HEADING CONTROL OF AN UNDERWATER VEHICLE

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a thesis submitted in partial fulfillment of the requirements for the degree of

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CERTIFICATE

This is to certify that the thesis entitled "Heading Control of an Underwater Vehicle" submitted by Suraj Mishra and Karanki Dinesh in partial fulfillment of the requirements for the award of Bachelor of Technology Degree in Electrical Engineering at the National Institute of Technology, Rourkela, is an authentic work carried out by them under my supervision.

To the best of my knowledge the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any degree or diploma

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ABSTRACT

In this thesis an overview of Autonomous Underwater Vehicles (AUV) is presented which covers the state of art in AUV technology, different components such as sensors and actuators of AUV and the applications of AUVs. This thesis describes the development and verification of six degree of freedom, non-linear simulation model. In this model, external forces and moments are defined in terms of vehicle coefficients. A nonlinear model of AUV is obtained through kinematics and dynamics equations which are linearized about an operating point to get a linearized horizontal plane model. The objective of the AUV control here is heading control i.e. to generate appropriate rudder angle position and thrust so that the desired heading is achieved.

For the above heading control we develop a controller that consists of two loops, one is controlled by a PD controller and the other loop by a P control action. The first and second order Nomoto model of the vehicle is formulated and studied for simpler qualitative analysis of complicated ship model.

Simulation studies were undertaken also for yaw control of a single AUV. The above controller is designed for effective tracking of desired trajectory of the AUV in horizontal plane. All the simulations were performed using both MATLAB and SIMULINK.

The results obtained for heading and yaw control of the AUV studied are presented and discussed in this thesis.

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CHAPTER 1:

INTRODUCTION

With the development of technology and applied sciences, the remotely operated vehicle (ROV) industry has made itself well established with thousands of ROVs having been assembled and deployed. The need for control and automation in robots, however, is becoming a more dominant issue in many situations and environmental conditions. One of the major deciding factor whether a vehicle is to be designed as an ROV or as an autonomous vehicle is its ability to communicate with the operator. Autonomous control is preferred to remote control in environments where communication with a vehicle is constrained.

One of the environments in which communication is inhibited is underwater. Underwater robots are playing a major role in underwater expedition and in exploring greater depths. Assembled and specialized vehicles for deepwater missions have been used in the offshore industry since late 1960s. However excessive dependence on a communications tether and a control platform has restricted their applications.

1.1 AUTONOMOUS UNDERWATER VEHICLE

Due to these limitations of ROVs there has been a surge in interest towards AUVs. AUVs are fully automatic and submersible platforms capable of performing underwater tasks and missions with their onboard sensor, navigation and payload equipments [6].The reason for this surge is many scientific and commercial tasks involve hazardous and inaccessible environments which can be performed by AUVs without any human guidance. The development of AUVs has been boosted by advancement in material science, sophisticated digital control and extremely accurate sensors. Present research on AUVs focuses on making them more compatible, automatic and intelligent. With an aim of achieving the above said goals, thorough research is going on worldwide with high priority on navigation, autonomy and control.

1.2 APPLICATIONS OF AUVs

Due to technical constraints, AUVs were engaged in limited tasks and missions but with rapid enhancement in technology nowadays AUVs are being deployed for many critical jobs with persistently evolving roles and missions.

- i. Commercial: Most of the oil and gas industry requires sea floor mapping and surveying before developing infrastructure. Nowadays AUVs are the most cost effective solution for this job with minimum environmental interference. With the help of AUVs we have an upper hand over the traditional bathymetric technique. Also post-lay pipeline surveys are now possible.
- **ii. Military**: Incorporating its sonar technology AUVs are capable of detecting manned submarines in anti-submarine warfare. They are also used to locate mines and detect unidentified objects to secure an area.
- **iii. Research:** To explore the ocean floor and microscopic lives in it scientist use AUVs equipped with special sensors for their detection and study.
- **iv. Environment:** For long term monitoring of radiation levels, leakage, pollution in aquatic habitats and inspection of underwater structures such as dams, pipelines and dykes AUVs are being utilized.

1.3 STATE OF ART

From the beginning human beings have always desired to explore the unexplored. This crave has empowered men to throw light on what lies deep down under sea. Due to physical limitations from the beginning automatic vehicles are preferred for deep sea exploration. The first Special Purpose Underwater Research Vehicle was developed by Stan Murphy and Bob Francois at the Applied Physics Laboratory at the University of Washington in 1957.

In the 1970s, first AUVs were built and they were put into commercial use in the 1990s. Today AUVs are mostly used for scientific studies, commercial purposes, military and survey operations. The HUGIN series was developed in cooperation between Kongsberg Maritime and the Norwegian Defense Research Establishment and it is the most commercially successful AUV series on the world market today.[1]

Challenges, now an AUV faces are navigation, communication, autonomy, and endurance issues. Automatic functioning is an important aspect of AUVs which deals with circuit configuration and controller strategy. In this project work, the main concern is on the autonomy. During a mission, an AUV may undergo different steering scenarios such as a complete turn at the end of a trajectory, a severe roll during avoiding an obstacle or frequent depth changes while following a tough seabed terrain. Different operations demand different controller strategy. A yaw and surge controller is used to guide the AUV on a particular direction without changing depth. A tracking controller is used to move the AUV on a predefined path.

1.4 MAIN COMPONENTS OF AN AUV

Generally AUV has a modular structure i.e. it is divided into three sections. The cylindrical body is the premier section as almost all the components are housed here. The frontal section has a nose cone shape while tapered tail at the rear end. Such structure hydro-dynamically more efficient and is capable of providing streamlined motion to the vehicle.



Fig 1.1: Components of AUV

The pressure hull provides the majority of the buoyancy for the vehicle and space for dry components such as batteries and control electronics. The thruster motor is connected to the tail. The tail section is designed in a conical fashion to reduce the drag which is caused by the pressure drop at the end of the robot. Scientific sensors like forward look sonar which helps in navigation are placed in the nose section. The cylindrical central section houses the microcontroller circuitry, power supply modules, Rate GYRO which is used to measure the yaw of the vehicle, main CPU, and Doppler Velocity Log (DVL) sensor(SONAR) which senses the

approximate distance travelled in direction of each coordinate axis. Fins help in changing the depth attained by vehicle. Rudder is movable vertically mounted fin, and performs the control of heading direction of the vehicle. Thruster motor is provided at stern end to provide necessary torque to move the vehicle in forward direction and rudder fins control the heading.

POWER MODULE

Pressure tolerant batteries are preferred for AUV. These special batteries do not require a special housing chamber for them. This eliminates the risk of fire hazard due to explosion of battery. the more deeper we go inside the water, the more pressure we are likely to observe. The pressure tolerant design provides better energy and strength to the battery.

GPS

Global Positioning System is a satellite navigation system maintained by the US government, which provides location and time information in all weather condition. With the help of GPS we can get the exact co-ordinates of any object and can locate it anywhere on the earth. To realize track following algorithm, coordinates of the vehicle must be determined precisely, which is provided by the GPS system.

SONAR

Sound Navigation and Ranging (SONAR) is a method that uses sound waves as a medium to navigate, communicate and detect objects in its path. In subsea environments it is very important to avoid collision with static obstacles which is done with the help of SONAR. The sound waves

sent from AUV are reflected back by obstacle and these signals are received by AUV to detect presence of object.

DOPPLER VELOCITY LOG

To provide navigation information Doppler Velocity Log (DVL) is used. DVL exploits Doppler's effect to calculate the velocity of the robot. By bouncing sound frequency in a direction it can determine the velocity of the vehicle in that direction. DVL is also quite useful in determining absolute displacement relative to a fixed frame.

INERTIAL MEASUREMENT UNIT

Inertial Measurement Unit (IMU) uses Gyroscope and Accelerometer to determine the velocity, orientation and the net gravitational force on the AUV. Using dead reckoning technique, the computer keeps a track of the AUV using the data obtained from the IMU. The Gyro being an integral member of the inertial navigational unit, helps in measuring and maintaining the orientation of the vehicle. Accelerometer provides data on velocity of the vehicle

1.4.1 SENSORS AND ACTUATORS

Different types of sensors are used depending upon the application of AUV e.g. whether we want to know the temperature of water, depth of seabed, concentration of any substance present or high quality photos for study.

TEMPERATURE SENSORS: Generally Platinum Resistance Thermometers (PRTs), which are suitable for use in all conditions, are used. PRTs and thermistors are both used for calculating any temperature level.

PRESSURE SENSORS: Strain gauge type pressure sensors are sensitive to temperature variation and hence the data obtained varies with all temperature level, irrespective of the extreme temperature. By the inclusion of a temperature sensor which is diffused into the silicon of the strain sensor, this problem can be overcome. The thermal characterization of the completed sensor is then obtained, which allows a performance of better accuracy over the full working range of temperatures.

CONDUCTIVITY SENSORS: It uses an epoxy formed body linked to a stainless steel base for standard applications.

OPTICAL SENSORS: Sensors such as transmisometers and fluorimeters operate by emitting a light beam (pulsed for the fluorimeters) through optical filters and into the sea water via a window set in the face of the sensor housing which has to be relatively thick to withstand high pressures. [5]

1.5 OBJECTIVES OF THE THESIS

The objectives of this project were to:

- design and model a prototype of the vehicle
- contrive a new control strategy for the vehicle
- Design a controller for track following of vehicle in 2D.

This project work was divided into two phases. First phase deals with modeling of the vehicle while the second phase deals with its controller design. This report doesn't deal with practical implementation as it deals only with theoretical realization.

1.6 ORGANISATION OF THESIS

This thesis is divided into five chapters.

Chapter 2, throws some light on the fundamental theories and concepts regarding underwater vehicles and their design. A brief discussion on kinematics and dynamics of the vehicle is given. The linearized model of the vehicle is considered and its linearized coefficients are calculated. Then a SIMULINK model of the vehicle is studied.

Chapter 3, presents an approximated NOMOTO model of the vehicle which is used for simpler heading control. The 1^{st} and 2^{nd} order model is studied along with their stabilizations. PD controller is used for stabilization of the model with optimum settling time and overshoot limitations.

Chapter 4, introduces the concept of error dynamics, which is optimally reduced for track following of the vehicle. The track following of an AUV is shown along with its MATLAB code. Trajectory tracking of a two thruster model is also shown using PD and LQR control technique.

CHAPTER 2:

KINEMATICS AND DYNAMICS OF AN AUV

To design any, vehicle knowledge of physical laws governing the system is essential. In this chapter modeling and design of AUV in horizontal plane is examined. Controller design of AUV using simple P-PD controller is presented. SIMULINK model for the system is designed and results are obtained to check stability of the obtained system.

2.1 COORDINATE FRAME ASSIGNMENT

To study the motion of marine vehicle 6 degrees of freedom are required since to describe independently the complete position and orientation of the vehicle we require 6 independent coordinates. To describe position and translation motion first three sets of coordinates and their time derivatives are required. While for orientation and rotational motion last three sets of coordinates and their time derivatives are required.

DOF	MOTION	Forces	Linear and angular velocity	Position
1	Motion in x-direction(surge)	X	и	x
2	Motion in y-direction(sway)	Y	ν	У
3	Motion in z-direction(heave)	Ζ	W	Z
4	Rotation in x-direction(roll)	K	р	φ
5	Rotation in y-direction(pitch)	М	q	θ
6	Rotation in z-direction(yaw)	N	r	Ψ

Table 2.1: Notation used for AUV modeling

To obtain a mathematical model of the AUV, its study can be divided into two sub-categories: Kinematics and Dynamics.

Kinematics deals with bodies at rest or moving with constant velocity whereas dynamics deals with bodies having accelerated motion.

2.2 KINEMATICS

For analyzing the motion of the vehicle in 6DOF, we choose two co-ordinate frames. The moving reference frame is fixed to the vehicle called as body-fixed reference frame. Motion of the body-fixed frame is described relative to an inertial frame. For marine vehicles, it is usually assumed that the acceleration of a point on the surface of Earth can be neglected. Thus, an Earth fixed frame can be considered to be an inertial frame. This suggests that the linear and angular velocities of the vehicle should be expressed in body-fixed frame while position and orientation should be described with respect to inertial frame.[8] In a very general form, the motion of vehicle in 6 DOF can be described by the following vectors

$$\eta = \begin{bmatrix} \eta_1^T & \eta_2^T \end{bmatrix}^T \qquad \eta_1 = \begin{bmatrix} x & y & z \end{bmatrix}^T \qquad \eta_2 = \begin{bmatrix} \phi & \theta & \psi \end{bmatrix}^T$$
$$v = \begin{bmatrix} v_1^T & v_2^T \end{bmatrix}^T \qquad v_1 = \begin{bmatrix} u & v & w \end{bmatrix}^T \qquad v_2 = \begin{bmatrix} p & q & r \end{bmatrix}^T$$
$$\tau = \begin{bmatrix} \tau_1^T & \tau_2^T \end{bmatrix}^T \qquad \tau_1 = \begin{bmatrix} X & Y & Z \end{bmatrix}^T \qquad \tau_2 = \begin{bmatrix} K & M & N \end{bmatrix}^T [8]$$

Where η describes the position and orientation of the vehicle with respect to the earth-fixed reference frame, v the translational and rotational velocities with respect to the body-fixed reference frame, and τ the total forces and moments acting on the vehicle with respect to the body-fixed reference frame.[8]



Fig 2.1 Body-fixed and inertial reference frames

Vehicle's path relative to the earth-fixed coordinate system is given [8]

$$\dot{\eta}_1 = J_1(\eta_2)v_1$$
 (2.1)

Where $J_1(\eta_2)$ is the transformation matrix as follows [8]:

$$J_{1}(\eta_{2}) = \begin{pmatrix} c\psi c\theta & -s\psi c\phi + c\psi s\theta s\phi & s\psi s\phi + c\psi s\theta c\phi \\ s\psi c\theta & c\psi c\phi + s\psi s\theta s\phi & -c\psi s\phi + s\psi s\theta c\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{pmatrix}$$
(2.2)

The body-fixed angular vector v_2 and the Euler rate vector $\dot{\eta}_2$ are related through transformation

matrix $J_2(\eta_2)$ by the relation [8]

$$\dot{\eta}_2 = J_2(\eta_2) v_2 \tag{2.3}$$

And
$$J_2(\eta_2) = \begin{bmatrix} 1 & s\phi t\theta & c\phi t\theta \\ 0 & c\phi & -s\phi \\ 0 & s\phi/c\theta & c\phi/c\theta \end{bmatrix}$$
 (2.4)

here, c^{\bullet} means cosine(\bullet), s^{\bullet} means sine(\bullet) and $t(\bullet)$ means tangent(\bullet).

2.3 DYNAMICS

Dynamics is further sub-divided into translational motion and rotational motion.

