COOPERATIVE SENSOR LOCALIZATION USING MAXIMUM LIKELIHOOD ESTIMATION ALGORITHM

A THESIS SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

Master of Technology
in
Telematics & Signal Processing

By
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CERTIFICATE

This is to certify that the thesis titled "**CO-OPERATIVE SENSOR LOCALIZATION USING MAXIMUM LIKELIHOOD ESTIMATION ALGORITHM**", submitted to the National Institute of Technology, Rourkela by **Mr. Upendra Kumar Sahoo**, Roll No. **20607024** for the award of the degree of Master of Technology in **Electronics & Communication Engineering *(Telematics & Signal Processing)***, is a bona fide record of research work carried out by him under my supervision and guidance.

The candidate has fulfilled all the prescribed requirements.

The thesis, which is based on candidate's own work, has not been submitted elsewhere for a degree/diploma.

In my opinion, the thesis is of standard required for the award of a Master of Technology degree in Electronics & Communication Engineering.

To the best of my knowledge, he bears a good moral character and decent behavior.

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UPENDRA KUMAR SAHOO

CONTENTS

ABSTRACT

In wireless sensor networks, Self-localization is an important application in wireless senor network. The positions of the sensor nodes spread in the environment of interest are very helpful in several applications like environmental monitoring, precision agriculture, geographical routing, and detection of hazardous material in an environment and manufacturing. Since it is impossible to measure the sensor data position manually and the conventional method based GPS require large amount cost, there is need of estimation of the position of the sensor node using in a cooperative manner. In this some sensor can be spread whose position is already known and it can be consider as the reference sensor node. Using the position of the reference sensor node and the relative sensor measurement the position of the sensor can be calculated.

The choice of sensor measurement technology also plays a major role in the network's localization accuracy, energy and bandwidth efficiency, and device cost. This thesis explains the theory of the sensor node localization for different pair wise measurements of time-of-arrival (TOA), received signal strength (RSS), quantized received signal strength (QRSS), and connectivity. We have taken the simulated data at different positions of the sensor node. From these different position the Cram´er-Rao lower bounds on the variance possible from unbiased location estimators are obtained. In this CRB calculation we have taken the RSS case only. Maximum Likelihood estimation algorithm is studied and applied for a particular node position.

LIST OF ABBREVIATIONS

RF	Radio Frequency
IC	Integrated circuit
RSS	Received signal strength
MEMS	Micro-electro mechanical system
GPS	Global positioning system
LPS	Local positioning system
WLAN	Wireless local area network
MS	Mobile station
VHF	Very high frequency
HVAC	High voltage alternating current
RFID	Radio frequency identification
AOA	Ange of arrival
TOA	Time of arrival
DWMDS	Distributed weighted multidimensional scaling
MSE	Mean squared error
QRSS	Quantized received signal strength
RSSI	Received signal strength indicator
TDOA	time difference of arrival
MLE	Maximum likelihood estimation

LOS	Line of sight
SCC	simple cross correlator
GCC	general cross correlator
CRB	Cramer-Rao bound
SNR	Signal to noise ratio
DS-SS	Direct sequence spread stectrum
UWB	Ultra wide band
NLOS	Non line of sight
FEC	Forward error correction
BPSK	Binary phase shift keying
CDF	Cumulative distributed function
PDF	Probability density function

List of figures

Chapter 1

INTRODUCTION

Networks, both wired and wireless, have been growing in size with years. As the size of the wireless sensor network node is decreasing, wireless sensor networks of thousands or tens of thousands of nodes have been proposed, and these numbers are continue to grow over time as wireless sensor nodes becoming cheaper. In most application of the sensor network the sensors are spread in the environment of interest and the data of the environment is measured by the sensor node. Using this data the condition of the environment can be calculated. Since the data is spread throughout the environment then without knowing the position from where it is measured the data become invaluable. In order to associate the position with the measured data the sensors should know their absolute position with respect to some reference point. Usually the sensors are spread randomly in a remote area where it is very difficult to measure their coordinate. For that purpose the sensor should find out their own position by cooperative communication among them. This means the sensors should transmit the signal among them and from the relative signal strength (RSS) and time of arrival (TOA) the relative position can be calculated. By placing some sensor nodes those know their absolute position the relative location can be change to some absolute value.

The thesis deals with the estimation of the positions of the sensor node present in environment of interest through cooperative communication among them which is already proposed in the paper [3-7] . The parameters considered during the position estimation are RSS and TOA.

Sensors are referred to a device which is connected, RF communication medium, to other devices present in the network. The sensors can be viewed as the nodes in a graph having some edges. These edges indicates the presence of communication link among different sensor nodes which is given in Fig. 1.1 In this scenario each node measures the environment parameter such as temperature, pressure and humidity and further transmit it to the other sensor through the communication link. In order to achieve this sensor nodes are equipped with transmission unit, measurement unit, processing unit and power supply unit. So all these units need to present in a single chip called integrated circuit (IC). Here the measurement parameter are considered to be the signal strength and time of arrival. These data need to be transmit to a central high power processor called fusion center and fusion center processes the entire data and estimated the position of the each node using some algorithm. Here the maximum likelihood estimation algorithm is used which is already explained in the paper [3].

Second, here position means the coordinate the sensor in x-,y- directions. These location coordinates to be estimated may be either physical location, or data location: Physical Location: The sensor's physical coordinates, that is, where it exists.

As discussed estimating a sensor's physical coordinates is very much important. For estimation of these physical coordinates, two types of sensor measurements are used in this thesis. First, the data is measured which is directly related to the relative sensor node position. For example if the one sensor measures the signal strength receives from the other sensor the change in signal strength between the two sensors is related to the distance between the two sensors.

Second, sensor data measurements can be directly taken into consideration, which are the direct measurements made at a single sensor of the environment near itself. For example, if two sensors are placed in every city of India for measurement of the rain, temperature throughout the year then the sensors present near to each other measures the data which are highly correlated to each other while the sensor present vary far from each other the data become more uncorrelated. This means the correlation between the data measurement empirically relates to the relative distance between the two sensors. However this situation is not considered in this thesis.

The motivation behind the study of this problem is introduced in the in Section 1.1 Following this, the location estimation problem in wireless sensor networks is explained in Section 1.3..

1.1 Wireless Sensor Networks

Advancement in radio frequency (RF) and micro-electro-mechanical systems (MEMS) IC design have made possible the use of networks of wireless sensors for a variety of new monitoring and control applications. For example, smart structures could actively respond to earthquakes to make safe buildings and bridges, and constantly monitor for cracks or structural problems. In precision agriculture the crops can be save from loss by supplying appropriate amount of water and fertilizer. In order to do this the WSNs need to be spread in the field of crop. In case of war appropriate action can be taken which only be possible due to the wireless sensor network. Traffic monitoring systems can be made intelligent by using the sensor network through out the city of interest. The source localization in the radar and sonar type problem can be made feasible. Environment monitoring application such as finding the intensity of the poisonous chemical and other varieties of chemical can be made. So there is large number of applications are related based upon the wireless sensor network [1, 26].

Since the data is distributed through out the environment so the data only can be made meaningful by knowing the position from where the data is observed. For example in precision agriculture the objective is to know the position where the humidity is very low. This low humidity indicates that the crop in that area is going to affect. So objective is to save that crop by supplying appropriate amount of water. Thus there is a need to embedded the data with the position otherwise it is very difficult to know the position where the humidity is less. Moreover for geographical routing of the data the sensor should know their own position and the position of the point to where the data is to be transmitted.

Since the sensors are spread in a random and remote area it is possible to manually measure the position of each sensor node. On the other hand this can be made feasible by using the GPS based sensor. Often the sensors are low power. Since GPS based system demands large amount of power so the tiny sensor will die down very fast. So that the lifetime of the sensor network will be very less. Usually the sensors are spread for long time interval. So the objective should be to increase the lifetime of the network. Moreover large amount of sensor nodes are required for these applications. Usually the GPS systems are very costly. So it may require large amount of cost to implement such system in real scenario. .

Instead, this thesis deals with small of sensors and some reference nodes, which obtain their coordinates - either via GPS, or by a system administrator during startup - and the rest, n unknown-location nodes, need to determine their position using the relative location between the known sensor node and unknown sensor node and the absolute position of the known sensor node. If sensors were capable of high-power transmission, they would be able to make measurements to multiple reference nodes and positioning techniques such as multilateration or multi-angulations could be applied. These direct techniques have been studied for a long time within and outside of the signal processing research group. However, low capability, energy-conserving devices are lack of a power amplifier, and also lack of the energy necessary for long-range communication, or it can be limited by some constraints on transmit power. Instead, wireless sensor networks, and localization techniques, can be applied using the multi-hop ‘cooperative’ localization, as shown in Fig. 1.2. Rather than solving each sensor’s position at one time, an algorithm needs to develop which can solve the position of the entire sensor node cooperatively. By this way the power of the sensor node can be saved and also it requires less amount cost.

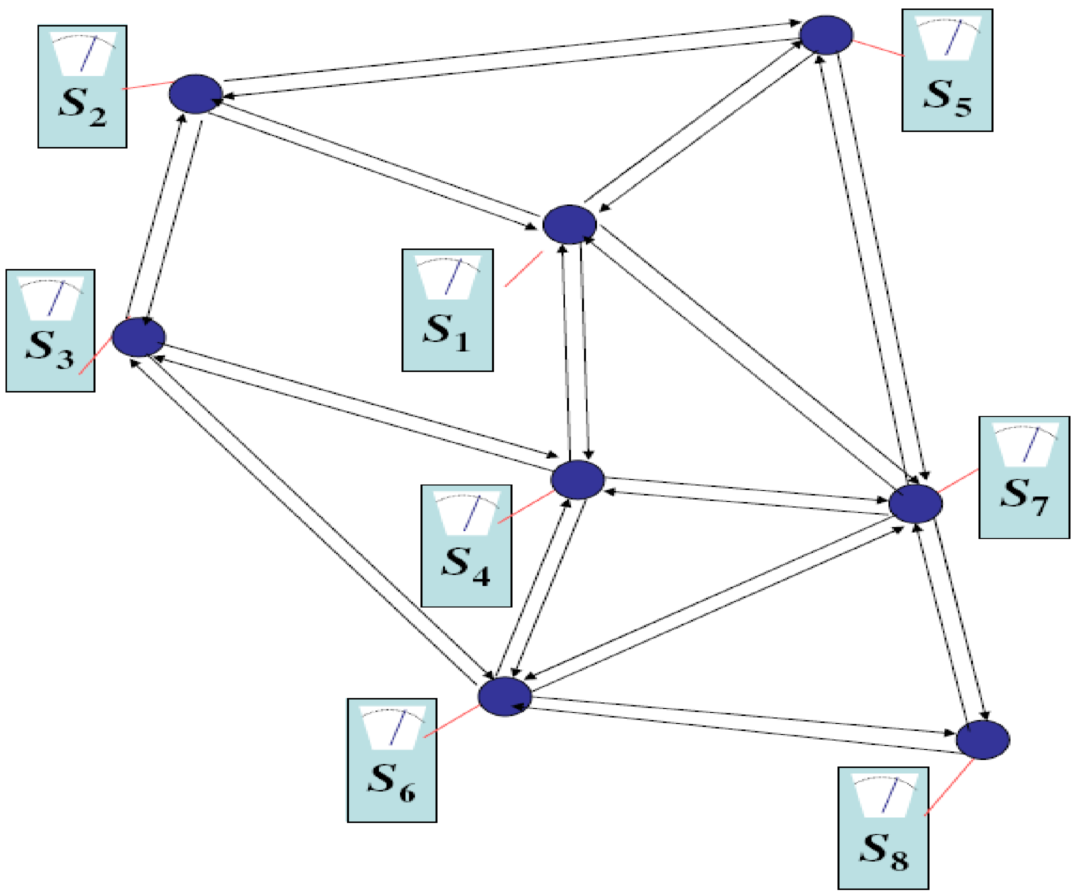

Figure 1.1: This thesis considers sensor networks, in which nodes have wired or wireless communication channels, shown as arrows, with some other nodes. Each node also has a sensor, shown as S1, . . . , S8, which may measure the physical environment, or the node itself.

1.1.1 Motivating Application Example: Animal Tracking

If cooperative localization is implemented in wireless sensor networks as described above, many compelling new applications can be enabled. This application can help the purposes of biological research and animal behavior studies community to track animals over time and over wide area [4]. Such tracking can able to answer questions about the behavior of animal and the interactions of the animal among their own and with other animals. It is very difficult for tracking of the animal using the current practices. A particular way is to attach VHF transmitter collars with the animals to track, and then triangulate their position by driving (or flying) to various locations with the use of a directional antenna. Alternatively, GPS-based system can be used with the animal but it requires large amount of cost and energy. Using wireless sensor networks this problem can be easily solved and the performance can be improved (as demonstrated by 'ZebraNet'). By fixing the wireless sensor network based tags to the individual animal and using multi-hop routing of location data through this sensor network enables low transmit powers. Further, the inter-animal distances, which are of measure interest of animal research group, can also be calculated using pair-wise measurements and cooperative localization methods. The end result of the requirement of longer battery lifetimes is less frequent in this case.

Motivating Application Example: Logistics

As another example, consider the deployment of a sensor network in an office building and manufacturing floor. Sensor can also play prominent role in automation of the manufacturing industry by spreading large number of sensors through out the area where the manufacturing process is going on. Sensors can measure the amount of things remain unused and where there is need of such amount. Monitoring all these and taking optimized action the production capability of the industry can increased drastically. Wired sensor has been used in monitoring and control of machinery. However the high cost can be reduced by using the wireless sensors. Furthermore the automation can be increased by using the automatic localization of the sensor nodes.

