Development of an Iterative Learning based Tip Position Controller of a Flexible Link Robot

A Thesis Submitted in Partial Fulfilment

of the Requirements for the Award of the Degree of

Master of Technology

in

Control & Automation

by

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Abstract

Apart from industrial robots there is another class of robots which are of interest to space industry for its lightweight structure. The lightweight flexible robots are advantageous compared to rigid ones in several fronts such as higher payload-to-arm weight ratio, faster execution, and low power actuator requirements. But with these advantages, there lies an array of control complexities in Flexible Robot manipulators, as the modelling and control of a flexible robot is complex and difficult due to under actuated behaviour, non linear time varying and distributed system parameters. In the past, many control strategies have been proposed for the tip position control of flexible link robots but most of these techniques have not considered actuator dynamics in modelling and experimental validation is not carried out.

The thesis proposes the use of a non linear model of a single link flexible robot manipulator obtained using Assumed Mode Method (AMM). The actuator dynamics has also been incorporated in the modelling of the single link flexible robot. The model thus obtained is experimentally validated using SIMULINK/MATLAB.

The objective of the thesis is to control the tip position of a single link flexible robot. In order to achieve a successful tip trajectory tracking mechanism, the thesis proposes the use of an adaptive control mechanism called Iterative Learning Control (ILC). Iterative Learning based controller design offers significant advantages over other techniques such as it improves transient response and tracking performance of the system. It also takes care of non-linear effects such as friction, actuator dynamics etc. It requires only superficial knowledge of the system dynamics. Finally, to illustrate the effectiveness of the proposed controller, the performance of the designed controller in terms of input tracking and vibration suppression is compared with an existing PD controller by simulations.

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CERTIFICATE

This is to certify that the thesis titled "***Development of an iterative learning based tip position controller of a flexible link robot***", submitted to the National Institute of Technology, Rourkela by **Nishant kumar**, Roll No. **210EE3293** for the award of Master of Technology in **Control & Automation**, is a bonafide record of research work carried out by him under our supervision and guidance.

The candidate has fulfilled all the prescribed requirements. The Thesis which is based on candidate's own work, has not submitted elsewhere for a degree/diploma. In our opinion, the thesis is of standard required for the award of a Master of Technology degree in Control & Automation. To the best of our knowledge, he bears a good moral character and decent behavior.

Prof. B.D. Subudhi

Prof. Subhojit Ghosh

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I would like to express my sincere gratitude to all the ***faculty members and staff*** of the Department for their unflinching support, inspiration, and cooperation and providing me with all sort of official facilities in various ways for the completion of the thesis work.

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Chapter 1

Introduction to flexible Robot Manipulator

1.1 Introduction

A Robot manipulator is a device used to operate some apparatus or machine by some mechanical means. Robots have a wide range of applications in industrial areas. Materials handling, spray painting and spot welding were some of the earlier applications. Robots were also applied to jobs that were hot, heavy, and hazardous such as die casting, forging, and spot welding. They were also used to deal with radioactive or biohazardous materials, using robotic arms, or they were used in remote places. Recently Robot manipulators have been used in other applications such as robotically-assisted surgery and in space. Robotics has been of great interest to mankind for over one hundred years. But the rigidity in their structure restricts their use in space applications. Flexible robots are preferred in those applications which require lightweight structures.

1.1.1 Brief description on flexible Robots

Flexible robots consist of manipulators that are made of flexible and lightweight materials such as a wear resistant 1095 spring steel used in the Flexible Manipulator Setup in our experiment. These manipulators are operated by using some actuator that may be a dc motor, some robots use electric motors and solenoids as actuators, while some have a hydraulic system, and some others may use a pneumatic system .Some may also use all these above mentioned actuator types. A power source is required to drive these actuators. When these flexible manipulators are actuated they undergo vibrations due to their flexibility. During the motion of a flexible link, at each point of its trajectory damped vibration exist which cause each point of the link to vibrate and thus the tip position does not come to the desired position quickly and once the torque is removed, the link takes some time to settle down to its final position. This can be easily understood from Fig. 1.1.

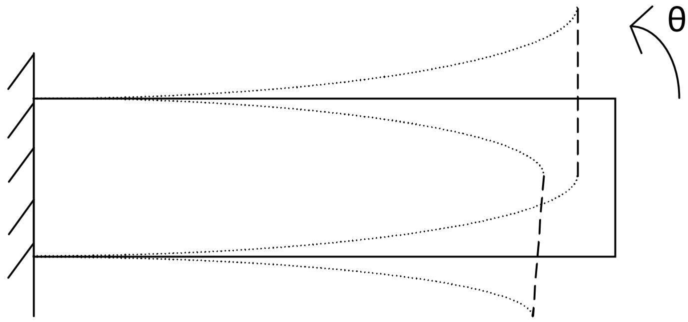

Fig1.1 A flexible link under vibration

From Fig. 1.1 shown, it is obvious that the link is vibrating through an angle θ. Had it been a rigid link, it would have settled down to its final position without any vibration but because of flexibility of the link it doesn't track the desired trajectory at once and the final tip position keeps on oscillating for some time due to damped vibration. Here each point of the link is under deformation. To derive model of the beam, the link cannot be simply considered to be concentrated at its centre of mass, rather it is important to consider the structure to posses infinite number of modes of vibrations Due to this reason to control its trajectory a more versatile and superior controller needs to be designed.

From mathematical point of view in case of the rigid link robot ordinary differential equations are sufficient to describe the dynamics assuming the total mass to be concentrated at centre of gravity of the body. In contrast to rigid links, due to infinite number of modes of vibration of a flexible link under vibration rigid body analysis would be no more valid and so to represent the distributed nature of position along the beam, Partial Differential Equation (PDE), known as Euler's Bernoulli equation is used.

1.1.2 Need for the flexible Robots

In earlier days the robots used were made up of rigid materials. But due to the rigidity of the flexible manipulator, the response was not so fast and power consumption was also very high, this led to the need for development of lightweight manipulators. Although lightweight manipulators offer several advantages over the rigid manipulators, but due to their flexibility they are difficult to control. Therefore the control mechanism of the flexible robot becomes more challenging with complex mathematical computations than that of their rigid counterparts. So to achieve precise trajectory tracking of a flexible robot manipulator, it is essential to first know the dynamic nature of the single link manipulator system and then develop a suitable model of the system, on which we can apply the controller.

1.1.3 Advantages of flexible robots

Fast response and light-weight structure are the two major requirements in the design and analysis of any system of interest. Initially robots were mainly used in industrial automation sectors and their application in other fields was limited. In space and medical fields the use of these bulky robots was not desired, more flexible structures was required for the operations to be carried out in those fields ,which led to the development of flexible manipulators and since then research on flexible structures control and modelling has increased rapidly.

Flexible manipulator robots offer several advantages in contrast to their traditional rigid manipulators. These include faster system response, lower energy consumption, low rated actuators, and lower overall mass and, in general, lower overall cost.

But, in addition to these benefits they are associated with serious control problem of vibration. As the structure is flexible when it is provided with an input torque it vibrates with low frequency and it take some time to damp it out. Therefore the control problem for the flexible robot is more complex than rigid link robots

1.2 Control Complexities in precise tip tracking of the flexible link robot.

The system-dynamics are significantly more complex. Problems arise due to precise positioning requirements, system flexibility which leads to vibration .Also due to infinite number of vibration modes, the system model is of infinite dimension .In order to control tip position we need to truncate the model into finite dimensions. The control objective of the flexible link robot is more complex than rigid link manipulator where vibration is suppressed quickly. In addition to this, there are many other physical limitations associated with the flexible link manipulator system:

- The control torque using dc motor can only be applied at the joints of the system.
- Only a finite number of sensors of limited bandwidth can be used and at few locations along the length of the manipulator.
- The actual tip position can only be calculated by using strain gauges installed at the clamped ends or the hubs, there is no provision of any sensor at the tip position of the manipulator in our system.

1.3 Motivation

Robot manipulators are now being widely used in industrial and medical fields. With the development of flexible link robots as assisting tools, which offer several advantages over their rigid link manipulators counterparts, the precise tip positioning has become an area of interest in recent time. In most robotic applications, the tip positioning/end-point control is crucial problem as the ultimate goal is to suppress the vibration more effectively. Many researchers have already provided invaluable contributions in this field but still because of difficulties and complexities in controller design, a new novel design of controller is needed. An Iterative learning based controller due to its additional features as discussed later can be expected to serve this purpose

1.4 Objectives of the thesis

The objectives of this thesis are as follows:

- To achieve precise tip-tracking of a single link flexible robot manipulator undergoing vibrations.
- To study the dynamics of a flexible beam and have a knowledge of Assumed mode method (AMM), for the modelling of a flexible robot manipulator system.
- To study Iterative Learning control technique.
- To tune a simple PD controller using Iterative learning based control technique so that vibration in the structure of a flexible link robot manipulator is minimized.

1.5 Problem Formulation

A tip feedback PID controller is applied to the single link flexible robot manipulator as shown in Fig.1.1.

The control law is given in equation (1.1)

$$u(t) = k_p e(t) + k_d \frac{de(t)}{dt} \tag{1.1}$$

Where, $e(t)=(\theta(t)-\theta_{ref})$ is the error in tip position in terms of hub angle of the link. θ_{ref} is the desired trajectory or the angle that the flexible link should follow and θ(t)is the actual hub angle of the link measured form the reference at a time instant 't'sec. K_p is the proportional gain, K_d is the derivative gain. We need to minimize this error using a tuned PID controller.

Fig.1.2 shows a feed-forward ILC design incorporated with the existing tip feedback PD controller in parallel. Fig.1.3 shows the equivalent ILC controller design.

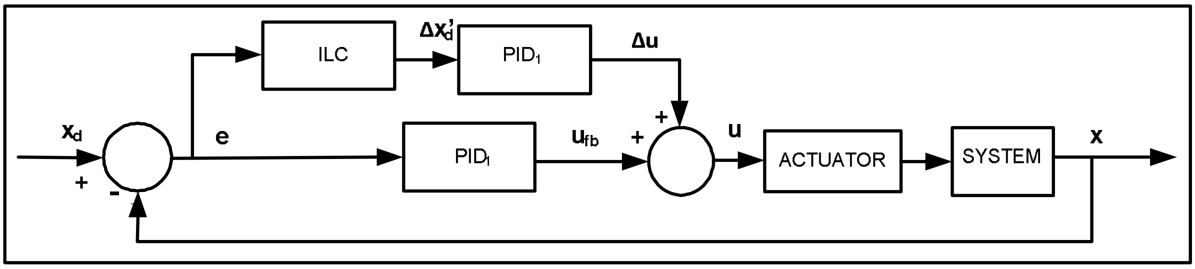

Fig.1.2 Proposed ILC design with actuator

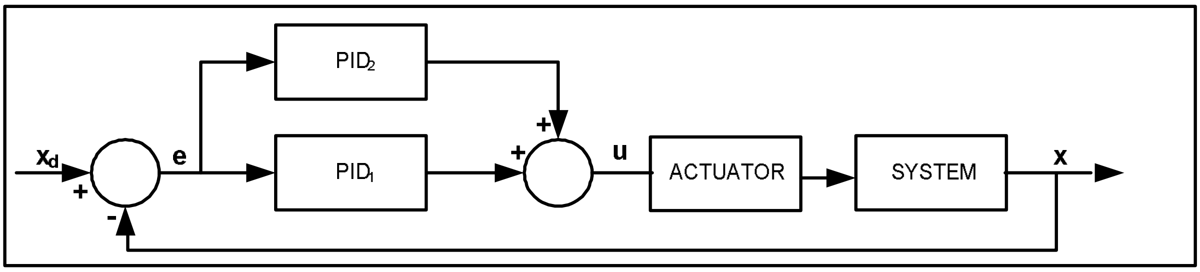

Fig.1.3 Modified PID controller design with actuator

To choose the optimum values of gain parameters k_p and k_d, we shall use Iterative learning based update law(P-type)[25] as discussed in further chapters. And with the help of Least Square (LS) algorithm and data of errors in tip position(e) and change in control input (Δu) generated over a number of iterations(N),we can get the modified PID gain parameters[6].

1.6 Literature Review of some past works

Researches on flexible link robot manipulator were very few during evolutions of robotics. But with their huge applications and advantages researchers found their modelling and control aspects as a boon to the development of human society. Recently many researches both in modelling and control aspects of the robot have been undergoing. But there is always a need for a better controller especially when one is talking about the flexible manipulator robots. Design and analysis of controller for flexible link systems is of prime concern for the researchers because the same controller can be applied to a wide class of system.

In late 80's research on flexible manipulator started. Many researchers have modelled the flexible robot system using both assumed mode and finite element methods. Linear Model of a single link flexible manipulator has been carried out in the works of Qian[1]. Dwivedy and Eberhard[10] reviewed both modelling and control aspects of flexible manipulator systems.

Single-link flexible manipulators using Lagrange's equation and the assumed mode method was studied in the works of Hastings and Book [17], Wang and Wei [18], Wang and Vidyasagar[19]. They followed their work with experimental analysis. In most of these cases joints were assumed to be stiff. A complete non linear model for single flexible link using assumed model is also carried in the works of Luca and Siciliana[4],[5]. Techniques based on different modes of vibration and an inversion based controller design has also been reported in their works [15]. Finite element approach based dynamical model for single link using is also proposed and compared with experimental results in the works of Tokhi and Mohamed [20],[21]. Dynamical model of a two link under actuated flexible joint and flexible link manipulator and a reduced-order controller based on singular perturbation method was studied by Subudhi and Morris [13]. Tan, Zhao and Xu proposed a new approach for closed-loop automatic tuning of a proportional integral derivative (PID) controller based on an iterative learning control (ILC) approach [6].They successfully applied ILC approach to a Permanent Magnet Linear Motor (PMLM) in precise tracking of the desired trajectory.

1.7 Organization of the Thesis

The work in this thesis is divided into four chapters which are as follows:

Chapter 1 provides brief introduction of flexible robots, their applications, their advantages, review of past works and control complexity

Chapter 2 provides the details about the flexible manipulator system used in this project and deals with the dynamics and modelling of the single link flexible robot

Chapter 3 provides the control mechanism for the tip positioning of the flexible manipulator. This is followed by simulation results with discussions.

