A Project Report on

Economic Design of X-bar Control Chart by Ant Colony Optimization

In partial fulfillment of the requirements of

Bachelor of Technology (Mechanical Engineering)

Submitted by

Mr. Vasanth Sai Viswakarma Anasuri

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Rourkela 769008

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Under the guidance of

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CERTIFICATE

This is to certify that the work in this thesis report entitled "**Economic Design of X-bar Control Chart by Ant Colony Optimization**" submitted by **Vasanth Sai Viswakarma Anasuri** in partial fulfillment of the requirements for the degree of Bachelor of Technology in Mechanical Engineering Session 2011-2012 in the department of Mechanical Engineering, National Institute of Technology Rourkela, is an authentic work carried out by him under my supervision and guidance.

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A C K N O W L E D G E M E N T

It gives me immense pleasure to express my deep sense of gratitude to my supervisor Prof. Saroj Kumar Patel for his invaluable guidance, motivation, constant inspiration and above all for his ever co-operating attitude that enabled me in bringing up this thesis in the present form.

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Abstract

Control charts are widely employed to monitor and maintain the statistical control of a process. Designing a control chart involves selecting a sample size, sample frequency and control limits for the control chart. The costs of sampling and testing, cost of detecting the out-of-control signal and possibly correcting it, and the cost of non-conforming items reaching the consumer give the control chart an economic aspect. In 1956, Duncan developed a Loss Cost Function for X-bar control chart with single assignable cause. The function has to be optimized using a non-conventional optimization technique. In this project, Ant Colony Optimization (ACO) has been employed to optimize Duncan's Loss Cost Function. Ants while searching for food deposit a chemical pheromone on their way back. The amount of pheromone deposited will be dependent on the quality and quantity of food. As the time progresses the ants become selective in choosing the path depending upon the pheromone deposited. Eventually, this leads the ants to choose the best possible path. An algorithm based on the traditional Ant Colony Optimisation technique developed by Niaki and Ershadi has been applied to the economic model of Duncan. The results were found to be on par with the results obtained by employing other non-conventional optimization techniques such as Genetic Algorithm.

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Nomenclature

a : Positive parameter associated with the pheromone level

a_1 : Fixed sampling cost

a_2 : Variable sampling cost

a_3 : Cost of finding assignable cause

a'_3 : Cost of investigating false alarm

a_4 : Hourly penalty cost associated with production in out-of-control state

b : Positive parameter associated with the attractiveness

D : Time required in finding an assignable cause following an action signal

$E(A)$: Net income per hour

$E(C)$: Net income per cycle

$E(L)$: Loss incurred per hour

$E(T)$: Expected length of production cycle

F_{njp} : Pheromone deposited in the stage corresponding to sample size by the j^{th} ant in p^{th} partition.

F_{hjp} : Pheromone deposited in the stage corresponding to sample frequency by the j^{th} ant in p^{th} partition.

F_{kjp} : Pheromone deposited in the stage corresponding to control limits width by the j^{th} ant in p^{th} partition.

g : Time required to take and interpret the results of a sample of size 1

h : Sampling frequency

k : Width of the control limits

m : Initial number of ants

n : Sample size

P : Pheromone deposit factor

q_0 : Probability that the ant choses a new path

T_{ij} : Pheromone associated with the path ij

V_0 : Income per hour of operation in the in-control state

V_1 : Income per hour of operation in the out-of-control state

α : Probability of the occurrence of type I error or false alarm

β	: Probability of the occurrence of type II error
δ	: Magnitude of the single assignable cause
η	: Weighting function
λ	: Expected number of occurrences for the Poisson distribution followed by single assignable cause
μ	: Mean of the sample
μ_0	: Process mean
μ_1	: Mean after the process shift
ρ	: Evaporation Constant
σ	: Standard deviation
σ^2	: Variance
τ	: Expected length of time after which assignable cause occurs

Chapter 1

Introduction

1.1 Quality Control and Statistical Process Control

Improving the quality of the output is a major factor for a successful and competitive business in the market. Quality can be defined as the fitness for use. It is inversely proportional to variability. In order to improve the quality, the variability in processes or products has to be reduced. Quality improvement or quality control is the set of activities that reduce of variability in processes and products.

Statistical process control (SPC) is a collection of statistical methods that monitor and control a process. SPC ensures that the process operates at its full potential to produce conforming product. Under SPC, a process desired to produce as much conforming product as possible with the least possible waste. Control chart is a key tool of SPC.

1.2 Control Charts

Statistical process control (SPC) and maintenance management are key tools in production process. They are used to achieve optimal product quality and cost reduction. Walter Shewhart is known as the father of modern control charts. He introduced the concept of Statistical Process Control in 1924. The main aim behind the idea of control charts is the need for perfection and elimination of non-conforming products. A control chart helps to differentiate between the *inherent variation* in a process and *variation due to assignable causes*. The inherent variation in a process is background noise due to several small unavoidable causes; assignable causes are considerably larger fluctuations when compared to the background noise. Variation from an assignable cause can only be removed from the process through human intervention.

One of the first control charts to receive attention was the X-bar chart, devised by Walter Shewhart. The X-bar chart provides an illustrative example for general control chart theory. This chart monitors the central tendency or mean of the process output. Under normal operating conditions the process yields a measurement of μ_0. The unavoidable random fluctuations occurring about μ_0 follow a Normal distribution. If a sample of n units is taken from a process and the mean is calculated then X-bar $\sim$ N ($\mu_0, \frac{\sigma}{\sqrt{n}}$), where σ is the standard deviation. The sample mean is then compared with the upper and lower control limits of the control chart to decide if the process is in-control or out-of-control. There are two possible errors: a process can be deemed in-control when in fact the process is out-of-control, and vice versa. When the process is judged to be out-of-control, there is an attempt to identify the special cause of variation which is called an Assignable Cause Search. An out-of-control state is often characterized by a shift in the process mean from μ_0 to $\mu_1 = \mu_0 + \delta\sigma$ (σ is the process standard deviation and δ is the size of the shift in the process mean).

1.3 Types of Control Chart

Control Charts are classified by the type of quality characteristic they are supposed to monitor. Control charts can be broadly classified as *Control Charts for Variables* and *Control Charts for Attributes*.

Control Chart for variables means the quality characteristic that can be measured on a numeric scale. Example of a variable is diameter, height, breaking strength, temperature etc.

Attributes deal with qualitatively such as whether an item is nonconforming or what the number of nonconformities in an item is. The attributes do not give us to extent to which the quality characteristic is non-conforming. Attribute control charts are advantageous as they give quick summaries i.e., we may simply classify products as acceptable or unacceptable, based on various quality criteria. These charts are easily understood by anyone.

Variable control charts are more efficient than attribute control charts. Variable control charts alert us about the problems in the process before any actual "unacceptable" (as detected by the attribute chart) will occur. The variable control charts are leading indicators of trouble that will sound an alarm before the number of scraps increases in the production process.

For controlling quality characteristics that represent variables of the product, the following charts are commonly used:-

1. **X-bar chart**: This chart is used to plot the sample means in order to control the mean value of a variable.

2. **R chart**: This chart is used to plot the sample ranges in order to control the variability of a variable

3. **S chart**: This chart is used to plot the sample standard deviations in order to control the variability of a variable.

4. **S^2 chart**: This chart is used to plot the sample variances in order to control the variability of a variable.

For controlling quality characteristics that represent attributes of the product, the following charts are commonly used:-

1. **c chart**: This chart is used to plot the number of defects (per batch, per day, per machine, per 100 feet of pipe, etc.). This chart assumes that defects of the quality attribute are rare, and the control limits in this chart are computed based on the Poisson distribution.

2. **u chart**: This chart is used to plot the rate of defects, that is, the number of defects divided by the number of units inspected (the n; e.g., feet of pipe, number of batches). Unlike the C chart, this chart does not require a constant number of units, and it can be used, for example, when the batches (samples) are of different sizes.

3. **np chart**: This chart is used to plot the number of defectives (per batch, per day, per machine) as in the C chart. However, the control limits in this chart are not based on the distribution of rare events, but rather on the binomial distribution.

Therefore, this chart should be used if the occurrence of defectives is not rare. For example, this chart can be used to control the number of units produced with minor flaws.

4. **p chart**: This chart is used to plot the fraction of defectives (per batch, per day, per machine, etc.) as in the u chart. However, the control limits in this chart are not based on the distribution of rare events but rather on the binomial distribution (of proportions). Therefore, this chart is most applicable to situations where the occurrence of defectives is not rare. All of these charts can be adapted for short production runs (short run charts), and for multiple process streams. If the percent of defectives is plotted, it is called 100p chart or percent defective chart.

1.4 Design of X bar Control Chart

An X bar Control Chart consists of

a) Central Line (CL)
b) Upper Control Limit (UCL)
c) Lower Control Limit (LCL)
d) Process Values plotted on the chart

If a plotted values lies in the limits of UCL and LCL, then the process is said to be "in control" or else the process is "out of control".

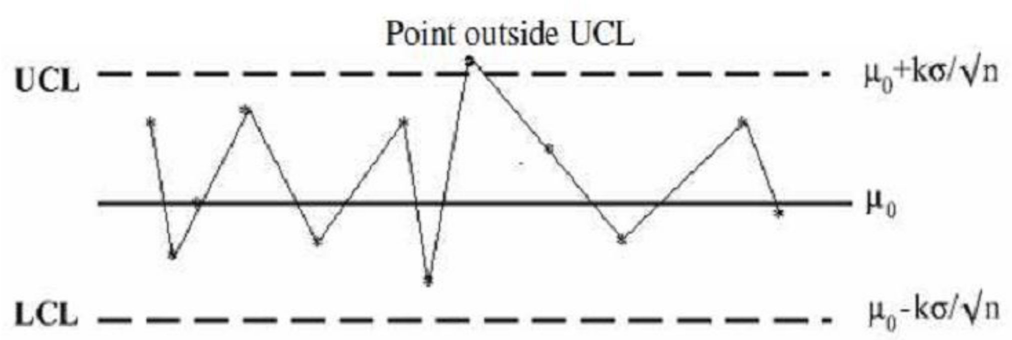


Fig.1.1 X bar Control Chart

Given a population of numbers $\{x_1, x_2, x_3 \ldots x_n\}$, the mean is given by

$$\text{Mean} = \mu = \frac{\sum_{i=1}^{n} x_i}{n}$$

Variance is given by

$$\sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}$$ σ is the standard deviation

To plot an X-bar Control Chart

- Samples of size *n* are chosen from the process at regular intervals and the mean for each sample(x-bar) is calculated.
- The distribution of x-bar is Normal Distribution with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$. The centre line is to be drawn at a height μ.
- The UCL and LCL are drawn at the heights of $\mu + k\frac{\sigma}{\sqrt{n}}$ and $\mu - k\frac{\sigma}{\sqrt{n}}$ respectively. When k=2, the probability that a given sample lies within the control limits is 95.44%. When k=3, the probability is 99.74% .

