Economic Design of $\overline{X}$ Control Chart using Particle Swarm Optimization

A THESIS SUBMITTED IN PARTIAL FULFILMENT

OF THE REQUIREMENTS FOR THE DEGREE OF

Master of Technology

in

Mechanical Engineering

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UNDER THE GUIDANCE OF

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CERTIFICATE

This is to certify that thesis entitled, **"Economic Design of $\overline{X}$ Control Chart using Particle Swarm Optimization**" submitted by **Mr. GAJANAND GUPTA** in partial fulfillment of the requirements for the award of **Master of Technology** in **Mechanical Engineering** with **"Production Engineering"** Specialization during session 2010-2011 in the Department of Mechanical Engineering National Institute of Technology, Rourkela.

It is an authentic work carried out by him under my supervision and guidance. To the best of my knowledge, the matter embodied in this thesis has not been submitted to any other University/Institute for award of any Degree or Diploma.

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ABSTRACT

Control chart is the most widely used tools for statistical process control. For detecting shift in process mean, $\overline{X}$ chart is the simplest and most commonly used. Control chart should be designed economically in order to achieve minimum quality control costs. The major function of control chart is to detect the occurrence of assignable causes so that the necessary corrective action can be taken before a large quantity of nonconforming product is manufactured. The $\overline{X}$ control chart dominates the use of any other control chart technique if quality is measured on a continuous scale. The design of a control chart refers to the selection of three parameters i.e., sample size, width of control limit, and interval between samples. Economic design of control chart has gained considerable importance in providing better quality of end products to customer at less cost. In the present work, a computer programme in C language based on a non-traditional optimization technique namely particle swarm optimization has been developed for the economic design of the $\overline{X}$ control chart giving the optimum values of the sample size, sampling interval and width of control limits such that the expected total cost per hour is minimized. The results obtained are found to be better compared to that reported in the literature.

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Chapter 1

Introduction

1.1 Quality Control

Quality is one of the most important consumer decision factors in the selection among competing products and services. The phenomenon is widespread, irrespective of the fact that the consumer is an individual, a retail store, an industrial organization or a defense program. Consequently, understanding and improving quality is a key factor leading to business success, an enhanced competitive position, and growth in the market. In the quality control a substantial return on investment from successfully employing quality and from improved quality is an integral part of business strategy. Thus, quality can be defined as the fitness for use, in other words quality is inversely proportional to variability or quality improvement is the reduction of variability in processes and products. Quality engineering is the set of managerial, operational, and engineering activities in a company uses to confirm that the quality characteristics of a product are at the required levels.

1.2 Statistical Process Control

Statistical process control (SPC) is a powerful collection of statistical methods to the monitoring and control of a process to ensure that it operates as its full potential to produce conforming product. Under SPC, a process desired to produce as much conforming product as possible with the least possible waste. Control chart is a key tool of SPC.

Most of the power of SPC lies in the ability to examine a process and the sources of variation in that process using tools that give weight to objective analysis over subjective opinions and that allow the strength of each source is to be determined numerically. Variations in the process may affect the quality of product. With its emphasis on early detection and prevention of problems, SPC has a distinct advantage over other quality methods, such as inspection, that apply resources to detecting and correcting problems after they have occurred.

In addition to that, SPC can lead to a reduction in the time required to produce the product. Basically SPC has seven following major tools:-

1. Scatter diagram
2. Check sheet
3. Histogram
4. Cause-and-effect diagram
5. Control chart
6. Pareto chart
7. Defect concentration diagram

1.3 Control Chart

Control chart is one of the widely used statistical process control (SPC) tools. It is used to statistically monitor the process through sampling inspection instead of 100% inspection. It only indicates whether the process is in-control or out-of-control but it cannot on its own rectify the process. If a point falls within the upper and lower control limits, the process is referred to as "in-control" whereas if it falls outside the control limits, the process is referred to as "out-of-control".

As illustrated in Fig. 1.1 a control chart consists of the following three lines :

i) a centre line (CL or μ_0) ,

ii) an upper control limit (UCL), and

iii) a lower control limit (LCL).

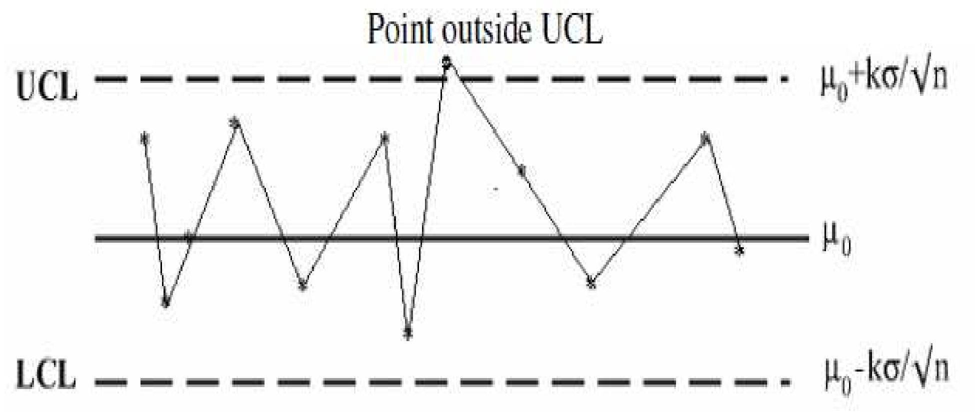

Fig.1.1. $\overline{X}$ Control Chart

Assuming that quality characteristics with process mean μ_0 and process standard deviation σ, is normally distributed with sample size n and width of control limit k, the upper and lower control limits can be given by following equations:

$$UCL = \mu_0+k\sigma/\sqrt{n} \quad (1.1)$$

$$LCL = \mu_0-k\sigma/\sqrt{n} \quad (1.2)$$

The upper and lower control limits on a control chart are usually set as ±3 standard deviations from the mean. If it is assumed that the data exhibit a normal distribution, these control limits will capture 99.74 percent of all the items produced as shown in Fig. 1.2. Similarly, if control limits are set at ±2 standard deviations from the mean, the two control limits would capture 95.44 percent of the values.

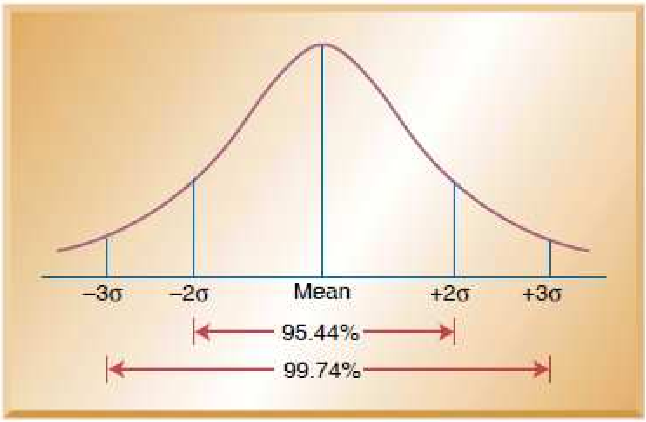

Fig.1.2. Area Under Normal Curve

1.4 Types of Control Chart

The types of charts are often classified according to the type of quality characteristic that they are supposed to monitor. Control charts can classify for variables and attributes.

