VIBRATION ANALYSIS OF BEAM WITH MULTIPLE CRACKS

A thesis submitted in the partial fulfillment of the requirements for the degree of

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IN

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BY

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2011

CERTIFICATE

This is to certify that the Project Report entitled, "**VIBRATION ANALYSIS OF BEAM WITH MULTIPLE CRACKS**" submitted by **Sonam Lakra(Roll-107CE011)**and **Pradeep Guria(Roll-107CE010)** in partial fulfillment for the requirements for the award of the Degree of Bachelor of Technology in Civil Engineering at National Institute of Technology, Rourkela(Deemed university) is an authentic work carried out by them under my supervision and guidance. To the best of my knowledge, the matters embodied in the thesis have not been submitted to any other university/Institute for the award of any Degree or Diploma .

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Abstract

The present work deals with the free vibration analysis of a cracked beam with multiple transverse cracks using finite element method. In this analysis, an 'overall additional flexibility matrix', instead of the 'local additional flexibility matrix' is added to the flexibility matrix of the corresponding intact beam element to obtain the total flexibility matrix, and from there the result is compared with previous studies. The natural frequencies of free vibration of the beam with multiple cracks are computed. It is observed that with increase in number of cracks the natural frequencies decreases. The effect of cracks is more pronounced when the cracks are near to the fixed end than free end. The natural frequency decreases with increase in relative crack depth.

Chapter 1

INTRODUCTION

Introduction

For the last several years, a considerable amount of research work has been undertaken to investigate the faults in structures. It has been observed that most of the structural members fail due to the presence of cracks. The cracks are developed mainly due to fatigue loading. Therefore the detection of cracks is an important aspect of structural design. A crack that occurs in a structural element causes some local variation in its stiffness, which affects the dynamic behavior of the element and the whole structure to a considerable degree. The frequencies of natural vibration, amplitudes of forced vibration, and areas of dynamic stability change due to the existence of such cracks. An analysis of these changes makes it possible to identify the magnitude and location of the crack. This information enables one to determine the degree of sustainability of the structural element and the whole structure. In this study, the presence of transverse and open crack in the structure has been considered. Also the crack depth This crack introduces new boundary conditions for the structure at the crack location. These boundary conditions are derived from the strain energy equation using Castigliano's theorem. Presence of crack also reduces the stiffness of the structure which has been derived from the stiffness matrix. For dynamic behaviors of beam with a transverse crack,Timoshenko beam theory with modified boundary conditions have been used to find out the theoretical expressions for the natural frequencies and the modes for the beam. For all the theoretical expressions as derived for dynamic characteristics of structure with a crack, respective numerical analysis was taken up with suitable numerical models with the help of the computer programme.

Chapter 2

LITERATURE REVIEW

Literature review:

A.D. Dimarogonas [i] :The fact that a crack or local defect affects the dynamic response of a structural member was known long ago. The first attempts to quantify local defects were by Kirmsher and A.D. Dimarogonas, Thomson. The effect of a notch on the structure flexibility was simulated by a local bending moment or reduced section, with magnitudes which were estimated by experimentation. The analysis of the local flexibility of a cracked region of structural element was quantified in the 1950s by Irwin, Bueckner, Westmann and Yang by relating local flexibility to the crack stress intensity factor (SIF). Based on this principle, a method was developed for the computation of the SIF based on the local bending stiffness (the inverse of the local flexibility) of a cracked prismatic beam, determined experimentally. Liebowitz and co-workers utilized existing results from fracture mechanics to calculate the local flexibility of a beam of rectangular cross-section b x h with a transverse surface crack of uniform depth a. Using the fracture mechanics relations between the strain energy release rate and stress intensity factor and the Castigliano theorem, they computed the cracked region local flexibility,for plane strain, as:

$C = M/\Delta Ø= (6\pi h/bEI)\ F1(s),$

Where h is the height, b=the width of the rectangular cross-section, EI= the flexural rigidity,

s = a/h, a = crack depth and

$F_l(s) = 1.86s^{2} - 3.95s^{3} + 16.37s^{4} + 37.22s^{5} + 76.81s^{6} + 126.9s^{7} + 172.5s^{8} - 144s^{9} + 66.6s^{l0}.$