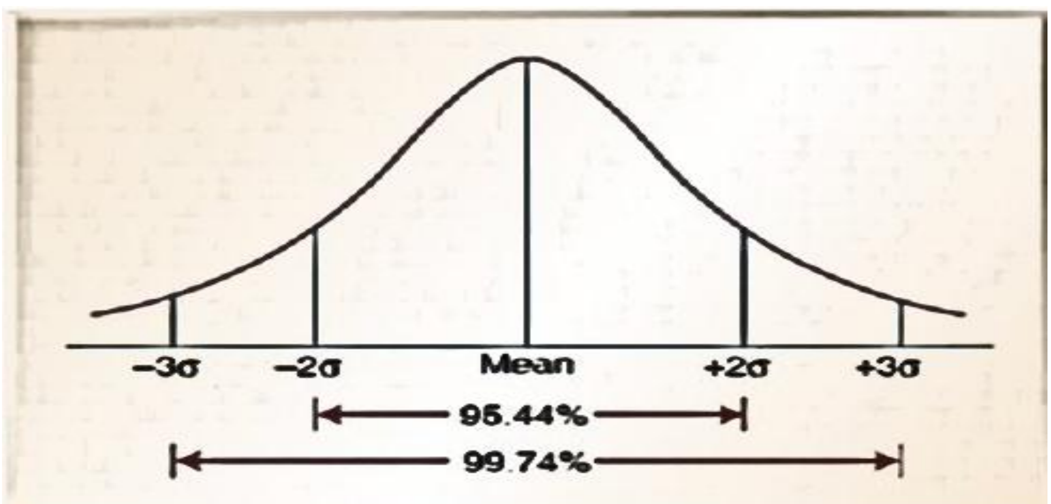


Fig.1.2 Area under a normal curve

There are two ways a process may be declared out of control. One is the data points plotted outside the control limits and the other is non-random pattern of data as in too many data points on one side of the central line or cyclic behavior of data points etc.

Montgomery [17] suggested the following decision rules to declare a process out of control.

- One or more values outside the control limits.
- 2 or 3 values outside the 2σ limits but within the control limits.
- 4 or 5 points on one side of the central line and outside the 1σ limits.
- 8 consecutive points on one side of central line.
- 6 or more consecutive points steadily increasing or decreasing.
- 15 consecutive points that are within 1σ limits.
- 14 points in a row alternating up and down.
- 8 consecutive points not within 1σ limits.

1.5 Errors in Control Charts

There are two types of errors in control chart: Type I and Type II. Type I error is when the process is concluded out of control even when the process is in control. Type I error is just a false alarm. The probability of type I error is given by α.

Type II error is when the process appears to be in control even though it is out of control. Type II error is caused by the shift in process mean. The control chart is not good enough to detect process shift. Thus this is a measure of inefficiency of the control chart. The probability of occurrence of type II error is given by β.

$\Phi(z) = (2\pi)^{-1/2} \exp(-z^2/2)$ is the standard normal density

- $\alpha = 2\int_{k}^{\infty} \Phi(z)dz$ (Probability of occurrence of type I error)

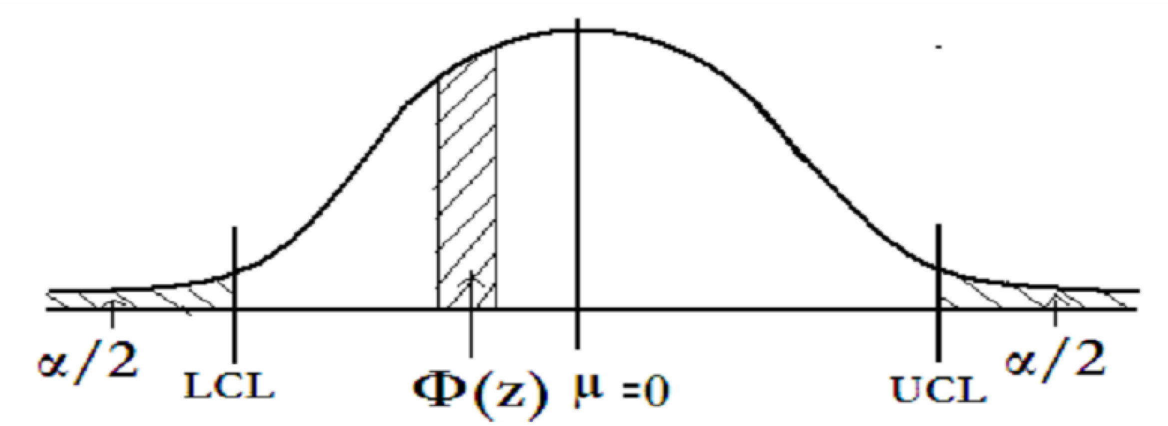


Fig.1.3 Type I error

- $\beta = 1 - \int_{-\infty}^{-k-\delta/\sqrt{n}} \Phi(z)dz - \int_{k-\delta/\sqrt{n}}^{\infty} \Phi(z)dz$ (Probability of occurrence of type II error)

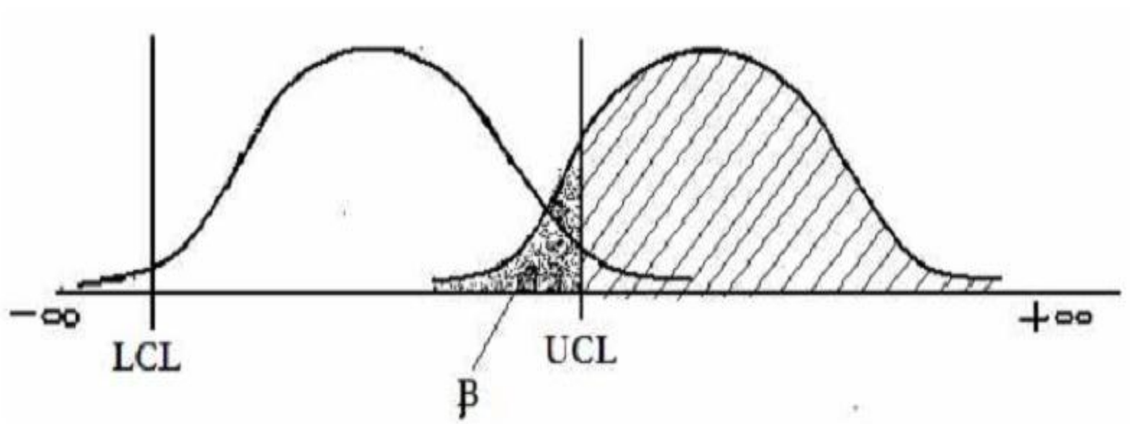


Fig.1.4 Type II error

1.6 Economic Design of Control Charts

Traditionally, the control charts are designed statistically i.e. the control charts are designed in order to minimize the α and β errors. But the design of a control chart also has an economic aspect. It involves the costs of sampling, inspection, checking for out of control signals, and cost of non-conforming units reaching the consumer. In an economic model, the control chart is designed with an objective to minimize the loss incurred by the control chart.

1.7 Ant Colony Optimization

A Combinatorial Optimization problem P= (S,f) is an optimization problem in which, given a finite set of solutions S (also called search space)and an objective function $f : S \rightarrow R^+$ that assigns a positive cost value to each of the solutions, the goal is either to find a solution of minimum cost value or—as in the case of approximate solution techniques—a good enough solution in a reasonable amount of time.

Ant Colony Optimization was introduced as a novel nature-inspired method for the solution of combinatorial optimization problems (COPs). The main idea of ACO is the foraging behavior of real ants. When searching for food, ants initially explore the area surrounding their nest in a random manner. After discovering the food source, the ant evaluates it and carries some food back to the nest. During the return trip, the ant deposits a pheromone trail on the ground. The amount of pheromone deposited depends on the quantity and quality of the food. It guides the other ants to the food source. Indirect communication among ants via pheromone trails enables them to find shortest paths between their nest and food sources. This ability of real ants has inspired the definition of artificial ants that can find approximate solutions to hard COPs.

1.7.1 Ant Colony Algorithm

```
while conditions not met do
        Schedule activities
                Ant Based Solution Construction ( )
                Pheromone update ( )
                Daemon activities ( )
        end Schedule activities
end while
```

1.7.2 Ant based solution construction

Artificial ants construct solutions from sequences of solution components taken from a finite set of *n* available solution components C={c_{ij}}. A solution construction starts with an empty partial solution $s^p = \Phi$. Then, at each construction step, the current partial solution s^p is extended by adding a feasible solution component from the set $N(s^p) \in C \mid s^p$, which is defined by the solution construction mechanism.

The choice of a solution component from $N(s^p)$ is done probabilistically at each construction step. The exact rules for probabilistic choice of solution components vary across different variants of ACO. The basic and the best known is that of Ant System (AS) proposed by Dorigo [12].

$$p(c_{ij}|s^p) = \frac{T_{ij}^{a} \cdot \eta(c_{ij})^{b}}{\sum_{c_{il \in N(s^p)}} T_{il}^{a} \cdot \eta(c_{il})^{b}}$$

where T_{ij} is the pheromone value associated with component c_{ij}, and η is a weighting function that assigns at each construction step a heuristic value to each feasible solution component $c_{ij} \in N(s^p)$. The values that are given by the weighting function are commonly called the heuristic information. Furthermore, a and b are positive parameters, whose values determine the relation between pheromone information and heuristic information [24].

1.7.3 Pheromone update

Pheromones are updated to increase the pheromone quantity of good or promising solutions and decrease the quantity associated with the bad ones. It is governed by the following equations suggested by Dorigo [12].

$$T_{ij} = (1-\rho)T_{ij} + \rho\Delta T \quad \text{if the ant passes through the path ij}$$

ρ is the evaporation factor. Pheromone evaporation is needed to avoid too rapid convergence of the algorithm. It implements a useful form of forgetting, favoring the exploration of new areas in the search space. In principle, algorithms update pheromone using either the iteration-best solution or the best-so-far solution. The best solution found from the start of the algorithm run. The best-so-far solution update leads to a faster convergence, while the iteration-best update allows for more diversification of the search [10].

