VIBRATION CONTROL OF FRAME STRUCTURE USING MULTIPLE TUNED MASS DAMPERS

A Thesis

submitted by

PARAMANANDA KUNDU

In partial fulfilment of the requirements for

the award of Degree of

MASTER OF TECHNOLOGY

In

STRUCTURAL ENGINEERING

DEPARTMENT OF CIVIL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY

ROURKELA, ORISSA-769 008

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CERTIFICATE

This is to certify that the thesis entitled **"VIBRATION CONTROL OF FRAME STRUCTURE USING MULTIPLE TUNED MASS DAMPERS"** submitted by **Mr. Paramananda Kundu** in partial fulfilment of the requirements for the award of Master of Technology Degree in Civil Engineering with specialization in Structural Engineering at the National Institute of Technology Rourkela is an authentic work carried out by him under my supervision.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any degree or diploma.

Dr. Kishore Chandra Biswal

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Date:

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ABSTRACT

Need for taller structure in construction and real estate industry is increasing all over the world. These structures are flexible and constructed as light as possible (as seismic load acts on a structure is a function of self-weight), which have low value of damping, makes them vulnerable to unwanted vibration. This vibration creates problem to serviceability requirement of the structure and also reduce structural integrity with possibilities of failure. Current trends use several techniques to reduce wind and earthquake induced structural vibration. Passive tuned mass damper (TMD) is widely used to control structural vibration under wind load but its effectiveness to reduce earthquake induced vibration is an emerging technique. Here a numerical study is proposed on the effectiveness of tuned mass damper to reduce translation structural vibration. Total three type of models, i.e., shear building with single TMD, 2D frame with single TMD and 2D frame with double TMD are considered. Total five numbers of loading conditions are considered named sinusoidal ground acceleration, EW component of 1940 El-Centro earthquake (PGA=0.2144g), compatible time history as per spectra of IS-1893 (Part -1):2002 for 5% damping at rocky soil (PGA=1.0g), Sakaria earthquake (PGA=1.238g), The Landers earthquake (1992) (PGA=1.029g) for time history analysis of considered model.

The effectiveness of single TMD to reduce frame vibration is studied for variation of mass ratio of TMD to frame. Also the effect of double tuned mass damper on the frame response is studied for uniform, non uniform distribution of mass ratio and variation of damping ratio of damper.

From the study it is found that effectiveness of TMD increases with increase in mass ratio. Use of double TMD is much more effective than single TMD of same mass ratio for vibration mitigation under earthquake as well as sinusoidal acceleration.

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ABBREVIATION

AMTMD	Active multiple tuned mass damper
DMF	Dynamic magnification factor
FEM	Finite element method
MTMD	Multiple tuned mass damper
MAPTMD	Multiple active–passive tuned mass dampers
MMD	Multiple mass damper
SSI	Soil structure interaction
SDOF	Single degree of freedom system
TMD	Tuned mass damper
1D	One dimension
2D	Two dimension
3D	Three dimensions

NOTATIONS

A	Area of the beam element
A_{max}	Maximum amplitude of acceleration of sinusoidal load
Ä	Sinusoidal acceleration loading
c	Damping of a single DOF system
[C]	The global damping matrix of the structure
$\underline{d}_0, \underline{\dot{d}}_0, \underline{\ddot{d}}_0$	Displacement, velocity, acceleration at time t=0 used in Newmarks Beta method
$\underline{d}_{i+1}, \underline{\dot{d}}_{i+1}, \underline{\ddot{d}}_{i+1}$	Displacement, velocity, acceleration at i^{th} time step used in Newmarks Beta method
E	Young's Modulus of the frame material
F_0	Maximum displacement amplitude of sinusoidal load
F(t)	Force vector.
$F(t)_I$, $F(t)_D$, $F(t)_S$	Inertia, damping and stiffness component of reactive force.
I	Moment of inertia of a beam element
k	Stiffness of a single DOF system
k^e	Stiffness matrix of a beam element
$[K^e]$	Transformed stiffness matrix of a beam element
[K]	The global stiffness matrix of the structure.
L	Length of the beam element
m	Mass of a single DOF system.
$m_L{}^e$	Lumped mass matrix
m^e	Consistent mass matrix of a beam element
$[M^e]$	Transformed consistent mass matrix of a beam element
[M]	The global mass matrix of structure

t	Time
$[T]$	Transformation matrix
$u(t)$	Displacement of a single DOF system
$\dot{u}(t)$	Velocity of a single DOF system
$\ddot{u}(t)$	Acceleration of a single DOF system
$U(t)$	Absolute nodal displacement.
$\dot{U}(t)$	Absolute nodal velocity.
$\ddot{U}(t)$	Absolute nodal acceleration.
$\ddot{U}_g(t)$	Ground acceleration due to earthquake.
ρ	Density of the beam material
β, γ	Parameters used in Newmarks Beta method
Δt	Time step used in Newmarks Beta method
M	Mass ratio of secondary to primary system in 2 DOF system
ω	Sinusoidal forcing frequency

CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

Vibration means to mechanical oscillation about an equilibrium point. The oscillation may be periodic or non-periodic. Vibration control is essential for machinery, space shuttle, aeroplane, ship floating in water. With the modernisation of engineering the vibration mitigation technique has find a way to civil engineering and infrastructure field.

Now-a-days innumerable high rise building has been constructed all over the world and the number is increasing day by day. This is not only due to concerned over high density of population in the cities, commercial zones and space saving but also to establish country land marks and to prove that their countries are up to the standards. As the seismic load acting on a structure is a function of the self-weight of the structure these structures are made comparatively light and flexible which have relatively low natural damping. Results make the structures more vibration prone under wind, earthquake loading. In many cases this type of large displacements may not be a threat to integrity of the structure but steady state of vibration can cause considerable discomfort and even illness to the building occupant.

In every field in the world conservation of energy is followed. If the energy imposed on the structure by wind and earthquake load is fully dissipated in some way the structure will vibrate less. Every structure naturally releases some energy through various mechanisms such as internal stressing, rubbing, and plastic deformation. In large modern structures, the total damping is almost 5% of the critical. So new generation high rise building is equipped with artificial damping device for vibration control through energy dissipation. The various vibration control methods include passive, active, semi-active, hybrid. Various factors that affect the selection of a particular type of vibration control device are efficiency, compactness and weight, capital cost, operating cost, maintenance requirements and safety.

A Tuned mass damper (TMD) is a passive damping system which utilizes a secondary mass attached to a main structure normally through spring and dashpot to reduce the dynamic response of the structure. It is widely used for vibration control in mechanical engineering systems. Now a days TMD theory has been adopted to reduce vibrations of tall buildings and other civil engineering structures. The secondary mass system is designed to have the natural frequency, which is depended on its mass and stiffness, tuned to that of the primary structure. When that particular frequency of the structure gets excited the TMD will resonate out of phase with the structural motion and reduces its response. Then, the excess energy that is built up in the structure can be transferred to a secondary mass and is dissipated by the dashpot due to relative motion between them at a later time. Mass of the secondary system varies from 1-10% of the structural mass. As a particular earthquake contains a large number of frequency content now a days multiple tuned mass dampers (MTMD) has been used to control earthquake induced motion of high rise structure where the more than one TMD is tuned to different unfavourable structural frequency.

1.2 METHODS OF CONTROL

A large numbers of technique have been tried to produce better control against wind and earthquake excitation.These can be classified into four broad categories: passive control, active control, semi-active control and hybrid control. Each of these will be discussed in following section.

1.2.1 Passive control

The most mechanically simple set of control schemes is enclosed in the passive control category, which has been widely accepted for civil engineering application.

Definition

Housner et al. have both provided brief overviews on structural control, including proper definitions for the various types of control practically implemented in structures. According to them a passive control system is one that does not require an external power source. All forces imposed by passive control devices develop as direct responses to the motion of the structure. Hence, sum of the energy of both the device and the primary system will be constant.

The main purpose of these systems is to efficiently dissipate vibrational energy, and the various methods of achieving this can be categorized in two ways. The first method includes converting kinetic energy directly to heat, such as through the yielding of metals, the deformation of viscoelastic solids and fluids, or the implementation of friction sliders. The second method works on transferring energy among two or more of the vibrational modes of the building, generally achieved by adding a supplemental oscillator that absorbs the vibrations of the primary structure.

Tune mass damper, tune liquid damper, base isolation are example of passive system.

Advantages and limitations

Passive control is the most widely-used method of controlling structural response under wind and earthquake loading, but it has some limitations. While it is reliable and relatively straight forward to design, passive control systems are generally only good for limited bandwidths of dynamics input. As a result, they are susceptible to the effects of off-tuning, de-tuning, or resonances of secondary modes.

1.2.2 Active control

Active control is a relatively upcoming subfield of structural engineering. It assures improved response to passive systems at the cost of energy and more complex systems.

Definition

Active control system has been as any control system in which an external power source is required to provide additional forces to the structure in a prescribed manner, by the use of actuators. The signals are sent to control the actuators and determine the feedback from the sensors provided on or through the structure. Due to the presence of an external power source, the force applied may either add or dissipate energy from the structure.

In order to maximize the performance of an active system, the actuator forces must be prescribed in real-time base on the inputs of the sensors. The direction and magnitude of these forces can be assigned in the variety of ways, all of which have their roots in the diverse and mathematically rich field of control engineering.

Advantage and limitations

The performance of active control is quite pronounced in some cases. Due to its capability to respond in real-time, active control eliminates most of the tuning drawbacks inherent in passive devices. However, active control has not been exuberantly embraced by the civil engineering community as a result of some significant limitations.

Most significant advantage of active control method is diminishes by their heavy reliance on external power supplies. The power consumption and cost is comparatively large for output of certain magnitude forces necessary to control large civil structures by the actuator. Additionally, there may be situation at which the control forces are needed coincides with the time when the power cut is the most likely, such as during an earthquake or large wind storm. This raises question on reliability concerns.

Beyond the issue of energy supply, engineers also hesitate to embrace non-traditional technologies for structures. The placement of sensors and the design of feedback schemes are also beyond the scope of most practicing engineers, and poorly designed active system may lead to deleterious energy inputs and destabilization of the primary system.

1.2.3 Semi-active control

Semi active control performed on the benefits of active control and the reliability of passive control, which makes it a much more appealing alternative to traditional control scheme in civil structures.

Definition

Semi active control systems act on the same principle of active control system but they differ in that their external energy requirement is smaller. These devices have an inherent stability in terms of bounded-input and output as these do not add mechanical energy to the primary system. Therefore, it may be viewed as controllable passive device.

Semi-active control relies on the reactive forces that develop due to variable stiffness or damping devices rather than application of actuator forces. That means, by changing the properties of these devices, using only nominal power the response of the system may be favourably modified. As a result, semi-active control methods appear to combine the best features of fully active and fully passive systems, leaving them as the best in term acceptance for structural control.

Advantages and limitation

The best advantage of semi-active systems is their ability to provide improved control forces with a low demand for power. As the power can be supplied by a battery, which ensures continued functionality even at power failure, adding reliability to any semi-active control method. Because of these benefits that enthusiasm towards the semi-active structural control schemes has increased in recent years, making it a viable alternative to proven passive devices.

While these advantages are in some case truly significant, semi-active control still has its detractors. Most relevant is the need for sensors technology and computer controlled feedback, which is as central to semi-active controls to active control.

1.2.4 Hybrid control

Hybrid systems act on the combined use of passive and active control system. For example, a base isolated structure which is equipped with actuator which actively controls the enhancement of its performance.

1.3 TUNED MASS DAMPER

A TMD is an inertial mass attached to the building location with maximum motion (generally near the top), through a properly tuned spring and damping element. Generally viscous and viscoelastic dampersare used. TMDs provide a frequency dependent hysteresis which increases damping in the frame structure attached to it in order to reduce its motion. The robustness is determined by their dynamic characteristics, stroke and the amount of added mass they employ. The additional damping introduced by the TMD is also dependent on the ratio of the damper mass to the effective mass of the building in a particular mode vibration. TMDs weight is varied between 0.25%-1.0% of the building's weight in the fundamental mode (typically around one third).

The frequency of a TMD is tuned to a particular structural frequency when that frequency is excited the TMD will resonate out of phase with frame motion and reduces its response.

Often for better response control multiple-damper configurations (MDCs) which consist of several dampers placed in parallel with distributed natural frequencies around the control tuning frequency is used. For the same total mass, a multiple mass damper can significantly increase the equivalent damping introduced to the system.

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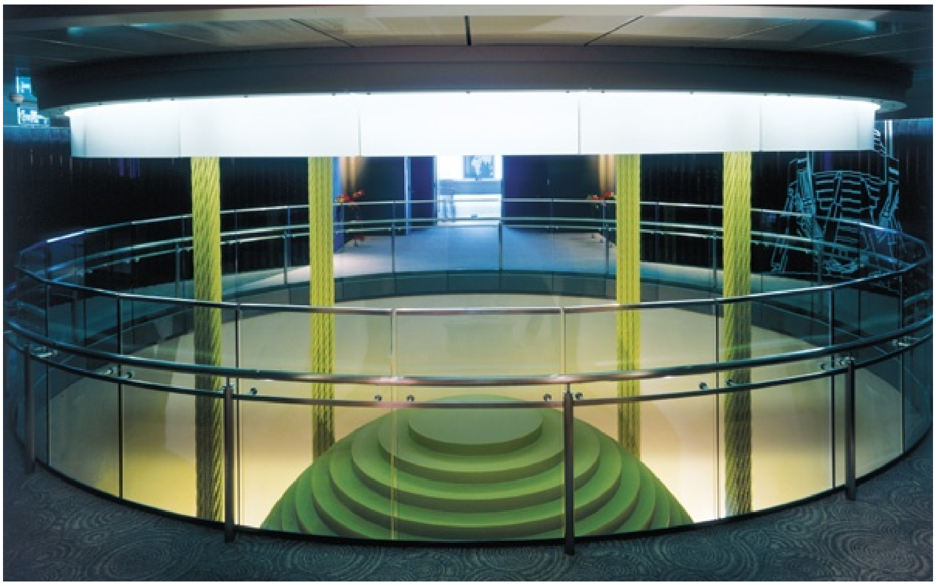


Fig. 1.1 Tuned mass damper in Taipei 101

1.4 REAL LIFE STRUCTURES EQUIPPED WITH TUNED MASS DAMPER

i) Citicrop Centre, New York

The first full-scale structural tuned mass damper was installed in the Citicorp Centre building in New York City. The height of the building is 279 m with fundamental period of around 6.5 s and damping ratio of 1% along both axes. It was finished in 1977 with a TMD placed on the sixty third floor in its crown having weight of 400 ton structure. That time the mass of the TMD was 250 times larger than any existing TMD. The damping of the overall building was increased from 1% to 4% of critical with a mass ratio of the TMD 2% of the first modal mass. Results in reduction of sway amplitude by a factor of 2. The TMD system consists of a large block of concrete bearing on a thin film of oil, with pneumatic spring which provides the structural stiffness.

ii) John Hancock Tower, Boston

Two dampers each having weight of 2700kN was added to the 60-storey John Hancock Tower in Boston to reduce the response to wind loading. The dampers were placed at opposite ends of the fifty-eighth story of the building with a spacing of 67 m. Due to typical shape of the building the damper was designed to counteract the sway and twisting of the building.

iii) CN Tower, Toronto

Due to uniqueness in the design perspective of the Canadian National Tower in Toronto adding TMD was compulsory to suppress the wind induced motion of the building in second and fourth modes. It was required to suppress dynamic wind loading effects of the 102 meter steel antenna at the top of the tower. The first and third modes of the antenna had the same vibrational characteristic as the more heavily damped concrete structure.

To reduce the vibrations, two doughnut-shaped steel rings with having mass of 9 tons were added at elevations corresponding to the peak vibration of the problematic modes. Each ring was mounted on a universal joint in such a way that could rotate in all directions and act as a tuned mass regardless of the direction of wind excitation. Four hydraulically activated dampers per ring were provided to dissipate the energy.

iv) Chiba Port Tower, Japan

Chiba Port Tower, a steel structure of 125 m in height 1950 tons weight and having a rhombus-shaped plan with a side length of 15 m (completed in 1986) was the first tower in Japan to be equipped with a TMD. The time period in the first and second mode of vibrations are 2.25 s and 0.51 s, respectively for the x direction and 2.7 sand 0.57 s for the y direction. Damping for the fundamental mode was computed at 0.5%. For higher mode of vibration damping ratios proportional to frequencies were assumed in the analysis. The use of the TMD was to increase damping of the first mode for both the x and y directions. The mass ratio of the damper with respect to the modal mass of the first mode was about 1/120 in the x direction and 1/80 in the y direction; periods in the x and y directions of 2.24 s and 2.72 s, respectively; and a damper damping ratio of 15%.

v) Taipei 101, Taiwan

Taipei 101, a steel braced building is the 3^{rd} tallest building in the world. Here the TMD was used for architectural purpose along with structural purpose. To reduce the vibration of the building sphere shaped TMD of weight 728 ton diameter 5.5 m between 88-92 floor is used. The enormous sphere was suspended by four set of cables, and the dynamic energy is dissipated by eight hydraulic pistons each having length of 2 m. The damper can reduce 40% of the tower movement. Another two tuned mass dampers, each weighing 6 metric tons sit at the tip of the spire. These prevent damage to the structure due to strong wind loads.

In Japan, to mitigate traffic-induced vibration for two story steel buildings under an urban expressway viaduct TMDs were used (Inoue et al. 1994). TMDs with mass ratio of approximately 1% result in the reduction of the peak values of the acceleration response of the two buildings by 71% and 64%, respectively.

In the world tallest high rise structure Burj Al Arab is equipped with 11 TMD at different storey to control wind induced vibration.

CHAPTER 2

LITERATURE REVIEW AND AIM OF WORK

2.1 REVIEW OF LITERATURE

Till date numerous works has been done on single as well as multiple tuned mass damper. The concept of TMD was first used by **Frahm** [1909] to reduce the rolling motion of ships as well as ship hull vibrations. Later **Hartog** [1940] developed analytical model for vibration control capabilities of TMDs. Later he optimized TMDs parameter for harmonic excitations as well as wide band input.

The main drawback of a single TMD is its sensitivity of the effectiveness to the error in the natural frequency of the structure and the damping ratio of the tuned mass damper. The effectiveness of a tuned mass damper is reduced significantly by mistuning. As a result of more than one TMD with different dynamic characteristics has been proposed to improve the effectiveness. **Iwanami and Seto** [1984] studied that two tuned mass dampers are more effective than a single tuned mass damper. Though the effectiveness is not that much significant.

Clark [1988] studied the methodology for designing multiple tuned mass dampers for reducing building response. The method used was based on extending Den Hartog work from a single degree of freedom to multiple degrees of freedom system. A significant motion reduction was achieved for a structure by using simplified linear mathematical models and design technique under 1940 El Centro earthquake excitation.

Manikanahally and Crocker [1991] considered MTMD system in which each TMD is tuned to different structural frequency.

