MODERN METHODS FOR POWER SYSTEM HARMONICS ESTIMATION

(STUDY OF LEAST MEAN SQUARE FILTER, RECURSIVE LEAST SQUARES FILTER, KALMAN FILTER AND EXTENDED KALMAN FILTER)

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

Bachelor in Technology

In

Electrical Engineering

By

ABHIJIT PRADHAN

Roll No: 107EE018

BIBHU PRASAD PANIGRAHI

Roll No: 107EE043

Under the guidance of

PROF. SANKARSAN RAUTA

Department of Electrical Engineering

National Institute of Technology

Rourkela

2011

NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA

May, 2010

National Institute of Technology

Rourkela

CERTIFICATE

This is to certify that the thesis entitled, "**MODERN METHODS FOR POWER SYSTEM HARMONICS ESTIMATION (Study of Least Mean Squares (LMS) Filter, Recursive Least Squares (RLS) Filter, Kalman and Extended Kalman Filter)**" submitted by Abhijit Pradhan and Bibhu Prasad Panigrahi in partial fulfillment of the requirements for the award of Bachelor of Technology Degree in Electrical Engineering in the National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by them under my supervision and guidance.

To the best of my knowledge, the matter embodied in this thesis has not been submitted to any other university /institute for the award of any Degree or Diploma.

Date:

Prof. Sankarsan Rauta

Dept. of Electrical Engineering

National Institute of Technology, Rourkela

Rourkela-769008

ACKNOWLEDGEMENT

It is a matter of great satisfaction and dignity for us to present our project undertaken during the seventh and eighth semesters, for partial fulfillment of our Bachelor of Technology degree at National Institute of Technology, Rourkela.

We would like to thank the afore-mentioned institute for giving us the opportunity to use their resources and work in such a challenging environment.

First and foremost, we would like to express our deepest gratitude to our guide **Prof. Sankarsan Rauta** for his able guidance during our project work. This project could never have been possible without his supervision and much needed help amidst his busy schedule.

We are also grateful to **Prof. B. D. Subudhi** of our Dept., whose lectures in the subject of State Space modeling gave us the concept of necessary simulations to be obtained in the project.

We would also like to thank our friends and seniors who have always encouraged and supported us during our project work.

Last but not the least we would like to thank all the staff members of Electrical Engineering Dept. who have been extremely co-operative with us.

Abhijit Pradhan — Roll No: 107EE018

Bibhu P. Panigrahi — Roll No:107EE043

Date:

Place: Rourkela

LIST OF ABBREVIATIONS

Abbreviation	Full Form
ASD	Adjustable Speed Drives
CBEMA	Computer and Business Equipment Manufacturer's Association
DFT	Discrete Fourier Transform
EKF	Extended Kalman Filter
FFT	Fast Fourier Transform
GRV	Gaussian Random Variable
Hz	Hertz
KF	Kalman Filter
KVA	Kilo Volt Ampere
LMS	Least Mean Square
MMSE	Minimum Mean-Squared Error
MSE	Mean Squared Error
p.u.	Per unit
RMS	Root Mean Square
THD	Total Harmonic Distortion
THDF	Transformer Harmonic De-rating Factor

LIST OF FIGURES AND TABLES

CONTENTS

ABSTRACT

Harmonics has been present for a long time and its presence shapes the performance of a power system. Therefore, estimation of harmonics is of paramount importance while analyzing a power system network. Following the inception of harmonics, various filters have been devised to achieve an optimal control strategy for harmonic alleviation. This thesis introduces various algorithms to analyze harmonics in the power system. The objective is to estimate the power system voltage magnitude in the presence distortions taking into account the noise by employing different estimation approaches. We have focused our attention towards the study of Least Mean Squares (LMS) based filter, Recursive Least squares (RLS) based filter, Kalman filter (KF) and Extended Kalman (EKF) filter. For a test signal LMS, RLS, KF and EKF based algorithms have been analyzed and results have been compared. The proposed estimation approaches are tested for only static signals.

CHAPTER 1

INTRODUCTION

1.1 HARMONICS

Every load today is capable of creating harmonics with the exception of the incandescent light bulb. But, the harmonic content varies from load to load and each load responds to the effects of harmonics.[5]

Harmonics became a buzzword in the early 1980s, making many people reassessed the effectiveness of their building's wiring system. Yet, many still view the concept as a relatively new occurrence. However, harmonics have been there since well before the early '80s.The associated problems existed in the electrical system way back when transistor tubes were first used in the 1930s. Aside from grounding, many consider harmonics as one of the biggest concerns for the power quality industry today. In this chapter, we'll discourse the fundamentals of harmonics and the problems it can cause in a power system.[5]

1.2 What are Harmonics

We define harmonics as voltages or currents at frequencies that are multiples of the fundamental frequency. In most systems, the fundamental frequency is 50 Hz. Therefore, harmonic order is 100 Hz, 150 Hz, and 200 Hz and so on.[5]

We usually specify these orders by their harmonic number or multiple of the fundamental frequency. For example, a harmonic with a frequency of 150 Hz is known as the third harmonic (50x3 = 150). In this case, for each cycle of the fundamental waveform, there are three complete cycles of the harmonic waveforms. The even multiples of the fundamental frequency are called as even-order harmonics while the odd multiples are called as the odd-order harmonics.[5]

1.3 Creation of Harmonics

Up until 1980, all loads were considered to be linear. This means if the voltage input to a device is a sinusoidal wave, the resultant voltage waveform generated by the load is also a sinusoidal wave.[5]

In 1981, manufacturers of electronic hardware switched to an efficient type of internal power supply known as a switch-mode power supply (SMPS). The SMPS converts the applied voltage sine wave to a distorted current waveform that resembles alternating current pulses, the original since the load no more exhibit constant impedance throughout the applied AC voltage waveform.[5]

Most electrical equipment today creates harmonics. If a device converts AC power to DC power (or vice versa) as part of its steady-state operation, it is considered to be a harmonic current-generating device. Such devices include uninterruptible power supplies, copiers, PCs, etc.[5]

1.4 Effects of Harmonics

The biggest problem with harmonics is voltage waveform distortion. We can calculate a relationship between the fundamental and distorted waveforms by finding the square root of the sum of the squares of all harmonics generated by a single load, and then dividing this number by the nominal 50 Hz waveform value. We do this by a mathematical calculation known as a **Fast Fourier Transform (FFT) Theorem**. This calculation method determines the Total Harmonic Distortion (THD) contained within a nonlinear current or voltage waveform.[5]

1.4.1 Triplen harmonics:

Electronic equipment generates more than one harmonic frequency. For example, computers generate 3rd, 9th, and 15th harmonics. These are known as triplen harmonics. They pose a bigger problem to engineers and building designers because they do more than distort voltage waveforms. They can overheat the building wiring, cause nuisance tripping, overheat transformer units, and cause random end-user equipment failure.[5]

1.4.2 Circuit overloading:

Harmonics cause overloading of conductors and transformers and overheating of utilization equipment, such as motors. Triplen harmonics can especially cause overheating of neutral conductors on 3-phase, 4-wire systems. While the fundamental frequency and even harmonics cancel out in the neutral conductor, odd-order harmonics are additive. Even under balanced load conditions, neutral currents can reach magnitudes as high as 1.73 times the average phase current.[5]

This additional loading creates more heat, which breaks down the insulation of the neutral conductor. In certain cases, it breaks down the insulation between windings of a transformer. In both cases, the result is a fire hazard. But, one can reduce this potential damage by using sound wiring practices.[5]

To be on the safe side, more engineers are doubling the size of the neutral conductor from feeder circuits to panel boards and branch circuit partition wiring to handle the additive harmonic currents.[5]

1.5 Methods of Elimination of Harmonics

Four of the most recommended solutions include:

- **Increasing the size of the neutral conductor**
- **Decreasing the load of delta-wye transformer**
- **Replacing the delta-wye transformer with a k-factor transformer**
- **Installing a harmonic filter at the power source or equipment location.**

The first three solutions help us cope with the problem; the fourth actually eliminates the problem.[5]

1.5.1 Neutral conductor sizing:

We know that harmonic currents affect the neutral conductor. Since these currents don't cancel out in a balanced 3-phase system, the neutral carries more current than you anticipate. When this happens, the neutral conductor path overheats.[5]

This is why we should double the size of the neutral conductor for feeders and branch circuits serving nonlinear loads. Office partition manufacturers have design requirements in place for doubling the neutral conductor in their partition wiring. Increased neutral currents sometimes cause many electrical fires within office partitions. Some OEM partition wiring schemes include a separate neutral per phase conductor while others use a shared neutral doubled in size.[5]

1.5.2 Transformer loading:

Transformers are more efficient when supplying linear loads. But, the majority feed nonlinear equipment, generating more copper losses in dry-type transformers than the fundamental current. These losses are associated with eddy currents and hysteresis in the core and skin effect losses in windings. The result is transformer overheating and winding insulation breakdown.[5]

1.5.3 K-factor transformers:

These transformers differ in construction from standard dry-type transformers. They can handle harmonic currents at near capacity without having to be de-rated. Construction features include:

- Electrostatic shield between the primary and secondary windings of each coil.
- Neutral conductor lug size that's twice that of the phase conductor lugs.
- Parallel smaller windings on the secondary to negate skin effect.
- Transposition of primary delta winding conductors (in large size units) to reduce losses.[5]

1.5.4 Harmonic filters: A harmonic filter can eliminate the potentially dangerous effects of harmonic currents created by nonlinear loads. It traps these currents and, through the use of a series of capacitors, coils, and resistors, shunts them to ground. A filter unit may contain several of these elements, each designed to filter a particular frequency.[5]

We can install filters either between the device we are trying to protect and the load's power source, or between the device causing the condition and its power source.[5]

There are two types of harmonic filters:

- Passive Filter
- Active Filter

Passive filters: These are inexpensive compared with most mitigating devices. Internally, they cause the harmonic current to resonate at its frequency. This prevents the harmonic current from flowing back to the power source and causing problems with the voltage waveform. A disadvantage of the passive filter is that it cannot be perfectly tuned to handle the harmonic current at a significant frequency.[5]

Active filters: These filters, on the other hand, can be tuned to the exact frequency of the harmonic current and do not cause resonance in the electrical system. They can also address more than one harmonic problem at the same time. Active filters can also provide mitigation for other power quality problems such as voltage sags and power line flicker. They use power electronics to replace part of the distorted current sine wave coming from the load, giving the appearance you're using only linear loads. As a result, the active filter provides power factor correction, which increases the efficiency of the load.[5]

1.6 Determination of presence of harmonic currents

If non-linear loads are a significant part of the total load in the facility (>20%), there is a chance of harmonics problem. So the amount of current produced by non-linear loads is calculated by calculating the THD.[6][24]

1. Electronic Ballasts come with current THD ranging from 60% to 100%. It is absolutely necessary to avoid electronic ballasts with more than 20% THD. Near to 100% THD is produced by PWM ASDs. This can be easily brought down to less than half by installing cheap 3% impedance line side reactors (chokes).[6][24]
2. A measurement of current in the neutral of a 3-phase 4-wire system gives us knowledge about the presence of harmonics. If neutral current is found substantially higher than the imbalance from the imbalance in the 3-phase currents, it can be safely assumed that there are harmonics present in the system.[6][24]
3. Another very important sign of presence of current harmonics in the system include inexplicable higher than normal temperatures in the transformer, voltage distortion and high crest factor.[6][24]

1.7 Crest Factor

Crest Factor of any waveform is defined as the ratio of the peak value to the RMS value. For a perfect sinusoidal wave it is equal to 1.414. Crest factors other than 1.414 indicate a distortion in the waveform. Typically distorted current waveforms have crest factor higher than 1.414 and distorted voltage waveforms have crest factor lower than 1.414.Distorted waveforms with crest factor lower than 1.414 lower than 1.414 are known as “Flat Top” voltage waveforms. The Computer and Business Equipment Manufacturers Association (CBEMA) recommend a method for de-rating transformers based on the current crest factor. CBEMA defines the transformer harmonic de-rating factor (THDF) as the ratio of 1.414 to the transformer current crest factor. The de-rated KVA of the transformer would be the nominal KVA of the transformer multiplied by the THDF. This method is however applied when the distortion caused in the current is caused by single phase non-linear loads.[6][24]

1.8 Measurement of distorted waveform

- A digital oscilloscope is required to measure the wave shape, THD and amplitude of each harmonic.[6][24]
- If only RMS value of the waveform is required, a "True RMS" multi-meter is enough. But this instrument is not used because all instruments do not give correct readings when measuring distorted waveforms.[6][24]
- The majority of low cost portable instruments are "average responding RMS calibrated" .These instruments give correct readings for distortion free sine waves and would probably read low when the current waveform is distorted.[6][24]

1.9 Characteristic and Non-Characteristic Harmonics

The characteristic harmonics are harmonics of those orders which are always present under ideal operations- balanced AC voltages, symmetric three phase network and equidistant pulses In the AC/DC converter the DC current is assumed to be constant. In this case there are harmonics in the AC current of the order $h=np+1$, where p= pulse number, n is any integer. The harmonics in the converter of DC voltage are of the order $h= np$. The harmonics of the order other than characteristic harmonics are termed as non-characteristic. These are due to [7][24]

- Imbalance in the operation of two bridges forming a 12 pulse converter.
- Firing angle errors.
- Imbalance and distortion in AC voltages.
- Unequal transformer leakage impedances.[7][24]

Filters can be designed to eliminate characteristic harmonics but the analysis and hence the elimination of non-characteristic harmonics is extremely difficult. So, it is necessary to take adequate precautions so that non-characteristic harmonics are not generated.