Chapter 4 concludes the work with suggestions for future work.

Chapter 2

Experimental Setup of a Flexible Link Manipulator System and Dynamic Modelling of a Flexible Link Robot

2.1 Flexible link manipulator experimental setup

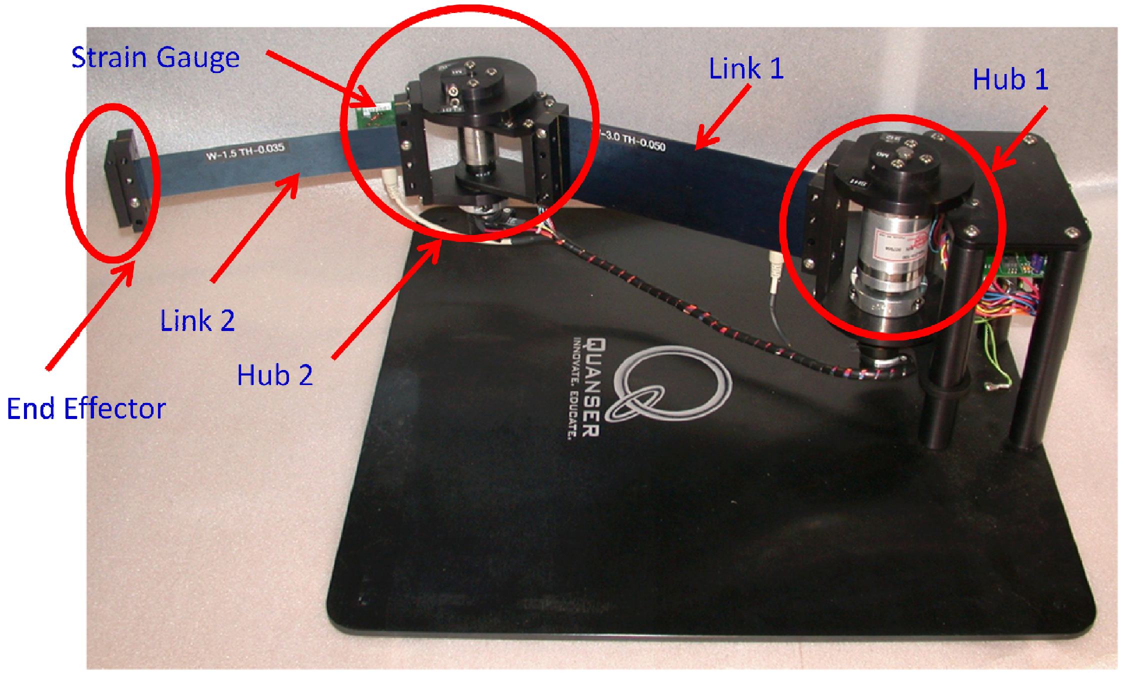

Fig.2.1 Experimental setup of Quanser two link flexible robot system

Fig.2.1 shows the experimental setup of a two link serial flexible manipulator robot. The setup consists of two serial flexible links manufactured by Quanser. There are two hubs or joints of the system where separate strain gauges are installed. At the end of the second link there is an end effector where additional mass or payload mass can be added. The linear amplifier, Q8 terminal board, DAQ system and different sensors like strain gauge, quadrature optical encoder, limit switches are the main components of the setup. The two serial flexible links are actuated by dc motor installed with strain gauges at the clamped end of the links for measurement of tip deflection.

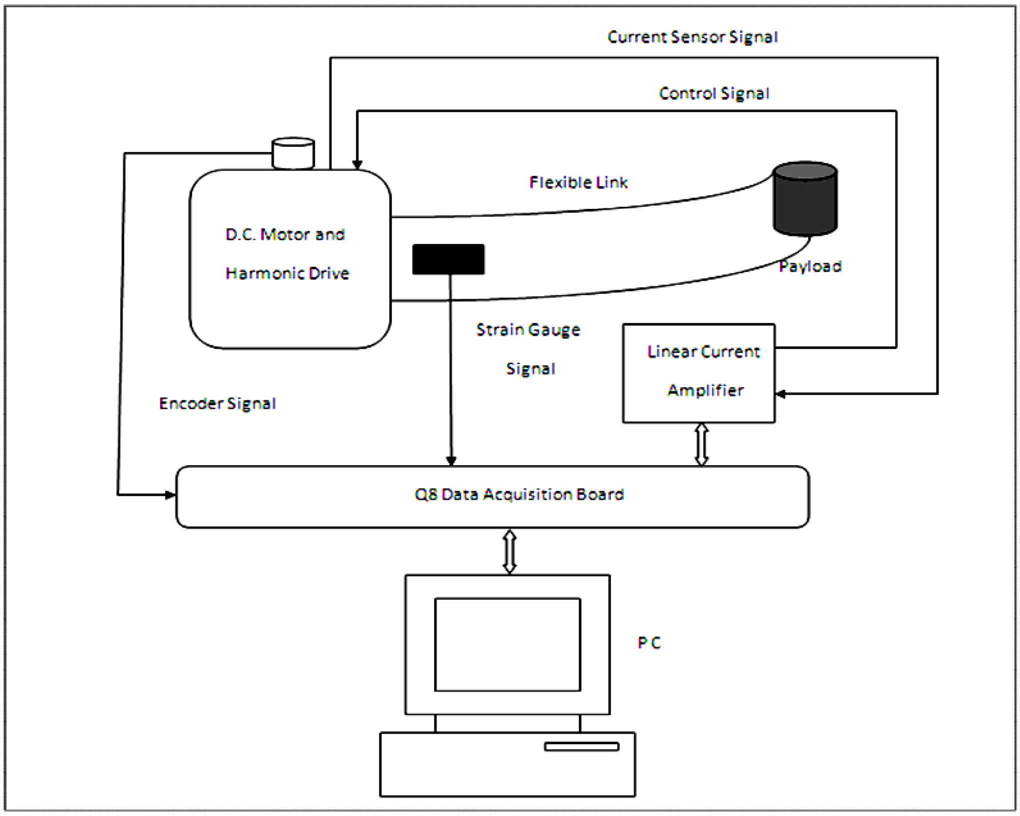

Fig.2.2 Flexible manipulator system schematic

The schematic diagram of the experimental setup of a two link flexible manipulator is shown in Fig. 2.2 .Quanser Q8 data acquisition board acts as the interface between the computer and the setup. The data from the computer to the system is passed through Q8 software which allows the SIMULINK models to run in real-time target.

We can give commands to the real-time set up through Computer running on Windows XP platform installed in the laboratory. The designed controller is then implemented using MATLAB/SIMULINK which is integrated with the real time plant through QuaRC Q8 interfacing software. In SIMULINK environment the experimental results obtained are analysed.

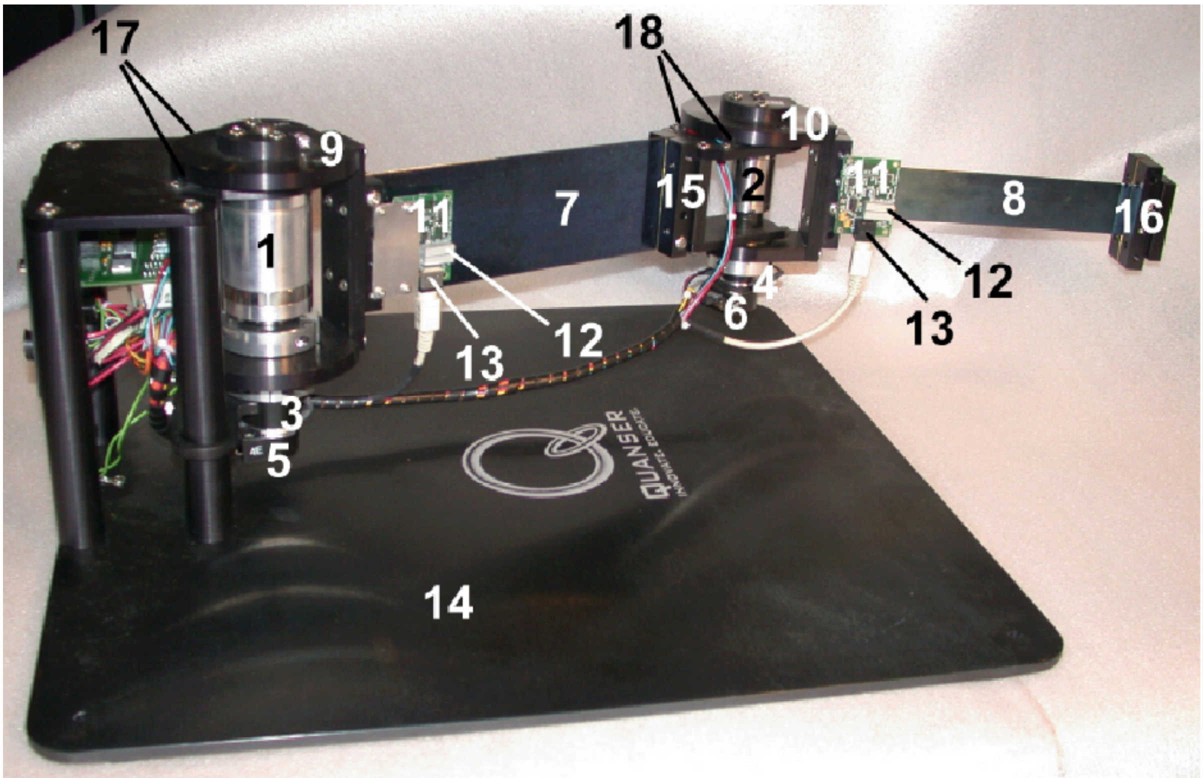

Fig 2.3 Component nomenclature ([3])

Table 2.1 2-DOF Serial Flexible Link Robot Component Nomenclature

Sl. No	Description	Sl. No.	Description
1	Harmonic Drive(link1)	2	Harmonic Drive(link 2)
3	DC Motor (link 1,Shoulder)	4	DC Motor (link2,Elbow)
5	Motor Encoder(link 1)	6	Motor Encoder(link 2)
7	Flexible Link (link 1)	8	Flexible Link (link 2)
9	Rigid Joint (link 1)	10	Rigid Joint (link 2)
11	Strain Gauge Amplifier Board	12	Strain Gauge Offset Potentiometer
13	Strain Gauge Connector	14	Base Plate
15	Link 1 End-Effector	16	Link 2 End-Effector
17	Joint 1 Limit Switches	18	Joint 2 Limit Switches

2.1.1 Flexible links

The flexible manipulator system is consists of two flexible links of the dimensions shown in table. This pair is made of one three-inch wide steel beam and another beam which is one-and-a-half-inch wide. Each beam has a different thickness (i.e., stiffness). These links are made of wear-resistant 1095 spring steel.

Table 2.2 Flexible link dimensions

Parameters	**Length (cm)**	**Width (cm)**	**Thickness (cm)**
Link 1	22	7.62	2.261
Link 2	22	3.81	0.127

The links are mounted to the actuator through the speed reducer. At the base of the links strain gauge is fabricated for tip deflection measurement.

2.1.2. Sensors

Different sensors are used for measurement of signals for example optical encoder for angular position measurement, strain gauge for strain measurement, limit switches for limiting maximum and minimum positions etc.

Strain gauge: There are two strain gauges for the Two-Degree-Of-Freedom Serial Flexible Link robot clamped at the base of each flexible beam. Both the strain gauge sensor is connected to its own signal conditioning and amplifier board. Each strain gauge has an amplifier board, which is equipped with two potentiometers with 20 turns each. The first one is gain potentiometer which supplies power to the circuitry and has a fixed maximum gain of 2000.While the other one is offset potentiometer and is used for zero tuning and is adjusted manually in order to eliminate any offset voltage present in the strain gauge measurement. A balanced Wheatstone bridge circuit is used with strain gauge forming one of its arms to measure the change in resistance caused due to change in length of the system. The basic principle of operation is that when a force is applied to an object the dimensions of the object changes which causes change in the resistance of Wheatstone bridge and finally the voltage is changed. The resistance is given by

$$R = \frac{l}{A} \wp \tag{2.1}$$

where, *l, a* and $\wp$ are the length, area of cross-section of the body and resistivity of the body. The voltage generated is given in terms of strain created by the body in mm/mm. The bending of the tip in either direction creates strain at the base, i.e. length of the gauge increases so resistance and voltage. This tip deflection y can be calibrated in terms of strain in meter [3] and is given by

$$y = \frac{2}{3} \frac{E_b L_b^{\ 2}}{T} \tag{2.2}$$

Where L_b is the length of the link measured from free end up to strain gauge, *T* is the thickness of the link and E_b, strain in m/m at base is given as:

$$E_b = \frac{6FL_b}{EXT^2} \tag{2.3}$$

where, *E* is young's modulus of elasticity; *X* is the width of the flexible link, *F* is load force at the tip in N.

Limit switches: Two limit switches are installed at maximum and minimum positions of rotation of the joints of each flexible link manipulator. Limit switches ensure the safe operation by limiting the movement of the flexible link. Each of them requires ±15VDC power supply. Specifically, these limit switches are made up of the Hamlin 55100 Mini Flange Mount Hall Effect Sensors.

Q-Optical encoder: Angular position of the load shafts are measured by the optical encoder. Specifically the encoder used is a US Digital S1 single–ended optical shaft encoder that offers a resolution of 4096 counts per revolution in quadrature mode having 1024 counts per revolution

2.1.3 DC motor actuator

The flexible manipulator link 1 is installed with a the maxon 273759 precision brush motor of 90 watts and link 2 is having maxon 118752 precision brush motor of 20 watts. This motor is highly efficient having a low rotor inductance offering low inductance. Thus it provides a much faster response than a simple DC motor. The top speed of the motor may be reduced by a speed limiter for safe execution. A harmonic drive, placed coaxial with the actuator is used for speed reduction. Lightweight, zero backlashes, not bulky and precise high gear ratio are some of the advantages of harmonic drives over conventional gear train box. Both optical encoder and harmonic drives are mounted on the motor shaft. Harmonic drive LLC are the manufacturers of the harmonic drives used in the setup.