1.7.4 Daemon actions

Daemon actions can be used to implement centralized actions which cannot be performed by single ants. Examples include the application of local search to the constructed solutions, or the collection of global information that can be used to decide whether it is useful or not to deposit additional pheromone to bias the search process from a non-local perspective [24].

Chapter 2

Literature Review

2.1 Economic Design of Control Charts

Control Charts are widely used to monitor and maintain the statistical control of a process. The design of a control chart involves the selection of the parameters like sample size *n*, sample frequency *h*, and control limits width *k*. The design of a control chart also has an economic aspect as it involves the costs of sampling, inspection, checking for out of control signals, and cost of non-conforming units reaching the consumer. Duncan [5] was the first to propose an economic model for the design of control chart. The model has to be further optimized using various meta-heuristics to obtain an optimal design.

Prabhu et.al [20] suggested an economic-statistic design for adaptive X bar control chart. An adaptive X-bar control chart is better than a traditional X-bar Control Chart. An adaptive X-bar control chart employs rapid mean sampling rate and large average sample size. This improves its efficiency. The control chart is designed using this model and optimizing its parameters. The average time to signal was constrained in the economic model in order to meet the required statistical properties.

Cai et.al [3] have proposed an economic model for the design of a control chart for a trended process. Traditional applications of the control charts are based on the assumption of process stability. But this is violated in many cases. The authors opine that the trended output resulting from a deteriorating factor like tool wear, material consumption, power consumption have to be interpreted differently. The researchers developed an economic model and tested the results.

Chou et.al [9] has given a method for economic statistical design of multivariate control charts for monitoring mean and covariance vectors. Their objective was to determine the optimum values of the test parameters economically such that the type I and type II errors of the control chart can be satisfied. The researchers have applied the test static -2*l*nL to the cost model established by Montgomery and Klatt.

Vijaya et.al [25] has provided a simple approach to the robust economic design of control charts. Robust economic designs are capable of incorporating in them robustness corresponding to the ambiguity of cost in the cost and process parameters. Robust economic designs are of two types. One type considers the uncertainty in the estimation of cost and makes the design suitable for any scenario. The second type considers different discreet scenarios for a single process and makes the design robust for all possible scenarios. The researchers have introduced a simple statistic for the robust economic design process with many different scenarios. Simple Genetic Algorithm has been employed to optimize the test parameters.

Chen and Yang [6] have adopted Banerjee and Rahim's cost structure to develop an economic design of a Moving Average control chart under a Weibull shock model in a continuous flow process.

Chen and Cheng [4] have investigated the non-normality effects on economic statistical design of X-bar control charts. The researchers have assumed that the sample mean follows Johnson's distribution. The McWilliams cost model was used to optimize the design parameters. The authors concluded from the sensitivity analysis that non-normality has a significant effect on the design parameters and hence should not be ignored.

Serel and Moskowitz [22] have suggested an economic model to EWMA (exponentially weighted moving average) control charts based on economic or both economic and statistical criteria. EWMA charts are used to jointly monitor the mean and variance of a process. Taguchi's quadratic cost loss function was used to calculate the losses. The average run length of the process is obtained by Markov's chain approach. The numerical values of smoothing constants, sample size, sampling frequency, control chart limits are determined from numerical search method.

Chen and Yang [7] have presented an economic design of X-bar control chart with a Weibull distributed process failure mechanism when there is a possibility of multiple assignable causes. A cost model based upon variable sampling intervals was developed and analyzed. Optimal values of the design parameters including the sample size, the sampling intervals, and control limit coefficient were solved by minimizing the expected total cost per unit time, based on the varieties of combinations of Weibull parameters. The comparative tests performed on a multiple cause model and single cause model show that the former provides a lower loss-cost than the latter when the process has an increasing hazard rate.

Lin et.al [14] developed the economic design of ARMA(autoregressive moving average) control chart to determine the optimal values of the test and chart parameters of the chart such that the expected total cost per hour is minimized. When designing control charts, we generally assume that the observations are independent of time i.e. the observations taken at different points of time are independent. This may not be true for certain processes like continuous chemical processes. Autocorrelation in the data has a significant effect on the statistical performance of control charts. The researchers used the ARMA control chart developed by Jiang, Tsui, and Woodall (2000). Genetic Algorithm was employed to optimize the design parameters.

Niaki et.al [19] presented an economic model using particle swarm optimization technique for the economic-statistic model of MEWMA (multivariate exponentially weighted moving average) control charts. MEWMA control chart is used to monitor several correlated quality characteristics simultaneously and where we have to detect small deviations of the characteristics. Particle swarm approach has been used for both economic model and economic-statistical model. The comparative tests between the economic and the economic-statistical models show better statistical performances of the economic-statistical design with negligible increase in cost.

Chih et.al [8] has applied particle swarm approach to the economic and economic-statistical design of X-bar control charts. This problem is a constrained optimization problem that involves the simultaneous use of the discreet and continuous variables. A numerical example in the study of Rahim and Banerjee(1993), was considered to indicate the procedure for solving the PSO algorithm performance. The results were compared with those from Genetic Algorithm, under the same conditions. PSO was found to be a promising method for solving the problems of inherent in the economic and economic statistical designs of an X bar control chart.

Chen et.al [5] have presented an integrated model for combining the preventive maintenance and the economic design of x-bar control charts using the Taguchi loss function. A preventive maintenance can reduce the failure rate to an out of control state by an amount proportional to the preventive maintenance level. The maintenance activities are coordinated with the statistical characteristics of the sampling results. Finally, a numerical experiment is conducted to investigate the model's working underlying the effect of preventive maintenance on the quality control costs.

2.2 Ant Colony Optimization

Ant Colony Optimization was developed in the early 90s by Marco Dorigo and his colleagues. This optimization technique was inspired by the foraging behavior of ants. After discovering the food source, the ant evaluates it and carries some food back to the nest. During the return trip, the ant deposits a pheromone trail on the ground. The amount of pheromone deposited depends on the quantity and quality of the food. It guides the other ants to the food source. Indirect communication among ants via pheromone trails enables them to find shortest paths between their nest and food sources. This ability of real ants has inspired the definition of artificial ants that can find approximate solutions to hard COPs such as TSP (travelling salesman problems), Job Shop Scheduling etc.

Dorigo and Blumb [11] provided a survey on theoretical results on ant colony optimization. Some convergent results were reviewed. The relation between ant colony optimization and other approximate methods of optimization were discussed and finally, certain areas of further research have been identified.

Aelterman et.al [2] suggested a new algorithm for manipulating the radiation pattern of Electronically Steerable Array Radiator Antennas. A continuous Ant Colony Optimization (ACO) technique is used to calculate the optimal impedance values of reactance loading different parasitic radiators placed in a circle around a center antenna. The results were compared to analytical techniques and optimization algorithms for calculating these reactance. Results show that the method is able to calculate near-optimal solutions for gain optimization and side lobe reduction.

Socha and Dorigo [24] have extended ant colony optimization for continuous domains. They have adapted the traditional Ant colony optimization technique to solve the continuous domain problems without changing the basic structure. The authors presented the general idea, implementation and results and compared them to the results reported in the literature.

Middendorf et.al [16] developed a new technique of multi colony ant algorithms. Here several colonies of ants cooperate in finding good solutions for an optimization problem. If the information to be exchanged is not too large, then different colonies of ants can be placed on different processors. Different kind of information exchange between the ant colonies was studied by the researchers in their work.

Zhu and Zhang [26] compared applications of the ACOEA (Ant Colony Optimisation with Elitist Ant), ACO (Ant Colony Optimization), Genetic Algorithm and Goal Chasing Algorithm. Optimization of sequence in the Mixed Model Assembly Line (MMAL) effectively balances the rate at which raw materials are used for production. The authors applied ACOEA algorithm with the taboo search and elitist strategy to form an optimal sequence of multi-product models which can minimize deviation between the ideal material usage rate and the practical material usage rate.

Abbas et.al [1] investigated optimum path planning for CNC drilling machines that is used in a special class of products involving a large number of holes arranged in a rectangular matrix like boiler plates, drum and trammel screens, connection flanges in steel structures, printed circuit boards etc. The researchers applied the ACO algorithm to the path planning of a CNC drilling tool. A slight modification in the basic algorithm was suggested in order to take advantage of the rectangular layout of holes.

Meng et.al [15] proposed a novel ant colony foraging simulation model named Direct Ant Colony Foraging (DACF). The technique combines the direct interaction mechanism and the indirect pheromone interaction mechanism. DACF2 and DACF3 are developed separately based on Wilensky's ant colony foraging model and Panait's model. These models describe the ant colony foraging behavior in nature. It also gives inspiration to the research of the emergent behaviors in complex systems. The simulation results demonstrate the superiority of DACF2 and DACF3.

Seo and Kim [21] developed an ant colony optimization algorithm with parameterized search space is developed for JSSP. The objective was to minimize the make-span. The problem is modeled as a disjunctive graph where arcs connect only pairs of operations related rather than all operations are connected in pairs. The proposed algorithm is compared with a multiple colony ant algorithm using 20 benchmark problems. The results show that the proposed algorithm is very accurate.

Niaki and Ershadi[18] applied the ant colony algorithm to the economic design of MEWMA (multivariate exponentially weighted moving average) control chart. They proposed that the path of an ant involves as many number of stages as there are variables in the loss function. The ACO parameters like pheromone deposit constant, evaporation factor, initial number of ants and probability of ants selecting a new path were tuned using RSM (response surface modeling).

2.3 Objective of Present Work

In the present work, we intend to design an economic model for an X-bar control chart with the objective of minimizing the loss from Duncan's cost loss function using the algorithm proposed by Niaki and Ershadi [18]. The optimization of Duncan's economic model is a single objective problem with no constraints. The objective is to minimize the loss incurred by the design. This involves the selection of n, h and k such that the loss expression is minimized. Sample size n, is a discreet set whereas the sample frequency h and control limit width k are continuous sets. In 2006, Socha and Dorigo extended the Ant Colony technique to the continuous domains. But not much research was done for applying the technique to mixed discreet-continuous variable problems.