1.10 Estimation of Harmonics

In order to provide the customers and electrical utilities a quality of power, it is imperative to know the harmonics parameters such as magnitude and phase. This is essential for designing the filter for eliminating or reducing the effects of harmonics in the power system.[8][24]

Many algorithms have been proposed for the evaluation of harmonics. To obtain the voltage and current frequency spectrum from discrete time samples, most frequency domain harmonic analysis algorithms are based on the Discrete Fourier Transform (DFT) or on the Fast Fourier Transform (FFT) [8] method. However these two methods suffer three pitfalls, namely, aliasing, leakage and picket fence effect. [9], [10] and [11]. Although other methods, including the proposed algorithm in this paper , suffer from these three problems, and this is because of existing high frequency components measured in the signal [9], however truncation of the sequence of sampled data , when only a fraction of the sequence of a cycle exists in the analyzed waveform, can boost leakage problem of the DFT method. So, the need of new algorithms that process the data, sample-by-sample, and not in a window as in FFT and DFT, is of paramount importance. [8]. One of the methods is that Kalman Filter. A more robust algorithm for estimating the magnitudes of sinusoids of known frequency embedded in an unknown measurement noise which can be a measure of both stochastic and signals was introduced by Dash and Sharaf [12]. But this algorithm is not able to track the abrupt and dynamic changes of signals and its harmonics.[8][24]

In this paper we have conducted a comprehensive study of the Kalman Filter, Extended Kalman Filter, Least Mean Squares (LMS) based filter and Recursive Least Squares (RLS) based filter. For simulations using MATLAB, a static power system signal is used .The characteristics and the algorithms of the afore-mentioned filters are thoroughly studied and the results are compared.

CHAPTER 2

FILTERS

2.1-Least Mean Square Filter:

The least-mean-square (LMS) algorithm is a linear adaptive filtering algorithm that consists of two basic processes: [4]

1. A filtering process which involves (a) computing the output of transversal filter produced by a set of tap inputs, and (b) generating an estimation error by comparing this output to desired response.[4]
2. An adaptive process which involves the automatic adjustment of the tap weights of the filter in accordance with the estimation error.[4]

Thus, the combination of these two processes working together constitutes a feedback loop around the LMS algorithm. First we have a transversal filter around which LMS algorithm is built: this component is responsible for performing the filtering process. Second we have a mechanism for performing adaptive control process on the tap weights of the transversal filter, hence the designation "adaptive weight-control mechanism". The tap inputs u(n), u(n-1), …., u(n-M+1) form the elements of M-by-1 tap input vector u(n), M-1 is the number of delay elements; these tap inputs span multidimensional space denoted by U_n. Correspondingly, the tap weights $\hat{w}_0(n)$, $\hat{w}_1(n)$,…..$\hat{w}_{M-1}(n)$ form the elements of M-by-1 tap weight vector $\hat{w}(n)$.[4]

During filtering process the desired response is supplied for processing, alongside the tap input vector u(n). Given this input the transversal filter produces an output $\partial(n/U_n)$ used an estimate of the desired response d(n). We also define the estimation error as the difference between e(n) as the difference between the desired response and actual filter output. The estimation error e(n) and the tap-input vector are applied to the control mechanism, the feedback loop around the tap weights is thereby closed. A scalar version of inner product of estimation error and tap input u (n-k) is computed for k = 1, 2, 3…..,M-2, M-1. The result defines the correction $\delta\hat{w}(n)$ applied to weight $\hat{w}(n)$ at n+1 iteration. The scaling factor used here is denoted by μ. It is called the step-size parameter. The LMS algorithm uses the product to u (n-k) e*(k) as an estimate of element k in the gradient vector $\nabla J(n)$ that characterizes the method of steepest descent. Accordingly the computation of each tap weight in the LMS algorithm suffers from gradient noise.[4]

The LMS algorithm involves feedback in its operation, which therefore raises the related issue of stability. In this context, a meaningful criterion is to require that [4]

$$J(n) \longrightarrow J(\infty) \qquad as\ n \longrightarrow \infty$$

where J(n) is the mean-squared error produced by the LMS algorithm at time n and its final value J(∞) is a constant. For LMS algorithm to satisfy this criterion, the step-size parameter μ has to satisfy certain conditions related to the Eigen structure of the correlation matrix of the tap inputs.[4]

<u>2.1.1Least Mean Square Algorithm</u>:

To develop an estimate of the gradient vector$\nabla J(n)$, the strategy is to substitute the estimates of correlation matrix R and the cross correlation vector p [4]

$$\nabla J(n) = -2p + 2Rw(n) \tag{2.1}$$

The simplest choice of estimators for R and p is to use instantaneous estimates that are based on sample values of the tap-input vector and desired response, as defined by, respectively [4]

$$\hat{R}(n) = u(n)u^{H}(n) \tag{2.2}$$

And

$$\hat{p}(n) = u(n)d^{*}(n) \tag{2.3}$$

Correspondingly, the instantaneous estimate of the gradient vector is [4]

$$\hat{\nabla}J(n) = -2u(n)d^{*}(n) + 2u(n)u^{H}(n)\,\hat{w}(n) \tag{2.4}$$

This estimate is biased because the tap weight estimator ŵ(n) is a random vector that depends upon tap-input vector u(n). Substituting the estimate for the gradient vector ∇J(n) in the steepest descent algorithm, we get the recursive relation for updating tap-weight vector: [4]

$$\hat{w}(n+1) = \hat{w}(n) + \mu\, u(n)[d^{*}(n) - u^{H}(n)\hat{w}(n)] \tag{2.5}$$

The result can be written in the form of three basic relations:

1. **Filter Output**: $y(n) = \hat{w}^H(n)\, u(n)$ (2.6)

2. **Estimation Error** : $e(n) = y(n) - d(n)$ (2.7)

3. **Tap Weight Adaptation :** $\hat{w}(n+1) = \hat{w}(n) + \mu\, u(n)\, e^*(n)$ (2.8)

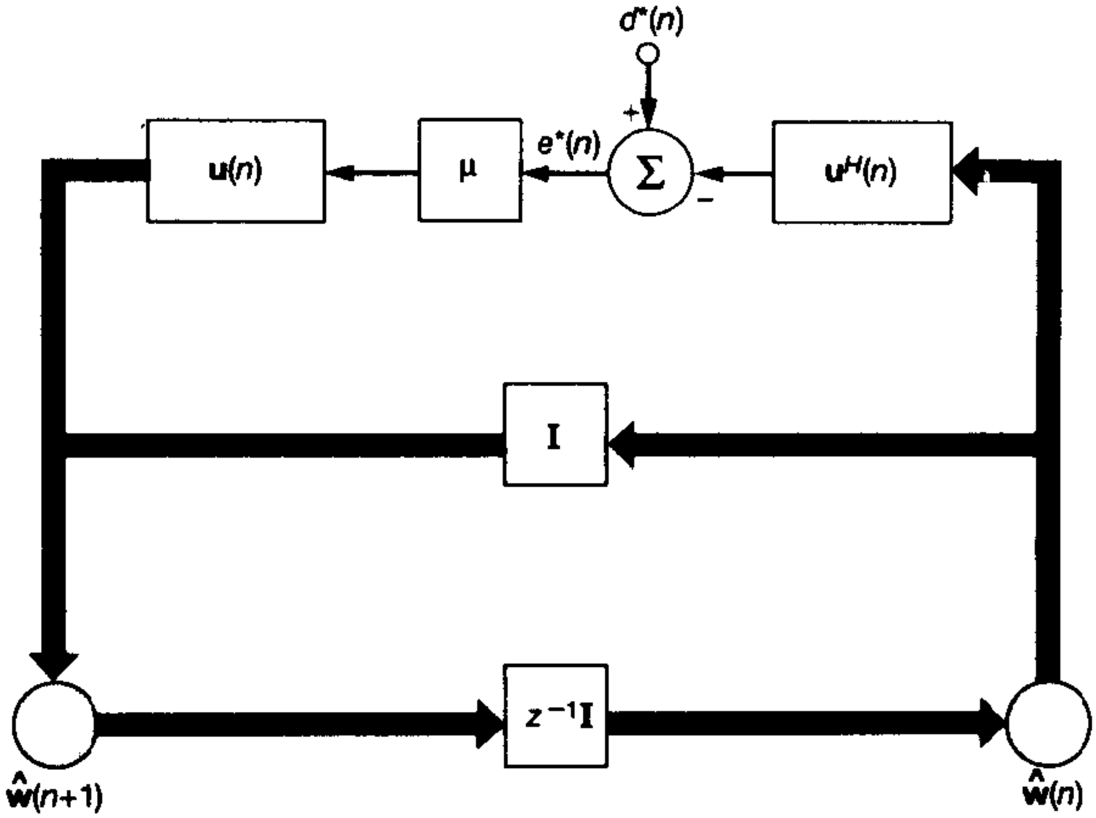


Fig 2.1: Signal-flow Graph Representation of the LMS Algorithm

This above signal-flow graph has been reproduced from [4]

The estimation vector error e (n) is based on the current estimate of the tap -weight vector, ŵ(n). The second term μu(n)e*(n) on the right hand-side of (2.8) represents the correction that is applied to the current estimate of the tap -weight vector, ŵ(n). The iterative procedure is started with an initial guess ŵ(0).[4]

The algorithm described by (2.6) to (2.8) is the complex form of adaptive least-mean-square (LMS) algorithm. At each iteration or time update , it also requires the knowledge of most recent values : u(n), d(n) and $\hat{w}(n)$. The LMS algorithm is the member of family of *stochastic-gradient-algorithms*. When the LMS algorithm operates on stochastic inputs, the allowed set of directions along which we "step" from one iteration cycle to the next is random and cannot therefore be thought of as being true gradient directions.[4]

The LMS algorithm requires only 2M + 1 complex multiplications and 2M complex additions per iteration, where M is the number of tap weights used in the adaptive transversal filter. In other words, the computational complexity of the LMS algorithm is O(M).[4]

2.2-Recursive Least Squares Filter:

An important feature of recursive least squares(RLS) algorithm is that it utilizes the information contained in the input data, extending back to the instant of time when the algorithm is initiated. The resulting rate of convergence is therefore typically an order of magnitude faster than the simple LMS algorithm. This improvement in performance is achieved at the expense of large increase in computational complexity.[4]

In the recursive implementations of the method of least squares, we start the computation with known initial conditions and use the information contained in new data samples to update the old estimates. So it is found that the length of the observable data is variable. Accordingly, we express cost function to be minimized as $\mathcal{E}(n)$, where n is the length of the variable data. Also, we are introducing weighting factor into the definition of cost function $\mathcal{E}(n)$. We thus write [4]

$$\mathcal{E}(n) = \sum_{i=1}^{n} \beta(n,i) \, |e(i)|^2 \qquad (2.9)$$

where e(i) is the difference between the desired response d(i) and the output y(i) produced by a transversal filter whose tap inputs (at time i) equal u(i), u(i-1),……u(i-M+1). That is, e(i) is defined by[4]

$$\begin{aligned} e(i) &= d(i) - y(i) \qquad (2.10) \\ &= d(i) - w^H(n)u(i) \end{aligned}$$

Where u(n) is the tap input vector at time n, is defined by [4]

$$u(n) = [u(i), u(i\text{-}1),......,u(i\text{-}M+1)]^T \qquad (2.11)$$

where w(n)is the tap weight vector at time n, is defined by [4]

$$w(n) = [w_0(n), w_1(n),.....w_{M-1}(n)] \qquad (2.12)$$

The tap weights of the transversal filter remain fixed during the observation interval 1≤ I ≤ n for which the cost function б(n) is defined.[4]

The weighting factor $\beta(n,i)$ in (2.9) has the property that

$$0 < \beta(n,i) \leq 1 \quad i = 1, 2,......,n \qquad (2.13)$$

A special form of weighting that is commonly used is the exponential weighting factor or forgetting factor defined by [4]

$$\beta(n,i) = \lambda^{n-i} \quad i = 1, 2,.....,n \qquad (2.14)$$

λ is a positive constant with value close to, but less than 1. When λ is 1, we have the ordinary method of least squares. The inverse of 1 – λ is a measure of the memory of the algorithm. The λ = 1 case, corresponds to infinite memory. Thus in the method of exponentially weighted least squares, we minimize the cost function [4]

$$б(n) = \sum_{i=1}^{n} \lambda^{n-1} \, |e(i)|^2 \qquad (2.15)$$

The optimum value of the tap weight vector, ŵ(n) for which the cost function attains its minimum value is defined by the normal equations written in matrix form: [4]

$$\phi(n)\hat{w}(n) = z(n) \qquad (2.16)$$

The M-by-M correlation matrix ϕ(n) is now defined by

$$\phi(n) = \sum_{i=1}^{n} \lambda^{n-1} \, u(i)u^H(i) \qquad (2.17)$$

The M-by-1 cross-correlation vector z(n) between the tap inputs of the transversal filter and the desired reponse is defined by [4]

$$z(n) = \sum_{i=1}^{n} \lambda^{n-1} \ u(i)d^*(i) \tag{2.18}$$

Isolating the term corresponding to i = n from the rest of the summation on the right hand side of (2.17), we may write [4]

$$\phi(n) = \lambda[\sum_{i=1}^{n-1} \lambda^{n-1-i} \ u(i)u^H(i)] + u(n)u^H(n) \tag{2.19}$$