The boxes and parts which is to be warehoused and factory and office equipment can all be tagged with wireless sensors when first brought into the facility. Storage conditions (temperature, humidity) can be monitored and the HVAC system can be controlled. So by using wireless sensor network the equipment can be saved from loss. The wireless sensors are placed in every equipment and the sensors measure the relative distance between them. If the relative distance is equal at the time when it was placed then there is indication that the equipment or thing is at the same place. But when the distance changed by large value at that time there is indication that the thing is displaced or someone has stolen. So that it can be said to the information security about the stolen of the thing. Then again using the relative position estimation algorithm the place where it is present can be traced and can be retrieved from that place.

These methods of using radio-frequency identification (RFID) tags, can also be extended for application like this. Suppose large number of packets is entering to a house and these are placed inside the house. So at the time of entering these packets can be tagged by RFID. So there is no need to remember where these packets are present. Using the cooperative sensor localization system the relative position of the packet can be found out and can be used at the time of requirement. So this relative position estimation system is having large number of potential application in real life world. Moreover this requires less amount cost so it can be made available for general people application.

Costs of Commercial Logistics Solutions:

Logistics applications can be made easy by using this local positioning systems (LPS)., Sensors can be made active using signals transmitted to or received from high-capability base stations to locate them. However the cost for the deployment of the base station with LPS is the to cover an area of interest is very high.

For example, for a personal security application system on a small college campus a company called Detection Systems, Inc. (now owned by Bosch Security Systems) deployed a LPS. Here the RSS is used to find the location of the person which pushes the alarm button of its radio tag. So the security officer can rush to that area for assistance. It helps for security inside a campus. However the system is very successful for giving protection to the people inside a campus. However the cost of the Detection System is very high that is around $400,000.

Another LPS system is provides by WhereNet Corp. This is marketed as an “active RFID” solution. In this case the time of arrival (TOA) is being used to locate the position. In this the the active tags transmit the signal to the multiple base station. Based on the time difference between the time of arrival of the received signal the position is estimated. Obviously this type of system suffers from multiple path fading and its cost is very high near to \$350,000 to \$500,000 for a 1 million square foot warehouse.

In addition, the theoretical accuracy of cooperative sensor localization increases with the density of sensors, which is similar to the Metcalfs Law, which holds that the important of network system increases with the increase of the number of nodes in the network. So if there is a large number of sensor nodes are present for different application then the cooperative algorithm can give good performance by using all the type of the sensors.

Pair-wise and Sensor Data Measurements

Usually the sensors are spread over an area of interest to measure some data. As discussed the measured data by the sensors present near to each has a high amount of correlation. So the sensor data can be used to roughly estimate the position of the sensor. By this way it can be easily find which sensor is near to the one particular sensor. By this way the life time of the sensor can further be increased because there is no need to used extra transmission of data for position estimation. This type of algorithm has already been used in different literatures. This type of method is only very much helpful for isotropic scenario which generally very less occur real time environment.

Such use of sensor data measurements for position estimation is restricted to the isotropic field. Since the isotropic field is very rare thus there is need to incorporate some other information for sensor position location. In this thesis some other methods are used which is already reported in some literatures for position estimation. These are pair wise signal measurements such as received signal strength, time of arrival, angle of arrival. In received signal strength each sensor transmits a signal having some value and is received by other sensors. From the difference between the transmitted power and the received power by the different sensors and the model of environment the relative position can be calculated. This type of solution affects by the small scale fading and shadowing effect. Similarly in case of time of arrival each sensor transmits the signal at a particular time and this time is stored and the receving time by the other sensor are also stored. Taking the difference between the transmitted time and receiving time the relative sensor position can be calculated. These measurements can be made using acoustic or RF signals.

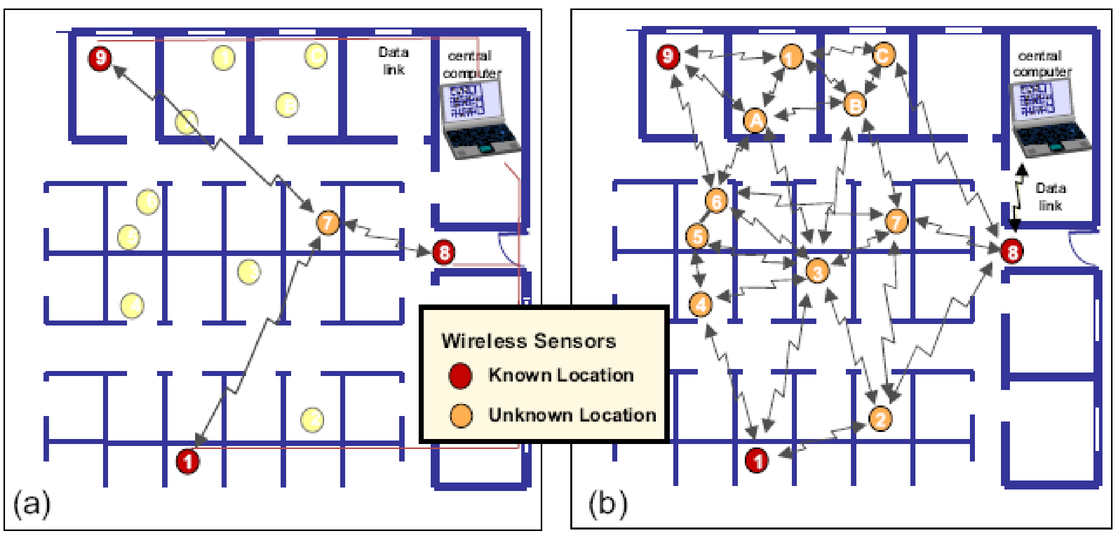

Figure 1.2: In cooperative localization (b), measurements made between any pairs of sensors can be used to aid in the location estimate. Traditional multi-lateration or multi-angulation (a) is a special case in which measurements are made only between an unknown-location sensor and known-location sensors.

1.2 Problem statement

Before plugging into the design it is useful to state the cooperative sensor location estimation problem [3]. So the objective is to estimate the positions of the sensors present the environment of interest. Suppose there are ‘N’ numbers of sensor nodes are present in the environment. From these the positions of ‘m’ number of sensor nodes are known to us. This can be achieved by manually or by using the GPS system. So there are ‘n=N-m’ numbers of sensors remain whose positions is not known. Thus the objective is to estimate the positions of these ‘n’ numbers of sensor by cooperative manner. Here the positions of the sensors are defined by two parameters x-coordinate and y-coordinate. Mathematically it can be represented as like below. In other way the two dimensional localization problem can be formulated like below. So total number 2n numbers of parameters is to be estimated, $\theta = [\theta_x, \theta_y]$, where

(1.1) $\theta_x = [x_1, x_2, ..., x_n]$, $\theta_y = [y_1, y_2, ..., y_n]$

The number of parameters known to us is the coordinates $[x_{n+1}, ..., x_{n+m}, y_{n+1}, ..., y_{n+m}]$, and at least one of the of location measurements among the TOA, RSS, AOA. The coordinates of the sensor i can be referred as z_i where $z_i=[x_i,y_i]^T$. Only the two dimension case is taken into account here. This can be extended into three dimension case. The measured data is the Pair-wise measurements X_{ij} between the sensor ‘i’ and ‘j’ which is related to the relative distance between the sensor nodes. Eg. Time-of-arrival (TOA), angle-of-arrival (AOA), received signal strength (RSS), or connectivity (whether or not two devices can communicate). Le the sensor measurement at sensor i at time t be denoted by $V_i(t)$.

In this case all the pairwise measurements are not taken into account only the pairwise measurements which can be possible are taken into account. If all the pairwise measurements are consider then it wil be (N/2) pairs of measurements. Let the sensor ‘i’ can communicate with H(i) number of sensors. This can be consider as a set. Obviously $i \notin H(i)$ and $H(i) \subset \{1...n+m\}$. The pairwise measurement can be different based upon the environment it is spread. If it is air it can be acoustic or electromagnetic signal. If it is water then it is only acoustic. Infrared signal can also be used as a communication between the two sensors. Moreover for different measurements like DOA,AOA,RSS different signaling technique can be used for example for TOA direct sequence spread spectrum (DS-SS) or ultra wideband [25] is more suitable. These are discussed in chapter II. The main theme of the chapter-II is to study the impairment introduced in the measurement method due to noise. The main cause of impairment in the environments effect such as additive white Gaussian noise, shadowing effect, impulse noise, multipath effect etc. However these effects cannot be changed and also it is not known to us in advance. So the objective is to design efficient algorithm to estimate the positions of the sensor with less error. So large number of sensor measurement can only be helped to increase the performance of the estimator algorithm. In order to study the distortion the first objective is to find the appropriate model for this which is given in Chapter II.

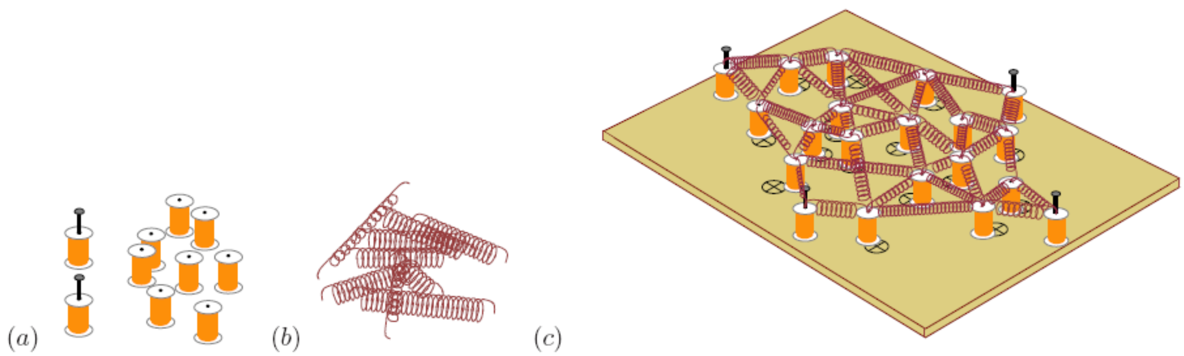

Figure 1.3: Cooperative localization is analogous to finding the resting point.

1.2.1 Imperfect Prior Knowledge

For some applications some nodes have some imperfect prior knowledge. This imperfect a priori knowledge can be used with the measurement data to rectify this coordinates. This can be used in designing the efficient algorithm and the performance can also be increased. The mean-squared errors can be taken as the appropriate performance bounds in the case of imperfect prior information [4].

Our location estimation problem is based upon the relative location estimation. Then relative location estimation will be used with the knowledge of the sensor nodes which positions are known to us to obtain the position of the unknown sensor node. So different measurement technique will be used in order to achieve this objective.

Notation	Description
$N=n+m$ n m	Total number of sensors present in the environment Number of sensors with unknown position. Number of sensors with known positions.
Zi	Actual distance between two sensor i, i=1,2,….,n+m
$P_{i,j}$ $\Pi_{0,}\nabla_0$ n_p σ_{dB}	Power received by sensor i which is transmitted by sensor j Π_0 is the free space received power at some reference distance ∇_0 RF is the path loss exponent Standard deviation of RSS at some node in dB
δ_{ij}	Measured /estimated raw distance between the sensors i and j
$\hat{z}_i$ Z	Estimated coordinates of the sensors i, i=1,…,n Actual coordinate matrix

Chapter 2

LOCATION MEASUREMENT AND MODEL

For the development of a good localization algorithm, the channel impairments methods need to study accurately. There is large number challenges are present in this. Propagation of RF signals in real world is obstructed by building, moving object, multipath fading etc. The pair-wise measurements need to accurately model in the scenario where it is done. This chapter discusses the study done by the Neal Patwari and his group to model the environmental factors. They have done a large extensive on this. This can be found out from the literatures [3]. These models can help to design efficient algorithms which are discussed in the later chapters. These models are presented and tested using the measured data.

Generally, the time varying errors such as (e.g. due to additive noise and interference) are responsible for the degradation in range and angle measurements. The effect of these time varying errors can be erased by taking the average measurements data. Where as environment dependent errors are due to the building, trees, obstacles, people etc. So the time varying errors can be modeled as random variable since these are varying with time where as the environment dependent errors effect is much more. So the main challenge is to model these environment dependent errors.

2.1 Measurement Characterization

In order to do the statistical characterization the measurement data the following procedure needs to be done. Suppose the statistical characterization is to be done for a network of ‘N’ number of sensor nodes those are arranged in a particular geometry. For this ‘K’ number of such network needs to be placed at different environmental condition such as in the building, in the open area. Then large number of data is to be measured between the different sensor nodes. Then there is to find the mean of all these data. For accurate characterization the measured data is to be taken much more. Then the joint probability function between the different sensor networks among the K number of sensor network is to be calculated. For different type of geometry this type of experiments need to be repeated for different environment scenario.

These measurements need to take some simplifying assumptions. These assumptions may be incorrect in real environment scenario. For example if large number of sensor nodes are spread densely in an environment then the different communication link are similar in terms of their correlation where as it is very difficult to model these measured data using these correlation factors inside the model. Thus it can be assumed that the different communication links are not correlated to each other. By this simplifying assumption it will be easy to mode the measured data. However this can be incorporated to design very efficient localization algorithm, which can be treated as the future work of this?

The second simplifying assumption is about the choice of the distribution function of the measurement data. For the measured data is to be subtracted from its mean and then the distribution of the residual error is to be modeled near to a famous distribution such as Gaussian, lognormal or mixture distribution. Then the likelihood function of the error can be calculated. Since this may add some other error and it will not give good performance still it is advisable to simplify this to a near famous distribution and it is more motivated at the time of the starting of the research in this field. Then in future it may be extended for distribution function estimation such as nonparametric distribution function. By using nonparametric distribution a large number of measured data can be easily approximated to a correct distribution. The main problem associated with this nonparametric distribution estimation is that it requires large number of computational complexity which may not be suitable for wireless sensor network. Due to development of low power VLSI and efficient VLSI architecture now-a-days it is not a problem from computational point of view.

