the valves. The user can regulate the inputs by adjusting the input volumetric flow rates and simultaneously, the output levels are adjusted and displayed at any instant of time in the simulation VI. The areas of cross-section of all the four tanks are same.

4.2 MATHEMATICAL MODELLING OF FIFO TANK SYSTEM

The mathematical model helps to explain a system and the effects of different components. The mathematical modelling of FIFO tank system is discussed here.

The flow of liquid through a valve is given by Eq. (2.11) as

$$q(t) = C'_v \sqrt{h(t)} \tag{4.1}$$

where

$C'_v = C_v Z$

C_v = Coefficient of the valve

The flow through the valve connected to the third tank is given as

$$q_3(t) = C'_{v3} \sqrt{h_3(t)} \tag{4.2}$$

where

$C'_{v3} = C_{v3} Z$

C_{v3} = Coefficient of the valve connected to the third tank

The interaction between the tanks is shown from the valve Eq. (4.1) for the flow, q_{23} is

$$q_{23}(t) = C'_{v23} \sqrt{h_2(t) - h_3(t)} \tag{4.3}$$

where

$C'_{v23} = C_{v23} Z$

C_{v23} = Coefficient of the valve connected between the second and third tank

Eq. (4.3) shows that the flow between the two tanks depends on the levels in both the tanks, each affecting the other. Hence the FIFO tank system is called as an interacting system.

The interaction between the tanks is shown from the valve Eq. (4.1) for the flow, q_{34} is

$$q_{34}(t) = C'_{v34} \sqrt{h_3(t) - h_4(t)} \tag{4.4}$$

where

$C'_{v34} = C_{v34} Z$

C_{v34} = Coefficient of the valve connected between the third and fourth tank

The flow through the valve connected to the fourth tank is given as

$$q_4(t) = C'_{v4}\sqrt{h_3(t)} \tag{4.4}$$

where

$C'_{v4} = C_{v4}\, Z$

C_{v4} = Coefficient of the valve connected to the fourth tank

An unsteady-state mass balance Eq. (2.6) around the first tank can be written as

$$\rho A_1 \frac{dh_1(t)}{dt} = \rho q_{i1}(t) - \rho q_{12}(t) - \rho q_1(t)$$

$$\Rightarrow A_1 \frac{dh_1(t)}{dt} = q_{i1}(t) - q_{12}(t) - q_1(t) \tag{4.5}$$

$$\Rightarrow A_1 \frac{dh_1(t)}{dt} = q_{i1}(t) - C'_{v12}\sqrt{h_1(t) - h_2(t)} - C'_{v1}\sqrt{h_1(t)} \tag{4.6}$$

An unsteady-state mass balance Eq. (2.6) around the second tank can be given by

$$\rho A_2 \frac{dh_2(t)}{dt} = \rho q_{i2}(t) + \rho q_{12}(t) - \rho q_{23}(t) - \rho q_2(t)$$

$$\Rightarrow A_2 \frac{dh_2(t)}{dt} = q_{i2}(t) + q_{12}(t) - q_{23}(t) - q_2(t) \tag{4.7}$$

$$\Rightarrow A_2 \frac{dh_2(t)}{dt} = q_{i2}(t) + C'_{v12}\sqrt{h_1(t) - h_2(t)} - C'_{v23}\sqrt{h_2(t) - h_3(t)} - C'_{v2}\sqrt{h_2(t)} \tag{4.8}$$

An unsteady-state mass balance Eq. (2.6) around the third tank can be given by

$$\rho A_3 \frac{dh_3(t)}{dt} = \rho q_{i3}(t) + \rho q_{23}(t) - \rho q_{34}(t) - \rho q_3(t)$$

$$\Rightarrow A_3 \frac{dh_3(t)}{dt} = q_{i3}(t) + q_{23}(t) - q_{34}(t) - q_3(t) \tag{4.9}$$

$$\Rightarrow A_3 \frac{dh_3(t)}{dt} = q_{i3}(t) + C'_{v23}\sqrt{h_2(t) - h_3(t)} - C'_{v34}\sqrt{h_3(t) - h_4(t)} - C'_{v3}\sqrt{h_3(t)} \tag{4.10}$$

An unsteady-state mass balance Eq. (2.6) around the fourth tank can be written as

$$\rho A_4 \frac{dh_4(t)}{dt} = \rho q_{i4}(t) + \rho q_{34}(t) - \rho q_4(t)$$

$$\Rightarrow A_4 \frac{dh_4(t)}{dt} = q_{i4}(t) + q_{34}(t) - q_4(t) \tag{4.11}$$

$$\Rightarrow A_4 \frac{dh_4(t)}{dt} = q_{i4}(t) + C'_{v34}\sqrt{h_3(t) - h_4(t)} - C'_{v4}\sqrt{h_4(t)} \tag{4.12}$$

Eqs. (4.6), (4.8), (4.10) and (4.12) are the nonlinear equations due to the presence of square root terms in the relations, i.e. the outputs will vary nonlinearly with the inputs.

4.3 LINEARIZATION

The nonlinear Eqs. (4.6), (4.8), (4.10) and (4.12) are derived for modelling the FIFO tank system. The nonlinearities of the processes are very difficult to model a system. Hence, the nonlinear equations of the process are linearized using Taylor series method. Then, a control system is developed and designed based on the linear model.