Sun et al. [1992] investigated analytically as well as experimentally the effect of large number of liquid oscillator attached to a main structure in which the oscillators were tuned to structural natural frequency.

Igusa and Xu [1994] examined the vibration control capability of multiple tuned mass dampers with natural frequency distributed over a certain frequency range for the structures subjected to wide band input. TMD'S design was optimize by using calculus of variations with a constraint on the total mass. Results showed that the optimal designed multiple TMD's are more robust than a single TMD with equal total mass in vibration mitigation of main structure.

Kareem and Kline [1995] studied the dynamic characteristics and effectiveness of multiple mass dampers (MMDs) (a collection of several mass dampers with distributed natural frequencies) under random loading. The random loads considered were narrow- and wide-banded excitations represented by wind and seismic load. In this regard two different buildings were taken. A 31m by 31m square building in plan with 93m height, having natural frequency and damping ratio equal to 0.01 Hz and 0.4, respectively, was used for seismic analysis. For wind loading a rectangular building 31m by 155m in plan with height 186m was considered. Response under wind and earthquake loading was found by changing different parameters like effect of number of damper, damping, and non-uniform distribution of mass. Result showed that the MMDs configuration is more effective in controlling the motion of the primary structure. Due to smaller size of individual than a single TMD it is very easily portable and installable in old as well as retrofitted structure.

Abe and Igusa [1995] studied the effectiveness of one or more TMDs to minimize the maximum structural response with closely spaced natural frequencies by analytical method. The input load considered was a harmonic load with a possible range of frequencies.

Perturbation theory was used with three sets of small parameters the ratio of TMD and structure modal masses, the damping of the system, and the differences between the structural and loading frequencies. Studies were carried for both lumped-mass and continuous structures (simply supported beam). It was concluded that the vibration mitigation of a structure depends upon correct placement of TMD along with the number of TMD, regardless of the spacing of the structures natural frequency.

Joshi and Jangid [1997] investigated the effectiveness of optimally designed multiple tuned mass dampers (MTMD) for reducing the dynamic response of a base excited structure in a particular mode of vibration. The base excitation was modelled as a stationary white noise random process. The parameters like damping ratio, the tuning frequency ratio and the frequency bandwidth of the MTMD system were optimised based on the minimization of the root mean square (r.m.s.) displacement of the main structure. The stationary responseof the structure with MTMD was analysed for the optimum parameters of the MTMD system. It was concluded that the optimally designed MTMD system is more effective for vibration control than the single tuned mass damper for same mass ratio, damping of the main system does not have any influence on the optimum damping ratio of both the single TMD and the MTMD system, number of TMDs also does not have much influence on the optimum tuning frequency and the corresponding effectiveness of the MTMD system.

Jangid and Datta [1997] conducted a parametric study to investigate the effectiveness of MTMDs for reducing the dynamic response of a simply torsionally coupled system subjected to lateral excitation, modelled as a broad-band stationary random process. MTMDs considered for this purpose having uniformly distributed frequencies and are arranged in a row covering the width of the system. The parameters considered were the eccentricity of the main system, its uncoupled torsional to lateral frequency ratio and the damping of MTMDs. It was concluded that the effectiveness of MTMDs in controlling the lateral response of the torsionally coupled system decreases with the increase in the degree of asymmetry.

Jangid [1999] investigated the optimum parameters of Multiple Tuned Mass Dampers (MTMD) for an undamped system under harmonic base excitation using a numerical searching technique. The criteria used for the optimality was the minimization of steady-state displacement response of the main system. Curve fitting technique was used to find the formulae for the optimum parameters of MTMD (i.e. damping ratio, bandwidth and tuning frequency) which can further be used for engineering applications. The optimum parameters of the MTMD system were calculated for different mass ratios and number of dampers. From numerical study it was concluded that the optimum damping ratio of MTMD system decreases with the increase of the number of MTMD and increases with the increase of mass ratio, optimum band-width of the MTMD system increases with the increase of both the mass and number of MTMD and optimum tuning frequency increases with the increase of the number of MTMD and decreases with the increases of the mass ratio.

Li [2000] studied the robustness of multiple tuned mass dampers (MTMDs) having a uniform distribution of natural frequencies for decreasing unwanted vibration of a structure under ground acceleration. The MTMD was fabricated by keeping the stiffness and damping constant and changing the mass. The structure was represented by its mode-generalized system to control a particular mode of vibration using the mode reduced-order method. The optimum parameters of the MTMD like: the frequency spacing, average damping ratio, mass ratio and total number of dampers were investigated for steel structure (whose damping ratio is 0.02) by conducting a numerical searching technique in two directions. Optimization was done by the minimization of the maximum value of the dynamic magnification factor (DMF) of the structure with MTMD. It was concluded from the study that the optimum average damping ratio of the MTMD decreases with the increase of the total number of the MTMD and increases with the increase of the mass ratio, the optimum frequency spacing of the MTMD increases with the increase of both the total number and mass ratio. It was also found that the optimum MTMD is more effective than the optimum MTMD (II) (mass constant and varying the stiffness and damping coefficient) and the optimum single TMD with equal mass.

Wu and Chen [2000] investigated the optimal placement and the seismic performance of MTMD whose frequency is tuned to different structural frequency. Optimization objective of the MTMD was to decrease the acceleration of the main structure. Numerical simulation was performed on a six-story shear building having identical floor mass of $4x10^{\wedge 4}$kg and the identical stiffness of $4x10^{\wedge 7}$N/m for each floor for four optimal location of MTMD. The 1st to 3^{rd}mode frequencies were respectively 7.624, 22.43 and 35.93 rad/s. A damping ratio of 3% was assumed for all modes. It was concluded that the optimal MTMD showed great advantage over conventional single TMD in acceleration control as well as in efficient usage of building spare space.

Chen and Wu [2001] studied the seismic ineffectiveness of a tuned mass damper on the modal response of a six storey shear building. Later he proposed multistage and multimode tuned mass dampers and its several optimal locations for practical design and placement of the dampers in seismically excited building structures to reduce its response. The effectiveness of the proposed procedure was checked under a stochastic seismic load and 13 earthquake records for different MTMD location. Numerical results showed that the multiple dampers can effectively reduce the acceleration of the uncontrolled structure by 10–25% more than a single damper. From time-history analysis it was found that the multiple dampers weighing 3% of total structural weight can reduce the floor acceleration up to 40%.

Park and Reed [2001] numerically evaluated the performance of multiple dampers with uniformly and linearly distributed masses, under harmonic excitation. A linearly elastic single degree of freedom system with damping ratio 0.01; and the total mass ratio of the MMD system 0.01 was taken. An algorithm was developed to identify the optimum tuning of the individual dampers, which evaluate the performance by effectiveness, robustness and redundancy. It was concluded that the uniformly distributed system is effective in reducing the peak dynamic magnification factor also slightly more reliable when an individual damper fails. The linearly distributed system is also more robust under mistuning. It was also found that the 11 and 21 mass system is optimum for both configurations (uniformly and linearly distributed masses) for harmonic excitationand the El Centro earthquake simulation respectively.

Li and Liu [2002] investigated the performance of active multiple tuned mass dampers (AMTMD) with a uniform distribution of natural frequencies to reduce the undesirable vibrations of structures under the ground acceleration. The multiple tuned mass dampers (MTMD) in the AMTMD were manufactured by keeping the stiffness and damping constant and varying the mass. The optimum parameters like frequency spacing, average damping ratio, tuning frequency ratio, total number and normalized acceleration feedback gain coefficient of the AMTMD were found by a numerical searching technique. The control forces in the AMTMD were generated through keeping the identical displacement and velocity feedback gain and varying the acceleration feedback gain. The structure was modelled as a single-degree-of-freedom (SDOF) system (the mode-generalized system for a particular mode to be controlled) using mode reduced-order method. It was concluded that the proposed AMTMD significantly reduce the oscillations of structures under the ground acceleration than MTMD as well as ATMD. Also the effectiveness increases with the increase in both the mass ratio and total number of ATMD.

Li [2002] studied and compared the performance of five number of TMD (MTMD-1 – MTMD-5) model, which comprise of various combinations of the stiffness, mass, damping coefficient and damping ratio with a uniform distribution of natural frequencies to reduce unenviable vibration of a single degree of freedom structure (having damping ratio 0.02) under the ground acceleration using a numerical searching technique. The structure was represented by its mode-generalized system in the specific vibration mode being controlled by adopting the mode reduced-order approach. The optimization was done by minimizing the maximum value of the displacement dynamic magnification factor (DDMF) and that of the acceleration dynamic magnification factor (ADMF) of the structure with the MTMD-1 – MTMD-5. It was concluded that the optimum MTMD-1 and MTMD-4 yield approximately the same control performance, and offer higher effectiveness and robustness than the optimum MTMD-2, MTMD-3, and MTMD-5 in reducing the displacement and acceleration responses of structures. It was further found that for both the best effectiveness and robustness and the simplest manufacturing, it is preferable to select the optimum MTMD-1.

Li and Liu [2003] investigated and compared the control performance of eight new MTMD models (the UM-MTMD1~UM-MTMD3, US-MTMD1~US-MTMD3, UD-MTMD1 andUD-MTMD2), with the system parameters (mass, stiffness and damping coefficient), uniformly distributed close to their average values for a single degree of freedom system. The structure was represented by the mode-generalized system corresponding to the specific vibration mode that needs to be controlled. The optimum parameters include the optimum mass spacing; stiffness spacing, damping coefficient spacing, frequency spacing, average damping ratio and tuning frequency ratio were calculated by numerical simulation. It was summarised that the average damping ratio, spacing of optimum frequency, mass, stiffness and damping coefficient as well as effectiveness of the six MTMD models increases with the increasing of the mass ratio.

Li [2003] studied the performance of multiple active–passive tuned mass dampers (MAPTMD) with a uniform distribution of natural frequencies to prevent oscillations of a single degree of freedom structures under the ground acceleration through numerical studies. The controlling forces in the MAPTMD are generated by keeping the identical displacement and velocity feedback gain and varying the acceleration feedback gain. To control a particular oscillation mode the structure was represented by the mode-generalized system. It was concluded that the optimum tuning frequency ratio of MAPTMD decreases with the increase of the mass ratio and it has better robustness and effectiveness than single APTMD which increases with the increase in mass ratio.

Chen and Wu [2003] studied the performance of MTMD systems and compared the result with the TMD systems numerically as well as through shake table tests on a 1/4-scale three-storey building structure under the white noise excitation (the scaled 1940 El-Centro earthquake and the scaled 1952 Taft earthquake). Experimental results showed that the multiple damper systems are better than a single tuned mass damper in reducing the floor accelerations. It was also found that the numerical and experimental results are in good agreement to validate the dynamic properties of the structure.

Wang and Lin [2005] investigated the influence of soil–structure interaction (SSI) effect on the robustness of multiple tuned mass dampers (MTMD) for vibration control of irregular buildings modelled as torsionally coupled structures (single storey building) due to ground motions by an efficient modal analysis methodology. The performance index of MTMD was established based on the foundation-induced building floor motions with and without the installation of MTMDs. It was concluded from numerical verifications that the increase in height–base ratio of an irregular building and the decrease in relative stiffness of soil to structure generally amplify both SSI and MTMD detuning effect, mainly for a building with highly torsionally coupled effect. Also detuning effect can be reduced with proper increase of the frequency spacing of the optimal MTMDs. Result also showed that if the SSI effect is significant, the MTMD is more effective than single TMD.

Hoang and Warnitchai [2005] developed a new method to design multiple tuned mass dampers (multiple TMDs) to reduce excessive vibration of structures using a numerical optimizer that follows the Davidon–Fletcher–Powell algorithm which can handle large number of design variables without any restriction before the analysis. The method was used to design multiple TMDs for SDOF lumped-mass structures subjected to wide-band excitation. It showed that the optimally designed multiple TMDs have distributed natural frequencies and distinct damping ratios at low damping level. It was concluded that, in case of uncertainties in the structural properties; increasing the TMD damping ratios along with enlarging the TMD frequency range make the system more robust. It is also mandatory to design TMDs for higher damping ratios and a narrower frequency range if TMD parameters themselves are uncertain.

Li and Qu [2006] studied the effectiveness of multiple tuned mass dampers (MTMD) with identical stiffness and damping coefficient but different mass to reduce translational and torsional responses for two-degree-of-freedom (2DOF) structure (which represents the dynamic characteristic of a general asymmetric structure) using numerical simulation. The 2DOF structure was a modelled as a 2DOF system of an asymmetric structure with prevalent translational and torsional responses under earthquake excitations using the mode reduced-order method. From the study it was concluded that MTMD is capable of reducing the torsional response of the torsionally flexible structures and the translational and torsional responses of the torsionally stiff structures.

Han and Li [2006] investigated the vibration control capacity of active multiple tuned mass damper (AMTMD) with identical stiffness and damping coefficient but varying mass and control force. A three storey steel structure model with three ATMDs which was subjected to several historical earthquakes implemented in SIMULINK. During numerical simulation, a stiffness uncertainty of 15% of its initial stiffness of the structure was considered. The optimization ATMD parameters were done in frequency domain by minimization of the minimum value of the maximum dynamic magnification factor for general structure. From numerical result it was concluded that AMTMD has better effectiveness than a single ATMD for structure subjected to historical earthquake and also in structure where there is a stiffness uncertainties of 15%.

Li and Ni [2007] studied a gradient-based method for optimizing non-uniformly distributed multiple tuned mass dampers (MTMD) and their effectiveness on a single degree of freedom of system. The main objective of optimization was to reduce the maximum displacement or frequency response of the main rather than the root-mean-square response. It was concluded that the effectiveness of optimal non-uniformly distributed MTMD is better than the optimal uniformly distributed MTMDs whose frequency spacing, stiffness or mass and damping sometimes has restrictions for simplicity. Due to the flexibility of the proposed method, other errors of estimate can be taken into account easily.

Guo and Chen [2007] speculated the reverberation matrix method (RMM) to perform dynamic analysis of space structures with multiple tuned mass dampers (MTMD). Theory of generalized inverse matrix was used to find the frequency of structures with and without damping along with the resonant frequency. To evaluate the medium and long time response of structures, the artificial damping technique was utilised. The proposed method was used for finding free vibration, frequency response, and transient response of structures (a continuous beam and a two storey space frame) with MTMD under harmonic load. To validate the proposed method both the example were solved numerically by ANSYS software. Numerical results showed that the use of MTMD can effectively modify the distribution of natural frequencies as well as decrease the frequency/transient responses of the structure. It was also found that as the element numbers in the structure increases the FEM (ANSYS) results approach to that of the RMM result.

Han and Li [2008] estimated the performance of general linearly distributed parameter-based multiple-tuned mass dampers (LDP-MTMD) with respect to the MTMD with identical damping coefficient and damping ratio but unequal stiffness and uniform distribution of masses (UM-MTMD3) on single degree freedom system. The optimization criterion was considered to minimize the minimum values of the maximum dynamic magnification factors of structures with four LDP-MTMD models. It was concluded that it is preferable to select the optimum UM-MTMD3 or the optimum MTMD with identical stiffness and damping coefficient but unequal mass and uniform distribution of natural frequencies. It was also found that optimum tuning frequency ratios of both general LDP-MTMD and UM-MTMD3 are close to each other.

Zuo [2009] studied the characteristics and optimization of a new type of TMD system, in which multiple TMDs are connected in series to the main structure. The parameters of spring stiffness and damping coefficients were optimized for mitigation of random and harmonic vibration. It was concluded that series multiple TMDs are more effective, robust and less sensitive to the parametric variation of the main structure than all the other types of parallel MTMDs and single TMD of the same mass ratio. It was also found that a series of two TMDs of total mass ratio of 5% can appear to have 31–66% more mass than the classical TMD, and it performs better than the ten TMDs in optimal parallel of the same total mass ratio.

2.2 AIM AND SCOPE OF THE PRESENT WORK

The aim of the present work is to study numerically the effect of TMD either single or multiple on the dynamic response of multi-storey frame structures.

It is proposed to model a 3D frame building as multi degree of freedom shear building (1D) as well as frame building (2D). TMD is modelled as 1D which will respond to horizontal translation only. Finite element method has been used as numerical tool to study the dynamic response of frame-TMD system. Linear time history analysis of multi-storey frame with and without TMD under sinusoidal and four past earthquake ground accelerations is carried out.

CHAPTER 3

FINITE ELEMENT FORMULATIONS

3.1 ELEMENT MATRIX OF PLANE FRAME IN LOCAL COORDINATE SYSTEM

3.1.1 Mass matrix

The mass matrix of individual elements is formed in local direction then it is transformed to global direction and finally it is substituted into main equation. The inertial property or mass of a structure in dynamic analysis can be taken by two methods.

i) Lumped mass

This is the simplest method for considering the inertial properties of the structure, where it is assumed that the mass of the structure is lumped at the nodal coordinate corresponding translation displacement. In this method inertial component associated with any rotational degree of freedom is considered as zero. This type of mass matrix is generally taken if the given structure is modelled as shear building. The lumped mass matrix is diagonal. The lumped mass matrix for a beam element is given below.

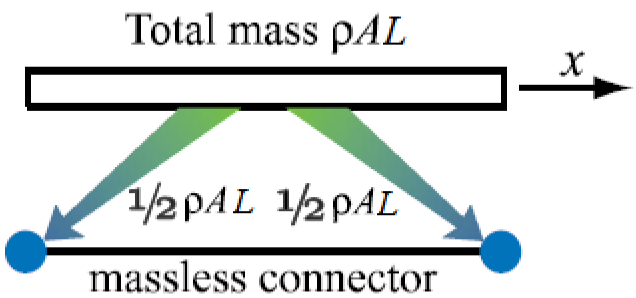


Fig. 3.1 Lumped mass for beam element

$$m_L^e = \frac{1}{2}\rho A L\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{3.1}$$

ii) Consistent mass

In this method inertial component associated with any rotational degree of freedom is considered. The mass matrix is symmetric but not diagonal. The consistent mass matrix for a beam element is given below.

$$[m^e] = \frac{\rho\, A\, L}{420}\begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 156 & 22L & 0 & 54 & -13L \\ 0 & 22L & 4L^2 & 0 & 13L & -3L^2 \\ 70 & 0 & 0 & 140 & 0 & 0 \\ 0 & 54 & 13L & 0 & 156 & -22L \\ 0 & -13L & -3L^2 & 0 & -22L & 4L^2 \end{bmatrix} \tag{3.2}$$

Where

ρ = Density of the beam material

3.1.2 Stiffness matrix

The stiffness matrix is also symmetric matrix. The elemental stiffness matrix for a beam or a frame element considering axial deformation is given below.

$$[k^e] = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & \frac{-EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & \frac{-12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & \frac{6EI}{L^2} & \frac{2EI}{L} \\ \frac{-EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & \frac{-12EI}{L^3} & \frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \tag{3.3}$$

Where,

E = Young’s Modulus of the frame material.

A = Cross sectional area of the element.