Neal Patwari and his group have conducted several measurement experiments in order to estimate accurate statistical models for RSS and TOA measurements data for indoor wireless sensor networks. Section 2.6 deals with these measurement experiments. However, it is required to have a general idea to the sources of the errors and difficulties associated with each type of measurements, before understanding to a particular measurement campaign. So, Sections 2.2 through 2.5 contains the introduction for various measurement models. Each measurement model section deals with the four sub-topics: ‘Major Sources of Error’, ‘Statistical Model’, and ‘Calibration and Synchronization’.

2.2 Method of Received Signal Strength

Received signal strength (RSS) is defined as the signal voltage measured at the receiver node by using some circuit. This RSS based position estimation is a very popular area especially in the mobile network. However this requires large amount of power needs to be transmitted by one sensor to avoid the noise present in the environment. For this purpose a high power short pulse needs to be transmitted in the environment. However it can be easily affected by the environment noise. In order to use this the environment noise and its implication to the RSS must be studied thoroughly. The main source is the noise present in the environment such as AWGN and impulse noise.

2.2.1 Major Sources of Error in RSS

The decays of signal power in free space is proportional to d^{-2}, where d is the distance between transmitter and receiver. So the multipath signals and shadowing are two major sources which belong to the environment-dependence in the measured RSS. Multiple signal with different phase which comes from reflection from different objects present in the real world added to each other at the receiver so it leads to fading at the receiver. Further different frequency with different phase reached at the receiver which leads to frequency selective fading. These two factor accounts for the distortion of RSS at the receiver. In order to avoid this wideband signals needs to be used having a high power. CDMA is one such solution to this.

The effect of frequency selective fading can be made decreases by using CDMA method. However due to the presence of wall, floor and different objects in the environment the shadowing effect come into picture. Since this objects positions are random this shadowing effect can be assumed to be random in nature. So there is need to find the distribution about this random variable. This can be obtained by taking large number of measured data and then subtract the mean from this measured data. Due to this the received signal is attenuated.

2.2.2 Statistical Model

Typically, the ensemble mean value power decays proportional to d^{-np} where n_p is the 'path-loss exponent'. This value is typically between 2 and 4. So the the ensemble mean power at some distance d can be modeled as

$$\bar{p}(d) = \pi_o - 10 n_p \log \frac{d}{\Delta_0} \tag{2.1}$$

where π_0 is the received power (dBm) at a short reference distance ∇_0. The difference between the measured received power and its mean, which is due to the randomness of the shadowing, can be modeled as log-normal distribution. This model is based on a wide variety of measurement result and analytical evidence [30]. This model is also tested by the experimental measurements done by Neal Patwari which is shown in Section 2.6. The standard deviation of received power (when received power is expressed in dBm), σ_{dB}, has units of (dB) and is constant with distance. Typically, σ_{dB} is between 4 and 12. Thus, the distribution of the received power (dBm) at any sensor i transmitted by j, Pi,j is

$$f(P_{ij} = p/\theta) = N(p; \bar{p}(\|z_i - z_j\|), \sigma_{dB}^2) \tag{2.2}$$

Where $N(x; y, z)$ is our notation for value of x of a Gaussian p.d.f. with mean y and variance z,θ is the coordinate parameter vector from(1.1), and the actual transmitter-receiver distance $\|z_i - z_j\|$ is given by

$$\|z_i - z_j\| = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{2.3}$$

For a two dimensional location coordinate $z_i = [x_i, y_i]^T$

2.2.3 Estimating Range from RSS

The estimated distance between any two sensor node devices i and j, can be found from $p_{i,j}$ given by $d_{i,j} = \|z_i - z_j\|$ is,

$$\log f(\ p_{i,j} | \theta) = c_1 - \frac{\left[P_{i,j} - \bar{P}(\|z_i - z_j\|)\right]^2}{2\sigma_{dB}^2} \tag{2.4}$$

where c_1 is a constant independent of θ. Due to the quadratic form of the distribution, it is found that the maximum of (2.4) occurs for $P_{i,j} = \overline{P}(\|z_i - z_j\|)$, where the $\overline{p}$ is shown in (2.1). Thus the estimated distance can be calculated as $\delta_{i,j}^{MLE}$ which best estimates $\|z_i - z_j\|$ in the maximum likelihood sense is given by

(2.5) $$\delta_{i,j}^{MLE} = \Delta_0 10^{\frac{\pi_0 - p_{i,j}}{10 n_p}}$$

Thus $\delta_{i,j}^{MLE}$ has a log-normal distribution since $\log \delta_{i,j}^{MLE}$ has a Gaussian distribution, and that

(2.6) $$E\left[\delta_{i,j}^{MLE}\right] = C\|z_i - z_j\|,$$

Where

(2.7) C=exp[γ/2], where $\gamma = \left(\frac{10 n_p}{\sigma_{dB} \log 10}\right)^2$

(2.8) $$\delta_{i,j}^{BC} = \frac{\Delta_0}{C} 10^{\frac{\pi_0 - p_{i,j}}{10 n_p}}$$

2.2.4 Problem of Calibration and Synchronization

It is found from (2.1) that the distribution of the RSS model is a function of the path loss exponent np. This need to be estimated along with the coordinates of the sensor, as an unknown parameter.

Moreover, the measured RSS is also depends upon the calibration of the transmitter as well as the receiver. Since the calibration and power of the sensors vary from device to device, there is a need that each sensor should report their calibration data to their neighborhood sensors. Otherwise the sensor power and calibration value can be taken as unknown parameter and needs to be estimated along the coordinate. Which further increases the complexity of the sensor node? This can be solved by using the method of estimation with some unknown parameter. This may also degrades the performance of the estimation process.

2.3 Method of Time-of-Arrival

The time of arrival is the time at which the received signal reach at the receiver. The time difference between the transmitted signal and the received signal depends upon the speed of the signal and the distance between the two sensor nodes. So as the distance between the two sensor increases the TOA increases. The main draw backs associated with this is the synchronization problem. Due to multiple reflection from the different objects present I the environment the objective is to find the time of arrival of the first signal. This type of distance estimation also affected by additive noise present in the environment. If the environment noise is more stronger that the received signal then it is very difficult to find the line of sight signal and the processor speed should be very fast to record the arrival time.

2.3.1 Additive Noise

Although assuming that the multipath signal is not present the TOA also affects by the noise present in the environment. If the noise is strong enough then it is become very difficult to find the line of sight signal. For this the correlation based TOA can be used. In this case the correlation of the received signal is found out with the known transmitted signal. Then the time for which this gives the maximum value that can be considered as the TOA. However the more advanced methods are available which is known as the general cross correlation in which the received signal is first prefilter and large noise value are attenuated and then the cross correlation is carried out.

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$$\mathrm{var}(TOA) \geq \frac{1}{8\pi^2 B T_s F_c^2 SNR}, \tag{2.9}$$

where Ts is the signal duration, and SNR is the signal to noise power ratio. By changing the system to find a very high SNR, the bound can be achieved for multipath-free environment. Thus it provides an intuition about how duration, bandwidth, and power affect parameters affects the ability to estimate the TOA.

2.3.2 Multipath Problem

The errors caused by the multipath system is very serious than the error caused by the additive noise. In this the late arriving signals can be added to give a high peak in the cross correlation. So there is need to design an alternative method for finding the TOA which will give less error. In order to do this the first peak of the cross correlation need to betaken into account than the maximum cross correlation value. For this there is need to fix a threshold value and when the correlation value will be greater than this value that time may be taken as the TOA. However this value is to fixed in advance by doing some experiment about the environment.

. Usually the TOA errors in estimation problem are caused by two major problems:

Early-Arriving Multipath: After the LOS signal a large number of multipath signals arrive at the receiver whose contributions in the calculation of the cross-correlation decrease to find the actual location of the peak from the LOS signal. If the late arriving multiple path signals are strong enough at that time the cross correlation due to the late arriving signal minimizes the effect of the real peak. This type of situation is very bad in TOA estimation. In this case the the late-arriving multipath components attenuates the LOS signal severely. It can be considered as a scenario of 'lost in the noise' and missed completely. It gives large amount of errors in the TOA estimate.

In the case dense sensor networks, any pair of sensors can able to measure the TOA. There is distinct advantage of this to measure the TOA. With the decrease of the length the path length decreases and the LOS signal power (relative to the power in the multipath components) generally increases. The measurement study is very much helpful in verification of this claim. Because it presents the synchronized indoor TOA measurements data which specifically measured the received power in the LOS signal and then it is compared with the received power measurement at later time. These measurements were done in a large number of links in an office building, and it was shown that the relative LOS signal power is high at low path lengths, and slowly decreasing with increasing path length. Thus, the severely attenuated LOS problem is especially severe in networks with large inter-sensor distances.

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2.3.3 Statistical Model

$$f(T_{i,j} = t|\theta) = N\left(t; \left\|z_i - z_j\right\| / v_p + \mu_T, \sigma_T^2\right), \quad (2.10)$$

The presence of large errors magnitude can invalidate the Gaussian model. These errors increase the tails of the measured signal distribution of measured TOA heavier than Gaussian. It can be modeled using a mixture distribution. In this mixture of distribution there is two distributions one with small variance distribution and the other is a large variance distribution. This can be considered as situation of outliers in a Gaussian noise. For this situation position estimation system needs to be designed which will be robust to these large errors. These errors can also be called as non-line-of-sight (NLOS) errors. For the situation where TOA measurements are made in a channel which is changing, the TOAs which consist of excess delays can be ignored. For the static channels, the redundancy can be used if the numbers of range measurements to a device are greater than the minimum number of device required, to identify likely NLOS errors [21, 5].

2.3.4 Problem of Calibration and Synchronization

If the clocks in the wireless sensors are accurately synchronized, then the time delay can easily determined by subtracting the known transmit time from the from the measured TOA. However the available clock synchronization algorithms are well suited for acoustic signals, not for RF signals.

For asynchronous sensor network a common practice to estimate the time of arrival is if a sensor send a signal to another sensor then the second sensor immediately reply to it. Thus time of arrival is half of the time difference between the difference of the reception time and the transmission time from the time taken by the second sensor to reply this. For this case the processor speed of the second sensor should be very fast to receive and reply the second sensor otherwise the second should also transmit the time taken by it to reply. If the first sensor is equipped with a sophisticated DSP processor then it can easily find out the receiving time from the multiple sensor using the muti use interference cancellation technique.

On the other way the state of the each sensor clock can be taken as an unknown parameter and it can be added with the parameter to be estimated. With this the number of parameters to be estimated increases. So the number of computational complexity also increases.

The difference between the arrival times of the same signal at two sensors is called the time-difference of arrival (TDOA). Usually the TDOA measurement between the two sensors does not depend on the bias of the clock. This has already been used in source localization and GPS system.

2.3.5 Method of Ultra-Wideband and TOA

UltraWideband (UWB) communication uses a narrow pulses having very short duration of time. This type of signal are broadly spread in frequency domain. This type of signal can be used for calculation of time of arrival. This type of signal can easily discriminate in the environment where the objects are present very far from each other. This type of signal can also be used to find the relative distance beteen the two sensor which are placed very near to each other and the objects are present far away from this. The article published by Gezici et. al. gives a detailed of UWB-based localization. A signal can be considered as a UWB signal if either its fractional bandwidth, which is the ratio of its bandwidth to its center frequency, is larger than 0.2, or the signal is a multiband signal having total bandwidth greater than 500 MHz.

However the generation of UWB signal is very tough. So a large amount of power is reuired for generation of the UWB signal. Thus the life time of the sensor node decreases by using this type of method. Hence this is not suitable for wireless sensor network scenario. Only it can be suitable for the scenario where power is not a constraint. So it is suitable for radar and sonar type application where power is not a problem.

2.4 Method of Angle-of-Arrival

In this case the angle information is used for localization of the senor. This made feasible by providing the information about the direction of the receiving of the signal to the other sensors, where as previous methods the sensors gives the distance to the neighbor sensor. In this case each sensor is equipped with three or four antennas that measures the time at which the signal is received. The inter antenna distance is already known to the sensor. Using the time of arrival of the signal and the geometry of the position of the antenna the angle of arrival is calculated. This is similar to the array signal processing. In which the antennas can be considered as an array whose position with respect to the center of the sensor node is already known. In The AOA is estimated from the differences in arrival times for a transmitted signal at each of the sensor array elements. The estimation is similar to time-delay estimation, but generalized to the case of more than two array elements. When the impinging signal is narrowband (that is, its bandwidth is much less than its center frequency), then a time delay τ relates to a phase delay φ by $\varphi = 2\pi f c \tau$ where fc is the center frequency. Narrowband AOA estimators are often formulated based on phase delay.

The second method of finding the angle of arrival is by using the three directional antennas. Due to the directional nature of the antenna the received signal strength is different at different antenna. By taking the ration of the signal strength and the positions of the antenna the direction of the source can be found out. Thus there is different ways to find out the received signal strength..

Since all these methods require large number of antennas in the sensor node thus the size and cost of the sensor node increases by large amount. In order to do this a high frequency signal is to be used. However the processing at this high frequency signal requires large amount of high power processor, which further increases the requirement of a large device. In order decrease all these millimeter wave can be used. In case of millimeter wave the oxygen absorption helps to decrease the multipath effect.

2.4.1 The Sources of Error and Statistical Model

The sources of the error in case of angle of arrival is similar to the case of TOA. In this case the multipath fading and aadditive noise are measure source of error. If the error is modeled as Gaussian then the mean is the accurate angle of arrival and having some standard deviation value. Theoretically the standard deviation of the angle of arrival is already calculated and in different literatures [24] [8]. So due to different ways of angle of arrival calculation standard deviation is also different.

2.4.2 Problem of Calibration and Synchronization

Since it is not priority known the orientation of the antennas. Then the calculation of angle of arrival is very difficult. In this case the sensor should tell the orientation of the angle to its neighbor sensor such that it will be easy. More over the since the sensor nodes are spread randomly through out the environment so there is need to calculate the orientation of the antennas. Otherwise as discussed previously the orientation of the antenna can be add as unknown parameter to estimate. These further increases the number of computation required at the sensor node.