Ratings of the DC motor are given below:

Table 2.3 Actuator specifications

Motor Specification	Link 1	Link 2
Armature resistance	11.5	2.32
Armature inductance	3.16	0.24
Torque constant	0.119	0.0234
Back emf constant	0.119	0.0234

2.1.4 Q8 terminal board and HIL board

Q8 terminal board and Quanser hardware-in-loop (HIL) board are interfacing terminal boards used for processing signals coming from different parts with the PC. Different signals are processed and sent through it, thus it forms a platform where a different control signals establish a connection between the user and the plant. Q8 terminal board is provided with analog I/O, digital I/O ports, encoder signals etc. The different signals are processed in the Q8 terminal board and converted to digital form, and then sent to the PC for the implementation of control algorithm. 32 bit digital I/O ports serve as a communication medium between the Q8 terminal board and the processor. While HIL board is used for the reason that the motor requires analog signals but the signal generated by the controller algorithm is digital in nature so the HIL board converts this digital signal to analog form and sends to the motor. The Q8 terminal board and Q8 HIL board is shown in Fig. 2.4 and Fig. 2.5:

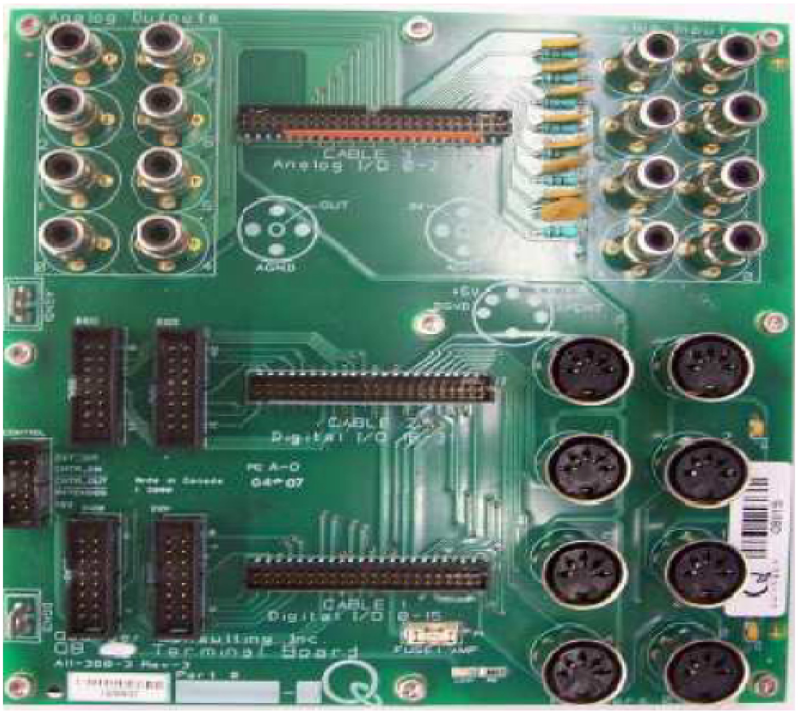

Fig.2.4 Q8 Terminal board

Fig.2.5Hardware-In-Loop (HIL) board

2.1.5 Linear current amplifier

A linear current amplifier with two channels is installed which are used to provide actuator currents to dc motor .There is a provision for current measurement in it, and user can enable or disable it also..

The specification of the linear current amplifier is given in table 2.4

Table 2.4 Amplifier specifications

Parameter	**Rating**
Maximum continuous current	3A
Peak current	5 A
Maximum continuous voltage	28 V
Peak power	300 W
Bandwidth	10 KHz
Gain	0.5 A/V

The control signal from Q8 terminal board to the motor passes through this amplifier. This amplifier has a constant current to voltage gain of 2V/A.

2.2 Dynamic modelling of a flexible link manipulator

In this section we discuss the dynamic modelling of a single link flexible robot. There are several methods of modelling of a single link flexible robot such as Assumed Mode Method and Finite Element Method. In this thesis the Assumed Mode Method (AMM) has been considered for the modelling and discussed in detail.

In the AMM modelling, the elastic deflection of the beam is represented by, infinite number of modes of vibration, but to avoid complex mathematical computations only finite number of modes is considered. In addition to this the model obtained using AMM considers several frequencies of vibration of the beam ,for simplicity only lower frequencies of vibrations are considered as they dominant in system's dynamic behaviour .

Before modelling of the single link flexible robot, we need to consider following assumptions for the link:

- The flexible link of the robot is an Euler –Bernoulli beam with uniform density
- The deflection in the beam is small compared to its length
- The payload mass attached is a concentrated mass
- The Flexible link manipulator operates in horizontal plane.
- Thickness of the beam is small compared to its free length.

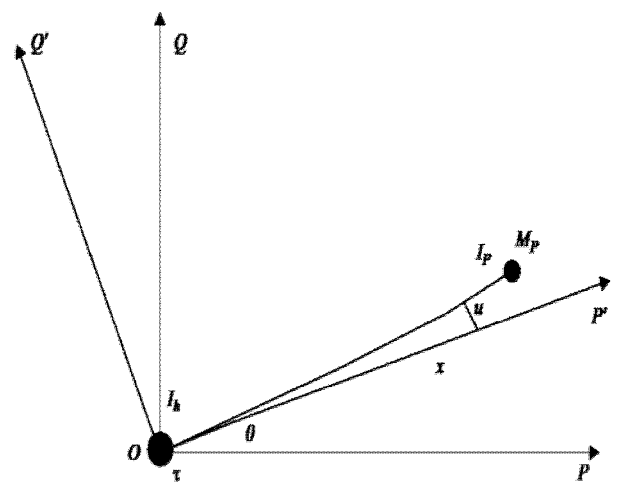

Fig.2.6 Deflection of a flexible link

Fig.2.6 shows a single link flexible robot manipulator under deformation. Some notations used in deriving the model of the single link flexible manipulator system are given as follows:

y(x,t): Position of the tip measured from reference ie. the P-axis in Fig.2.6.

u(x,t): Elastic deflection of the beam measured from undeformed structure.

l: Length of the flexible link.

M_P: Payload mass attached to the tip.

ρ: uniform linear mass density in kg/m .

I_h: the hub inertia.

$\tau(t)$ the torque applied by the motor to the hub.

$\theta(t)$: the joint rotation angle of the beam.

EI: the uniform flexural rigidity of the beam.

Displacement $y(x, t)$ of a point along the manipulator at a distance x from the hub is given as follows:

$$y(x, t) = x\theta(t) + u(x, t) \tag{2.4}$$

To obtain equations of motion of the manipulator, the associated energies have to be obtained. Using extended Hamilaton's principle as discussed in following section we get the equation of motion of planar n-link flexible manipulator.

$$E_K = \frac{1}{2} I_H \dot{\theta}^2 + \frac{1}{2}\int_0^l \left(\frac{\partial u}{\partial t} + x\dot{\theta}\right)^2 \rho dx + \frac{1}{2} M_p \left(\frac{\partial u}{\partial t} + x\dot{\theta}\right)^2_{x=l} \tag{2.5}$$

$$E_P = \frac{1}{2} EI \int_0^l \left(\frac{\partial^2 u}{\partial x^2}\right) dx \tag{2.6}$$

The work done for a given input torque (τ) is

$$W = \tau\theta \tag{2.7}$$

The Euler Bernoulli equation for a beam is given as follows:

$$EI \frac{\partial^4 u(x,t)}{\partial x^4} + \rho \frac{\partial^2 u(x,t)}{\partial t^2} = -\rho x\ddot{\theta} \tag{2.8}$$

Using equations (2.4) & (2.8), we get

$$EI\frac{\partial^4 y(x,t)}{\partial x^4}+\rho\frac{\partial^2 y(x,t)}{\partial t^2}=0 \tag{2.9}$$

The boundary conditions are given as:

$$y(0,t)=0, I_h\frac{\partial^3 y(0,t)}{\partial t^2\partial x}-EI\frac{\partial^2 y(0,t)}{\partial x^2}=\tau(t) \tag{2.10}$$

$$y(x,0)=0,\ \frac{\delta y(x,0)}{\delta x}=0$$

$$\textit{while the mass boundary condition leads:} \tag{2.11}$$

$$M_p\frac{\partial^2 y(l,t)}{\partial x^2}-EI\frac{\partial^3 y(l,t)}{\partial x^3}=0, I_p\frac{\partial^3 y(l,t)}{\partial t^2\partial x}+EI\frac{\partial^2 y(l,t)}{\partial x^2}=0$$

2.2.1 Non linear Model of multi link flexible robot using assumed mode method (AMM)

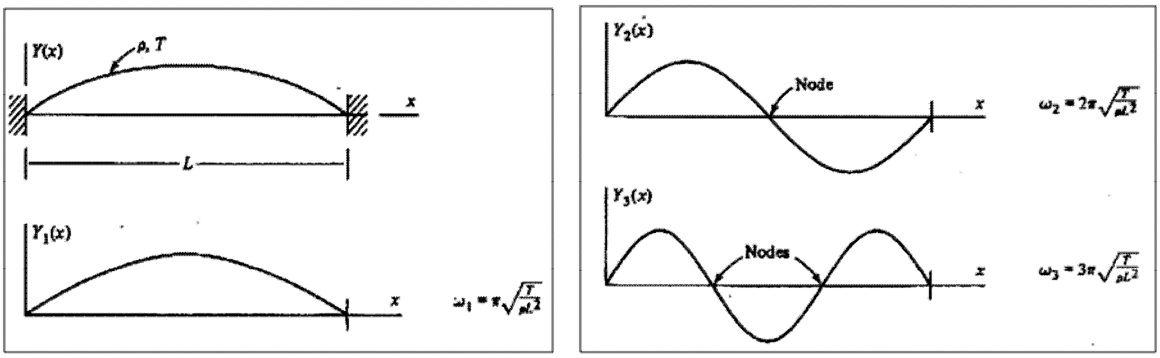

Fig.2.7(a) Mode shapes of a flexible beam under vibration Fig 2.7 (b)

Fig.2.7(a) shows the fundamental frequency of vibration of a Euler beam with fundamental frequency given as follows:

$$\omega_1=\pi\sqrt{\frac{T}{\rho L^2}}$$

where, T is the tension in the beam, ρ is the linear density of the beam and L is the free length of the beam

Fig.2.7(b) shows other frequencies of vibration of the beam known as the overtones and is given as follows:

$$\omega_n = n\pi\sqrt{\frac{T}{\rho L^2}}$$

The overtones are integral multiples of the fundamental frequency and hence known as higher harmonics with fundamental frequency known as fundamental harmonic.

Using the assumed modes method (AMM) solution of the dynamic equation of motion for the tip position of the manipulator can be obtained as a linear combination of the product of admissible functions ϕ_i and ∂_i follows:

$$y(x,t) = \sum_{i=0}^{n} \phi_i(x)\partial_i(t) \tag{2.12}$$

Where the admissible function, ϕ_i also called the mode shape, and is purely a function of the displacement along the length of the manipulator and ∂_i is purely a function of time and includes an arbitrary, multiplicative constant, known as generalized co ordinates of the beam.

Using equations (2.9) and (2.12) we get two ordinary differential equations as:

$$\begin{aligned} &\frac{d^4\phi_i(x)}{dx^4} - \beta_i^4\phi_i(x) = 0,\ \frac{d^2\partial_i(t)}{dt^2} + \omega_i^2\partial_i(t) = 0 \\ &where\ \omega_i^2 = \frac{EI}{\rho}\beta_i^4 \end{aligned} \tag{2.13}$$

Solving equation 2.13 we get the expression for the mode shapes and the generalized coordinates:

$$\partial_j(t) = \exp(j\omega_j t) \tag{2.14}$$

$$\begin{aligned} &\phi_j(x) = C_{1,j}\sin(\beta_j x) + C_{2,j}\cos(\beta_j x) + C_{3,j}\sinh(\beta_j x) + C_{4,j}\cosh(\beta_j x) \\ &\beta_j^4 = \omega_j^2\rho / EI \end{aligned} \tag{2.15}$$

where, ω_j is the j^{th} angular frequency of the link undergoing vibration

Applying boundary conditions we get:

$$C_{3,j} = -C_{1,j} \; and \; C_{4,j} = -C_{2,j} \tag{2.16}$$

While, the mass boundary conditions leads to

$$\begin{bmatrix} F_{1j} & F_{1j} \\ F_{2j} & F_{2j} \end{bmatrix} \begin{bmatrix} C_{1,j} \\ C_{2,j} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$|F| = 0$ leads to a transcendental equation as follows:

$$\begin{aligned} &\left(1+\cos(\beta_j L)\cosh(\beta_j L)\right) - \frac{M_L \beta_j}{\rho}\left(\sin(\beta_j L)\cosh(\beta_j L) - \cos(\beta_j L)\sinh(\beta_j L)\right) \\ &-\frac{J_L \beta_j^{\,3}}{\rho}\left(\sin(\beta_j L)\cosh(\beta_j L) + \cos(\beta_j L)\sinh(\beta_j L)\right) + \frac{M_L J_L \beta_j^{\,4}}{\rho^2}\left(1-\cos(\beta_j L)\cosh(\beta_j L)\right) = 0 \end{aligned} \tag{2.17}$$

Substituting the values of the known parameters from Table 2.2 and Table 3.1 in equation (2.17), we found the values of β as follows:

$\beta_1 = 3.5513$, $\beta_2 = 7.72748$

and the values of coefficients are as follows:

$C_{11} = 0.437$; $C_{21} = -0.31$; $C_{12} = 0.255$; $C_{22} = -0.3257$;

Thus the modal shapes at the tip position obtained are as follows:

$\Phi_1 (L) = 0.3116$, $\Phi_2 (L) = 0.3066$

where, L is the length of the link. L=0.22m.

Higher values of β have been neglected to avoid mathematical complexity. As mentioned earlier also only two modes of vibration are sufficient to take care of the dynamic behaviour of the single link flexible manipulator. Hence we have considered only two frequencies of vibration of flexible link under vibration.