The nonlinear terms are linearized with respect to h_1, h_2, h_3 and h_4. The linearization is done around the steady-state values $\bar{h}_1, \bar{h}_2, \bar{h}_3$ and $\bar{h}_4$ respectively.
The nominal steady-state value of $q_1(t)$ is

$$q_1(t) = \bar{q}_1 + \left.\frac{\partial q_1(t)}{\partial h_1}\right|_{ss} (h_1(t) - \bar{h}_1)$$

$$\Rightarrow q_1(t) = \bar{q}_1 + C_1(h_1(t) - \bar{h}_1) \tag{4.13}$$

where

$$C_1 = \frac{C'_{v1}}{2}(\bar{h}_1)^{-1/2} \tag{4.14}$$

The nominal steady-state value of $q_{12}(t)$ is

$$q_{12}(t) = \bar{q}_{12} + \left.\frac{\partial q_{12}(t)}{\partial h_1}\right|_{ss} (h_1(t) - \bar{h}_1) + \left.\frac{\partial q_{12}(t)}{\partial h_2}\right|_{ss} (h_2(t) - \bar{h}_2)$$

$$\Rightarrow q_{12}(t) = \bar{q}_{12} + C_{12}(h_1(t) - \bar{h}_1) - C_{12}(h_2(t) - \bar{h}_2) \tag{4.15}$$

where

$$C_{12} = \frac{C'_{v12}}{2}(\bar{h}_1 - \bar{h}_2)^{-1/2} \tag{4.16}$$

Substituting Eqs. (4.13) and (4.15) into the Eq. (4.5), we get

$$A_1 \frac{dh_1(t)}{dt} = q_{i1}(t) - \bar{q}_{12} - C_{12}(h_1(t) - \bar{h}_1) + C_{12}(h_2(t) - \bar{h}_2) - \bar{q}_1 - C_1(h_1(t) - \bar{h}_1)$$

$$\Rightarrow A_1 \frac{dh_1(t)}{dt} = q_{i1}(t) - \bar{q}_{12} - \bar{q}_1 - C_{12}(h_1(t) - \bar{h}_1) + C_{12}(h_2(t) - \bar{h}_2) - C_1(h_1(t) - \bar{h}_1) \tag{4.17}$$

A steady-state mass balance around the tank 1 can be written as

$$\rho \bar{q}_{i1} - \rho \bar{q}_{12} - \rho \bar{q}_1 = 0$$

$$\Rightarrow \bar{q}_{i1} - \bar{q}_{12} - \bar{q}_1 = 0 \tag{4.18}$$

Subtracting Eq. (4.18) from Eq. (4.17) and simplifying, we get

$$A_1 \frac{dH_1(t)}{dt} = Q_{i1}(t) - (C_1 + C_{12})H_1(t) + C_{12}H_2(t) \tag{4.19}$$

where $Q_{i1}(t)$, $H_1(t)$ and $H_2(t)$ are the deviation variables of $q_{i1}(t)$, $h_1(t)$ and $h_2(t)$, respectively.

$$\left.\begin{aligned} Q_{i1}(t) &= q_{i1}(t) - \bar{q}_{i1} \\ H_1(t) &= h_1(t) - \bar{h}_1 \\ H_2(t) &= h_2(t) - \bar{h}_2 \end{aligned}\right\} \tag{4.20}$$

The nominal steady-state value of $q_2(t)$ is

$$q_2(t) = \bar{q}_2 + \left.\frac{\partial q_2(t)}{\partial h_2}\right|_{ss} (h_2(t) - \bar{h}_2)$$

$$\Rightarrow q_2(t) = \bar{q}_2 + C_2(h_2(t) - \bar{h}_2) \tag{4.21}$$

where

$$C_2 = \frac{C'_{v2}}{2}(\bar{h}_2)^{-1/2} \tag{4.22}$$

The nominal steady-state value of $q_{23}(t)$ is

$$q_{23}(t) = \bar{q}_{23} + \left.\frac{\partial q_{23}(t)}{\partial h_2}\right|_{ss} (h_2(t) - \bar{h}_2) + \left.\frac{\partial q_{23}(t)}{\partial h_3}\right|_{ss} (h_3(t) - \bar{h}_3)$$

$$\Rightarrow q_{23}(t) = \bar{q}_{23} + C_{23}(h_2(t) - \bar{h}_2) - C_{23}(h_3(t) - \bar{h}_3) \tag{4.23}$$

where

$$C_{23} = \frac{C'_{v23}}{2}(\bar{h}_2 - \bar{h}_3)^{-1/2} \tag{4.24}$$

Substituting Eqs. (4.15), (4.21) and (4.23) into the Eq. (4.7), we get

$$A_2 \frac{dh_2(t)}{dt} = q_{i2}(t) + \bar{q}_{12} + C_{12}(h_1(t) - \bar{h}_1) - C_{12}(h_2(t) - \bar{h}_2) - \bar{q}_{23} -$$
$$C_{23}(h_2(t) - \bar{h}_2) + C_{23}(h_3(t) - \bar{h}_3) - \bar{q}_2 - C_2(h_2(t) - \bar{h}_2)$$

$$\Rightarrow A_2 \frac{dh_2(t)}{dt} = q_{i2}(t) + \bar{q}_{12} - \bar{q}_{23} - \bar{q}_2 + C_{12}(h_1(t) - \bar{h}_1) - C_{12}(h_2(t) - \bar{h}_2)$$
$$- C_{23}(h_2(t) - \bar{h}_2) + C_{23}(h_3(t) - \bar{h}_3) - C_2(h_2(t) - \bar{h}_2) \quad (4.25)$$

A steady-state mass balance around the tank 2 can be written as

$$\rho \bar{q}_{i2} + \rho \bar{q}_{12} - \rho \bar{q}_{23} - \rho \bar{q}_2 = 0$$

$$\Rightarrow \bar{q}_{i2} + \bar{q}_{12} - \bar{q}_{23} - \bar{q}_2 = 0 \quad (4.26)$$

Subtracting Eq. (4.26) from Eq. (4.25) and simplifying, we get

$$A_2 \frac{dH_2(t)}{dt} = Q_{i2}(t) + C_{12} H_1(t) - (C_{12} + C_2 + C_{23}) H_2(t) + C_{23} H_3(t) \quad (4.27)$$

where $Q_{i2}(t)$ and $H_3(t)$ are the deviation variables of $q_{i2}(t)$ and $h_2(t)$ respectively.