Translational equations of motion are given by:

$$m \left[\dot{u} - vr + wq - x_g (q^2 + r^2) + z_g (pr + \dot{q}) \right] = \sum X$$

$$m \left[\dot{v} - wp + ur - z_g (qr - \dot{p}) + x_g (qp + \dot{r}) \right] = \sum Y$$

$$m \left[\dot{w} - uq + up - z_g (p^2 + q^2) + x_g (rp - \dot{q}) \right] = \sum Z$$
(2.5)

$$I_{xx}\dot{p} + (I_{zz} - I_{yy})qr + m\left[-z_{g}(\dot{v} - wp + ur)\right] = \sum K$$

$$I_{yy}\dot{q} + (I_{xx} - I_{zz})rp + m\left[z_{g}(\dot{u} - vr + wq) - x_{g}(\dot{w} - uq + vp)\right] = \sum M$$

$$I_{zz}\dot{r} + (I_{yy} - I_{xx})pq + m\left[x_{g}(\dot{v} - wp + ur)\right] = \sum N$$
(2.6)

The first three equations correspond to translational motion of the vehicle while the last three equations deal with the rotational motion of the vehicle. The centre of body fixed frame is taken as the centre of earth fixed frame. So the centre of buoyancy is same as the centre of body fixed frame which makes the off diagonal elements of the I_0 matrix, zero.

where

$$I_{0} = \begin{pmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{pmatrix}$$

This technique is of great help as it reduces the complexity by making $x_g = y_g = z_g = 0$

2.4 LINEARIZED KINEMATICS AND DYNAMICS

We are assuming pure horizontal plane motion of the vehicle, we only consider body relative surge u, sway v, yaw rate r and earth relative position x, heading y and yaw angle ψ . Neglecting all other out of plane terms. Equations 3.1 and 3.2 along with above assumptions result in :

$$\dot{x} = u\cos\psi - v\sin\psi$$
$$\dot{y} = u\sin\psi + v\cos\psi$$
$$\dot{\psi} = r$$

We linearize these equations by assuming that there are small perturbations around a steady point. Let U be the steady state forward velocity about which u is linearized. Sway and yaw rate are linearized about zero. Using Maclaurin expansion of the trigonometric terms and neglecting higher order terms, the linearized kinematic equations are

$$\dot{x} = u - v\psi$$
$$\dot{y} = U\psi + v$$
$$\dot{\psi} = r$$

As in kinematics equation derivation, we simplify the dynamic equation of motion by considering only body relative surge, sway, yaw rate and earth relative position, heading and yaw angle. And again neglecting all the out of plane terms results in:

$$m\left[\dot{u} - vr - r^{2}x_{g} + \dot{r}y_{g}\right] = X$$
$$m\left[\dot{v} + ur - y_{g}r^{2} + \dot{r}x_{g}\right] = Y$$
$$I_{zz}\dot{r} + m\left[x_{g}\left(\dot{v} + ur\right) - y_{g}\left(\dot{u} - vr\right)\right] = N$$

Now, using linearization conditions given above and dropping out any higher order terms we arrive at:

$$X = m \left[\dot{u} + y_g \dot{r} \right]$$
$$Y = m \left[\dot{v} + x_g \dot{r} + Ur \right]$$
$$N = I_{zz} \dot{r} + m \left[x_g \left(\dot{v} + Ur \right) - y_g \dot{u} \right]$$

Here, X, Y and N are vehicle parameters and are combination of various external forces such as added mass, hydrodynamic damping, hydrostatics etc. Above equation along with linearized coefficient derived later results in following linearized vehicle equation of motion:

$$(m - X_{\dot{u}})\dot{u} - X_{u}u - X_{r}r = X_{prop}$$

$$(m - Y_{\dot{v}})\dot{v} + (mx_{g} - y_{\dot{r}})\dot{r} + (mU - Y_{r})r - Y_{v}v = Y_{\delta_{r}}\delta_{r}$$

$$(I_{zz} - N_{\dot{r}})\dot{r} + (mx_{g} - N_{\dot{v}})\dot{v} + (mx_{g} - N_{r})r - N_{v}v = N_{\delta_{r}}\delta_{r}$$

Where $X_{\dot{u}}, Y_{\dot{v}}, Y_{\dot{r}}, N_{\dot{v}}, N_{\dot{r}}$ are the added masses, X_{prop} is thruster force, δ_r is rudder angle.

2.5 STEERING CONTROLLER

Change in rudder angle causes yaw moment on vehicle causing change in heading direction of vehicle.

$$\begin{bmatrix} m - Y_{\dot{v}} & mx_g - Y_{\dot{r}} & 0 & 0\\ mx_g - N_{\dot{v}} & I_{zz} - N_{\dot{r}} & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} - \begin{bmatrix} Y_v & Y - m_r U & 0 & 0\\ N_v & N_r - mx_g & 0 & 0\\ 1 & 0 & 0 & U\\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ y \\ \psi \end{bmatrix} = \begin{bmatrix} Y_{\delta_r} \\ N_{\delta_r} \\ 0 \\ 0 \end{bmatrix} [\delta_r]$$

If sway velocity is assumed to be less relative to other terms, we can neglect it with respect to other terms and $(x_g, y_g, z_g) = (x_b, y_b, z_b)$ so that the kinematics and dynamics equations can be written into following matrix form:

$$\begin{pmatrix} I_{zz} - N_{\dot{r}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} \dot{r} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} + \begin{pmatrix} N_{r} & 0 & 0 \\ 0 & 0 & U \\ 1 & 0 & 0 \end{pmatrix} \begin{bmatrix} r \\ y \\ \psi \end{bmatrix} = \begin{bmatrix} N_{\delta_{r}} \\ 0 \\ 0 \end{bmatrix} [\delta_{r}]$$

From the above matrix representation, the transfer function for the inner yaw loop is found as

$$G_{\psi}(s) = \frac{\psi(s)}{\delta_r(s)} = \frac{\frac{N_{\delta_r}}{I_{zz} - N_r}}{s^2 - \frac{N_r}{I_{zz} - N_r}s}$$

Here the denominator contains root at s = 0 this is due to the fact that hydrostatic forces i.e. gravitational force and buoyancy are zero.

We now develop a simple controller design, using the above derived state equation. This controller consists of inner PD yaw loop and outer P heading loop.

2.6 LINEARIZED COEFFICENT DERIVATION

2.6.1 HYDROSTATICS

As the vehicle is steering controlled, the forces and moments due to gravity and buoyancy in horizontal plane are zero.[9]

2.6.2 HYDRODYNAMIC DAMPING

AXIAL DRAG

Vehicle axial drag is given by [9]

$$X = -\left(\frac{1}{2}\rho C_d A_f\right) u \left| u \right|$$

 $\Rightarrow X_{u|u|} = -\frac{1}{2}\rho C_d A_f$

where,

 $\rho = 1030 \text{ kg/m}^3 = \text{density of surrounding fluid},$

 $A_f = 0.0285 \text{ m}^2 = \text{vehicle frontal area},$

 $C_d = 0.3 =$ axial drag coefficient

Linearizing,

$$X = -\left(\frac{1}{2}\rho C_{d}A_{f}\right)(u+U)|U+u|$$

$$= -\frac{1}{2}\rho C_{d}A_{f}\left(U^{2}+2Uu+u^{2}\right) \qquad [\because u \ll U \& U > 0]$$

$$\Rightarrow X_{u} = -\rho C_{d}A_{f}U = -17.6kg/s \qquad [U = 2m/s]$$

CROSS FLOW DRAG

To linearize quadratic cross-flow drag, we linearize sway and yaw perturbation velocity about 0.[9]

so,
$$v^2 = m_v v$$
$$r^2 = m_r r$$

where $m_v = 0.12 \text{ m/s}$ =sway coefficient

 $m_r = 0.3 \text{ rad/s} = \text{yaw coefficient}$

$$Y_{vc} = -\frac{1}{2} \rho C_{dc} m_{v} \int_{x_{t}}^{x_{b_{2}}} 2R(x) dx - 2\left(\frac{1}{2} \rho S_{fin} C_{df} m_{v}\right)$$

$$N_{vc} = -\frac{1}{2} \rho C_{dc} m_{v} \int_{x_{t}}^{x_{b_{2}}} 2xR(x) dx + 2x_{fin} \left(\frac{1}{2} \rho S_{fin} C_{df} m_{v}\right)$$

$$Y_{rc} = -\frac{1}{2} \rho C_{dc} m_{r} \int_{x_{t}}^{x_{b_{2}}} 2x |x| R(x) dx - 2x_{fin} |x_{fin}| \left(\frac{1}{2} \rho S_{fin} C_{df} m_{r}\right)$$

$$N_{rc} = -\frac{1}{2} \rho C_{dc} m_{r} \int_{x_{t}}^{x_{b_{2}}} 2x^{3} R(x) dx - 2x_{fin}^{3} \left(\frac{1}{2} \rho S_{fin} C_{df} m_{r}\right)$$

here , C_{dc} = drag coefficient of cylinder=1.1

R(x) is hull radius

for nose section,

$$R(x) = \frac{1}{2}d\left[1 - \left(\frac{x + a_{offset} - a}{a}\right)^2\right]^{\frac{1}{n}}$$

for tail section

$$R(x) = \frac{1}{2}d - \left[\frac{3d}{2c^2} - \frac{\tan\theta}{c}\right]\left(x - l\right)^2 + \left[\frac{d}{c^3} - \frac{\tan\theta}{c^2}\right]\left(x - l_f\right)^2$$

where,

$$a =$$
 full length of nose section = 0.191 m

b = full length of center section = 0.654 m

$$c =$$
 full length of tail section = 0.541 m

n = exponential parameter = 2

 2θ = included angle at the tail of tip =0.872 rad/s

d = maximum body diameter

 $S_{fin} =$ platform area = 0.00665 m²

 C_{df} = fin cross flow drag coefficient =0.558

 $a_{offset} = 0.0165 \text{ m}$

 $c_{offset} = 0.0368 \text{ m}$

 l_f = vehicle forward length = $a+b-a_{offset}$ = 0.828 m

calculating we get,

$$Y_{vc} = -15.7 \text{ kg/s}$$

 $Y_{rc} = -0.120 \, \text{kgm/s}$

 $N_{vc} = 0.403 \text{ kgm/s}$

 $N_{rc} = -2.16 \, \mathrm{kgm^2/s}$

2.6.3 ADDED MASS

$$\begin{split} X_A &= X_{\dot{u}} \dot{u} - Y_{\dot{r}} m_r r \\ Y_A &= Y_{\dot{v}} \dot{v} + Y_{\dot{r}} \dot{r} + X_{\dot{u}} U r \\ N_A &= -N_{\dot{v}} \dot{v} + N_{\dot{r}} \dot{r} - (X_{\dot{u}} - Y_{\dot{v}}) U v + Y_{\dot{r}} U r \end{split}$$

for REMUS AUV

$$X_{ii} = -0.93 \text{ kg}$$

CROSS FLOW ADDED MASS

From Newman, the added mass per unt length of a single cylindrical slice is given as:-

$$m_a(x) = \pi \rho R(x)^2$$

Added mass of the fins

$$m_{af}(x) = \pi \rho (a_{fin}^2 - R(x)^2 - \frac{R(x)^4}{a_{fin}^2})$$

where, a_{fin} =maximum lift above centre line =5.14m

$$Y_{i} = -\int_{x_{t}}^{x_{f}} m_{a}(x)dx - \int_{x_{f}}^{x_{f_{2}}} m_{af}(x)dx - \int_{x_{f_{2}}}^{x_{b_{2}}} m_{a}(x)dx$$
$$N_{i} = -\int_{x_{t}}^{x_{f}} xm_{a}(x)dx + \int_{x_{f}}^{x_{f_{2}}} xm_{af}(x)dx + \int_{x_{f_{2}}}^{x_{b_{2}}} xm_{a}(x)dx$$
$$Y_{i} = N_{i}$$
$$N_{i} = -\int_{x_{t}}^{x_{f}} x^{2}m_{a}(x)dx - \int_{x_{f}}^{x_{f_{2}}} x^{2}m_{af}(x)dx - \int_{x_{f_{2}}}^{x_{b_{2}}} x^{2}m_{a}(x)dx$$

where,

 x_t =aft end of tail section=-0.721m

 x_{t2} =forward end of tail section=-0.218m

 x_f = aft end of fin section = -0.685m

$$x_{f2}$$
 =forward end of fin section =-0.611m

 $x_b = aft end of bow section = 0.437m$

 x_{b2} =forward end of bow section=0.610m

Calculating

$$Y_{v} = -35.5 \ kg$$

 $N_{i} = 1.93 \ kgm$

 $Y_{r} = 1.93 \ kgm$

$$N_{\dot{r}} = -4.88 kg m^2$$

The cross term results from added mass coupling

$$X_{ra} = -m_r Y_{\dot{r}} = -0.597$$

$$Y_{ra} = X_{\dot{u}}U = -1.86$$

$$N_{va} = -(X_{\dot{u}} - Y_{\dot{v}})U = -69.14$$

$$N_{ra} = Y_{\dot{r}}U = 3.86$$

2.6.4 FIN LIFT FORCES AND MOMENTS:

Formula for fin lift is:

$$L_{fin} = \frac{1}{2} \rho C_l S_{fin} \delta_e v_e^2$$

where,

 $C_l =$ fin lift coefficent

 S_{fin} = fin platform area

 δ_e = effective fin angle in radians

 v_e =effective fin velocity

$$M_{fin} = x_{fin} L_{fin}$$

where x_{fin} = axial position of fin post in body reference

$$u_{fin} = u$$

 $v_{fin} = v + x_{fin}r$

Effective fin angle $\delta_e = \delta_r - \beta_{re}$

$$\beta_{re} = \frac{v_{fin}}{u_{fin}} = \frac{1}{u} (v + x_{fin} r)$$

Equation for fin lift and moment are :

$$Y_{\delta_{r}} = \rho C_{L\alpha} S_{fin} U^{2} = 85.482 kgm / s^{2}$$

$$Y_{vf} = -\frac{1}{2} \rho C_{L\alpha} S_{fin} U = -21.37 kgm / s$$

$$Y_{rf} = -\frac{1}{2} \rho C_{L\alpha} x_{fin} S_{fin} U = 13.634 kgm / s$$

$$N_{\delta_{r}} = \rho C_{L\alpha} x_{fin} S_{fin} U^{2} = -54.54 kgm^{2} / s^{2}$$

$$N_{vf} = -\frac{1}{2} \rho C_{L\alpha} x_{fin} S_{fin} U = 13.364 kgm / s$$

$$N_{rf} = -\frac{1}{2} \rho C_{L\alpha} x_{fin}^{2} S_{fin} U = -8.7 kgm / s^{2}$$