Applying Lagrange -Euler equation,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial \dot{q}_i}=W_i \tag{2.18}$$

where, L=E$_k$-E$_p$,

We get the dynamic equations of motion for a planar n-link flexible arm which can be written in the closed form as:

$$M(q)\ddot{q}+h(q,\dot{q})+Kq=T \tag{2.19}$$

where, $q=(\theta,\partial)^T$ represent the state vector of the model, There are seven state variables taken in the model given as follows:

$$q=\begin{bmatrix}\theta & \partial_1 & \partial_2 & \dot{\theta} & \dot{\partial}_1 & \dot{\partial}_2 & i_a\end{bmatrix}$$

M is a positive-definite symmetric inertia matrix,

h is a vector of coriolis and centripetal forces,

K is the diagonal stiffness matrix.

i_a is the actuator current.

T is a column vector consisting of control torque at the joint location.

Substituting all the four boundary conditions from (2.10), (2.11) one may get the expression for the modal shape as:

$$\phi(x)=D\left(\sinh\beta x-\sin\beta x-\frac{(\sin\beta L+\sinh\beta L)}{(\cos\beta L+\cosh\beta L)}\left(\cosh\beta x-\cos\beta x\right)\right) \tag{2.20}$$

The coefficients of mass matrix (M) lare:

$$M_{11}(\partial)=J_0+J_L+M_L L^2+I_0+M_L(\phi^T{}_e\partial)^2 \tag{2.21}$$

$$M_{1j}=M_L L\phi_{j-1,e}+J_L\phi'_{j-1,e}+\sigma_{j-1} \quad , j=2,...,m+1 \tag{2.22}$$

$$M_{ii}=m_b+M_L\phi^2{}_{i-1,e}+J_L\phi'^2{}_{i-1,e}, \qquad i=2,...,m+1 \tag{2.23}$$

$$M_{ij=}M_L\phi_{i-1,e}\phi_{j-1,e}+J_L\phi^{'}_{i-1,e}\phi^{'}_{j-1,e},\quad i=2,...,m+1 \tag{2.24}$$

where,

$$\phi_e^{\ T}=\phi^T\,|_{x=l}=\left[\phi_1\ ...\ \phi_m\right]|_{x=l}\quad \phi_{ie}=\phi_i(l) \tag{2.25}$$

$$\sigma_i=\rho AL^2\int_0^l\phi_i(x)xdx\quad i=1,2...,m+1 \tag{2.26}$$

The non-linear terms h_l and $h_{2,}$ known as Coriolis and Centrifugal forces respectively are given below:

$$\begin{aligned} h_1 &= 2M_L\dot{\theta}(\phi_e^{\ T}\partial)(\phi_e^{\ T}\dot{\partial}) \\ h_2 &= -M_L\dot{\theta}^2(\phi_e\phi_e^{\ T})\partial \end{aligned} \tag{2.27}$$

Equivalent spring constant matrix K is given below as

$$K=diag\left\{0\ ,\ \omega^2{}_1m_b\ ,\omega^2{}_2m_b\right\} \tag{2.28}$$

and damping matrix F is given as:

$$F=diag\left\{0\ ,\ \sqrt{\omega^2{}_1m_b}\ ,\sqrt{\omega^2{}_2m_b}\right\} \tag{2.29}$$

where, m_b is the link mass,

I_o is the joint actuator inertia,

J_o is the link inertia relative to the joint,

M_L and J_L is load mass and load inertia, respectively.

2.2.2 Non-linear model with actuator dynamics

The actuator used in this setup is a dc motor and is placed at the hub of link .The DC motor is connected through a gear-box which ensures the safe operation of the setup. The control input to motor is fed from the linear current amplifier and harmonic drive limits the speed of operation.

Let T_m, T_1, T_2 and T_L be the torque developed by the motor, torque at motor shaft, torque transmitted to the load and load torque respectively, θ_m and θ_L (θ, as assumed earlier) be the speed of motor at motor shaft and load shaft respectively and J_m and J_L are the inertias of motor and load respectively, R_a, L_a, K_t, K_b and N_r are the armature resistance, inductance, motor torque constant, back emf constant and gear ratio, respectively.

Applying KVL one may get the voltage equation for the armature circuit as:

$$u = L_a \frac{di_a}{dt} + R_a i_a + e_b \tag{2.30}$$

where, $e_b = K_b \dot{\theta}_m$ is the back-emf generated in the armature circuit. Given the motor voltage u the current i_a flow through the armature circuit and develops electro-magnetic torque as

$T_m = K_t I_a$.

The harmonic drive is integrated with motor shaft and the flexible link is mounted on the harmonic drive. The ratio of speed can be given as follows:

$$N_r = \frac{\theta_m}{\theta_L} = \frac{T_2}{T_1} \tag{2.31}$$

The torque balance equation can be written in equation (2.32) as follows:

$$T_m = J_m \ddot{\theta}_m + T_1 \tag{2.32}$$

and torque transmitted to load is given as follows:

$$T_2 = J_L \ddot{\theta}_L + T_L \tag{2.33}$$

From equations (2.31)-(2.33), the load torque can be given as

$$T_L = N_r T_m - J_h \ddot{\theta} \tag{2.34}$$

where, hub inertia of the flexible link robot $J_h = J_L + N_r^2 J_m$ is the total inertia referred to the load side of the motor. Substituting for T_m in equation (2.34), we get expression for load torque as follows:

$$T_L = N_r K_t i_a - J_h \ddot{\theta}_L \tag{2.35}$$

The torque T_L developed as given in equation (2.35) is used to drive the flexible link through the speed reducer. The actuator current i_a is taken as one of the states in the model obtained using AMM. Thus the model obtained using AMM takes care of the actuator dynamics also. Thus the actuator dynamics increases the system order by one.

2.3 Model Validation

To test the validity of the non linear model proposed we have executed the model on the experimental setup with same input signals and got the response in terms of deflection of the tip from its undeformed structure and the hub angle of the base of the beam undergoing vibrations. Here the model validation is done using a bang-bang input with amplitude of 0.5 N-m (Fig. 2.8, Fig. 2.9 & Fig. 2.10).

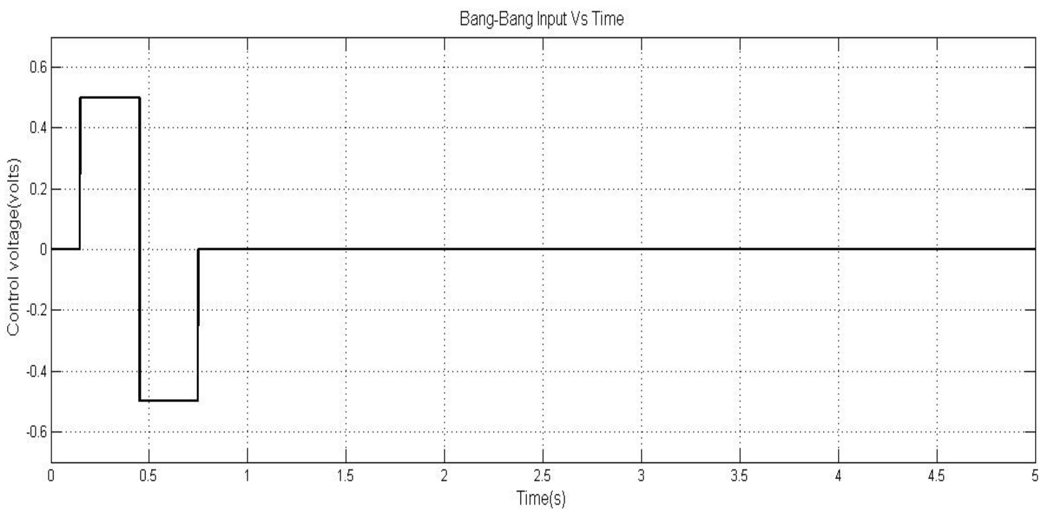

Fig. 2.8 Bang-Bang input

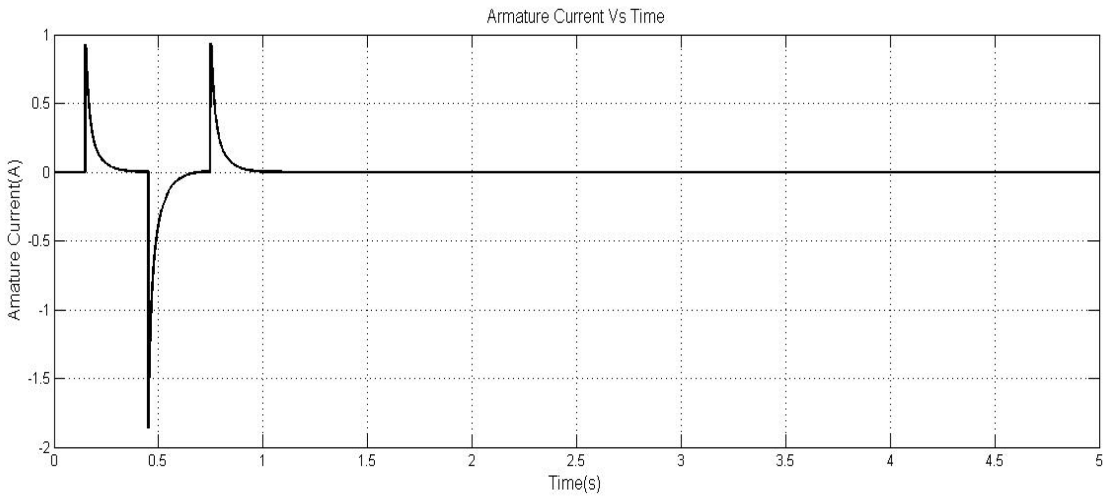

Fig.2.9 Actuator current (i_A)

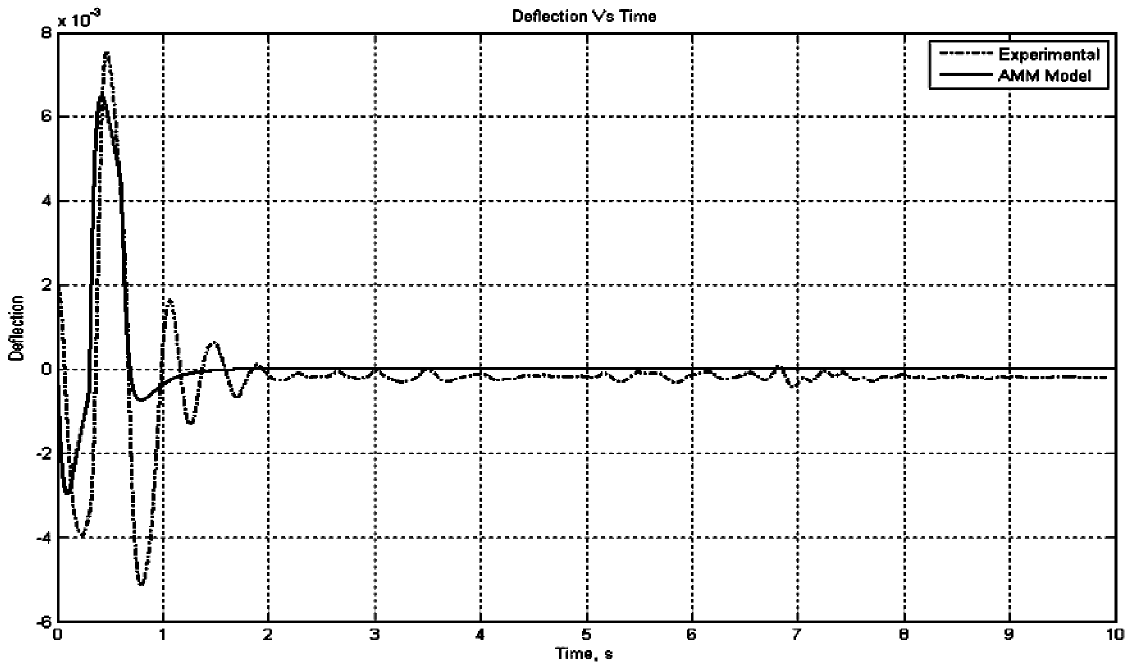

Fig.2.10 Deflection (m) of the tip

Fig.2.10 shows the Experimental and AMM model deflection obtained using simulation. Fig.2.9 shows the plot of actuator current. The plot of actuator current shows a similar pattern to that of the bang bang input provided to the system. Although it does not match exactly in direct proportionality as described in section 2.2.2.