L = Length of the element.

3.2 ELEMENT MATRIX OF PLANE FRAME IN GLOBAL COORDINATE SYSTEM

The matrices formulated in the above section are for a particular element in local coordinate system (along the length of each element). A frame element consists of number of node and element. Hence each element matrix will vary according to its local axes orientation. To assemble the matrices each element matrix is transformed to global coordinate system. It is clear that the plane frame element has six degree of freedom – three at each node (two displacements and a rotation). The sign convention used is that displacements are positive if they point upwards and rotations are positive if they are counter clockwise. Consequently for a structure with n nodes, the global stiffness and mass matrix ([K], [M]) will be 3n X 3n (since it has three degrees of freedom at each node). The global stiffness and mass matrix ([K], [M]) is formed by assembling the transformed elemental stiffness and mass matrix ([K^e], [M^e]) by making calls to the MATLAB function PlaneFrameAssemble which is written specially for this purpose.

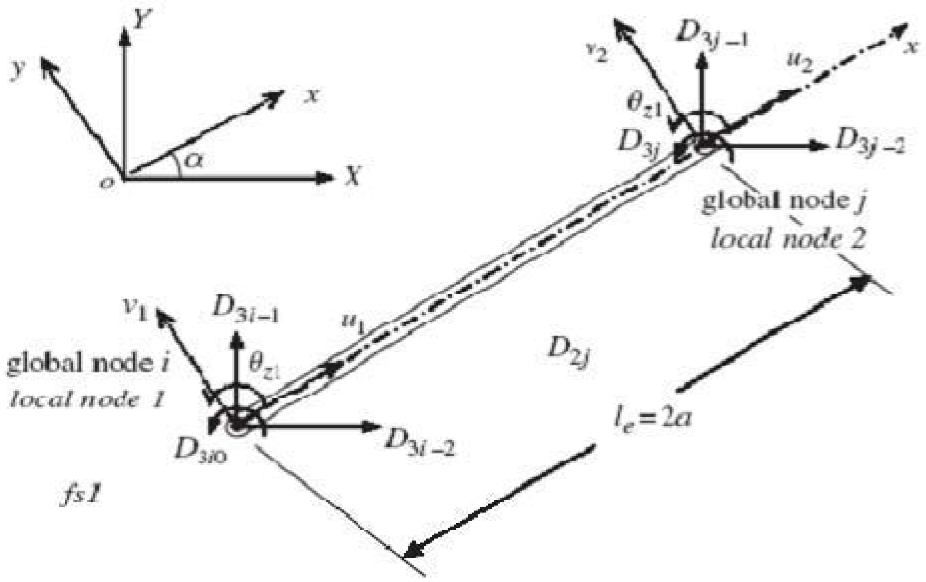


Fig.3.2. Co-ordinate transformation for 2D frame elements

In the fig.the local and global nodes are 1,2and i, j respectively. Similarly local and global axes are x, y and X, Y respectively.

Let T be the transformation matrix and C=Cosα, S=Sinα for the frame element, which is given by

$$[T] = \begin{bmatrix} C & S & 0 & 0 & 0 & 0 \\ -S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C & S & 0 \\ 0 & 0 & 0 & -S & C & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3.4)$$

Using the transformation matrix, [T] the matrices for the frame element in the global coordinate system become

$$[K^e] = [T^T][k^e][T] \qquad (3.5)$$

$$[M^e] = [T^T][m^e][T] \qquad (3.6)$$

3.3 DYNAMIC EQUILIBRIUM EQUATION OF STRUCTURE

The dynamic response of a structure at any instant of time t under an excitation force is defined by its displacement u(t), velocity $\dot{u}(t)$ and acceleration $\ddot{u}(t)$. The total force acting on a structure is resisted by its inertia $F(t)_I$, damping $F(t)_D$ and stiffness $F(t)_S$ component of reactive force. The force equilibrium equation of a structure at any instant of time of t, subjected to dynamic load F(t) can be expressed by the following equation

$$F(t)_I + F(t)_D + F(t)_S = F(t) \tag{3.7}$$

Where,

$$F(t)_I = m.\ddot{u}(t) \tag{3.8}$$

$$F(t)_D = c.\dot{u}(t) \tag{3.9}$$

$$F(t)_S = k.\ u(t) \tag{3.10}$$

Where, m= mass of the system.

c = damping of the system

k= stiffness of the system

For multi degree of freedom system corresponding equation of motion become

$$[M]\ddot{U}(t) + [C]\dot{U}(t) + [K]U(t) = \{F(t)\} \tag{3.11}$$

Where, [M] = The global mass matrix of structure.

[C] = The global damping matrix of the structure.

[K] = The global stiffness matrix of the structure.

U(t) = Absolute nodal displacement.

$\dot{U}(t)$ = Absolute nodal velocity.

$\ddot{U}(t)$ = Absolute nodal acceleration.

F(t) = Force vector. (For earthquake loading $F(t) = -[M].\ddot{U}_g(t)$)

$\ddot{U}_g(t)$ = Ground acceleration due to earthquake.

The effect of TMD can be considered by adding extra opposite nature force to forcing function.

3.4 DYNAMIC ANALYSIS OF STRUCTURE

The main three factors that govern the particular type of analysis process to be applied to structural depend upon the type of externally applied loads, the behaviour of the structure/or structural materials and the type of structural model selected. Dynamic analysis is two types, linear and nonlinear analysis. The building frame has been analysed by linear time history analysis.

If non-linear behaviour is not involved in structure, the linear time history analysis is the best method to find out the response of a structure than any other method. In this method the response of a structure is find out at discrete time interval which require a great computational effort. Another interesting advantage of this method is that the relative values of response quantities are saved in the response histories.

3.4.1 Steps for the dynamic analysis of 2D frame

1. Discretising the domain: Dividing the each element into number of small part connected by nodes and numbering them globally.
2. Formulation of the Element matrices: The element or local stiffness and mass matrix is found for all elements, which is symmetric of size 6×6.

3. Assembling the global stiffness matrices: The element stiffness matrices are then transformed to global coordinate system and combined globally based on their degrees of freedom values.
4. Applying the boundary condition: The boundary condition is applied by suitably deleting the rows and columns of the mass and stiffness matrix corresponding zero force or displacements.
5. Solving the equation: The equation is solved in MATLAB to get the value of U by using Newmark's Beta method.

3.5 SOLUTION OF DYNAMIC EQUILIBRIUM EQUATION BY NUMERICAL INTEGRATION

The analytical solution of the dynamic equilibrium equation of the structure is not possible if the applied force F(t) or ground acceleration $\ddot{U}_g(t)$ varies arbitrarily with time or the system is nonlinear. The most general approach to tackle such problem is the direct numerical integration of the dynamic equilibrium equations for each time step. There are various numerical integration methods for solution of differential equation. All approaches can fundamentally be classified as either explicit or implicit integration methods. Most methods use equal time intervals at Dt, 2Dt, 3Dt........NDt.

3.5.1 Newmark's Beta method

Here Newmark's Beta method has been used for solution of differential equations. Because of its general versatility, it has been adopted into numerous commercially available computer programs for purposes of structural dynamics analysis. Newmark's equations are given by

$$\underline{\dot{d}}_{i+1} = \underline{\dot{d}}_i + (\Delta t)\left[(1-\gamma)\underline{\ddot{d}}_i + \gamma\underline{\ddot{d}}_{i+1}\right] \tag{3.12}$$

$$\underline{\dot{d}}_{i+1} = \underline{\dot{d}}_i + (\Delta t)\underline{\dot{d}}_i + (\Delta t)^2\left[\left(\frac{1}{2}-\beta\right)\underline{\ddot{d}}_i + \beta\underline{\ddot{d}}_{i+1}\right] \tag{3.13}$$

Where, β and γ are parameters chosen by the user. The parameter β is generally chosen between 0 and ¼, and γ is often taken to be ½. For instance, choosing γ = ½ and β = 1/6, are chosen, eq. 3.12 and eq. 3.13 correspond to those for which a linear acceleration assumption is valid within each time interval. For γ = ½ and β = ¼, it has been shown that the numerical analysis is stable; that is, computed quantities such as displacement and velocities do not become unbounded regardless of the time step chosen.

To find $\underline{d}_{i+1}$, first multiply eq. 3.13 by the mass matrix [M] and then substitute the value of $\ddot{\underline{d}}_{i+1}$ into this eq. to obtain

$$\underline{[\mathrm{M}]}\,\ddot{\underline{d}}_{i+1} = \underline{[\mathrm{M}]}\,\underline{d}_i + (\Delta t)\underline{[\mathrm{M}]}\,\dot{\underline{d}}_i + (\Delta t)^2\underline{[\mathrm{M}]}\left(\frac{1}{2} - \beta\right)\ddot{\underline{d}}_i + \beta(\Delta t)^2\left[\underline{F}_{i+1} - \underline{[\mathrm{K}]}\,\underline{d}_{i+1}\right] \quad (3.14)$$

Combining the like terms of eq. 3.14 we obtain

$$\left(\underline{[\mathrm{M}]} + \beta(\Delta \mathrm{t})^2\underline{[\mathrm{K}]}\right)\underline{d}_{i+1} = \beta(\Delta t)^2\underline{F}_{i+1} + \underline{[\mathrm{M}]}\underline{d}_i + (\Delta t)\underline{[\mathrm{M}]}\dot{\underline{d}}_i + (\Delta t)^2\underline{[\mathrm{M}]}\left(\frac{1}{2} - \beta\right)\ddot{\underline{d}}_i \quad (3.15)$$

Finally, dividing above eq. by $\beta(\Delta t)^2$, it is obtained

$$\underline{K'}\underline{d}_{i+1} = \underline{F'}_{i+1} \quad (3.16)$$

$$\underline{K'} = \underline{[\mathrm{K}]} + \frac{1}{\beta(\Delta t)^2}\underline{[\mathrm{M}]} \quad (3.17)$$

$$\underline{F'}_{i+1} = \underline{F}_{i+1} + \frac{\underline{[\mathrm{M}]}}{\beta(\Delta t)^2}\left[\underline{d}_i + (\Delta t)\dot{\underline{d}}_i + \left(\frac{1}{2} - \beta\right)(\Delta t)^2\ddot{\underline{d}}_i\right] \quad (3.18)$$

The solution procedure using Newmark's method is as follows:

1. Starting at time t=0, $\underline{d}_0$ is known from the given boundary conditions on displacement, and $\dot{\underline{d}}_0$ is known from the initial velocity conditions.
2. Solve eq. at t=0 for $\ddot{\underline{d}}_0$ (unless $\ddot{\underline{d}}_0$ is known from an initial acceleration condition); that is,

$$\underline{\ddot{d}}_0 = \underline{[M]}^{-1}(\underline{F}_0 - \underline{K}\underline{d}_0)$$

3. Solve eq. 3.16 for $\underline{d}_1$, because $\underline{F'}_{i+1}$ is known for all time steps and , $\underline{d}_0$, $\underline{\dot{d}}_0$, $\underline{\ddot{d}}_0$ are known from steps 1 and 2.
4. Use eq. 3.13 to solve for $\underline{\ddot{d}}_1$ as

$$\underline{\ddot{d}}_1 = \frac{1}{\beta(\Delta t)^2}\left[\underline{d}_1 - \underline{d}_0 - (\Delta t)\underline{\dot{d}}_0 - (\Delta t)^2\left(\frac{1}{2} - \beta\right)\underline{\ddot{d}}_0\right]$$

5. Solve eq. 3.12 directly for $\underline{\dot{d}}_1$
6. Using the results of steps 4 and 5, go back to step 3 to solve for $\underline{d}_2$ and then to steps 4 and 5 to solve for $\underline{\ddot{d}}_2$ and $\underline{\dot{d}}_2$. Use steps 3-5 repeatedly to solve for $\underline{d}_{i+1}$, $\underline{\dot{d}}_{i+1}$ and $\underline{\ddot{d}}_{i+1}$.

CHAPTER-4

RESULTS AND DISCUSSION

4.1 RANDOM EARTHQUAKE GROUND ACCELEROGRAM

Total four numbers of past random accelerogram named EW component of 1940 El-Centro earthquake (PGA=0.2144g) fig. 4.4.(a), compatible time history as per spectra of IS-1893 (Part -1):2002 for 5% damping at rocky soil (PGA=1.0g) fig. 4.4.(b), Sakaria earthquake (PGA=1.238g) fig. 4.4.(c), The Landers earthquake (1992) (PGA=1.029g) fig. 4.4.(d) are taken into consideration for time history analysis of the proposed 1D shear building and 2D frame building model with and without Single and multiple TMD .

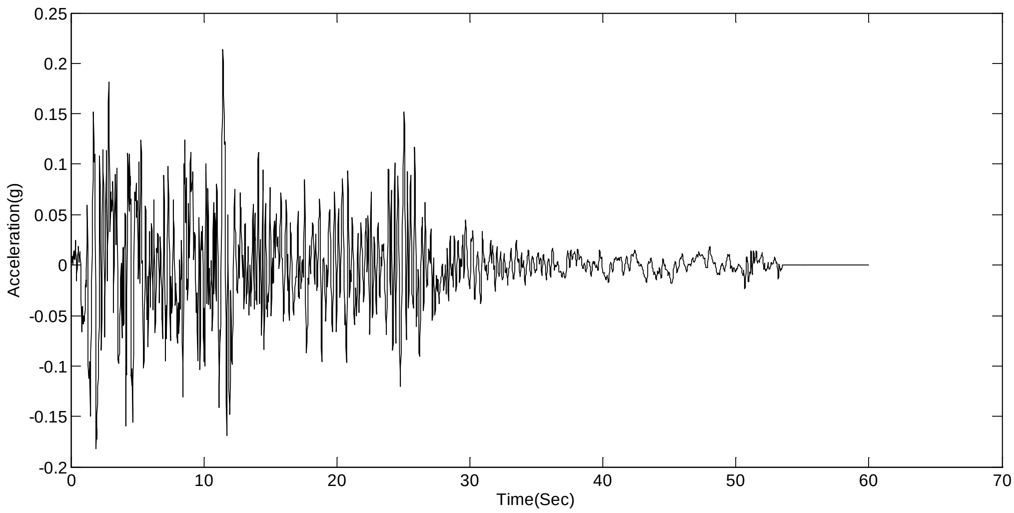


Fig. 4.1.(a): EW component of El-Centro earthquake accelerogram (1940)

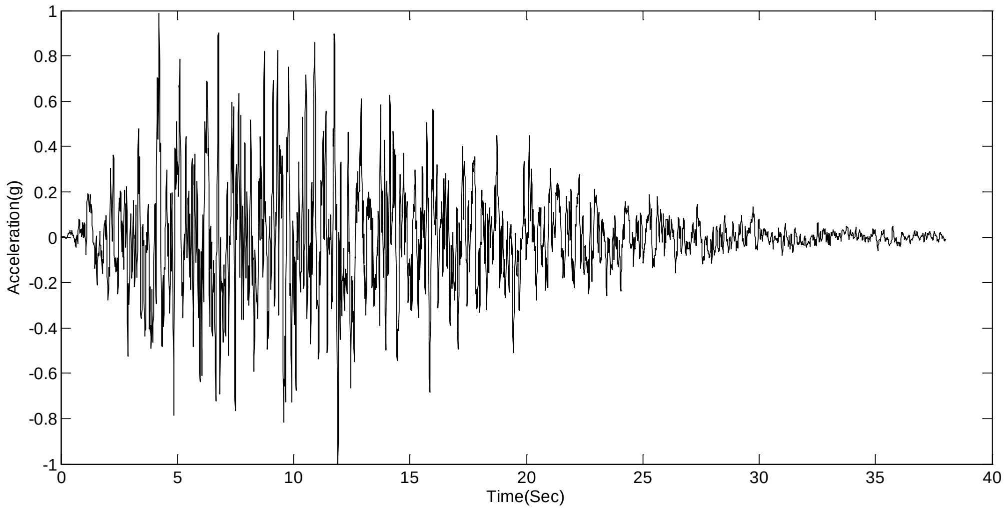


Fig. 4.1.(b): Compatible Earthquake ground acceleration time history as per spectra of IS-1893 (Part -1):2002 for 5% damping atrocky soil

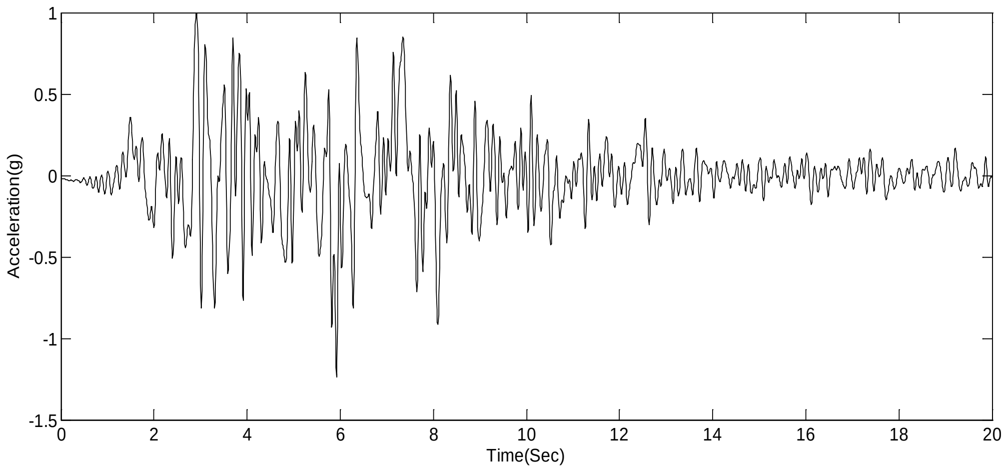


Fig. 4.1.(c): Sakaria earthquake accelerogram

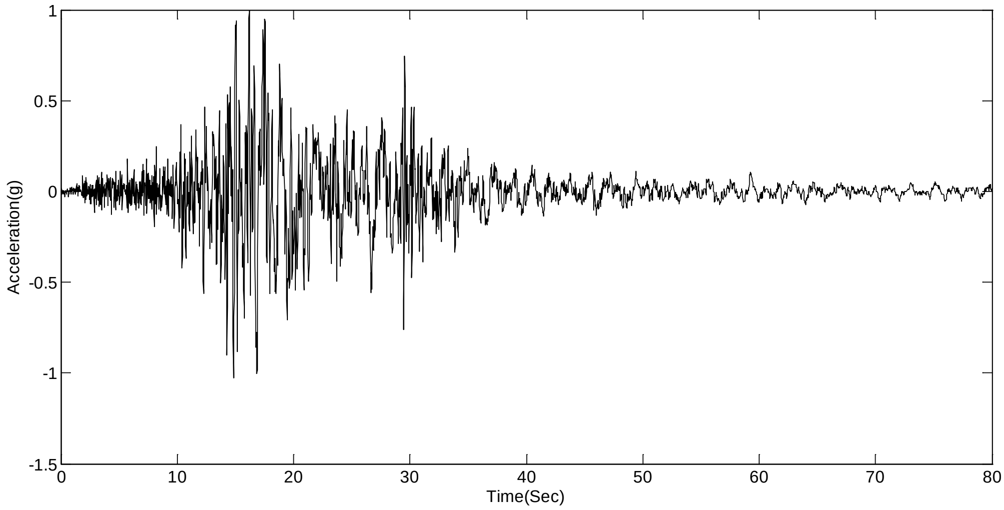
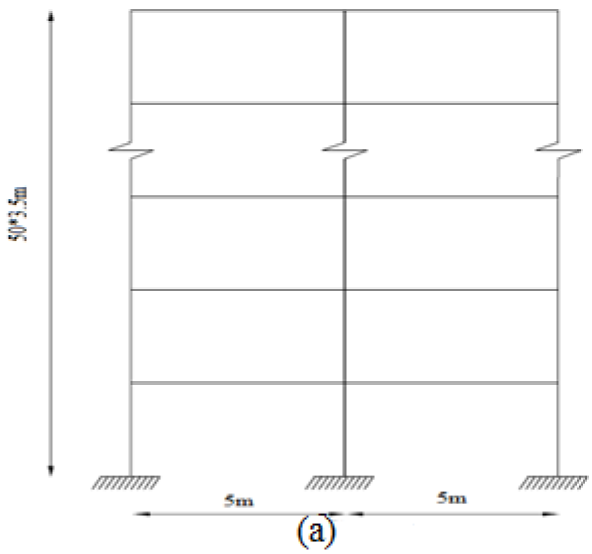


Fig. 4.1.(d): The Landers earthquake accelerogram (1992)

4.2 1D SHEAR BUILDING MODEL



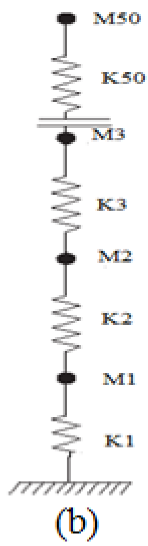


Fig.4.2.(a): Actual building frame

Fig. 4.2.(b): Idealized shear building

Plane concrete building frame can be idealized as shear building, which is modeled as one dimensional multi-degree of freedom system with one degree of freedom at each node. It is assumed that the axial stiffness of the beam at the floor level is very high so there will be no rotation at the floor level between any beam column joint. The nomenclature came from the reason of presence of constant shear force across the height of the column. Whole the shear building is discretised as 50 number of element. The preliminary dimension of the frame, member size and material properties are given below.