2.5 Method of Quantized RSS

Connectivity measurement is an important method to find whether two sensors are connected to each other or not. If two sensors are not connected then there is no way to study the distance between these two sensors. In this the first objective is to find out whether two sensors are connected to each other or not. This can be done by using the method of quantized receive signal. In this case the data is first sent from one sensor to the other sensor making it packet. Then if two sensors are connected then the other sensor can easily demodulate the packet without any error if it is not connected then it is not possible. This method can be termed as the quantized received signal. The signal is first quantized and then it is transmitted to the neighbor senor.

This can be modeled as binary signal strength receiver ike beow. In this case the mathematical representation is given by

$$Q_{i,j} = \begin{cases} 1, P_{i,j} > P_1 \\ 0, P_{i,j} < P_1 \end{cases} \tag{2.11}$$

where *Pi,j* is power received at the sensor I send by the sensor j in (dBm) and p1 is the threshold value. If the received signal is above this threshold value then it ca be considered as 1 other wise it can be consider as 0.

In many cases the received data is not a step function. In this case the error is to be calculated by sending large number of packets through the channel. So here the number of packets are erroneous is to be calculated. This method can also be implemented in different sensors to find out the position and the relative distance between then . However the main problem is associated with this the large number of packets needs to be transmit through the channel and then the error can be calculated. It takes large amount of time. Moreover it requires large amount of power. So this will helps to decrease the lifetime of the sensor network by a large amount. So the life time of the sensor network deceases very fast. The objective should be to use very less power for localization so that the other energy can be used for other applications. For which the sensor network is meant. Thus The number of received packets can be modeled as a random variable. This can be given by

$$P[Q_{i,j} = 1 / P_{i,j}] = P[no \quad packet \quad error / P_{i,j}] \tag{2.12}$$

This can also be modeled as the Gaussian process having some mean and the variance which is shown latter.

The probability of error depends upon the signaling method being used, packet length and the forward error correction. The lower and the upper bound can be calculated using some methods. This also depends upon the type of the received detection is being used. Taking all these account this method can be think as a complicated method for position estimation.

Finally, it can be noted that the proximity is a step function of RSS assumption cannot loose the upper and lower bound significantly. In case of digital receivers in typical fading channels. In case of digital receivers, the received power is very large so that the error is very small and similarly the probability of getting the real power is one. The range of power which provide probability equal to the zero and one is very less hence the error occurs in this method is very less. The multipath fading is the measure error in this case which degrades the accuracy of the method to large amount. This error is very large than packet error. The log normal standard deviation in this case is of the value 8 dB. As the power level of the transmitted signal increases then the error value goes on decreasing. So more power and large number of packets helps to design an efficient method of position estimation in this case.

This can be modeled mathematically like below.

(2.13) $$P[Q_{i,j} = s \mid Z_i, Z_j] = s + (-1)^S \Phi[g_{i,j}(1)],$$

(2.14) $$g_{i,j}(s) = \sqrt{\gamma} \ln \frac{\|Z_i - Z_j\|}{d_s},$$

(2.15) $$\gamma = \left(\frac{10\eta_p}{\sigma_{dB} \log 10} \right)^2$$

where $s \in (0,1)$, and d_s is the range value which provides the actual distance between the two signal. Specifically, from (2.1),

(2.16) $$d_s = \nabla_0 10^{\frac{\Pi_0 - P_s}{10\eta_P}}$$

The function $\phi(s)$ gives the CDF of a Gaussian random variable having mean zero and variance one.

2.5.1 Method of Quantized RSS

In the previous case only one bit quantization is used. It can be extended for large number of quantization level such that the error can be decreases. Connectivity measurements method is just a binary quantization of the case of RSS measurements method. In more general case large number of quantization level can be considered. Suppose it is divided into 'k' number of quantization levels. This value should be known at the other sensor nodes. In this case the number of errors increases because of the increase of the number of level and decrease of the step size. So the error can only be decreases by increasing the size of the quantization level and decrease of the number of quantization level. So more power is required in this case. So power amplifier needs to be used in the transmitter. This situation also requires more powers so that the lifetime of the sensor network also decreases. So the sophisticated scenario needs to be done.

There is also another problem that occurs in QRSS method. That problem can be called as the processor problem. If the number of quantization level increases, the processor needs to be very fast to implement. In this case the fast processor is required so that the cost of the device also increases.

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This can be modeled like below $P_s, s \in \{1,...,k-1\}$ are the number of quantization levels Similarly, let us define d_s as the path length. This path length correspond to the the mean received power. This is equal to P_s, as given in (2.16). So a measurement of $Q_{i,j} = s$ for s $\in \{0,...,k-1\}$ would occur if $P_{i,j} \in [P_s, P_{s+1}]$.

Similarly, $Q_{i,j} = s$ would occur if $\delta_{i,j} \in (d_{s+1}, d_s]$, where

$$\delta_{i,j} = \Delta_0 10^{\frac{\pi_0 - p_{i,j}}{10\eta_p}} \qquad (2.17)$$

There is no lower bound for the 'out-of-range' power (the lower bound of level s=0), For this case let us define $P_0 = -\infty$(dBm). There is also no term to define the maximum power (the upper bound of level s=k-1), For this case there is also need to define $P_k = \infty(dBm)$. Using(2.16), this it can be that $d_0 = \infty$ and d_k=0. It may be noted that the P_s are increasing with s, but d_s are decreasing with s.

Now the probability density function of QRSS measurements can be written as,

$$P[Q_{i,j} = s | z_i, z_j] = \phi[g_{i,j}(s+1)] - \phi[g_{i,j}(s)], \qquad (2.18)$$

Here $g_{i,j}(k)$ is given by (2.14) and the convention is being used for $0<d<\infty$, $\ln\frac{d}{0} = \infty$ and that $\ln\frac{d}{\infty} = -\infty$

2.5.2 Problem of Calibration and Synchronization

The QRSS and connectivity also posses the same calibration and synchronization problem as in RSS. It may be noted that, the QRSS is a more realistic scenario which may in real position estimation since the data is quantized. Since RSS need to be quantized so that it can be easily detected at the receiver. Moreover in real case a technique should be their which will indicate the receiver the proper value of the quantized. In real scenario these are not known in that case a sub-optimal algorithm can be used to estimate the quantized function.

2.6 Channel Measurement Experiments done by 'Neal Patwari'

This section deals with the pair wise measurements done (M2M) by using wideband channel. These measurements were done in the Motorola laboratory facility in Plantation, Florida. The term 'multipoint-to-multipoint' indicates that the every link between the each sensor nodes pair is done. That means the every channel between every pair of sensors is measured. The conventional measurement method deals with the pair wise measurement between the base station with the each small station present in the environment. This is valid for WLN deployment or mobile tower deployment. This can be called as point-to-multipoint measurements.

The objective of these was to test how the real world environment is following the measurement model considered in the thesis how these algorithms will perform if it will be used in real world environment. This can also help to get the real data of the environment. This data can be directly tested with the localization algorithm to find how accurate the algorithm can work in real environment.

In both campaigns the following environment is taken into account. The measurement scenario was an office building partitioned by 1.8m high cubicle walls. There was also presence of hard partitioned offices, external glass windows and cement walls on the outside of the area. There also metal and concrete support beams were present within and outside the area where the experiment was done. The offices premises were occupied by desks, bookcases, metal and wooden filing cabinets, computers and some equipment. Since the areas was an open plan, hence it may be difficult to define the dimension of the 'room',. This was a typical of office environments.

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The log-normal distribution of the RSS measurements can be verified by quantile-quantile plot using the examining the residuals $r_{i,j}^{R}$. If the data is lognormal then the plot will give a straight line.

2.7 Conclusion:

The models presented in Sections 2.2 through 2.5 have been verified by the experiment done by Neal Patwari. Experiments data has been shown in the site, which has been conducted by the Neal Patwari and his group. Wireless sensor networks has been designed only to use these pair-wise measurements. Cramer-Rao bounds can be efficiently obtained using the mode given in the literature.

Chapter 3

LOCALIZATION BOUND

3.1 Limits on Localization Covariance

The Cramer-Rao lower bound gives a mean for calculation of the lower bound for one particular estimator. This is only valid for the case of unbiased estimator. Since in our case the random variable is modeled as of mean zero and variance finite so it is a unbiased estimator. The number lower bound can be calculated for different methods such as AOA, TOA, RSS and QRSS. The lower bound indicates the performance of the estimator that means if the lower bound is very low then that estimator is very good. On the other hand it can be think that the lower bound gives one reference to judge whether the performance of a method is good or not. And moreover it also helps to judge if the performance is good how much good is it. In this thesis the mathematics done by Neal Patwari et. al. [3] is reported to calculate the lower bound. It is found that the lower bound depends upon the following factors.

Number of unknown-location and known-location sensors,

1. Sensor geometry,
2. Whether localization is in two or three dimensions,
3. Measurement type(s) implemented (i.e., RSS, TOA, or AOA),

5. Channel parameters (such as σ_{dB} and n_p in RSS, σ_T in TOA, or $\sigma_\_$ in AOA measurements),
6. Which pairs of sensors make measurements (network connectivity),
7. 'Nuisance' (unknown) parameters which must also be estimated (such as clock bias for TOA or orientation for AOA measurements).

This section deals with the analytical results obtained for the CRB. Firstly it is indicated that the in cooperative sensor location the priori information about the sensor node position and the pair wise measurements are used. So the lower bound depends upon the priori information about the sensor node position and the measurements. In this case also it is shown that as the number of priori information increases and the lower bound decreases. ,It also depends upon the self calibration of the sensors present in the environment. Different lower bound has been obtained for different measurement. This lower bound is shown to calculate the relative performance between the different methods.

In order to make the analysis simple two types of assumptions are taken into account the device parameters and the channel parameters are assumed to be known. This is requires in the first of the calculation of the lower bound. If the unknown parameters goes on increasing then the lower bound goes on increasing. Because as the unknown parameter is need to be estimated first then the actual parameter is to be estimate. In this case the unknown parameter provide some uncertainty or bound which goes on adding with the increase of the unknown parameter. For this case to make the formulation short the assumption need to be incorporated inside the formulation.

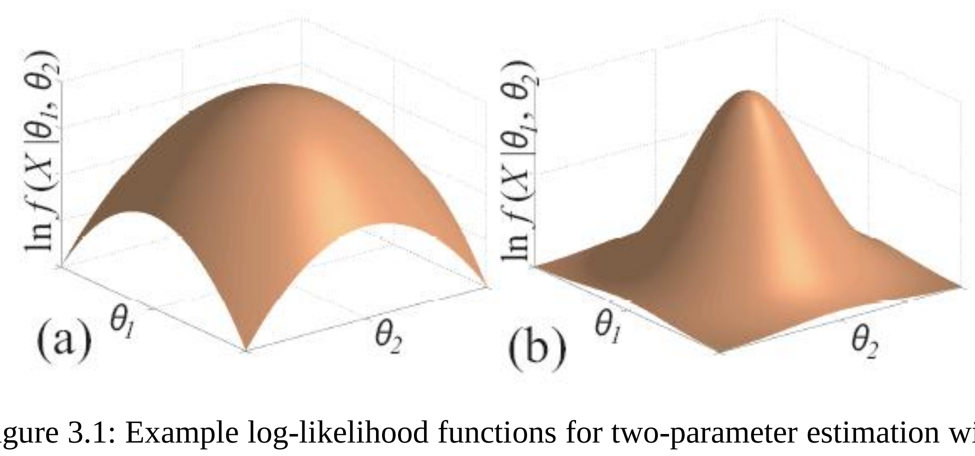
3.1.1 What is the Cram´er-Rao lower Bound?

The Cram´er-Rao bound (CRB) gives the lower bound of the performace estimation. The bound is very useful for comparison purpose between different methods . This can be treated as an uncertainty principle for location estimation. A detailed analytical result is given in the thesis which is taken from Neal Patwari's thesis. This can be treated as an overview. This is found out by using cooperative method All this methods are based upon the statistical distribution assumption. The statistical methods are given in the previous sections. The statistical distribution is a function of the unknown parameter. So this can be consider as conditional likelihood function of the distribution function on the parameter to be estimated, which can be given by f (X|θ), where X represents the random measurement, and the other value is the parameters that are to be estimated. The lower bound can be given by (3.1)

$$Cov(\hat{\theta}) \geq \left\{ E\left[-\nabla_{\theta} \left(\nabla_{\theta} \ln f(x|\theta) \right)^{T} \right] \right\}^{-1}$$

This is the general forlumation of the lower bound

Here the $Cov(\hat{\theta})$ is the covariance of the parameter estimator, E[.] gives the expected value, ∇_{θ} is the gradient operator w.r.t. the vector θ,subscript T indicate transpose.



. Figure 3.1: Example log-likelihood functions for two-parameter estimation with (a) small and (b) large curvature. The variance bound will be higher in example (a) than in (b).

The bound can be treated as the sensitive analysis for any random measurements. That means this indicate how the distribution function is sensitive to the parameter to be estimated. If it is more sensitive then the bound is very less. Hence the bound can be defined in terms of the curvature of the distribution function with respect to any parameter. If the curve is more then sensitivity id more and the bound is less. Usuallay the bound is calculated taking logarithm of the distribution function. As shown in figure if the curve is more the bound is less and the estimation performance is more. However this is only limited to the unbiased estimator. The CRB is very limited to the unbiased estimators. Such an estimator provide coordinate estimates that, if averaged over enough realizations, are equal to the true coordinates.

3.2 Decreasing Bound on Self-Calibration Estimators

A fundamental question arises regarding the 'cooperative' sensor localization is the adding of some unknown-location sensors to the entire network increases the performance of the position estimation in the entire network or not. This is because theory says more dense network should provide the accurate estimation of the entire sensor network. This question can be considered as a critical question. Because it relates to the theory that more dense sensor networks should provide better accuracy in the sensor localization.