2.6.5 BODY LIFT AND MOMENTS:

To calculate Body lift, formula is :

$$L_{_{body}} = -\frac{1}{2}\rho d^2 C_{yd} u^2$$

where $C_{yd} = \text{body lift coefficient}$

this equation on simplification as done by hoener

$$L_{body} = -\frac{1}{2}\rho d^2 C_{yd\beta} uv = -\frac{1}{2}\rho d^2 C_{yd\beta} Uv \qquad \text{[after linearization]}$$

which results in body lift coefficient :

$$Y_{vl} = -\frac{1}{2}\rho d^2 C_{yd\beta} U = -45.09 kgm / s$$

and body lift moments :

$$N_{vl} = \frac{1}{2} \rho d^2 C_{yd\beta} U x_{cp} = -14.474 kgm / s$$

2.6.6 COMBINED TERMS

$$\begin{split} & \sum X = X_{\dot{u}}\dot{u} + X_{u}u + X_{r}r \\ & \sum Y = Y_{\dot{v}}\dot{v} + Y_{v}v + Y_{\dot{r}}\dot{r} + Y_{r}r + Y_{\delta_{r}}\delta_{r} \\ & \sum N = N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} + N_{v}v + N_{r}r + N_{\delta_{r}}\delta_{r} \end{split}$$

now

$$\begin{split} Y_{v} &= Y_{vc} + Y_{vl} + Y_{vf} = -82.16 kg \ / \ s \\ N_{v} &= N_{vc} + N_{va} + N_{vl} + N_{vf} = -69.577 kgm \ / \ s \\ Y_{r} &= Y_{rc} + Y_{ra} + Y_{rf} = 11.654 kgm \ / \ s \\ N_{r} &= N_{rc} + N_{ra} + N_{rf} = -7 kgm^{2} \ / \ s \end{split}$$

Table 2.2 Linearized Vehicle Coefficents

X_u	-17.6 kg / s
$X_{\dot{u}}$	-0.93 <i>kg</i> kg
X _r	-0.579 kgm/s
Y_{v}	-82.16 kg/s
Y_r	11.654 kgm/s
$Y_{\dot{ u}}$	-35.5 kg
$Y_{\dot{r}}$	1.93 kgm
$Y_{\delta r}$	85.482 kgm/s^2
N_{v}	-69.577 kgm/s
N_r	-7 kgm^2 / s
$N_{\dot{v}}$	1.93 kgm
$N_{\dot{r}}$	$-4.88 kgm^2$
$N_{\delta r}$	$-54.54 kgm^2 / s^2$

So,

$$G_{\psi}(s) = \frac{\frac{-54.54}{3.45 + 4.88}}{s^2 - \frac{(-7)}{3.45 + 4.88}s}$$
$$= \frac{-6.547}{s^2 + 0.84s}$$

Yaw control is done by PD controller with general transfer function given by

$$\frac{\delta_r(s)}{e_{\psi}(s)} = K_p(1 + \tau_d s)$$

where, e_{ψ} (error in yaw) = ψ_{des} (desired yaw) – ψ (actual yaw). K_p is the proportional gain, τ_d is the derivative time constant.

Outer heading loop transfer function relates the ψ_{des} to y. As inner yaw loop is very fast compared to outer heading loop, we can assume that ψ_{des} is nearly equal to ψ so that the transfer function

$$G_{y}(s) = \frac{y(s)}{\psi(s)} = \frac{U}{s}$$

For heading control, a proportional controller (P controller) is used whose gain,

$$G = \frac{\psi(s)}{e_v(s)}$$

where e_{v} is the error in position of the vehicle.

Adjusting in MATLAB, the controller gains are found out to be $K_p = -10$ and $K_d = -2.5$



2.7 SIMULINK implementation for heading and yaw control of an AUV:

Fig 2.2 SIMULINK model with fin angle saturation

in the above figure the SIMULINK model for yaw and heading control are shown ,considering the fin angle saturation. The saturation block represents the constraints on fin rotation.

After developing the SIMULINK model the next objective is to guide the AUV in a desired path. This process is known as tracking.in this process a given destination or path to be tracked by the AUV is given to it through MATLAB code.by implementing these codes the AUV will track the desired path. The MATLAB codes used here and the results are shown in Appendix and Results section respectively. We have derived a linear model of AUV in x-y plane. While trying to implement AUV dynamics in MATLAB we use a non-linear model as the controller will try to operate at the desired velocity and will linearize it about that point. So the controller will be effective on non-linear plant model.

2.8 RESULTS AND DISCUSSIONS



Fig 2.3 yaw angle~ time

the above figure represents variation of yaw angle with time. Due to an unit step input the vehicle tries to track the required path, hence its yaw changes abruptly. But after completion of task as the time progresses the yaw angle tends to zero.



Fig 2.4 y ~ time

As shown in the SIMULINK model, the input to the vehicle is y-desired which is 3 here. With progression in time the vehicle tries to reach its required location. There is an overshoot associated with this response which quickly reduces and the vehicle attains its required location within stipulated time period..



Fig 2.5 ψ_{des} ~ time

Due to the step input, the value of ψ_{des} changes accordingly. At the beginning a sudden change in the value of ψ_{des} is observed. As time progresses the vehicle achieves its required ψ_{des} value, hence ψ_{des} decreases and reaches zero value.



Fig 2.6 $\delta_R \sim \text{time}$

Due to Fin angle saturation i.e.-2<fin angle<2, the above response is obtained. The fin angle saturation represents the constraint we are having on fin angle, i.e. it should lie in between (-2, 2) due to mechanical constraints. From the above figure, the rudder angle first became -2 radians and remains the same for some time. Then it becomes 2 radians and remains there for some instants. Then a continuous variation in rudder angle value is observed which remains throughout the time period.

Due to the application of step input, the value of y_{des} changes abruptly. Hence the discrepancy caused is shown as error in y, in the figure shown below. But with progress in time as the actual value of y tends close to the value of y_{des} , the error in y reduces and reaches zero finally.



with the progress in time the error in ψ decreases and reaches zero. Due to fin angle saturation some overshoot is obtained.



Fig 2.8 error in $\psi \sim$ time



Dynamics Simulation of the AUV is obtained assuming a fixed value of propeller thrust and rudder angle. The vehicle has to follow a circular path which is given as an input to the vehicle in parametric fashion. The above figure shows the vehicle is tracking a perfect circle as the input is provided to the vehicle.

2.9 MATLAB CODE

```
Main Program
1 -
        tspan=0:0.1:100;
 2
        x0=[0 0 0.01 0 0.5 0.2];
 3
        xref=10*sin(0.2*tspan);
 4
        yref=10*cos(0.2*tspan);
 5
        [t,x]=ode45(@auv_dynamics11,tspan,x0);
 6
        plot(x(:,1),x(:,2),'g','Linewidth',2)
 7
        hold on
  -
 8
        plot (xref, yref)
 9 -
        axis square
```