Total height of the building = 175 m

Height of each floor = 3.5 m

Each bay width = 5 m

Number of storey =50

Number of bay =2

Size of beam = (0.25×0.35) m

Size of column = (0.3×0.5) m

Grade of concrete = M_{20}

Modulus of elasticity = 22360.6×10^6 N/m^2

Total mass of shear building = 304280 kg

First natural frequency = 3.0637 rad/s

4.3 LINEAR TIME HISTORY ANALYSIS OF SHEAR BUILDING WITH AND WITHOUT SINGLE TMD

A comparison study is done on the effectiveness of single tuned mass damper for vibration control by linear time history analysis of shear building under a sinusoidal load and the above mentioned four numbers past earthquake data. The response is calculated in term of displacement at the top floor with and without single TMD. The damping ratio of the shear building as well as damper is taken as 0.05 for every mode. In each case fundamental frequency of the building without TMD is tuned to the frequency of the damper.

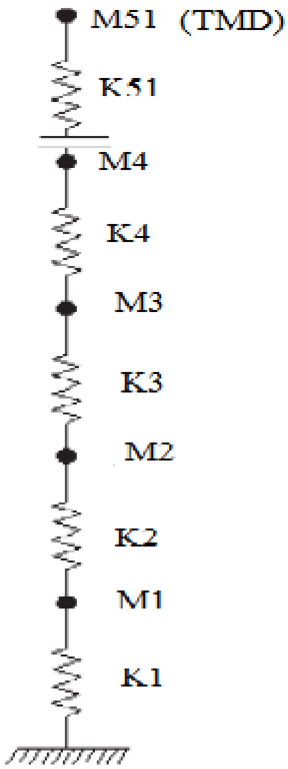


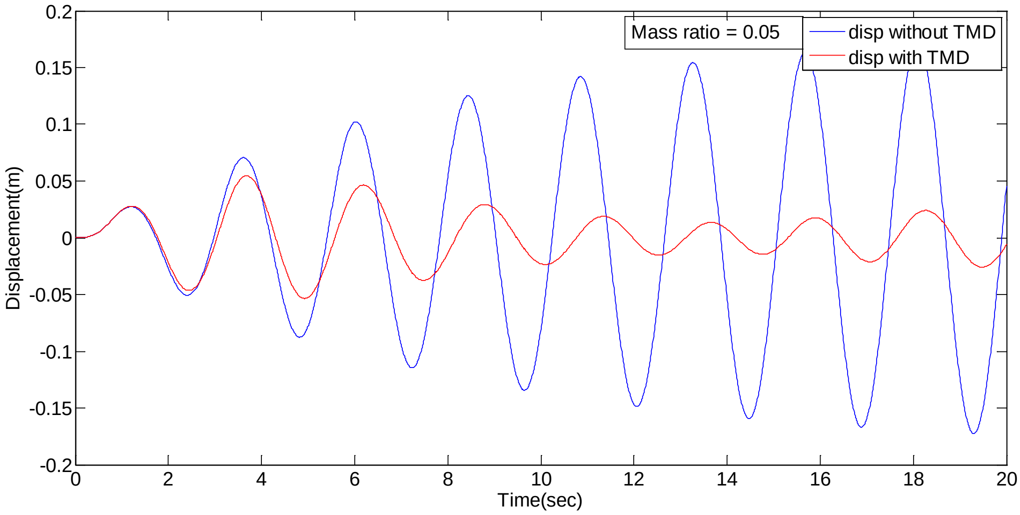
Fig. 4.3: Idealized shear building with TMD

4.3.1 Effect of TMD mass ratio variation on the response of the shear building

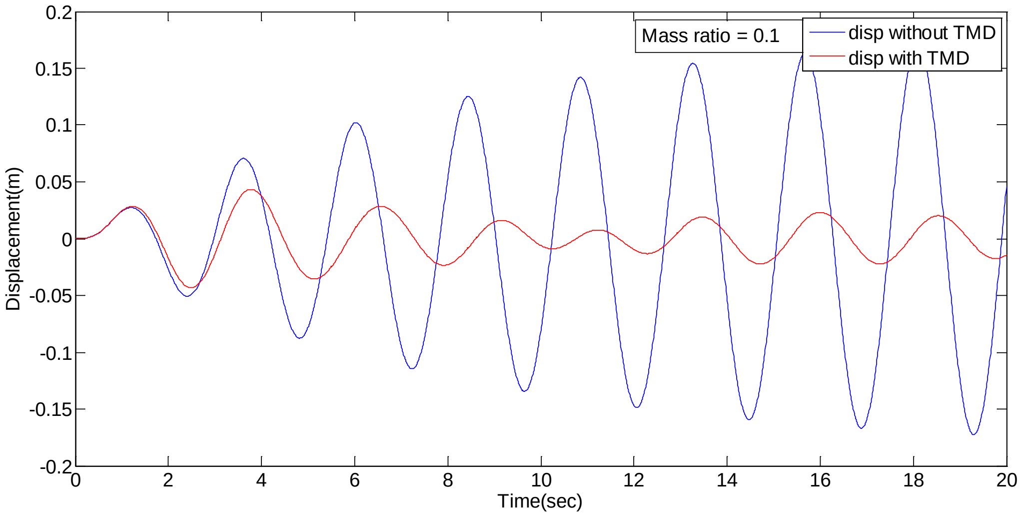
Two different mass ratios of 0.05 and 0.1 are taken in analysis.

(a) Sinusoidal acceleration

First case the shear building is subjected to sinusoidal acceleration $\ddot{A}=A_{max}\sin(\omega.t)$ at ground. Where, A_{max} and ω are the maximum amplitude of acceleration and frequency of the sinusoidal acceleration respectively. The parameters A_{max} and ω are 0.1 m/s^2 and 3.0637 rad/s (considering resonance condition) respectively.



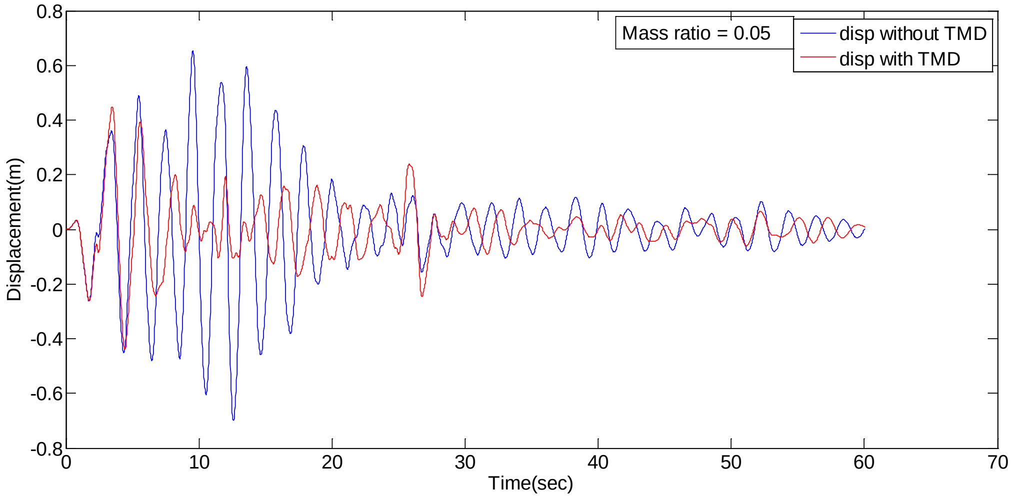
(a)



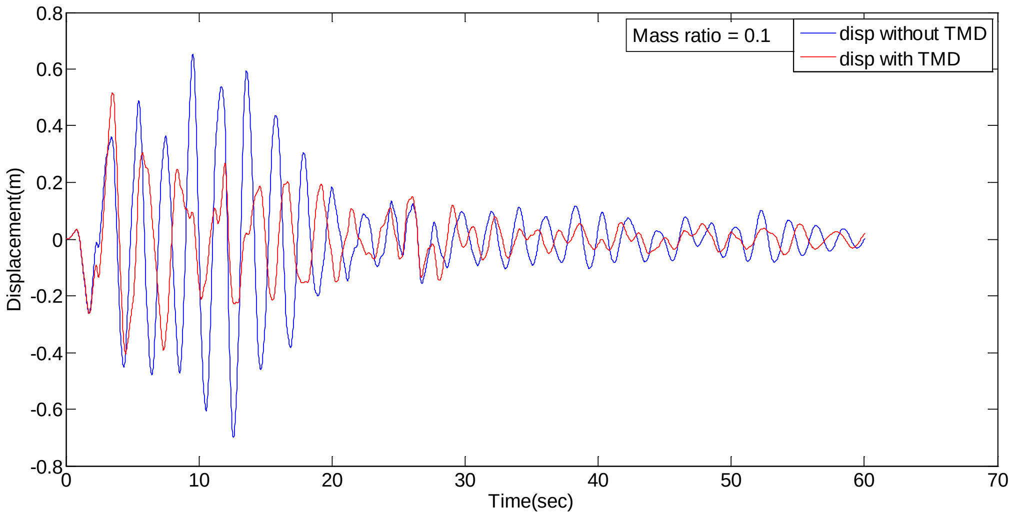
(b)

Fig. 4.4: Displacement of the shear building with and without single TMD at 50th floor under sinusoidal ground acceleration. For (a) Mass ratio 0.05, (b) Mass ratio 0.1

(b) Random earthquake ground acceleration

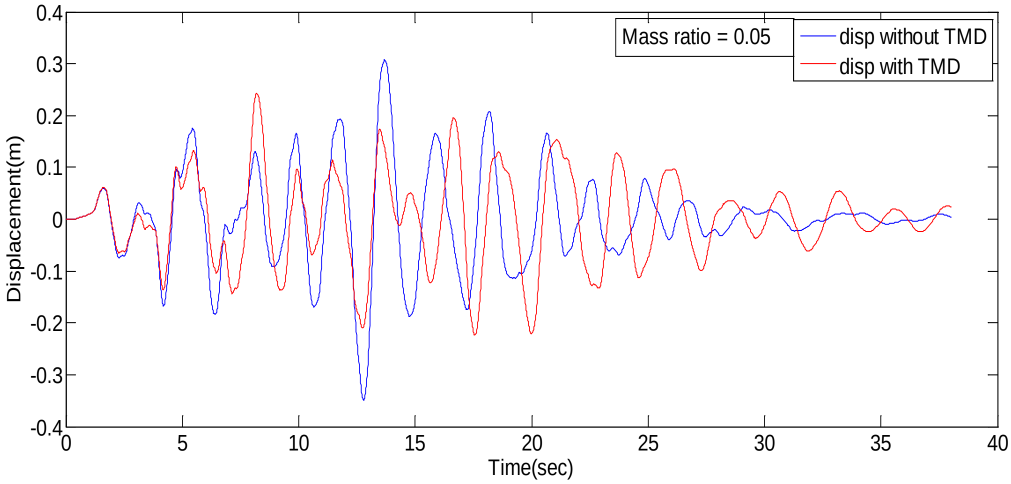


(a)

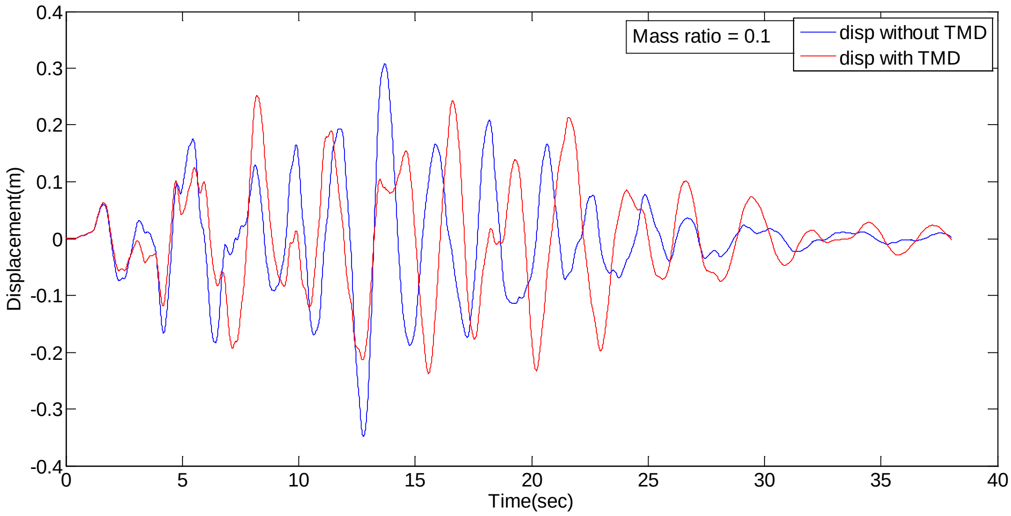


(b)

Fig. 4.5: Displacement of the shear building with and without single TMD at 50th floor under EW component of 1940 El-Centro earthquake. For (a) Mass ratio 0.05, (b) Mass ratio 0.1

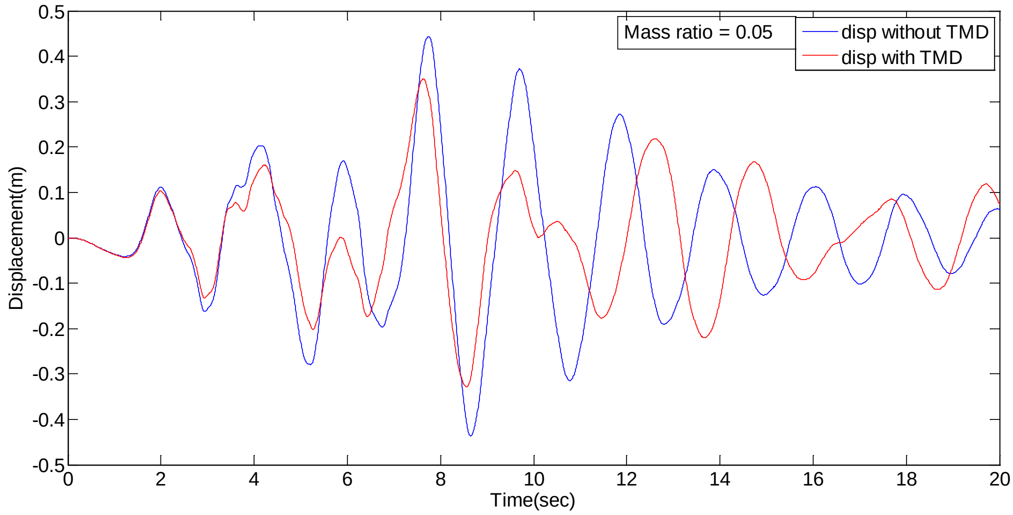


(a)

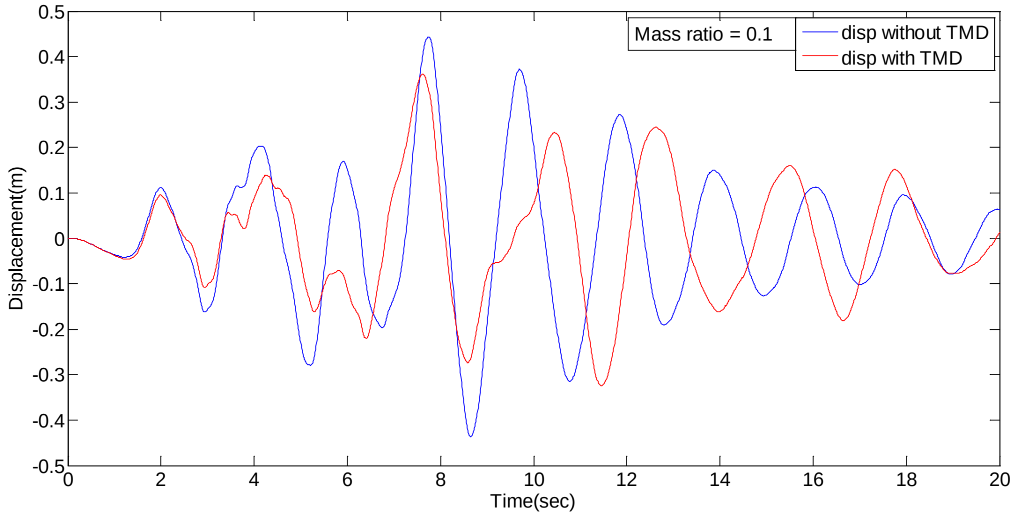


(b)

Fig. 4.6: Displacement of the shear building with and without single TMD at 50th floor under Compatible time history as per spectra of IS-1893 (Part -1):2002 for 5% damping at rocky soil. For (a) Mass ratio 0.05, (b) Mass ratio 0.1