In order to verify this , a single sensor node is added to the unknown-location sensor in an existing sensor network. It is found that the given sufficient condition presents the lower bound of the sensor node localization decreases. This is very good information for cooperative sensor node localization. However it cannot be made zero. So more sensor means more measurements and a very good position estimation.

In fact, this theorem can be applied to more generally to a larger class of network estimation problems called 'self-calibration' estimators. In this case each sensor in the network has a parameter which is to be estimated. So the information in this case is the measurements between the sensor nodes and the number of positions which is already known.

In fact, this theorem can be applied to more generally to a larger class of network estimation problems called 'self-calibration' estimators. In this case each sensor in the network has a parameter which is to be estimated. So the information in this case is the measurements between the sensor nodes and the number of positions which is already known.

3.2.1 The problem of Self-Calibration Estimation

Specifically, let a vector of device parameters $\gamma = [\gamma_1, \gamma_2, \dots, \gamma_{n+m}]$. where, it is assumed that each device to have one parameter, but note that the results can be equally applied if γ is a vector of parameters. Sensor node 1...n are unknown nodes whose position is not known to us (in previous section it is called unknown device parameters, but now it can be consider as the location of the sensors) and sensors n+1…n+m can be called as the references sensor nodes, Thus the unknown parameters are to be estimated is $\theta = [\theta_1, \dots, \theta_n]$ where $\theta_i = \gamma_i$ for i=1…n. It can be noted that $\{\gamma_i : i = n+1 \dots n+m\}$ are known values. Sensors i and j make pair-wise observations $X_{i,j}$ having probability density function $f_{X/\gamma}(X_{i,j}|\gamma_i, \gamma_j)$. The device can be permitted to make some incomplete observations. This is because two devices may present out of the range of each other or is having some limited processing capacity. Further consider *H(i)*={*j*:sensor *j* makes a pair-wise measurement with the sensor *i*}. Obviously a sensor cannot make pair wise measurement with it. So it can be shown by $i \notin H(i)$. By the law of symmetry, if $j \in H(i)$ then $i \in H(j)$.

By the law of reciprocity, further it can be assumed that $X_{i,j} = X_{j,i}$. This is sufficient to be considered only the lower triangle part of the observation matrix X = $((X_{i,j}))_{i,j}$ during the formulation of the joint likelihood function. In general if it may be possible to make independent measurements between the links of sensor i and sensor j. It can also be assumed that a scalar sufficient statistic can also be found. Finally, it can be assumed that $\{X_{i,j}\}$ are statistically independent of each other for j < i. This is a very good simplification but it is necessary for analysis. Using the pair wise measurements similar to those presented in Section 2.6 is very important for verification of true performance.

The log of the joint conditional PDF is

$$l(X|\gamma) = \sum_{i=1}^{m+n} \sum_{\substack{j \in H(i) \\ j<i}} l_{i,j} \tag{3.2}$$

where

$$l_{i,j} = \log f_{X|\gamma}(X_{i,j}|\gamma_i, \gamma_j).$$

where the CRB on the covariance matrix for any unbiased estimator $\hat{\theta}$ is cov($\hat{\theta}$) is $\text{cov}(\hat{\theta}) \geq F_{\theta}^{-1}$, . The Fisher information matrix (FIM) F_θ is defined as,

$$F_{\theta} = -E\left(\nabla_{\theta}\left(\nabla_{\theta} l\left(X|\gamma\right)\right)\right)^{T} = \begin{bmatrix} f_{1,1} & \cdots & f_{1,n} \\ \vdots & \ddots & \vdots \\ f_{n,1} & \cdots & f_{n,n} \end{bmatrix} \quad (3.3)$$

3.2.2 Conditions for a decreasing CRB

As it is found that more the number of sensor device in the environment more performance so if the number of known device or unknown device increases the performance increases. So as more number of sensor devices are used for eatimation of the location parameter, the accuracy of the estimation of the postion of all the sensor device increases. So for an N number of sensor nodes network, there may be O(N) number of parameters, and $O(N^2)$ number of measurement variables $\{X_{i,j}\}$. The analysis which is done in this section provides some sufficient conditions make to ensure that the CRB decreases as the number of sensor devices are goes on increases in the wireless sensor network . Further let a wireless sensor network consisting of n number of unknown sensor nodes and m number of reference sensor nodes whose position is already known to us. It may be define that the total number of sensor device is N = n+m. Here take a consideration that addition of one singe device to the sensor network., the objective is to know the effect of it in the performance of it. How the position estimation changes and the performance increases. Consider F and G are the FIMs which is defined in (3.3), respectively. (on the other hand F is a n× n matrix, and G is an n+ 1×n+ 1 matrix.)

Consider $[G^{-1}]_{ul}$ is thee upper left n*n block of G^{-1}. So for the (n+1) device case:

Condition 1: $\frac{\partial}{\partial \theta_{n+1}} l_{k,n+1} = \pm \frac{\partial}{\partial \theta_k} l_{k,n+1}, \forall k = 1..n$ and

Condition2: Sensor node n+1 makes a pair wise measurements between it and at least one neighbor sensor which is unknown sensor and at least two sensor nodes. So in total;

Then two properties hold:

Property 1: $F^{-1}-[G^{-1}]_{ul} \geq O$ in the positive semi-definite sense, and

Property 2: $tr F^{-1} > tr[G^{-1}]_{u}l$.

Property (1) indicates that the additional unknown sensor position parameters which is introduced by the (n+1)st unknown sensor node does not change the estimation of the original n unknown sensor position parameters. Further, Property (2) indicates that the sum of the CRB variance bounds for the n unknown sensor position parameters goes on decreasing. Thus when an unknown sensor node enters a wireless network and adds some pair-wise observations with at least one unknown sensor node and at least two sensor node in total, then the bound on the average estimation variance of the for total n number of sensor nodes coordinate estimates is reducing. This is also satisfied by the data processing theorem

CRB for network self calibration:

The diagonal elements, $f_{k,k}$, of F is given by

$$f_{k,k} = E\left(\frac{\partial}{\partial\theta_k} l(X|\theta)\right)^2 = E\left(\sum_{j\in H(k)} \frac{\partial}{\partial\theta_k} l_{k,j}\right)^2$$

$$f_{k,k} = \sum_{j\in H(k)} \sum_{p\in H(k)} E\left(\frac{\partial}{\partial\theta_k} l_{k,j}\right)\left(\frac{\partial}{\partial\theta_k} l_{k,p}\right)$$

If $X_{k,j}$ and $X_{k,p}$ are independent random variables, and $E\left[\frac{\partial}{\partial\theta_k} l_{k,j}\right] = 0$, the expectation of the product is only nonzero for *p*=*j*. Thus $f_{k,k}$ simplifies to the *k*=*l* result in (3.4). The off-diagonal elements similarly simplify,

$$f_{k,l} = \sum_{j\in H(k)} \sum_{p\in H(l)} E\left(\frac{\partial}{\partial\theta_k} l_{k,j}\right)\left(\frac{\partial}{\partial\theta_l} l_{l,p}\right)$$

Here, due to independence and zero mean of the two terms, the expectation of product will be zero unless both *p*=*k* and *j*=*l* .Thus the $k \neq l$ result in (3.4).

There is need to compare F, the FIM for the n unknown sensor nodes problem, and **G**, the FIM of the n+1 unknown sensor node devices case. Partitioning the G blocks by the way given below,

$$G = \begin{bmatrix} G_{ul} & g_{ur} \\ g_{ll} & g_{lr} \end{bmatrix}$$

where G_{ul} is an n*n matrix, g_{lr} is the scalar Fisher information for the node having parameters θ_{n+1} , and

$g_{ur} = g_{ur}^{T}$ are n*1 vectors with kth element,

$$g_{ur}(k) = I_{H(n+1)}(k) E\left(\frac{\partial}{\partial\theta_k} l_{k,n+1}^{n+1} \right)\left(\frac{\partial}{\partial\theta_{n+1}} l_{k,n+1}^{n+1} \right)$$

$$g_{lr} = \sum_{j \in H(n+1)} E\left(\frac{\partial}{\partial\theta_{n+1}} l_{n+1,j}^{n+1} \right)^2$$

Here, the log-likelihood function can be denoted by the observation between sensor node i and j in as $l_{i,j}^{n}$ and $l_{i,j}^{n+1}$

For the unknown sensor node n and n+1 cases, respectively. Similarly, consider $l^{n}(X|\gamma_n)$ and $l^{n+1}(X|\gamma_{n+1})$

The joint likelihood function in (3.2) for the unknown sensor node n and n+1 devices cases, respectively. Then

$$l^{n+1}(X|\gamma_{n+1}) = \sum_{i=1}^{m+n+1} \sum_{\substack{j \in H(i) \\ j<i}} l_{i,j}^{n+1} = l^{n}(X|\gamma_n) + \sum_{j \in H(n+1)} l_{n+1,j}^{n+1}$$

Since $l_{n+1,j}^{n+1}$ is a function only of parameters $\gamma_{n+1} = \theta_{n+1}$ and γ_j ,

$$\frac{\partial^2}{\partial\theta_k\partial\theta_l}\sum_{j\in H(n+1)} l_{n+1,j}^{n+1} = \begin{cases} I_{H(n+1)}(k)\dfrac{\partial^2}{\partial\theta_k^2} l_{n+1,k}^{n+1}, & l=k \\ 0, & l\neq k \end{cases}$$

Thus $\mathbf{G}_{ul}$=F+diag(h), where h={h_1,h_2,…,h_n} and $h_k = I_{H(n+1)}(k)E\left(\frac{\partial}{\partial\theta_k} l_{n+1,k}^{n+1}\right)^2$.

Comparing the cramer-rao lower bound of the covariance matrix of the first sensor node n n and the second sensor node n+1 device cases, given by F^{-1} and $[G^{-1}]_{ul}$,. Now, $[G^{-1}]_{ul}$ is the upper left n*n submatrix of G^{-1},

$$[G^{-1}]_{ul}=\{G_{ul}-g_{ur}g_{lr}^{-1}g_u\}^{-1}=\{F+J\}^{-1}$$

where $J = diag(h) - \dfrac{g_{ur}g_{ur}^T}{g_{lr}}$

Both the F and G matrices are Hermitian matrices. It is clear that the F is positive semidefinite. Consider the $\lambda_k(F)$, k=1,..,n are the eigenvalues of the matrix F and $\lambda_k(F+J)$, k=1…n are the eigen values of the both matrices. These are arranged in the increasing order. It can be shown that J is positive semidefinite, then it is known that:

(A.1) $$0\le\lambda_k(F)\le\lambda_k(F+J), \forall k=1...n$$

Since the eigenvalues of a matrix inverse are the inverses of the eigenvalues of the matrix,

(A.2) $$\lambda_k\left(\{F+J\}^{-1}\right)\le\lambda_k\left(F^{-1}\right), \forall k=1...n,$$

Which proves property ! of theorem III.5. If in addition, it can be shown that tr(J)>0, then tr(F+J)>tr(F), and therefore $\sum_{k=1}^n\lambda_k(F+J) > \sum_{k=1}^n\lambda_k(F)$.

This with (A.1) implies that $\lambda_j(F+J)>\lambda_j(F)$ for at least one $j\in 1...n$. Thus in addition to (A.2),

A.2.1 Showing positive semi definiteness and positive trace of J

The diagonal elements of J, $[J]_{k,k}$ are, (A.3) $$[J]_{k,k} = h_k - g_{ur}^2(k)/g_{lr}.$$

If $k \notin H(n+1)$ then h_k=0 and g_{ur}(k)=0, thus $[J]_{k,k}$=0. Otherwise, if $k \in H(n+1)$

$$[J]_{k,k} = E\left(\frac{\partial l_{n+1,k}^{n+1}}{\partial \theta_k}\right)^2 - \frac{\left[E\left(\frac{\partial l_{n+1,k}^{n+1}}{\partial \theta_k}\right)\left(\frac{\partial l_{n+1,k}^{n+1}}{\partial \theta_{n+1}}\right)\right]^2}{\sum_{j \in H(n+1)} E\left(\frac{\partial l_{n+1,j}^{n+1}}{\partial \theta_{n+1}}\right)^2}$$

Because of reciprocity, the numerator is equal to the square of the j=k term in the sum in the denominator. Thus

$$[J]_{k,k} \geq E\left(\frac{\partial l_{n+1,k}^{n+1}}{\partial \theta_k}\right)^2 - E\left(\frac{\partial l_{n+1,k}^{n+1}}{\partial \theta_k} \quad \frac{\partial l_{n+1,k}^{n+1}}{\partial \theta_{n+1}}\right) = 0$$

The equality will hold if k is the only member of the set *H(n+1)*. When condition (2) of theorem III.5 holds, $[J]_{k,k}$ will be strictly greater than zero. Thus tr J>0.