For controlling quality characteristics that represent variables of the product, the following charts are commonly used:-

1.4.1 $\overline{X}$ chart: This chart is used to plot the sample means in order to control the mean value of a variable.

1.4.2 R chart: This chart is used to plot the sample ranges in order to control the variability of a variable.

1.4.3 S chart: This chart is used to plot the sample standard deviations in order to control the variability of a variable.

1.4.4 S^2 chart: This chart is used to plot the sample variances in order to control the variability of a variable.

For controlling quality characteristics that represent attributes of the product, the following charts are commonly used:-

1.4.5 c chart: This chart is used to plot the number of defects (per batch, per day, per machine, per 100 feet of pipe, etc.). This chart assumes that defects of the quality attribute are rare, and the control limits in this chart are computed based on the Poisson distribution.

1.4.6 u chart: This chart is used to plot the rate of defects, that is, the number of defects divided by the number of units inspected (the *n*; e.g., feet of pipe, number of batches). Unlike the C chart, this chart does not require a constant number of units, and it can be used, for example, when the batches (samples) are of different sizes.

1.4.7 np chart: This chart is used to plot the number of defectives (per batch, per day, per machine) as in the C chart. However, the control limits in this chart are not based on the distribution of rare events, but rather on the binomial distribution. Therefore, this chart should be used if the occurrence of defectives is not rare. For example, this chart can be used to control the number of units produced with minor flaws.

1.4.8 p chart: This chart is used to plot the fraction of defectives (per batch, per day, per machine, etc.) as in the u chart. However, the control limits in this chart are not based on the distribution of rare events but rather on the binomial distribution (of proportions). Therefore, this chart is most applicable to situations where the occurrence of defectives is not rare. All of these charts can be adapted for short production runs (short run charts), and for multiple process streams. If the percent of defectives is plotted, it is called 100p chart or percent defective chart.

1.5 Control Charts for Variables vs. Charts for Attributes

Sometimes, the quality control engineer has a choice between variable control charts and attributes control charts.

1.5.1 Advantages of attribute control charts

Attribute control charts have the advantage of allowing for quick summaries of various aspects of the quality of a product, that is, the engineer may simply classify products as acceptable or unacceptable, based on various quality criteria. Thus, attribute charts sometimes the need for expensive, precise devices and time consuming measurement procedures. Also, this type of chart tends to be more easily understood by persons those are unfamiliar with quality control procedures; therefore, it may provide more persuasive (to management) evidence of quality problems.

1.5.2 Advantages of variable control charts

Variable control charts are more sensitive than attribute control charts. Therefore, variable control charts may alert us to quality problems before any actual "unacceptable" (as detected by the attribute chart) will occur. The variable control charts are leading indicators of trouble that will sound an alarm before the number of scraps increases in the production process.

1.6 Other Specialized Control Charts

The types of control charts mentioned so far are as the "workhorses" of quality control, and they are probably the most widely used methods. However, with the advent of inexpensive desktop computing, procedures requiring more computational effort have become increasingly popular. Based on these some specialized control charts are following also:-

1. Control charts for non-normal Data
2. Cumulative sum (CUSUM) control chart
3. Moving average (MA) control chart
4. Exponential weighted moving average (EMWA) control chart

1.7 Important Parameters of Control Chart

A control chart has the following three important parameters:-

(1) Sample size (n)

(2) Variable sampling interval or sampling frequency (h)

(3) Width of Control limit (k)

1.8 Design of Control Chart

Control Charts are widely used to establish and maintain statistical control of a process. They are also effective devices for estimating process parameters, particularly in process capability studies. The use of a control chart requires that the engineer or analyst select a sample size, a sampling frequency or interval between samples, and the control limits for the chart. Selection of these three parameters is usually called the design of the control chart.

Basically design of control chart is of following three types:-

1. Statistical design of control chart
2. Economic design of control chart
3. Statistical economic design of control chart

1.8.1 Statistical design of control chart

Since control chart is based on sampling inspection, it is always associated with two types of statistical errors namely Type-I error and Type-II error. These two errors cannot be completely eliminated since 100% inspection is not carried out. However, these two errors can be minimized which serves as the basic principle of statistical design of control chart.

1.8.1(a) Type-I error :- Type-I error is committed when control chart tells that the process is out-of-control when it is actually in-control. It is same as giving a false signal or alarm.

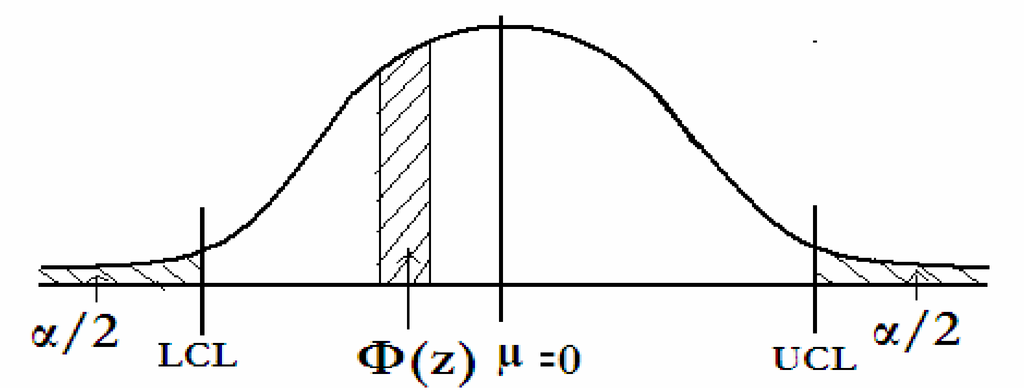

Fig.1.3. Type-I Error

As illustrated in Fig. 1.3, the probability of committing Type-I error or the rate of false alarm (α) is given by :-

$$\alpha = 2\int_{k}^{\infty} \phi(z)dz \tag{1.3}$$

1.8.1(b) Type-II error :- Type-II error comes when a control chart indicates that the process is in-control when it is actually out-of-control. In other words this error comes when control chart is not prompt enough to detect the process shift. Thus, this is a measure of inefficiency of control chart. As illustrated in Fig. 1.4 if the probability of committing Type-II error is β, then 1-β is called power of detecting the process shift or simply power which is calculated as given below:

$$1-\beta = \int_{-\infty}^{-k-\delta\sqrt{n}} \phi(z)dz + \int_{k-\delta\sqrt{n}}^{\infty} \phi(z)dz \tag{1.4}$$