Niaki and Ershadi [18] suggested that a path of an ant consists of as many number of stages as there are variables in the objective function. They considered that an ant reaching a stage is equivalent to selecting a value for one of the variables of the model. The stages h and k being continuous sets, have infinite number of solutions. So the stages h and k are further divided into sub stages.

By increasing the number of paths between two stages, the total number of paths an ant can travel increases exponentially. So the number of sub-stages each stage is divided is to be chosen reasonably.

Chapter 3
Economic Model of X-bar Chart

3.1 Introduction

Sample size (n), sampling frequency or interval between the samples (h), and width of the control limits (k) - selection of these three parameters is usually called the Design of the Control Chart.

The design of a control chart also has an economic aspect as it involves the costs of sampling, inspection, checking for out of control signals, and cost of non-conforming units reaching the consumer. Duncan [13] was the first to propose an economic model for the design of control chart. This project deals with the economic model proposed by the researcher. The following assumptions have been made by Duncan in his research.

(1) The process is either in-control or out-of-control state and in-control at the beginning.

(2) There is one assignable cause in the production process.

(3) The process will have a shift '$\delta\sigma$', if assignable cause occurs.

(4) The process shift is instant and cannot go back to in-control state automatically.

(5) The distribution of X is normal.

(6) Production is continuous during the search and repairing the process.

(7) The assignable cause is assumed to occur according to Poisson's ratio with an intensity of λ occurrences per hour.

3.2 Production Cycle

A production cycle is defined as the interval of time from start (in the in-control state) followed by a detection and correction of the assignable cause. The cycle consists of four periods: a) in-control period; b) out-of-control period; c) time taken for sampling and interpreting the results; d) time taken to find the assignable cause [17].

a) The in-control period is expected to be $1/\lambda$

b) The time interval the process remains in control is an exponential random variable with mean $1/\lambda h$. If an assignable cause occurs between j^{th} and $(j+1)^{th}$ samples, the expected time of occurrence within this time interval is

$$\tau = \frac{\int_{jh}^{(j+1)h} e^{-\lambda t} \lambda(1-jh)dt}{\int_{jh}^{(j+1)h} e^{-\lambda t} \lambda\, dt} = \frac{1-(1+\lambda h)e^{-\lambda h}}{\lambda(1-e^{-\lambda h})}$$

When the assignable cause occurs, the probability that it will be detected on any subsequent sample is given by power of the test i.e. $(1-\beta)$ where β is the probability of type II error. The probability of false alarm is given by α.

The number of samples required to produce an out-of-control signal given that the process is actually out of control is a geometric mean of $1/(1-\beta)$. So, the length of out-of-control period is given by $\{\frac{h}{(1-\beta)} - \tau\}$.

c) The time required to take samples and interpret them is proportional to the sample size. So the length of this time period is given by *'gn'*.

d) The time required to find an assignable cause is a constant D.

So, the expected length of the cycle is

$$E(T) = \frac{1}{\lambda} + \frac{h}{(1-\beta)} - \tau + gn + D$$

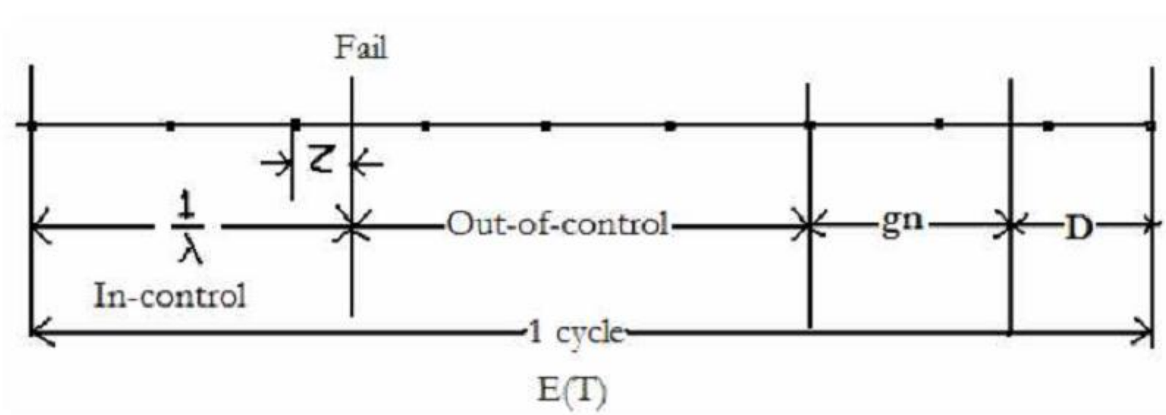


Fig.3.1 Stages of a Production Cycle

3.3 Loss Function

The derivation of Duncan's Loss Cost Function was explained by Montgomery [17]. The net income per hour of operation in the in-control state is V_0 and the net income per hour of operation in the out-of-control state is V_1. The cost of taking a sample of size n is assumed to be of the form $a_1 + a_2 n$; that is a_1 is the fixed cost and a_2 is the varying cost. The expected number of samples taken within a cycle is the expected cycle length divided by the sample frequency. The cost of finding the assignable cause is a_3 and the cost of investigating a false alarm is a_3'. The expected number of false alarms generated during a cycle is α times the expected number of samples taken before the shift, or

$$\alpha \sum_{j=0}^{\infty} \int_{jh}^{(j+1)h} j e^{-\lambda t} dt = \frac{\alpha e^{-\lambda h}}{1-e^{-\lambda h}}$$

So, the expected net income per cycle is given by

$$E(C) = V_0\left(\frac{1}{\lambda}\right) + V_1\left(\frac{h}{(1-\beta)} - \tau + gn + D\right) - a_3 - \frac{a_3' e^{-\lambda h}}{1-e^{-\lambda h}} - (a_1 + a_2 n)\frac{E(T)}{h}$$

The net income per hour E(A) is obtained by dividing E(C) with E(T).
Let a_4 be the hourly penalty cost for the production in out-of-control state. Then $a_4 = V_0 - V_1$
So the equation E(A) becomes

$$E(A) = V_0 - \frac{(a_1 + a_2 n)}{h} - \frac{a_4\left[\frac{h}{(1-\beta)} - \tau + gn + D\right] + a_3 + a_3' \alpha e^{-\lambda h}/(1-e^{-\lambda h})}{\frac{1}{\lambda} + \frac{h}{(1-\beta)} - \tau + gn + D}$$

or $E(A) = V_0 - E(L)$ where

$$E(L) = \frac{(a_1 + a_2 n)}{h} + \frac{a_4\left[\frac{h}{(1-\beta)} - \tau + gn + D\right] + a_3 + a_3' \alpha e^{-\lambda h}/(1-e^{-\lambda h})}{\frac{1}{\lambda} + \frac{h}{(1-\beta)} - \tau + gn + D}$$

E(L) represents the loss incurred per hour in the process. E(L) is a function of the control chart parameters n, h and k. So maximizing the net income per hour is equivalent to the minimization of the loss function [17].

Chapter 4

Methodology

4.1 Example

The following example that was earlier solved by Montgomery [17] and Shiau [23] was solved using the Ant Colony optimization and the results are compared.

A manufacturer produces a nonreturnable glass bottles for packaging a carbonated soft drink beverage. The wall thickness of the bottles is an important quality characteristic. If the wall is too thin, internal pressure generated during filling will cause the bottle to burst. The manufacturer has used x-bar control charts for process surveillance for some time. These control charts have been designed with respect to statistical criteria. However, in an effort to reduce costs, the manufacturer wishes to design an economically optimum x-bar control chart for the process.

Based on an analysis of quality control technicians' salaries and the costs of the test equipment, it is estimated that the fixed cost of taking a sample is $1. The variable cost of taking a sample is estimated to be $0.01 per bottle. It takes approximately 1 minute (0.0167 h) to measure and record the wall thickness of the bottle.

The process is subject to several different types of assignable causes. However when the process goes out-of-control, the magnitude of shift is approximately two standard divisions. Process shift occurs at random with a frequency of about one every 20h operation. Thus the exponential distribution with parameter λ = 0.05 is a reasonable model of the run length in control. The average time required to investigate an out-of-control signal is 1h. The cost of investigating an action signal that results in the elimination of the assignable cause is $25, while the cost of investigating a false alarm is $50.

The manufacturer estimates that the penalty cost of operating in the out-of-control state for one hour is $100.

An economic model for the x bar control chart had to be designed.

The loss expected per hour is given by Duncan's equation with

a_1 = $1

a_2 = $0.10

a_3 = $25

a'_3 = $50

a_4 = $100

λ = 0.05

δ = 2.0

g = 0.0167

D = 1.0

$$E(L) = \frac{(a_1 + a_2 n)}{h} + \frac{a_4\left[\frac{h}{(1-\beta)} - \tau + gn + D\right] + a_3 + a'_3 \alpha e^{-\lambda h}/(1-e^{-\lambda h})}{\frac{1}{\lambda} + \frac{h}{(1-\beta)} - \tau + gn + D}$$

A computer program in C language was written for the optimization of the above equation. As discussed earlier, this is a single objective and unconstrained problem. Ant colony algorithm as discussed in the previous chapter was used in the optimization of the equation.

4.2 Proposed Ant Colony Algorithm

Niaki and Ershadi [18] applied a modified ant colony optimization technique to optimize statistically constrained MEWMA control charts (Multivariate Exponentially Weighted Moving Average Control Charts). A similar algorithm is applied in the present work to optimize Duncan's cost loss expression.

Niaki and Ershadi suggested that a path in the proposed algorithm consists of as many number of stages as there are number of variables in the objective function and reaching each stage is equivalent to selecting a value to a variable. Since the objective function i.e. Duncan's loss cost expression has three variables involved, there are three stages in the path of each ant. Since the variables h and k assume continuous values there are infinite number of solutions possible. So it is necessary to breakdown these stages into limited number of sub-stages as suggested by Niaki.