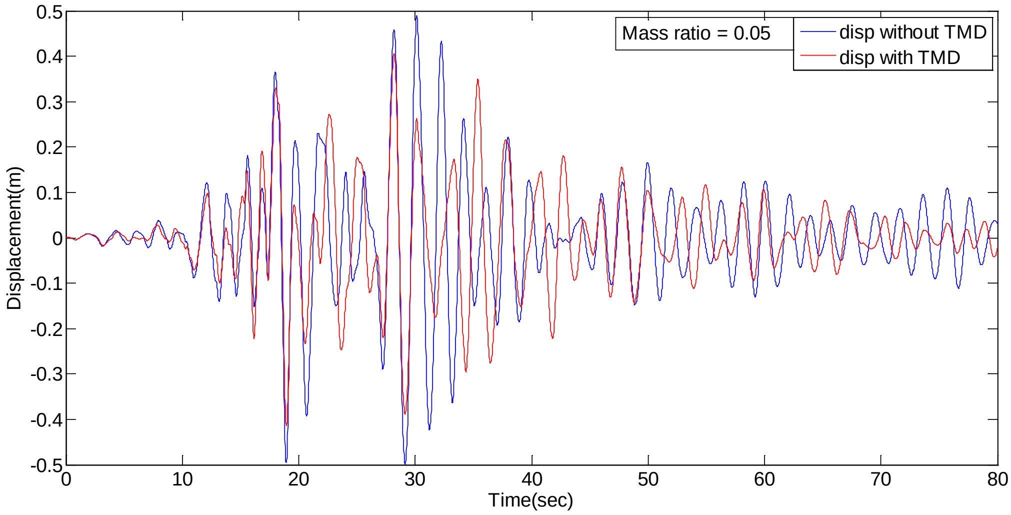


(a)

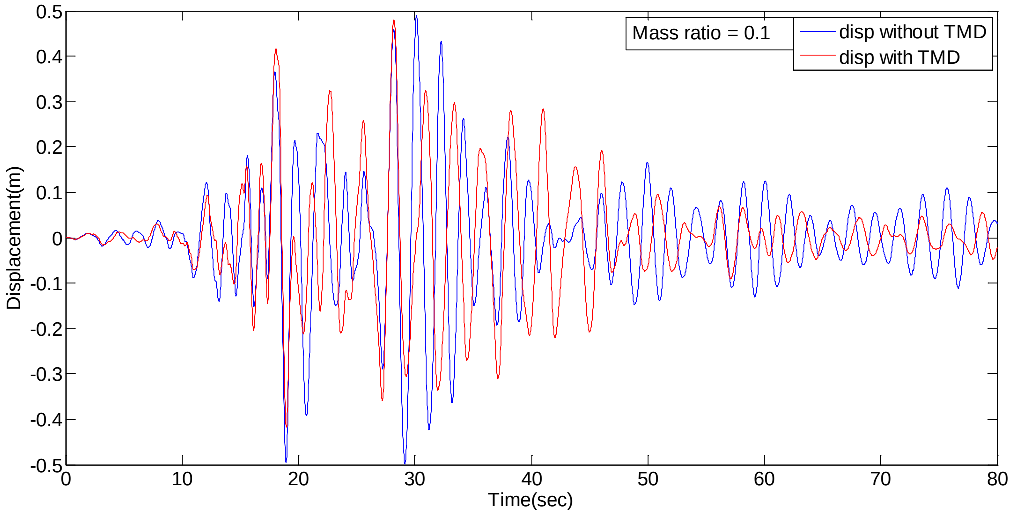


(b)

Fig. 4.7: Displacement of the shear building with and without single TMD at 50th floor under Sakaria earthquake. For (a) Mass ratio 0.05, (b) Mass ratio 0.1



(a)



(b)

Fig. 4.8: Displacement of the shear building with and without single TMD at 50th floor under the Landers earthquake 1992. For (a) Mass ratio 0.05, (b) Mass ratio 0.1

Table 4.1

Comparison study on the Maximum displacement (m) at the top floor of the shear building with and without single TMD (with variation of mass ratio)

Type of loading	without single TMD **(A)**	with TMD of mass ratio 0.05 **(B)**	with TMD of mass ratio 0.1 **(C)**	**(A-B)**	**(A-B)×100/A**	**(A-C)**	**(A-C)×100/A**	**(B-C)**	**(B-C)×100/B**
Sinusoidal acceleration	0.1288	0.0394	0.0313	0.0894	69.4099	0.0975	75.6988	0.0081	20.5584
El-Centro earthquake accelerogram 1940	0.7001	0.4402	0.4057	0.2599	37.1233	0.2944	42.0511	0.0345	7.8373
spectra of IS-1893 (Part -1):2002 for 5% damping at rocky soil	0.3487	0.2240	0.2376	0.1247	35.7614	0.1111	31.8612	-0.0136	-6.0714
Sakaria earthquake accelerogram	0.4428	0.3503	0.3618	0.0925	20.8898	0.0810	18.2927	-0.0115	-3.2829
The Landers earthquake accelerogram (1992)	0.4988	0.4137	0.4184	0.0851	17.0609	0.0804	16.1187	-0.0047	-1.1361

From the table 4.1 it can be concluded that the effectiveness of a single TMD for vibration suppression of a shear building increases with the increase in mass ratio of the TMD to structure. Maximum percentage of displacement reduction for TMD mass ratio of 0.05 and 0.1 under sinusoidal acceleration is 69.41 and 75.7. As the considered sinusoidal load contain single known frequency it is very easy to tune with the frequency of single TMD. Maximum percentage of displacement reduction for same TMD under earthquake loading is found for El-Centro earthquake 1940. Again maximum increase in percentage of displacement reduction is higher for sinusoidal acceleration than earthquake ground acceleration. But under some earthquake acceleration maximum displacement of the shear building increases in certain amount with increase in TMD mass ratio as shown in table 4.1.

4.4 2D FRAME MODEL

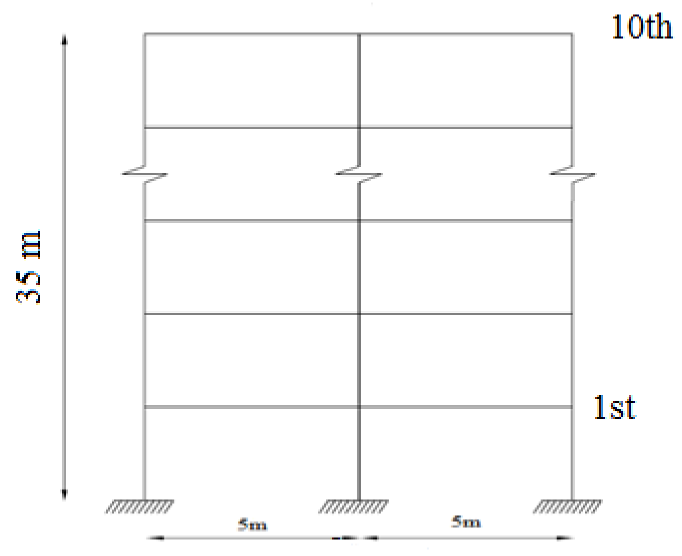


Fig. 4.9: 2D frame model

The frame is now modelled as two dimensional multi degree of freedom systems with each node having three degree of freedom. Whole the frame is discretised into 100 number of element. Basic data of the problem is given below.

Total height of the building = 35 m

Height of each floor = 3.5 m

Each bay width = 5 m

Number of storey = 10

Number of bay =2

Size of beam = (0.25×0.35) m

Size of column = (0.3×0.5) m

Grade of concrete = M_{20}

Modulus of elasticity = 22360.6×10^6 N/m^2

Live load on slab = 3500 N/m^2

Total mass of 2D frame = 61250 kg

First and second natural frequency = 5.5349 rad/s and 17.5789 rad/s

4.5 LINEAR TIME HISTORY ANALYSIS OF 2D FRAME WITH AND WITHOUT SINGLE TMD

The effectiveness of single tuned mass damper for vibration control is studied by linear time history analysis of the frame building under a sinusoidal load and the four numbers past earthquake data. The damping ratio of the frame building as well as damper is taken as 0.05 for every mode. In each case fundamental frequency of the building without TMD is tuned to the frequency of the damper. The response is calculated in term of displacement at the 10th floor.

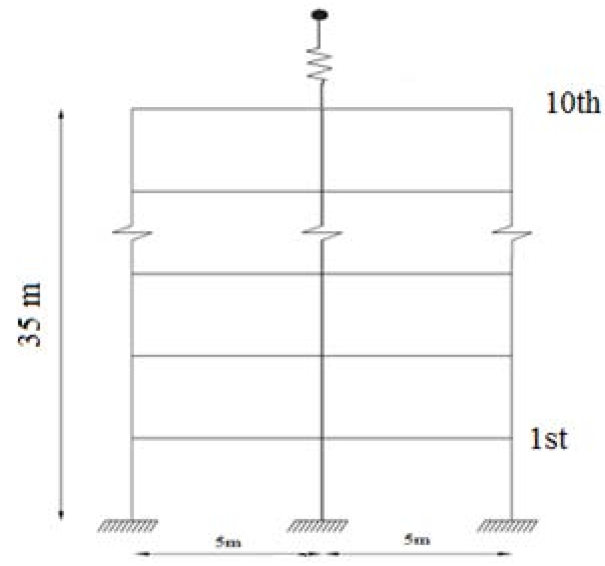
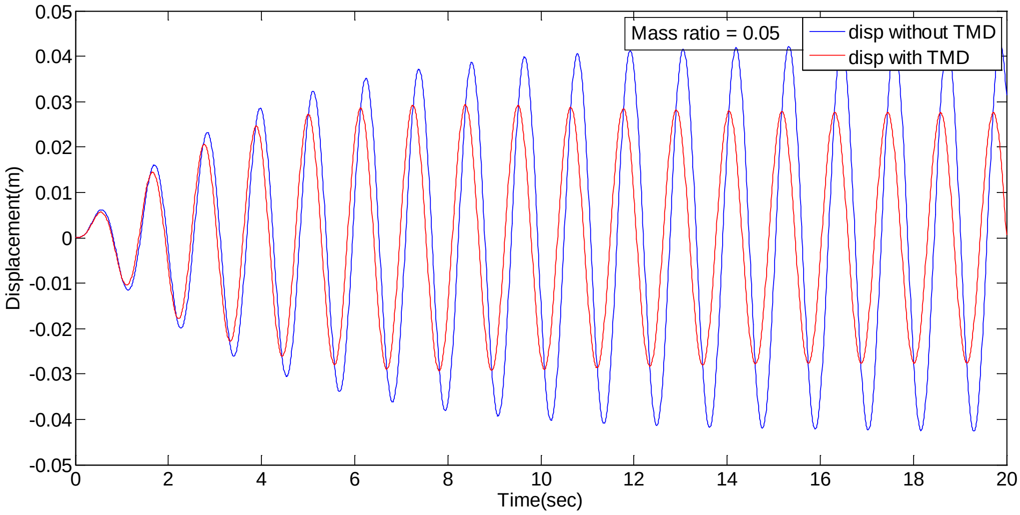


Fig. 4.10:2D frame model with single TMD

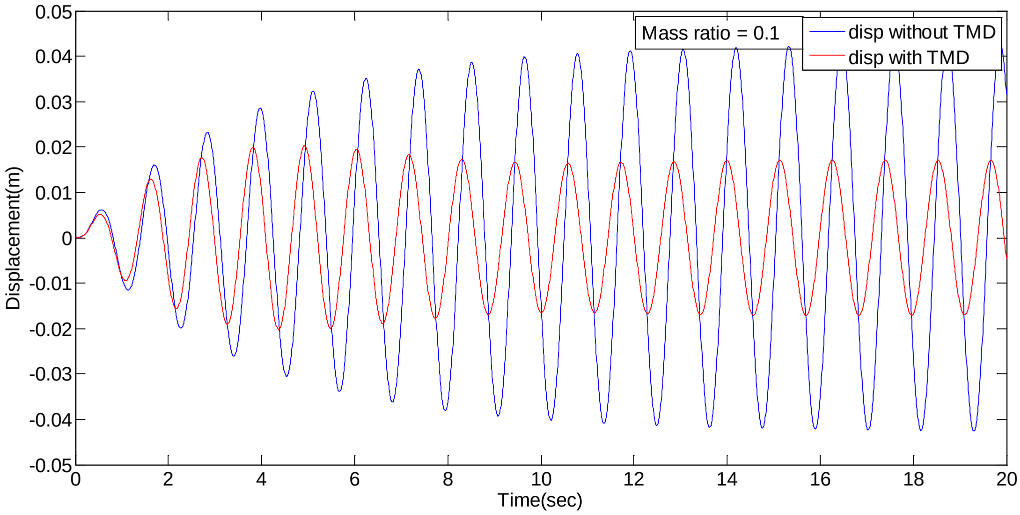
4.5.1 Response of the 2D frame with variation of TMD mass ratio

(a) Sinusoidal acceleration

Two different mass ratios of 0.05 and 0.1 are taken in analysis. Frame building is subjected to sinusoidal acceleration $\ddot{A}=A_{max}\sin(\omega.t)$ at ground. Where, A_{max} and ω are the maximum amplitude of acceleration and frequency of the sinusoidal acceleration respectively. The parameters A_{max} and ω are 0.1 m/s^2 and 5.5349 rad/s (considering resonance condition) respectively.



(a)

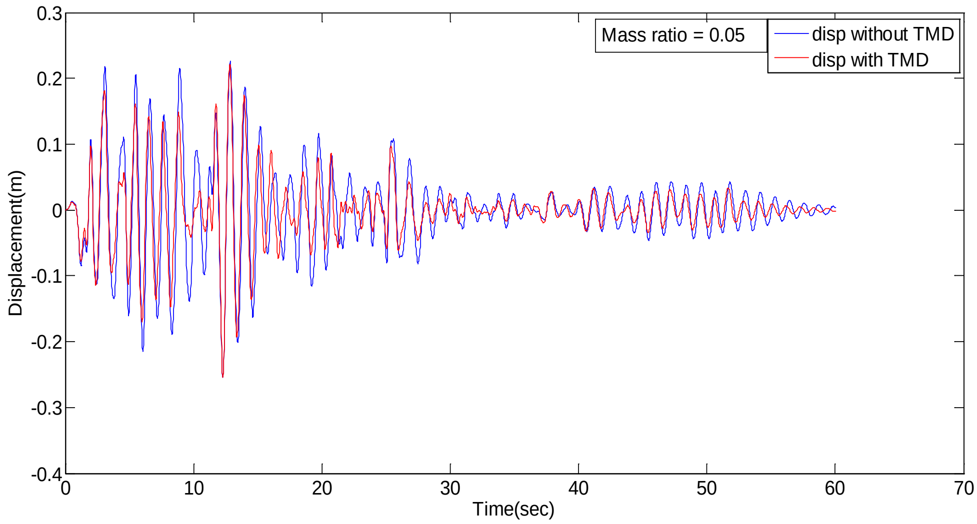


(b)

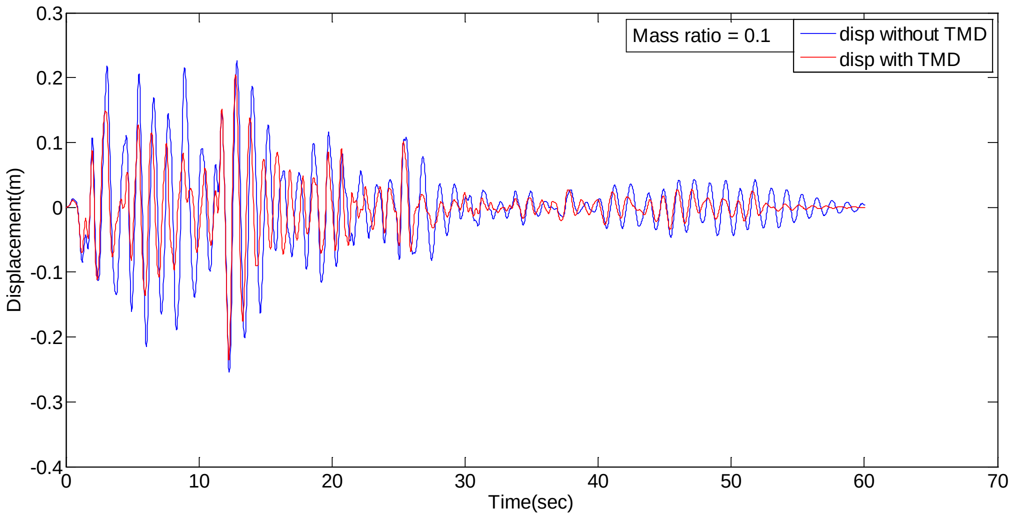
Fig. 4.11: Displacement of the 2D frame with and without single TMD at 10th floor under sinusoidal ground acceleration. For (a) Mass ratio 0.05, (b) Mass ratio 0.1

(b) Random earthquake ground acceleration

Here response of the 2D frame (in term of displacement) calculated with two different mass ratio of 0.05 and 0.1 for the TMD under the above mentioned random earthquake ground acceleration.