$$\left.\begin{aligned} Q_{i2}(t) &= q_{i2}(t) - \bar{q}_{i2} \\ H_3(t) &= h_3(t) - \bar{h}_3 \end{aligned}\right\} \quad (4.28)$$

By using the procedure already discussed, Eqs. (4.10) and (4.12) can be written in terms of deviation variables as

$$A_3 \frac{dH_3(t)}{dt} = Q_{i3}(t) + C_{23} H_2(t) - (C_{23} + C_3 + C_{34}) H_3(t) + C_{34} H_4(t) \quad (4.29)$$

$$A_4 \frac{dH_4(t)}{dt} = Q_{i4}(t) + C_{34} H_3(t) - (C_{34} + C_4) H_4(t) \quad (4.30)$$

where $Q_{i3}(t)$, $Q_{i4}(t)$ and $H_4(t)$ are the deviation variables of $q_{i3}(t)$, $q_{i4}(t)$ and $h_4(t)$, respectively.

$$\left.\begin{aligned} Q_{i3}(t) &= q_{i3}(t) - \bar{q}_{i3} \\ Q_{i4}(t) &= q_{i4}(t) - \bar{q}_{i4} \\ H_4(t) &= h_4(t) - \bar{h}_4 \end{aligned}\right\} \quad (4.31)$$

Eqs. (4.19), (4.27), (4.29) and (4.30) are the linearized forms of the nonlinear models of the FIFO tank system. The control system for these models can be easily designed. The linearized equations are in terms of deviation flow rates and deviation level outputs. The solutions of these equations give the deviation level of outputs in tank versus time for a certain inflow rate. If the actual output level is desired, the steady-state value must be added to the corresponding deviation level outputs.

4.4 RESPONSES OF FIFO TANK SYSTEM

The step response of a FIFO tank system gives the information on the stability of the system and it has the ability to reach the steady state.

Taking the Laplace Transform of Eq. (4.19) ,we get

$$A_1\, s\, H_1(s) \;=\; Q_{i1}(s) \;-\; (C_1 + C_{12})\, H_1(s) \;+\; C_{12}\, H_2(s)$$

$$\Rightarrow\; H_1(s) \;=\; \frac{K_1}{\tau_1 s + 1}\, Q_{i1}(s) \;+\; \frac{K_2}{\tau_1 s + 1} H_2(s) \tag{4.32}$$

where

$$K_1 \;=\; \frac{1}{C_1 + C_{12}} \quad , \quad K_2 \;=\; \frac{C_{12}}{C_1 + C_{12}}$$

$$\tau_1 \;=\; \frac{A_1}{C_1 + C_{12}}$$

Eq. (4.32) relates the level in tank 1 with the input flow into the tank 1 and the level in tank 2. The parameter τ_1 is the time constant and K_1 is the gain or sensitivity gives the amount of change of levels in both the tanks per unit change of flow into the tank 1. This change takes place while a constant opening is kept in the outlet valves of both the tanks. The parameter K_2 gives the amount of change of level in tank 1 per change of level in tank 2.

Similarly, Laplace Transform of Eq. (4.27) gives

$$H_2(s) \;=\; \frac{K_3}{\tau_2 s + 1}\, Q_{i2}(s) \;+\; \frac{K_4}{\tau_2 s + 1} H_1(s) + \frac{K_5}{\tau_2 s + 1} H_3(s) \tag{4.33}$$

where

$$K_3 \;=\; \frac{1}{C_{12} + C_2 + C_{23}} \quad , \quad K_4 \;=\; \frac{C_{12}}{C_{12} + C_2 + C_{23}}$$

$$K_5 \;=\; \frac{C_{23}}{C_{12} + C_2 + C_{23}} \quad , \quad \tau_2 \;=\; \frac{A_2}{C_{12} + C_2 + C_{23}}$$

The Laplace Transform of Eqs. (4.29) and (4.30) give the solution as

$$H_3(s) \;=\; \frac{K_6}{\tau_3 s + 1}\, Q_{i3}(s) \;+\; \frac{K_7}{\tau_3 s + 1} H_2(s) + \frac{K_8}{\tau_3 s + 1} H_4(s) \tag{4.34}$$

$$H_4(s) = \frac{K_9}{\tau_4 s+1} Q_{i4}(s) + \frac{K_{10}}{\tau_4 s+1} H_3(s) \tag{4.35}$$

where

$$K_6 = \frac{1}{C_{23}+C_3+C_{34}} \quad , \quad K_7 = \frac{C_{23}}{C_{23}+C_3+C_{34}}$$

$$K_8 = \frac{C_{34}}{C_{23}+C_3+C_{34}} \quad , \quad \tau_3 = \frac{A_3}{C_{23}+C_3+C_{34}}$$

$$K_9 = \frac{1}{C_{34}+C_4} \quad , \quad K_2 = \frac{C_{34}}{C_{34}+C_4}$$

$$\tau_4 = \frac{A_4}{C_{34}+C_4}$$

Eqs. (4.32), (4.33), (4.34) and (4.35) are simplified and we get

$$H_1(s) = G_{11}\ Q_{i1}(s) + G_{12}\ Q_{i2}(s) + G_{13}\ Q_{i3}(s) + G_{14}\ Q_{i4}(s) \tag{4.36}$$

$$H_2(s) = G_{21}\ Q_{i1}(s) + G_{22}\ Q_{i2}(s) + G_{23}\ Q_{i3}(s) + G_{24}\ Q_{i4}(s) \tag{4.37}$$

$$H_3(s) = G_{31}\ Q_{i1}(s) + G_{32}\ Q_{i2}(s) + G_{33}\ Q_{i3}(s) + G_{34}\ Q_{i4}(s) \tag{4.38}$$

$$H_4(s) = G_{41}\ Q_{i1}(s) + G_{42}\ Q_{i2}(s) + G_{43}\ Q_{i3}(s) + G_{44}\ Q_{i4}(s) \tag{4.39}$$