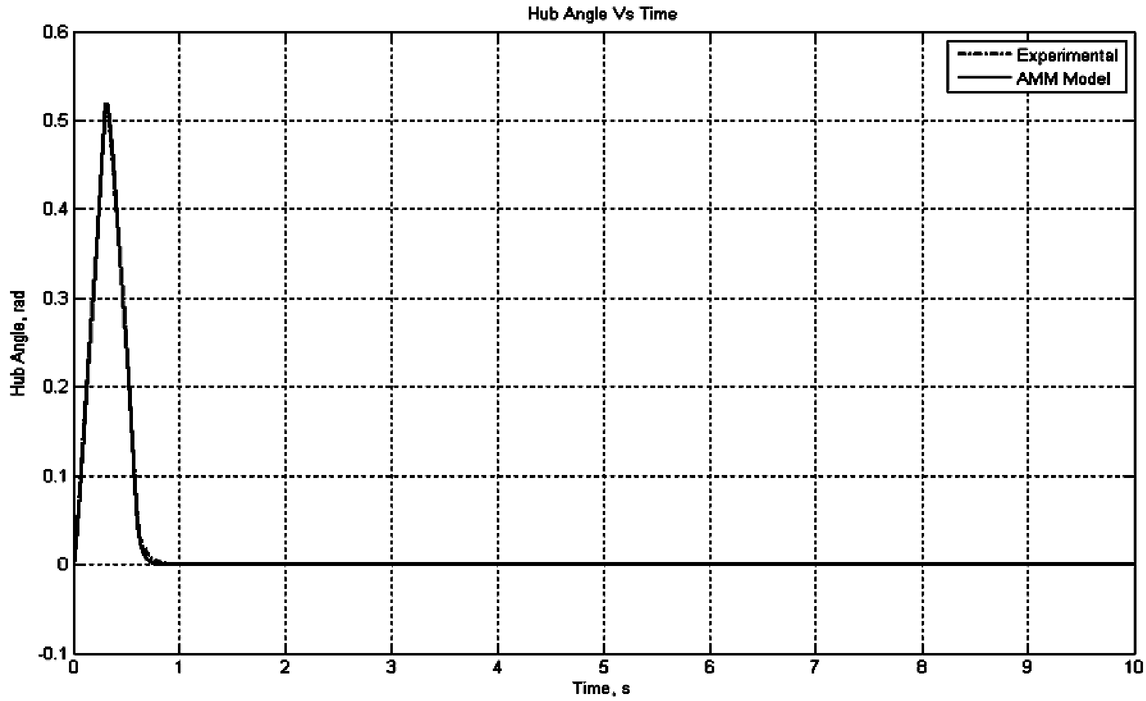

Fig. 2.11 Hub angle

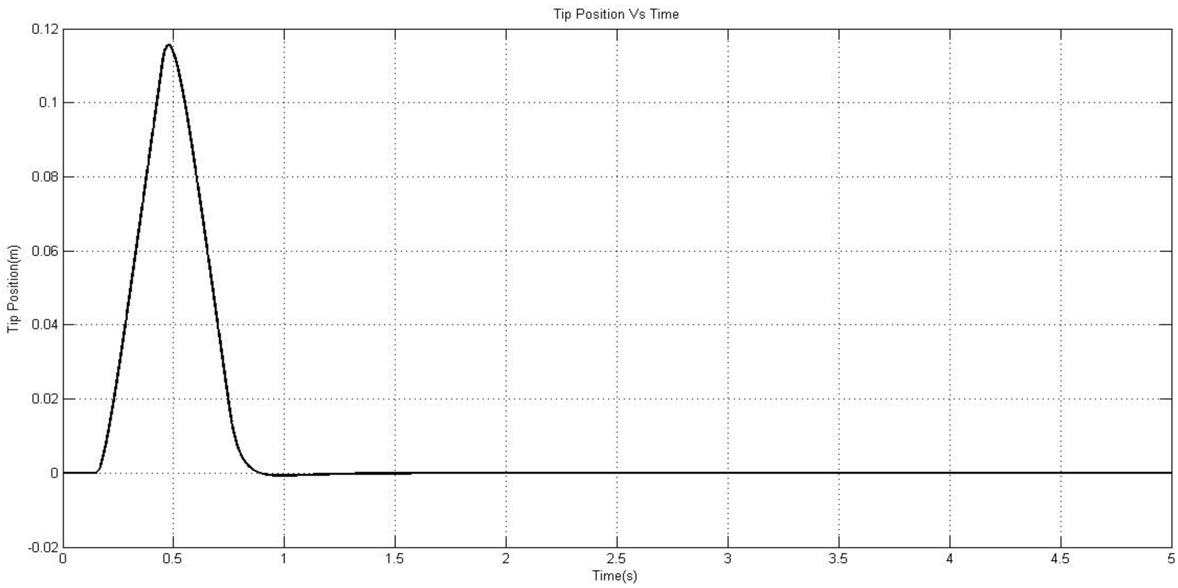

Fig.2.12 Tip position of the flexible link

2.4 Results of model validation and Summary of the Chapter

From the model validation results as shown in Fig. 2.8 -Fig. 2.12 the simulation results agrees with the experimental results. There is almost no variation in the hub angle obtained by the experiment and simulation results (Fig.2.11). The deflection also shows similar nature having an acceptable accuracy of the order of 0 4 mm at 0.8th second of its operation Fig. 2.10. While Fig. 2.12 shows the tip position obtained by simulation results, the result obtained from the plant was also same with no visible error which is obvious from Fig.2.11, and hence has not been shown. It is clear that the deflection in the tip position obtained experimentally is sustained even after a time span of 1.4s in comparison to our simulation results which dies out at the same time instant (Fig 2.10). Nevertheless the magnitude of that deflection is of the order of 0.0008m which can easily be neglected. More importantly the hub angle (Fig. 2.11) of the flexible link under running operation is almost the same as that obtained from the simulation result. Thus experimentally we have validated the non linear model obtained using assumed mode method (AMM).

In this chapter the major components of a single link flexible robot system installed in the lab has been discussed. It is followed by the method of modelling of a single link flexible Robot using AMM in which actuator dynamics is also taken into consideration. The chapter ends with the model validation of the single link flexible manipulator setup installed in lab. The results obtained agree with the simulation results (Fig. 2.10, Fig. 2.11).

Chapter 3

Iterative Learning based Controller Design

3.1 Introduction

There is a number of different control techniques already developed for multilink flexible robots. Initial works on control strategy of flexible robots include linear control techniques based on the transfer function model. In the works of Oakley and Canon [23] Canon and Schimtz [24], a non linear model using assumed mode method was derived. A PD controller was then designed and was applied to the linear model as mentioned in their works. They also did their model validation for a step command. Recently many new controller designs have been developed for multilink flexible robots by the researchers in this field. Some of the noted work of A De Luca and Siciliano [15] addressed an inversion based control technique applied for the assumed mode method model. Gopinath and Kar have proposed an ILC based controller design for industrial robot manipulator that performs repeated tasks[7].

3.2 Iterative Learning based controller design

In recent developments ILC technique has become a desirable methodology for the control of real time systems operating in repetitive manner. A variety of real-life control-engineering problems, for example electrical systems such as electrical drives, mechanical systems such as robotic manipulators, bioengineering systems, chemical process systems such as batch reactors, as well as aerodynamic systems, and others can be solved using Iterative learning control (ILC) techniques. Iterative Learning Control (ILC) is a control method designed for the system showing repetitiveness in its operations. Iterative learning based control technique is used to enhance tracking performance, using the error inputs obtained from each trial.

3.3 Advantages of ILC:

Some advantages of using Iterative learning based control technique are as follows:

- Simplicity of the structure,
- Good output tracking,
- Model-independent design,
- Improves transient response and tracking performance of processes or system that executes the same operation over and over.
- Takes care of non-linear effects such as friction, actuator dynamics etc.

Thus ILC possesses greater advantages over other traditional controller designs employed for the plants which are having uncertainty and delay.

3.4 Iterative Learning based Controller design for a single link flexible Robot system.

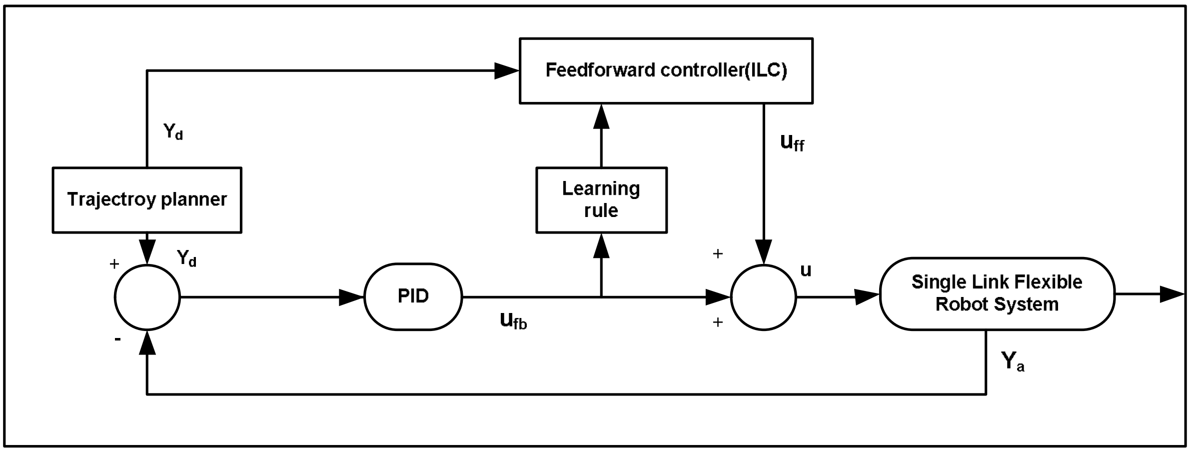

Fig.3.1. Structure of ILC based controller for flexible manipulator system

Fig. 3.1 shows a feed forward Iterative learning based controller structure. The learning rule as discussed in section 3.4 updates the reference input in each trial of its operation. The entire procedure is essentially carried out over two phases [6]. In the first phase, a modified ILC procedure which consists of ILC update law is carried out to yield the ideal input and output signals of the overall ILC-augmented control system. The second phase uses these signals 'e'(error) and Δu(the change in reference input) to identify the best-fitting PID parameters using a standard least-squares (LS) algorithm .Strain gauge measures the tip deflection in meter as given in equations (2.1)-(2.3) and optical encoder provides the joint angle. These sensor signals are directly used in the controller design. Initially, a repetitive signal is used to find out the tuned PID_2 parameters. Finally the controller is also tested for a non repetitive signal viz. a step input also.

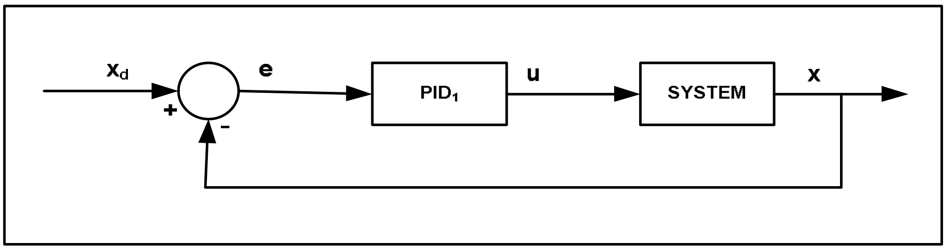

Fig.3.2 Basic PID controller for the system

First a simple PD controller was chosen. The controller output of the PD controller is given as:

$$u(t) = K_p e(t) + K_d \frac{de(t)}{dt} \tag{3.1}$$

where, $e(t)=(\theta(t)-\theta_{ref})$ is the error in the hub angle of the single link flexible robot manipulator ,the proportional term K_p produces a controller output u that is proportional to the error value. The proportional response can be adjusted by multiplying the error by a constant K_p, called the proportional gain constant. The derivative term K_d slows the change in transient response .It also reduces the magnitude of the overshoot produced by the integral component.

Fig.3.3 shows that an ILC is employed to modify the output of the controller PID_1 to enable the output x to track x_d not x_d'.

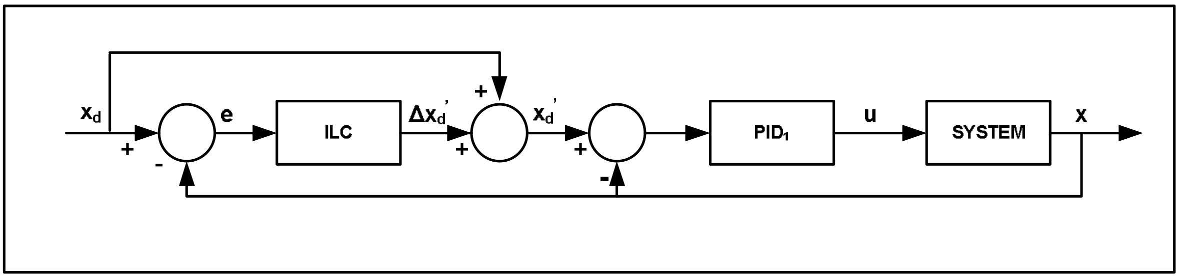

Fig.3.3. ILC controller structure

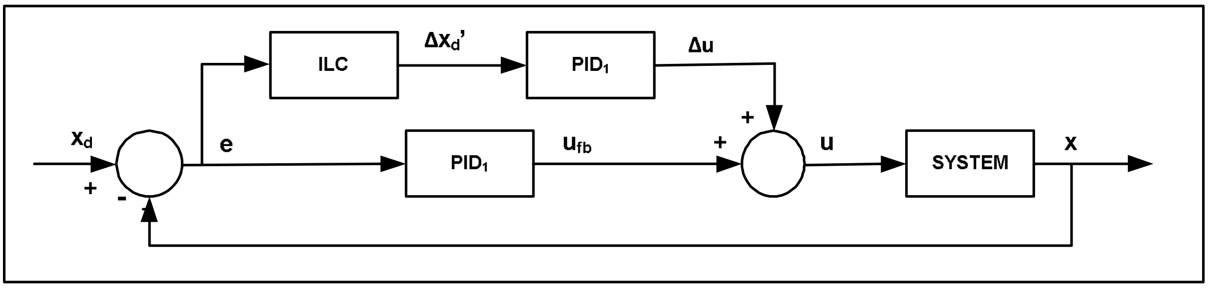

Fig. 3.4 Modified ILC diagram

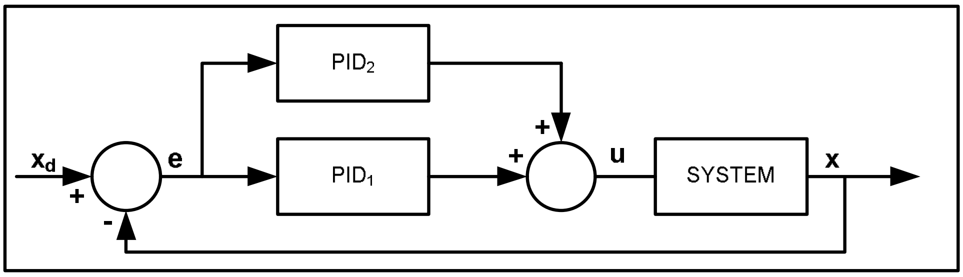

Fig.3.5 Tuned PID controller design

Fig.3.4 is the same derived form of the ILC structure as shown in Fig.3.3.The structure of the modified ILC diagram (Fig. 3.4) is easier to implement on the SIMULINK and therefore has been considered in the project. Using this structure we obtain the tuned PID controller structure (Fig.3.5) based on Least Square (LS) estimation method.

3.4.1 ILC Update law and convergence condition

A P-type update law[25] has been adopted for ILC. In this update law the change in the reference input is directly proportional to the error in each trial and hence the term P-type. During the i_{th} iteration of the operation, the modified desired trajectory is given by:

$$x_{d,i}'(t) = x_d(t) + \Delta x_{d,i}'(t) \tag{3.2}$$

The update law for ILC is

$$\Delta x_{d,i+1}'(t) = \Delta x_{d,i}'(t) + \lambda e_i(t+1) \tag{3.3}$$

where, λ is the learning rate.

Convergence condition: The actual error 'e' becomes very small within bound of 0.002m and ILC stops updating. The ideal input e and output Δu for a cycle of the reference signal is now available for the next phase.