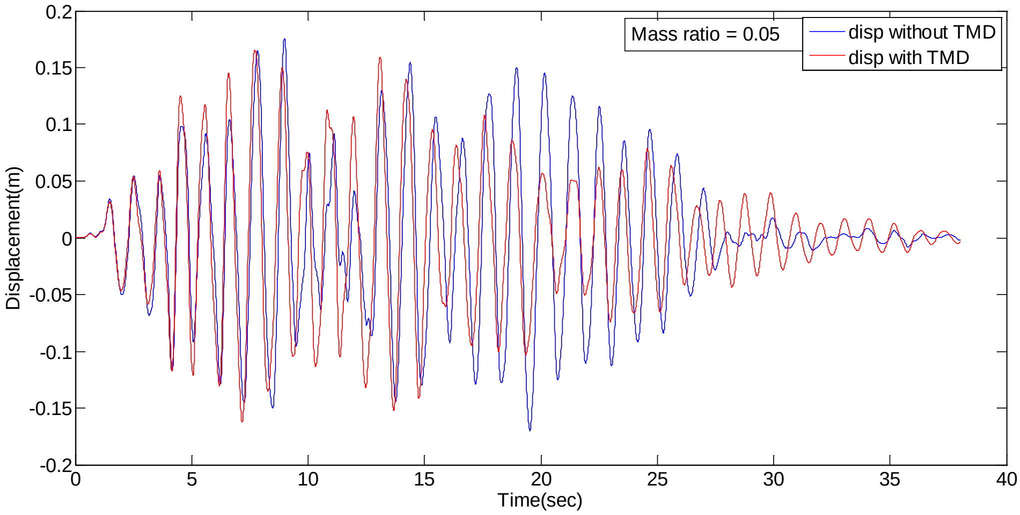


(a)

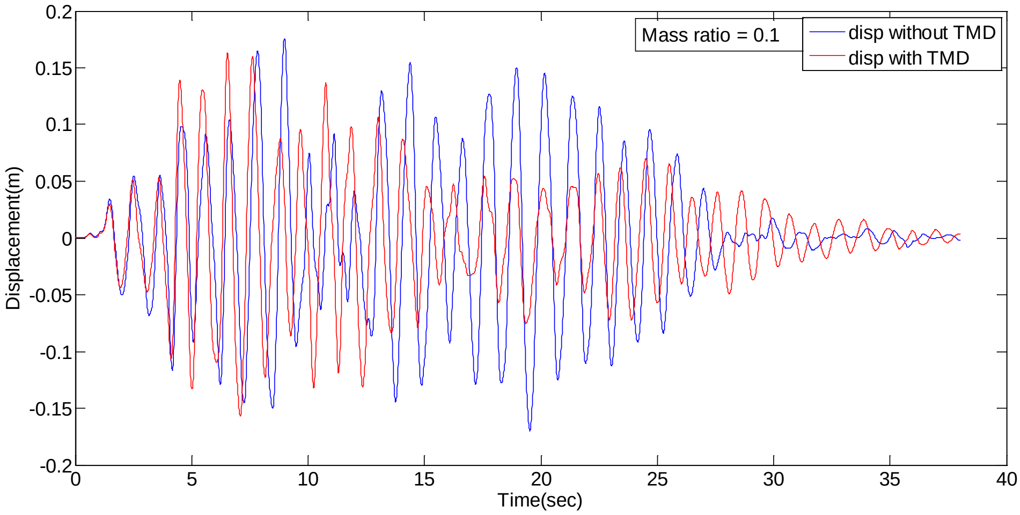


(b)

Fig. 4.12: Displacement of the 2D frame with and without single TMD at 10th floor under EW component of 1940 El-Centro earthquake. For (a) Mass ratio 0.05, (b) Mass ratio 0.1

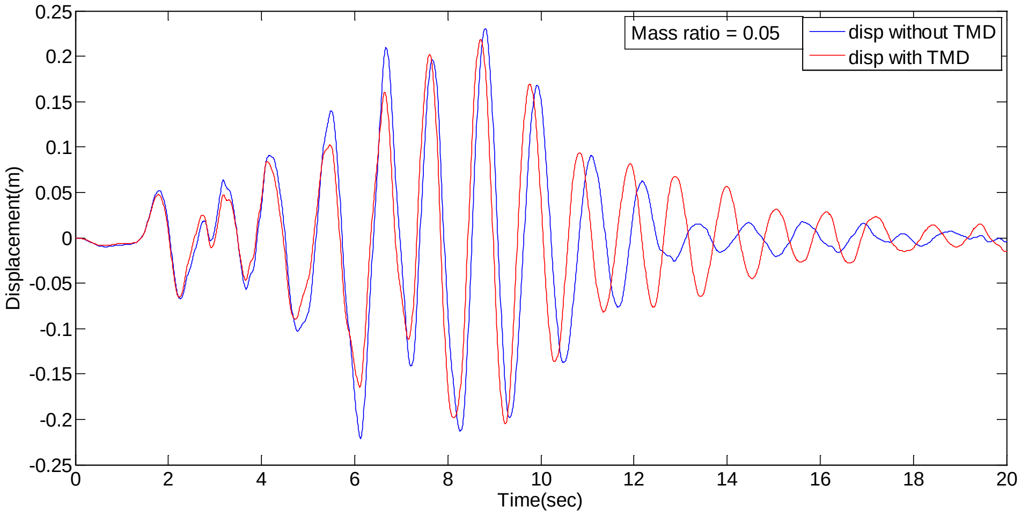


(a)

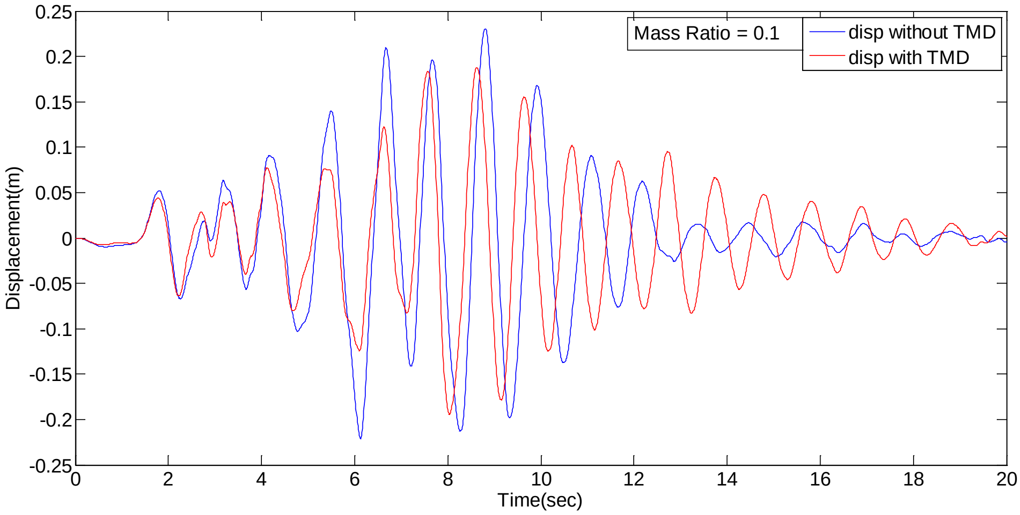


(b)

Fig. 4.13: Displacement of the 2D frame with and without single TMD at 10th floor under Compatible time history as per spectra of IS-1893 (Part -1):2002 for 5% damping at rocky soil. For (a) Mass ratio 0.05, (b) Mass ratio 0.1.

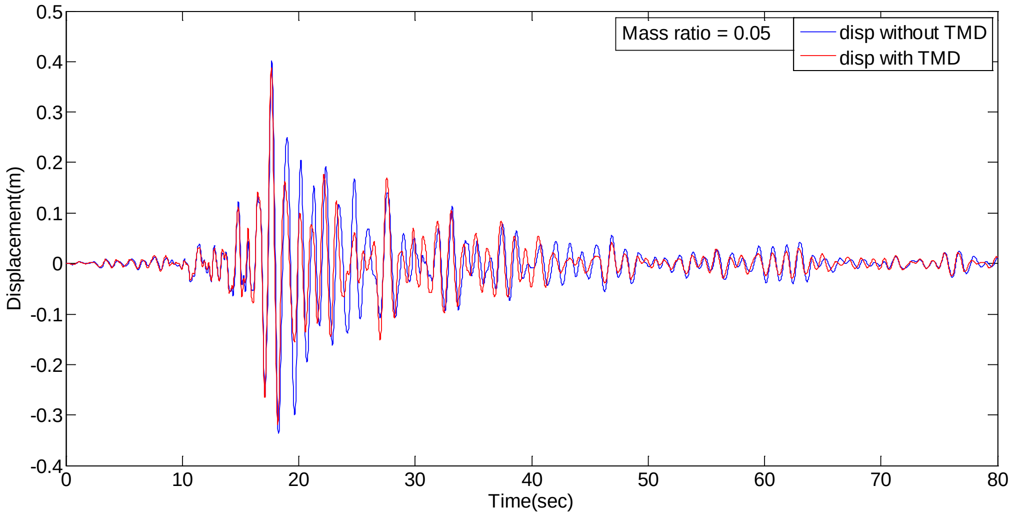


(a)

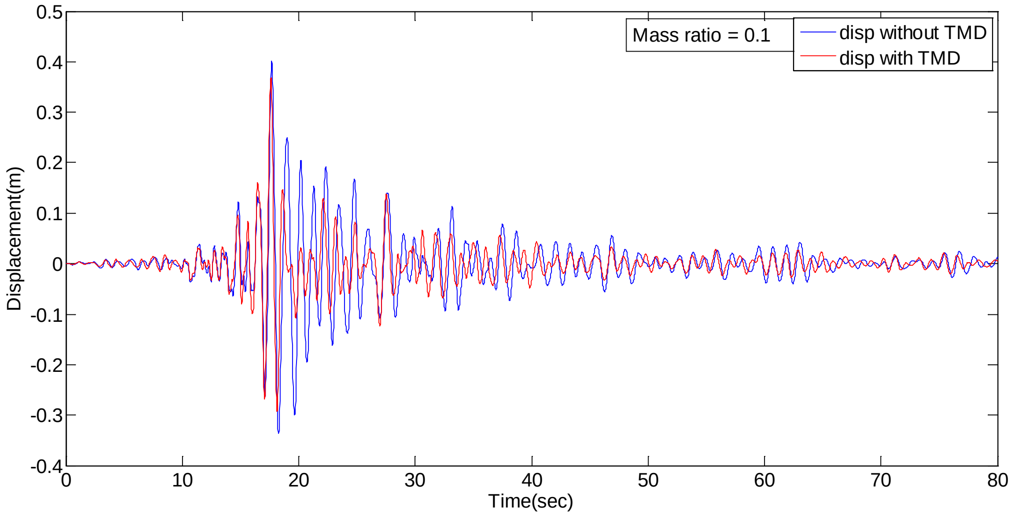


(b)

Fig. 4.14: Displacement of the 2D frame with and without single TMD at 10th floor under Sakaria earthquake. For (a) Mass ratio 0.05, (b) Mass ratio 0.1



(a)



(b)

Fig. 4.15: Displacement of the 2D frame with and without single TMD at 10th floor under The Landers earthquake 1992. For (a) Mass ratio 0.05, (b) Mass ratio 0.1

Table 4.2

Comparison study on the Maximum displacement (m) at the top floor of the 2D frame with and without single TMD (with variation of mass ratio)

Type of loading	Without TMD **(D)**	With TMD of mass ratio 0.05 **(E)**	With TMD of mass ratio 0.1 **(F)**	**(D-E)**	**(D-E)×100/D**	**(D-F)**	**(D-F)×100/D**	**(E-F)**	**(E-F)×100/E**
Sinusoidal acceleration	0.0426	0.0293	0.0202	0.0133	31.2207	0.0224	52.5822	0.0091	31.0580
El-Centro earthquake accelerogram 1940	0.2542	0.2540	0.2358	0.0002	0.0787	0.0184	7.2384	0.0182	7.1654
spectra of IS-1893 (Part -1):2002 for 5% damping at rocky soil	0.1757	0.1658	0.1633	0.0099	5.6346	0.0124	7.0575	0.0025	1.5078
Sakaria earthquake accelerogram	0.2305	0.2158	0.1876	0.0147	6.3774	0.0429	18.6117	0.0282	13.0677
The Landers earthquake accelerogram (1992)	0.4011	0.3877	0.3676	0.0134	3.3408	0.0335	8.3520	0.0201	5.1844

It can be concluded from table 4.2, that like shear building the response of 2D frame decreases with the increase in mass ratio of the TMD. The maximum response reduction under earthquake loading is very insignificant than under sinusoidal loading. The maximum percentage of response reduction takes place under Sakaria earthquake for both mass ratio of 0.05 and 0.1.

4.6 LINEAR TIME HISTORY ANALYSIS OF 2D FRAME WITH AND WITHOUT DOUBLE TMD

The effectiveness of double tuned mass damper for vibration control is studied by linear time history analysis of the 2D frame under a sinusoidal load and the four numbers past earthquake data. The damping ratio of the 2D frame is taken as 0.05 for every mode. First and second frequency of the frame without TMD is tuned to the frequency of the first and second damper respectively. The response is calculated in term of displacement at the 10th floor.

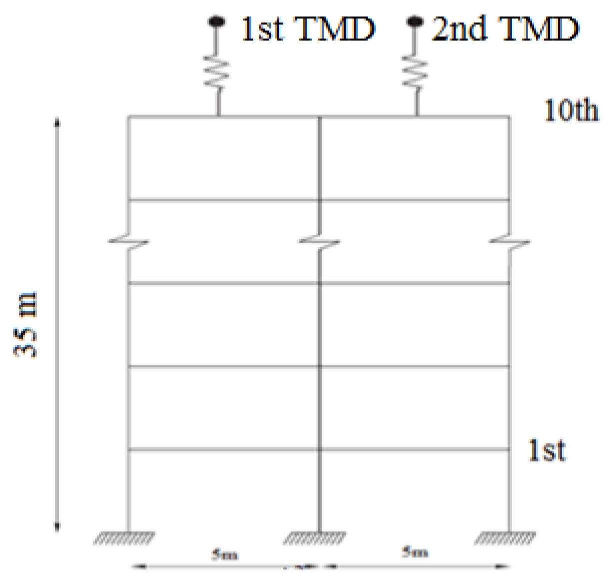


Fig. 4.16: 2D frame model with double TMD

4.6.1 Effect of non-uniform mass ratio of both TMD on the response of the 2D frame

Here response of the 2D frame (in term of displacement) is calculated with mass ratio of 0.075 and 0.025 for first and second TMD under sinusoidal acceleration and the above mentioned four random earthquake ground acceleration**.** The damping ratio of the damper is taken as 0.05.

(a) Sinusoidal acceleration

2D frame is subjected to sinusoidal acceleration $\ddot{u}=A_{max}(\sin(\omega_1.t)+\sin(\omega_2.t))$ at ground. Where, A_{max} and ω are the maximum amplitude of acceleration and frequency of the sinusoidal acceleration respectively. The parameters A_{max}, ω_1 and ω_2 are $0.1 m/s^2$, 5.5349 rad/s and 17.5789 rad/s respectively (considering resonance condition) respectively.

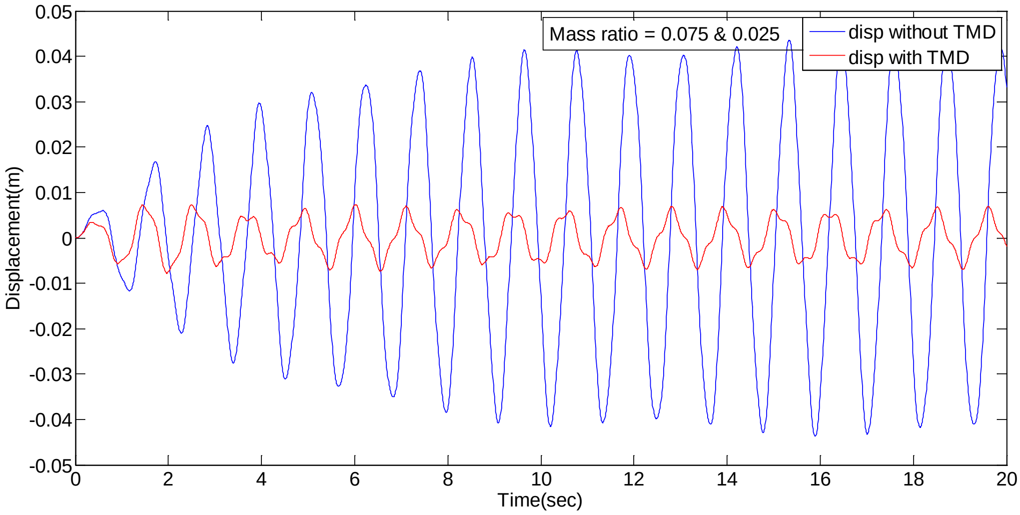


Fig. 4.17: Displacement of the 2D frame with and without double TMD (non-uniform TMD mass ratio as 0.075 and 0.025) at 10th floor under sinusoidal ground acceleration.

(b) Random earthquake ground acceleration

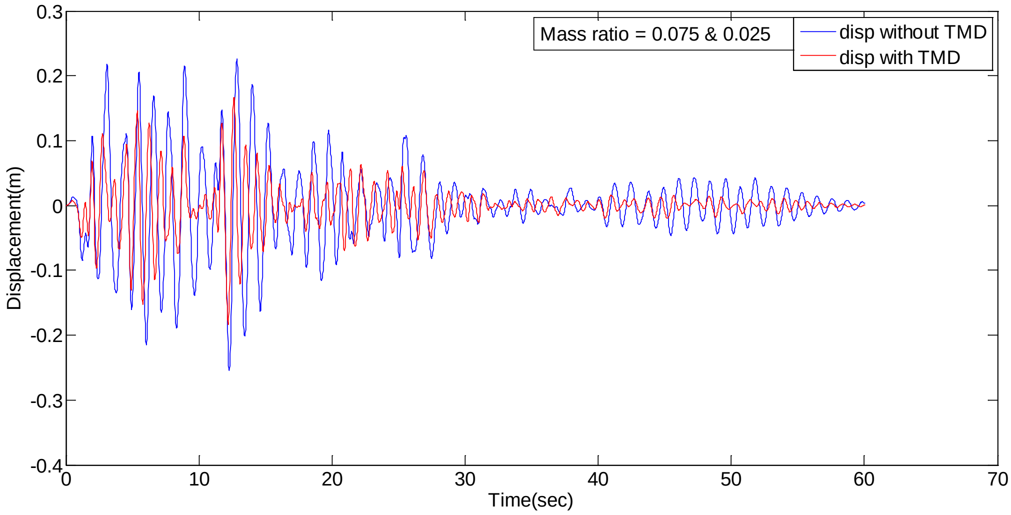


Fig. 4.18: Displacement of the 2D frame with and without double TMD (non-uniform TMD mass ratio as 0.075 and 0.025) at 10th floor under EW component of 1940 El-Centro earthquake

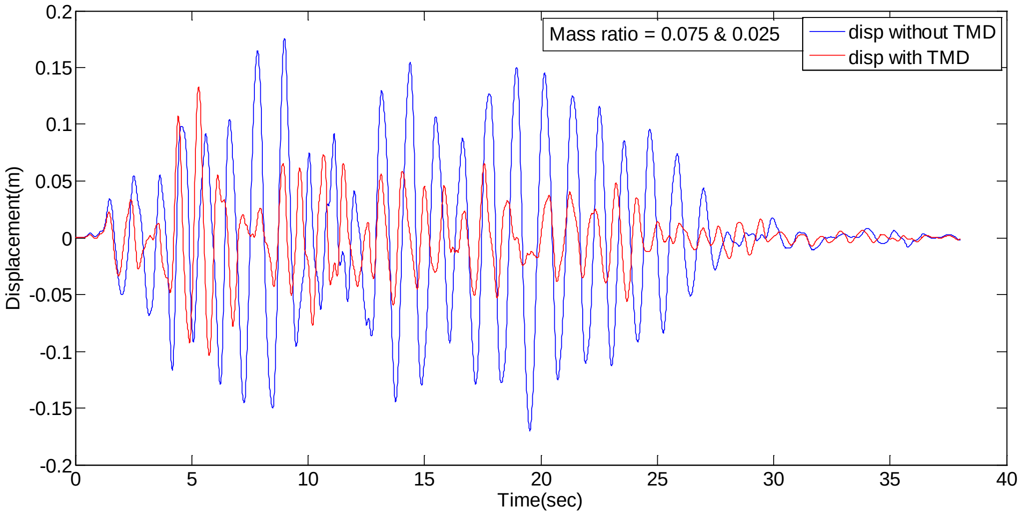


Fig. 4.19: Displacement of the 2D frame with and without double TMD (non-uniform TMD mass ratio as 0.075 and 0.025) at 10th floor under Compatible time history as per spectra of IS-1893 (Part -1):2002 for 5% damping at rocky soil