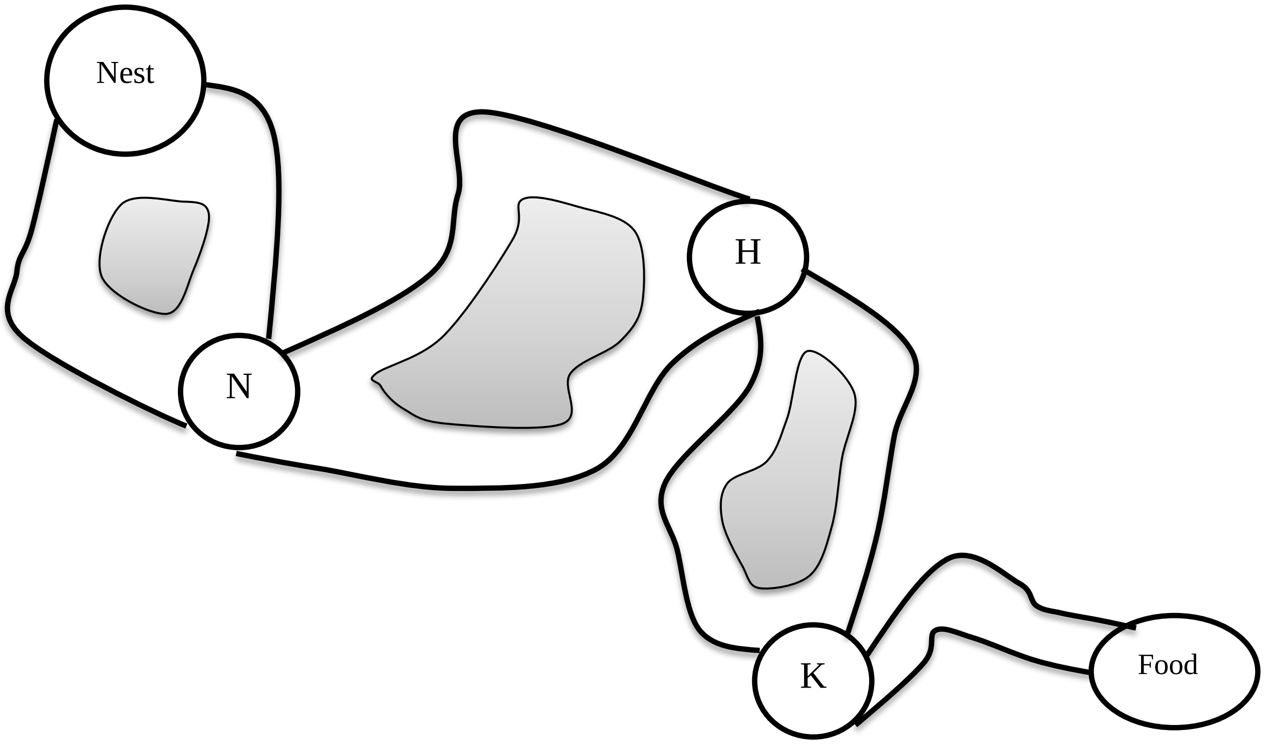


Fig.4.1 Path of the ant across the three stages

The above figure shows the paths of two different ants. There are two different paths to reach stage 1 from the nest and there are two different paths from stage 1 to stage 2 and so on. So the total number of paths increases exponentially when the numbers of paths between two stages are increased. This leads to a complexity in the solution and increases the time taken to achieve it. To avoid this complexity, each bounded variable is segmented into reasonable partitions. The ACO is then applied to the model after segmentation.[18]

The proposed algorithm consists of four steps:

1. Generating initial artificial ants
2. Estimating initial deposits
3. Generating individual ants
4. Stopping the algorithm

4.2.1 Generating initial ants

In the first step, *m* artificial ants are simultaneously generated and they start travelling the four stages. Each stage is segmented into a limited number of reasonable partitions. Each partition corresponds to the path chosen by a particular ant. A uniformly generated random number between the bounds of the variable decides the path chosen by the ant. For example, when an ant starts from its nest, generates a random number 3 corresponding to the #3rd partition, the ant travels through the 3rd partition. From first stage to second stage, suppose the ant generates a random number 0.78 corresponding to #4th partition, this results in the selection of the 4th partition. A similar process is followed to reach the food source. The number of partitions each variable is divided is a function of time taken by the algorithm to attain a near optimum solution and solving conditions. [18]

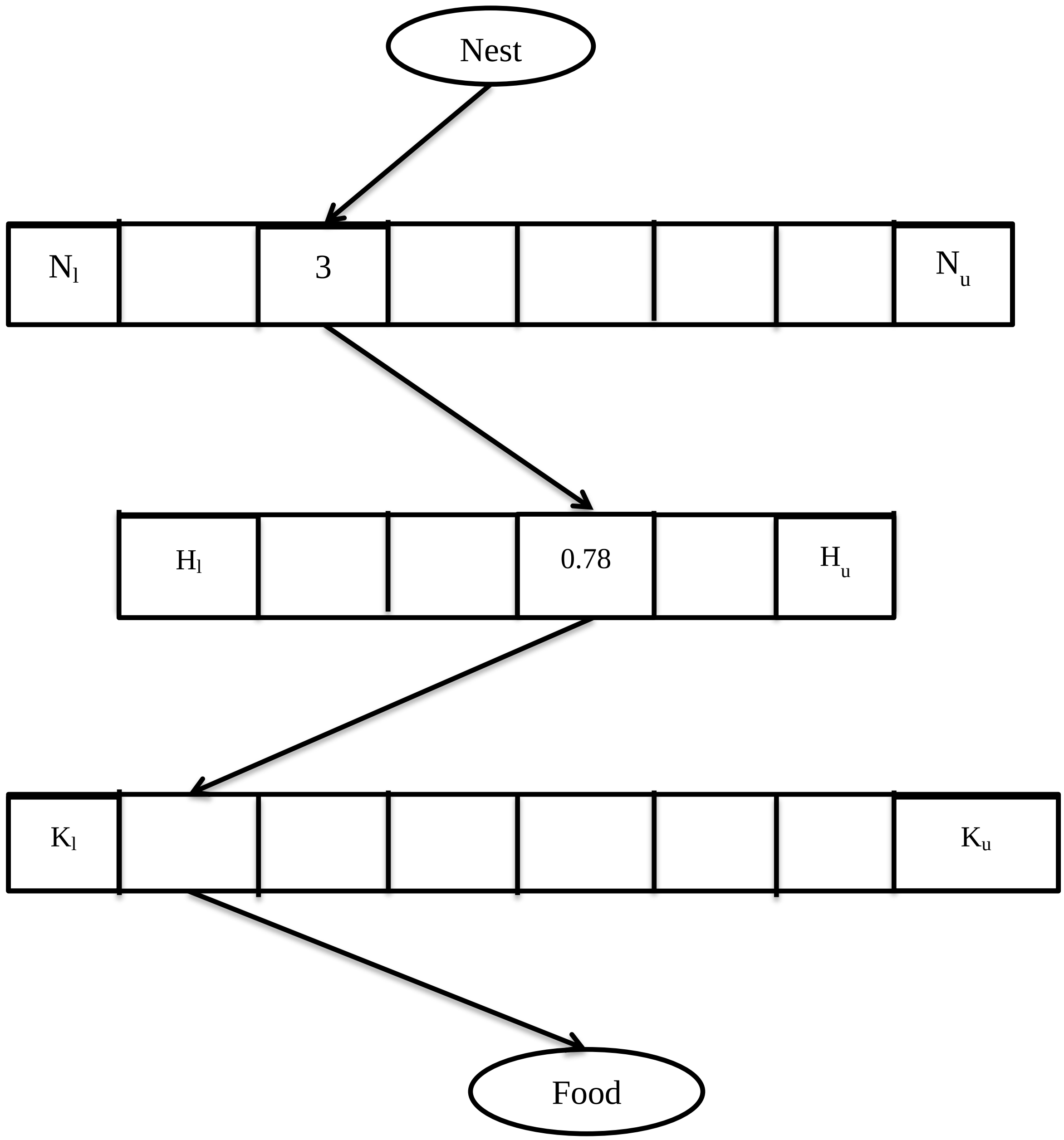


Fig.4.2 Ant selecting a partition for each stage

4.2.2 Initial Deposit Estimation

Every ant while travelling leaves a pheromone trail that helps the remainder ants to search the solution space effectively. This calculation is similar to the estimation of initial deposit pheromone by Niaki and Ershadi. For the stage 1 i.e. the stage corresponding to the selection of sample size *n*, the amount of pheromone deposited by ant *j* in the partition *p* can be calculated by the following formula.

$$F_{njp} = P_{np} \frac{E(L)_j}{E(L)_{min}}$$

Where, P_{np} is a parametric constant

$E(L)_{min}$ is the minimum $E(L)$ obtained by the *m* ants

$E(L)_j$ is the $E(L)$ corresponding to the j^{th} ant

Similarly for stage 2, i.e. the stage corresponding to the selection of sample frequency *h,* the pheromone depositions by the j^{th} ant in the p^{th} partition is given by the equation

$$F_{hjp} = P_{hp} \frac{E(L)_j}{E(L)_{min}}$$

where P_{hp} is a parametric constant.

The pheromone deposited in the stage 3, i.e the stage corresponding to the selection of control limits width *k* by ant *j* in the partition *p* is given by F_{kjp}.

$$F_{kjp} = P_{kp} \frac{E(L)_j}{E(L)_{min}}$$

where P_{kp} is a parametric constant.

The parametric constants P_{np} ,P_{hp} ,P_{kp} are assumed to be

$$P_{np}, P_{hp}, P_{kp} = \begin{cases} P & \text{if the ant passes through the partition p} \\ 0 & \text{if otherwise} \end{cases}$$

The total pheromone in the partition *p* of stage *n* is given by

$$F_{np} = \sum_{j=1}^{m} F_{njp}$$

Similarly the total pheromone deposited by *m* ants in the partition *p* of stages *h* and *k* are given by F_{hp} and F_{kp}

$$F_{hp} = \sum_{j=1}^{m} F_{hjp}$$

$$F_{kp} = \sum_{j=1}^{m} F_{kjp}$$

After the estimation of the total pheromone deposited by all the ants in each partition of the three stages, pheromone evaporation must take place in order to increase the pheromone associated with good solutions and reduce the pheromone associated with bad solutions.

The pheromone updating is done according to the following equation

$$F_{np} = \rho \times F_{np} + (1-\rho)\, F_0$$

Where ρ is the evaporation factor and F_0 is the inverse of the total value of the pheromone deposited in the stage *n*.

Similar equations are employed in updating the pheromones in *h* and *k* stages.

4.2.3 Generating individual ants

Now, the ants are generated individually. For each ant, a random number q is generated and compared to a constant number q_0. If q is greater than or equal to q_0, the ant chooses a random path as described in the section 6.1. But if q is less than q_0, the ant chooses the best so far path i.e. the partition containing the maximum pheromone level in each stage.