For a P-type update law, the learning rate λ must satisfy the following convergence condition [25]

$$\| I - CB\lambda \|_{\infty} < 1 \tag{3.4}$$

3.4.2 Least square algorithm

Using e and Δu Fig.3.4 we have calculated new PID_2 parameters as per the LS method given as follows:

$$\begin{aligned} &\Delta u(t) = \varphi^T(t)\theta \\ &where\ \theta = [k_{p2}\ k_{d2}]^T\ and\ \varphi^T(t) = \left[e(t) \frac{de(t)}{dt} \right] \end{aligned} \tag{3.5}$$

where, k_{p2} and k_{d2} are the tuned gain parameters or the PID_2 gain parameters (Fig.3.5)

Using LS algorithm we have

$$\begin{aligned} &\hat{\theta} = (\phi^T\phi)^{-1}\phi^T U, \quad where\ \phi = \left[\varphi^T(1)\ \varphi^T(2).....\varphi^T(N)\ \right]^T \\ &U = \left[\Delta u(1)\Delta u(2)....\Delta u(N)\right]^T \end{aligned} \tag{3.6}$$

and N is the number of data used in estimation. From phase 1 of the ILC implementation we get the number of estimates used in LS algorithm as follows:

N=166 (approx), using this value and equation (3.6) we get the tuned gain parameters in phase two of ILC. Thus the tuned PID controller design is obtained (Fig.3.5).

3.5 Simulation Results for a Single Link Flexible Robot

System parameters of a two link flexible robot manipulator are shown in table 3.1.We have considered only link 1 in our experiment.

Table 3.1 System parameters

Parameters	Link 1	Link 2
Maximum continuous current	0.944 A	1.21 A
Gear ratio	100	50
Moment of inertia at motor shaft	$6.28X10^{-6}$ Kg m^2	1.03 $X10^{-6}$ Kg m^2
Moment of inertia of drive mounting bracket	$7.361X10^{-4}$ Kg m^2	$444.55X10^{-6}$ Kg m^2
Moment of inertia of compounded end effector system	0.17043 Kg m^2	0.0064387 Kg m^2
Torsional stiffness constant	22 Nm/rad	2.5 Nm/rad
Mechanical time constant	5 ms	4 ms
Young's modulus of elasticity	$2.0684X10^{11}$ Pa	$2.0684X10^{11}$ Pa

1. Desired Trajectory (Fig.3.6): First a sinusoidal input has been applied as an input signal. The frequency of the signal is 0.5Hz and amplitude 0.5 radians.

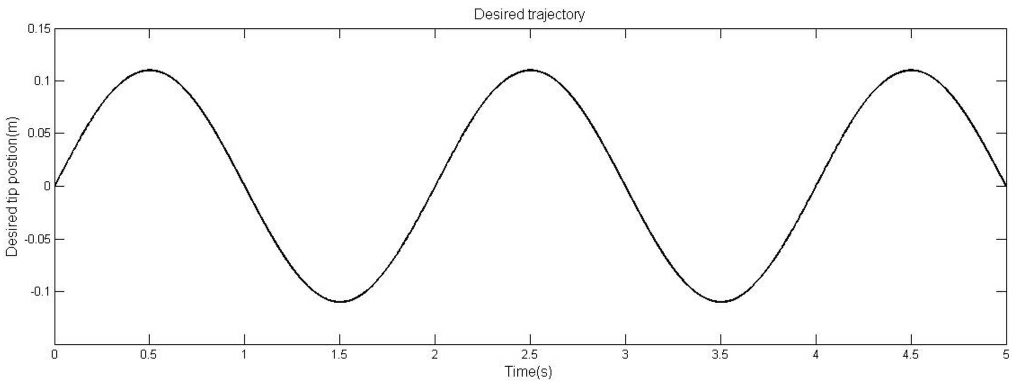

Fig.3.6 Desired tip position in meters

2. Actual Tip position without any controller (Fig.3.7): The simulation is carried out for a time interval of 5 sec. The curve of tip position without any controller shows that because of the link inertia, the tip undergoes a higher deflection than the desired trajectory in the first quarter of the cycle and at each point of its trajectory it never reaches its desired position.

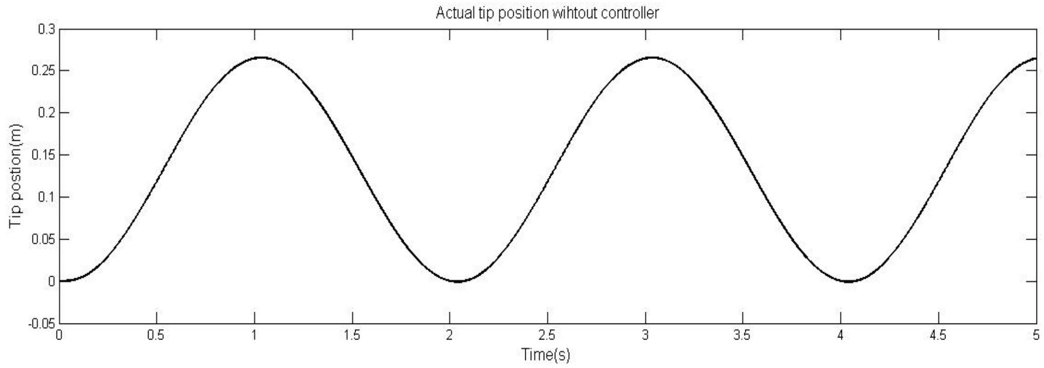

Fig.3.7 Actual tip position in meters without any controller

3. Error in tip position without any controller (Fig.3.8): The error plot shows that it is also repetitive in nature having a maximum absolute value of 0.32 m .This value is very high and needs to be rectified. So we need to apply a controller to remove this unwanted error. For this we have applied a PD controller which will be tuned using an ILC approach.

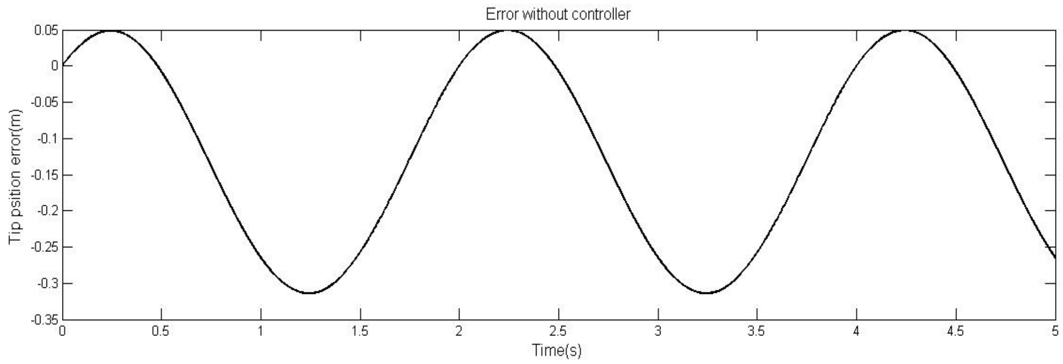

Fig.3.8 Tip position error without any controller

4. Tip position under initial PD controller (Fig.3.9): Initially we have taken a simple PD controller with gain parameters as: k_p =0.01 and k_d = 0.1.The gain parameters is chosen arbitrarily. But using Iterative learning control technique we can tune these parameters to achieve vibration suppression more effectively, which are discussed later.

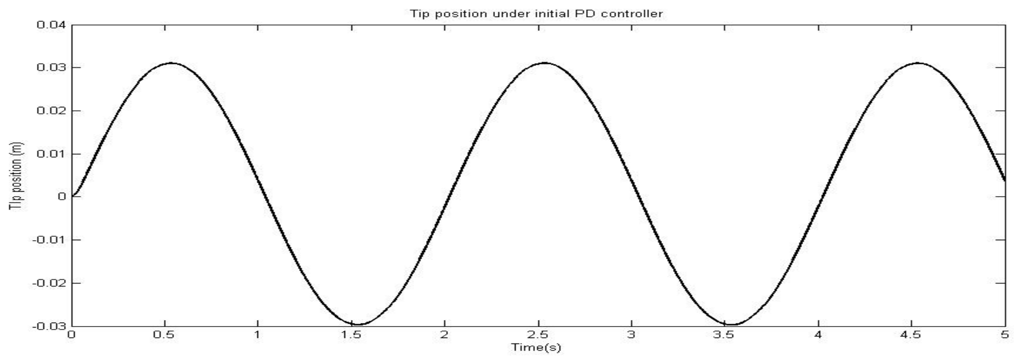

Fig.3.9 Actual tip position under Initial PD controller

5. The error plot shows that using a PD controller the error in tip position is reduced having a maximum value of .08 meters as compared to 0.32 meters previously under no controller, but still this error is not desired.

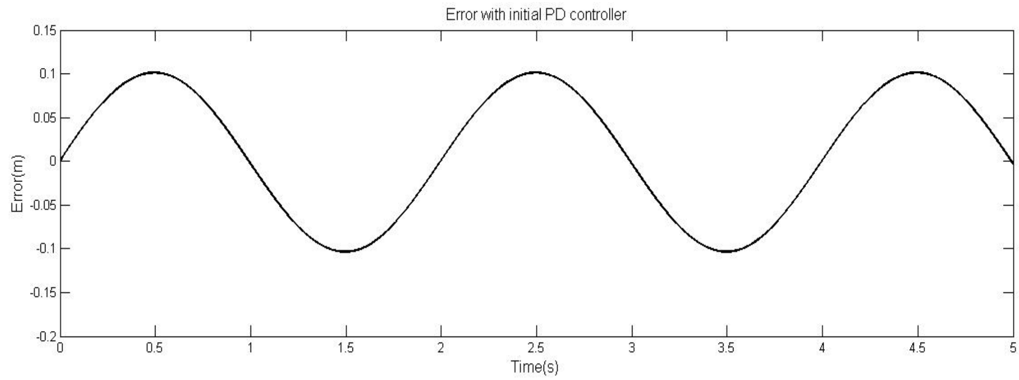

Fig.3.10 Tip position error under $k_{p}=0.01$ and $k_d = 0.1$

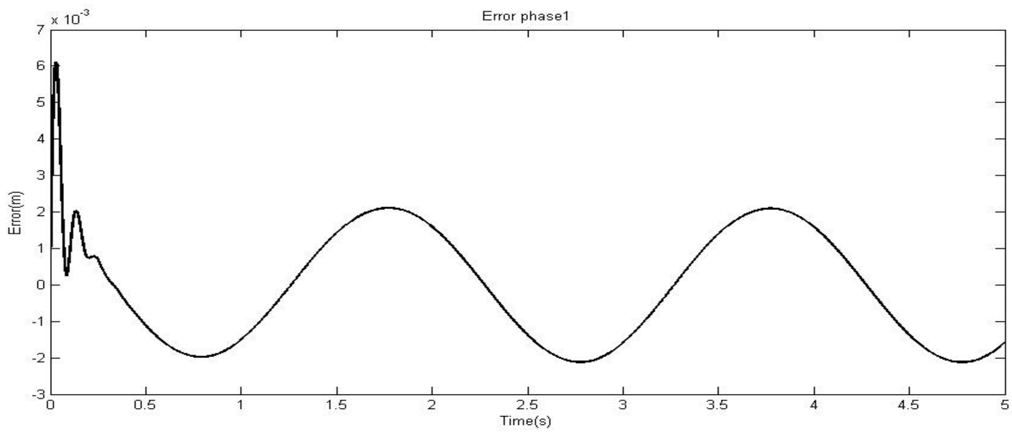

Fig.3.11 Error in tip position obtained from ILC under phase1

6. Fig.3.11 shows the error plot obtained at the end of phase 1 of ILC. An Iterative learning based controller has been employed to tune the arbitrarily chosen gain parameters using equations (3.2) and (3.3).The ideal input e and output Δu as shown in Fig 3.4, are obtained from this phase. With each trial the value of error e is reduced and is found to exist within an acceptable bound of 0.002m of absolute value(Fig.3.11). The values of error in each iteration (*e(i)*) along with the change in reference input, *Δu(i)* are used in the LS algorithm to tune the existing PD parameters to generate new PD gain parameters. The data of tuned PD gain parameters is given in the Table 3.2.

Table.3.2 ILC based tuned PD controller gains

Learning rates(λ)	New Proportional gain(k_{p2})	New derivative gain(k_{d2})	Mean error(e_{rms})
0.1	100.0504	-0.1333	0.0006864
0.3	300.1521	-0.0593	0.0005206
0.5	500.7875	-0.0939	0.002143
0.8	800.3845	-0.0592	0.02527
0.05	50.0405	-0.1696	0.001342

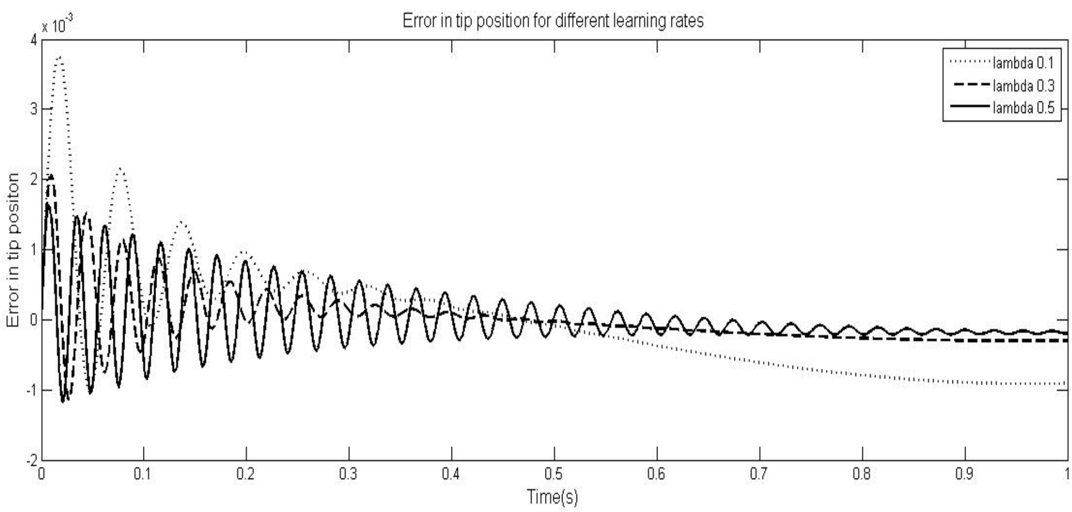

Fig.3.12 Tip position error (m) under tuned PD controller

7. Error in tip position for learning rates of 0.1, 0.3 and 0.5:It is obvious from the plot of error in Fig.(3.12) that error reduced significantly for the three learning rates 0.1, 0.3 and 0.5.For a learning rate of 0.5 the error plot initially showed high oscillations which can cause the beam to vibrate at high frequency. But for learning rate of 0.1 the initial vibrations was not so huge having e_{rms} of 0.0006864 and it reduced to an acceptable bound having an absolute value of 0.001 the error shown by a learning rate of 0.3 is also within acceptable bound.