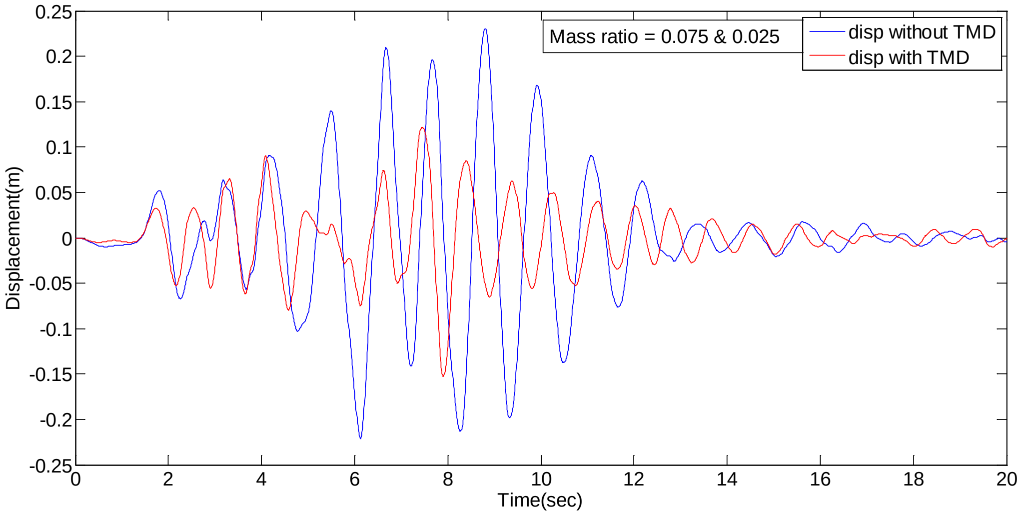


Fig. 4.20: Displacement of the 2D frame with and without double TMD (non-uniform TMD mass ratio as 0.075 and 0.025) at 10th floor under Sakaria earthquake

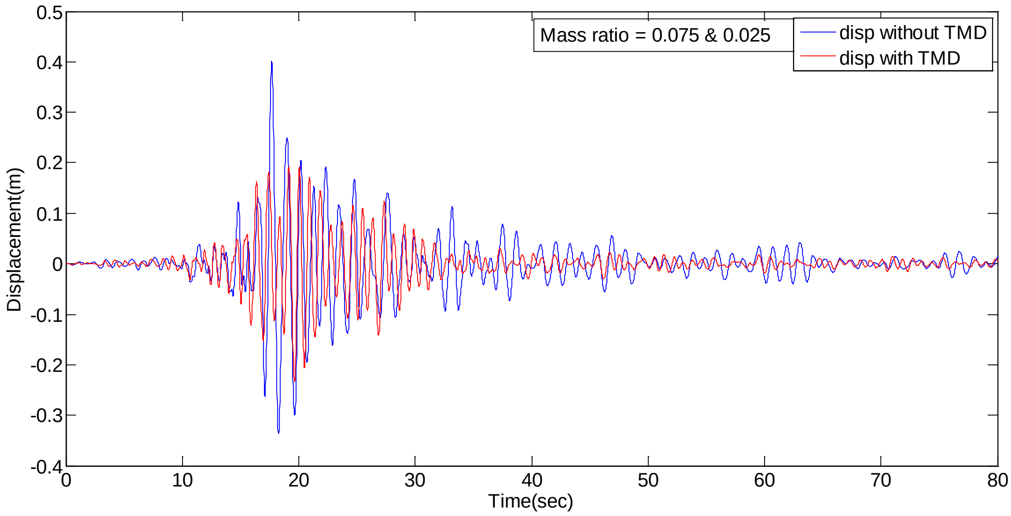


Fig. 4.21: Displacement of the 2D frame with and without double TMD (non-uniform TMD mass ratio as 0.075 and 0.025) at 10th floor under The Landers earthquake 1992

Table 4.3

Comparison Study on the maximum displacement (m) of the 2D frame without and with single or double TMD (for non-uniform mass ratio)

Type of loading	Without TMD **(G)**	with single TMD for mass ratio 0.1 **(H)**	with double TMD for mass ratio 0.075 and 0.025**(I)**	**(G-H)**	**(G-H)×100/G**	**(G-I)**	**(G-I)×100/G**	**(H-I)**	**(H-I)×100/H**
Sinusoidal acceleration	0.0438	0.0202	0.0078	0.0236	53.8813	0.0360	82.1918	0.0124	61.3861
El-Centro earthquake accelerogram 1940	0.2542	0.2358	0.1832	0.0184	7.2384	0.0710	27.9308	0.0526	22.3070
spectra of IS-1893 (Part -1):2002 for 5% damping at rocky soil	0.1757	0.1633	0.1327	0.0124	7.0575	0.0430	24.4735	0.0306	18.7385
Sakaria earthquake accelerogram	0.2305	0.1876	0.1219	0.0429	18.6117	0.1086	47.1150	0.0657	35.0213
The Landers earthquake accelerogram(1992)	0.4011	0.3676	0.1927	0.0335	8.3520	0.2084	51.9571	0.1749	47.5789

Here two different cases are taken response of the 2-D frame with single TMD and double TMD with different mass ratio (ratio of first TMD and second TMD mass to total structural mass is kept as 0.075, 0.025). But the sum of the total mass ratio of double TMD is kept constant as single TMD. It is found from the table 4.3 that for random earthquake acceleration, the maximum percentage of response reduction (51.96%) takes place under The Landers earthquake 1992. Also the difference in response reduction is higher for Landers earthquake. Percentage of response reduction for sinusoidal acceleration is 82.2%.

4.6.2 Effect of uniform mass ratio of both TMD on the response of the 2D frame

Here response of the 2D frame (in term of displacement) calculated with equal mass ratio of 0.05 for each TMD under sinusoidal acceleration andrandom earthquake ground acceleration**.** The damping ratio of the damper is taken as 0.05.

(a) Sinusoidal acceleration

Loading condition considered same as given in section 4.6.1 (a)

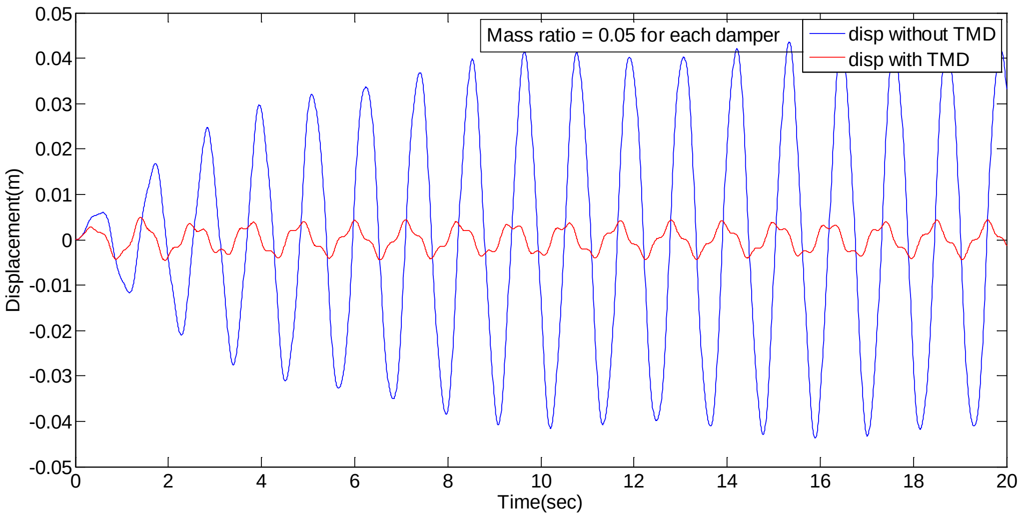


Fig. 4.22: Displacement of the 2D frame with and without double TMD at 10th floor under sinusoidal ground acceleration with uniform mass ratio of 0.05

(b) Random earthquake ground acceleration

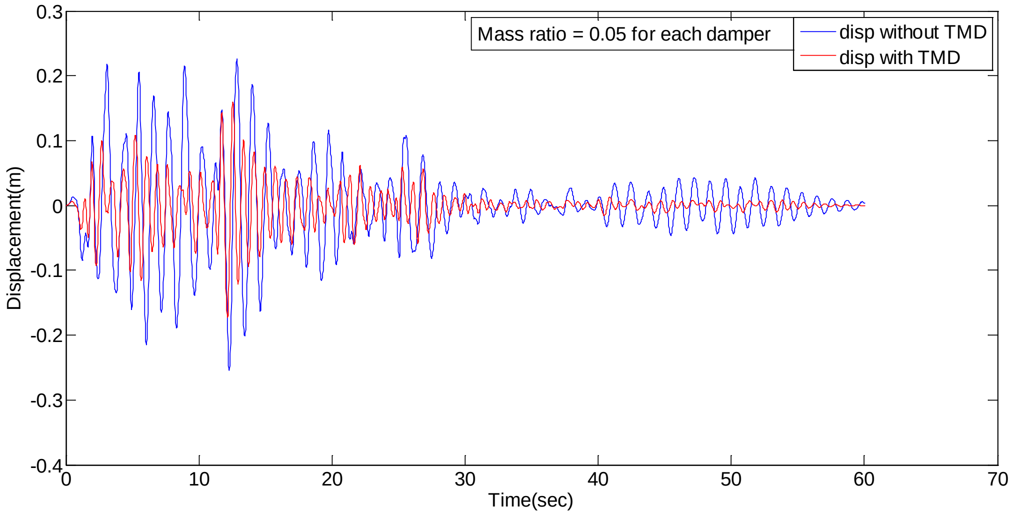


Fig. 4.23: Displacement of the 2D frame with and without double TMD at 10th floor under EW component of 1940 El-Centro earthquake with uniform mass ratio of 0.05

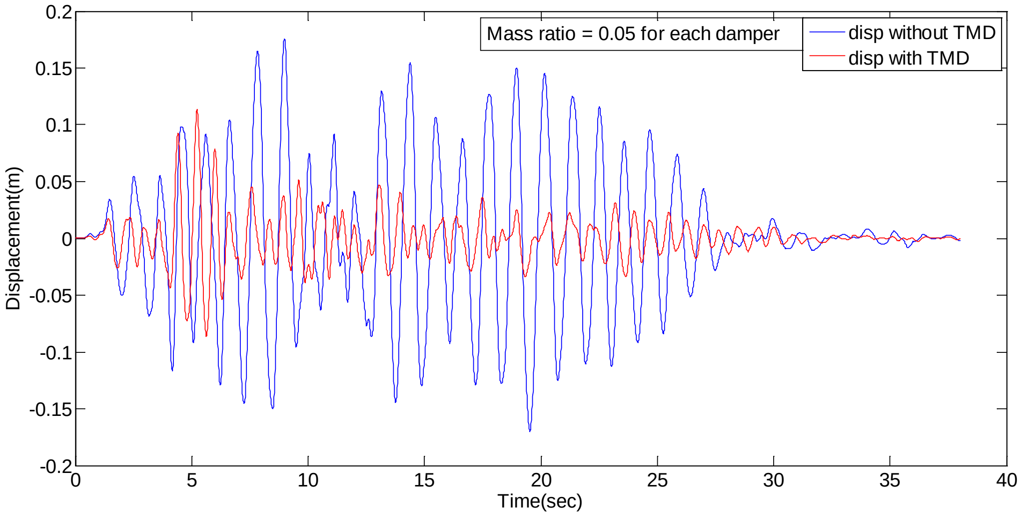


Fig. 4.24: Displacement of the 2D frame with and without double TMD at 10th floor under Compatible time history as per spectra of IS-1893 (Part -1):2002 for 5% damping at rocky soil with uniform mass ratio of 0.05

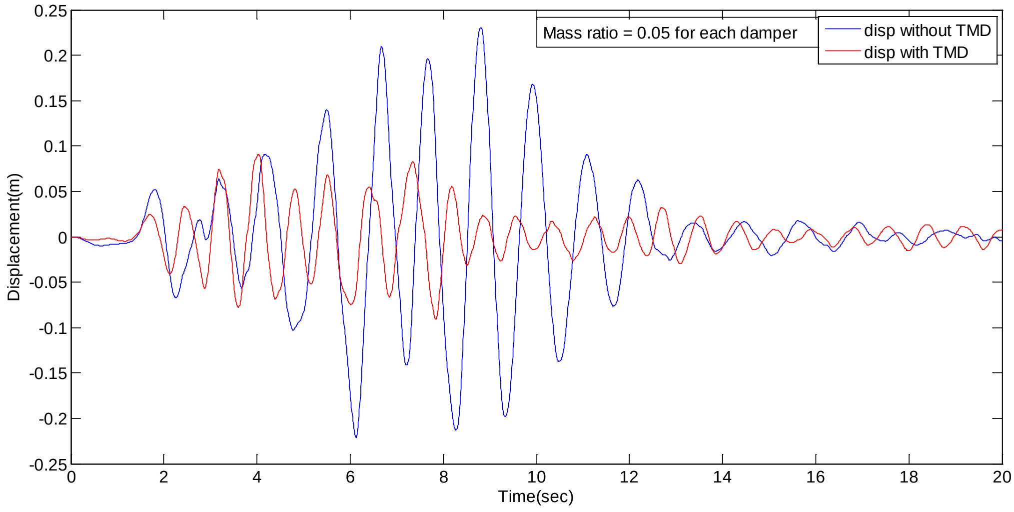


Fig. 4.25: Displacement of the 2D frame with and without double TMD at 10th floor under Sakaria earthquake with uniform mass ratio of 0.05

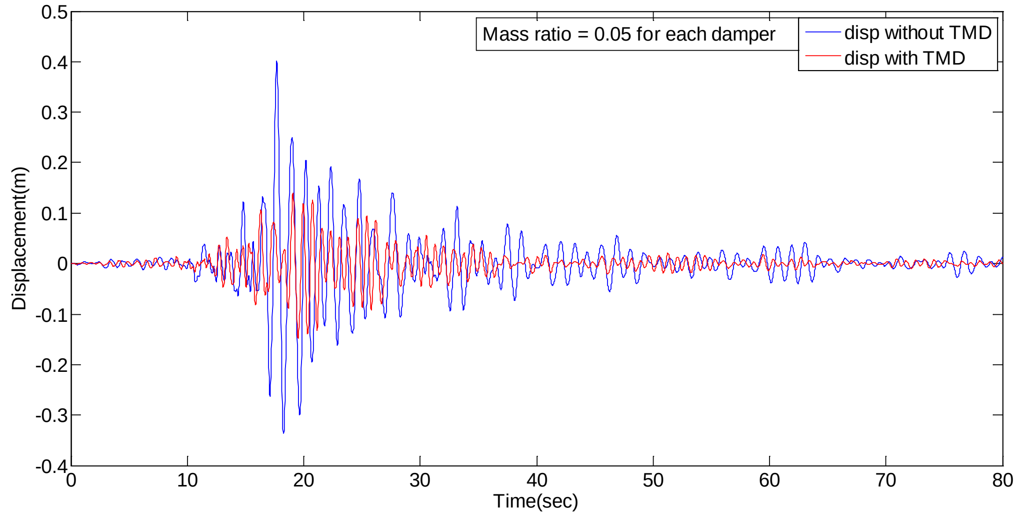


Fig. 4.26: Displacement of the 2D frame with and without double TMD at 10th floor under The Landers earthquake 1992 with uniform mass ratio of 0.05

Table 4.4

Comparison study on the maximum displacement (m) of the 2D frame without and with single or double TMD (uniform mass ratio of 0.05 for each damper)

Type of loading	without TMD **(J)**	with single TMD for mass ratio 0.1 **(K)**	with double TMD for each mass ratio 0.05 **(L)**	**(J-K)**	**(J-K)×100 /J**	**(J-L)**	**(J-L)×100/J**	**(K-L)**	**(K-L)×100/K**
Sinusoidal acceleration	0.0438	0.0202	0.0046	0.0236	53.8813	0.0392	89.4977	0.0156	77.2277
El-Centro earthquake accelerogram 1940	0.2542	0.2358	0.1732	0.0184	7.2384	0.0810	31.8647	0.0626	26.5479
Time history spectra of IS-1893 (Part - 1):2002 for 5% damping at rocky soil	0.1757	0.1633	0.1132	0.0124	7.0575	0.0625	35.5720	0.0501	30.6797
Sakaria earthquake accelerogram	0.2305	0.1876	0.0910	0.0429	18.6117	0.1395	60.5206	0.0966	51.4925
The Landers earthquake accelerogram(1992)	0.4011	0.3676	0.1394	0.0335	8.3520	0.2617	65.2456	0.2282	62.0783

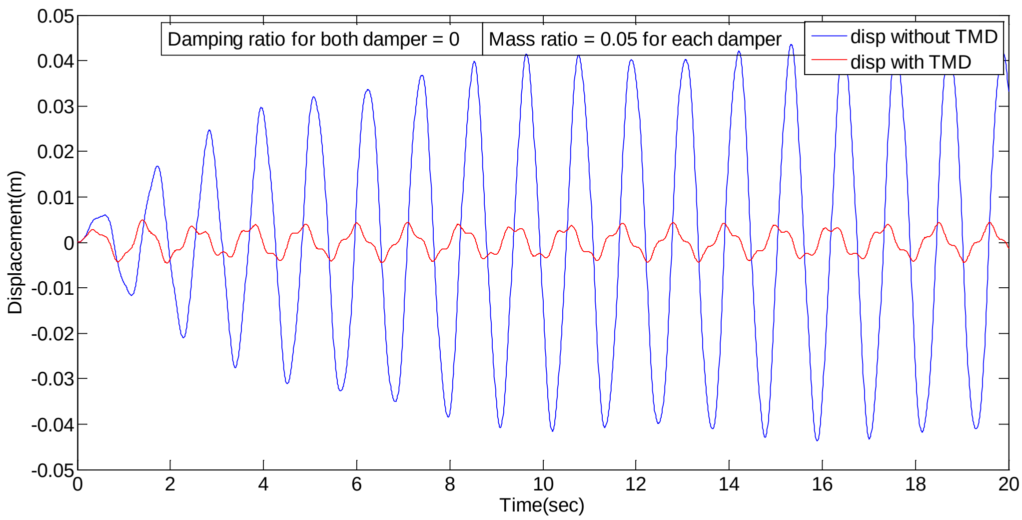
Effectiveness of double TMD with uniform TMD to structural mass ratio is considered here. From table 4.4 it is found that double TMD with uniform mass ratio are much more effective in vibration control than a single TMD of same mass ratio or double TMD with non-uniform mass ratio. Maximum response reduction of the 2D frame is also increase with increase in TMD mass to structural mass ratio. Here under almost all earthquake significant response reduction takes place but not at that much rate as in case of sinusoidal load. The maximum response reduction is 89.55 % for sinusoidal ground acceleration and 65.25% for the Landers earthquake acceleration.