The recursion for updating the value of correlation matrix of the tap inputs: [4]

$$\phi(n) = \lambda \ \phi(n\text{-}1) + u(n)u^H(n) \tag{2.20}$$

where ϕ(n-1) is the old value of correlation matrix, and the matrix product $u(n)u^H(n)$ plays the role of "correction" term in the updating operation.[4]

We may use (2.18) to derive the following recursion for updating the cross-correlation vector between the tap inputs and the desired response: [4]

$$z(n) = \lambda \ z(n\text{-}1) + u(n)d^*(n) \tag{2.21}$$

<u>2.1.2Recursive Least Square Algorithm</u>:

With the correlation matrix ϕ(n) assumed to be positive definite and therefore nonsingular, we may apply the matrix inversion lemma to the recursive equation (2.20). We first make the following identifications: [4]

$$A = \phi(n)$$

$$B^{-1} = \lambda \ \phi(n\text{-}1)$$

$$C = u(n)$$

$$D = 1$$

Applying the matrix inversion lemma, we obtain the following recursive equation for the inverse of correlation matrix: [4]

$$\phi^{-1}(n) = \lambda^{-1}\phi^{-1}(n-1) - \frac{\lambda^{-2}\phi^{-1}(n-1)u(n)u^{H}(n)\phi^{-1}(n-1)}{1+\lambda^{-1}u^{H}(n)\phi^{-1}(n-1)u(n)} \tag{2.22}$$

For convenience of computation, let

$$P(n) = \phi^{-1}(n) \tag{2.23}$$

And

$$k(n) = \frac{\lambda^{-1}P(n-1)\,u(n)}{1+\lambda^{-1}u^{H}(n)P(n-1)u(n)} \tag{2.24}$$

$$P(n) = \lambda^{-1}P(n-1) - \lambda^{-1}k(n)u^{H}(n)P(n-1) \tag{2.25}$$

The M-by-M matrix P(n) is referred to as the inversion correlation matrix.[4]

We have

$$k(n) = \lambda^{-1}P(n-1)u(n) - \lambda^{-1}k(n)u^{H}(n)P(n-1)u(n)$$

$$= [\lambda^{-1} P(n-1) - \lambda^{-1}k(n)u^{H}(n)P(n-1)]u(n) \tag{2.26}$$

So, we get

$$k(n) = P(n)u(n) \tag{2.27}$$

or, $$k(n) = \phi^{-1}(n)u(n) \tag{2.28}$$

The gain vector k(n) is defined as the tap input vector u(n) transformed by the inverse of the correlation matrix ϕ(n).[4]

To develop recursive equation for developing the least squares estimate $\hat{w}$ (n) for the tap weight vector we use equation (2.21), (2.23) and (2.24) to express the least squares estimates $\hat{w}$(n) for the tap weight vector at iteration at n as follows: [4]

$$\hat{w}(n) = \phi^{-1}(n)\, z(n)$$

$$= P(n)\, z(n) \qquad (2.29)$$

$$= \lambda\, P(n)z(n\text{-}1) + P(n)u(n)\, d^*(n)$$

Substituting (2.25) for P(n) in the first term only in the right-hand side of (2.29) we get

$$\hat{w}(n) = \hat{w}(n\text{-}1) - k(n)u^H(n)\hat{w}(n\text{-}1) + P(n)u(n)d^*(n) \qquad (2.30)$$

Using P(n)u(n) equals the gain factor k(n), we get the desired recursive equation for updating the tap weight vector [4]

$$\hat{w}(n) = \hat{w}(n\text{-}1) + k(n)[d^*(n) - u^H(n)\hat{w}(n\text{-}1)] \qquad (2.31)$$

$$\hat{w}(n) = \hat{w}(n\text{-}1) + k(n)\xi^*(n)$$

where ξ(n) is the priori estimation error defined by

$$\xi(n) = d(n) - u^T(n)\hat{w}^*(n\text{-}1) \qquad (2.32)$$

$$= d(n) - \hat{w}^H(n\text{-}1)u(n)$$

The inner product $\hat{w}^H(n\text{-}1)u(n)$ represents the estimate of the desired response d(n), based on the old least squares estimate of the tap weight vector that is made at time n-1.[4]

The a priori estimation error ξ(n) is different from the posteriori estimation error [4]

$$e(n) = d(n) - \hat{w}^H(n)u(n) \qquad (2.33)$$

2.3 KALMAN FILTER:

The important feature of the kalman filtering is the recursive processing of the noise measurement data. In power system applications, kalman filter is used to estimate voltage and frequency variations. The kalman filtering has also been used for dynamic estimation of voltage and current phasors. This filtering technique is used to obtain the optimal estimate of the power system voltage magnitudes at different harmonic levels.[1] The Kalman filter is an estimator which is used to estimate the state of a linear dynamic system influenced by Gaussian White noise, using measurement that are linear functions of the system state, but corrupted by additive Gaussian white noise. The Kalman Filter allows to estimate the state of dynamic systems with certain types of random behavior by using these statistical information.[2]

2.3.1Estimation of a process:

The Kalman filter deals with the general problem of trying to estimate the state of a discrete-time controlled process that is governed by the linear stochastic difference equation [3]

$$X_k = Ax_{k-1} + Bu_{k-1} + w_{k-1} \tag{2.34}$$

With a measurement **z ϵ R^m** that is

$$Z_k = Hx_k + v_k \tag{2.35}$$

The random variables w_k and v_k represent the process and measurement noise and are assumed to be independent of each other. They are white noise with normal probability distributions.[3]

$$p(w) \sim N(0,R) \tag{2.36}$$

$$p(v) \sim N(0,Q) \tag{2.37}$$

With each time step or measurement the process noise covariance R and measurement noise covariance Q matrix may change. But, here we are assuming they are constant. The matrix n x n matrix A in the difference equation and the n x l matrix B refer to the state at the previous time step k-1 to the state at the current step k. Here, both A and H are assumed to be constant.[3]

2.3.2The computational origins of the filter:

We define $\hat{x}_k^-$ as our *a priori* state estimate at step k given knowledge of the process prior to step k, and $\hat{x}_k$ as our *a posteriori* state estimate at step k given measurement z_k. Then, we can write *a priori* and *a posteriori* estimate errors as[3]

$e_k^- = x_k - \hat{x}_k^-$, and

$$e_k = x_k - \hat{x}_k$$

Then, *a priori* estimate error covariance is

$$P_k^- = E[e_k^- e_k^{-T}] \qquad (2.38)$$

And the *a posteriori* estimate error covariance is

$$P_k = E[e_k e_k^T] \qquad (2.39)$$

Our goal is to find an equation that computes an *a posteriori* state estimate $\hat{x}_k$ as a linear combination of an *a priori* estimate $\hat{x}_k^-$ and a weighted difference between an actual measurement z_k and a measurement prediction $H\hat{x}_k$.[3]

$$\hat{x}_k = \hat{x}_k^- + K(z_k - H\hat{x}_k^-) \qquad (2.40)$$

The difference $(z_k - H\hat{x}_k^-)$ is called the measurement innovation, or the residual. The residual reflects the inconsistency between the predicted the measurement $H\hat{x}_k^-$ and the actual measurement z_k. If the residual is zero then, the two are in complete concurrence.[3]

The n x m matrix *K* in (2.40) is the gain or blending factor that minimizes the *a posteriori* error covariance (2.39). This minimization can be achieved by first substituting (2.40) into the above definition for e_k, substituting it into (2.39), performing the mentioned expectations, taking the derivative of the trace of the result w.r.t *K*, adjusting that result equal to zero, and then solving for K.[3]

The kalman gain calculated that minimizes P_k is given by

$$K_k = P_k^- H^T (H P_k^- H^T + R)^{-1}$$

$$= \frac{P_k^- H^T}{R + H P_k^- H^T} \qquad (2.41)$$

We can observe that as the measurement error covariance approaches zero, the gain K weights the residual more heavily. Specifically,[3]

$$lim_{R_k \to 0} K_k = H^{-1}$$

On the other hand, as the a priori estimate error covariance approaches zero, the gain K weights the residual less heavily. Specifically,[3]

$$lim_{P_k^- \to 0} K_k = 0$$

So, we can see that as the measurement error covariance R approaches zero, the actual measurement z_k is trusted more and more, while the predicted measurement $H\hat{x}_k^-$ is trusted less and less. On the other hand, as the a priori estimate error covariance P_k^- approaches zero the actual measurement z_k is trusted less and less, while the predicted measurement $H\hat{x}_k^-$ is trusted more and more.[3]

2.3.3Kalman Filtering Algorithm:

The kalman filter incorporates a form of feedback control by estimating the process state at some time and then obtaining the feedback in the form of noisy measurements. The kalman filter equations can be divided into two groups: *time update* equations and *measurement update* equations. The time update equations are accountable for extrapolating forward (in time) the current state and error covariance estimates to obtain the *a priori* estimates for the next time step. The measurement update equations are accountable for incorporating a new measurement into the *a priori* estimate to obtain an improved *a posteriori* estimate. The time update equations can also be considered as *predictor* equations, while the measurement update equations can be considered as *corrector* equations. So, the final estimation algorithm acts as a *predictor-corrector* algorithm for solving various numerical problems.[3]

The time update equations are:

$$\hat{x}_k^- = A\hat{x}_{k-1}^- + Bu_{k-1} \tag{2.42}$$

$$P_k^- = AP_{k-1}A^T + Q \tag{2.43}$$

The measurement update equations are:

$$K_k = P_k^- H^T (H P_k^- H^T + R)^{-1} \qquad (2.44)$$

$$\hat{x}_k = \hat{x}_k^- + K(z_k - H\hat{x}_k^-) \qquad (2.45)$$

$$P_k = (I - K_k H) P_k^- \qquad (2.46)$$

The first step in the measurement update is to compute the Kalman gain, K_k. The next step is to measure the process to obtain z_k, and then to generate an *a posteriori* state estimate by incorporating the measurement as in (2.45). The final step is to obtain an *a posteriori* error covariance estimate using (2.46). After each time and measurement update pair, the process is repeated with the previous *a posteriori* estimates used to predict the new *a priori* estimates.[3]

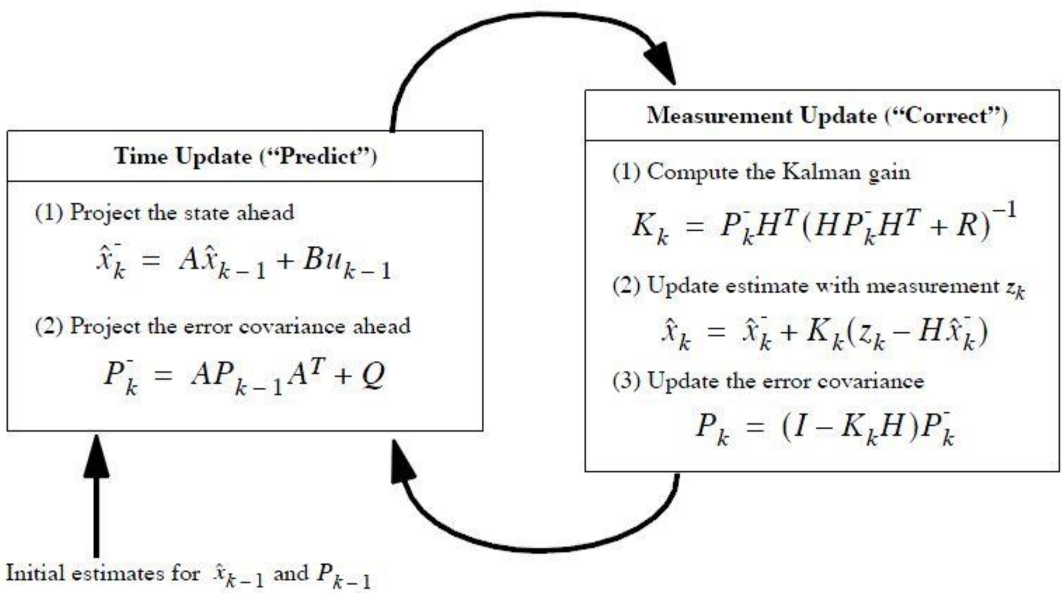


Fig 2.2: Kalman Filter Prediction Estimation Cycle

This figure has been reproduced from [3]

2.4Extended Kalman filter

In estimation theory, extended kalman filter (EKF) is the non-linear version of the kalman filter i.e., it can be used for state estimation in system that is non-linear. This non-linear filter linearizes about the current mean and variance.[23]

2.4.1The process to be estimated

The Kalman Filter addresses the general problem of trying to estimate the state $x \in R^n$ of a discrete time-controlled process that is governed by a *linear* stochastic difference equation. But some of the most interesting and successful applications of Kalman Filtering have been the ones when the process to be estimated and the measurement relationship to the process is non-linear. A Kalman Filter that linearizes about the current mean and covariance is known as an Extended Kalman Filter or EKF.[3]

In something similar to the Taylor Series, we can linearize the estimation around the current estimate using the partial derivatives of the process and the measurement functions to compute the estimates even in the case of non-linear relationships. This is done by modifying some of the material presented in the kalman filtering algorithm. Let us consider that the process has a state vector $x \in R^n$, but that process is now governed by the non-linear stochastic difference equation.[3]

$$x_k = f\left(x_{k-1}, u_{k-1}, w_{k-1}\right), \tag{2.47}$$

with a measurement $z \in R^m$ that is

$$z_k = h(x_k, v_k) \tag{2.48}$$

where the random variables w_k and v_k again represent the process and measurement noise as in (2.36) and (2.37). In this case the non-linear function f in the difference equation (2.47) relates the state at the previous time step k-1 to the current time step k. It includes parameters as a driving function u_{k-1} and the zero mean process noise w_k. The *non-linear* function h in the measurement equation (2.48) refers to the state x_k to the measurementz_k.[3]

Actually, one does not know the individual values of the noise w_k and v_k at each time step. However, one can approximate the state and measurement vector without considering them [3]

$$\tilde{x}_k = f(\hat{x}_{k-1}, u_{k-1}, 0) \tag{2.49}$$

$$\tilde{z}_k = h(\tilde{x}_k, 0), \tag{2.50}$$

Where $\tilde{x}_k$ is some *a posteriori* estimate of the state (from previous time step *k*).