AUV Dynamics

2 - xdotrseros(size(x)); 3 - xartul=x(1); 4 - yartul=x(2); 5 - psiatual=x(3); 6 - wartul=x(5); 7 - yartul=x(5); 9 - xartul=x(6); 9 - xartul=x(6); 10 - xur=1.62; 11 - xrr=1.93; 12 - n1=91.44; 13 - yvr=0.03; 15 - yur=26.6; 16 - nd2=66.0; 17 - wr=2; 18 - mr=2; 19 - xur=2; 10 - xur=2; 11 - xr=2; 12 - nd1=are.6.15; 13 - yur=0; 14 - yur=0; 15 - xur=2; 16 - nd2=c6.0; 17 - wdot(1=x(4) fros (psiatual]=x(5) fros (psiatual]; 18 - xur=0; 19 - xur=1, 19 - xur=1, 10 - xur=2; 11 - xur=2; 12 - xub(1 = x(4) fros (psiatual] = x(5) fros (psiat	1	E	<pre>function [xdot,t] = auv_dynamics11(t,x)</pre>
3 - xactual=x(1);	2	7	<pre>xdot=zeros(size(x));</pre>
4- γactual=x(2);	3	-	<pre>xactual=x(1);</pre>
S- piactualrx(3);	4	-	<pre>yactual=x(2);</pre>
6 - uactual = x(4) 7 - vactual = x(5); 8 - ractual = x(5); 9 - - 10 - xue=-1.62; 11 - xre=-1.93; 12 - mlt=11.44; 13 - yve=-1010; 14 - yre=-0.62; 15 - yve=-28.6; 16 - md2-66.01; 17 - mve=-3.18; 18 - mre=-94; 19 - mue=-2; 10 - uue=-2; 11 - mve=-3.18; 12 - moltar=-6.15; 13 - mve=-24; 14 - uu=-10; 15 - mue=-2; 16 - mue=-2; 17 - mue=-2; 10 - mue=-2; 11 - moltar=-6.15; 12 - moltar=-6.15; 13 - mue=-2; 14 - uu=100; 15 - dutar=10; 16 - moltar=-10; 17 - xobt(1 =xx(4) + sos)[5	-	<pre>psiactual=x(3);</pre>
1 vacuul=x(5);	6	77	uactual=x(4)
<pre>8 ractual = x (6); 9 ractual = x (6); 10 xu=-1.62; 11 xr=-1.93; 12 n1=31.44; 13 yrv=-1310; 14 yrr=0.632; 15 yu=-28.6; 16 n02=66.01; 17 nv=-3.18; 18 nr=-94; 19 nu=-2; 10 nu=-2; 11 nu=-2; 12 ndeltar=6.15; 12 ndeltar=6.15; 13 nu=-24; 14 tu=100; 15 deltar=10; 16 nu=-21; 17 ndeltar=6.15; 18 nu=-21; 19 nu=-21; 10 nu=-24; 11 ndeltar=6.15; 12 ndeltar=6.15; 13 nu=-24; 14 tu=100; 15 deltar=10; 15 deltar=10; 16 nu=-21; 17 xdot(1)=x(4)*cos(psiactual)-x(5)*sin(psiactual); 18 xdot(2)=x(4)*sin(psiactual)-x(5)*sin(psiactual); 19 xdot(2)=x(4)*sin(psiactual)-x(5)*sin(psiactual); 19 xdot(3)=x(6); 10 xdot(4)=(xuu/m11*x(4)*abs(x(4))+(xrr/m11*x(6)*abs(x(6))+(m2/m11*x(5)*x(6)+(tu/m11); 11 xdot(5)=-(m11/m22)*x(4)*abs(x(5))+(mr/m23)*x(4)*x(5)+(ndeltar/m33)*x(</pre>	7	-	<pre>vactual=x(5);</pre>
9 xu=-1.62; 10 - xu=-1.62; xu=-1.62; 11 - xrr=-1.93; xr=-1.93; 12 - m1=91.44; xr=-1.93; 13 - yvv=-1310; xr=-1.93; 14 - yrr=0.632; xr=-1.93; 15 - yuv=-28.6; xr=-1.93; 16 - m02=66.01; xr=-94; 17 - wvv=-3.18; xr=-94; 18 - mr=-94; xr=-2; 20 - mu=-24; xr=-24; 21 - mdeltar=-6.15; xr=-24; 22 - m03=8.33; xr=-24; 23 - deltar=10; xr=-1.14***********************************	8	-	<pre>ractual=x(6);</pre>
10 xum=1.62; 11 xrr=1.93; 12 ml=31.44; 13 yv=1310; 14 yr=0.62; 15 yu=28.6; 16 m22=66.01; 17 m2=46.01; 18 mr=-9; 19 mu=-2; 20 mu=-2; 21 mdeltar=6.15; 22 m3=8.3; 23 - 24 tu=100; 25 deltar=10; 26 - 27 xdot(1=x(4)*sos[stactual]=x(5)*sin(pstactual); 28 xdot(1=x(4)*sos[stactual]=x(5)*sin(pstactual); 29 xdot(1=x(4)*sos[stactual]=x(5)*sin(pstactual); 29 xdot(1=x(4)*sos[stactual]=x(5)*sin(pstactual); 29 xdot(1=x(4)*sos[stactual]=x(5)*sin(pstactual); 21 xdot(1=x(4)*sos[stactual]=x(5)*sin(pstactual); 21 xdot(1=x(4)*sin(pstactual]=x(5)*sin(pstactual); 22 xdot(1=x(4)*sin(pstactual]=x(5)*sin(pstactual); 23 - 24 tu=100; 25 xdot(1=x(4)*sin(pstactual]=x(5)*sin(pstactual); <td>9</td> <td></td> <td></td>	9		
<pre>11 - xrr=1.93;</pre>	10	7	xuu=-1.62;
12 - m11=31.44; 13 - yvv=-1310; 14 - yrr=0.632; 15 - yuv=-28.6; 16 - m22=66.01; 17 - mvv=-3.18; 18 - mrr=-94; 19 - mvv=-2, 20 - muv=-2; 21 - mdeltar=-6.15; 22 - m33=8.33; 23 - ettar=10; 24 - tu=100; 25 - deltar=10; 26 - xdot(1]=x(4)*cos[psiactual)-x(5)*sin[psiactual); 26 - xdot(2)=x(4)*sin[psiactual)+x(5)*cos(psiactual); 27 - xdot(1]=x(4)*cos[psiactual)-x(5)*sin(psiactual); 28 - xdot(1]=x(4)*cos[psiactual)-x(5)*sin(psiactual); 29 - xdot(2)=x(4)*sin[psiactual)+x(5)*cos(psiactual); 29 - xdot(1]=x(4)*cos[psiactual)-x(5)*sin(psiactual); 29 - xdot(1]=x(4)*cos[psiactual)+x(5)*cos(psiactual); 29 - xdot(1]=x(4)*cos[psiactual)+x(5)*cos(psiactual); 29 - xdot(2)=x(4)*sin[psiactual)+x(5)*cos(psiactual); 20 - xdot(2)=x(4)*sin[psiactual)+x(5)*cos(psiactual); 21 - xdot(3)=x(6); 22 -	11	-	xrr=-1.93;
13 - yvv-1310; 14 - yrr=0.632; 15 - yuv=28.6; 16 - b02=66.01; 17 - mvv=3.18; 18 - nrr=94; 19 - nur=2; 20 - nuv=24; 21 - ndeltar=6.15; 22 - m33=8.33; 23 - ettar=10; 24 - tu=100; 25 - deltar=10; 26 - xdot (1=x(4)*cos(psiactual)-x(5)*sin(psiactual); 28 - xdot (2=x(4)*sin(psiactual)+x(5)*cos(psiactual); 29 - xdot (1=x(4)*cos(psiactual)+x(5)*sin(psiactual); 29 - xdot (2=x(4)*sin(psiactual)+x(5)*sin(psiactual); 29 - xdot (1=x(uu)mi1 *x(4)*abs(x(4))+(xrr/mi1 *x(6)*abs(x(6))+(m2/mi1)*x(5)*x(6)+(tu/mi1); 31 - xdot (2=x(4)*sin(psiactual)+(x(5)*abs(x(6))+(m2/mi1)*x(5)*x(6)+(tu/mi1); 31 - xdot (5=-(mi1/m22)*x(4)*x(6)+(myr/m2)*x(6)*abs(x(6))+(m2/mi3)*x(4)*x(5); 32 - end	12	-	m11=31.44;
14 yrr=0.632;yrr=0.632;15 yuv=28.6;n22=66.01;16 n22=66.01;nvv=3.18;17 nvv=3.18;nrr=-94;18 nrr=-94;nur=-2;20 nuv=-24;nuv=-24;21 ndeltar=-6.15;n33=8.33;22 deltar=10;n33=8.33;23 constraintsn4110;24 cu=100;tu=100;25 cu=xdot(1)=x(4)*cos(psiactual)-x(5)*sin(psiactual);26 cu=xdot(2)=x(4)*sin(psiactual)+x(5)*cos(psiactual);27 cu>xdot(2)=x(4)*sin(psiactual)+x(5)*cos(psiactual);28 cu>xdot(2)=x(4)*sin(psiactual)+x(5)*cos(psiactual);29 cu>xdot(3)=x(6);30 - cu>xdot(4)=(xuu/m11)*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m22/m11*x(5)*x(6)+(tu/m11);31 - xdot(6)=(mv/m33)*x(5)*abs(x(5))+(mrr/m33)*x(4)*x(6)*abs(x(6))+(mu/m33)*x(4)*x(5)+(ndeltar/m33)*x(4)*x(4)*ab(4)*x(4)*deltar;32 - endend	13	-	yvv=-1310;
<pre>15 gruw=-28.6; 16 m22=66.01; 17 mv=-3.18; 18 mr=-94; 19 mr=-2; 20 mu=-24; 21 moletar=-6.15; 22 moletar=-6.15; 23 moletar=-6.15; 24 dettar=10; 25 dettar=10; 26 moletar=10; 27 xdot(1)=x(4)*cos(psiactual)-x(5)*sin(psiactual); 28 xdot(2)=x(4)*sin(psiactual)-x(5)*sin(psiactual); 29 xdot(2)=x(4)*sin(psiactual)+x(5)*cos(psiactual); 29 xdot(4)=(xuu/m11)*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m22/m11)*x(5)*x(6)+(tu/m11); 31 xdot(4)=(xuu/m11)*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m22/m11)*x(6)*abs(x(6))+(yuv/m22)*x(4)*x(5); 32 xdot(6)=(mu/m33)*x(5)*abs(x(5))+(mrr/m33)*x(4)*x(6)+(mur/m33)*x(4)*x(6)+(mur/m33)*x(4)*x(5)+(mdettar/m33)*x(4)*x(4)*x(6)+(mur/m33)*x(4)*</pre>	14	7	yrr=0.632;
16 m22=66.01; 17 mvv=-3.18; 18 mrre-94; 19 mure-2; 20 muv=-24; 21 ndeltar=-6.15; 22 m33=6.33; 23 - 24 tu=100; 25 deltar=10; 26 - 27 xdot(1)=x(4)*cos(psiactual)-x(5)*sin(psiactual); 28 xdot(2)=x(4)*sin(psiactual)-x(5)*sin(psiactual); 29 xdot(2)=x(4)*sin(psiactual)+x(5)*cos(psiactual); 29 xdot(3)=x(6); 30 xdot(4)=(xuu/m11)*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m2/m11)*x(5)*x(6)+(tu/m11); 31 xdot(5)=-(m1/m02)*x(4)*x(6)+(yvv/m22)*x(5)*abs(x(5))+(yrr/m02)*x(6)*abs(x(6))+(yuv/m22)*x(4)*x(5); 32 xdot (6)=(mvv/m33)*x(5)*abs(x(5))+(mrr/m33)*x(4)*x(6)+(muv/m33)*x(4)*x(5)+(ndeltar/m33)*x(4)*x(4)*deltar; 32 -end -	15	-	yuv=-28.6;
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	16	-	m22=66.01;
18 - nrr=-94; 19 - nur=-2; 20 - nuv=-24; 21 - ndeltar=-6.15; 22 - nd3=8.33; 23 - - 24 - tu=100; 25 - deltar=10; 26 - - 27 - xdot (1)=x (4) *cos (psiactual) -x (5) *sin (psiactual); 28 - xdot (2)=x (4) *sin (psiactual) -x (5) *sin (psiactual); 29 - xdot (2)=x (4) *sin (psiactual) +x (5) *cos (psiactual); 29 - xdot (3)=x (6); 30 - xdot (4) = (xuu/m11) *x (4) *abs (x (4)) + (xrr/m11) *x (6) *abs (x (6)) + (m22/m11) *x (5) *x (6) + (tu/m11); 31 - xdot (5)=-(m11/m22) *x (4) *x (6) + (yvv/m22) *x (5) *abs (x (5)) + (yrr/m22) *x (6) *abs (x (6)) + (yuv/m22) *x (4) *x (5) ; 32 - -end	17	-	nvv=-3.18;
19 -nur-2;20 -nuv-24;21 -ndeltar=-6.15;22 -m33=8.33;2324 -tu=100;25 -deltar=10;2627 -xdot(1)=x(4)*cos(psiactual)-x(5)*sin(psiactual);28 -xdot(2)=x(4)*sin(psiactual)+x(5)*cos(psiactual);29 -xdot(2)=x(6);30 -xdot(4)=(xuu/m11*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m22/m11*x(5)*x(6)+(tu/m11);31 -xdot(5)=-(m11/m22)*x(4)*x(6)+(pvv/m22)*x(5)*abs(x(5))+(prr/m23)*x(4)*x(5)+(ndeltar/m33)*x(4)*x(4)*deltar;32 -end	18	7	nrr=-94;
20 - nuv=-24; 21 - ndeltar=-6.15; 22 - m33=8.33; 23 - tu=100; 24 - tu=100; 25 - deltar=10; 26 - xdot(1)=x(4)*cos(psiactual)-x(5)*sin(psiactual); 27 - xdot(2)=x(4)*sin(psiactual)+x(5)*sin(psiactual); 28 - xdot(2)=x(4)*sin(psiactual)+x(5)*cos(psiactual); 29 - xdot(3)=x(6); 30 - xdot(4)=(xuu/m11)*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m22/m11)*x(5)*x(6)+(tu/m11); 31 - xdot(5)=-(m11/m22)*x(4)*x(6)+(yvv/m22)*x(5)*abs(x(5))+(yrr/m22)*x(6)*abs(x(6))+(yuv/m23)*x(4)*x(5); 32 - xdot(6)=(nvv/m33)*x(5)*abs(x(5))+(nrr/m33)*x(6)*abs(x(6))+(nur/m33)*x(4)*x(6)+(nuv/m33)*x(4)*x(5)+(ndeltar/m33)*x(4)*x(4)*deltar; 33 - -end	19	-	nur=-2;
21 - ndeltar=-6.15; 22 - m33=8.33; 23 - u=100; 24 - tu=100; 25 - deltar=10; 26 - - 27 - xdot(1)=x(4)*cos(psiactual)-x(5)*sin(psiactual); 28 - xdot(2)=x(4)*sin(psiactual)+x(5)*cos(psiactual); 29 - xdot(3)=x(6); 30 - xdot(4)=(xuu/m11)*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m22/m11)*x(5)*x(6)+(tu/m11); 31 - xdot(5)=-(m11/m22)*x(4)*x(6)+(yyy/m22)*x(5)*abs(x(5))+(yrr/m22)*x(6)*abs(x(6))+(yyy/m23)*x(4)*x(5); 32 - xdot(6)=(nvv/m33)*x(5)*abs(x(5))+(nrr/m33)*x(6)*abs(x(6))+(nur/m33)*x(4)*x(5)+(ndeltar/m33)*x(4)*x(4)*deltar; 33 - -end	20	-	nuv=-24;
<pre>22 - m33=8.33; 23 24 - tu=100; 25 - deltar=10; 26 27 - xdot(1)=x(4)*cos(psiactual)-x(5)*sin(psiactual); 28 - xdot(2)=x(4)*sin(psiactual)+x(5)*cos(psiactual); 29 - xdot(3)=x(6); 30 - xdot(4)=(xuu/m11)*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m22/m11)*x(5)*x(6)+(tu/m11); 31 - xdot(5)=-(m11/m22)*x(4)*x(6)+(yvv/m22)*x(5)*abs(x(5))+(yrr/m22)*x(6)*abs(x(6))+(yuv/m22)*x(4)*x(5); 32 - xdot(6)=(nvv/m33)*x(5)*abs(x(5))+(nrr/m33)*x(6)*abs(x(6))+(nur/m33)*x(4)*x(5)+(ndeltar/m33)*x(4)*x(4)*deltar; 33 - end</pre>	21	-	ndeltar=-6.15;
<pre>23</pre>	22	7	m33=8.33;
<pre>24 - tu=100; 25 - deltar=10; 26</pre>	23		
<pre>25 - deltar=10; 26 27 - xdot(1)=x(4)*cos(psiactual)-x(5)*sin(psiactual); 28 - xdot(2)=x(4)*sin(psiactual)+x(5)*cos(psiactual); 29 - xdot(3)=x(6); 30 - xdot(4)=(xuu/m11)*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m22/m11)*x(5)*x(6)+(tu/m11); 31 - xdot(5)=-(m11/m22)*x(4)*x(6)+(yvv/m22)*x(5)*abs(x(5))+(yrr/m22)*x(6)*abs(x(6))+(yuv/m22)*x(4)*x(5); 32 - xdot(6)=(nvv/m33)*x(5)*abs(x(5))+(nrr/m33)*x(6)*abs(x(6))+(nur/m33)*x(4)*x(6)+(nuv/m33)*x(4)*x(5)+(ndeltar/m33)*x(4)*x(4)*deltar; 33 - end</pre>	24	-	tu=100;
<pre>26 27 - xdot(1) = x(4) *cos(psiactual) - x(5) *sin(psiactual); 28 - xdot(2) = x(4) *sin(psiactual) + x(5) *cos(psiactual); 29 - xdot(3) = x(6); 30 - xdot(4) = (xuu/m11) *x(4) *abs(x(4)) + (xrr/m11) *x(6) *abs(x(6)) + (m22/m11) *x(5) *x(6) + (tu/m11); 31 - xdot(5) = - (m11/m22) *x(4) *x(6) + (yvv/m22) *x(5) *abs(x(5)) + (yrr/m22) *x(6) *abs(x(6)) + (yuv/m22) *x(4) *x(5); 32 - xdot(6) = (nvv/m33) *x(5) *abs(x(5)) + (nrr/m33) *x(6) *abs(x(6)) + (nur/m33) *x(4) *x(5) + (ndeltar/m33) *x(4) *x(4) *deltar; 33 - end</pre>	25	-	deltar=10;
<pre>27 - xdot(1) = x(4) *cos(psiactual) - x(5) *sin(psiactual); 28 - xdot(2) = x(4) *sin(psiactual) + x(5) *cos(psiactual); 29 - xdot(3) = x(6); 30 - xdot(4) = (xuu/m11) *x(4) *abs(x(4)) + (xrr/m11) *x(6) *abs(x(6)) + (m22/m11) *x(5) *x(6) + (tu/m11); 31 - xdot(5) = - (m11/m22) *x(4) *x(6) + (yvv/m22) *x(5) *abs(x(5)) + (yrr/m22) *x(6) *abs(x(6)) + (yuv/m22) *x(4) *x(5); 32 - xdot(6) = (nvv/m33) *x(5) *abs(x(5)) + (nrr/m33) *x(6) *abs(x(6)) + (nur/m33) *x(4) *x(5) + (ndeltar/m33) *x(4) *x(4) *deltar; 33 - end</pre>	26		
<pre>28 - xdot(2) = x(4) *sin(psiactual) + x(5) *cos(psiactual); 29 - xdot(3) = x(6); 30 - xdot(4) = (xuu/m11) * x(4) *abs(x(4)) + (xrr/m11) * x(6) *abs(x(6)) + (m22/m11) * x(5) * x(6) + (tu/m11); 31 - xdot(5) = - (m11/m22) * x(4) * x(6) + (yvv/m22) * x(5) *abs(x(5)) + (yrr/m22) * x(6) *abs(x(6)) + (yuv/m22) * x(4) * x(5); 32 - xdot(6) = (nvv/m33) * x(5) *abs(x(5)) + (nrr/m33) * x(6) *abs(x(6)) + (nur/m33) * x(4) * x(5) + (ndeltar/m33) * x(4) * x(4) * deltar; 33 - end</pre>	27	-	xdot(1) = x(4) * cos(psiactual) - x(5) * sin(psiactual);
<pre>29 - xdot(3) = x(6); 30 - xdot(4) = (xuu/m11) * x (4) * abs(x(4)) + (xrr/m11) * x (6) * abs(x(6)) + (m22/m11) * x (5) * x (6) + (tu/m11); 31 - xdot(5) = - (m11/m22) * x (4) * x (6) + (yvv/m22) * x (5) * abs(x (5)) + (yrr/m22) * x (6) * abs(x (6)) + (yuv/m22) * x (4) * x (5); 32 - xdot(6) = (nvv/m33) * x (5) * abs(x (5)) + (nrr/m33) * x (6) * abs(x (6)) + (nur/m33) * x (4) * x (6) + (nuv/m33) * x (4) * x (5) + (ndeltar/m33) * x (4) * x (4) * deltar; 33 - end</pre>	28	-	xdot(2) = x(4) * sin(psiactual) + x(5) * cos(psiactual);
<pre>30 - xdot(4) = (xuu/m11) *x (4) *abs(x(4)) + (xrr/m11) *x (6) *abs(x (6)) + (m22/m11) *x (5) *x (6) + (tu/m11); 31 - xdot(5) = - (m11/m22) *x (4) *x (6) + (yvv/m22) *x (5) *abs(x (5)) + (yrr/m22) *x (6) *abs(x (6)) + (yuv/m22) *x (4) *x (5); 32 - xdot(6) = (nvv/m33) *x (5) *abs(x (5)) + (nrr/m33) *x (6) *abs(x (6)) + (nur/m33) *x (4) *x (6) + (nuv/m33) *x (4) *x (5) + (ndeltar/m33) *x (4) *x (4) *deltar; 33 - end</pre>	29	-	xdot(3)=x(6);
<pre>31 - xdot(5) =- (m11/m22) *x (4) *x (6) + (yvv/m22) *x (5) *abs(x (5)) + (yrr/m22) *x (6) *abs(x (6)) + (yuv/m22) *x (4) *x (5); 32 - xdot(6) = (nvv/m33) *x (5) *abs(x (5)) + (nrr/m33) *x (6) *abs(x (6)) + (nur/m33) *x (4) *x (6) + (nuv/m33) *x (4) *x (5) + (ndeltar/m33) *x (4) *x (4) *deltar; 33 - end</pre>	30	7	xdot(4) = (xuu/m11) * x(4) * abs(x(4)) + (xrr/m11) * x(6) * abs(x(6)) + (m22/m11) * x(5) * x(6) + (tu/m11);
32 - xdot(6) = (nvv/m33) *x(5) *abs(x(5)) + (nrr/m33) *x(6) *abs(x(6)) + (nur/m33) *x(4) *x(6) + (nuv/m33) *x(4) *x(5) + (ndeltar/m33) *x(4) *x(4) *deltar; 33 - end	31	-	xdot (5) = -(m11/m22) * x (4) * x (6) + (yvv/m22) * x (5) * abs (x (5)) + (yrr/m22) * x (6) * abs (x (6)) + (yuv/m22) * x (4) * x (5);
33 end	32	-	xdot(6) = (nvv/m33) * x(5) * abs(x(5)) + (nrr/m33) * x(6) * abs(x(6)) + (nur/m33) * x(4) * x(6) + (nuv/m33) * x(4) * x(5) + (ndeltar/m33) * x(4) * x(4) * deltar;
	33	-	- end

2.10 Summary

A brief discussion on the kinematics and dynamics of the underwater vehicle is given in this chapter. The nonlinear dynamics equations of the vehicle are formulated and linearized. The linear coefficients of the vehicle are calculated and used to form the transfer function model of the vehicle. The SIMULINK model is used to study the heading control of the vehicle along with the MATLAB simulation for vehicle dynamics.