We can write Eqs. (4.36), (4.37), (4.38) and (4.39) in matrix form as

$$\begin{bmatrix} H_1(s) \\ H_2(s) \\ H_3(s) \\ H_4(s) \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & G_{34} \\ G_{41} & G_{42} & G_{43} & G_{44} \end{bmatrix} \begin{bmatrix} Q_{i1}(s) \\ Q_{i2}(s) \\ Q_{i3}(s) \\ Q_{i4}(s) \end{bmatrix} \tag{4.40}$$

The transfer functions $G_{ij}(s)$ are as follows

$$G_{11}(s) = \frac{0.0247s^6+0.26s^5+1.125s^4+2.577s^3+3.293s^2+2.22s+0.616}{0.0247s^7+0.296s^6+1.495s^5+4.106s^4+6.633s^3+6.306s^2+3.269s+0.7132}$$

$$G_{12}(s) = \frac{0.0185s^4+0.129s^3+0.329s^2+0.356s+0.14}{0.037s^6+0.389s^5+1.659s^4+3.67s^3+4.443s^2+2.793s+0.7132}$$

$$G_{13}(s) = \frac{0.0278s + 0.0416}{0.111s^4 + 0.778s^3 + 1.946s^2 + 2.0563s + 0.7779}$$

$$G_{14}(s) = \frac{0.0139}{0.111s^4 + 0.778s^3 + 1.946s^2 + 2.0563s + 0.7779}$$

$$G_{21}(s) = \frac{0.0185s^4 + 0.129s^3 + 0.329s^2 + 0.356s + 0.14}{0.037s^6 + 0.389s^5 + 1.659s^4 + 3.67s^3 + 4.443s^2 + 2.793s + 0.7132}$$

$$G_{22}(s) = \frac{0.037s^5 + 0.315s^4 + 1.0472s^3 + 1.7003s^2 + 1.35s + 0.4203}{0.037s^6 + 0.389s^5 + 1.659s^4 + 3.67s^3 + 4.443s^2 + 2.793s + 0.7132}$$

$$G_{23}(s) = \frac{0.0556s^2 + 0.167s + 0.125}{0.111s^4 + 0.778s^3 + 1.946s^2 + 2.0563s + 0.7779}$$

$$G_{24}(s) = \frac{0.0278s + 0.0417}{0.111s^4 + 0.778s^3 + 1.946s^2 + 2.0563s + 0.7779}$$

$$G_{31}(s) = \frac{0.0278s + 0.0417}{0.111s^4 + 0.778s^3 + 1.946s^2 + 2.0563s + 0.7779}$$

$$G_{32}(s) = \frac{0.0556s^2 + 0.167s + 0.125}{0.111s^4 + 0.778s^3 + 1.946s^2 + 2.0563s + 0.7779}$$

$$G_{33}(s) = \frac{0.111s^3 + 0.723s^2 + 1.473s + 0.917}{0.111s^4 + 0.778s^3 + 1.946s^2 + 2.0563s + 0.7779}$$

$$G_{34}(s) = \frac{0.0556s^2 + 0.195s + 0.153}{0.111s^4 + 0.778s^3 + 1.946s^2 + 2.0563s + 0.7779}$$

$$G_{41}(s) = \frac{0.0139}{0.111s^4 + 0.778s^3 + 1.946s^2 + 2.0563s + 0.7779}$$

$$G_{42}(s) = \frac{0.0278s + 0.0417}{0.111s^4 + 0.778s^3 + 1.946s^2 + 2.0563s + 0.7779}$$

$$G_{43}(s) = \frac{0.0555s^2 + 0.194s + 0.153}{0.111s^4 + 0.778s^3 + 1.946s^2 + 2.0563s + 0.7779}$$

$$G_{44}(s) = \frac{0.074s^4 + 0.519s^3 + 1.316s^2 + 1.436s + 0.57}{0.074s^5 + 0.63s^4 + 2.076s^3 + 3.317s^2 + 2.575s + 0.7779}$$

4.5 LABVIEW IMPLEMENTATION OF FIFO TANK SYSTEM

LabVIEW is used to simulate the FIFO tank system. The FIFO tank system is designed according to the mathematical modeling derived previously. The FIFO tank system is designed for both the nonlinear and linear cases. The FIFO tank systems are explained as follows.

- Front panel of nonlinear FIFO tank system
- Front panel of linear FIFO tank system
- Block diagram of linear FIFO tank system

For simulating the FIFO tank system, the following assumptions are made.

- Coefficient of the valves used in the models are kept constant
- Area of cross-sections of the tanks A_1, A_2, A_3 and A_2 are constant

4.5.1 Front Panel of Nonlinear FIFO Tank System

The front panel of nonlinear FIFO tank system is shown in Fig. 4.2. The user can adjust the input volumetric flow rates q_{i1}, $q_{i2,}$ q_{i3} and q_{i4} to get the desired output levels h_1, h_2, h_3 and h_4, respectively. The output levels change nonlinearly with the input flow rates. The user can either change one input volumetric flow keeping the other inputs constant or change all the four input volumetric flow rates simultaneously. The output levels change accordingly to the input volumetric flows.