One of the fundamental flaws of EKF is that the distributions (or densities in the continuous case) of the random variables remain no longer normal after undergoing respective non-linear transformations. The EKF is simply a specific state estimator that only approximates the optimality of Bayes' Rule by linearization.[3]

2.4.2The computational origins of the filter

To estimate a process with non-linear difference and measurement relationships, we begin by writing new governing equations that linearize an estimate about (2.49) and (2.50), [3]

$$x_k \approx \tilde{x}_k + A(x_{k-1} - \hat{x}_{k-1}) + W w_{k-1}, \tag{2.51}$$

$$z_k \approx \tilde{z}_k + H(x_k - \tilde{x}_k) + V v_k. \tag{2.52}$$

where,

- x_k and z_k are the actual state and measurement vectors,
- $\tilde{x}_k$ and $\tilde{z}_k$ are the approximate state and measurement vectors form (2.49) and (2.50),
- $\hat{x}_k$ is an *a posteriori* estimate of the state at step k
- The random variables w_k and v_k represent the process and measurement noise as in (2.36) and (2.37),
- A is the Jacobian matrix of partial derivatives of *f* with respect to *x*, that is ,

$$A_{[i,j]} = \frac{\partial f_{[i]}}{\partial x_{[j]}}(\hat{x}_{k-1}, u_{k-1}, 0),$$

- W is the Jacobian matrix of partial derivatives of *f* with respect to *w*, that is ,

$$W_{[i,j]} = \frac{\partial f_{[i]}}{\partial w_{[j]}}(\hat{x}_{k-1}, u_{k-1}, 0),$$

- H is the Jacobian matrix of partial derivatives of *h* with respect to *x*, i.e.,

$$H_{[i,j]} = \frac{\partial h_{[i]}}{\partial x_{[j]}}(\tilde{x}_k, 0)$$

- V is the Jacobian Matrix of partial derivatives of *h* with respect to *v*, i.e.,

$$V_{[i,j]} = \frac{\partial h_{[i]}}{\partial v_{[j]}}(\tilde{x}_k, 0)$$

It is to be noted that for simplicity in the notation we don't use the time step subscript k with the Jacobians *A*, *W*, *H* and *V* even though they are different in fact at each time step.[3]

Now we define a new notation for the prediction error,

$$\tilde{e}_{x_k} \equiv x_k - \tilde{x}_k \tag{2.53}$$

And the measurement residual,

$$\tilde{e}_{z_k} \equiv z_k - \tilde{z}_k \tag{2.54}$$

In practice, one does not have access to x_k in (2.53), it is the actual state vector, i.e., the quantity one is trying to estimate. On the other hand, one does have access to z_k in (2.54), it is the actual measurement that one is trying to estimate x_k . Using (2.53) and (2.54) we can write the governing equations for an *error process* as [3]

$$\tilde{e}_{x_k} \approx A(x_{k-1} - \hat{x}_{k-1}) + \varepsilon_k \tag{2.55}$$

$$\tilde{e}_{z_k} \approx H\tilde{e}_{x_k} + \eta_k \tag{2.56}$$

Where ε_k and η_k represent new independent random variables with zero mean and covariance matrices WQW^T and VRV^T, with *Q* and *R* as in (2.36) and (2.37) respectively.[3]

It is to be noted that the equations (2.55) and (2.56) are linear, and that they closely represent the difference and measurement equations (2.34) and (2.35) from the Kalman Filter. This motivates us to use the actual measurement residual $\tilde{e}_{z_k}$ in (2.54) and a second (hypothetical) Kalman Filter to estimate the prediction error $\tilde{e}_{x_k}$ given by (2.55). This estimate, call it $\hat{e}_k$, could then be used along with (2.53) to obtain a posteriori state estimates for the original non-linear process as [3]

$$\hat{x}_k = \tilde{x}_k + \hat{e}_k \tag{2.57}$$

The random variables of (2.55) and (2.56) have approximately the following probability distributions:

$$p\left(\tilde{e}_{x_k}\right) \sim N\left(0, E\left[\tilde{e}_{x_k}\tilde{e}_{x_k}^T\right]\right)$$

$$p(\varepsilon_k) \sim N(0, WQ_kW^T)$$

$$p(\eta_k) \sim N(0, VR_kV^T)$$

Given these approximations and letting the predicted value of $\hat{e}_k$ to be zero, the Kalman filter equation used to estimate $\hat{e}_k$ is:[3]

$$\hat{e}_k = K_k\tilde{e}_{z_k} \tag{2.58}$$

By substituting (2.58) back into (2.57) and making use of (2.54) we see that we do not actually need a second (hypothetical) Kalman Filter:[3]

$$\hat{x}_k = \tilde{x}_k + K_k\tilde{e}_{z_k} \tag{2.59}$$

$$= \tilde{x}_k + K_k(z_k - \tilde{z}_k)$$

Equation (2.59) can now be used for the measurement update in the Extended Kalman Filter, with $\tilde{x}_k$ and $\tilde{z}_k$ coming from (2.49) and (2.50) and the Kalman gain K_k coming from (2.44) with the appropriate substitution for the measurement error covariance.[3]

The complete set of equations for the EKF is shown below in the table 2-1 and table 2-2. Here $\hat{x}_k^-$ is substituted for $\tilde{x}_k$.[3]

Table 2-1 EKF Time Update Equations:

$$\hat{x}_k^- = f(\hat{x}_{k-1}^-, u_{k-1}, 0) \quad (2.60)$$

$$P_k^- = A_{k-1} P_{k-1} A_k^T + W_k Q_{k-1} W_k^T \quad (2.61)$$

As with the basic discrete Kalman Filter the time update equations in Table 2-1 project the state and covariance estimates from the previous time step *k-1* to the current time step k. Also, f in (2.60) comes from (2.49), A_k and W_k are the process Jacobians at the step k, and Q_k is the process noise covariance (2.36) at step *k*.[3]

Table 2-2 EKF Measurement Update Equations:

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + V_k R_k V_k^T)^{-1} \quad (2.62)$$

$$\hat{x}_k = \hat{x}_k^- + K_k (z_k - h(\hat{x}_k^-, 0)) \quad (2.63)$$

$$P_k = (I - K_k H_k) P_k^- \quad (2.64)$$

As with the basic discrete Kalman Filter, the measurement update equations in Table 2-2 correct the state and covariance estimates with the measurementz_k. Also, h in (2.63) comes from (2.50) H_k and V are the measurement Jacobians at step *k*, R_k is the measurement noise covariance (2.37) at step *k*.

The basic operation of the EKF is same as that of the linear discrete Kalman Filter. Figure below shows a complete picture of the operation of the EKF, combining the high-level diagram of Fig 2.1 with the equations from Tables 2-1 and 2-2.[3]

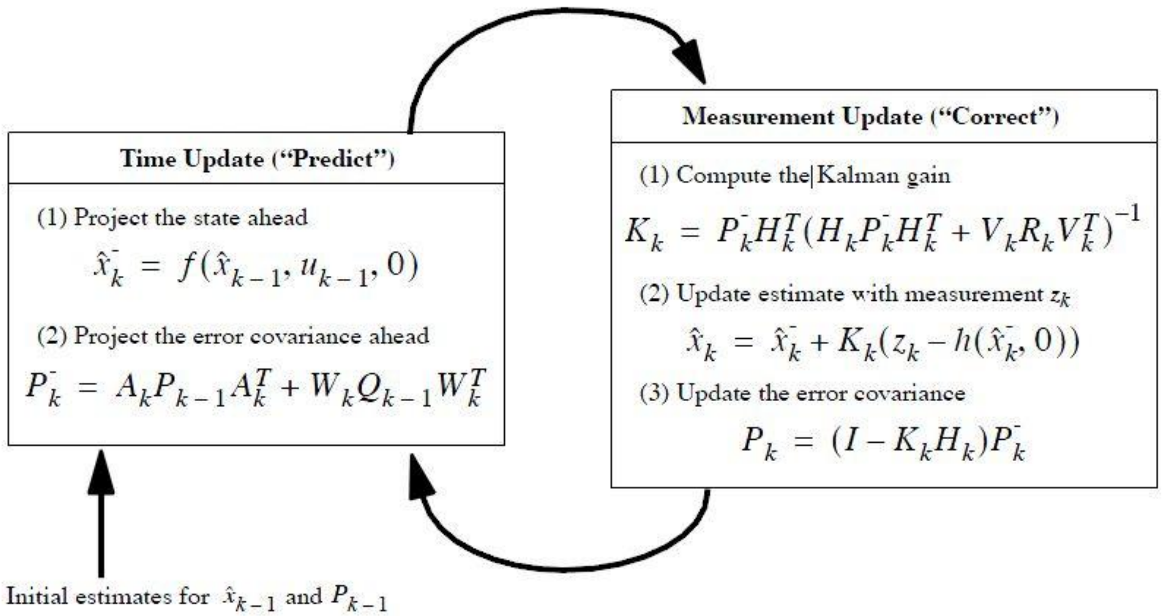


Fig 2.3: A complete picture of the operation of the Extended Kalman Filter

This figure has been reproduced from [3]

CHAPTER 3

COMPARISON BETWEEN LMS, RLS, KF AND EKF ESTIMATION METHOD

3.1 COMAPRISON BETWEEN RLS AND LMS ALGORITHMS

3.2 COMPARISON BETWEEN RLS, LMS AND KF ALGORITHMS

3.3 COMPARISON OF KF AND EKF ALGORITHMS

3.1 Comparison between Recursive Least Squares (RLS) and Least Mean Squares (LMS) algorithms

- In the LMS algorithm, the correction applied to the previous estimate comprises of product of three factors: the (scalar) step-size parameter μ, the error signal $e(n\text{-}1)$ and the tap-input vector $\mathbf{u}$(n-1). On the other hand in the RLS algorithm this correction comprises of the product of two factors: the true estimation error $\eta(n)$and the gain vector $\mathbf{k}(n)$. The gain vector consists of the inverse deterministic correlation matrix$\phi^{-1}(n)$, multiplied by the tap input vector $\mathbf{u}(n)$. The major difference between the LMS and RLS algorithms is therefore the presence of $\phi^{-1}(n)$ in the correction term of the RLS algorithm that has the effect of de-correlating the successive tap inputs, which makes the RLS algorithm *self-orthogonalizing*. Because of this property, the RLS algorithm is independent of the eigenvalue spread of the correlation matrix of the filter input.[13]
- In the LMS algorithm, the correction that is applied in updating the old estimate of the coefficient vector is based on the instantaneous sample value of the tap-input vector and the error signal. Whereas, in the RLS algorithm, we have to utilize the past available information to make this computation.[13]
- The LMS algorithm requires approximately *20M* iterations to converge in mean square, where M is the number of tap coefficients. But the RLS algorithm converges in mean square in less than *2M* iterations. The rate of convergence of the RLS algorithm is thus faster than the LMS algorithm.[13]
- No approximations are made in the RLS algorithm unlike the LMS algorithm. Thus, as the no of iterations reaches infinity, the least-squares estimate of the coefficient vector approaches the optimum Weiner value, and correspondingly the mean square error approaches the minimum value possible. Thus the RLS algorithm, in theory, exhibits zero misadjustment. The LMS always exhibits non-zero maladjustment, but this can be made quite small by using a sufficiently small step-size parameter.[13]
- Although the RLS algorithm is superior in performance to the LMS algorithm, it is by far more complex in nature as compared to its counterpart. The complexity of an algorithm for real time operation is determined by two principal factors :[13]

- The number of multiplications(with divisions counted as multiplications) per iteration, and
- The precision required to perform mathematical operations.[13]

The RLS algorithm requires a total of $3M(3+M)/2$ multiplications, which increases as the size of M , the number of filter coefficients. The LMS algorithm requires a total of $2M+1$ multiplications, increasing linearly with M. e.g. when M=35, the RLS algorithm requires 1995 multiplications, but the LMS algorithm requires only 71.[13]