Next, it is shown that J is diagonnaly dominant, i.e.,

$$[J]_{k,k} \geq \sum_{\substack{j=1 \\ j \neq k}}^{n} \left|[J]_{k,j}\right| = \sum_{\substack{j=1 \\ j \neq k}}^{n} \frac{|g_u(k) g_u(j)|}{g_{lr}},$$

Where $[J]_{k,k}$ is given in(A.3). Since $H(n+1) \neq \Phi$, thus g_{lr}>0, and an equivalent condition is,

(A.4) $$g_{lr} h_k \geq |g_{ur}(k)| \sum_{j=1}^{n} |g_{ur}(j)|.$$

If $k \notin H(n+1)$ then h_k=0 and g_{ur}(k)=0, and the equality holds. If $k \in H(n+1)$, then

$$g_{lr} h_k = E\left(\frac{\partial l_{k,n+1}^{n+1}}{\partial \theta_k}\right)^2 \sum_{j \in H(n+1)} E\left(\frac{\partial l_{n+1,j}^{n+1}}{\partial \theta_{n+1}}\right)^2.$$

Because of the condition (1) of theorem III.5,

$$E\left(\frac{\partial l_{k,n+1}^{n+1}}{\partial \theta_k}\right)^2 = \left|E\left(\frac{\partial l_{k,n+1}^{n+1}}{\partial \theta_{n+1}}\frac{\partial l_{k,n+1}^{n+1}}{\partial \theta_k}\right)\right|$$

Thus $$g_{lr}h_k = |g_{ur}(k)|\left[\sum_{\substack{j\geq 1\\ j\in H(n+1)}} |g_{ur}(j)| + \sum_{\substack{j\leq 0\\ j\in H(n+1)}} \left|E\left(\frac{\partial l_{j,n+1}^{n+1}}{\partial \theta_{n+1}}\frac{\partial l_{j,n+1}^{n+1}}{\partial \theta_j}\right)\right|\right]$$

Since $g_{ur}(j)=0$ if $j \notin H(n+1)$, the first sum can include all $j \in 1...n$. Since the 2nd sum is >0, (A.4) is true.

Diagonal elements in the J is positive semidefinite, which is already give. It can be noted that the matrix H(n+1) includes ≥ 1 reference sensor nodes and the 2nd sum is >0 and the inequality. This implies a positive definiteness of J and it can be assured that the CRB is strictly decreasing.

This section deals with the self-calibration lower bound analysis in Section 3.2 which already applied specifically to the localization estimation problem originally stated in Section 1.3. In the particular case , 2-D coordinate estimation problems are considered, when the measurements $X_{i,j}$ are RSS, QRSS, connectivity, TOA, or AOA. It can be turned out that the formulation of the variance bounds for these various measurements is remarkably similar. The particular differences can also be marked that the show how localization performance varies by measurement type.

CRB for RSS:

For the elements of $\mathbf{F_R}$, using (2.2) and (3.2),

$$l_{i,j} = \log\left(\frac{10\log 10}{\sqrt{2\pi\sigma_{dB}^2}}\frac{1}{P_{i,j}}\right) - \frac{\gamma}{8}\left(\log\frac{\|z_i - z_j\|^2}{\delta_{i,j}^2}\right)^2$$

Recall $\|z_i - z_j\| = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$. Thus

$$\frac{\partial}{\partial x_j} l_{i,j} = -\frac{\gamma}{2}\left(\log\frac{\|z_i - z_j\|^2}{\delta_{i,j}^2}\right)\frac{x_j - x_i}{\|z_i - z_j\|^2}.$$

Note that $\frac{\partial}{\partial x_j} l_{i,j} = -\frac{\partial}{\partial x_i} l_{i,j}$, thus the log-normal distribution of RSS measurements meets condition(1) of theorem III.5. The 2nd partials differ based on whether or not i=j and if the partial is taken w.r.t. y_i or x_i. For example

$$\frac{\partial^2 l_{i,j}}{\partial x_i \partial y_j} = -b \frac{(x_i - x_j)(y_i - y_j)}{\left\| z_i - z_j \right\|^4} \left[-\log\left(\frac{\left\| z_i - z_j \right\|^2}{\delta_{i,j}^2} \right) + 1 \right]$$

$$\frac{\partial^2 l_{i,j}}{\partial x_i \partial y_j} = -b \frac{(x_i - x_j)(y_i - y_j)}{\left\| z_i - z_j \right\|^4} \left[-\log\left(\frac{\left\| z_i - z_j \right\|^2}{\delta_{i,j}^2} \right) - 1 \right]$$

Note that $E\left[\log\left(\left\| z_i - z_j \right\|^2 / \delta_{i,j}^2\right)\right] = 0.$ Thus the FIM simplifies to take the form in (3.6) with s=4 and $h_{k,l} = 0 \forall k,l.$

TOA:

For the TOA case,

$$l_{i,j} = \left(-\log\sqrt{2\pi\sigma_T^2} - \frac{\left(T_{i,j} - \left\| z_i - z_j \right\| / v_p\right)^2}{2\sigma_T^2} \right)$$

Taking the partial w.r.t. x_j

$$\frac{\partial}{\partial x_j} l_{i,j} = -\frac{1}{\sigma_T^2} \left(\frac{v_p T_{i,j}}{\left\| z_i - z_j \right\|} - 1 \right) (x_j - x_i),$$

Note that in the TOA case it is also true that $\frac{\partial}{\partial x_j} l_{i,j} = -\frac{\partial}{\partial x_i} l_{i,j}$, meeting the condition (1) of theorem III.5. two examples of the second partial derivatives are given by,

$$\frac{\partial^2}{\partial x_i \partial y_j} = -\frac{1}{\sigma_T^2 v_p^2} \frac{cT_{i,j}}{\|z_i - z_j\|} \frac{(x_i - x_j)(y_i - y_j)}{(x_i - x_j)^2 + (y_i - y_j)^2}$$

$$\frac{\partial^2}{\partial x_i \partial x_j} = -\frac{1}{\sigma_T^2 v_p^2} \left[\frac{v_p T_{i,j}}{\|z_i - z_j\|} - 1 - \frac{v_p T_{i,j}}{\|z_i - z_j\|} \frac{(x_i - x_j)^2}{\|z_i - z_j\|^2} \right]$$

The 2nd partial derivatives depend on the term, $v_p T_{i,j} / \|z_i - z_j\|$, which has an expected value of 1, and the terms of F_R take the form in (3.6) with s=2 and $h_{k,l} = 0 \forall k, l.$

QRSS:

The derivation of the CRB for the case when measurements are k-level QRSS. It is already been indicated that the CRB for any self-calibration estimator is a function if the expected value of the second partial derivatives of the terms {li,j} where,

$$l_{i,j} = \log p[Q_{i,j} | z_i, z_j].$$

The first partial derivatives for the QRSS case of $l_{i,j}$ with respect to x_i are

$$\frac{\partial}{\partial x_i} l_{i,j} = \frac{\frac{\partial}{\partial x_i} P[Q_{i,j} | z_i, z_j]}{P[Q_{i,j} | z_i, z_j]}$$

Similarly,

$$\frac{\partial^2}{\partial x_i^2} l_{i,j} = \frac{\frac{\partial^2}{\partial x_i^2} P[Q_{i,j} | z_i, z_j]}{p[Q_{i,j} | z_i, z_j]} - \left(\frac{\frac{\partial}{\partial x_i} P[Q_{i,j} | z_i - z_j]}{P[Q_{i,j} | z_i, z_j]} \right)^2$$

Thus

$$-E\left[\frac{\partial^2}{\partial x_i^2} l_{i,j}\right] = -\sum_{s=0}^{k-1} \frac{\partial^2}{\partial x_i^2} p\left[Q_{i,j} = s \middle| z_i, z_j\right] + \sum_{s=0}^{k-1} \frac{\left(\frac{\partial}{\partial x_i} P\left[Q_{i,j} = s \middle| z_i, z_j\right]\right)^2}{P\left[Q_{i,j} = s \middle| z_i z_j\right]}.$$

The first term is the telescoping sum of $\frac{\partial^2}{\partial {x_i}^2}\Phi[.]$ terms,

$$\sum_{s=0}^{k-1} \frac{\partial^2}{\partial x_i^2} P\left[Q_{i,j} = s \middle| z_i, z_j \|\right] = \sum_{s=0}^{k-1} \frac{\partial^2}{\partial x_i^2} \Phi\left[g_{i,j}(s+1)\right] - \sum_{s=0}^{k-1} \frac{\partial^2}{\partial x_i^2} \Phi\left[g_{i,j}(s)\right] = \frac{\partial^2}{\partial x_i^2} \Phi\left[g_{i,j}(k)\right] - \frac{\partial^2}{\partial x_i^2} \Phi\left[g_{i,j}(0)\right] = 0$$

To further evaluate, note that

$$\frac{\partial}{\partial x_i} P\left[Q_{i,j} = s \middle| z_i, z_j \|\right] = \frac{\sqrt{\gamma}}{\sqrt{2\pi}} \frac{x_i - x_j}{\|z_i - z_j\|^2} \left[\exp\left(-\frac{\gamma}{2}\ln^2 \frac{\|z_i - z_j\|}{d_{s+1}}\right) - \exp\left(-\frac{\gamma}{2}\ln^2 \frac{\|z_i - z_j\|}{d_{s+1}}\right)\right]$$

As a result it simplifies to

$$\text{-E}\left[\frac{\partial^2}{\partial x_i^2} l_{i,j}\right] = \frac{\gamma}{2\pi} \frac{(x_i - x_j)^2}{\|z_i - z_j\|^4} h_{i,j},$$

3.3 The Cooperative Localization CRB

This section deals with the self-calibration lower bound analysis which is given in Section 3.2 and is applied specifically for the sensor node position estimation problem originally stated in Section 1.3. Here only the 2-D coordinate estimation is taken into account, when the measurements $X_{i,j}$ are RSS, QRSS, connectivity, TOA, or AOA. the formulation of the variance bounds of the CRLB for these various measurements is same. The particular differences can be found out for the sensor node localization problem which varies by measurement type.

3.3.1 Calculate Fisher information sub-matrices:

First, form these $n \times n$ matrices: $\mathbf{F_{xx}}$,$\mathbf{F_{xy}}$,$\mathbf{F_{yy}}$. As it is already introduced in section 1.3, n is the number of unknown sensor node location. The k,l element, for the sensor node $k, l \in \{1, ..., n\}$ of each matrix can be calculated by

(3.6)

$$[F_{xx}]_{k,l} = \begin{cases} \gamma \sum_{i \in H(k)} h_{k,i} (x_k - x_i)^2 / \|z_k - z_i\|^s & k = l \\ -\gamma I_{H(k)}(l) h_{k,l} (x_k - x_l)^2 / \|z_k - z_l\|^s & k \neq l \end{cases}$$

$$[F_{xy}]_{k,l} = \begin{cases} \gamma \sum_{i \in H(k)} h_{k,i} (x_k - x_i)(y_k - y_i) / \|z_k - z_i\|^s & k = l \\ -\gamma I_{H(k)}(l) h_{k,l} (x_k - x_l)(y_k - y_l) / \|z_k - z_l\|^s & k \neq l \end{cases}$$

$$[F_{yy}]_{k,l} = \begin{cases} \gamma \sum_{i \in H(k)} h_{k,i} (y_k - y_i)^2 / \|z_k - z_i\|^s & k = l \\ -\gamma I_{H(k)}(l) h_{k,l} (y_k - y_l)^2 / \|z_k - z_l\|^s & k \neq l \end{cases}$$

where, γ is a channel constant, and s is an exponent. These both parameters are function of the measurement type and are given, and IH(k)(l) is a indicator function which is used for calculation, (this allows to include the information if sensor node k made a measurement with sensor node l), IH(k)(l) = 1 if l H(k), or 0 if not. Also, $h_{k,l}$ is consider to be a loss term which occurs due to quantization od the signal. This may be equal to any other methods that are given by TOA, RSS, and AOA. Since all these are assumed to use unquantized measurements, and for QRSS, hk,l it is given by

(3.7) $$h_{i,j} = \frac{1}{2\pi} \sum_{s=0}^{k-1} \frac{\left[\exp\left(-\frac{1}{2} g_{i,j}^2 (s+1) \right) - \exp\left(-\frac{1}{2} g_{i,j}^2 (s) \right) \right]^2}{\Phi(-g_{i,j}(s+1)) - \Phi(-g_{i,j}(s))}$$

Where $g_{i,j}(s)$ was given in (2.14), _(x) is the CDF of the standard normal, and the distance thresholds ds can be given as (2.16). It may be noted that it can be used for any K_{level} QRSS measurements methods but in particular case, 2-level QRSS represents the connectivity measurements. For K = 2, (3.7) . It can be simplified to

$$h_{i,j} = \frac{\exp\left[-g_{i,j}^{2}(1)\right]}{2\pi\Phi\left[-g_{i,j}(1)\right]\left\{1-\Phi\left[-g_{i,j}(1)\right]\right\}}. \tag{3.8}$$

3.3.2 Merge sub-matrices to form the FIM:

Now the objective is to form the $2n \times 2n$ Fisher information matrix (FIM) F which is of theo the 2n coordinates of the unknown sensor node in θ that is need for estimation. For the case of TOA, RSS, or QRSS, select **F=F$_{TR}$**, while for AOA, select **F=F$_{A}$**, where

$$F_{TR} = \begin{bmatrix} F_{xx} & F_{xy} \\ F_{xy}^{T} & F_{yy} \end{bmatrix}, \; F_{A} = \begin{bmatrix} F_{yy} & -F_{xy} \\ -F_{xy}^{T} & F_{xx} \end{bmatrix}, \tag{3.9}$$

Where F_{xx},F_{xy}, and F_{yy} are given in (3.6), and the superscript ^{T} indicates matrix transposition.

3.3.3 Invert the FIM to get the CRB

As it is known that the CRB matrix which is the inverse of the fisher information matrix So it is equal to F^{-1}, which is the inverse of the FIM. The diagonal represents the values which are the variance bounds for the unknown sensor node position that is 2n number of parameters of θ. It can be more precisely indicate that the an estimator of sensor *i*'s coordinates be $\hat{z}_i = \left[\hat{x}_i, \hat{y}_i\right]^{T}$. Thus the location variance of the estimator can be obtained to be σ_i^2,

$$\sigma_i^2 \equiv tr\left\{\mathrm{cov}_{\theta}\left(\hat{z}_i\right)\right\} = Var_{\theta}\left(\hat{x}_i\right) + Var_{\theta}\left(\hat{y}_i\right), \tag{3.10}$$

Then the Cramer-rao bound can be found out ,

$$\sigma_i^2 \geq \left(F^{-1}\right)_{i,i} + \left(F^{-1}\right)_{i+n,i+n} \tag{3.11}$$

	Channel constants γ	Exponent s	FIM F
TOA	$\gamma = 1/(v_p \sigma_T)^2$	S=2	$F=F_{TR}$
RSS	$\gamma = \left(\frac{10\eta_p}{\sigma_{dB} \log 10} \right)^2$	S=4	$F=F_{TR}$
QRSS	$\gamma = \left(\frac{10\eta_p}{\sigma_{dB} \log 10} \right)^2$	S=4	$F=F_{TR}$
Connectivity	$\gamma = \left(\frac{10\eta_p}{\sigma_{dB} \log 10} \right)^2$	S=4	$F=F_{TR}$
AOA	$\gamma = 1/\sigma_\alpha^2$	S=4	$F=F_{TR}$

3.3.4 Results Seen from the CRB:

Here the scaling parameter is calculated. This indicates that if the geometry of the sensor network remains constant and the scale increases how it changes the FIM. This can be found without calculating the CRLB. So the objective is without calculating the CRB for the scaling characteristics of the variance bound can be explored or not. Another question arises what will happen if the geometry and the connectivity remains constant and the network is scaled up .