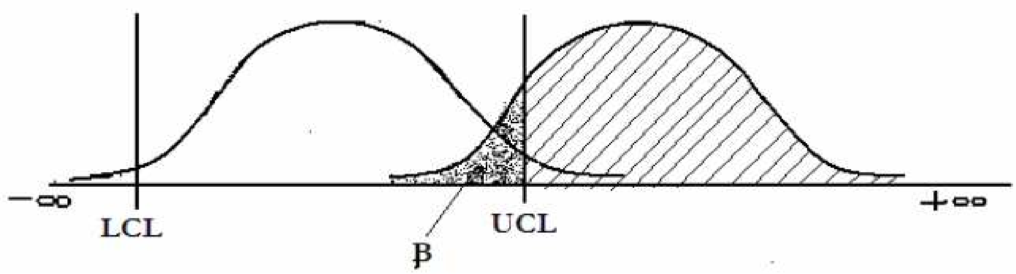

Fig.1.4. Type-II Error

1.8.2 Economic design of control chart

In economic design of control chart the objective is to reduce the total cost of maintaining the control chart as minimum as possible. It is used to determine the values of various design parameters i.e. sample size (n), sampling interval (h), and control limit coefficient (k) that minimizes total economic cost.

1.8.3 Statistical economic design of control chart

Statistical-economic design is basically a combination of statistical and economic design of control chart. In this type of design, the total cost of maintaining the control chart need to be minimized and at the same time Type-I and Type-II errors are not allowed to exceed their permissible level.

1.9 Particle Swarm Optimization

Particle swarm optimization (PSO) is a population based stochastic optimization technique developed by Dr. Eberhart and Dr. Kennedy in 1995, inspired by social behavior of bird flocking or fish schooling.

PSO shares many similarities with evolutionary computation techniques such as Genetic Algorithms (GA). The system is initialized with a population of random solutions and searches for optima by updating generations. However, unlike GA, PSO has no evolution operators such as crossover and mutation. In PSO, the potential solution, called particles, fly through the problem space by following the current optimum particles.

Compared to GA, the advantages of PSO are that PSO is easy to implement and there are few parameters to adjust. PSO has been successfully applied in many areas i.e. function optimization, artificial neural network training, fuzzy system control, and other areas where GA can be applied.

1.9.1 Comparisons between genetic algorithm and PSO

Most of evolutionary Techniques have the following procedure:

1. Random generation of an initial population
2. Reckoning of a fitness value for each subject. It will directly depend on the distance to the optimum.
3. Reproduction of the population based on fitness values.
4. If requirements are met, then stop. Otherwise go back to step 2.

From the procedure, PSO shares many common points with GA. Both algorithms start with a group of a randomly generated populated, both have fitness values to evaluate the population, both update the population and search for the optimum with random techniques.

However, PSO does not have genetic operators like crossover and mutation. Particles update themselves with the internal velocity. They also have memory, which is important to the algorithm.

Compared with genetic algorithms, the information sharing mechanism in PSO is significantly different. In Gas, chromosomes share information with each other. So the whole population moves like a one group towards an optimal area. In PSO, only gbest gives out the information to others. IT is a one–way information sharing mechanism. The evolution only looks for the best solution.

Chapter 2

Literature Survey

Introduction

In this chapter few selected research paper related to economic design of $\overline{X}$ control chart has considered. On economic design lot of work has been done already, based on this some research work some papers has classified into four category i.e. based on economic design, statistical design, types of distribution and particle swarm optimization for economic design.

2.1 Economic Design

Weiler [1] represented that the optimum sample size should minimize the total amount of inspection required to detect a specified shift for $\overline{X}$ control chart. If the shift is from an in-control state μ_0 to out-of-control state $\mu_1=\mu_0+\delta\sigma$, then he shows that the optimal sample size is

$$n=\frac{12.0}{\delta^2}$$ when ± 3.09 sigma control limits are used

$$n=\frac{11.1}{\delta^2}$$ when ± 3 sigma control limits are used

$$n=\frac{6.65}{\delta^2}$$ when ± 2.58 sigma control limits are used

$$n=\frac{4.4}{\delta^2}$$ when ± 2.33 sigma control limits are used

He did not consider costs, the implication is that minimizing the total inspection will minimize total costs.

Duncan [2] proposed the optimum economic design of the $\overline{X}$ control chart. This paper was the first to deal with a fully economic model of a Shewhart-type control chart and to incorporate formal optimization methodology into determining the control chart parameters. He assumed that the process is characterized by an in-control state μ_0 and that a single assignable cause of magnitude δ, which occurs at random, results in a shift in the mean from μ_0 to either $\mu_0 + \delta\sigma$ or $\mu_0 - \delta\sigma$.

Woodall [3] presented that economic models assign a cost to passing defective items, which would include customer dissatisfaction costs and liability claims, among other components, and this is counter to Deming's philosophy that these costs cannot be measured and that customer satisfaction is necessary to staying in business.

Saniga and Shirland [4] presented that very few practitioners have implemented economic models for the design of control charts. They told that as most quality engineers claim that a major objective in the use of statistical process control procedures is to reduce costs.

Makis and Fung [5] proposed that Economic Manufacturing Quantity (EMQ) model i.e. all items produced are of perfect quality and the production facility never breaks down. However, in real production, the product quality is usually a function of the state of the production process which may deteriorate over time and the production facility may fail randomly. In this paper, they presented the effect of machine failures on the optimal lot size and on the optimal number of inspections in a production cycle. The formula for the long-run expected average cost per unit time is also obtained for a generally distributed time to failure.

Kim and Hong [6] presented an EMQ model which determines an optimal lot size in a failure prone machine. They assumed that time between failures of a machine is generally distributed, and a machine is repaired instantaneously when it fails. Depending on various types of failure rate function of a machine, they discussed how to determine an EMQ and proved its uniqueness. Variations of an EMQ depending on repair cost are also examined. Through numerical experiments, extensive investigations has been carried out on the effects of repair cost and setup cost to an EMQ as well as average cost, and some interesting behaviors was observed.

Bendaya and Rahim [7] presented an integrated model of the economic production quantity, the economic design of $\overline{X}$ control chart and the optimal maintenance level. In the proposed model, Preventive Maintenance (PM) activities reduce the shift rate to the out-of-control state proportional to the PM level. Compared to this case without PM the extra cost of maintenance results in lower quality control cost which may lead to lower overall expected cost. These issues are illustrated using an example of a Weibull shock model with an increasing hazard rate.

Shiau et al [8] proposed economic design of $\overline{X}$ control chart using genetic algorithm with the realistic monitoring error model embedded. They considered that Statistical process control (SPC), is developed to detect the occurrence of assignable causes so that unnecessary quality costs can be avoided. However, it is necessary to provide a practical way to evaluate the monitoring capability of an automatic gauge for its application. That is, the monitoring capability study should be conducted before applying any automatic gauge on SPC application. As introduced in this paper, the effects of manufacturing capability, gauge capability, control limits should be concurrently considered to remodel the monitoring error model. The decision of economic design of $\overline{X}$ control chart will be practically and appropriately made.