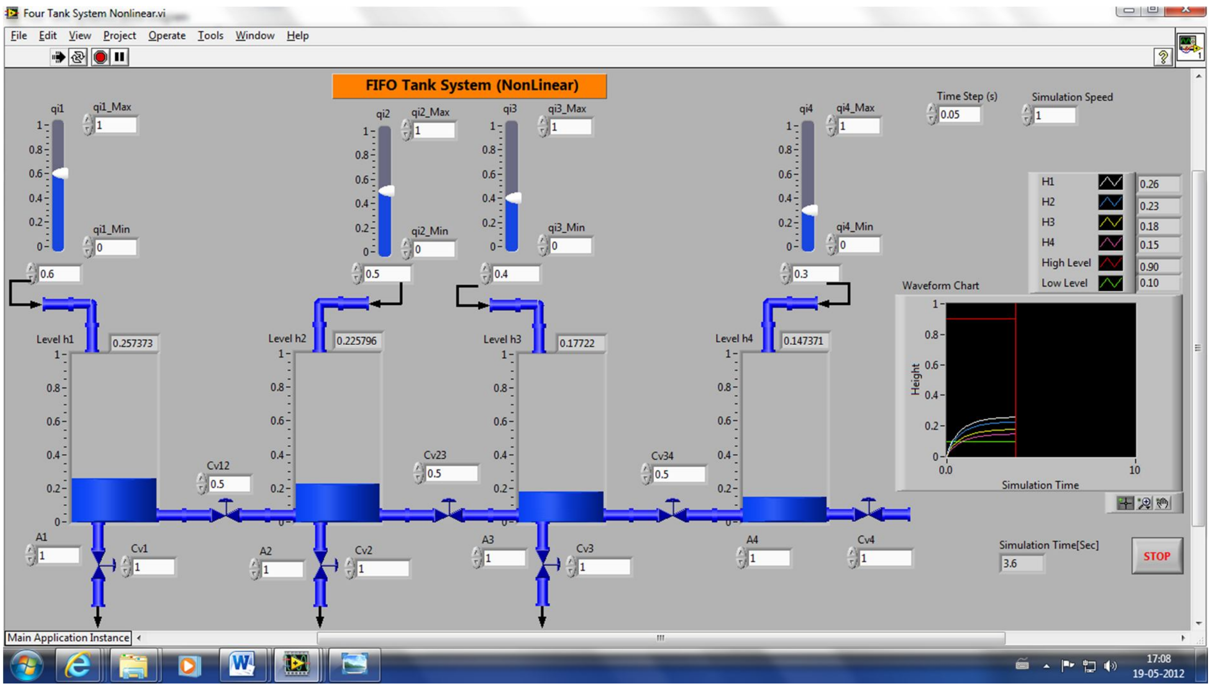


Fig. 4.2: Front panel of nonlinear FIFO tank system

4.5.2 Front Panel of Linear FIFO Tank System

The front panel of linear FIFO tank system is shown in Fig. 4.3. The FIFO tank system is designed based on the deviation variables. The deviation variables of input volumetric flow rates Q_{i1}, Q_{i2}, Q_{i3} and Q_{i4} are adjusted to get the desired deviation variables of output levels H_1, H_2, H_3 and H_4, respectively. The output levels change linearly with the input flow rates as clearly observed from the Eqs. (4.19), (4.27), (4.29) and (4.30), respectively. We can either change one input flow keeping the other inputs constant or change all the four input volumetric flow rates simultaneously. The output levels are in terms of the deviation variables which are obtained from the simulation VI. The steady-state value $\bar{h}_1$ must be added to $H_1(t)$ to get the actual output level $h_1(t)$. Similarly, if the actual output level $h_2(t)$ is required, the steady state value $\bar{h}_2$ is added to the deviation level $H_2(t)$ and so on.

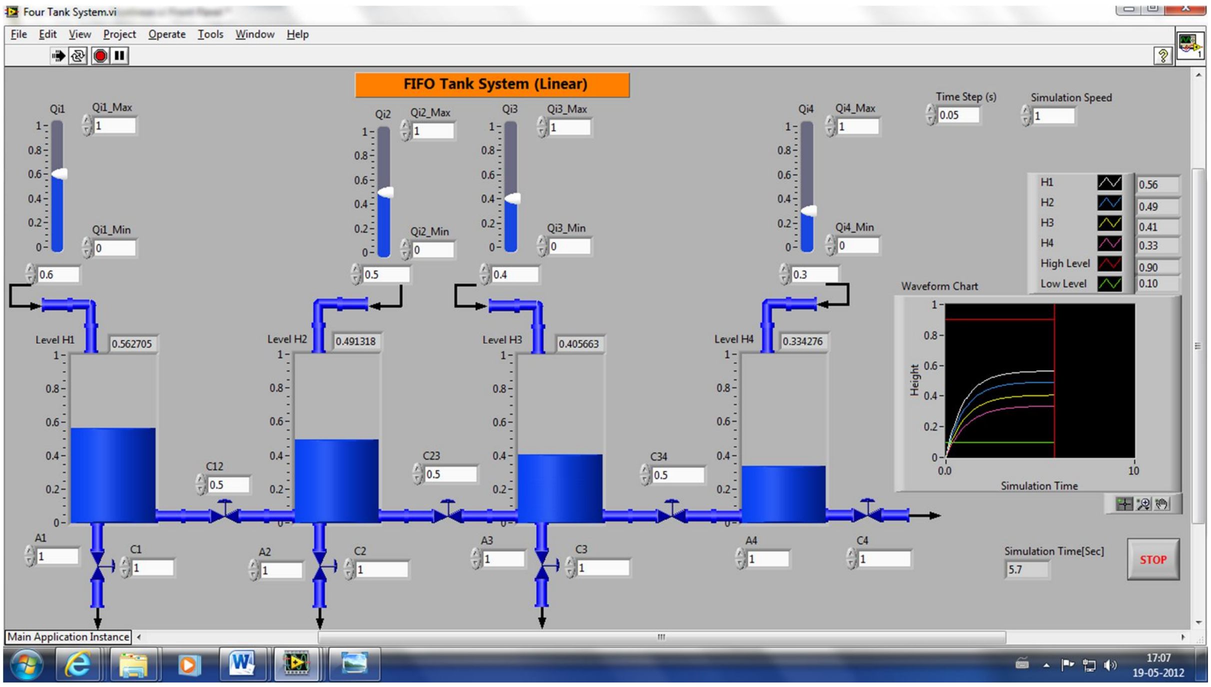


Fig. 4.3: Front panel of linear FIFO tank system

4.5.3 Block Diagram of Linear FIFO Tank System

The block diagram of linear FIFO tank system is shown in Fig. 4.4. The block diagram for linear FIFO tank system is designed based on the mathematical models in Eqs. (4.19), (4.27), (4.29) and (4.30), respectively. In the block diagram, the programming can be written with graphics instead of text codes.