CHAPTER 3

HEADING CONTROL OF UNDERWATER VEHICLE

A six degree-of-freedom rigid body motion in space is typically considered for determining ship response in waves. For ship maneuvering study a three degree-of-freedom plane motion is usually considered sufficient. However, for high speed vessels like the container ships, turning motion induced roll is very high so cannot be neglected. Hence a four degree-of-freedom description that includes surge, sway, yaw and roll modes is needed. Since the hydrodynamics involved in ship steering is highly nonlinear, coupled nonlinear differential equations are needed to fully describe the complicated ship maneuvering dynamics. A simple transfer function model description is usually preferred when a qualitative prediction capability is all we need from the model. This is the case in a model-based controller design, since the feedback controller itself tolerates certain amount of modeling error and a complicated model might result in a controller too complicated to implement. The popularity of the first order Nomoto model in ship steering autopilot design is due to its simplicity and relative accuracy in describing the course-keeping yaw dynamics, where typically, small rudder angles are involved. Extension to large rudder angle yaw dynamics basing upon the Nomoto model has been proposed to better describe the nonlinear behavior of yaw dynamics.

3.1 NOMOTO MODEL

NOMOTO model is a linear ship steering model which uses first order or second order models for ship steering autopilot design. The advantage of this model over others is its simplicity and accuracy in tracking yaw dynamics.

Linear theory suggests we can write equations of motion as :

$$M\dot{v} + N(U)v = b\delta_r$$

Where,

$$M = \begin{pmatrix} m - Y_{\psi} & m_{x_g} - Y_{\dot{r}} \\ m_{x_g} - N_{\psi} & I_{zz} - N_{\dot{r}} \end{pmatrix}$$
$$N = \begin{pmatrix} -Y_{\psi} & mU - Y_{r} \\ -N_{\psi} & m_{x_g}U - N_{r} \end{pmatrix}$$
$$\psi = \begin{pmatrix} v \\ r \end{pmatrix}$$
$$b = \begin{pmatrix} Y_{\delta_r} \\ N_{\delta_r} \end{pmatrix}$$

Nomoto's 2nd order model:

By eliminating sway velocity v Nomoto transfer function between r and δ_r is obtained as:

$$\frac{r}{\delta_r}(s) = \frac{K_R(1+T_3)}{(1+T_1s)(1+T_2s)}$$

The parameters in the above equation are related to coefficients as :

$$T_1 T_2 = \frac{\det(M)}{\det(N)}$$

$$=\frac{(m-Y_{\psi})(I_{zz}-N_{\dot{r}})-(m_{x_g}-N_{\psi})(m_{x_g}-Y_{\dot{r}})}{-Y_{\psi}(m_{x_g}U-N_{r})+N_{\psi}(mU-Y_{r})}$$

$$T_1 + T_2 = \frac{n_{11}m_{22} + n_{22}m_{11} - n_{12}m_{21} - n_{21}m_{12}}{\det(N)}$$

 $K_{R} = \frac{n_{21}b_{1} - n_{11}b_{2}}{\det(N)}$

$$\begin{split} K_{R}T_{3} &= \frac{m_{21}b_{1} - m_{11}b_{2}}{\det(N)} \\ T_{1} + T_{2} &= \frac{-V_{v}(I_{zz} - N_{v}) + (m_{x_{z}}U - N_{v})(m - Y_{v}) - (mU - Y_{v})(m_{x_{z}} - N_{v}) + N_{v}(m_{x_{z}} - Y_{v})}{\det(N)} \\ K_{R} &= \frac{-N_{v}Y_{\delta_{v}} + Y_{v}N_{\delta_{v}}}{\det(N)} \\ K_{R}T_{3} &= \frac{(m_{x_{z}} - N_{v})Y_{\delta_{v}} - (m - Y_{v})N_{\delta_{v}}}{\det(N)} \\ \frac{r(s)}{\delta_{v}(s)} &= \frac{K_{R}(1 + T_{3}s)}{(1 + T_{1}s)(1 + T_{2}s)} = \frac{K_{R} + K_{R}T_{3}s}{T_{1}T_{2}s^{2} + (T_{1} + T_{2})s + 1} \\ K_{R}T_{3} &= -1.9103 \\ \det(M) &= (m - Y_{v})(I_{zz} - N_{v}) - (m_{x_{z}} - N_{v})(m_{x_{z}} - Y_{v}) \\ &= 546.1384 \\ \det(N) &= -Y_{v}(m_{x_{z}}U - N_{v}) + N_{v}(mU - Y_{v}) \\ &= -1798.221 \\ T_{1}T_{2} &= \frac{\det(M)}{\det(N)} = -0.3037 \\ T_{1} + T_{2} &= -0.74884 \quad K_{R} = -5.799 \\ \frac{r(s)}{\delta_{v}(s)} &= \frac{1.9103s + 5.799}{0.3037s^{2} + 0.7488s - 1} \\ Nomoto's 1^{st} \text{ order model :} \end{split}$$

In this model approximation is found out by using a effective time constant

$$T = T_1 + T_2 - T_3$$

This first order model works well only for low frequency operation as it gives good results but as the frequency is increased the approximation starts giving erronious results hence second order model is preferred in such cases.

$$H(s) = \frac{K}{s(1+Ts)}$$

where $T = T_1 + T_2 - T_3 = -1.0782$

 $=\frac{5.799}{1.0782s^2-s}$

3.2 NOMOTO MODEL ANALYSIS USING SIMULINK



Fig 3.1 SIMULINK model of 2nd order Nomoto model

1st order Nomoto model of the vehicle is an approximation of the 2nd order model which is applicable only in low frequency scenarios. The bode plot and step response of the 1st order Nomoto model is shown in fig 4.15, fig 4.16. All the plots correspond to an unstable system. The negative phase margin and the unbounded step response represents the instability.

1st ORDER



Fig 3.2 Bode plot of 1st order Nomoto model for heading control of an AUV



Fig 3.3 Step response of 1st order Nomoto model for heading control of an AUV

2nd ORDER UNSTABLE

The second order Nomoto model of the vehicle is studied with its root locus plot and bode plot. These plots reveal that the system model is unstable as there is a pole on the right hand side of the s-plane. Hence the step response is unbounded due to instability.



Fig 3.4 Bode plot of 2nd order Nomoto model for heading control of an AUV



Fig 3.5 Step response of 2nd order Nomoto model for heading control of an AUV



Fig 3.6 Root locus of 2nd order Nomoto model for heading control of an AUV

2nd ORDER STABLE

A PD controller is used to the 2^{nd} order Nomoto model to stabilize the system. The response of the closed loop stable system is plotted in the root locus and in the bode plots. For obtaining a optimum settling time of 1sec and a maximum overshoot of 5% the controller is designed. The ζ line is plotted in the root locus plot for obtaining the value of gain which is shown in the figure below.



Fig 3.7 Bode plot of 2nd order Nomoto model for heading control of an AUV



Fig 3.8 Step response of 2nd order Nomoto model for heading control of an AUV



Fig 3.9 Root locus of 2nd order Nomoto model for heading control of an AUV

3.3 SUMMARY

A brief discussion on the Nomoto model of linear ship steering is proposed in this chapter. Simple transfer function analysis approach is used for qualitative study of vehicle steering. The 1^{st} order and 2^{nd} order Nomoto models are studied. Using a PD controller the unstable open loop 2^{nd} order model is stabilized.

CHAPTER 4:

PATH TRACKING OF AN AUV

Here the objective is to drive a robot along a desired trajectory. As the name suggests an Autonomous Underwater vehicle should be capable of task completion without using any external input. In tracking a desired path is to be followed by the AUV. Error dynamics is considered for tracking purpose here. The error associated with heading and surge is calculated and constantly minimized for required system performance.

An AUV can be designed in various different ways. Each of these designs have their own advantages and disadvantages. For example such designs can be categorized as

- I. Torpedo shape
- II. One hull, four thrusters
- III. Two hulls, four thrusters
- IV. Two hulls, two rotating thrusters

Here the trajectory tracking for a torpedo shaped and a two hulls, two rotating thrusters type AUV is studied.

TORPEDO SHAPED AUV

4.1 ERROR DYNAMICS

We first derive error dynamics of the vehicle from its dynamics.

 $u_e = u - u_d$

 $r_e = r - \alpha_r$

Where u, r are actual surge and heading rate

 u_d, α_r are desired surge and heading rate

Assuming constant desired surge velocity $u_d = 2$ m/s

$$\dot{u}_e = \dot{u}$$
 and

$$\dot{u}_{e} = \frac{X_{u}}{(m - X_{\dot{u}})}u + \frac{X_{r}}{(m - X_{\dot{u}})}r + \frac{X_{prop}}{(m - X_{\dot{u}})}$$

So the forward speed control law is

$$X_{prop} = 31.44(-f_u - k_u e_u)$$

And similarly yaw dynamics is defined as

$$\dot{r} = f_r - 2.952\delta r$$

So steering control law is

$$\delta r = 0.33875(f_r + K_r r)$$

Using these control laws propeller thrust force and rudder angle control the dynamics of the vehicle and a circular path is followed by the vehicle.

4.2 SIMULATION RESULT OF TRAJECTORY TRACKING OF AN AUV

The results obtained and the MATLAB program used for tracking are shown below. As an input

to the AUV a parametric input is given for circular trajectory tracking.



Fig 4.1 Tracking control of AUV in horizontal plane

4.3 MATLAB CODE

```
function [ xdot,t,ex ] = auv dynamics15( t,x )
 1
 2
     xdot=zeros(size(x));
 3
     xactual=x(1);
 4 vactual=x(2);
 5 psiactual=x(3);
 6 uactual=x(4);
 7 vactual=x(5);
 8 ractual=x(6);
10 xuu=-1.62;
11
     xrr=-1.93;
12
     m11=31.44;
13
     yvv=-1310;
     yrr=0.632;
14
      yuv=-28.6;
15
16
     m22=66.01;
17
     nvv=-3.18;
18
     nrr=-94;
     nur=-2;
19
20
     nuv=-24;
21 ndeltar=-6.15;
22 m33=8.33;
23 ku=2;
24 kr=1.2;
26 xref=10*sin(0.2*t);
27 yref=10*cos(0.2*t);
28 eu=x(1)-2;
29 r=x(6);
30 fu=(xuu/m11) *x(4) *abs(x(4)) + (xrr/m11) *x(6) *abs(x(6)) + (m22/m11) *x(5) *x(6);
31 fr = (nvv/m33) * x (5) * abs(x(5)) + (nrr/m33) * x (6) * abs(x(6)) + (nur/m33) * x (4) * x (6) + (nuv/m33) * x (4) * x (5);
32 tu=31.25*(-fu-ku*eu);
33 deltar=0.338*(fr+kr*r);
34
35 -
     xdot(1) = x(4) * cos(x(3)) - x(5) * sin(x(3));
36 -
     xdot(2) = x(4) * sin(x(3)) + x(5) * cos(x(3));
37 -
     xdot(3) = x(6);
38 -
     xdot(4) = (xuu/m11) *x(4) *abs(x(4)) + (xrr/m11) *x(6) *abs(x(6)) + (m22/m11) *x(5) *x(6) + (tu/m11);
39 -
     x \det (5) = -(m11/m22) * x (4) * x (6) + (\nabla v v/m22) * x (5) * abs (x (5)) + (\nabla v v/m22) * x (6) * abs (x (6)) + (\nabla v v/m22) * x (4) * x (5);
40 -
     xdot(6) = (nvv/m33) *x (5) *abs(x (5)) + (nrr/m33) *x (6) *abs(x (6)) + (nur/m33) *x (4) *x (6) + (nuv/m33) *x (4) *x (5) + (ndeltar/m33) *x (4) *deltar;
41 -
     -end
Main program
 1 - tspan=0:.1:8
 2 -
          [t,x]=ode45('auv dynamics15 copy',tspan,[0 0 0 0.1 0]);
 3 -
          plot(x(:,1),x(:,2));
```

TWO THRUSTERS MODEL

The trajectory tracking of two thrusters model is comparatively simpler than the torpedo shaped model. The controller required for trajectory tracking is proposed in two different methods i.e. using PD controller and LQR based control.

The simulation result and the MATLAB program used for achieving trajectory tracking is shown below.

PD CONTROL

A Proportional-Derivative based control is used to control the thrust upon the vehicle. Due to the presence of two thrusters there will be two thrust force in different direction. Here our objective is to manipulate these thrust forces to achieve the required trajectory.