4.6.3 Effect of damping ratio variation of both TMD on response of the 2D frame for uniform mass ratio

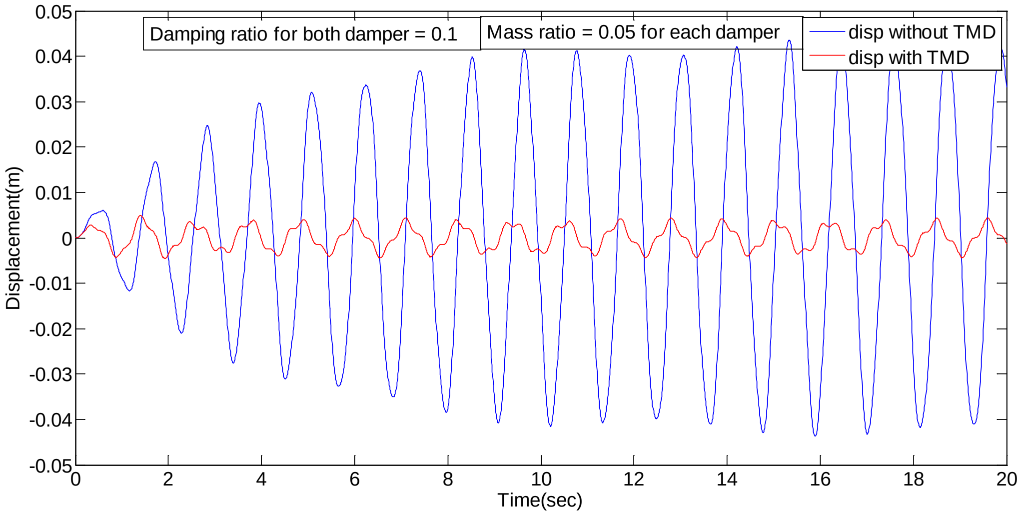
The effect of variation of damping ratio of both TMD is studied through the response of the 2D frame (in term of displacement). Equal mass ratio of 0.05 for each TMD is considered under sinusoidal acceleration and random earthquake ground acceleration.

(a) Sinusoidal acceleration

Loading condition considered same as section 4.6.1 (a)

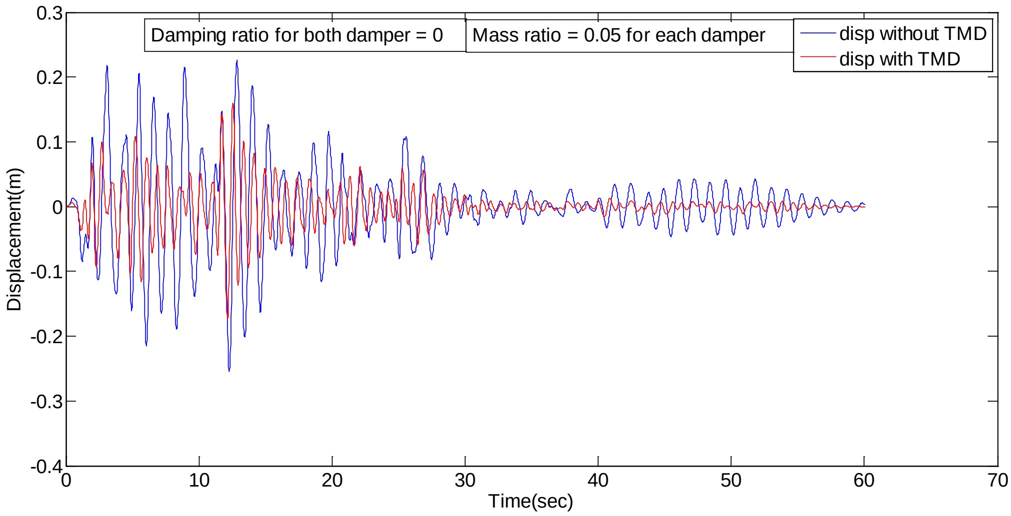


(a)

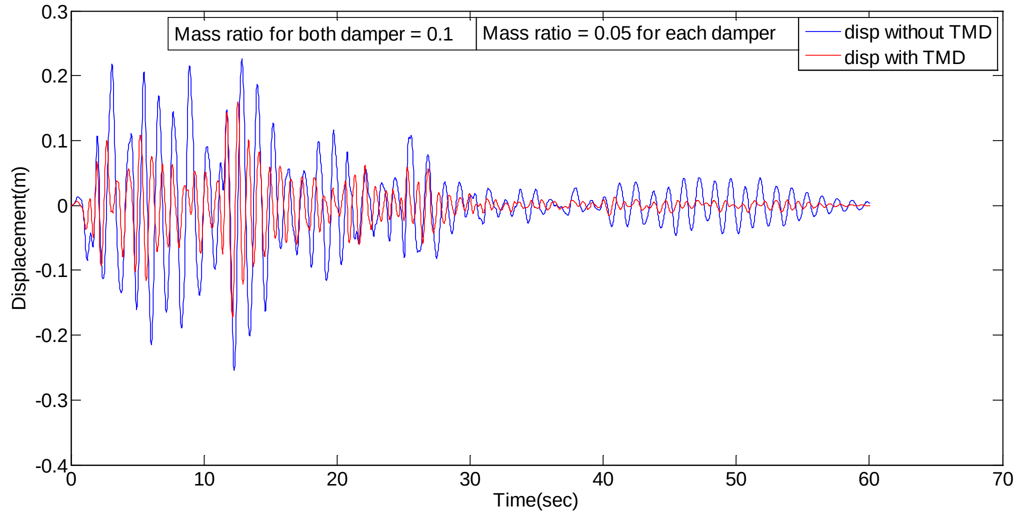


(b)

Fig. 4.27: Displacement of the 2D frame with and without double TMD at 10th floor under sinusoidal ground acceleration with uniform mass ratio of 0.05. For both TMD damping ratio (a) 0, (b) 0.1

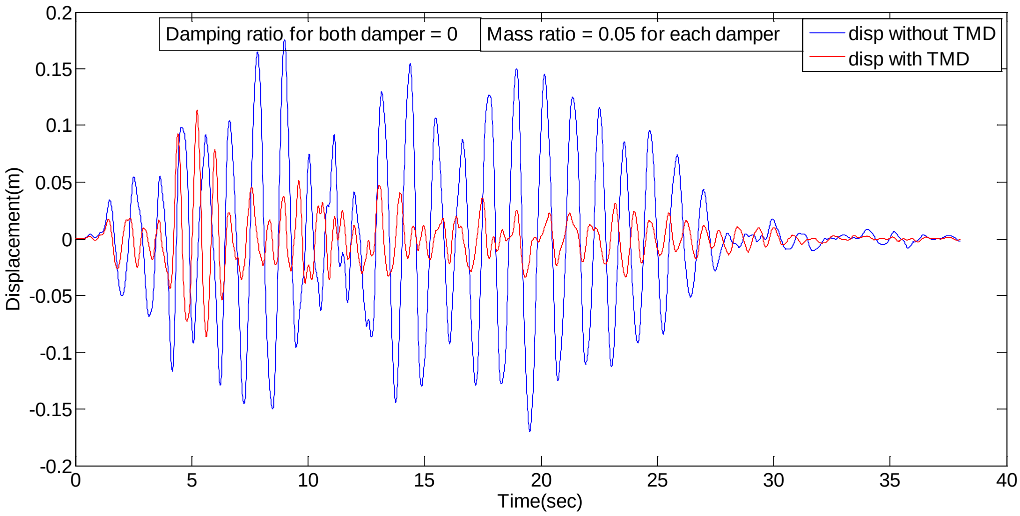


(a)

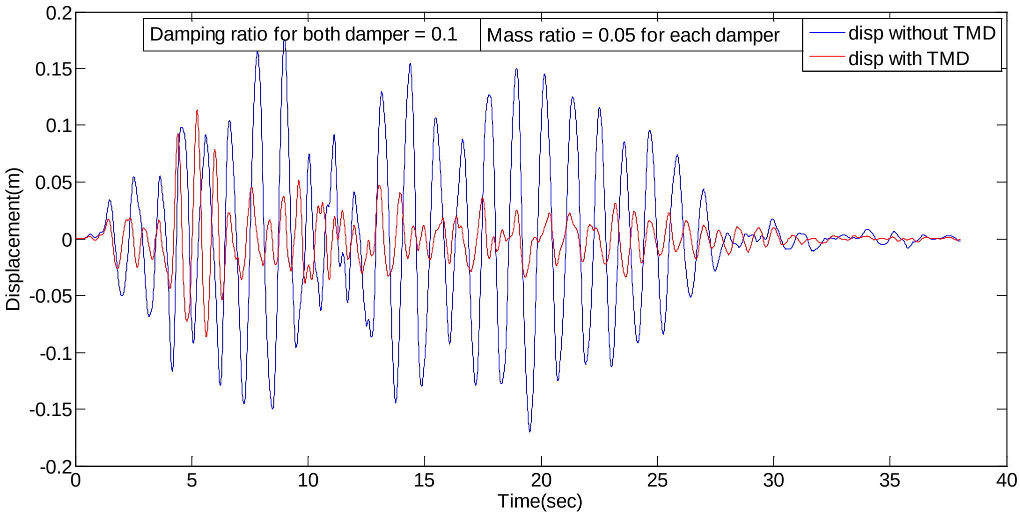


(b)

Fig. 4.28: Displacement of the 2D frame with and without double TMD at 10th floor under EW component of 1940 El-Centro earthquakewith uniform mass ratio of 0.05. For both TMD damping ratio (a) 0, (b) 0.1

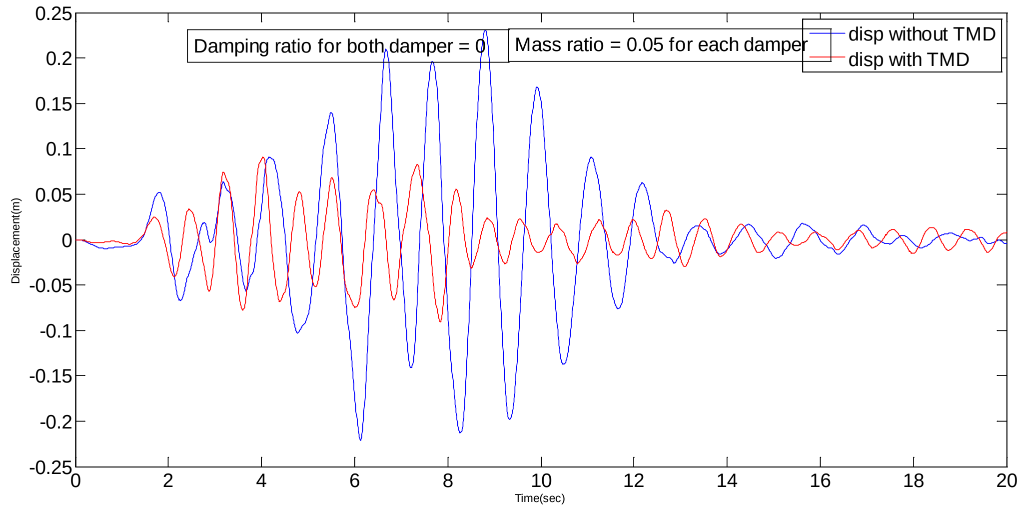


(a)

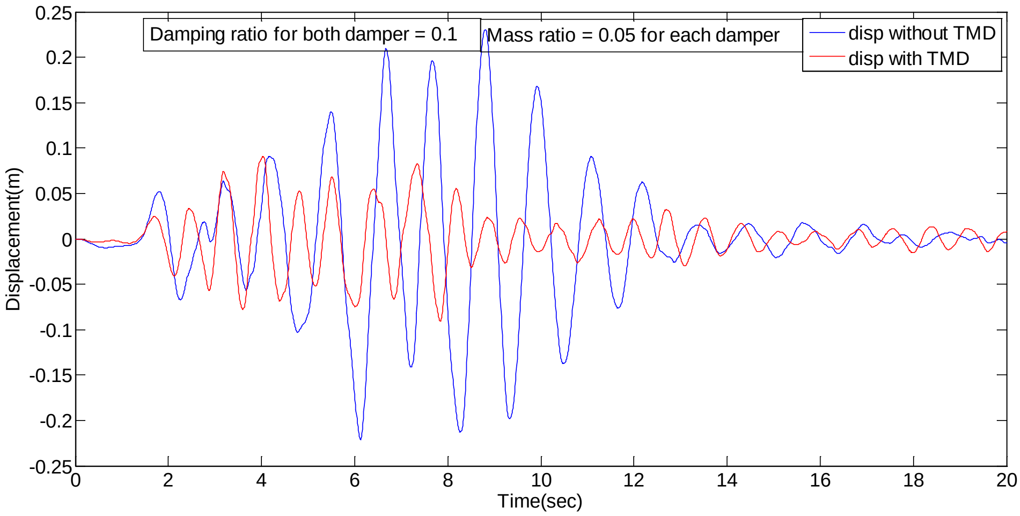


(b)

Fig. 4.29: Displacement of the 2D frame with and without double TMD at 10th floor under Compatible time history as per spectra of IS-1893 (Part -1):2002 for 5% damping at rocky soilwith uniform mass ratio of 0.05. For both TMD damping ratio (a) 0, (b) 0.1

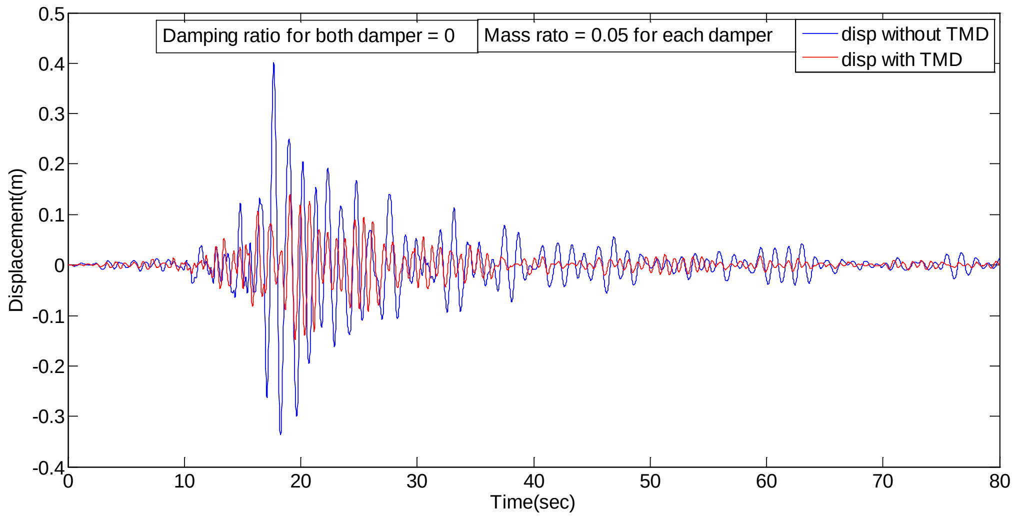


(a)

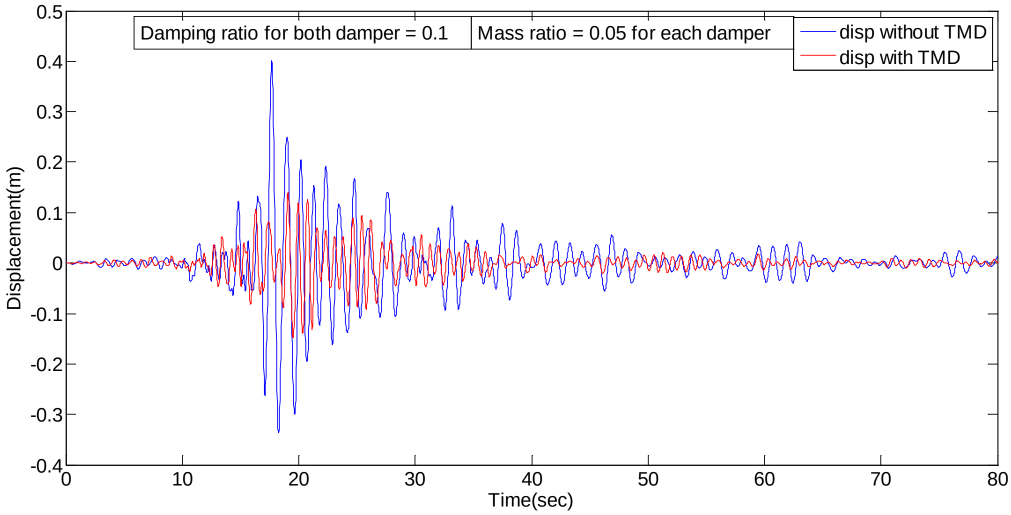


(b)

Fig. 4.30: Displacement of the 2D frame with and without double TMD at 10th floor under Sakaria earthquake with uniform mass ratio of 0.05. For both TMD damping ratio (a) 0, (b) 0.1



(a)



(b)

Fig. 4.31: Displacement of the 2D frame with and without double TMD at 10th floor under The Landers earthquake 1992 with uniform mass ratio of 0.05. For both TMD damping ratio (a) 0, (b) 0.1

From the fig.4.27 to fig 4.31 it is found that the response of the 2D frame does not change with change in damping ratio of the damper and even maximum values of response remain constant. Hence damping ratio of the damper has no or zero effect on the response of the 2D frame under sinusoidal as well as random ground acceleration.

CHAPTER 5

CONCLUSION AND FUTURE SCOPE OF STUDY

5.1 CONCLUSION

Present study focused on the ability of MTMD to reduce earthquake induced structural vibration. The frame is discretised using finite element technique. Linear time history analysis of the frame has been done at each time step by using Newmarks Beta method. Numerical simulation has been performed to compare the frame response with effect of uniform, non-uniform variation of mass ratio and variation of damping ratio of MTMD. From study it can be concluded that.

1) Response of the frame building reduces with the increase in mass ratio of the single TMD.
2) TMDs are much more effective to reduce structural vibration when subjected to sinusoidal ground acceleration.
3) The MTMD with non-uniform distribution of mass ratio is more effective than single TMD same mass ratio.
4) The MTMD with uniform distribution of mass ratio is most effective in vibration control in the present study.
5) The frame has same response with single and multiple TMD if multiple TMD with uniform or non-uniform distribution of mass ratio is tuned to same structural frequency.
6) The response of the frame building has no effect on the variation of damping ratio of the damper.

5.2 FUTURE SCOPE OF STUDY

1) The frame model considered here is as one and two-dimensional. A further study can be done including three-dimensional structure model.

2) In current study both the frame and Damper has been modelled as linear one.Thus a further study of this problem can be carried out using a nonlinear model for frame or TMDs or both.

3) A further study includes using MTMD tuned with all the unfavourable structural frequency as well as placing them in different level of the frame.

4) A future study can be done with active multiple tuned mass dampers.

5) Optimal MTMD can also be designed considering unfavourable frequency of past earth data.

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