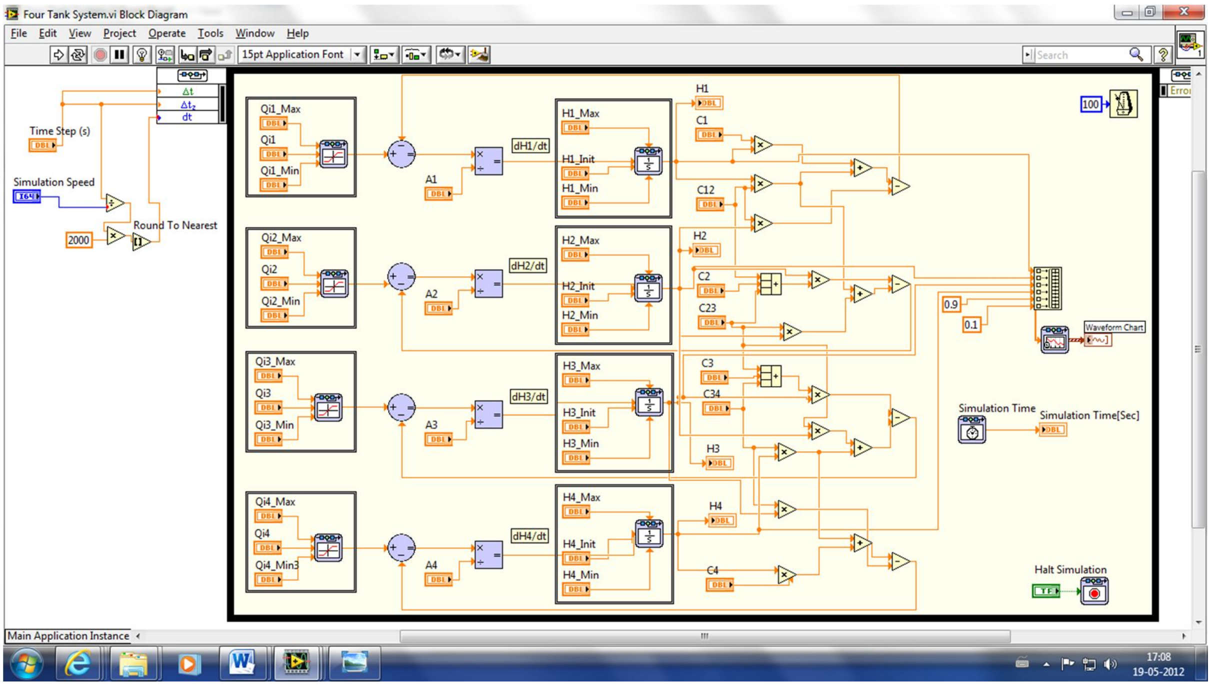


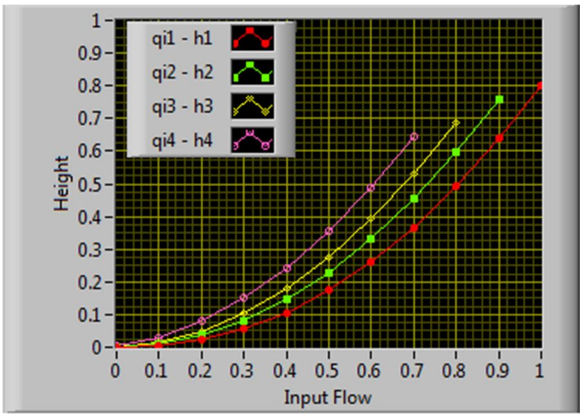
Fig. 4.4: Block diagram of linear FIFO tank system

4.6 SIMULATION RESULTS AND DISCUSSIONS

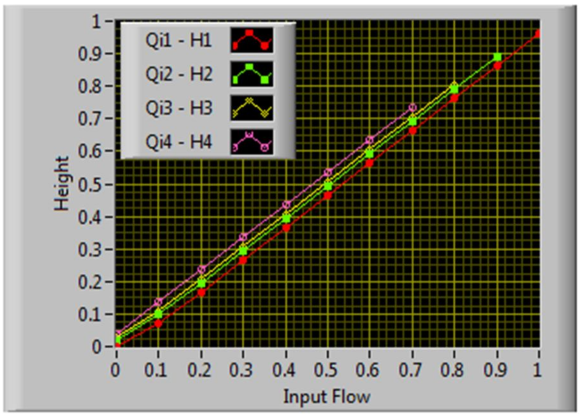
The linearization of the nonlinear equations and its response graphs are analysed and discussed here.