3.2 Comparison between Kalman, Recursive Least Squares(RLS) and Least Mean Squares(LMS) Filter

A significant number of Active Noise Control (ANC) systems use some form of the LMS [17] [18] algorithm due its relatively reduced complexity. However, it suffers from problems namely slow convergence and high sensitivity to the eigenvalue spread [18] [4]. To overcome these problems we generally use a RLS based filter, but it is widely known that the RLS loses many of its good properties for a forgetting factor lower than one. Namely, in some of the applications, the LMS filter is better in tracking non-stationery signals than the RLS algorithm [4] [19]. One approach, which works considerably well with the non-stationery signals, is to use a Kalman Filter, which is a generalised version of the RLS filter [4][20][21]. But, similar to the RLS algorithm, the Kalman Filter has high computational complexity, which makes it expensive to use for some applications.[22]

The convergence of all the three algorithms can be interpreted as the convergence of several modes corresponding to the eigenvectors of the autocorrelation matrix of the reference signal. If the frequency response of the path to be identified is somewhat flat then the models with lower energy (eigen values) in the reference signal will also have low energy in the desired signal. In such cases, the step size and the forgetting factors of the RLS and the LMS algorithms can be so adjusted that the filters corresponding to higher energy modes have similar bandwidth to the ones generated by the Kalman Filter. Thus, a good tracking response can be achieved. However, this changes when the path to be identified has resonance or the desired noise reduction is high.[22]

But, if the real purpose is system identification and not noise reduction, all the modes are equally important. Therefore, in this case the KF has a clear advantage over the RLS and LMS algorithms.[23]

3.3 Comparison between Kalman Filtering and Extended Kalman Filtering Algorithms

Although the Kalman filtering algorithm is one of the most widely used methods for tracking and estimation due to its simplicity, robustness and optimality, its application to non-linear systems is virtually impossible. One of the most common approaches for non-linear systems is the use of Extended Kalman Filter (EKF) [15] [16]. The EKF linearizes all the non-linear models so that traditional Kalman filtering can be applied. But, whereas the EKF scores over the KF as far as the non-linear systems are concerned, it suffers from certain disadvantages as it is difficult to tune, implement and its algorithm is significantly complex in nature. [14]

The EKF applies the KF to non-linear systems by simply linearising all the non-linear models so that e we can apply the Kalman Filter equations to the non-linear systems. But it suffers from two well-known drawbacks:

- Linearisation can lead to highly unstable filters if the assumptions of local linearity are violated.
- The derivation of Jacobian matrices is non-trivial in most applications and often leads to significant implementation difficulties. [14]

CHAPTER 4

ESTIMATION OF POWER SYSTEM HARMONICS

4.1 ESTIMATION OF A TEST SIGNAL USING LEAST MEAN SQUARE

4.2 ESTIMATION OF A TEST SIGNAL USING RECURSIVE LEAST SQUARES

4.3 ESTIMATION OF A TEST SIGNAL USING KALMAN FILTERING

4.4 EXTENDED KALMAN FILTER OUTPUT SHOWING MEAN SQUARE ERROR

ESTIMATION OF POWER SYSTEM HARMONICS USING LEAST MEAN SQUARE, RECURSIVE LEAST SQUARES, KALMAN FILTERING AND EXTENDED KALMAN FILTERING:

A signal containing N sinusoids has been considered as given below:

$$X = \sum_{i=1}^{N} A_i \sin(w_i t_k + \varphi_i) + v_k, \qquad k = 1,2,3,.....,N$$

Where A_{ik}, w_i and φ_i is the amplitude, frequency and phase of the i^{th} sinusoid respectively. t_k is the k^{th} sample of the sampling time and v_k is a zero mean Gaussian white noise. In this thesis, we have used this signal having amplitude of 1p.u (for LMS & RLS) and 20p.u (for KF) and frequency of 50Hz. The process noises are generated using the random number generator with the help of matlab command "randn". The amplitude estimation of the signal (where the estimated is the filtered one in case of LMS and RLS) has been carried out at different harmonic levels starting from fundamental to 5^{th} harmonic signal. The simulation results have been shown in the subsequent pages which compare between the original signal and estimated signal.

A static test signal corrupted with non-linearities and Gaussian noise has been used and the estimation of amplitude is done using Extended Kalman filtering algorithm. The original signal and estimated signal and the comparison between the two have been shown in fig 4.10. The mean square error for the estimated signal has been found out and shown in fig 4.11.

SIMULATION RESULTS:

- **ESTIMATION OF A TEST SIGNAL USING LEAST MEAN SQUARE**
- **ESTIMATION OF A TEST SIGNAL USING RECURSIVE LEAST SQUARES**
- **ESTIMATION OF A TEST SIGNAL USING KALMAN FILTER**
- **EXTENDED KALMAN FILTER OUTPUT SHOWING MEAN SQUARE ERROR**

4.1 Least Mean Square Simulation Results:

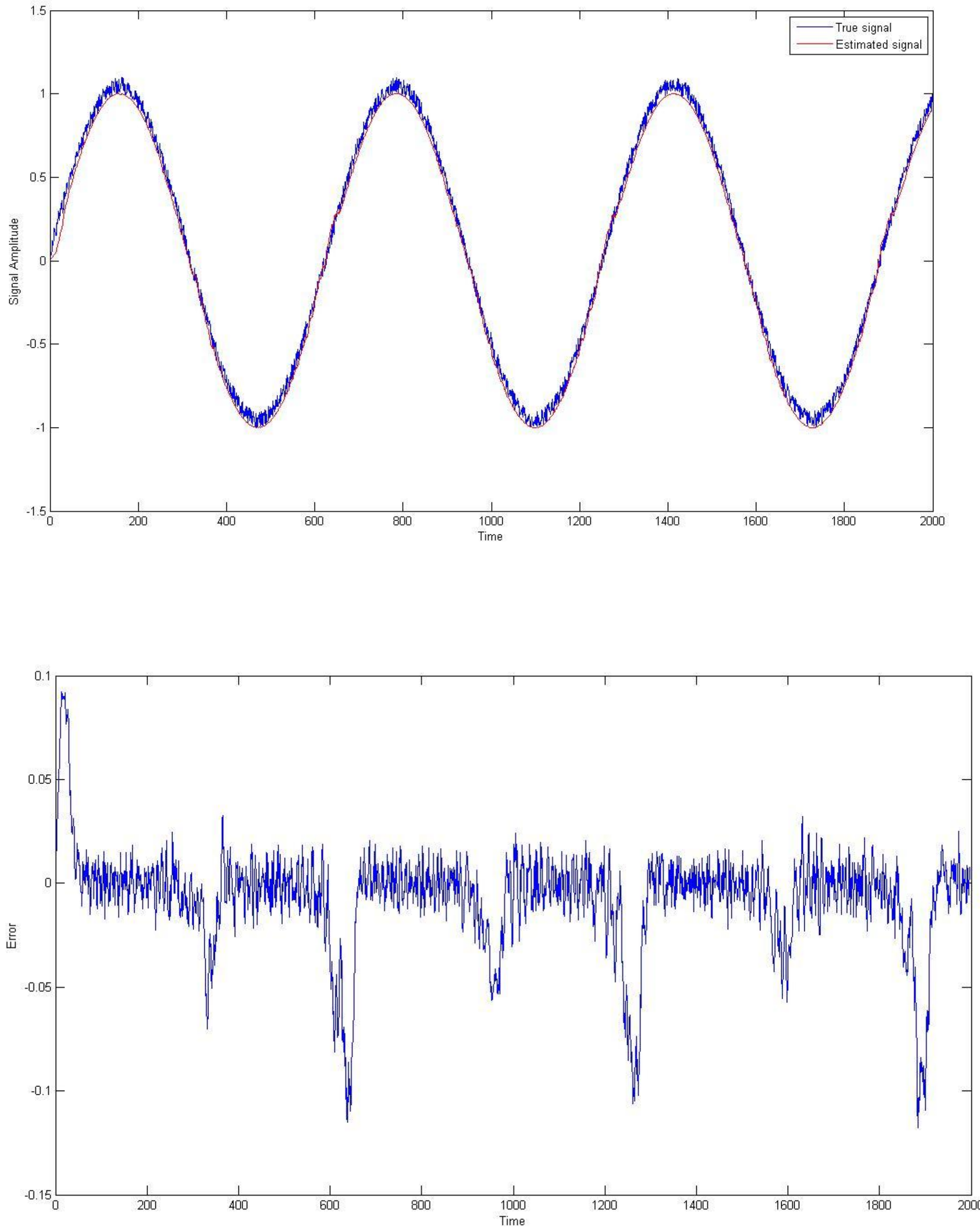


Fig 4.1: Fundamental Amplitude Estimation Using LMS

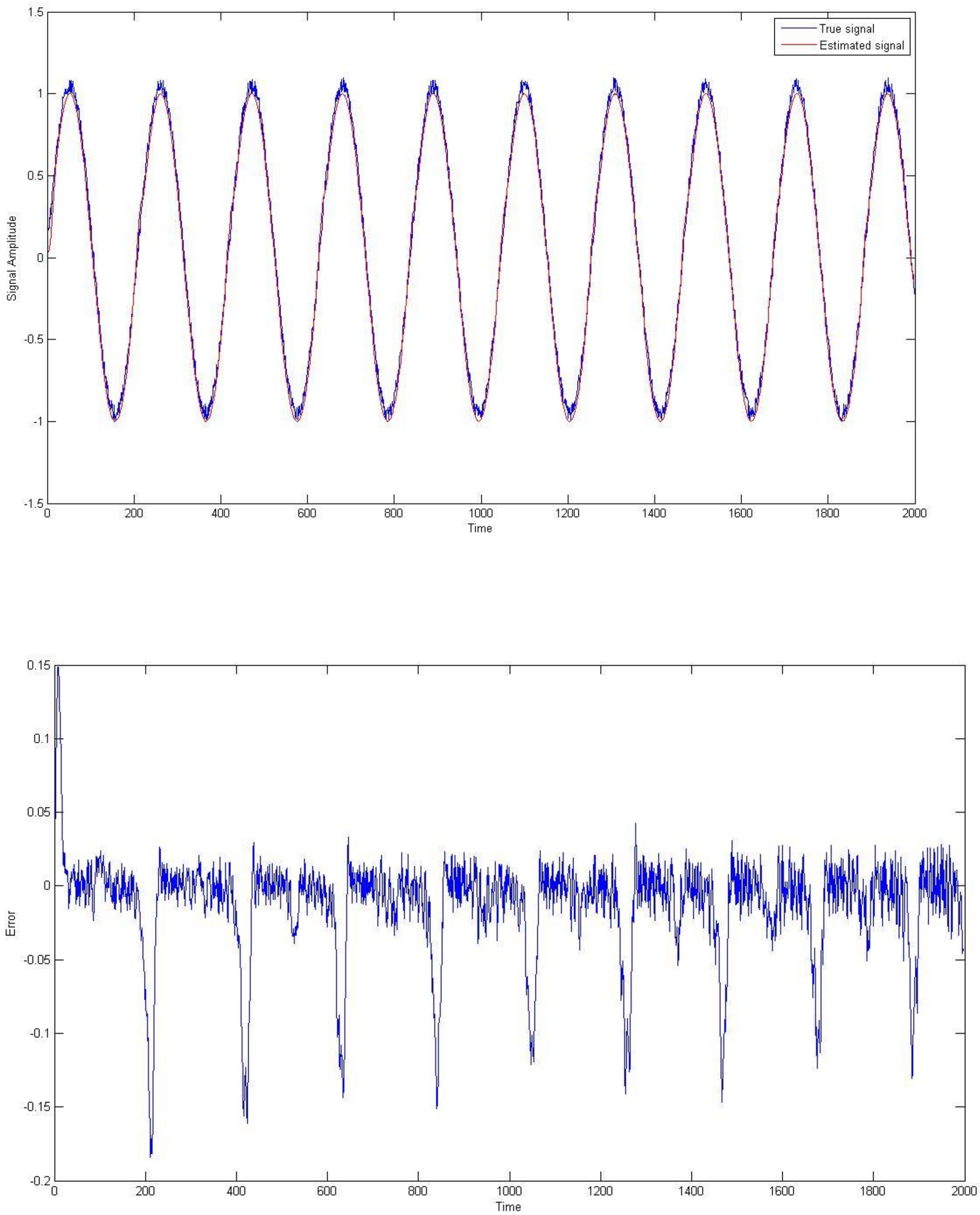
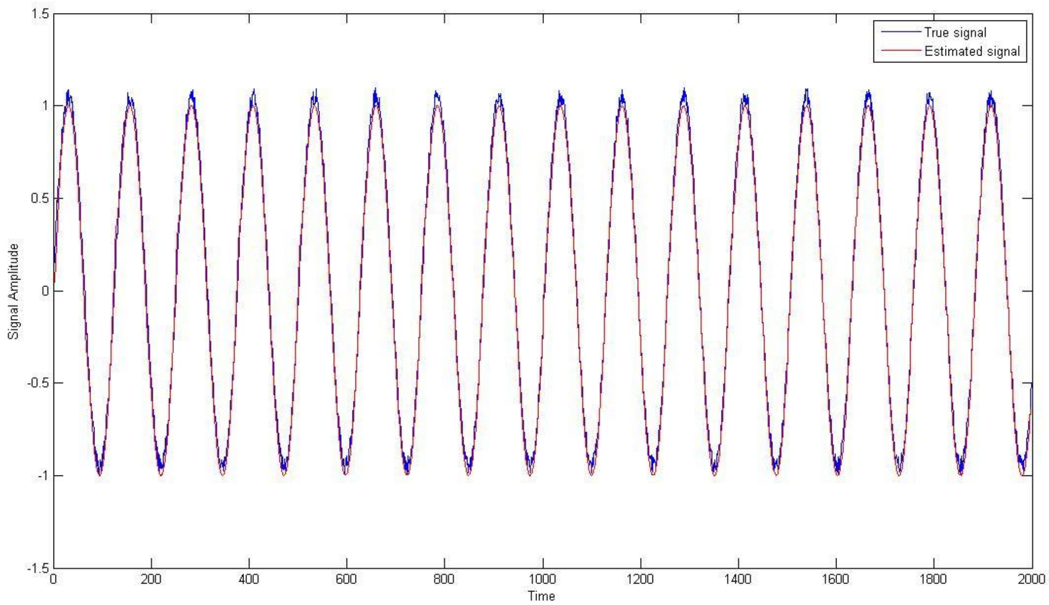