4.2.4 Stopping the algorithm

The algorithm should be stopped after a sufficient number of iterations. The number of iterations is the number of the individual ants generated in the third step of the algorithm. After the algorithm stops, the minimum $E(L)$ reached so far would give us the optimum solution

4.2.5 Flow Chart

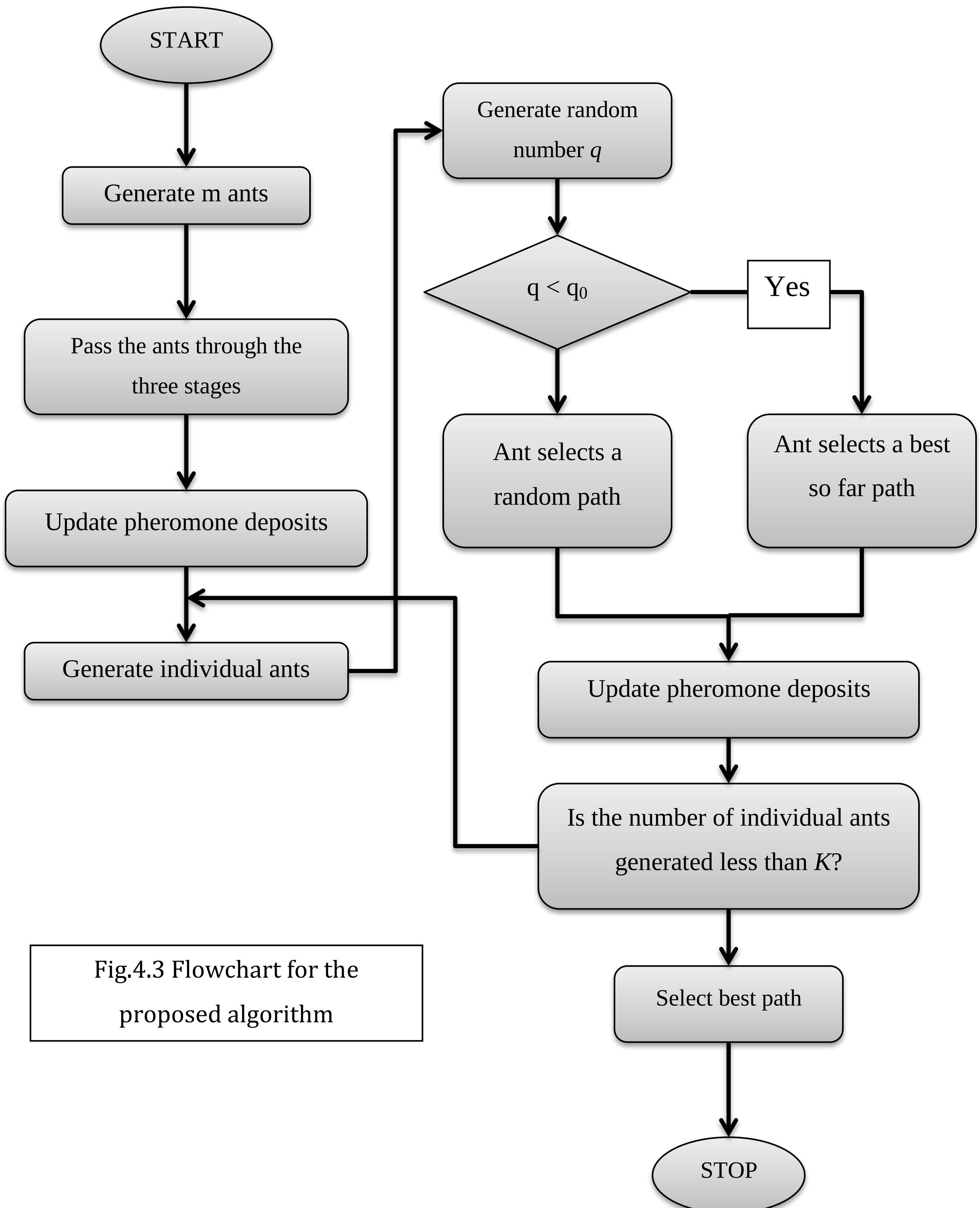


Fig.4.3 Flowchart for the proposed algorithm

4.2.6 Algorithm

```
START
Initialise N[ ], H[ ], K[ ];
for m ants
  {
        generate E(L);
  }
estimate min{E(L)}
deposit pheromone( );
pheromone evaporate( );
for x individual ants
        {
        generate q
        if  (q>q0)
                {
                generate E(L);
                if  E(L) < E(L)min
                        E(L)min = E(L)
                deposit pheromone( );
                pheromone evaporate( );
                }
        else
                {
                for max{N[ ]}, max{H[ ]}, max{K[ ]}
                deposit pheromone
                }
        }
        STOP
```

Chapter 5

Results and Discussion

A computer program in C language was written based on Ant Colony Optimization algorithm to get the optimum values of the design parameters of the control chart. The output from the C program gives the result of optimum solution of the example taken from Montgomery [17] giving the economic design of $\bar{X}$ chart.

This program calculates the optimum values of sampling frequency *h* and width of control limit *k* for different values of sample size *n*, and also computes the corresponding minimum values of the cost function E(L).

The minimal value of E(L) for different parameters of the ant colony algorithm is found to be 10.367

The values of sample size *n*, sampling frequency *h*, and control limits width *k* corresponding to this E(L) are:

$n = 5$

$h = 0.808$

$k = 2.98$

The feasible ranges of the ACO parameters as suggested by Niaki and Ershadi [22] are as follows:

Parameter (factor)	**Levels**
Initial artificial ants (m)	45 and 55
Evaporation coefficient (ρ)	0.25 and 0.35
Deposit parameter (P)	0.4 and 0.6
Probability of new path selection (q_0)	0.4 and 0.6

The program was run for different values of the parameters.

The output for the first step of the algorithm i.e. generating *m* ants that randomly select their path is given in the below table for 30 ants.

Table 5.1 E(L) generated by 30 ants randomly

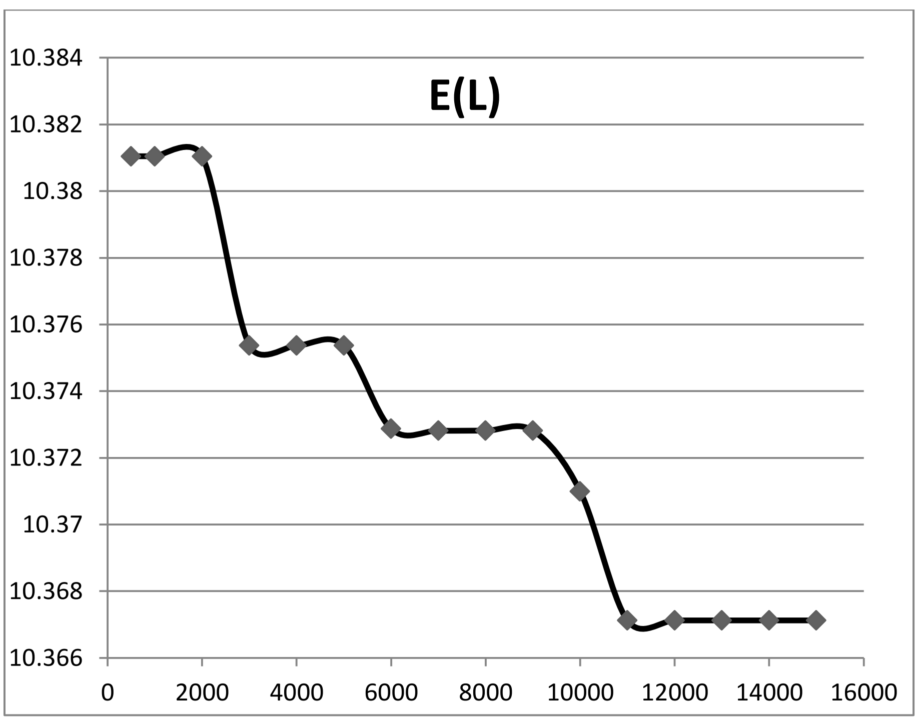
n	h	k	Alpha	Power	Cost
1	0.515979	3.314423	0.000918	0.094352	26.899031
13	0.687915	3.352985	0.0008	0.999943	11.804629
6	0.893689	3.912683	0.000091	0.838006	10.878015
12	0.91535	2.613385	0.008965	0.999992	11.67005
11	0.582391	3.224335	0.001263	0.999674	11.744335
1	0.618665	2.46452	0.01372	0.321134	15.490204
3	0.993097	2.598608	0.00936	0.806616	11.088272
7	0.402802	2.514343	0.011925	0.997258	12.959506
1	0.718988	2.980164	0.002881	0.163502	22.866
9	0.764288	3.383142	0.000717	0.995563	10.831503
3	0.670465	3.493445	0.000477	0.488295	12.56289
6	0.7646	2.402667	0.016276	0.993725	11.160124
12	0.71192	3.744647	0.000181	0.999273	11.492684
5	0.835992	3.876703	0.000106	0.724223	11.298058
15	0.723602	3.966241	0.000073	0.999921	12.085844
3	0.541193	3.13172	0.001738	0.630199	11.280176
13	0.86778	2.677271	0.007423	0.999997	11.830321
13	0.999799	4.094177	0.000042	0.999086	11.381049
10	0.559723	3.006366	0.002644	0.999547	11.708999
5	0.414246	3.812213	0.000138	0.745348	11.488202
6	0.806311	2.466718	0.013636	0.992498	10.946193
1	0.951257	2.31582	0.020568	0.376062	16.478498
5	0.752734	2.791199	0.005251	0.953612	10.445767
11	0.835883	3.807654	0.00014	0.99764	11.112371
8	0.846228	2.669635	0.007593	0.998592	10.933488
8	0.969476	3.124304	0.001782	0.994338	10.621872
12	0.759418	2.494897	0.012599	0.999995	12.141911
6	0.765369	3.622974	0.000291	0.899023	10.550925
9	0.49093	2.950391	0.003174	0.998854	11.867426
4	0.881714	3.065253	0.002175	0.825041	10.676272

Consider the following values: $m = 50$; $\rho = 0.25$; $P = 0.4$; $q_0 = 0.4$

The $E(L)_{optimum}$ obtained for number of iterations is given in the below table:

Table 5.2 $E(L)_{optimum}$ with number of iterations for $m = 50$; $\rho = 0.25$; $P = 0.4$; $q_0 = 0.4$