4.6.1 Nonlinear and Linear Graphs

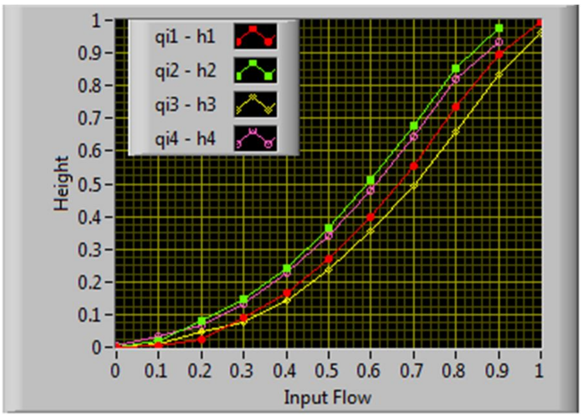
The level graph of FIFO tank system for both nonlinear and linear case are shown in Fig. 4.5. When the input flow rates q_{i1}, q_{i2}, q_{i3} and q_{i4} change, the output levels h_1, h_2, h_3 and h_4 change. Here either one input can be varied while keeping other inputs constant or all the four inputs can be varied simultaneously. The variation of input flows as well as output levels in case of nonlinear system is shown in Fig. 4.5 (a) and (b) where as in case of linear system is shown in Fig. 4.5 (c) and (d), respectively. From the figures, there are two things clearly observed. First one is that the nonlinearities of the system are eliminated and are converted into linear form using Taylor's series method. The second one is that the variations of one input while keeping other inputs constant, all the four output levels are changed. Hence, it shows the interacting nature of the system. The interactions occurred in the system are eliminated by using the cross controllers.



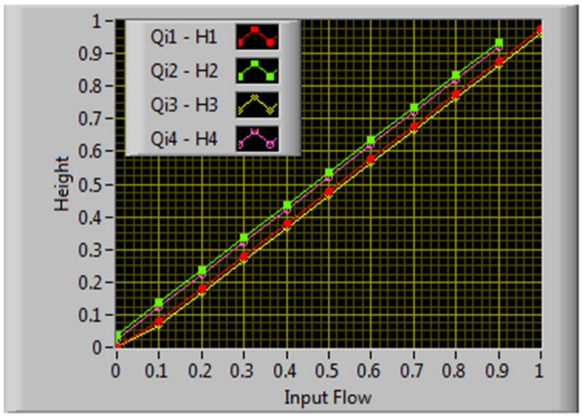
(a)



(c)



(b)

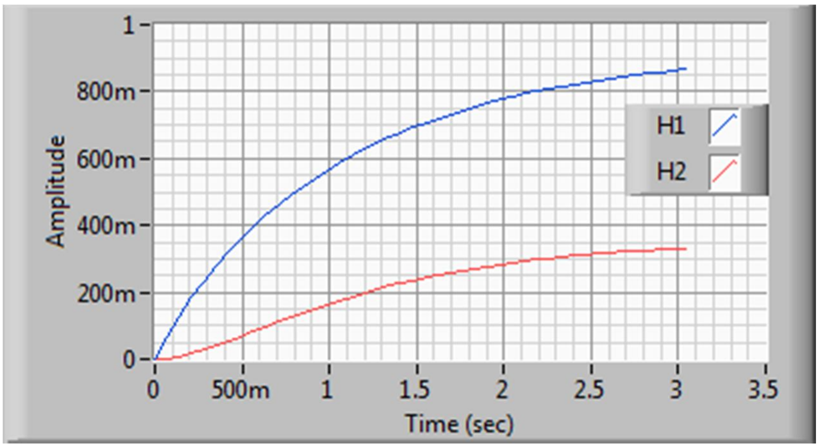


(d)

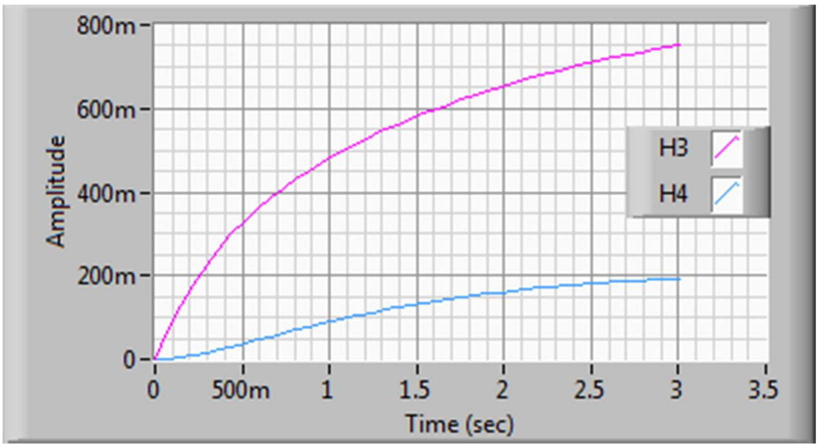
Fig. 4.5: Graph between input flow versus level (height) of FIFO tank system: (a) nonlinear, (b) nonlinear, (c) linear and (d) linear

4.6.2 Responses of FIFO Tank System

The step responses of (4 x 4) order system are analyzed and discussed here. The responses for FIFO tank interacting system are shown in Fig. 4.6 (a) and (b). From the figures, it can be observed that change of only one input variable will affect the four outputs. From the response graph, it is clearly analyzed that the change in Q_{i1} and Q_{i3} affects all the four output levels H_1, H_2, H_3 and H_4 as shown in Fig. 4.6 (a) and (b), respectively. Fig. 4.6 (a) shows the effect on H_1 and H_2 w.r.t. the input flow rates. The effect on H_3 and H_4 w.r.t. the input flow rates are shown in Fig. 4.6 (b). Due to the change in input flow rates Q_{i1}, Q_{i2}, Q_{i3} and Q_{i4}, the change in output levels H_1 and H_2 is shown in Fig. 4.7 (a). Similarly, the effect on H_3 and H_4 due to change in the input flow rates Q_{i1}, Q_{i2}, Q_{i3} and Q_{i4} is shown in Fig. 4.7 (b).