Woodall [9] presented the economic design of control chart and also told that in many economic designs the Type-I error of the control chart is considerably higher than in a statistical design, and told that it will lead to more always a problem, as managers will be reluctant to shut down a process if the control scheme has a history of many false alarms. Furthermore, if the Type-I error is high, then this could lead to excessive process adjustment, which often increases the variability of the quality characteristic.

2.2 Economic Statistical Design

Taylor [10] has reported that control procedures based on taking a sample of constant size at fixed intervals of time is non-optimal. He suggests that sample size and sampling frequency should be determined at each point in time based on the posterior probability that the process is in an out-of-control state. Dynamic programming methods were utilized extensively in the development.

Saniga [11] has investigated a study relating to the joint economic statistical design of $\overline{X}$ and R charts. Saniga used constraints on Type-I error, power and the average time to signal for the charts. His economic statistical designs have higher cost than the pure economic designs, but give superior over a wider range of process shifts and also have statistical properties that are as good as control charts designed entirely from statistical consideration.

Chen and Cheng [12] presented the economic–statistical design of $\overline{X}$ control charts for non-normal quality measurements. This cost model is used to determine the optimal design parameters the sample size, time between successive samples, and number of standard deviations away from the center line. This analysis shows that non-normality has a significant effect on the design parameters and hence should not be ignored. Sensitivity to the Weibull shape and the process-mean shift are also considered.

2.3 Types of Distribution

Girshick and Rubin [13] have investigated a process model in which a machine producing items characterized by a measurable quality characteristic can be in one of four states. States 1 and 2 are production states. In state 1 the output quality characteristic is described by the probability density function $f_i(x)$, i=1, 2, State 1 is the "in-control" state. While in state 1, there is a constant probability of a shift to state 2. They treat both 100% inspection and periodic inspection rules. The economic criterion is to maximize the expected income from the process. Their work is of significant theoretical value. They were the first researchers to propose the expected cost per unit time criterion and rigorously show its appropriateness for the problem.

Taylor [14] reported the optimal control rule for a two-state process with a normally distributed quality characteristic. Taylor's work has indicated that non-optimality, fixed sample size, fixed sampling interval control rules are widely used in practice because of administrative simplicity.

Chen and Yang [15] presented an economic design of $\overline{X}$ control charts with a Weibull distributed process-failure mechanism when there is an occurrence of possible multiple assignable causes and assumed that once an assignable cause occurs, no further assignable causes will occur. A cost model based on the variable sampling intervals was formulated and analyzed.

Optimal values of the design parameters including the sample size, the sampling intervals, and width of control limit was solved by minimizing the expected total cost per unit time, based on the varieties of combinations of Weibull parameters. Comparisons between a multiplicity-cause model and a single-cause model are performed under both having same time and cost parameter and numerical results show that the former provides a lower loss-cost than the latter when the process has an increasing hazard rate.

Chen and Yang [16] investigated a model of a moving average control chart (MA control chart) with a Weibull failure mechanism from an economic point. When the process-failure mechanism follows a Weibull model or other models having increasing hazard rates, it is desirable to have the decreasing sampling interval with the age of the system. The MA control chart is used to monitor quality characteristics of raw material or products in a continuous process. A cost model utilizing a variable scheme instead of fixed sampling lengths in a continuous flow process is studied in this research. The variable sampling scheme is used to maintain a constant integrated hazard rate over each sampling interval. Optimal values for the design parameter, the moving subgroup size, the sampling interval, and the width of control limit are determined by minimizing the loss-cost model. The performance of the loss cost with various Weibull parameters was studied.

Chen and Yeh [17] reported an approach which simultaneously considered the properties of cost and quality based on the Burr distribution. The objective was to determine three parameters, namely, sample size, sampling interval between successive samples, and control limits, when an $\overline{X}$ chart monitors a manufacturing process with Gamma (λ, 2) failure characteristic and non-normal data. The design parameters of the $\overline{X}$ control charts can be obtained through the genetic algorithm (GA) method. An example was also adopted to indicate the solution procedure and sensitivity analyses. The results show that an increase of skewness coefficient (a_3) results in a slight decrease for sample size (n) while an increase of kurtosis coefficient (a_4) leads to a wider control limit width.

2.4 Particle Swarm Optimization

Zee-Lee [18] proposed a particle swarm optimization (PSO) method for solving the economic dispatch (ED) problem in power systems. Many nonlinear characteristics of the generator, such as ramp rate limits, prohibited operating zone, and non smooth cost functions are considered using the proposed method in practical generator operation. The feasibility of the proposed method is demonstrated for three different systems, and it is compared with the GA method in terms of the solution quality and computation efficiency. The experimental results show that the proposed PSO method was indeed capable of obtaining higher quality solutions efficiently in ED problems.

Feng et al [19] proposed how to solve problems such as low global search capability and insufficient diversity of Pareto optimal set existing in MOPSO, a multi objective particle swarm optimization algorithm based on crowding distance sorting is proposed. An external population is preserved to store the non-dominated individuals during the evolution process. The shrink of the external population is achieved based on individuals' crowding distance sorting by descending order, which deletes the redundant individuals in the crowding area. An individual with relatively big crowding distance is selected as the global best to lead the particles evolving to the disperse region. The dominant relation between individuals is compared with the constraint Pareto dominance to embody the constraints without external parameters

Abido [20] proposed a new multi-objective particle swarm optimization (MOPSO) technique and applied to environmental/economic power dispatch optimization problem. The proposed MOPSO technique presents a multi-objective version of the conventional PSO technique and utilizes its effectiveness to solve the multi-objective optimization problems. The MOPSO technique evolves a multi-objective version of PSO by proposing redefinition of global best and local best individuals in multi-objective optimization domain. The MOPSO technique has been implemented to solve the EED problem with competing and non-commensurable cost and emission objectives. The comparison with the different reported techniques demonstrates the superiority of the proposed MOPSO in terms of the diversity of the Pareto-optimal solutions obtained.

Immanue et al [21] presents an efficient and reliable particle swarm optimization (PSO) algorithm based technique for solving the emission and economic dispatch problems. The performance of the PSO is compared with conventional method, real coded genetic algorithm and hybrid genetic algorithm.

2.5 Objective of the Present Work

From the literature review, it is observed that lot of work has been done on design of $\overline{X}$ control chart of various types following various approaches and optimization techniques. The particle swarm optimization technique has been observed to have application in wide variety of fields. However, this technique has not been tried in the economic design of control chart. With this motivation, the objective of the present work is to use the PSO technique in economic design of $\overline{X}$ control chart where quality characteristic is normally distributed and compare the results with that reported in literature.