Fig 4.2: 3^{RD} Harmonic Amplitude Estimation Using LMS



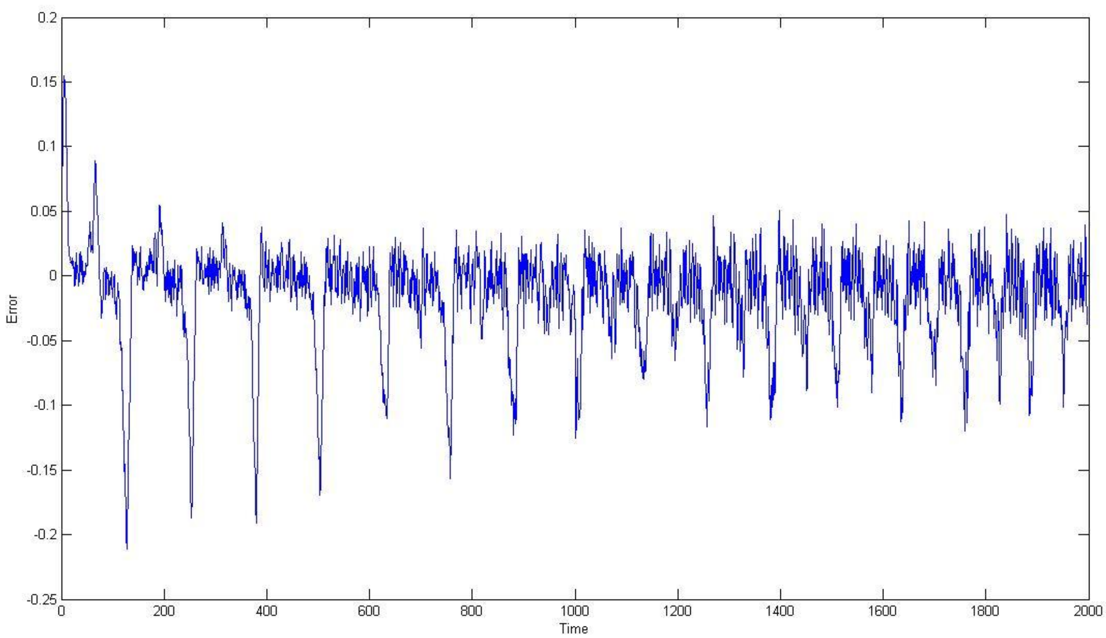


Fig 4.3: 5^{TH} Harmonic Amplitude Estimation Using LMS

4.2 Recursive Least Squares Simulation Results:

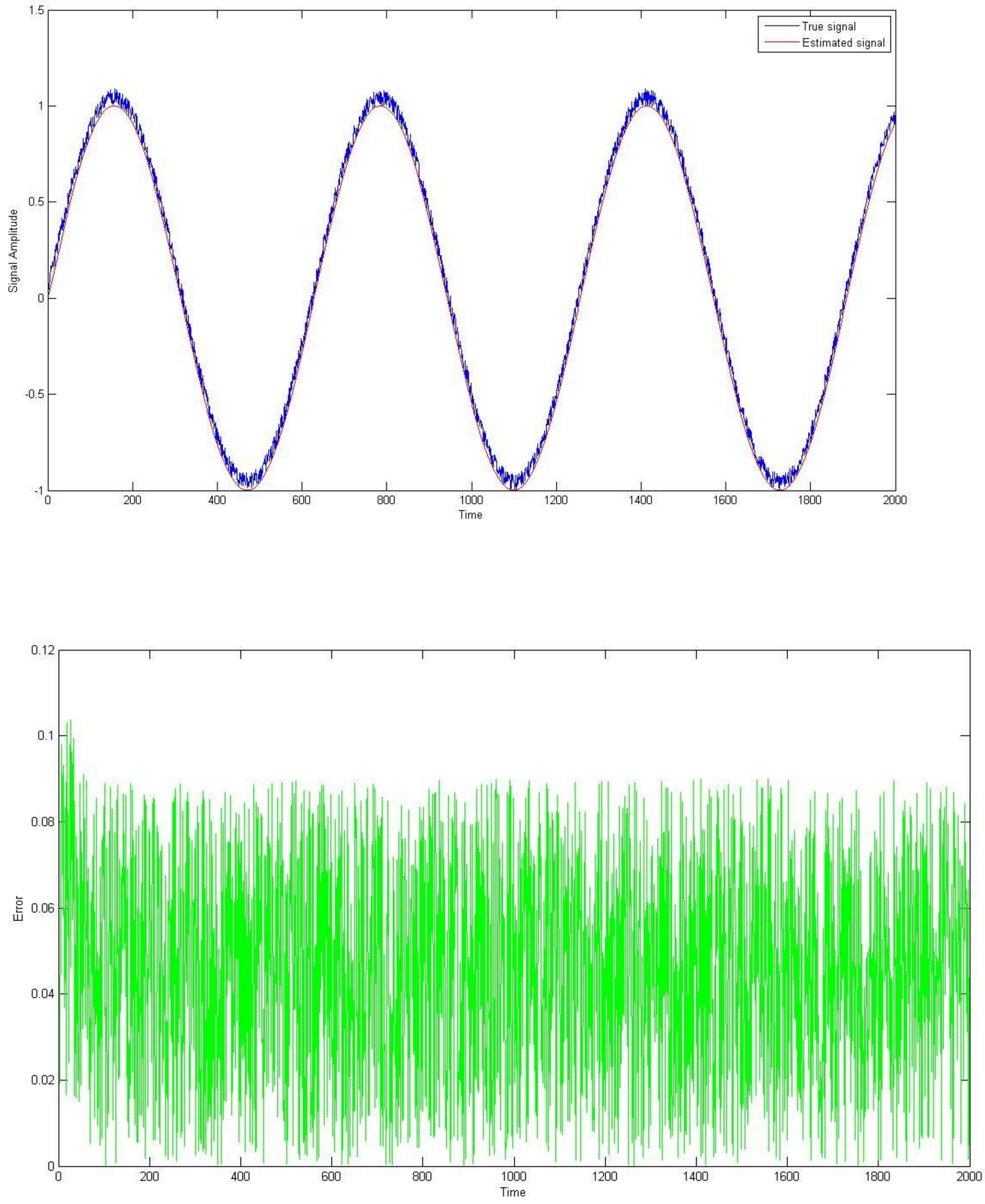


Fig 4.4: Fundamental Amplitude Estimation Using RLS

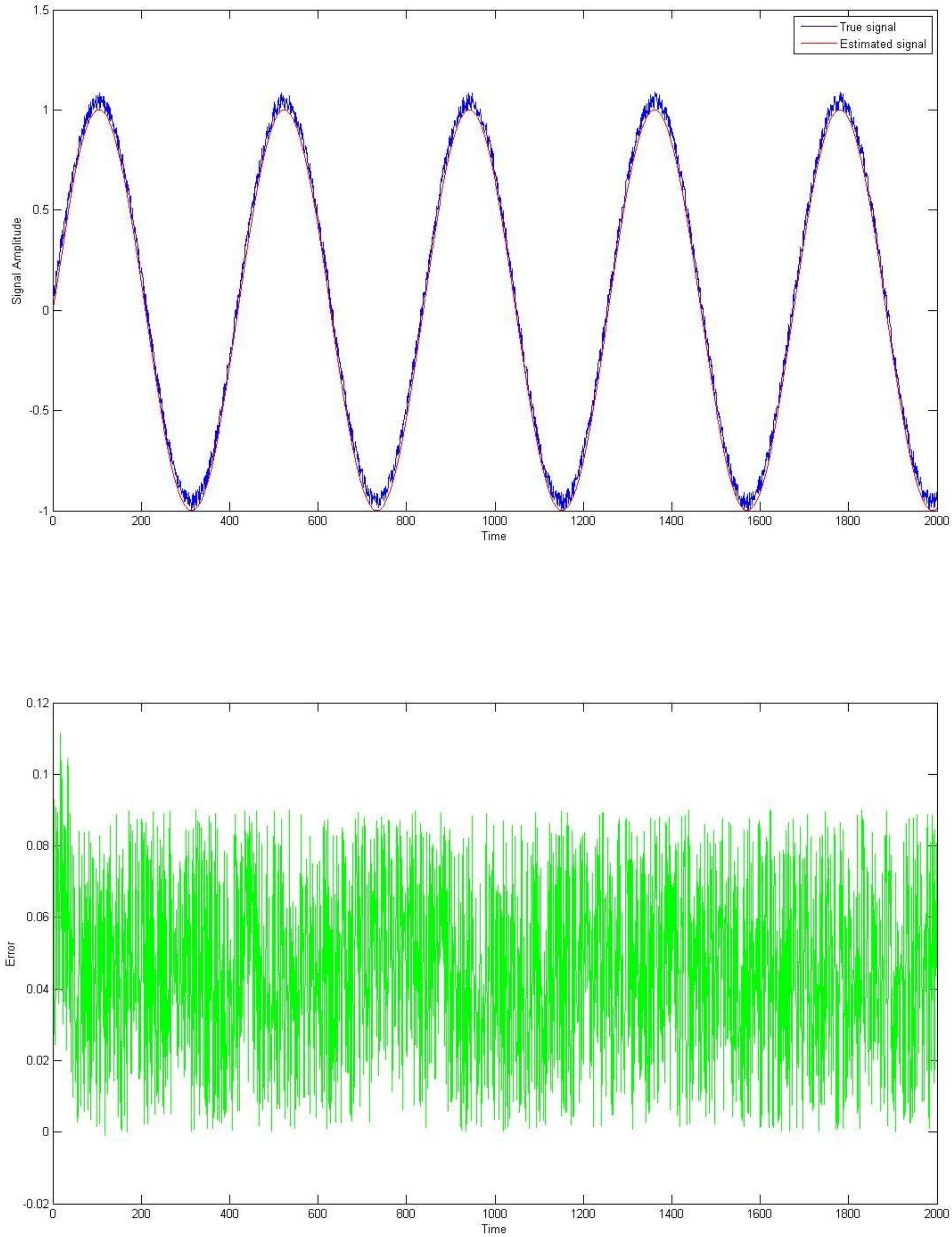


Fig 4.5: 3^{RD} Harmonic Amplitude Estimation Using RLS

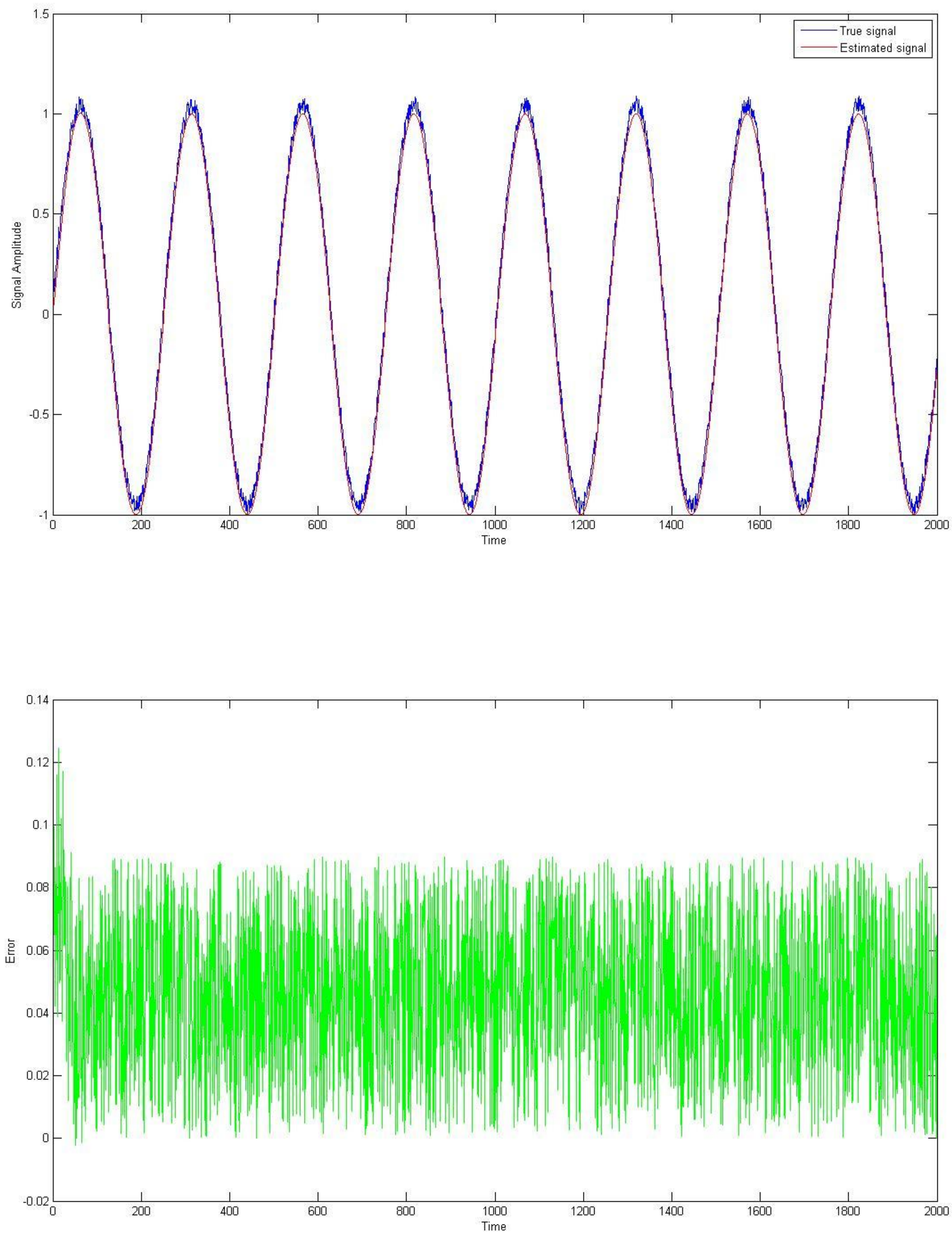
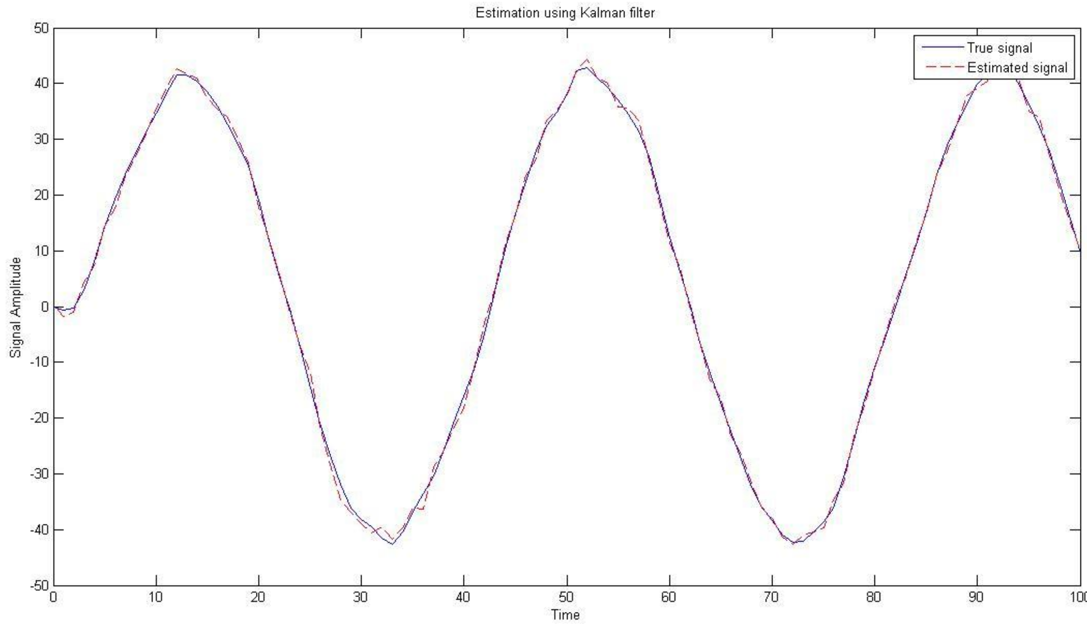