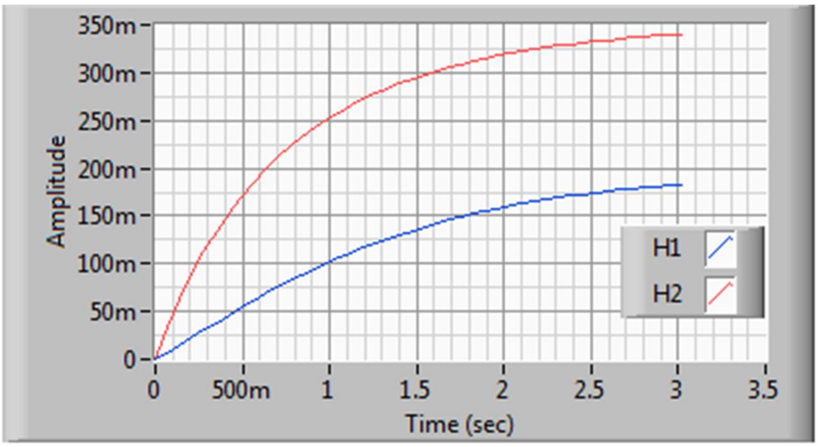


(a)

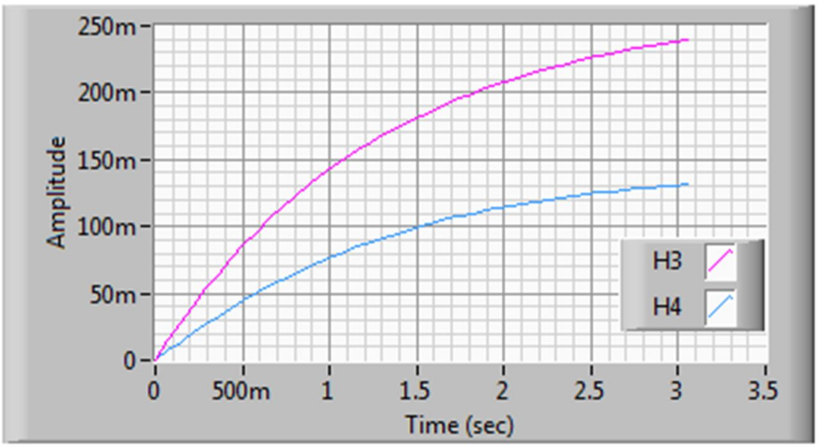


(b)

Fig. 4.6: Responses of FIFO tank system: (a) H_1-H_2, (b) H_3-H_4 with Q_{i1} = 1/*s*, Q_{i2} = 0, Q_{i3} = 1/*s* and Q_{i4} = 0



(a)



(b)

Fig. 4.7: Responses of FIFO tank system: (a) H_1-H_2, (b) H_3-H_4 with Q_{i1} = 0.1/*s*, Q_{i2} = 0.5/*s*, Q_{i3} = 0.2/*s* and Q_{i4} = 0.1/*s*

4.7 SUMMARY

The fundamentals of FIFO tank system and their mathematical modeling are explained in this chapter. The nonlinear model of the FIFO tank system is converted into linear form and a control system is developed for this linear model. The step responses of (4 x 4) process are analyzed thoroughly and give the acceptable results. The LabVIEW implementations of FIFO tank system using the mathematical models have discussed here.

CHAPTER 5

Conclusion

CONCLUSION

This chapter provides the conclusion about the work and gives the suggestions for future work.

5.1 CONCLUSION

During this part of the project work, the analysis and linearization of the MIMO process i.e. level process, thermal process and FIFO tank system are described.

The work presents the linearization of multivariable nonlinear processes that makes the system linear. The multivariable nonlinear processes i.e. level process, thermal process and FIFO tank system are discussed here. The models of the multivariable nonlinear processes are nonlinear and these nonlinear models are linearized using Taylor series method. The nonlinear system is linearized using Taylor series method. The interaction in an MIMO system makes control and stability analysis of the system very complicated compared to a SISO system. The interaction in an MIMO system is eliminated by using cross controllers. The time constant in case of interacting system is greater than for the non-interacting system which results a slower responding system.

The thermal control systems are explained thoroughly. This work concerns the control of temperature of process liquid in the tank and analyses the step responses of the thermal process. The outlet process liquid temperature changes with the change of inlet process liquid temperature, the inlet cooling water temperature and the inlet cooling water flow rate. The nonlinear thermal system is linearized using Taylor series method and then the responses of the process to a step change in $T_i(t)$, $T_{Ci}(t)$ and $q_C(t)$ are obtained.

Here, the responses of (2 x 2), (3 x 2), (3 x 3) and (4 x 4) order multivariable nonlinear processes are analyzed.

Major Contribution

- Design and simulate the SISO tank system using LabVIEW
- Design and simulate the TITO tank system using LabVIEW
- The thermal system is designed and simulated using LabVIEW
- Design and simulate the FIFO tank system using LabVIEW

5.3 SUGGESTIONS FOR FUTURE WORK

The control of level and temperature of liquid in tanks and flow between tanks are the basic problem in the process industries. Future scope of the work can be made as follows.

- Hardware implementation of level process and thermal process
- Model Predictive Controller (MPC), Fuzzy Logic Controller can be used to improve the performance of the MIMO system.
- Linearization of MIMO system can be done using Newton's method and Feedback Linearization method.

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PUBLICATIONS FROM THIS WORK

Journal:

1. **S. B. Prusty** and U. C. Pati, "Analysis of Higher-Order Thermal Process using LabVIEW", **Accepted** for publication in *Journal of Instrument Society of India,* June 2012.

Conference:

2. **Sankata B. Prusty** and Umesh C. Pati; "Linearization of a MIMO Process using LabVIEW", Proceedings of *IEEE* Students' Conference on Electrical, Electronics and Computer Sciences, MANIT, Bhopal, pp. 1-5, 1-2 March 2012. (**Published**)