No. of Iterations	E(L)	*n*	*h*	*k*
500	10.381046	5	0.858	3.045
1000	10.381046	5	0.858	3.045
2000	10.381046	5	0.858	3.045
3000	10.375372	5	0.844	3.036
4000	10.375372	5	0.844	3.036
5000	10.375372	5	0.844	3.036
6000	10.372871	5	0.769	3
7000	10.372817	5	0.791	2.931
8000	10.372817	5	0.791	2.931
9000	10.372817	5	0.791	2.931
10000	10.370998	5	0.815	3.038
11000	10.367125	5	0.808	2.981
12000	10.367125	5	0.808	2.981
13000	10.367125	5	0.808	2.981
14000	10.367125	5	0.808	2.981
15000	10.367125	5	0.808	2.981



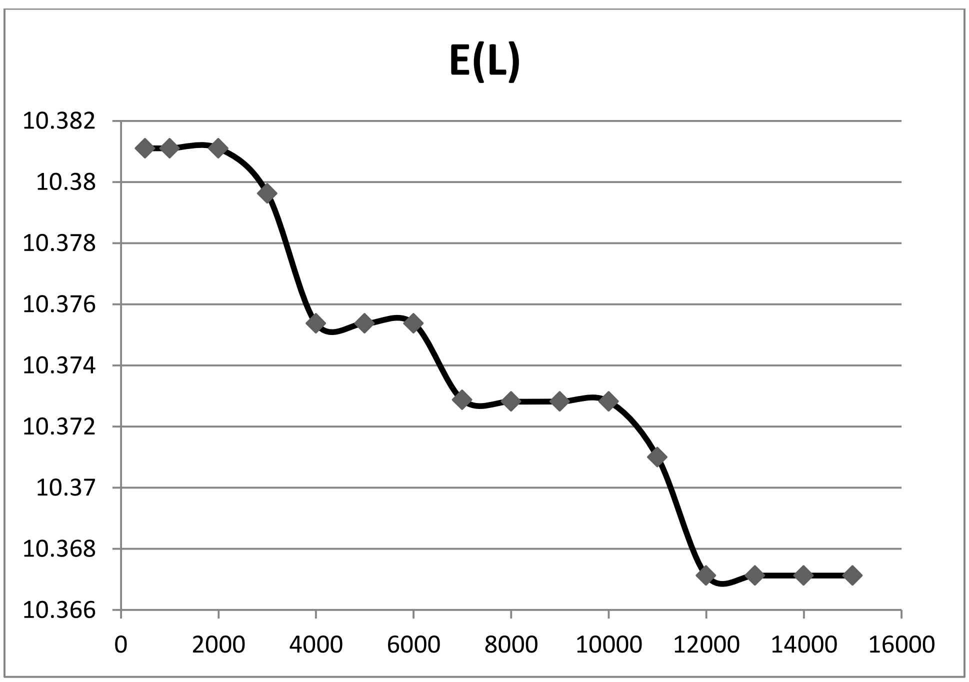
Graph 5.1 E(L) vs No. of iterations

Consider the following values: $m = 50$; $\rho = 0.25$; $P = 0.5$; $q_0 = 0.5$

The $E(L)_{optimum}$ obtained for number of iterations is given in the below table:

Table 5.3 $E(L)_{optimum}$ with number of iterations for $m = 50$; $\rho = 0.25$; $P = 0.5$; $q_0 = 0.5$

No. of Iterations	E(L)	*n*	*h*	*k*
500	10.381101	5	0.888	2.972
1000	10.381101	5	0.888	2.972
2000	10.381101	5	0.888	2.972
3000	10.379627	5	0.748	3.016
4000	10.375372	5	0.844	3.036
5000	10.375372	5	0.844	3.036
6000	10.375372	5	0.844	3.036
7000	10.372871	5	0.769	3
8000	10.372817	5	0.791	2.931
9000	10.372817	5	0.791	2.931
10000	10.372817	5	0.791	2.931
11000	10.370998	5	0.815	3.038
12000	10.367125	5	0.808	2.981
13000	10.367125	5	0.808	2.981
14000	10.367125	5	0.808	2.981
15000	10.367125	5	0.808	2.981



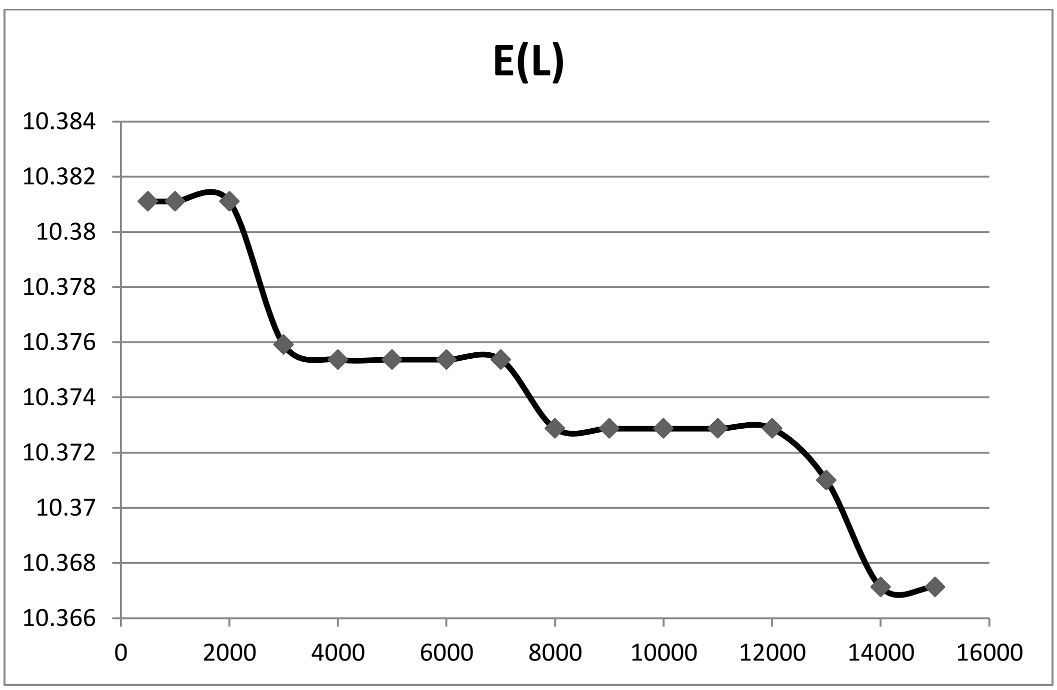
Graph 5.2 E(L) vs No. of iterations

Consider the following values: m = 50 ; ρ = 0.25 ; P = 0.6 ; q_0 = 0.6

The $E(L)_{optimum}$ obtained for number of iterations is given in the below table:

Table 5.4 $E(L)_{optimum}$ with number of iterations for m = 50 ; ρ = 0.25 ; P = 0.6 ; q_0 = 0.6

No. of Iterations	E(L)	n	h	k
500	10.381101	5	0.888	2.972
1000	10.381101	5	0.888	2.972
2000	10.381101	5	0.888	2.972
3000	10.375912	5	0.782	2.929
4000	10.375372	5	0.844	3.036
5000	10.375372	5	0.844	3.036
6000	10.375372	5	0.844	3.036
7000	10.375372	5	0.844	3.036
8000	10.372871	5	0.769	3
9000	10.372871	5	0.769	3
10000	10.372871	5	0.769	3
11000	10.372871	5	0.769	3
12000	10.372871	5	0.769	3
13000	10.370998	5	0.815	3.038
14000	10.367125	5	0.808	2.981
15000	10.367125	5	0.808	2.981



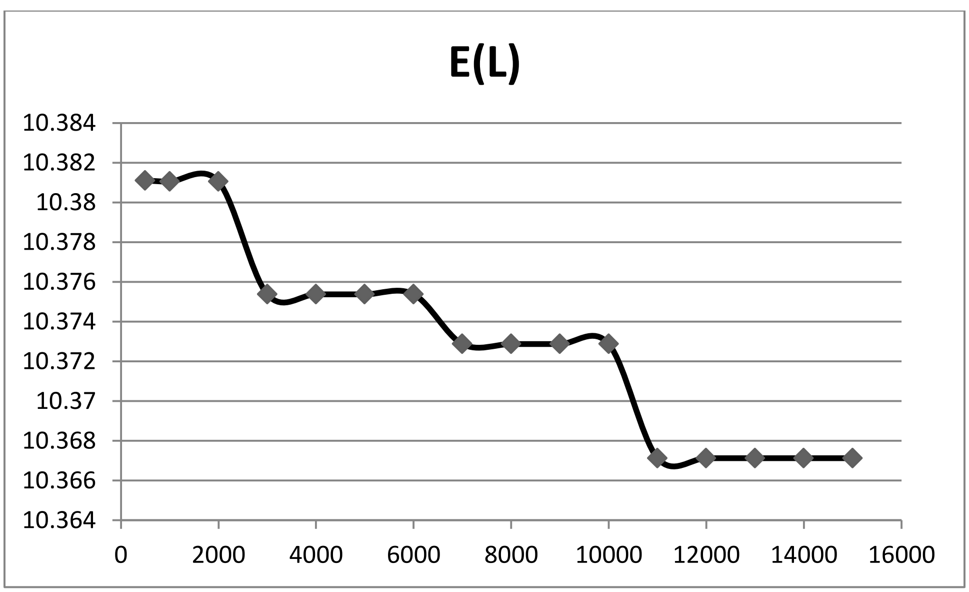
Graph 5.3 E(L) vs No. of iterations

Consider the following values: m = 50 ; ρ = 0.35 ; P = 0.4 ; q_0 = 0.45

The $E(L)_{optimum}$ obtained for number of iterations is given in the below table:

Table 5.5 $E(L)_{optimum}$ with number of iterations for m = 50 ; ρ = 0.35 ; P = 0.4 ; q_0 = 0.45

No. of Iterations	E(L)	n	h	k
500	10.381101	5	0.888	2.972
1000	10.381046	5	0.858	3.045
2000	10.381046	5	0.858	3.045
3000	10.375372	5	0.844	3.036
4000	10.375372	5	0.844	3.036
5000	10.375372	5	0.844	3.036
6000	10.375372	5	0.844	3.036
7000	10.372871	5	0.769	3
8000	10.372871	5	0.769	3
9000	10.372871	5	0.769	3
10000	10.372871	5	0.769	3
11000	10.367125	5	0.808	2.981
12000	10.367125	5	0.808	2.981
13000	10.367125	5	0.808	2.981
14000	10.367125	5	0.808	2.981
15000	10.367125	5	0.808	2.981