Chapter 3

Economic Model

3.1 Introduction

Control charts are mainly used to establish and maintain statistical control of a process. It is always required to consider the design of a control chart from an economical point of view because the choice of control chart parameters affect the whole cost. The three main control chart parameters are:

(i) sample size (**n**),
(ii) sampling frequency or interval between the samples (**h**), and
(iii) width of the control limits (**k**).

Selection of these three parameters is usually called *the Design of the Control Chart.*

In any production process, regardless of how well designed or carefully maintained it is, a certain amount of inherent or natural variability will always exist. This is called the variation due to *chance causes* and the process is said to be *statically under control.* Other kinds of variability which arise mainly from three sources such as improperly adjusted or controlled machines, operator errors and defective raw material, are called *assignable causes* and the process is said to be *out-of-control*. The above three parameters n, h, and k control the costs of sampling and testing, cost associated with investigating out-of-control signals and possibly correcting assignable causes and costs of allowing non-conforming units to reach the required quality.

3.2 Process Characteristics

To formulate an economic model for the design of a control chart, certain assumptions about the behavior of the process are required to be made. For simplicity, the process is assumed to be characterized by one quality characteristics of variable type that follows normal distribution. The process is assumed to be under in-control state at the beginning. The in-control state will correspond to the mean of these quality characteristics when no assignable causes are present. To determine the nature of the transition between the in-control and out-of-control states requires certain assumptions. At random, the process may shift to out-of-control state when an assignable cause affects the process. The process shifting to out-of-control state is assumed to be due to the shift in process mean only and not due to the shift in process standard deviation. Each out-of-control state is usually associated with a particular type of assignable cause. It is assumed to be affected by only a single assignable cause at any time which means that the out-of-control state is detected by the control chart and the process is rectified and brought back to in-control state before the occurrence next assignable cause of same or different type. According to Poisson process, an assignable cause occurs during a certain interval of time. It indicates that the length of time the process remains in the in-control state, given that it begins in-control, is an experimental random variable. This assumption simplifies the development of economic models. The nature in which process shifts occur is sometimes called the process-failure mechanism. So it implies that process transitions between states are instantaneous. Moreover the process is not self-correcting. That is, once a transition to an out-of-control state has occurred, the process can be returned to the in-control condition only by management intervention following the out-of-control signal on the control chart.

3.3 Cost Parameters

In the design of control charts, the following three categories of costs are considered.

i. costs of sampling and testing,
ii. costs associated with investigating an out-of-control signal and with the repair or correction of any assignable causes found, and
iii. costs associated with the production of non-conforming items.

Usually the cost of sampling and testing is assumed to consist of both fixed and variable components, say a_1 & a_2 respectively such that the total cost of sampling and testing is:

$$a_1 + a_2 * n \tag{3.1}$$

The second one is the costs of investigating the assignable cause and possibly correcting the process following an out-of-control state. Cost of investigating false alarm will be different from the costs of correcting assignable cause. These two costs are represented in the model by two different cost coefficients. The costs associated with producing non-conforming items consist of typical failure costs i.e. the costs of rework or scrap for internal failures, or replacement or repair costs for units covered by warranties in the case of external failure.

Economic models are generally formulated using a total cost function, which expresses the relationships between the control chart design parameters and the above three types of costs. The production, monitoring and adjustment process are a series of independent cycles over time. Each cycle begins with the production process in the in-control state and continues until the process monitoring via the control chart results in an out-of-control signal. A new cycle starts after an adjustment in which the process is returned to the in-control state.

Let E(T) be the expected length of a cycle and E(C) be the expected total cost incurred during a cycle. Then the expected cost per unit time is

$$E(A) = \frac{E(C)}{E(T)} \tag{3.2}$$

where C, T are dependent random variables and E(A) is expected value of the ratio of expected cost.

The Equation 3.2 is optimized to design economically optimal control chart. The sequence of production-monitoring-adjustment, with accumulation of costs over the cycle, can be represented by a particular type of stochastic process called Renewal Reward Process where time cost is given by the ratio of the expected reward per cycle to the expected cycle length.

3.4 An Economic Model of the Control Chart

The design of an $\overline{X}$ control chart requires the determination of various design parameters. These include the size of the sample *(n)* drawn in each interval, the sampling interval *(h)* and the width of control limit *(k)*. So it can be concluded that cost is a function of three independent variables i.e., n, h and k. The following costs are important in determining the decision variables in the economic design of $\overline{X}$ control charts: sampling cost, search cost and the cost of operating both in-control and out-of-control. It is assumed that output quality is measurable on a continuous scale and is normally distributed. When the process is in-control, the initial process mean is μ_0. However, due to the occurrence of an assignable cause, the initial mean may be shifted from μ_0 to $\mu_0 + \delta\sigma$ or $\mu_0 - \delta\sigma$ (out-of-control state), where δ is the shift parameter and σ is the process standard deviation. The control limits of the $\overline{X}$ control charts are set at $\mu_0 \pm k$ times the standard deviation of the sample means, where k is known as the control limit coefficient, such that

$$\text{UCL} = \mu_0 + k\,\sigma/\sqrt{n} \qquad (3.3)$$

$$\text{LCL} = \mu_0 - k\,\sigma/\sqrt{n} \qquad (3.4)$$

The assignable cause is assumed to occur according to a Poisson distribution process with an intensity of λ occurrences per hour. Assuming that the process begins in the in-control state, the time interval that the process remains in-control is an exponential random variable with mean $1/\lambda$ h. Therefore, if the occurrence of the assignable cause is between the j^{th} and $(j+1)^{st}$ samples, the expected time of occurrence within this interval is given by following equation

$$\tau = \frac{\int_{jh}^{(j+1)h} \exp(-\lambda t)(t - jh)\,dt}{\int_{jh}^{(j+1)h} \exp(-\lambda t)\lambda\,dt} = \frac{1-(1+\lambda h)\exp(-\lambda h)}{\lambda(1-\exp(\lambda h))} \qquad (3.5)$$

When the assignable cause occurs the probability that it will be detected on any subsequent sample is given by following

$$1-\beta = \int_{-\infty}^{-k-\delta\sqrt{n}} \phi(z)dz + \int_{k-\delta\sqrt{n}}^{\infty} \phi(z)dz \tag{3.6}$$

where $\phi(z)=(2\pi)^{-1/2}\exp(-z^2/2)$ is the standard normal density. The quantity 1-β is the power of the test if β is the Type-II error probability. The probability of occurrence of false alarm is given by

$$\alpha = 2\int_{k}^{\infty} \phi(z)dz \tag{3.7}$$

A production cycle is defined as the interval of time from the start of production where the process is assumed to start in in-control state following an adjustment to the detection and elimination of the assignable cause. As illustrated in Fig. 3.1 the cycle consists of four periods:

(i) In-control period

(ii) Out-of-control period

(iii) Time to take a sample and interpret the result i.e. gn

(iv) Time to find the assignable cause i.e. D

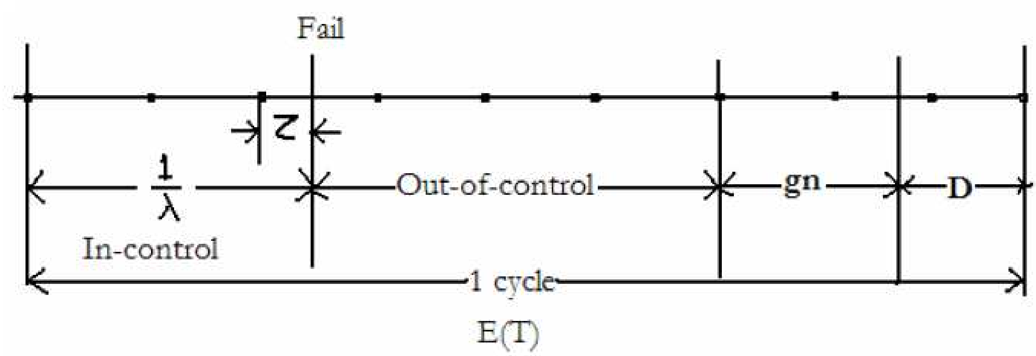

Fig. 3.1. Stages of a Production Cycle

The expected length of in-control period is 1/λ. If the number of samples required to produce an out-of-control signal when the process actually out-of-control is a geometric random variable with mean 1/(1-β), then the expected length of out-of-control period can be given by h/(1-β)-τ. The time required to take a sample and interpret the result is a constant g proportional to the sample size, so that gn is the length of this segment of the cycle. The time required to find the assignable cause following an action signal is a constant D**.** Therefore, the expected length of a cycle is given by following expression

$$E(T) = \frac{1}{\lambda} + \frac{h}{1-\beta} - \tau + gn + D \tag{3.8}$$

Let the net income per hour of operation in the in-control state is V_0 and the net income per hour of operation in the out-of-control state is V_1. The cost of taking a sample of size n is assumed to be of the form (a_1+a_2*n) where a_1 and a_2 represent the fixed and variable components of sampling cost respectively. The expected number of samples taken within a cycle is the expected cycle length divided by the interval between samples i.e. E(T)/h. The cost of finding an assignable cause is a_3 and the cost of investigating a false alarm is $a_3^{'}$. The expected number of false alarms generated during a cycle is α times the expected number of samples taken before the shift, or

$$\alpha \sum_{j=0}^{\infty} \int_{jh}^{(j+1)h} je^{-\lambda t} dt = \frac{\alpha e^{-\lambda t}}{1 - e^{-\lambda t}} \tag{3.9}$$

Therefore, the expected net income per cycle is

$$E(C) = V_0 \frac{1}{\lambda} + V_1 (\frac{h}{1-\beta} - \tau + gn + D) - a_3 - \frac{a_3^{'} e^{-\lambda h}}{1-e^{-\lambda h}} - (a_1 + a_2 n) \frac{E(T)}{h} \tag{3.10}$$

The expected net income per hour can be found by dividing the expected net income per cycle i.e. Equation 3.10 by the expected cycle length i.e. Equation 3.8**.** So the result becomes

$$E(A)=\frac{E(C)}{E(T)}$$

$$=\frac{V_0(1/\lambda)+V_1[h/(1-\beta)-\tau+gn+D]-a_3-a_3^{'}\alpha e^{-\lambda h}/(1-e^{-\lambda h})}{\frac{1}{\lambda}+\frac{h}{1-\beta}-\tau+gn+D}-\frac{a_1+a_2n}{h} \quad (3.11)$$

Let $a_4=V_0 - V_1$, where a_4 is the hourly penalty cost associated with production in the out-of-control state. So E(A) can also be written as

$$E(A)=V_0-\frac{(a_1+a_2*n)}{h}-\frac{a_4[h/(1-\beta)-\tau+gn+D]-a_3-a_3^{'}\alpha e^{-\lambda h}/(1-e^{-\lambda h})}{1/\lambda+h/(1-\beta)-\tau+gn+D}-\frac{a_1+a_2n}{h} \quad (3.12)$$

Or, $E(A)=V_0-E(L)$ where

$$E(L)=\frac{(a_1+a_2*n)}{h}+\frac{a_4[h/(1-\beta)-\tau+gn+D]+a_3+a_3^{'}\alpha e^{-\lambda h}/(1-e^{-\lambda h})}{1/\lambda+h/(1-\beta)-\tau+gn+D} \quad (3.13)$$

The expression E(L**)** represents the expected loss per hour by the process. E(L) is a function of three control parameters n, k and h. Equation 3.13 clearly shows that by minimizing E(L) expected net income per hour can be maximized**.** So Equation 3.13 is the objective cost function which is to be minimized by particle swarm optimization to find out the optimum values of sample size (n), sampling interval (h), and width of control limit (k) for minimum cost value of function E(L).

Chapter 4

Methodology

For comparing the results of the economic design of control chart using particle swarm optimization the following example earlier solved by Montgomery [22] and Shiau [8] has been considered in this work.

Example: A manufacturer produces nonreturnable glass bottles for packaging a carbonated soft-drink beverage. The wall thickness of the bottles is an important quality characteristic. If the wall is too thin, internal pressure generated during filling will cause the bottle to burst. In an effort to reduce costs, the manufacturer wishes to design an economically optimum $\overline{X}$ chart for the process.

Based on an analysis of quality control technicians' salaries and the costs of test equipment, it is estimated that the fixed cost of taking a sample is \$1. The variable cost of sampling is estimated to be \$0.10 per bottle, and it takes approximately 1 min (0.0167h) to measure and record the wall thickness of a bottle.

The process is subjected to several different types of assignable causes. However, on the average, when the process goes out-of-control, the magnitude of the shift is approximately two standard deviations. Process shifts occur at random with a frequency of about one every 20 h of operation. Thus, the exponential distribution with parameter $\lambda = 0.05$ is a reasonable model of the run length in-control. The average time required to investigate an out-of-control signal is 1 h. The cost of investigating an action signal that results in the elimination of an assignable cause is \$25, whereas the cost of investigating a false alarm is \$50.

The bottles are sold to a soft-drink bottler. If the walls are too thin, an excessive number of bottles will burst when they are filled. When this happens, the bottler's standard practice is to backcharge the manufacturer for the costs of cleanup and lost production. Based on this practice, the manufacturer estimates that the penalty cost of operating in the out-of-control state for one hour is \$100.

The expected cost per hour associated with the use of an $\overline{X}$ chart for this process is given by equation 3.13, with a_1 = \$1, a_2 = \$0.10, a_3 = \$25, $a_3^{'}$ = \$50, a_4 = \$100, λ = 0.05, δ = 2.0, g = 0.0167 and D= 1.0.