8. Actual tip position for a learning rate of 0.1

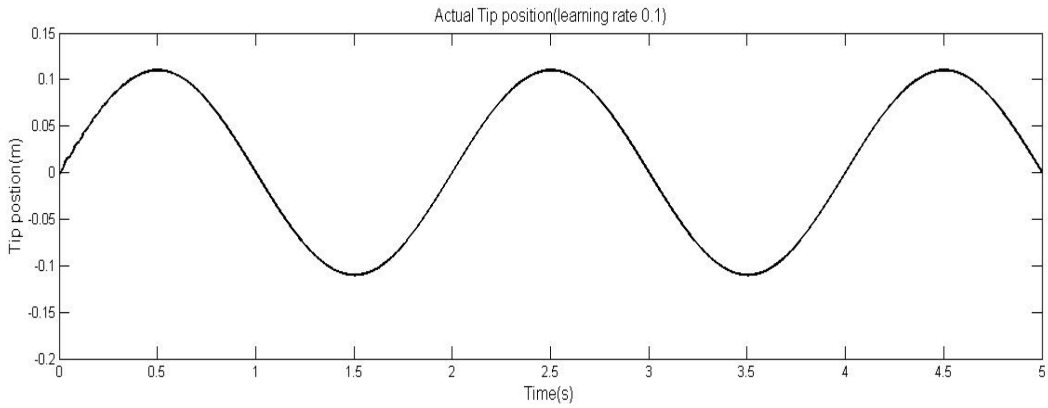

Fig.3.13 Tip position for a learning rate of 0.1

Fig. 3.13 shows the actual tip position of the single link flexible robot for a learning rate of 0.1.From Fig.3.11 the oscillations in the tip position increase rapidly with increase in the learning rate making the system unstable. Thus till now we have achieved the minimization in the tip position error of a vibrating flexible link for a sinusoidal input. Learning rates of 0.1 and 0.3 have given better results as compared to learning rate of 0.5 and above. Now to check the versatility of the proposed ILC controller we need to apply the controller for non-repetitive input. We have given a step input to the Single link flexible robot manipulator system.

9. Desired trajectory for a step input: Step size 0.22m(Fig.3.14).

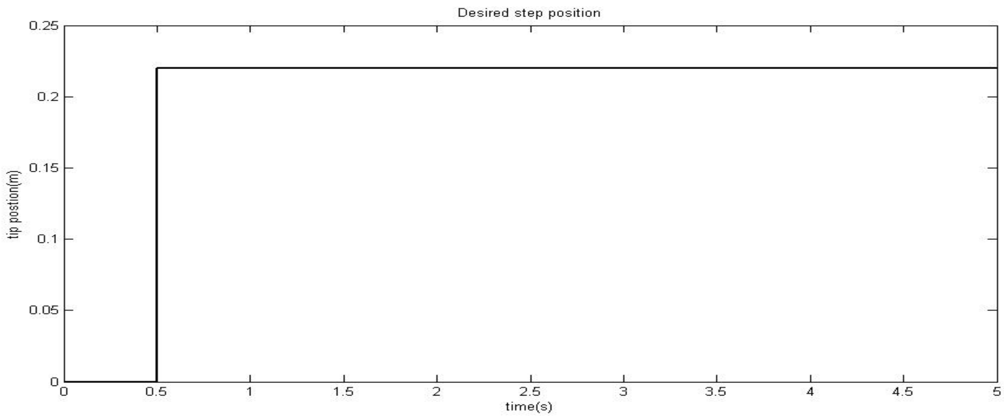

Fig. 3.14 Desired trajectory, a step function as an input

In the simulation we have given a desired hub angle with step size 1 radian.This hub angle multiplied by the length of the link(0.22m) gives the desired tip position in meters(Fig.3.14).

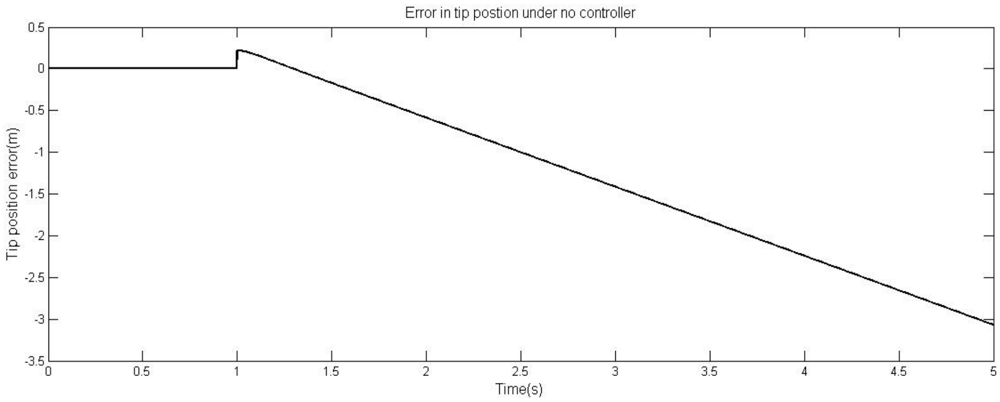

Fig. 3.15 Error in tip position without any controller

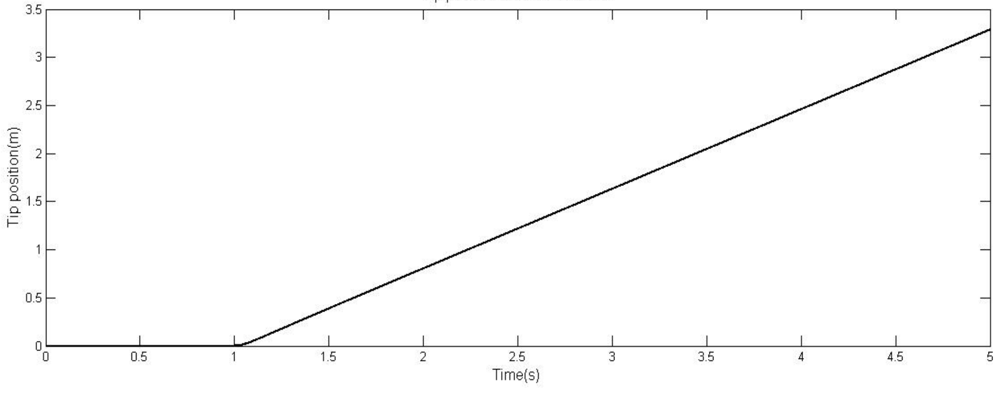

Fig.3.16 Tip position of the flexible link under no controller

From Fig.3.15-3.16 it can be easily understood that without any controller the tip position keeps on increasing due to inertia of the link when is provided with a step torque.

10. For a PD controller having gain parameters as $k_p = 0.01$ and $k_d = 0 .1$, the tip position is shown in Fig.3.17.

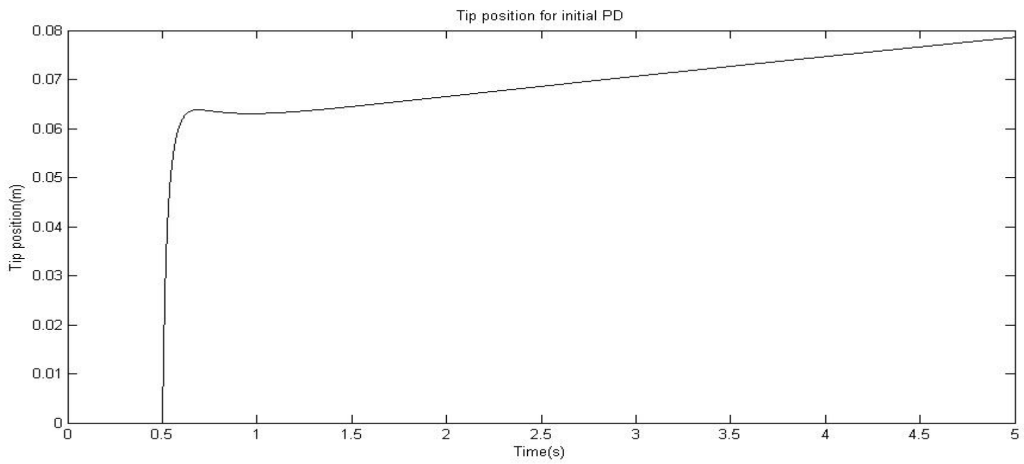

Fig. 3.17 Tip position under initial PD controller

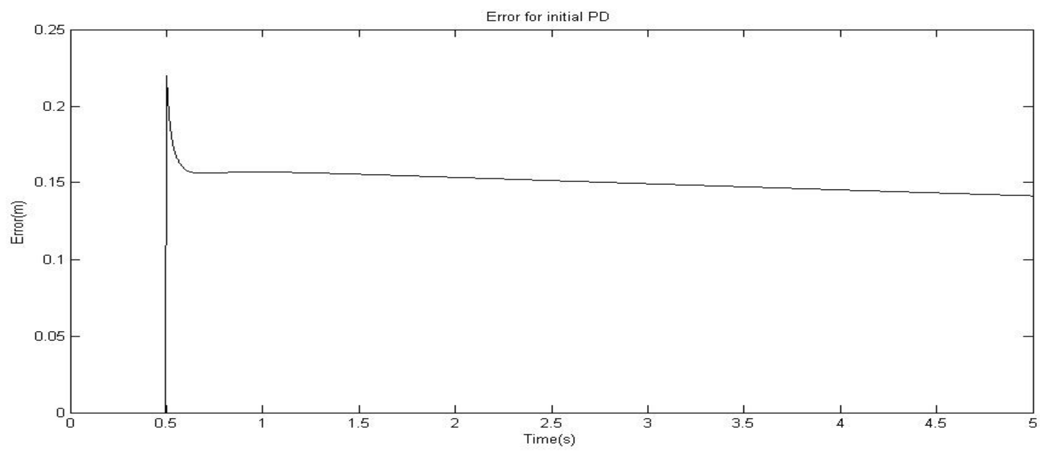

Fig. 3.18 Error in tip positions under initial PD controller

From Fig.3.18 it is clear that the tip position never reaches its desired trajectory in the time span of 5 seconds of its simulation. The error plot (Fig.3.18) shows a positive error in the tip position of the flexible link. This error shows that the chosen PD controller is not enough to make the tip of the link to track its desired trajectory. In fact, this error is substantial in magnitude having a maximum value of 0.2 meters and with passage of time it does not reduce quickly.

11. When the tuned PD controller is applied, the error reduces significantly. For different learning rates the result are as shown in Fig.3.20.The tip position of the flexible link for learning rates of 0.3 and 0.1 are shown in Fig.3.19 and Fig.3.21.

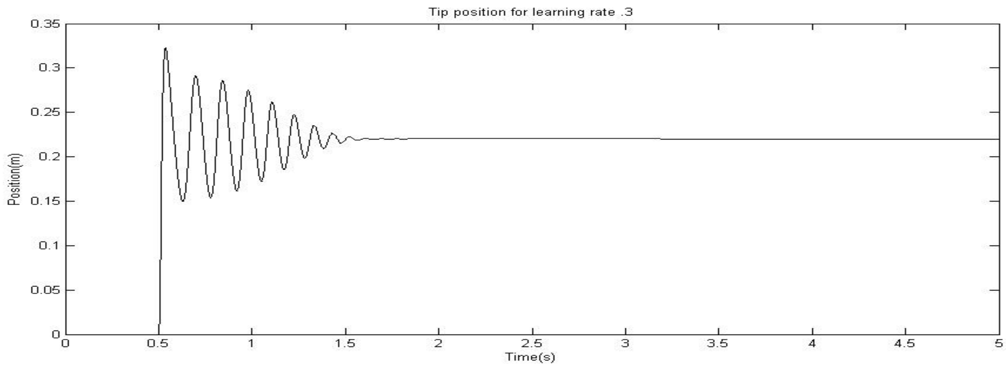

Fig. 3.19 Tip position for a learning rate of 0.3

Fig.3.19 shows that although small in magnitude, but there are initial oscillations in the tip position of the single link flexible robot manipulator. These oscillations fade out within 1 sec of its operation and the desired trajectory is achieved. But we need to check the response for other learning rates also. This can be clarified from the plot of errors for different learning rates as shown in Fig.3.20.

From Fig.3.20 on comparing the result of errors in tip position for the learning rates of 0.3, 0.1 and 0.2, the response of learning rate 0.1 seems to be most desirable. We have observed in case of a sinusoidal input also that the response of the learning rate 0.1 in terms of precise tip tracking was having negligible error.