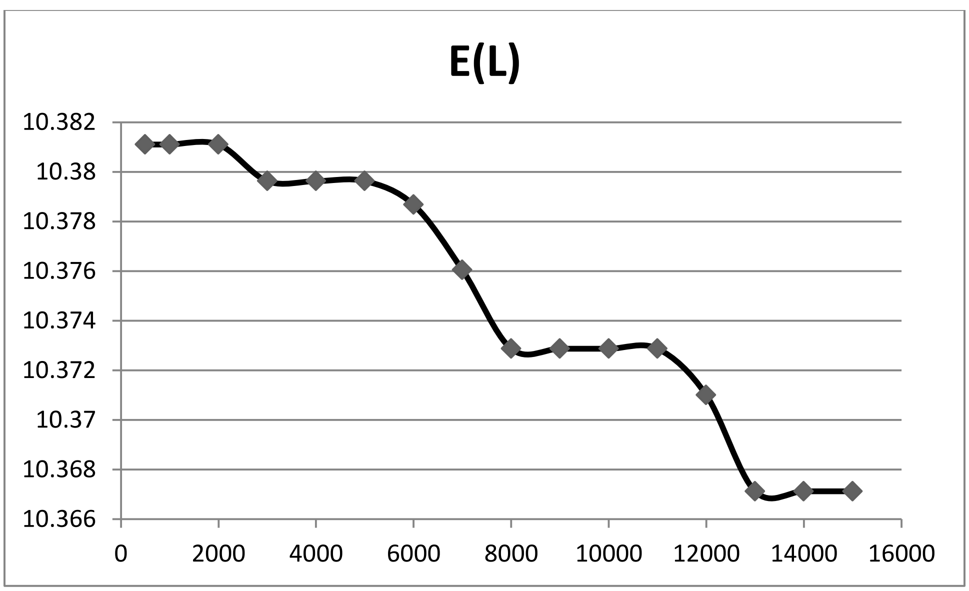
Graph 5.4 E(L) vs No. of iterations

Consider the following values: m = 50 ; ρ = 0.35 ; P = 0.5 ; q_0 = 0.55

The $E(L)_{optimum}$ obtained for number of iterations is given in the below table:

Table 5.6 $E(L)_{optimum}$ with number of iterations for m = 50 ; ρ = 0.35 ; P = 0.5 ; q_0 = 0.55

No. of Iteration	E(L)	n	h	k
500	10.381101	5	0.888	2.972
1000	10.381101	5	0.888	2.972
2000	10.381101	5	0.888	2.972
3000	10.379627	5	0.748	3.016
4000	10.379627	5	0.748	3.016
5000	10.379627	5	0.748	3.016
6000	10.378669	5	0.867	3.017
7000	10.376045	5	0.779	3.068
8000	10.372871	5	0.769	3
9000	10.372871	5	0.769	3
10000	10.372871	5	0.769	3
11000	10.372871	5	0.769	3
12000	10.370998	5	0.815	3.038
13000	10.367125	5	0.808	2.981
14000	10.367125	5	0.808	2.981
15000	10.367125	5	0.808	2.981



Graph 5.5 E(L) vs No. of iterations

It is evident from the above results that the ACO parameters have a significant effect on the number of iterations required for the convergence.

Table 5.7 No. of iterations required to converge for a selected parameter values

Loss Cost	m (Initial number of ants)	ρ (evaporation constant)	P (Pheromone deposit factor)	q_0 (Probability thet the ant finds a new path)	No. of iterations required to converge
10.367125	50	0.25	0.4	0.4	11000
10.367125	50	0.25	0.5	0.5	12000
10.367125	50	0.25	0.6	0.6	14000
10.367125	50	0.35	0.4	0.45	11000
10.367125	50	0.35	0.5	0.55	13000

However, when the number of iterations is sufficiently large, the effect of ACO parameters i.e. initial number of ants m, pheromone deposit factor P, evaporation constant ρ, and the probability of finding a new path q_0 will be negligible.

For 15000 iterations, the algorithm converges to E(L) = 10.367125

The corresponding sample size n = 5 ; sample frequency h = 0.808 ; control limits width k = 2.98

Furthermore, the results are compared to the results obtained by Montgomery [17] and Shiau [23]. The result obtained by Ant Colony Algorithm is found to be superior to that obtained by Montgomery. The result is however on par with that obtained by Shiau.

Chapter 6

Conclusion and Recommendation

The minimum value of the loss cost function was found to be 10.367125. The corresponding *sample size* value is *5*.The corresponding *sampling frequency* is *0.808* hours. The corresponding *control limit width* value is *2.98*.The *number of iterations* required is *15000*.

It can be concluded that Ant Colony Algorithm is a good optimization technique to be used for the optimization of economic models of X-bar control chart.

Scope of future work

According to Montgomery [17], the statistical relationship between the response and the variables is unknown and must be estimated. For the tuning of ACO parameters, RSM (Response Surface Methodology) should be used. Response surface methodology is a mathematical-statistical procedure in finding optimal values of some random factors that have significant effects on a response. So RSM can be used in order to find out the best value for the ACO parameters.

References

1. Abbas, A.T., Aly, M.F., Hamza, K., International Journal of Production Research, Volume 49(19), (2011), Page 5877–5891.
2. Aelterman, J., Goossens, R., Declercq, F., Rogier, H., Ant Colony Optimization-based radiation pattern manipulation algorithm for Electronically Steerable Array Radiator Antennas, IET Science, Measurement and Technology, Volume 3(4),(2009), Page 302 – 311.
3. Cai, D.Q., Xie, M., Goh, T.N., Tang, X.Y., Economic design of control chart for trended processes, International Journal of Production Economics, Volume 79, (2002), Page 85 -92.
4. Chen, H., Cheng, Y., Non-normality effects on the economic–statistical design of X charts with Weibull in-control time, European Journal of Operational Research, Volume 176, (2007), Page 986–998.
5. Chen, W.S., Yu, F.J., Guh, R.S., Lin, Y.H., Economic design of x-bar control charts under preventive maintenance and Taguchi loss functions, Journal of Applied Operational Research, Volume 3(2), (2011), Page 103–109.
6. Chen, Y.S., Yang, Y.M., An extension of Banerjee and Rahim's model for economic design of moving average control chart for a continuous flow process, European Journal of Operational Research, Volume 143, (2002), Page 600–610.
7. Chen, Y.S., Yang, Y.M., Economic design of x-bar control charts with Weibull in-control times when there are multiple assignable causes, International Journal of Production Economics, Volume 77, (2002), Page 17–23.
8. Chih, M., Yeh, L.L., Li, F.C., Particle swarm optimization for the economic and economic statistical designs of the X-bar control chart, Applied Soft Computing , Volume 11, (2011), Page 5053–5067.
9. Chou, C.Y., Chen, C.H., Liu H.R., Huang X.R., Economic-statistical design of multivariate control charts for monitoring the mean vector and covariance matrix, Journal of Loss Prevention in the Process Industries, Volume 16, (2003), Page 9–18.
10. Dorigo, M., and Stutzle, T., Ant Colony Optimization, MIT Press, Cambridge, 2004.
11. Dorigo, M., Blumb, C., Ant colony optimization theory: A survey, Theoretical Computer Science, Volume 344, (2005), Page 243–278.
12. Dorigo, M., Maniezzo, V., Colorni, A., 1996. Ant System: Optimization by a colony of cooperating agents, IEEE Transactions on Systems, Man, and Cybernetics – Part B 26(1), Page 29–41.
13. Duncan, A.J., The economic design of x-chart used to maintain current control of a process, Journal of the American Statistical Association, Volume 51, (1956), Page 228-242.

14. Lin, S.N., Chou, C.Y., Wang, S.L., Liu, H.R., Economic design of autoregressive moving average control chart using genetic algorithms, Expert Systems with Applications, Volume 39, (2012), Page 1793–1798.
15. Meng, Z., Zou, B., Zeng, Y., Considering direct interaction of artificial ant colony foraging simulation and animation, Journal of Experimental & Theoretical Artificial Intelligence, (2011), Page 1–13, iFirsts.
16. Middendorf, M., Reischle, F., Schmeck, H., Multi Colony Ant Algorithms, Journal of Heuristics, Volume 8, (2002), Page 305–320.
17. Montgomery, D.C., Introduction to Statistical Quality Control, 5th edition, New York, John Wiley & Sons, 2005.
18. Niaki, S.T.A., Ershadi, M.J., A hybrid ant colony, Markov chain, and experimental design approach for statistically constrained economic design of MEWMA control charts, Expert Systems with Applications, Volume 39, (2012), Page 3265–3275.
19. Niaki, S.T.A., Malaki, M., Ershadi, M.J., A particle swarm optimization approach on economic and economic-statistical designs of MEWMA control charts, Scientia Iranica E, Volume 18(6), (2011), Page 1529–1536.
20. Prabhu, S.S., Montgomery, D.C., Runger, G.C., Economic-statistical design of an adaptive X-bar chart, International Journal of Production Economics, Volume 49, (1997), Page l-l5.
21. Seo, M., Kim, D., Ant colony optimisation with parameterised search space for the job shop scheduling problem, International Journal of Production Research Volume 48(4), (2010), Page 1143–1154.
22. Serel, D.A., Moskowitz, H., Joint economic design of EWMA control charts for mean and variance, European Journal of Operational Research, Volume 184, (2008), Page 157–168.
23. Shiau, Y.R., Lin, M.H., Chung, I.P., Pan, Y.L., Monitoring capability study and genetic economic design of X-bar control chart, Int Journal of Advanced Manufacturing Technology, Volume 30, (2006), Page 273–278.
24. Socha, K., Dorigo, M., Ant colony optimization for continuous domains, European Journal of Operational Research, Volume 185, (2008), Page 1155–1173.
25. Vijaya V.B., Murty S.S.N., A simple approach for robust economic design of control charts, Computers & Operations Research, Volume 34, (2007), Page 2001 – 2009.
26. Zhu, Q., Zhang, J., Ant colony optimisation with elitist ant for sequencing problem in a mixed model assembly line, International Journal of Production Research, Volume 49(15), (2011), Page 4605–4626.