In the present work, a computer program based on PSO algorithm (see Section 4.1) was written in “C” language which is used to find a optimum solution to the above problem.

4.1 PSO Algorithm

PSO simulates the behaviors of bird flocking. Let a group of birds are randomly searching food in an area and there is only one piece of food in the area being searched. The group behavior learned by the birds while flying to reach their destination in minimum possible time is simulated in solving the optimization problems. In PSO, each single solution is a “bird” in the search space i.e. called a “particle”. All the particles have fitness values which are evaluated by the fitness function to be optimized, and have velocities which direct the flying of the particle. The particles fly through the problem space by following the current optimum particles.

PSO is initialized with a group of random particles (solutions) and then searches for optima by updating generations. In every iteration, each particle is updated by following two best values. The first one is the best solution, it has achieved so far. This value is called *pbest*. Another best value that is tracked by the particle swarm optimizer is the best value obtained so far by any particle in the population. This best value is a global best and is called *gbest*. When a particle takes part of the population as its topological neighbors, the best value is a local best and is called *lbest*.

After finding the two best values, the particle updates its velocity and positions with the following equations

$$V_i^{k+1} = V_i^k + c_1 rand * (pbest_i - s_i^k) + c_2 rand * (gbest - s_i^k) \quad (4.1)$$

$$S_i^{k+1} = S_i^k + V_i^{k+1} \quad (4.2)$$

where

V_i^k = velocity of particle i in iteration k

rand = random number between 0 and 1

S_i^k = current position of particle i in iteration k

$pbest_i$= best value of particle i

gbest= best value of the group

c_1 and c_2 are learning factors. In this work, the values of c_1 and c_2 are both taken as 2.05.

The first term of right-hand side of Equation 4.1 is corresponding to global search. The second and third terms of that are corresponding to local search. So**,** this method has a well balanced mechanism to utilize global and local search efficiently. On the basis of this algorithm following steps has been generated:-

1. The particles are randomly generated between the maximum and minimum operating limits of the generators.
2. The particle velocities are generated.
3. Objective function values of the particles are evaluated. These values are set the pbest value of the particles.
4. The best value among all the pbest values (gbest) is identified.
5. New velocities for the particles are calculated using Equation 4.1.
6. The positions of each particle are updated using Equation 4.2.
7. New objective function values are calculated for pbest. If the stopping criterion is met, the current position is better than the previous pbest, then the new value is set to pbest representing the optimal solution, otherwise the procedure is repeated from step 4.

Chapter 5

Results and Discussion

The output from the C program based on PSO algorithm give the result of optimum solution of the example taken from Montgomery [22] giving the economic design of $\overline{X}$ chart. This program calculates the optimum values of sampling frequency h and width of control limit k for different values of sample size n, and also computes the corresponding minimum values of the cost function E(L) as defined in Equation 3.13.

Most of the $\overline{X}$ charts used in practice are based on 3-sigma limits. Therefore, keeping the value of k constant at 3, the cost function was minimized with respect to h for various integer values of n varying from 1 to 10 using PSO. The optimum values of h and the corresponding minimum values of cost function E(L) are tabulated in Table 5.1 and graphically plotted in Fig. 5.1. and it is observed that as the sample size n increases the cost decreases to a minimum value i.e. 9.419 at n = 8 and then it increases. Thus, the best economic design of $\overline{X}$ chart based on 3-sigma limits is obtained at n = 8 and h = 0.937.

Table 5.1. Economic Design of $\overline{X}$ Chart for 3-sigma Limits (k=3)

Sample size, n	**Width of control limit, k**	**Optimum sampling interval, h**	**Optimum cost, E(L)**
1	3	0.231	16.794
2	3	0.418	12.232
3	3	0.586	10.622
4	3	0.717	9.910
5	3	0.807	9.592
6	3	0.86	9.463
7	3	0.906	9.422
8	**3**	**0.937**	**9.419**
9	3	0.963	9.431
10	3	0.988	9.446

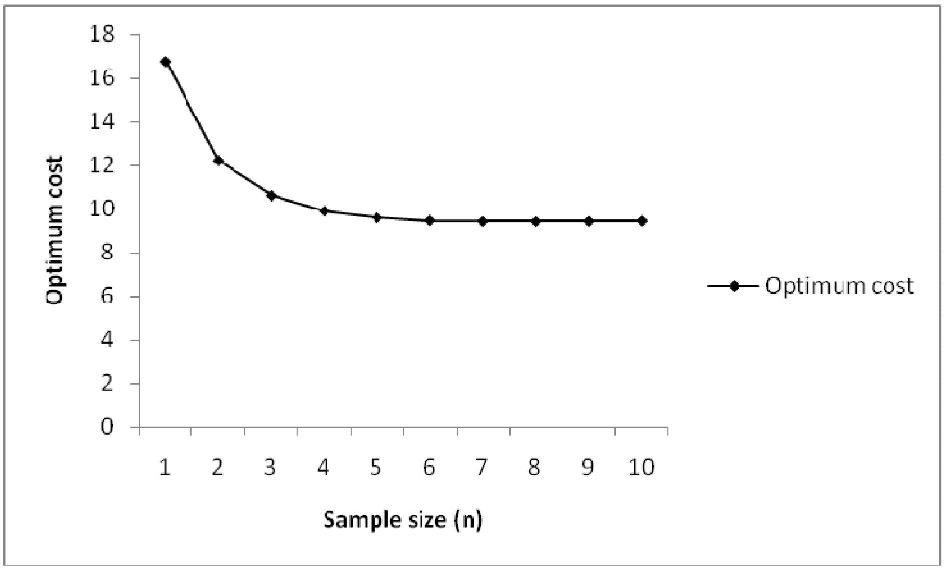

Fig.5.1. Optimum Cost for 3-sigma Limit (k=3)

Assuming the width of control limit as a variable, the cost function will have two independent variables h and k for a given value of n. This function was similarly minimized using PSO and the corresponding optimum values of h and k along with the value of minimum cost for various integer values of n from 1 to 10 have been shown in Table 5.2. Here also the same trend was observed as shown in Table 5.1 and also from Fig. 5.2 it is observed that as the sample size n increases the cost decreases to a minimum value at n = 9 and then it increases. Thus, the best economic design of $\overline{X}$ chart is obtained at n = 9, h= 0.963, and k=3.360 and the corresponding optimum minimum cost is 9.352.