Fig 4.2: Trajectory tracking of two thruster model

MATLAB Program for Trajectory Tracking

<pre>2 -</pre>	1	<pre>function [xdot,t] = auv_dynamics19(t,x)</pre>
<pre>3 - xactual=x(1); 4 - yactual=x(3); 6 - uactual=x(3); 6 - uactual=x(5); 7 - vactual=x(5); 8 - Factual=x(6); 9 - xuu=-1.62; 11 - xrr=-1.93; 12 - mil=31.44; 13 - yvv=-28.6; 14 - yrr=0.632; 15 - yuv=-28.6; 16 - m22=66.01; 17 - mvv=-3.18; 18 - nrr=-94; 19 - nur=-2; 20 - nuv=-24; 11 - xref=10*sin(0.1*t); 22 - ysef=10*sin(0.1*t); 23 - yref=10*sin(0.1*t); 24 - xref=10*sin(0.1*t); 25 - yref=10*sin(0.1*t); 26 - dxref=1cos(0.1*t); 27 - dyref==sin(0.1*t); 28 - y=x(1)-xref; 30 - ey=x(2)-yref; 31 - dex=x(4)-dxref; 32 - dey=x(5)-dyref; 33 - tu=kx*tex+kkx*dex; 41 - kyp=-00; 34 - kyp=-10; 35 - kdx=-60; 36 - ey=x(5)-dyref; 37 - dyref=x(1)-xref; 38 - tu=kx*tex+kkx*dex; 40 - fy=kyf*tey+kdy*dey; 41 - xdot(1)=x(1); 42 - xdot(2)=x(5); 43 - xdot(2)=x(5); 44 - xdot(1)=x(1); 45 - xdot(2)=x(5); 45 - xdot(2)=x(5); 46 - add 47 - kdy=-10; 47 - xdot(2)=x(5); 47 - xdot(2)=x(5); 48 - xdot(3)=x(1);x(4)*k(5)+krr/m1)*x(5)*k(5)+k(2/m1)*x(5)*k(5)+k(2/m22);x(4)*x(5)+k(2/m22);x(4)*x(5)+k(2/m22);x(5)*k(5)+k(2/m22);x(4)*x(5)+k(2/m22);x(4)*x(5)+k(2/m22);x(4)*x(5)+k(2/m22);x(4)*x(5)+k(2/m22);x(4)*x(5)+k(2/m22);x(4)*x(5)+k(2/m22);x(4)*x(5)+k(2/m22);x(4)*x(5)+k(2/m22);x(4)*x(5)+k(2/m22);x(4)*x(5)+k(2/m22);x(4)*x(5)+k(2/m22);x(4)*x(5)+k(2/m22);x(4)*x(5)+k(2/m22);x(4)*x(5)+k(2/m22);x(5)*k(2)+k(2/m22</pre>	2 -	<pre>xdot=zeros(size(x));</pre>
<pre>4 - yactual=x(2); 5 - psiactual=x(3); 6 - uactual=x(5); 7 - vactual=x(5); 8 - ractual=x(5); 9 - ractual=x(5); 10 - xuu=-1.62; 11 - xrr=-1.93; 11 - xrr=-1.93; 12 - m1=31.44; 13 - yvr=0.632; 14 - yrr=0.632; 15 - yuv=-28.6; 16 - m2=66.01; 17 - mvv=-3.18; 18 - mr==94; 19 - nur=-24; 19 - nur=-2; 20 - nuv=-24; 21 - mdeltar=-6.15; 22 - m3=8.33; 24 - xref=10*sin(0.1*t); 25 - yref=10*cos(0.1*t); 25 - yref=10*cos(0.1*t); 26 - dxref=trcos(0.1*t); 27 - dyref=-sin(0.1*t); 28 - ex=x(1)-xref; 31 - dex=x(4)-dxref; 32 - dey=x(5)-dyref; 33 - dey=x(5)-dyref; 34 - tu*by*ex+kk*tex; 45 - tu*by*ex+kk*tex; 46 - fy*p*ex+kk*tex; 47 - tu*by*ex+kk*tex; 49 - fy*p*ex+kk*tex; 40 - fy*p*ex+kk*tex; 41 - xdot(1)=x(4); 42 - xdot(2)=x(5); 43 - xdot(3)=x(6); 44 - xdot(3)=x(6); 45 - xdot(3)=x(6); 46 - xdot(3)=x(6); 47 - xdot(3)=x(6); 48 - xdot(3)=x(6); 49 - xdot(3)=x(6); 40 - xdot(3)=x(6); 41 - xdot(3)=x(6); 42 - xdot(3)=x(6); 43 - xdot(3)=x(6); 44 - xdot(3)=x(6); 45 - xdot(3)=x(6); 45 - xdot(6)=(mu/m3)*x(6)*abs(x(6)+[mu/m3)*x(6)*x(6)+[m</pre>	3 -	<pre>xactual=x(1);</pre>
<pre>S = pslactual=x(3); wactual=x(5); rectual=x(6); 10 = xuu=-1.62; 11 = xtr=-1.93; 12 = m11=31.44; 13 = yvv=-1310; 14 = yrr=0.632; 15 = yuv=-28.6; 16 = m22=66.01; 17 = nvv=-3.16; 18 = nrr=-94; 19 = nur=-2; 20 = nuv=-24; 21 = ndeltar=-6.15; 22 = m33=8.33; 23 = xref=10*sin(0.1*t); 25 = yref=10*cos(0.1*t); 26 = dxref=10*sin(0.1*t); 27 = dyref=-sin(0.1*t); 28 = ex=x(1)-xref; 30 = ey=x(2)-yref; 31 = dex=x(4)-dxref; 32 = dex=x(4)-dxref; 33 = tu*ky*exkkx*dex; 4 = xkbt(1*x)(6); 4 = xkbt(1*x)(6); 4 = xkbt(1*x)(6); 4 = xkbt(1)*x(6); 5 = xkbt(1*x)(6); 5 = xkbt(1*x)(6); 5 = xkbt(1*x)(6); 5 = xkbt(1*x)(6); 5 = xkbt(1*x)(6); 5 = xkbt(1*x)(6); 5 = xkbt(1)*x(6); 5 = xkbt(1)*x(5); 5 = xkbt(1)*x(5)*x(5)*x(5)+x(5)*x(5)*x(5</pre>	4 -	<pre>yactual=x(2);</pre>
<pre>6 - uactual=x(4) 7 - vactual=x(5); 8 - ractual=x(6); 9 10 - xuu=-1.62; 11 - xrr=-1.93; 12 - mi1=31.44; 13 - yvv=-1310; 14 - yrr=0.632; 15 - yuv=-28.6; 16 - m2=66.01; 17 - nvv=-3.18; 18 - nrr=-94; 19 - nur=-2; 20 - nuv=-24; 21 - ndeltar=-6.15; 22 - m33=8.33; 24 - xref=10*sin(0.1*t); 25 - yref=10*cos(0.1*t); 26 - dxref=1cos(0.1*t); 27 - dyref=-sin(0.1*t); 28 - ex=x(1)-xref; 30 - ey=x(2)-yref; 31 - dey=x(5)-dyref; 32 - dey=x(5)-dyref; 33 - ey=x(5)-dyref; 34 - xdot(3)=x(6); 44 - xdot(3)=x(6); 45 - xdot(1)=x(4);x(4)+(xr/m1)*x(6)*abs(x(6))+(m2/m1)*x(6)*x(6)+tu/m1); 45 - xdot(2)=x(5); 46 - dx; 47 - dyref=-sin(0.1*t); 47 - dyref=-sin(0.1*t); 48 - ey=x(1)-xref; 49 - ey=x(2)-yref; 41 - xdot(3)=x(6); 46 - tyv=x(4)-tyv=x(4)+tyv=x(4)+tyv=x(4)+tyv=x(4)+tyv=x(4)+ty=</pre>	5 -	<pre>psiactual=x(3);</pre>
<pre>7 - vactual=x(5); 8 - ractual=x(6); 9 - vuu=-1.62; 10 - xuu=-1.62; 11 - xr=-1.93; 12 - m11=31.44; 13 - yvv=-310; 14 - yr=0.63; 15 - yuv=-28.6; 16 - m22=66.01; 17 - nvv=-3.10; 18 - nr==-94; 19 - nuu=-2; 20 - nuv=-24; 21 - ndeltar=-6.15; 22 - m3=8.3; 23 - xref=10*sin(0.1*t); 25 - yref=10*cos(0.1*t); 26 - dxref=1*cos(0.1*t); 27 - dyref=-sin(0.1*t); 28 - yref=10*cos(0.1*t); 29 - ex=x(1)-xref; 30 - ey=x(2)-yref; 31 - dex=x(4)-dxref; 32 - dey=x(5)-dyref; 33 - dey=x(5)-dyref; 34 - kxx=60; 35 - kxx=60; 36 - kyy=-100; 37 - kdy=-100; 38 - uthpy*exkdx*dex; 40 - f=kyx=0; 39 - tuthpy*exkdx*dex; 40 - f=kyx=0; 30 - ey=x(2)-yref; 31 - dex=x(4)-dxref; 32 - dey=x(5)-dyref; 33 - dey=x(5)-dyref; 34 - kxx=60; 35 - kxx=60; 36 - kyy=-100; 37 - kdy=-100; 38 - uthpy*exkdx*dex; 40 - f=kyx=0; 39 - tuthpy*exkdx*dex; 40 - f=kyy=rethy*exkdx*dex; 41 - xdot(3)=x(6); 42 - xdot(1)=x(4); 43 - xdot(2)=x(5); 44 - xdot(3)=x(6); 45 - kdx+(4)-(xum/m1)*x(6)*abs(x(6))+(m2/m1)*x(6)*x(6)+(mu/m3)*x(4)*x(5)+(fy/m22); 47 - xdot(5)=-(m1/m22)*x(4)*x(5)+(mr/m3)*x(6)*abs(x(6))+(mu/m3)*x(4)*x(5); 48 - end</pre>	6 -	uactual=x(4)
<pre>8 - reactual=x(6); 9 - xuu=-1.62; 11 - xrr=-1.93; 12 - mli=31.44; 13 - yvv=-1310; 14 - yrr=0.632; 15 - yuv=-28.6; 16 - m22=66.01; 17 - mvv=-3.18; 18 - mrr=-94; 19 - nur=-2; 20 - nuv=-24; 21 - mdeltar=-6.15; 22 - m33=0.33; 24 - xref=10*sin(0.1*t); 25 - yref=10*cos(0.1*t); 26 - dxref=1*cos(0.1*t); 27 - dyref=-sin(0.1*t); 28 - yref=0; 29 - ex=x(1)-xref; 29 - ex=x(1)-xref; 29 - ex=x(1)-xref; 30 - ey=x(2)-yref; 31 - de=x*(4)-dxref; 32 - dey=x(5)-dyref; 33 - kdy=-100; 34 - kyx=-00; 35 - ky=-100; 36 - kyy=-100; 37 - kdy=-100; 37 - kdy=-100; 38 - u=yx(5)-dyref; 39 - u=yx(5)-dyref; 31 - de=x*(4)-dxref; 42 - xdot(1)=x(4); 43 - xdot(2)=x(5); 44 - xdot(1)=x(4); 45 - xdot(2)=x(5); 46 - xdot(2)=x(5); 47 - xdot(2)=x(5); 47 - xdot(2)=x(5); 48 - end</pre>	7 -	<pre>vactual=x(5);</pre>
<pre>9</pre>	8 -	<pre>ractual=x(6);</pre>
<pre>10 - xuu=-1.62; 11 - xrr=1.93; 12 - m11=31.44; 13 - yvv=-1310; 14 - yrv=0.63; 15 - yuv=-28.6; 16 - m22=66.01; 17 - nvv=-3.18; 18 - nrr=-94; 19 - nuv=-24; 20 - nuv=-24; 21 - ndeltar=-6.15; 22 - m33=8.33; 24 - xref=10*cos(0.1*t); 25 - yref=10*cos(0.1*t); 26 - dxref=1*cos(0.1*t); 27 - dyref=-sin(0.1*t); 28 - ex=x(1)-xref; 30 - ey=x(2)-yref; 31 - dex=x(4)-dxref; 32 - dey=x(5)-dyref; 33 - ey=x(5)-dyref; 34 - xkot(1=xu/k1)*x(4)*abs(x(4))+(xr/m11)*x(6)*abs(x(6))+(m2/m11)*x(5)*x(6)+(tu/m11); 45 - xdot(3)=x(6); 46 - xdot(3)=x(6); 47 - xdot(3)=x(6); 48 - xdot(3)=x(6); 49 - xdot(3)=x(6); 40 - xdot(3)=x(6); 41 - xdot(3)=x(6); 42 - xdot(3)=x(6); 43 - xdot(3)=x(6); 44 - xdot(5)=-(m1/m22)*x(4)*x(5)+(mr/m31)*x(4)*x(6)+(mr/m33)*x(4)*x(6)+(ty/m22)*x(6)*abs(x(6))+(mr/m33)*x(4)*x(6)+(ty/m22)*x(4)*x(6)+(ty/m22)*x(4)*x(6)+(ty/m22)*x(4)*x(6)+(ty/m32)*x(4)*x(6)+(ty/m32)*x(4)*x(6); 40 - xdot(3)=x(6):+(ty/m31)*x(5)*abs(x(5))+(tyr/m22)*x(6)*abs(x(6))+(tyr/m32)*x(4)*x(6)+(ty/m32)*x(4)*x(5); 45 - xdot(5)=-(m1/m22)*x(4)*x(5)+(tyr/m32)*x(6)*abs(x(6))+(tyr/m33)*x(4)*x(6)+(ty/m33)*x(4)*x(5); 46 - xdot(5)=(tyr/m33)*x(5)*abs(x(5))+(tyr/m33)*x(6)*abs(x(6))+(tyr/m33)*x(4)*x(5); 47 - xdot(6)=(tyr/m33)*x(5)*abs(x(5))+(tyr/m33)*x(6)*abs(x(5))+(tyr/m33)*x(4)*x(5); 48 - end </pre>	9	
$ \begin{array}{ll} 11 & = & xrr = -1.93; \\ 11 & = & yrr = -0.63; \\ 13 & = & yrr = -0.63; \\ 14 & = & yrr = -0.63; \\ 15 & = & yrr = -2.66; \\ 16 & = & m22 = 66.01; \\ 17 & = & nvr = -3.18; \\ 18 & = & nrr = -94; \\ 19 & = & nur = -2; \\ 20 & = & nur = -2; \\ 21 & = & ndeltar = -6.15; \\ 22 & = & m33 = 0.33; \\ 23 & & \\ 24 & = & xref = 10^* sin (0.1^*t); \\ 25 & = & yref = 10^* cos (0.1^*t); \\ 26 & = & dxref = 10^* cos (0.1^*t); \\ 27 & = & dyref = -sin (0.1^*t); \\ 28 & & \\ 29 & = & ex = x(1) - xref; \\ 29 & = & ex = x(1) - xref; \\ 21 & = & dex = x(4) - dxref; \\ 32 & - & dey = x(5) - dyref; \\ 31 & - & dex = x(4) - dxref; \\ 32 & - & dey = x(5) - dyref; \\ 33 & & \\ 39 & - & tue kx^* + kkx^* dex; \\ 40 & - & tyr^* - too; \\ 41 & & xdot(1) = x(6); \\ 42 & x dot(1) = x(6); \\ 43 & x dot(2) = x(5); \\ 44 & x dot(1) = x(6); \\ 14 & x dot(1) = x(6);$	10 -	xuu=-1.62;
<pre>12 - m11=31.44; 33 - yvv=-1310; 44 - yrr=0.632; 55 - yuv=-28.6; 16 - m22=66.01; 17 - nvv=-3.18; 18 - nrr=-94; 19 - nur=-2; 20 - nuv=-24; 21 - ndeltar=-6.15; 22 - m33=8.33; 23 - xref=10*sin(0.1*t); 25 - yref=10*cos(0.1*t); 26 - dxref=1*cos(0.1*t); 27 - dyref=-sin(0.1*t); 28 - ex=x(1)-xref; 30 - ey=x(2)-yref; 31 - de=x(4)-dxref; 32 - dey=x(5)-dyref; 33 - v=y=x(2)-yref; 34 - kpx=-20; 35 - kdx=-60; 36 - kpy=-500; 37 - kdy=-100; 38 - tu=kpx*exkdx*dex; 40 - fr=kpy*ey+kdy*dey; 41 - xdot(1)=x(4); 42 - xdot(1)=x(4); 43 - xdot(2)=x(5); 44 - xdot(3)=x(6); 45 - kdx+(4) - kdy+(4) + krr/m11)*x(6)*abs(x(6)) + (m2/m11)*x(5)*x(6) + (tu/m11); 46 - xdot(5)=-(m1/m22)*x(4)*x(6) + (mr/m23)*x(6)*abs(x(6)) + (mr/m33)*x(4)*x(5) + (fy/m22); 47 - xdot(6) = (mu/m31*x(5)*abs(x(5)) + (mr/m33)*x(4)*x(5) + (mu/m33)*x(4)*x(5) + (mu/m33)*x(4)*x(5); 48 - ed</pre>	11 -	xrr=-1.93;
<pre>13 - yvv=-1310; 14 - yrr=0.632; 15 - yuv=-28.6; 16 - m22=66.01; 17 - nvv=-3.18; 18 - nrr=-94; 19 - nur=-2; 20 - nuv=-24; 21 - ndeltar=-6.15; 22 - m33=8.33; 23 - xref=10*sin(0.1*t); 25 - yref=10*cos(0.1*t); 26 - dxref=1*cos(0.1*t); 27 - dyref=-sin(0.1*t); 28 - ex=x(1)-xref; 30 - ey=x(2)-yref; 31 - dex=x(4)-dxref; 32 - dey=x(5)-dyref; 33 - dey=x(5)-dyref; 33 - tukky*exkdx*dex; 40 - fy=kpy*exkdx*dex; 40 - fy=kpy*exkdx*dex; 41 - xdot(1)*x(4); 42 - xdot(1)*x(4); 43 - xdot(2)*x(5); 44 - xdot(3)*x(5); 45 - xdot(4)=(xu/m11)*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m2/m11)*x(5)*x(6)+(tu/m11); 46 - xdot(5)=-(m11/m22)*x(4)*x(6)+(mr/m33)*x(6)*abs(x(6))+(mr/m33)*x(4)*x(5)+(fy/m22); 47 - xdot(6)=(nur/m31)*x(5)*abs(x(5))+(prr/m22)*x(6)*abs(x(6))+(nur/m33)*x(4)*x(5); 48 - ed</pre>	12 -	m11=31.44;
14 - yrr=0.632; $yu=-28.6;$ $16 - m22=66.01;$ $17 - mvv=-3.18;$ $19 - nur=-2;$ $20 - nuv=-24;$ $19 - nur=-2;$ $m3=8.33;$ $24 - xref=10*sin(0.1*t);$ $25 - yref=10*cos(0.1*t);$ $26 - dxref=1*cos(0.1*t);$ $27 - dyref=-sin(0.1*t);$ $28 - ex=x(1)-xref;$ $29 - ex=x(1)-xref;$ $dey=x(2)-yref;$ $31 - dex=x(4)-dxref;$ $32 - dey=x(5)-dyref;$ $33 - tukpx*ex+kdx*dex;$ $40 - fyky*ex+kdx*dex;$ $40 - fyky*ex+kdx*dex;$ $41 - tukpx*ex+kdx*dex;$ $40 - fyky*ex+kdx*dex;$ $41 - tukpx*ex+kdx*dex;$ $41 - tukpx*ex+kdx*dex;$ $42 - xdot(1)=x(4);$ $42 - xdot(1)=x(4);$ $43 - xdot(2)=x(5);$ $44 - xdot(3)=x(6);$ $45 - xdot(6)=(xuu'nu1)*x(4)*abs(x(4))+(xrr/m1)*x(6)*abs(x(6))+(m2/m1)*x(5)*x(6)+(tu/m1);$ $46 - xdot(6)=(xuu'nu1)*x(4)*x(6)+(mv/m2)*x(6)+(mr/m3)*x(4)*x(5);$ $47 - xdot(6)=(xuu'nu3)*x(4)*x(5)+(mr/m3)*x(6)*abs(x(6))+(mr/m3)*x(4)*x(5);$	13 -	yvv=-1310;
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	14 -	yrr=0.632;
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	15 -	yuv=-28.6;
$ \begin{array}{rcl} 1 & - & nvr=-3 & i & i & i & i & i & i & i & i & i & $	16 -	m22=66.01;
<pre>10 - nur=-2; 10 - nur=-2; 10 - nuv=-24; 11 - ndEltar=-6.15; 12 - ndS=8.33; 13 - dref=1*cos(0.1*t); 14 - xref=1*cos(0.1*t); 15 - dref==sin(0.1*t); 16 - dref==sin(0.1*t); 17 - dyref==sin(0.1*t); 18 - ey=x(2)-yref; 19 - ey=x(2)-yref; 10 - ey=x(2)-yref; 11 - dex=x(4)-dref; 12 - dey=x(5)-dyref; 13 - dey=x(5)-dyref; 13 - dey=x(5)-dyref; 14 - xdot(1)=x(4); 15 - kdx=-60; 16 - kpy=-500; 17 - kdy=-100; 18 - tu=kpx*ex+kdx*dex; 10 - fy=kpy*ey+kdy*dey; 11 - tu=kpx*ex+kdx*dex; 10 - fy=kpy*ey+kdy*dey; 11 - tu=kpx*ex+kdx*dex; 10 - fy=kpy*ey+kdy*dey; 11 - xdot(1)=x(4); 13 - xdot(2)=x(5); 14 - xdot(3)=x(6); 15 - xdot(4)=(xuu/m11)*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m22/m11)*x(5)*x(6)+(tu/m11); 16 - xdot(5)=-(m11/m22)*x(4)*x(6)+(yvr/m22)*x(6)*abs(x(6))+(mr/m23)*x(4)*x(5)+(fym22); 17 - xdot(6)=(mv/m33)*x(5)*abs(x(5))+(mr/m33)*x(6)*abs(x(6))+(mr/m33)*x(4)*x(5); 18 - end</pre>	10 -	nvv5.10;
<pre>14</pre>	10 -	nrr=-2;