• TOA: TOA bounds will remain constant with a scaling of the dimensions. Note that since s = 2 for TOA, the fractions in (3.6) are unitless - if units of the coordinates were (ft) or even (cm) instead of (m), the ratios would be identical. Instead, the units come from the standard deviation of ranging error, $v_p\sigma^T$.

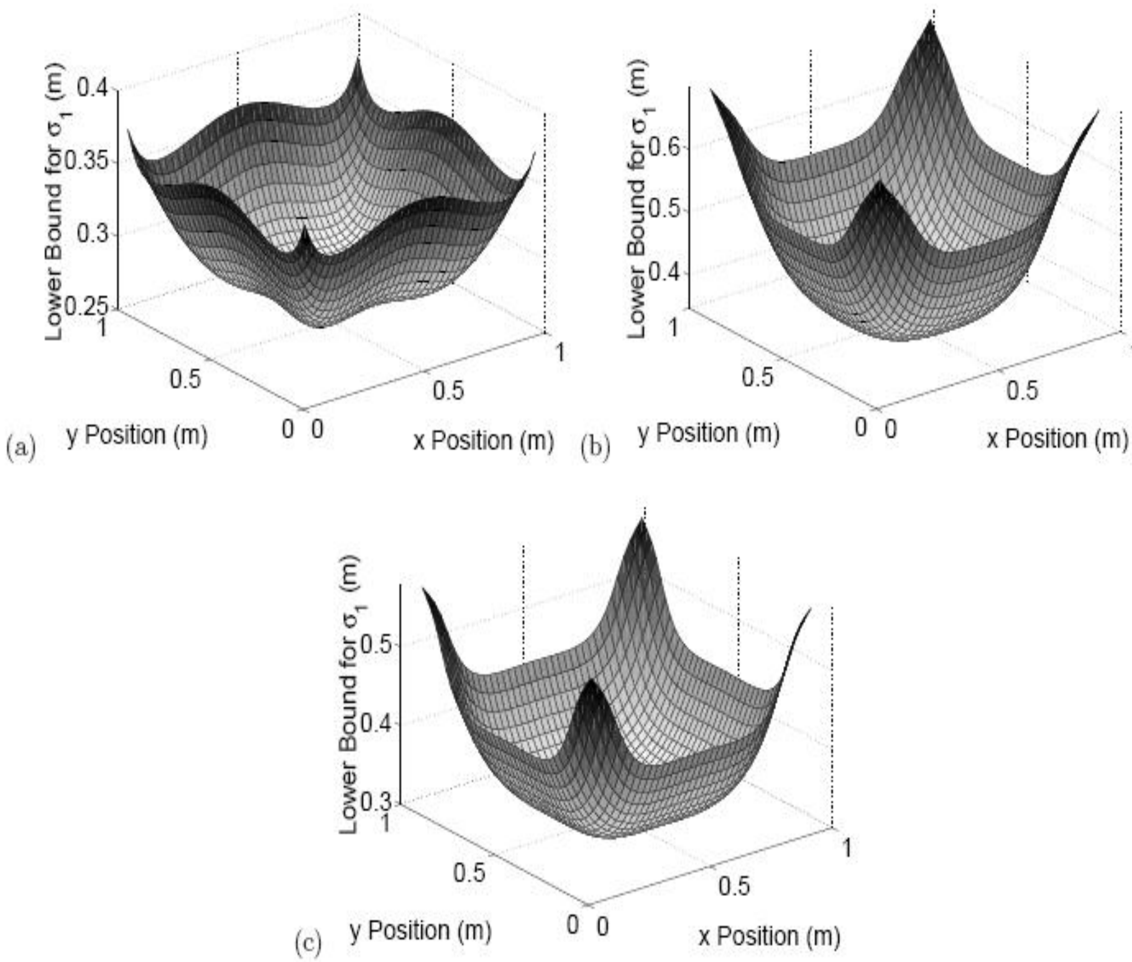


Figure 3.3: Lower bound for σ1 (m) for the single unknown-location device system vs. the coordinates of the unknown-location device, in a channel with σdB/np = 1.7, for (a) RSS, (b) proximity with d1 = 1/√2 m and (c) 3-level QRSS with d1 = 0.90 m and d2 = 0.56 m.

Three-Level QRSS

Next, consider the performance of the system in the case of K = 3 QRSS measurements. Again, the system is optimized to minimize the CRB when the unknown location device is located at z1 = [0.5, 0.5]T m. It can be shown that the CRB as a

function of the two threshold distances, d1 and d2, is given by,

(3.13)

$$\sigma_i^2 \geq \frac{2}{f_{1,1}}$$

$$f_{1,1} = \frac{2b}{\pi}\left\{ \frac{\exp\left[-b\ln^2\frac{d_\alpha}{d_1}\right]}{\Phi\left(-\sqrt{b}\ln\frac{d_\alpha}{d_1}\right)} + \frac{\exp\left[-b\ln^2\frac{d_\alpha}{d_2}\right]}{\Phi\left(-\sqrt{b}\ln\frac{d_\alpha}{d_2}\right)} + \frac{\left[\exp\left(-\frac{b}{2}\ln^2\frac{d_\alpha}{d_2}\right)-\exp\left(-\frac{b}{2}\ln^2\frac{d_\alpha}{d_1}\right)\right]^2}{\Phi\left(-\sqrt{b}\ln\frac{d_\alpha}{d_2}\right)-\Phi\left(-\sqrt{b}\ln\frac{d_\alpha}{d_1}\right)} \right\}$$

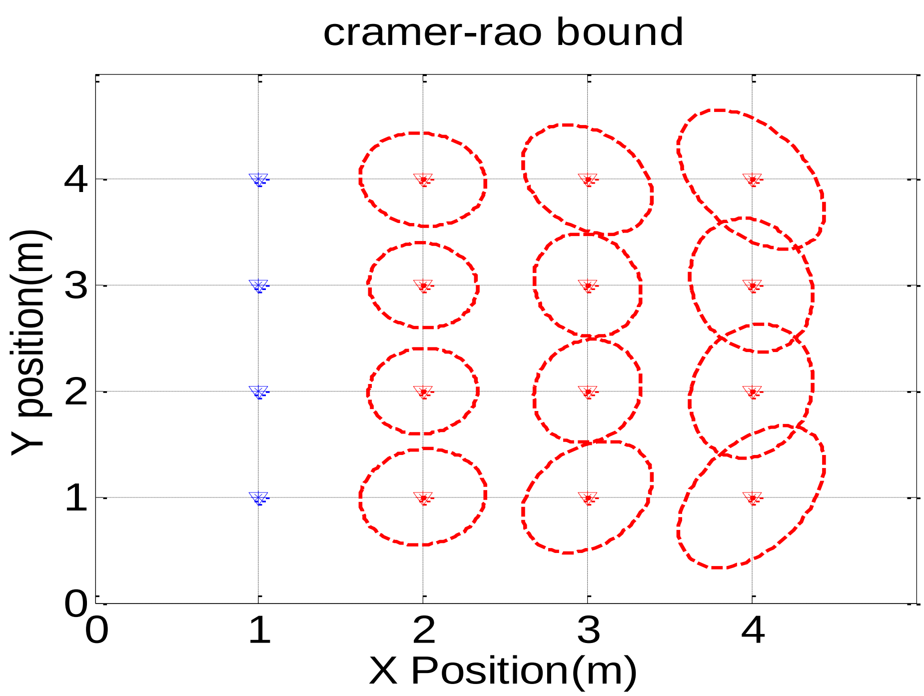


Fig.(3.5) result-1

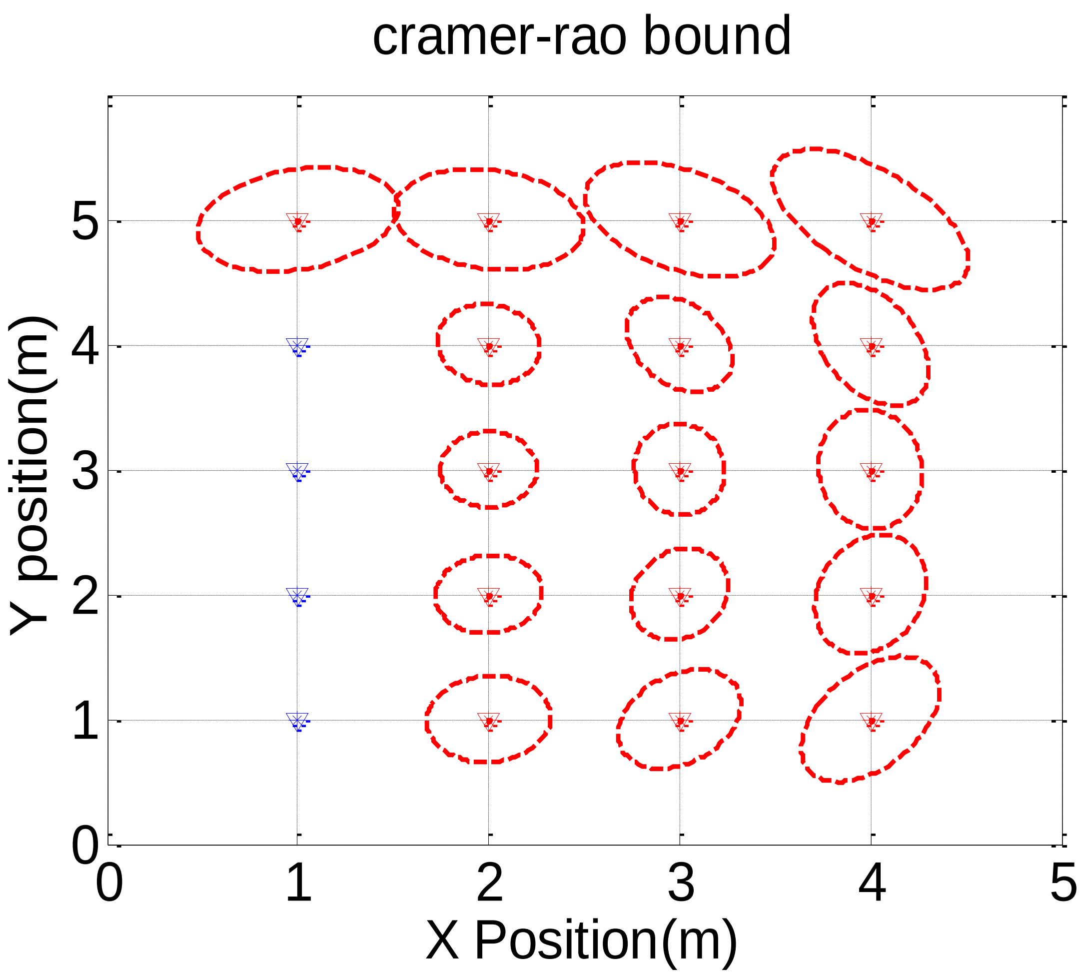


Fig-(3.6) result-2

This result shows how keeping reference node constant only increasing the total number of node we can able to decrease the CRB.