Fig 4.6: 5^{TH} Harmonic Amplitude Estimation Using RLS

4.3 Kalman Filtering Simulation Results:



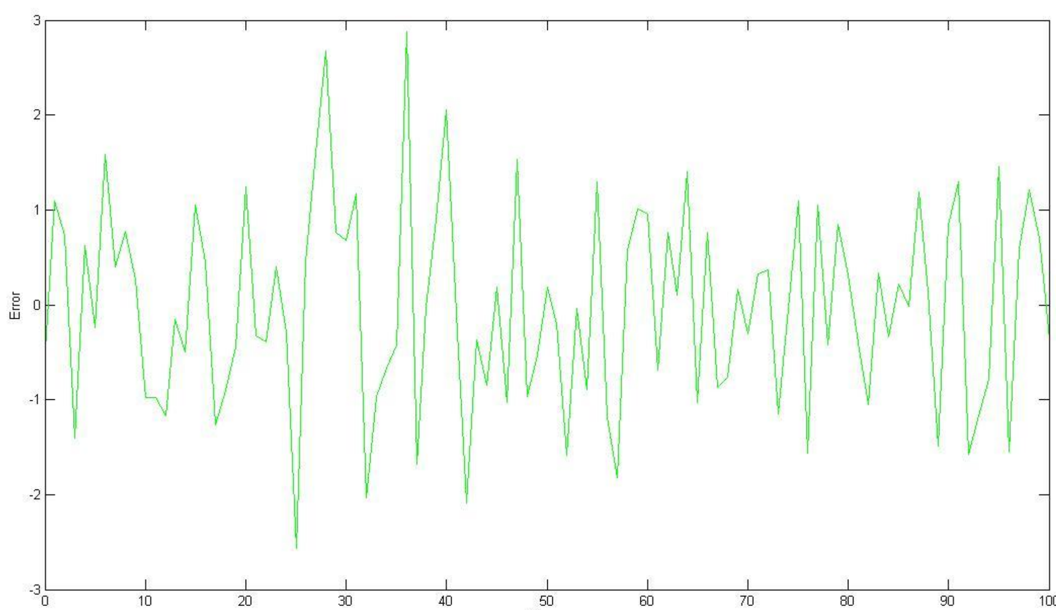


Fig 4.7: Fundamental Amplitude Estimation Using Kalman Filter

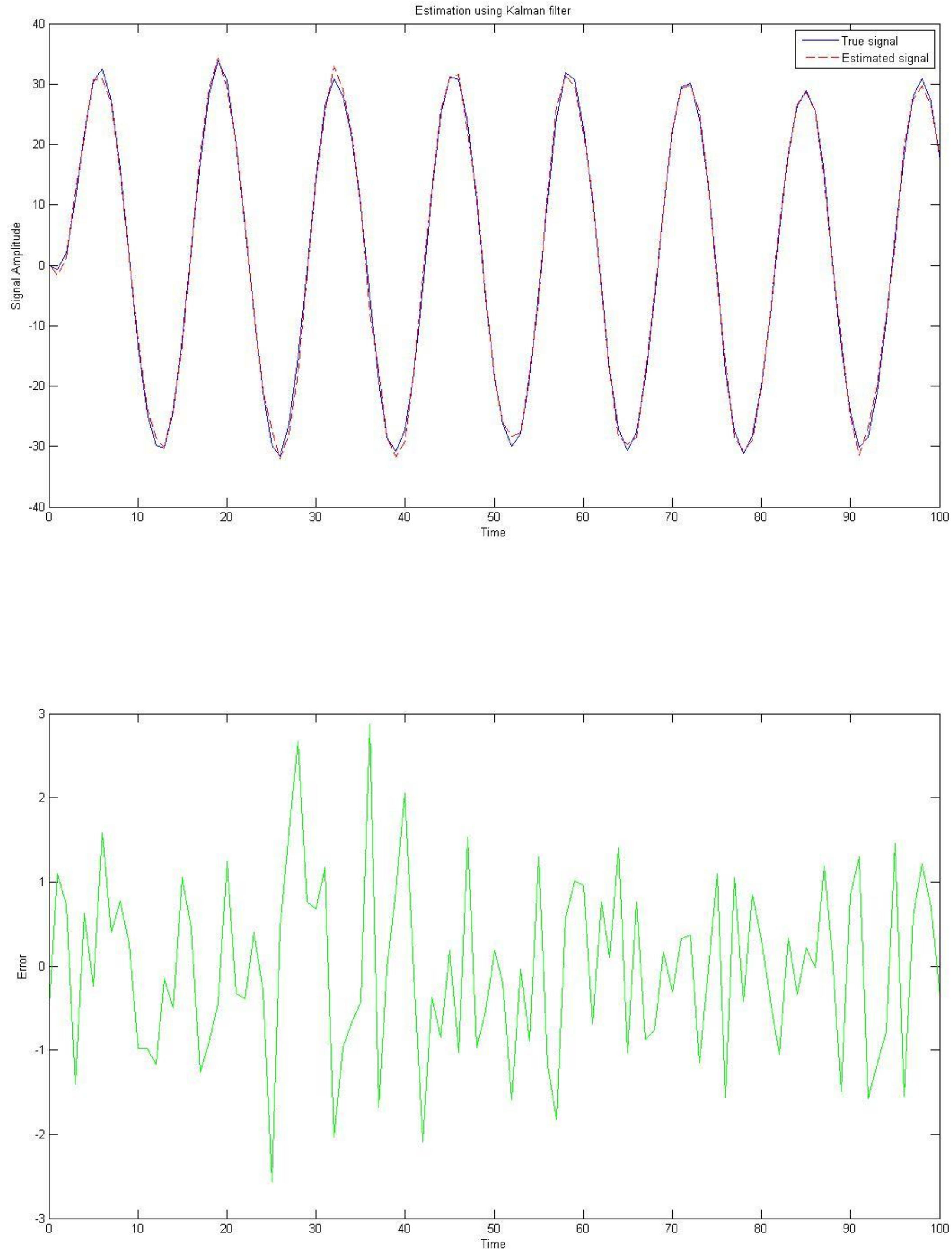


Fig 4.8: 3^{RD} Harmonic Amplitude Estimation Using Kalman Filter

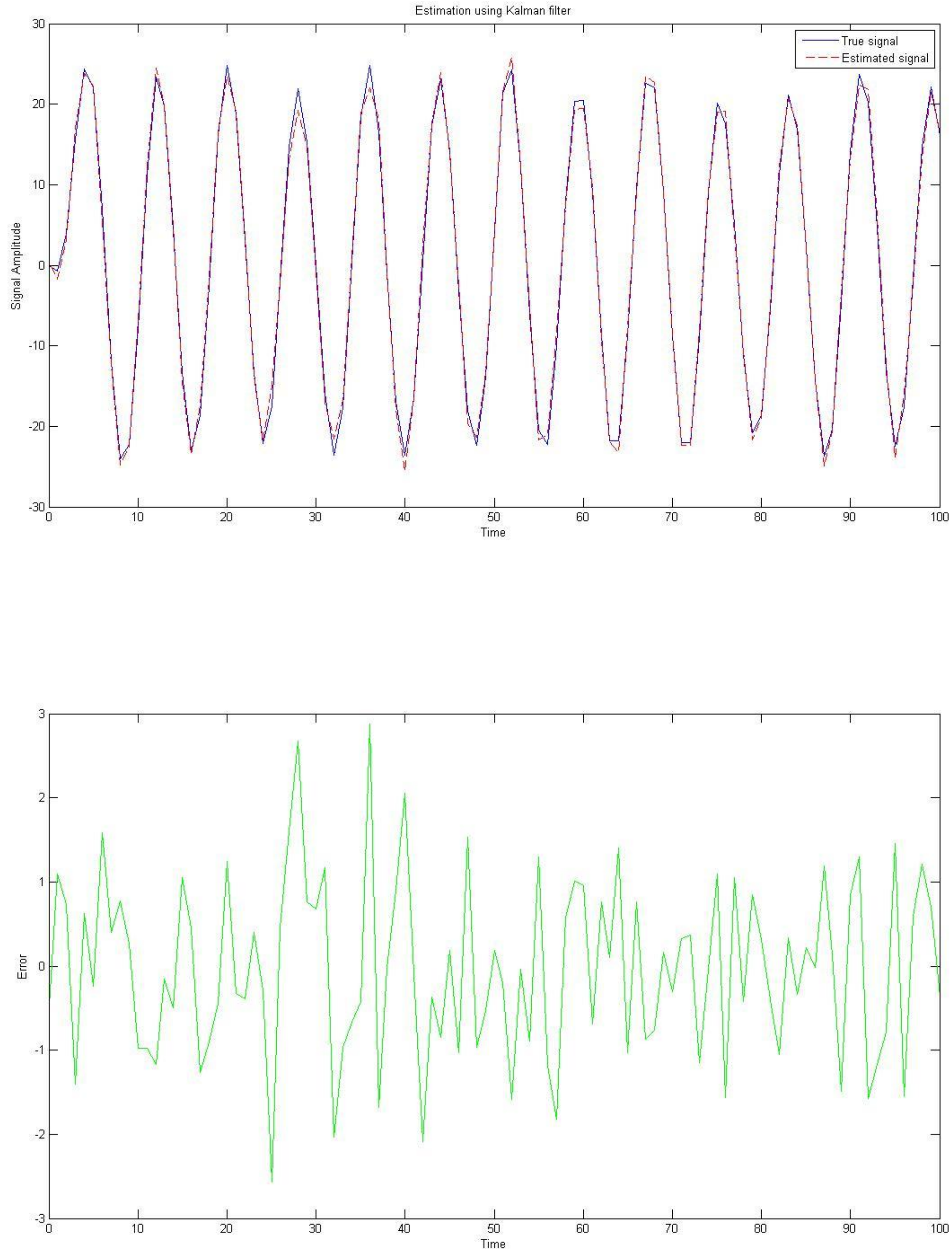


Fig 4.9: 5TH Harmonic Amplitude Estimation Using Kalman Filter

4.4 Extended Kalman Filter Simulation Results:

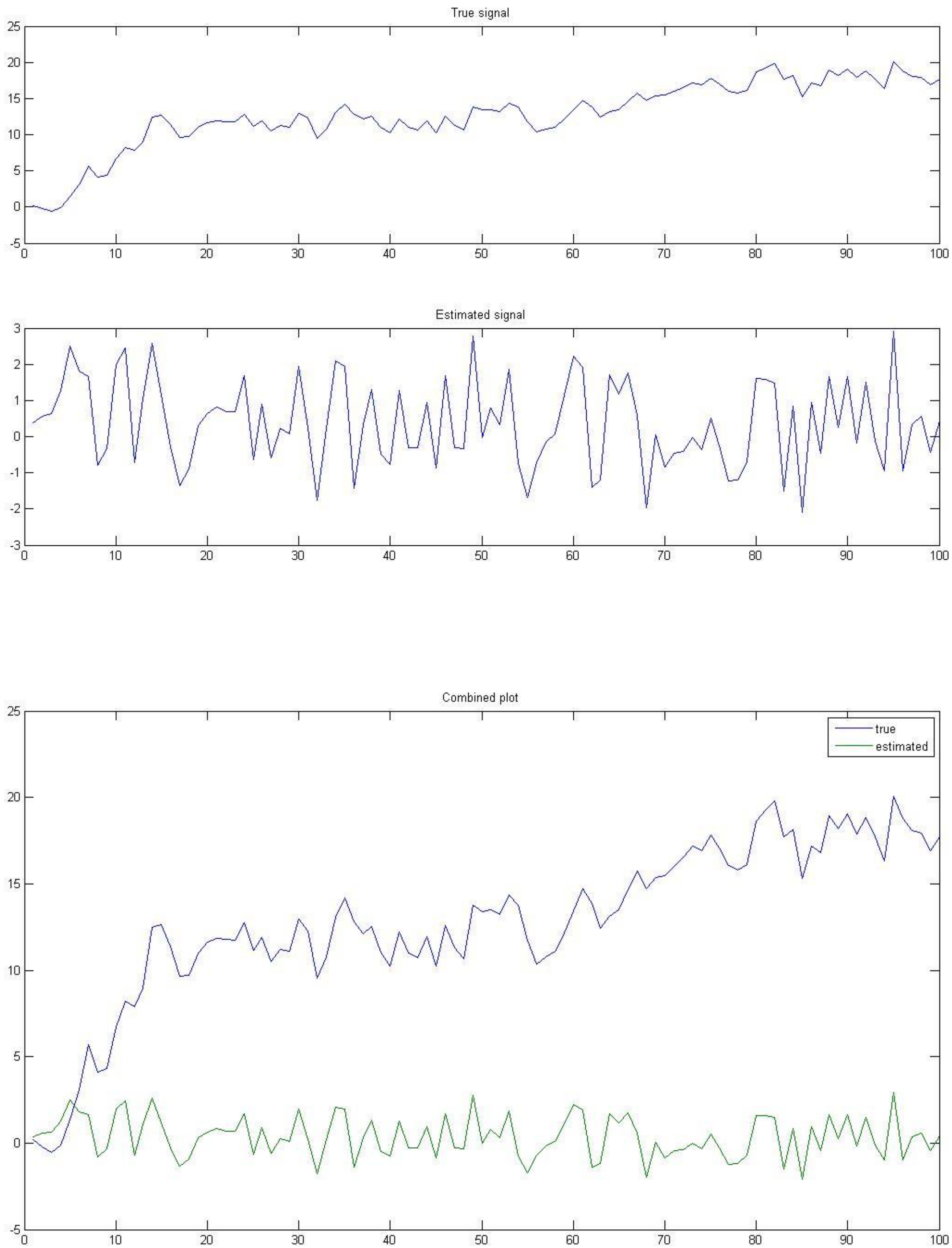


Fig 4.10: Extended Kalman Filter Output

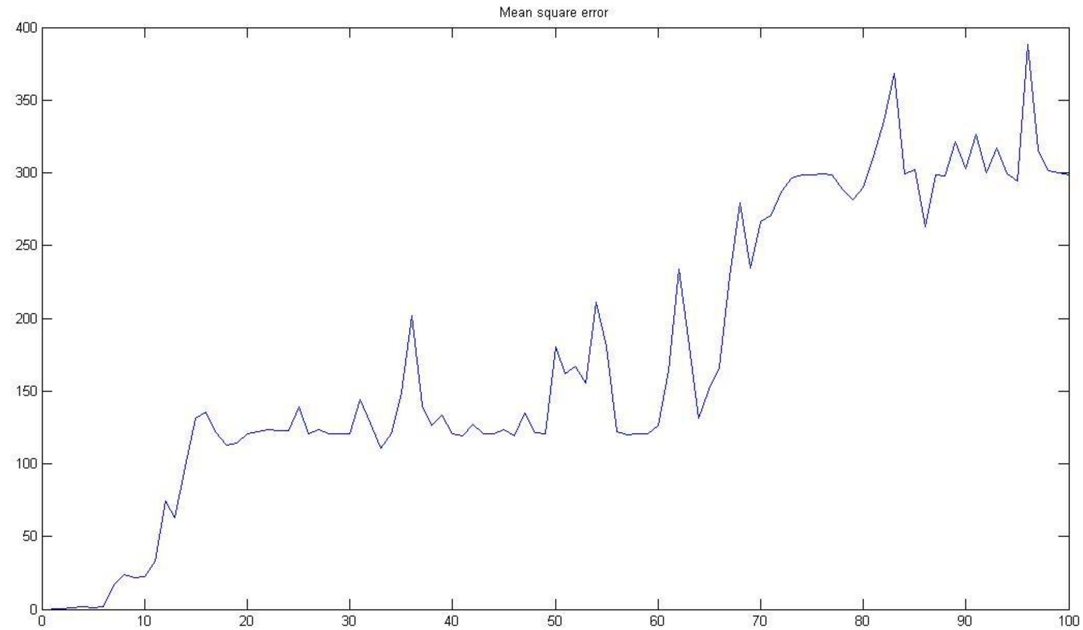


Fig 4.11: Mean Square Error in EKF

CONCLUSION

Harmonic distortion is one of the different aspects that affect the power system efficiency. Since the harmonic content of the power circuit depends upon the load, the presence of a non-linear load and electronic converters in the system are the main cause of harmonics. Harmonics can be broadly divided into two categories: characteristic and non-characteristic. The production of non-characteristic harmonics in the circuit should be avoided as far as the technical aspects are concerned. Characteristic harmonics are the integral multiples of the fundamental frequency and their amplitude is directly proportional to the fundamental frequency and inversely proportional to the order of the harmonic. Since it is essential to filter out those harmonics, we require an estimator to estimate the parameters of the harmonics.

There are various methods for the estimation of the parameters. We have discussed about the Least Mean Squares(LMS), Recursive Least Squares (RLS), Kalman Filter(KF) and Extended Kalman Filter (EKF) algorithms in this thesis.

The LMS is the most commonly used algorithm used for estimation. It is a gradient descent algorithm which adjusts the adaptive filter taps changing them by a quantity proportional to the instantaneous estimate of the gradient of the error surface.

The RLS algorithm performs an exact minimization of the sum of the squares of the desired signal estimation errors at each instant.

The Kalman Filter is basically a recursive estimator and its algorithm is also based on the least square error. Since all the algorithms produce a noisy estimate of the filter taps, we need a low pass filter which would then process this noisy signal. The filter bandwidth of this filter should be so chosen that it compromises between eliminating the noise from the noisy estimate and preserving the original signal. This feature is only provided by the KF. The RLS algorithm is not capable of doing this since its filter bandwidth is fixed. The LMS algorithm has this feature but its quantitative values are not adequate. But one limitation of KF is that it cannot be used for non-linear systems.

To work with non-linear systems we proposed the Extended Kalman Filter (EKF) .In this algorithm we need to compute the matrix of partial derivatives (Jacobians) in order to linearize the non-linear system about the current mean and co-variance. We found out that although this filter is able to estimate the parameters for non-linear systems significantly, it suffers from strategic disadvantage as the computation of Jacobians is very difficult.

Our main objective has been to compare between the various afore-mentioned algorithms and decide upon the most appropriate algorithm to be used depending upon the need of the situation.

REFERENCES:

[1] Dynamic state estimation of power system harmonics using kalman filter methodology, Husam M. Beides(Student member) and G.T Heydt(Senior Member), School of Electrical Engineering, Purdue University, West Lafayette, IN 47907, October 1991