Table 5.2. Optimum Design of $\overline{X}$ Chart

Sample size, n	**Optimum sampling interval, h**	**Optimum width of control limit, k**	**Optimum cost, E(L)**
1	0.470	2.260	14.552
2	0.772	2.467	11.669
3	0.957	2.45	10.587
4	0.977	2.654	9.979
5	0.973	3.017	9.666
6	0.994	3.034	9.498
7	0.894	3.500	9.412
8	0.920	3.651	9.375
9	**0.963**	**3.360**	**9.352**
10	0.963	3.360	9.362

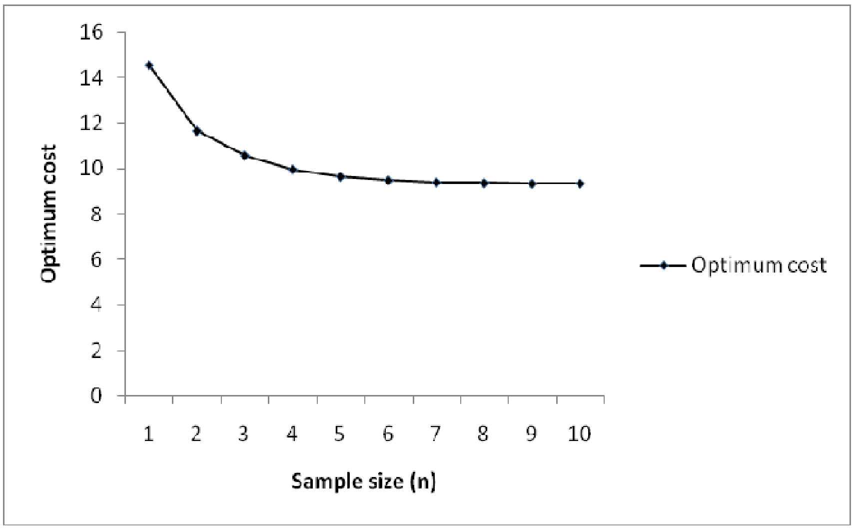

Fig.5.2. Optimum Cost for Variable k

Furthermore, the results are compared with the optimum design of $\overline{X}$ chart for the same example reported in literature as shown in Table 5.3 from which it is observed that for each value of n the cost obtained using PSO is less. Compared to other sample sizes, the minimum cost was obtained with sample size of 5 as reported by both Montgomery [22] i.e., cost 10.38 and Shiau [8] i.e. 10.367. But the minimum cost i.e., 9.352 is obtained with a different sample size i.e. 9 using PSO. Even at sample size of 5, the minimum cost i.e., 9.666 obtained using PSO is less than 10.38 and 10.367 as obtained by Montgomery and Shiau respectively and also from Fig. 5.3 and Fig.5.4 shows that cost obtained by PSO is less than that Montgomery and Shiau cost for each value of sample size.

Table 5.3.Comparison of Results

Sample size, n	**Optimum cost, E(L)**		**PSO optimum cost, E(L)**
	Montgomery [22]	**Shiau [8]**	
1	14.71	-	14.552
2	11.91	-	11.669
3	10.90	-	10.587
4	10.51	-	9.979
5	**10.38**	**10.367**	9.666
6	10.39	10.413	9.498
7	10.48	-	9.412
8	10.60	-	9.375
9	10.75	11.248	**9.352**
10	10.90	-	9.362

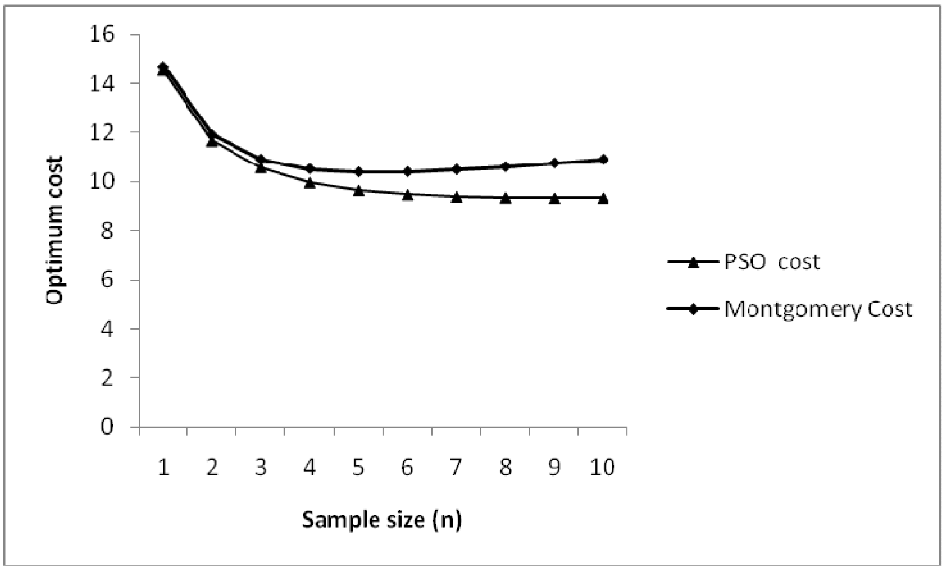

Fig.5.3. Comparison of Cost (PSO & Montgomery)

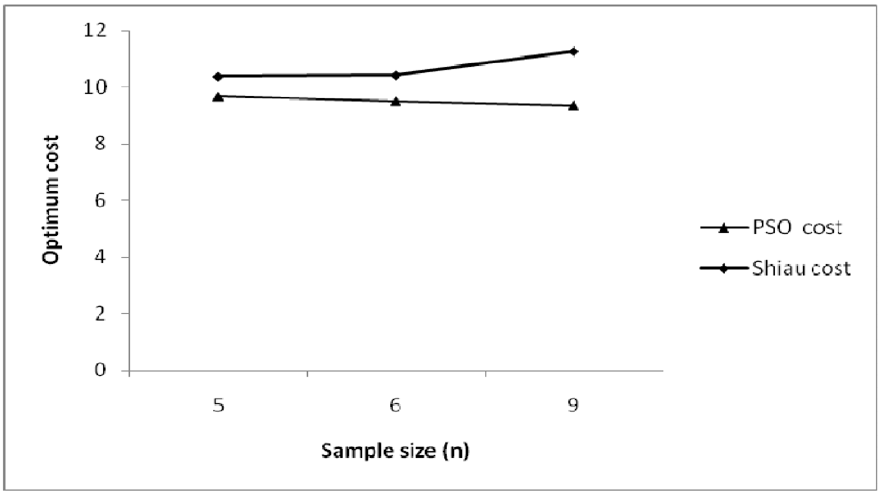

Fig.5.4 Comparison of Cost (PSO & Shiau)

Chapter 6

Conclusions

In this project work, the $\overline{X}$ control chart has been designed for economic purpose by using Particle Swarm Optimization and based on the results the following conclusions are made.

- The economic design of $\overline{X}$ chart using PSO is found to be superior to that obtained earlier in the literature.
- For $\overline{X}$ chart with 3-sigma limits (k = 3), minimum cost (i.e., 9.419) is obtained with sample size n = 8 and sampling interval h = 0.937.
- With variable k, minimum cost (i.e., 9.352) is obtained with sample size n=9, sampling interval h = 0.963 and width of control limit k = 3.360.
- The best economic design is thus found to be at sample size 9, width of control limits 3.36 sigma and sampling interval 0.963 h

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