<pre>21 - ndeltar=-6.15; 22 - m33=8.33; 23 - xref=10*sin(0.1*t); 25 - yref=10*cos(0.1*t); 26 - dxref=1*cos(0.1*t); 27 - dyref=-sin(0.1*t); 27 - dyref=-sin(0.1*t); 28 - ex=x(1)-xref; 30 - ey=x(2)-yref; 31 - dex=x(4)-dxref; 32 - dey=x(5)-dyref; 33 - dey=x(5)-dyref; 33 - dyref=-sin(0,1*t); 34 - kpx=-20; 35 - kdx=-60; 36 - kpy=-500; 37 - kdy=-100; 38 - dyref=-sin(0,1*t); 40 - fy=kpy*ey+kdy*dey; 41</pre>	20 -	nut = -2,
<pre>m33=8.33; m33=8.33; rref=10*sin(0.1*t); cf dxref=1*cos(0.1*t); dr ef=-sin(0.1*t); ex=x(1)-xref; dr ef=-sin(0.1*t); ex=x(1)-xref; dey=x(2)-yref; dey=x(3)-dyref; dey=x(5)-dyre</pre>	21 -	ndeltar=-6.15:
<pre>xref=10*sin(0.1*t); yref=10*cos(0.1*t); z5 - yref=10*cos(0.1*t); z6 - dxref=1*cos(0.1*t); z7 - dyref=-sin(0.1*t); z8 = ex=x(1)-xref; 30 - ey=x(2)-yref; 31 - dex=x(4)-dxref; dey=x(5)-dyref; 33 - dey=x(5)-dyref; 33 - kdx=60; kdx=60; kdy=100; z7 - kdy=100; z8 = tu*kpx*ex+kdx*dex; fy=kpy*ey+kdy*dey; 4 4 2 - xdot(1)=x(4); xdot(2)=x(5); xdot(4)=(xuu/m11)*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m22/m11)*x(5)*x(6)+(tu/m11); xdot(5)=(m11/m21)*x(4)*x(6)+(yvv/m22)*x(5)*abs(x(6))+(m22/m11)*x(5)*x(6)+(tu/m12)*x(4)*x(5)+(fy/m22); xdot(5)=(m11/m21)*x(4)*x(6)+(yvv/m21)*x(5)*abs(x(6))+(mur/m33)*x(4)*x(6)+(muv/m33)*x(4)*x(5); 48 - end</pre>	22 -	m33=8.33:
<pre>24 - xref=10*sin(0.1*t); 25 - yref=10*cos(0.1*t); 26 - dxref=1*cos(0.1*t); 27 - dyref=-sin(0.1*t); 28 - ex=x(1)-xref; 30 - ey=x(2)-yref; 31 - dex=x(4)-dxref; 32 - dey=x(5)-dyref; 33 - dey=x(5)-dyref; 33 - kdx=-60; 36 - kpy=-500; 37 - kdy=-100; 38 - tu=kpx*ex+kdx*dex; 40 - fy=kpy*ey+kdy*dey; 41 - xdot(1)=x(4); 42 - xdot(2)=x(5); 44 - xdot(3)=x(6); 45 - xdot(4)=(xuu/m11)*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m22/m11)*x(5)*x(6)+(tu/m11); 46 - xdot(5)=-(m11/m22)*x(4)*x(6)+(pyv/m22)*x(5)*abs(x(6))+(mr/m22)*x(4)*x(5)+(fy/m22); 47 - xdot(6)=(mv/m33)*x(5)*abs(x(5))+(mr/m33)*x(6)*abs(x(6))+(nur/m33)*x(4)*x(5); 48 - end</pre>	23	The Contest of the
<pre>25 - yref=10*cos(0.1*t); 26 - dxref=1*cos(0.1*t); 27 - dyref=-sin(0.1*t); 28 - ex=x(1)-xref; 30 - ey=x(2)-yref; 31 - dex=x(4)-dxref; 32 - dey=x(5)-dyref; 33 - dey=x(5)-dyref; 33 - kdx=-60; 36 - kpy=-500; 37 - kdy=-100; 38 - tu=kpx*ex+kdx*dex; 40 - fy=kp*ey+kdy*dey; 41 - xdot(1=x(4); 42 - xdot(2)=x(5); 44 - xdot(3)=x(6); 45 - xdot(4)=(xuu/m11)*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m22/m11)*x(5)*x(6)+(tu/m11); 46 - xdot(5)=-(m11/m22)*x(4)*x(6)+(yvv/m22)*x(5)*abs(x(5))+(mr/m32)*x(4)*x(5)+(fy/m22); 47 - xdot(6)=(nvv/m33)*x(5)*abs(x(5))+(nrr/m33)*x(6)*abs(x(6))+(nur/m33)*x(4)*x(5); 48 - end</pre>	24 -	<pre>xref=10*sin(0.1*t);</pre>
<pre>26 - dxref=1*cos(0.1*t); 27 - dyref=-sin(0.1*t); 28 - ex=x(1)-xref; 30 - ey=x(2)-yref; 31 - dex=x(4)-dxref; 32 - dey=x(5)-dyref; 33 - kdx=-60; 36 - kpy=-500; 37 - kdy=-100; 38 - dy=xx+kdx*dex; 40 - fy=kpy*ex+kdx*dex; 40 - fy=kpy*ey+kdy*dey; 41</pre>	25 -	<pre>yref=10*cos(0.1*t);</pre>
<pre>27 - dyref=-sin(0.1*t); 28 29 - ex=x(1)-xref; 30 - ey=x(2)-yref; 31 - dex=x(4)-dxref; 32 - dey=x(5)-dyref; 33 34 - kpx=-20; kdx=-60; 35 - kdy=-100; 36 - kpy=-500; 37 - kdy=-100; 38 39 - tu=kpx*ex+kdx*dex; 40 - fy=kpy*ey+kdy*dey; 41 42 - xdot(1)=x(4); 43 - xdot(2)=x(5); 44 - xdot(3)=x(6); 45 - xdot(4)=(xuu/m11*x(4)*abs(x(4))+(xrr/m11*x(6)*abs(x(6))+(m22/m11*x(5)*x(6)+(tu/m11); 46 - xdot(5)=(m11/m22)*x(4)*x(6)+(yvv/m22)*x(5)*abs(x(5))+(yrr/m22)*x(6)*abs(x(6))+(yuv/m22)*x(4)*x(5)+(fy/m22); 47 - xdot(6)=(nvv/m33)*x(5)*abs(x(5))+(nrr/m33)*x(6)*abs(x(6))+(nur/m33)*x(4)*x(6)+(nuv/m33)*x(4)*x(5); 48 end</pre>	26 -	dxref=1*cos(0.1*t);
<pre>28 29 - ex=x(1)-xref; ey=x(2)-yref; 31 - dex=x(4)-dxref; 32 - dey=x(5)-dyref; 33 34 - kpx=-20; kdx=-60; 35 - kdx=-60; 36 - kpy=-100; 37 - kdy=-100; 38 39 - tu=kpx*ex+kdx*dex; 40 - fy=kpy*ey+kdy*dey; 41 42 - xdot(1)=x(4); xdot(2)=x(5); xdot(4)=(xuu/m11)*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m22/m11)*x(5)*x(6)+(tu/m11); xdot(3)=x(6); 44 - xdot(3)=x(6); 45 - xdot(4)=(xuu/m11)*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m22/m11)*x(5)*x(6)+(tu/m11); xdot(6)=(mvv/m33)*x(5)*abs(x(5))+(mr/m33)*x(6)*abs(x(6))+(mur/m33)*x(4)*x(5); 48 - end</pre>	27 -	dyref=-sin(0.1*t);
<pre>29 - ex=x(1)-xref; 30 - ey=x(2)-yref; 31 - dex=x(4)-dxref; 32 - dey=x(5)-dyref; 33 - ktx=-20; ktx=-60; 36 - kty=-500; 37 - kdy=-100; 38 - tu=ktyx*ex+kdx*dex; 40 - fy=kty*ey+kdy*dey; 41</pre>	28	
<pre>30 - ey=x (2) -yref; 31 - dex=x (4) - dxref; dey=x (5) - dyref; 33</pre>	29 -	<pre>ex=x(1)-xref;</pre>
<pre>31 - dex=x (4) -dxref; 32 - dey=x (5) -dyref; 33 - ktx=60; ktx=60; ktx=60; kty=-500; kty=-100; 34 - tu=kpx*ex+kdx*dex; 40 - fy=kpy*ey+kdy*dey; 41 - xdot(1)=x(4); xdot(2)=x(5); xdot(3)=x(6); xdot(4)=(xuu/m11)*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m22/m11)*x(5)*x(6)+(tu/m11); xdot(5)=-(m11/m22)*x(4)*x(6)+(yvv/m22)*x(5)*abs(x(5))+(yrr/m22)*x(6)*abs(x(6))+(mv/m33)*x(4)*x(5)+(fy/m22); xdot(6)=(mvv/m33)*x(5)*abs(x(5))+(mrr/m33)*x(6)*abs(x(6))+(mur/m33)*x(4)*x(5); 48 - end</pre>	30 -	ey=x(2)-yref;
<pre>32 -</pre>	31 -	<pre>dex=x(4)-dxref;</pre>
<pre>33 34 - kpx=-20; 35 - kdx=-60; 36 - kpy=-500; 37 - kdy=-100; 38 99 - tu=kpx*ex+kdx*dex; 40 - fy=kpy*ey+kdy*dey; 41 42 - xdot(1)=x(4); 43 - xdot(2)=x(5); 44 - xdot(3)=x(6); 45 - xdot(4)=(xuu/m11)*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m22/m11)*x(5)*x(6)+(tu/m11); 46 - xdot(5)=-(m11/m22)*x(4)*x(6)+(yvv/m22)*x(5)*abs(x(5))+(yrr/m22)*x(6)*abs(x(6))+(yuv/m22)*x(4)*x(5)+(fy/m22); 47 - xdot(6)=(nvv/m33)*x(5)*abs(x(5))+(nrr/m33)*x(6)*abs(x(6))+(nur/m33)*x(4)*x(6)+(nuv/m33)*x(4)*x(5); 48 - end</pre>	32 -	<pre>dey=x(5)-dyref;</pre>
$\begin{array}{llllllllllllllllllllllllllllllllllll$	33	
<pre>34 - kpx=-20; 35 - kdx=-60; 36 - kpy=-500; 37 - kdy=-100; 38</pre>		
<pre>35 - kdx=-60; 36 - kpy=-500; 37 - kdy=-100; 38 39 - tu=kpx*ex+kdx*dex; 40 - fy=kpy*ey+kdy*dey; 41 42 - xdot(1)=x(4); 43 - xdot(2)=x(5); 44 - xdot(3)=x(6); 45 - xdot(4) = (xuu/m11)*x(4)*abs(x(4)) + (xrr/m11)*x(6)*abs(x(6)) + (m22/m11)*x(5)*x(6) + (tu/m11); 46 - xdot(5)=-(m11/m22)*x(4)*x(6) + (yvv/m22)*x(5)*abs(x(5)) + (yrr/m22)*x(6)*abs(x(6)) + (yuv/m22)*x(4)*x(5) + (fy/m22); 47 - xdot(6) = (nvv/m33)*x(5)*abs(x(5)) + (nrr/m33)*x(6)*abs(x(6)) + (nur/m33)*x(4)*x(6) + (nuv/m33)*x(4)*x(5); 48 - end</pre>	34 -	kpx=-20;
<pre>36 - kpy=-500; 37 - kdy=-100; 38 39 - tu=kpx*ex+kdx*dex; 40 - fy=kpy*ey+kdy*dey; 41 42 - xdot(1)=x(4); 43 - xdot(2)=x(5); 44 - xdot(3)=x(6); 45 - xdot(4)=(xuu/m11)*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m22/m11)*x(5)*x(6)+(tu/m11); 45 - xdot(4)=(xuu/m11)*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m22/m11)*x(5)*x(6)+(tu/m11); 46 - xdot(5)=-(m11/m22)*x(4)*x(6)+(yvv/m22)*x(5)*abs(x(5))+(yrr/m22)*x(6)*abs(x(6))+(yuv/m22)*x(4)*x(5)+(fy/m22); 47 - xdot(6)=(nvv/m33)*x(5)*abs(x(5))+(nrr/m33)*x(6)*abs(x(6))+(nur/m33)*x(4)*x(6)+(nuv/m33)*x(4)*x(5); 48 end</pre>	35 -	kdx=-60;
<pre>37 - kdy=-100; 38 39 - tu=kpx*ex+kdx*dex; 40 - fy=kpy*ey+kdy*dey; 41 42 - xdot(1)=x(4); 43 - xdot(2)=x(5); 44 - xdot(3)=x(6); 45 - xdot(4)=(xuu/m11)*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m22/m11)*x(5)*x(6)+(tu/m11); 46 - xdot(5)=-(m11/m22)*x(4)*x(6)+(yvv/m22)*x(5)*abs(x(5))+(yrr/m22)*x(6)*abs(x(6))+(yuv/m22)*x(4)*x(5)+(fy/m22); 47 - xdot(6)=(nvv/m33)*x(5)*abs(x(5))+(nrr/m33)*x(6)*abs(x(6))+(nur/m33)*x(4)*x(6)+(nuv/m33)*x(4)*x(5); 48 end</pre>	36 -	kpy=-500;
<pre>38 39 - tu=kpx*ex+kdx*dex; 40 - fy=kpy*ey+kdy*dey; 41 42 - xdot(1)=x(4); 43 - xdot(2)=x(5); 44 - xdot(3)=x(6); 45 - xdot(4)=(xuu/m11)*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m22/m11)*x(5)*x(6)+(tu/m11); 46 - xdot(5)=-(m11/m22)*x(4)*x(6)+(yvv/m22)*x(5)*abs(x(5))+(yrr/m22)*x(6)*abs(x(6))+(yuv/m22)*x(4)*x(5)+(fy/m22); 47 - xdot(6)=(nvv/m33)*x(5)*abs(x(5))+(nrr/m33)*x(6)*abs(x(6))+(nur/m33)*x(4)*x(6)+(nuv/m33)*x(4)*x(5); 48</pre>	37 -	kdy=-100;
<pre>39 - tu=kpx*ex+kdx*dex; 40 - fy=kpy*ey+kdy*dey; 41 42 - xdot(1)=x(4); 43 - xdot(2)=x(5); 44 - xdot(3)=x(6); 45 - xdot(4)=(xuu/m11)*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m22/m11)*x(5)*x(6)+(tu/m11); 46 - xdot(5)=-(m11/m22)*x(4)*x(6)+(yvv/m22)*x(5)*abs(x(5))+(yrr/m22)*x(6)*abs(x(6))+(yuv/m22)*x(4)*x(5)+(fy/m22); 47 - xdot(6)=(nvv/m33)*x(5)*abs(x(5))+(nrr/m33)*x(6)*abs(x(6))+(nur/m33)*x(4)*x(6)+(nuv/m33)*x(4)*x(5); 48 - end</pre>	38	
<pre>40 - fy=kpy*ey+kdy*dey; 41 42 - xdot(1)=x(4); 43 - xdot(2)=x(5); 44 - xdot(3)=x(6); 45 - xdot(4)=(xuu/m11)*x(4)*abs(x(4))+(xrr/m11)*x(6)*abs(x(6))+(m22/m11)*x(5)*x(6)+(tu/m11); 46 - xdot(5)=-(m11/m22)*x(4)*x(6)+(yvv/m22)*x(5)*abs(x(5))+(yrr/m22)*x(6)*abs(x(6))+(yuv/m22)*x(4)*x(5)+(fy/m22); 47 - xdot(6)=(nvv/m33)*x(5)*abs(x(5))+(nrr/m33)*x(6)*abs(x(6))+(nur/m33)*x(4)*x(6)+(nuv/m33)*x(4)*x(5); 48 end</pre>	39 -	<pre>tu=kpx*ex+kdx*dex;</pre>
<pre>41 42 - xdot(1) = x(4); 43 - xdot(2) = x(5); 44 - xdot(3) = x(6); 45 - xdot(4) = (xuu/m11) * x(4) * abs(x(4)) + (xrr/m11) * x(6) * abs(x(6)) + (m22/m11) * x(5) * x(6) + (tu/m11); 46 - xdot(5) = - (m11/m22) * x(4) * x(6) + (yvv/m22) * x(5) * abs(x(5)) + (yrr/m22) * x(6) * abs(x(6)) + (yuv/m22) * x(4) * x(5) + (fy/m22); 47 - xdot(6) = (nvv/m33) * x(5) * abs(x(5)) + (nrr/m33) * x(6) * abs(x(6)) + (nur/m33) * x(4) * x(6) + (nuv/m33) * x(4) * x(5); 48 - end</pre>	40 -	fy=kpy*ey+kdy*dey;
<pre>42 - xdot(1) = x(4); 43 - xdot(2) = x(5); 44 - xdot(3) = x(6); 45 - xdot(4) = (xuu/m11) *x(4) *abs(x(4)) + (xrr/m11) *x(6) *abs(x(6)) + (m22/m11) *x(5) *x(6) + (tu/m11); 46 - xdot(5) = - (m11/m22) *x(4) *x(6) + (yvv/m22) *x(5) *abs(x(5)) + (yrr/m22) *x(6) *abs(x(6)) + (yuv/m22) *x(4) *x(5) + (fy/m22); 47 - xdot(6) = (nvv/m33) *x(5) *abs(x(5)) + (nrr/m33) *x(6) *abs(x(6)) + (nur/m33) *x(4) *x(6) + (nuv/m33) *x(4) *x(5); 48 - end</pre>	41	
<pre>43 - xdot(2) = x(5); 44 - xdot(3) = x(6); 45 - xdot(4) = (xuu/m11) *x(4) *abs(x(4)) + (xrr/m11) *x(6) *abs(x(6)) + (m22/m11) *x(5) *x(6) + (tu/m11); 46 - xdot(5) = - (m11/m22) *x(4) *x(6) + (yvv/m22) *x(5) *abs(x(5)) + (yrr/m22) *x(6) *abs(x(6)) + (yuv/m22) *x(4) *x(5) + (fy/m22); 47 - xdot(6) = (nvv/m33) *x(5) *abs(x(5)) + (nrr/m33) *x(6) *abs(x(6)) + (nur/m33) *x(4) *x(6) + (nuv/m33) *x(4) *x(5); 48 - end</pre>	42 -	xdot(1) = x(4);
<pre>44 - xdot(3) = x(6); 45 - xdot(4) = (xuu/m11) *x(4) *abs(x(4)) + (xrr/m11) *x(6) *abs(x(6)) + (m22/m11) *x(5) *x(6) + (tu/m11); 46 - xdot(5) = - (m11/m22) *x(4) *x(6) + (yvv/m22) *x(5) *abs(x(5)) + (yrr/m22) *x(6) *abs(x(6)) + (yuv/m22) *x(4) *x(5) + (fy/m22); 47 - xdot(6) = (nvv/m33) *x(5) *abs(x(5)) + (nrr/m33) *x(6) *abs(x(6)) + (nur/m33) *x(4) *x(6) + (nuv/m33) *x(4) *x(5); 48 - end</pre>	43 -	xdot(2) = x(5);
45 - xdot (4) = (xuu/m11) *x (4) *abs (x (4)) + (xrr/m11) *x (6) *abs (x (6)) + (m22/m11) *x (5) *x (6) + (tu/m11); 46 - xdot (5) = - (m11/m22) *x (4) *x (6) + (yvv/m22) *x (5) *abs (x (5)) + (yrr/m22) *x (6) *abs (x (6)) + (yuv/m22) *x (4) *x (5) + (fy/m22); 47 - xdot (6) = (nvv/m33) *x (5) *abs (x (5)) + (nrr/m33) *x (6) *abs (x (6)) + (nur/m33) *x (4) *x (6) + (nuv/m33) *x (4) *x (5); 48 - end	44 -	xdot(3) = x(6);
46 - xdot (5) = - (m11/m22) *x (4) *x (6) + (yvv/m22) *x (5) * abs (x (5)) + (yrr/m22) *x (6) * abs (x (6)) + (yuv/m22) *x (4) *x (5) + (fy/m22); 47 - xdot (6) = (nvv/m33) *x (5) * abs (x (5)) + (nrr/m33) *x (6) * abs (x (6)) + (nur/m33) *x (4) *x (6) + (nuv/m33) *x (4) *x (5); 48 - end	45 -	xdot(4) = (xuu/m11) * x(4) * abs(x(4)) + (xrr/m11) * x(6) * abs(x(6)) + (m22/m11) * x(5) * x(6) + (tu/m11);
$\frac{47 - x dot(6) = (nvv/m33) * x(5) * abs(x(5)) + (nrr/m33) * x(6) * abs(x(6)) + (nur/m33) * x(4) * x(6) + (nuv/m33) * x(4) * x(5);}{end}$	46 -	xdot(5) =- (m11/m22) *x(4) *x(6) + (yvv/m22) *x(5) *abs(x(5)) + (yrr/m22) *x(6) *abs(x(6)) + (yuv/m22) *x(4) *x(5) + (fy/m22);
48 - end	47 -	xdot(6) = (nvv/m33) * x(5) * abs(x(5)) + (nrr/m33) * x(6) * abs(x(6)) + (nur/m33) * x(4) * x(6) + (nuv/m33) * x(4) * x(5);
	48 -	end

```
1 -
      x0=[0,0,0,0,0,0];
2 -
      tspan=0:0.1:200;
3 -
      [t,x]=ode45('auv_dynamics19',tspan,x0);
4 -
      xref=10*sin(0.1*tspan);
5 -
      yref=10*cos(0.1*tspan);
6 -
      plot(x(:,1),x(:,2))
7 -
      hold on
8 -
      plot(xref, yref)
```