[2] Comparison of Estimation Techniques using Kalman Filter and Grid-based Filter for Linear and Non-linear System, Subrata Bhowmik (NIT Rourkela) and Chandrani Roy(MCKV Institute of engineering), *Proceeding of the International on Computing: Theory and Applications*(ICCTA'07).

[3] An introduction to Kalman filter, Greg Welch and Gary Bishop TR 95-041, Department of Computer Science, University of North Carolina at Chapel Hill, Chapel Hill, NC 27599-3175, Updated: Monday, July 24, 2006

[4] Haykin Simon, Adaptive Filter Theory, Prentice Hall, 1996 (3RD Edition)

[5] Michaels Ken, Bell South Corp., Fundamental of harmonics, Jun1,1999, part 1-3
http://ecmweb.com/mag/electric_fundamentals_harmonics/

[6] http://www.pge.com/includes/docs/pdfs/mybusiness/customerservice/energystatus/powerquality/harmonics.pdf

[7] Padiyar K.R, HVDC Power Transmission Systems, Technology and System Interactions, New. Age International (P) Ltd. 1990

[8] Harmonic estimation in a power system using a novel hybrid Least Square- Adaline algorithm. M. Joorabian, S. S. Montazavi, A. A. Khayyami, Electrical Engineering Department, Shahid Chamran University, Ahwaz, 61355, Iran, 30TH May 2006

[9] A.A. Girgis, W. B. Chang and E. B. Makram, A digital recursive measurement scheme for online tracking of harmonics, *IEEE Trans. Power Delivery* ***6***(July(3)) (1991), pp. 153-1160.

[10] Y.N. Chang, Y.C. Hsieh and C. S. Moo, Truncation effects of FFT on estimation of dynamic harmonics on power system, *Power system technology*, 2000, *Proceedings, Power Conference 2000, International conference on, vol.3, 4-7 December(2000)*, pp. 1155-1160.

[11] A.A. Girgis and F. Ham, A qualitative study of pitfalls in FFT, *IEEE Trans. Aerospace Electron. System* **AES 16** (July(4)) (1980) pp. 434-439.

[12] P.K. Dash and A. M. Sharaf, A Kalman filtering approach for estimation of power system harmonics, *Proceedings of the 3RD International Conference On Harmonics in Power System* Nashville, Indiana, Sep.28 – Oct.1 (1998), pp.34-80.

[13] www.medialab.ch/ds/vorlesung/adaptive_filtering/adaptive_filtering_chapter4.ppt

[14] Julier J.Simon and Uhlmann K.Jeffrey, *A new extension of Kalman Filter to non-linear systems*, The Robotics Research Group, Department of Engineering Science, University of Oxford, pp1-2.

[15] Sorenson H.W., editor, *Kalman Filtering : theory and application*, IEEE press 1985.

[16] Uhlmann J.K. Algorithms for multiple target tracking, *American Scientist,* 80(2):128-141, 1992.

[17] Lopes P., Santos B., Bento M., Piedade M, Active Noise Control System, Recpad 98 Proceedings, IST, Lisbon, Portugal, March 1998.

[18] Kuo M.Sen and Morgan R.Denis, *Active Noise Control Systems, Algorithms and DSP Implementations*, John Wiley and Sons, Inc., 1996.

[19] Eleftheiriou E., Falconer D.D., *Tracking the properties and Steady State Properties of RLS Adaptive Filter Algorithms*, IEEE Trans. Acoust Speech Signal Process., vol. ASSP-34, pp.1097-1110.

[20] Anderson B.D.O., Moore B.J., *Optimal Filtering*, Prentice Hall, Inc, 1979.

[21] Haykin S., Sayed S.H., Zeidler J.R., Yee P., Wei P.C., Adaptive Tracking of Linear Time Variant Systems by Extended RLS Algorithms, Signal Processing, IEEE Transactions on, Volume:45 5 ,Page-1118-1128, May 1997.

[22] Lopes Paulo A.C., Piedade S.Moises, The Kalman Filter in Active Noise Control, 1000 Lisboa, Portugal, December 1999.

[23] www.wikipedia.org/extended kalman filter/

[24] Study of kalman, Extended kalman and Unscented kalman filter, Mamata Madhumita, Soumya Ranjan Aich, NIT Rourkela, May 2010