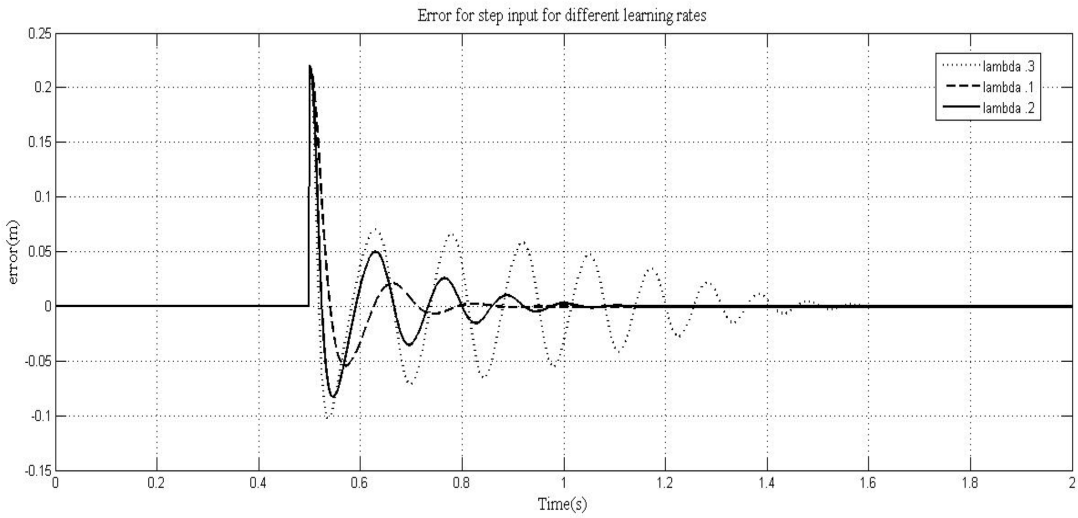

Fig.3.20 Error in tip position for different learning rate

The tip position for a learning rate of 0.1 is shown in Fig.3.21 as follows:

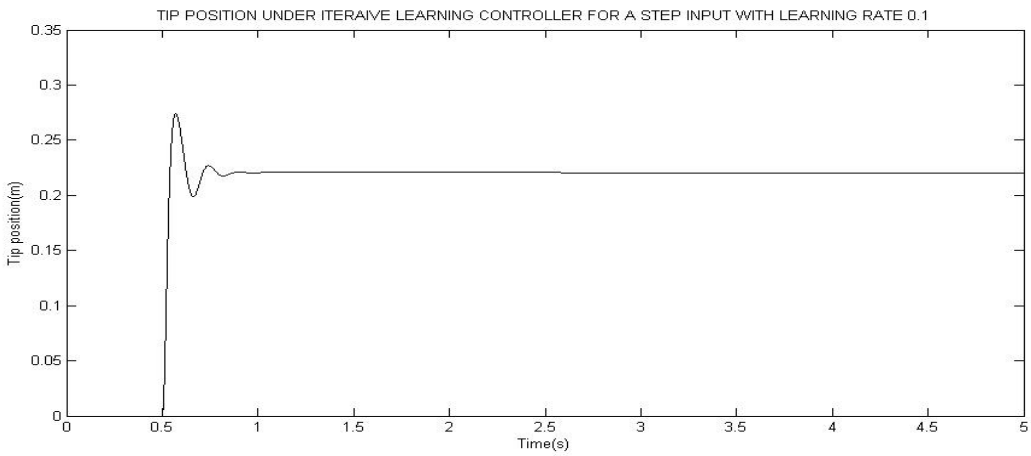

Fig. 3.21 Tip position for a learning rate of 0.1

Fig.3.21 shows that the position of the tip undergoes small oscillations and settles down to its desired trajectory very quickly. We have checked the ILC based controller for certain learning rates which have shown satisfactory results. Now to check the superiority of the above chosen learning rates over other learning rates we have simulated the same controller design with new learning rates of 0.05 and then with 0.5.

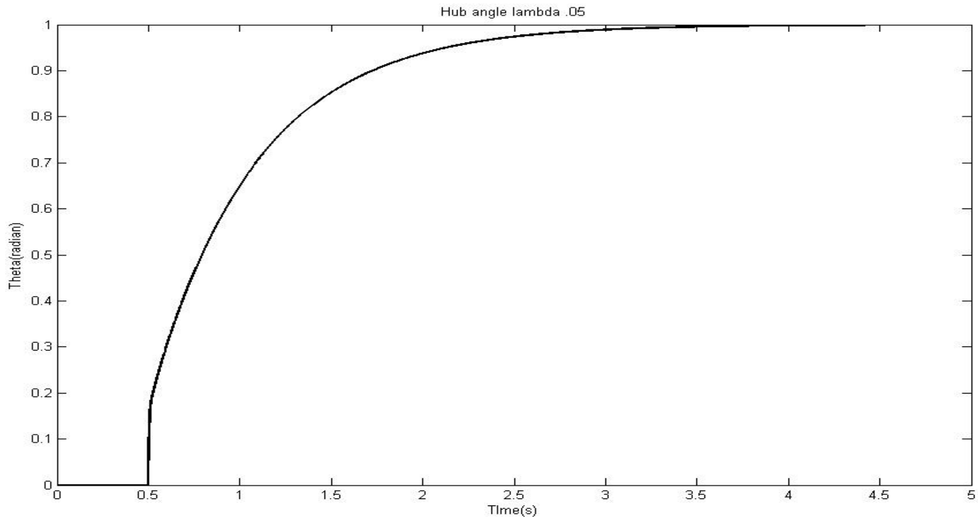

Fig.3.22 Hub angle of the single link flexible robot for learning rate 0.05

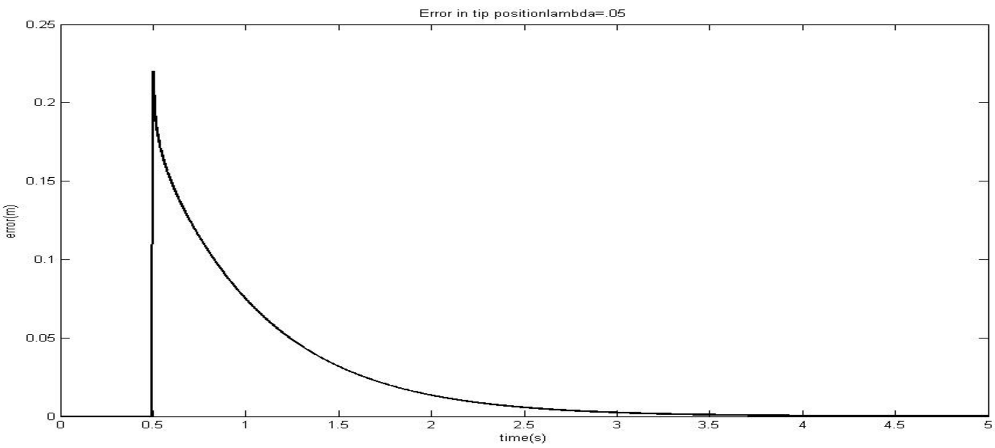

Fig.3.23 Error in tip position for learning rate 0.05

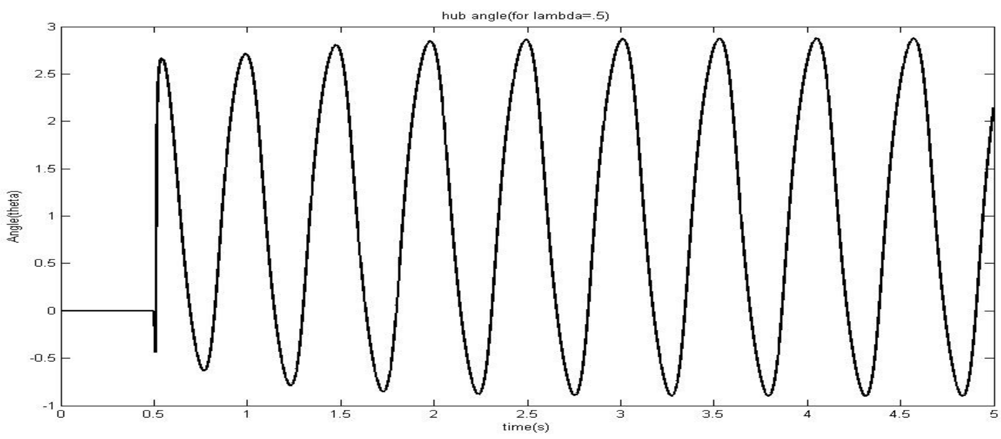

Fig.3.24 Hub angle for a learning rate 0.5

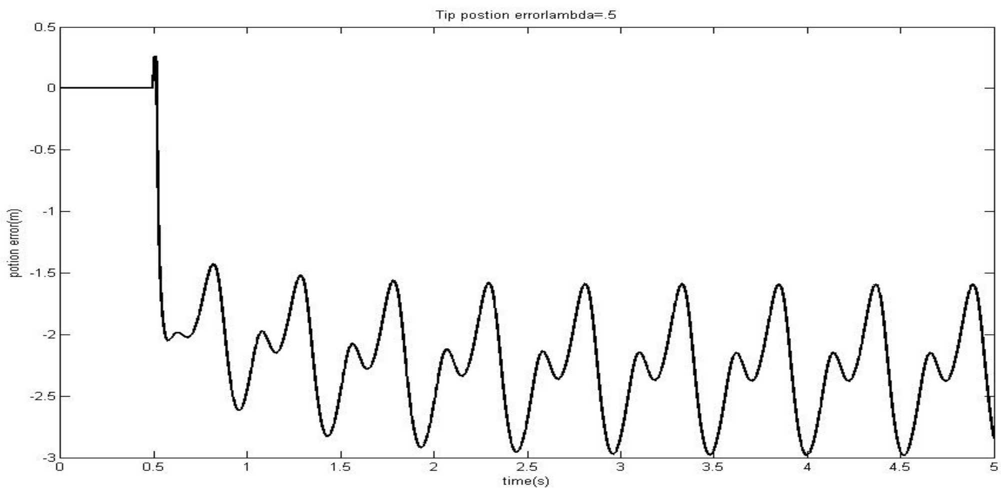

Fig.3.25 Error in tip position for a learning rate 0.5

Fig.3.22-Fig.3.23 shows that a learning rate of 0.05 is too slow to rectify the error in the tip position. The error reduces to zero at 4th second of its operation. While Fig.3.24 and Fig.3.25 show that a higher learning rate of 0.5 produces oscillations of high frequency in beam and the error in tip position does not reducing with time.

3.6 Conclusions of Simulation Results and Chapter summary:

A tip feedback PD controller law with the gain parameters (k_p=0.01 and k_d=0.1) chosen arbitrarily has not shown good results for both the sinusoidal and a step input. This can be easily understood from the Fig.3.9 and Fig.3.10, which show the actual tip position and the error in tip position, respectively. The error of 0.1m in tip position for a desired trajectory having amplitude of 0.05m is not acceptable. Using Iterative Learning based control technique in first phase the error comes out to be bounded the data of error and the change in reference input has been used to tune the existing PD parameters. Fig.3.12 shows that the error has been reduced to the order of 0.003m as compared to 1m with the initially chosen PD controller. The tuned PD controller thus obtained shows better results for sinusoidal input. The learning rate 0.1 and 0.3 shows better results in the terms of suppression of vibration in the flexible link structure. Fig.3.13 shows the tip position of the single link flexible manipulator for a learning rate of 0.1.It shows minimum error in tip position. Now to check the versatility of this Iterative learning based controller design, a step input is given. Fig.3.17 and Fig.3.18 shows the tip position and error in the tip position for the step input under initially chosen PD controller. For a step size of 0.22m, the error in tip position is coming out to be of the order of 0.15m, so it is unacceptable. Applying the Iterative learning based controller structure reduces the error to zero within one second of its operation for both the learning rates of 0.1 and 0.2. The learning rate of 0.1 shows best result as compared to the learning rate of 0.2 an 0.3 and hence is considered for the tuned PD control structure .The corresponding gain parameters of the tuned PD_2 controller design for different learning rates can be found form Table 3.2.

Thus in this chapter some past works on controllers applied for flexible robots are discussed in the beginning. This is followed by the discussions on Iterative Learning based controller design, its advantages, and the update laws used in P-type, followed by the convergence condition. The chapter ends with the simulation results carried out over a single link flexible robot manipulator. The results obtained have verified that Iterative Learning based controller design is better than the existing tip feedback PD controller design.

Chapter 4

Conclusions and Future work

4.1 Conclusions of the Thesis

In this thesis, an Iterative learning based controller structure is designed for a single link flexible robot manipulator. First the modelling of the system is carried out using assumed mode method (AMM). The dynamics of a flexible beam in the form of a partial differential equation (PDE) known as Euler Bernoulli equation is derived. Using Lagrange's approach the partial differential equation (PDE) is changed to ordinary differential equation (ODE) AMM considers infinite number of modes of vibration of the flexible link, but to avoid complex mathematical computations, we have considered only the first two modes of vibration in our modelling which truncates the dimension of the system model to the order of six.. The model thus obtained is non linear and time varying in nature. We have also considered the actuator dynamics in the modelling which increases the order of system by one. We have validated the obtained AMM model using a bang-bang input. The experimental results obtained agree with the simulation results.

A simple PD controller (Fig.3.19, Fig. 3.10) is not enough to suppress the vibration in the link having nonlinearity and time varying in nature. An Iterative learning based controller (fig.3.12, fig. 3.13) takes care of non linearity in the system, it requires minimum system knowledge. It improves the system response significantly iteratively. However, it can be said that in the tracking problem the tip feedback controller may have a delayed response, which our proposed controller does not take care of, for that another time delay controller can be designed. Under Iterative learning based control technique, we have tested the system for various learning rates. ILC controller having learning rate of 0.1 and 0.3 gives desirable response for a sinusoidal input. Actual tip position for a learning rate of both 0.1 and 0.3 almost agrees with the desired trajectory. On simulating the same controller for a step input, the response obtained is also satisfactory with a learning rate of 0.1 showing the best result in which the error in tip position is reduced to zero within a time span of 0 1 sec as compared to 1.5s and 2.5 sec for learning rates of 0.2 and 0.3 respectively(Fig.3.20). The proposed iterative learning based controller structure thus shows better results than the existing PD controller. The proposed controller is more versatile than other controllers in terms of tip position control for different input signals. Better tracking results are obtained for both, a repetitive and a non-repetitive signal.

Thus the model obtained using AMM method has been successfully applied to vibration suppression in the tip position of the flexible link with the help of an Iterative learning based controller design. It is found that better tracking performance has been achieved over the existing PID methods by using the proposed controller.

4.2 Suggestions for future work

In this thesis various control aspects of a single link robot manipulator have been studied which can be extended for multi-link also.

Some suggestions for future work are as follows:

- The modelling techniques AMM and FEM can be extended to two link manipulator considering the actuator dynamics also.
- The ILC based controller design may be applied to both the joints for the control of hub angles to achieve precise tip tracking of two link manipulator also.

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