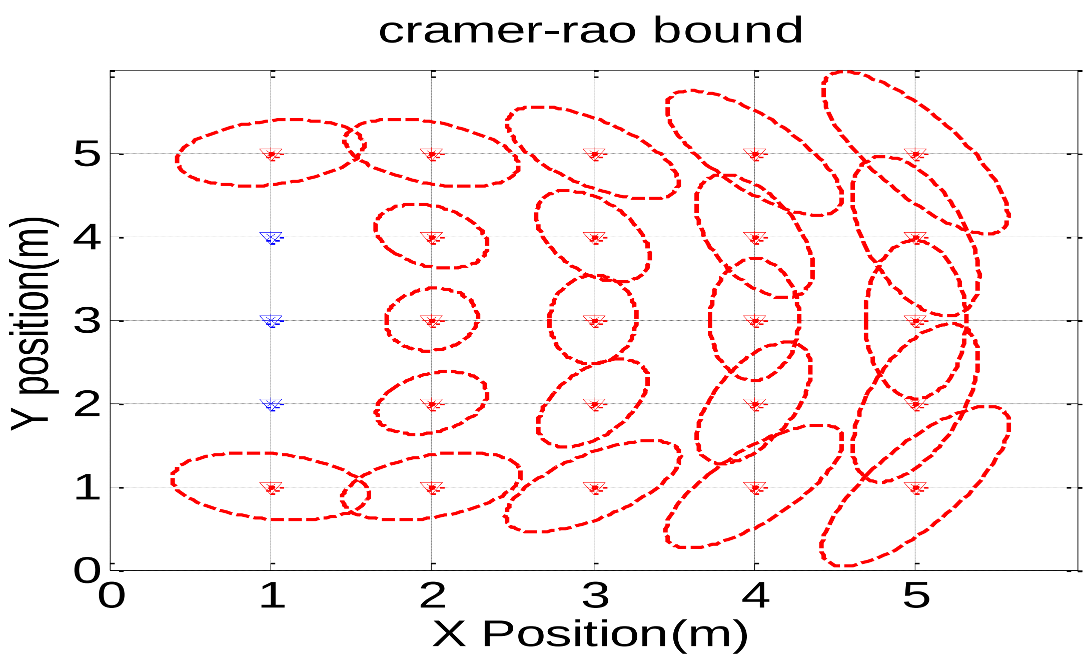


Fig-(3.7) result-3

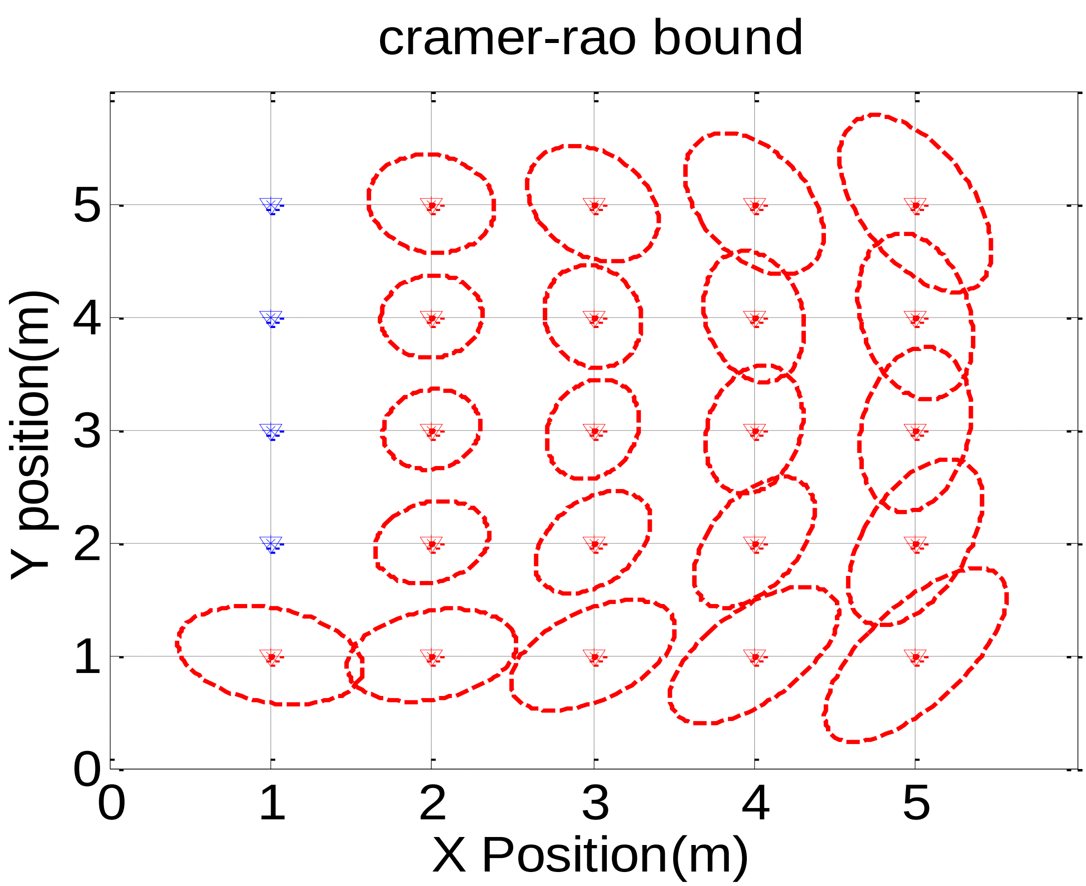


Fig-(3.8) result-4

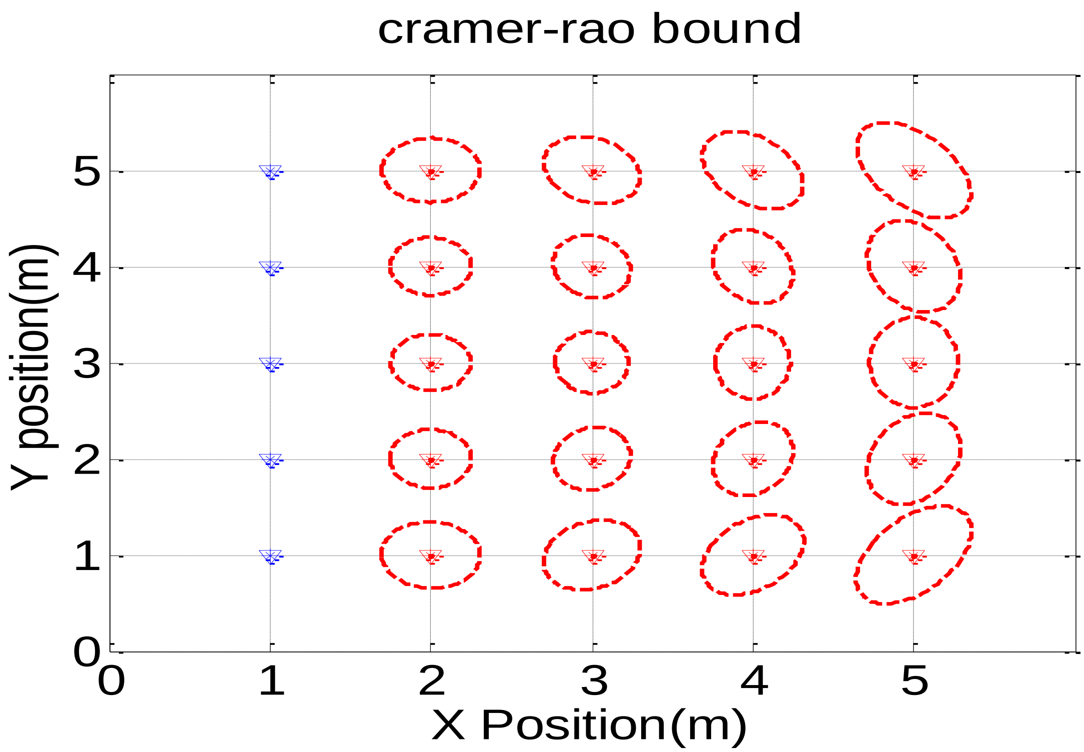


Fig-(3.9) result-5

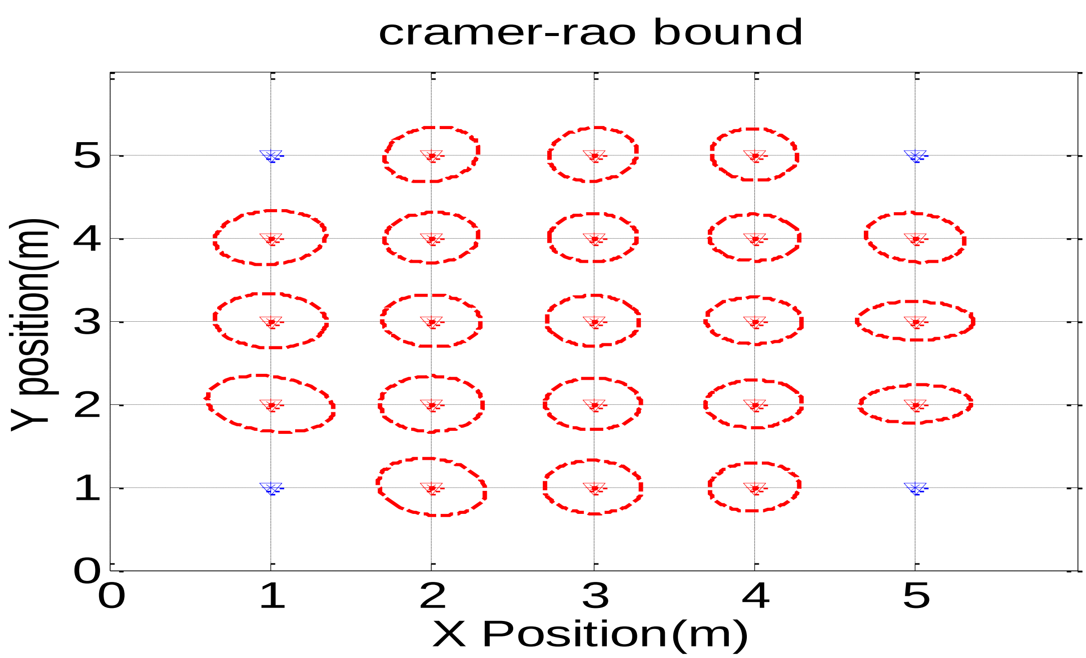


Fig-(3.11) result-6

This result shows how changing the number of reference node and position of the node affect the CRB.

Chapter 4

LOCALIZATION ALGORITHM

4.1 Localization Algorithms

In the previous section the problem is formulated and the lower bound is given. In this chapter on of the famous algorithm is used. It is called maximum likelihood estimation algorithm for the calculation of the position estimation. This algorithm is given in [3]. We have used here only for verification. This is already proposed in [3] for distributed sensor localization. This algorithm can be applied over any model of pair wise measurements. This is based upon the assumption used during the calculation of the distribution function of the measured data.

This is centralized solution. In this case all the sensor nodes need to measure the data and to send the data to a fusion center. The fusion center uses the entire data for the calculation of the position of the sensor node. Thus large number of energy will be used during the transmission. In order to avoid this distributed algorithm needs to be used. So that the life time of the sensor network can be increased and the will also be robust against the sensor positions. In this case there is large number of multi hop communication is required. In this case the gradient descent algorithm is used to find the optimum solution. Here the TROA or RSS can be used to find the optimum solution.

4.1 .1 TOA:

Since the TOA measurements, between any two sensor node can be modeled as Gaussian So

$$\{\hat{z}_i\} = \arg\min_{\{z_i\}} \sum_{i=1}^{N} \sum_{\substack{j \in H(i) \\ j<i}} \left[v_p \left(T_{i,j} - \mu_T \right) - \left\| z_i - z_j \right\| \right]^2 .$$

Recalling that zi represents the coordinate of the sensor node I and H(i) is the set of sensor nodes with which sensor i made the pair wise measurements and vp is the speed of the propagation of the signal , and μT is the mean of the TOA residual error.

4.1.2 RSS:

The MLE based estimation of the RSS case is

$$\{\hat{z}_i\} = \arg\min_{\{z_i\}} \sum_{i=1}^{N} \sum_{\substack{j \in H(i) \\ j<i}} \left(\ln \frac{\left(\delta_{i,j}^{MLE}\right)^2}{\left\| z_i - z_j \right\|^2} \right)^2$$

Here $\delta_{i,j}^{MLE}$ represents the function of the measured received power $P_{i,j}$ between the sensor node I and j as given in (2.17) (Esspecilly, the MLE of the distance is given by $P_{i,j}$). The RSS MLE is already biased than TOA based position estimation. (4.3)

$$E[\delta_{1,2}] = C\|z_1 - z_2\|$$

Here C is the multiplicative bias factor which is given in (2.7). For some of the typical channels (like those reported in paper), C ≈ 1.2. This adds 20% bias to the range of the measured data. .

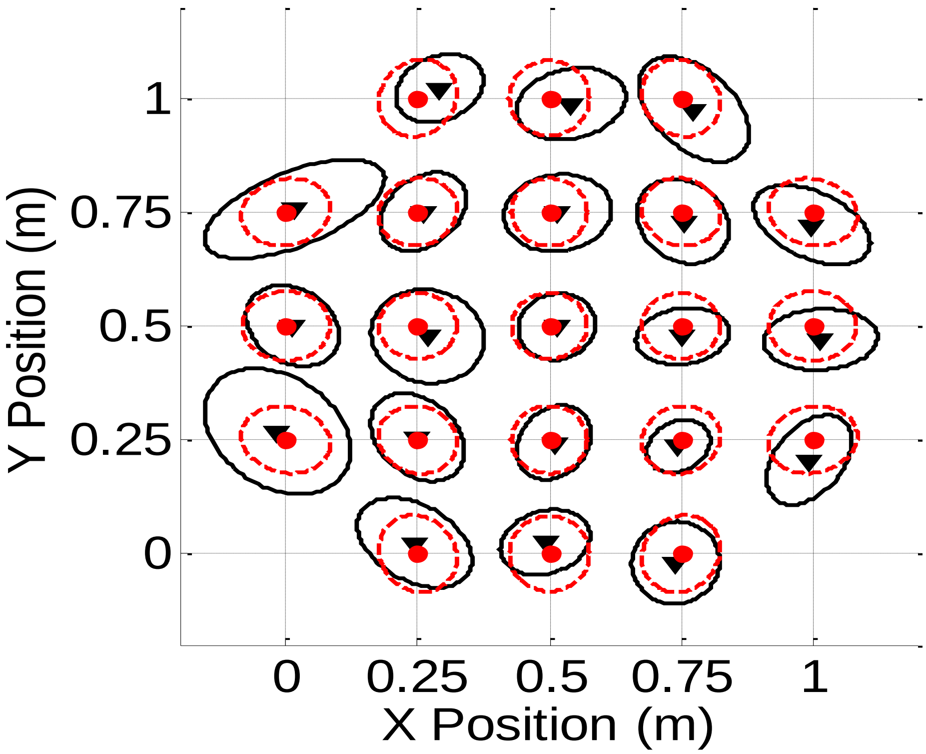


fig-(4.1) result-7

Chapter 5

CONCLUSION

Conclusion:

In this thesis we have explained some of the pair wise measurement techniques (TOA, AOA, RSS and QRSS) that can be used as sensor localization. A study of the CRB and how it affects by the number of nodes, number of reference nodes and the number of unknown wireless sensor node is also studied. This thesis gives some information of the paper [3]. There is need to find out some results which are already given. This work is very little. large number of work need to be done about how large number of sensor nodes can be auto configurable to provide the measured data to the fusion center and the sensor node can be estimated. In order to increase the lifetime of the sensor network the algorithm is to be distributed and robust. This work is very little for the wireless sensor network. The maximum likelihood estimation technique can be combined with clustering technique for efficient estimation

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