LQR CONTROL

Linear Quadratic Regulator control is applied to the linearised dynamics equation of the vehicle.

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The state space equation for the linearized model can be written as :

 $\dot{x} = Ax + Bu$

Where the state matrix
$$A = \begin{pmatrix} N_r & 0 & 0 \\ I_{zz} - N_r & 0 & 0 \\ 0 & 0 & U \\ 1 & 0 & 0 \end{pmatrix}$$
 and
the input matrix $B = \begin{pmatrix} N_{\delta_r} \\ 0 \\ 0 \end{pmatrix}$
suitably choosing $Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $R = (1)$

substituting the values of the linearized vehicle coefficients

$$A = \begin{pmatrix} 0.84 & 0 & 0 \\ 0 & 0 & 2 \\ 1 & 0 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} -54.54 \\ 0 \\ 0 \end{pmatrix}$$

The solution for control problem using optimal control can be stated as

$$K = \begin{pmatrix} -1.0563 & -1 & -2.2724 \end{pmatrix}$$

CHAPTER 5:

CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORK

In this thesis modeling of AUV is done and kinematics and dynamic equations of motion were obtained. Using geometrical parameters and relevant empirical formulas various hydrodynamic coefficients for the dynamic model are determined. Various methods for heading control of AUV were developed and controllers were designed.

Transfer function for heading control of AUV using simple P-PD controller was designed and simulation results were obtained to see the accuracy of the model obtained. Also, a Nomoto model approximation was presented and both 1st order and 2nd order models were obtained. Control of Nomoto model using PD controller is done with some design parameters preassigned. The stability of model obtained is visible in results obtained.

Path following problem of AUV in horizontal plane is considered and using error dynamics reduction technique on the surge velocity u and heading rate r, it was implemented. The required path is given in parametric form to the AUV.

Depth control and obstacle avoidance are major requirements for an AUV. Then for more complex operations a leader follower model has to be implemented. Their formation control is a major research field now-a-days.

REFERENCES:

- Breivik, Morten and Fossen. Thor I. "Guidance Laws for Autonomous Underwater Vehicles". Norwegian University of Science and Technology, Norway.
- [2] Blidberg, D Richard. "The Development of Autonomous Underwater Vehicles (AUV);A Brief Summary". Autonomous Undersea Systems Institute, Lee New Hampshire, USA.
- [3] Desa, Elgar., Madhan, R. and Maurya, P. "Potential of autonomous underwater vehicles as new generation ocean data platforms". National Institute of Oceanography, Dona Paula, Goa, India.
- [4] Ching-Yaw Tzeng, Ju-Fen Chen." Fundamental properties of linear ship steering dynamic models ", Journal of Marine Science and Technology, Vol. 7, No. 2(1999): pp. 79-88
- [5] http://robotics.ee.uwa.edu.au/auv/usal.html
- [6] O. Xu. Autonomous underwater vehicles (auvs). Report, The University of Western Australia, 2004.
- [7] http://ise.bc.ca/design_sensors.html
- [8] Fossen, Thor I. "Guidance and Control of Ocean Vehicles". Wiley, New York, 1994
- [9] Prestero, Timothy. "Verification of a Six-Degree of Freedom Simulation Model of REMUS
- [10] Autonomous Underwater Vehicle". MIT and WHOI. 2001.