Fine-mesh finite element techniques were used to compute local flexibilities by Gudmunson , Rauch, Chen and Wang, Haisty et al., and Krawczuk, Ostachowicz, Schmalhorst, and Qian et al. A local flexibility will reduce the stiffness of a structural member, thus reducing its natural frequency. Dimarogonas has noticed that eq. (1) suggests that for small crack depth the local flexibility Ac is proportional to $(a/h)^2$. Since this flexibility is added to the flexibility c of the uncracked member, the total flexibility will be $C = c + \Delta c$ and the corresponding stiffness

$K = 1/C$

The most popular parameter applied in identification methods is change in natural frequencies of structure caused by the crack. It is due to the fact that estimates of the natural frequencies can be obtained from measurement of the vibration at only one point on the structure. The drawback, that escaped the attention of most investigators, is that the change of natural frequency due to the crack is proportional to the square of the relative crack depth or for redundant systems, as shown by Dimarogonas, thus insignificant for most practical identification needs. The identification problem was discussed by Chang and Petroski, Kozlow and Shatoff. It was formally introduced for cracked beams by Chondros and Chondros and Dimarogonas, who developed nomograms for the calculation of the crack depth for different beams and locations of the crack (assumed known). Adams et al. have developed an experimental technique to estimate the location and depth of the crack from changes in the natural frequencies. Cawley and Adams and Anifantis et al. further expanded the spectral method for identification of defects in beams and frames by analyzing the changes in the vibration frequency spectrum. Yuen presented a systematic study of the relationship between damage location and size, and the changes in the eigenvalues and eigenvectors of a cantilever beam. Anifantis et al. and Rizos and co-workers developed an identification technique for location and magnitude of the crack in a cantilever beam with a rectangular cross-section having a transverse surface crack on the basis of more than one natural frequency change.

M Kisa et al [ii]:The vibrational characteristics of a cracked Timoshenko beam are analysed. The study integrates the finite element method and component mode synthesis. The beam divided into two components related by a flexibility matrix which incorporates the interaction forces. These forces can be derived from fracture mechanics theory as the inverse of the compliance matrix calculated using stress intensity factors and strain energy release rate expressions. Each substructure is modelled by Timoshenko beam finite elements with two nodes and 3 degrees-of-freedom (axial, transverse and rotation) at each node.]

E. I. SHIFRIN [iii]:A new technique is proposed for calculating natural frequencies of a vibrating beam with an arbitrary finite number of transverse open cracks.The main feature of this method is related to decreasing the dimension of the matrix involved in the calculation, so that reduced computation time is required for evaluating natural frequencies compared to alternative methods which also make use of a continuous model of the beam.]

N. T. KHIEM et al [iv]: A new method for natural frequency analysis of beam with an arbitrary number of cracks is developed on the bases of the transfer matrix method and rotational spring model of crack. The resulted frequency equation of a multiple cracked beam is general with respect to the boundary conditions including the more realistic (elastic) end supports and can be constructed analytically by using symbolic codes. The procedure proposed is advanced by elimination of numerical computation of the high order determinant so that the computer time for calculating natural frequencies in consequence is significantly reduced. Numerical computation has been carried out to investigate the effect of each crack, the number of cracks and boundary conditions on the natural frequencies of a beam.

D.Y. Zheng et al [v]: The natural frequencies and mode shapes of a cracked beam are obtained using the finite element method. An 'overall additional flexibility matrix', instead of the 'local additional flexibility matrix', is added to the flexibility matrix of the corresponding intact beam element to obtain the total flexibility matrix, and therefore the stiffness matrix. Compared with analytical results, the new stiffness matrix obtained using the overall additional flexibility matrix can give more accurate natural frequencies than those resulted from using the local additional flexibility matrix.In addition, the authors constructed a shape function that can perfectly satisfy the local flexibility conditions at the crack locations, which can give more accurate vibration modes.

Chapter 3

THEORY AND FORMULATION

Theory and Formulation:

The equation of motion for vibration of beam under load 'P' in the form of matrix is written[ii] as

$$[M]\{\ddot{u}\} + \left|[K] - P\lfloor K_g \rfloor\right| \{u\} = 0 \qquad (1)$$

Where, $[M]$= Consistent mass matrix

$[K]$= Bending stiffness matrix of the beam

$\lfloor K_g \rfloor$= Geometric stiffness matrix

The above equation for free vibration can be written as ,

$$[M]\{\ddot{u}\} + [K]\ \{u\} = 0 \qquad (2)$$

Above equation represents an eigen value problem and the solution for the equation is square of the natural frequency given by the equation[v],

$$[K] - (w_n)^2[M] = 0 \qquad (3)$$

Stiffness matrix for cracked beam element

The stiffness matrix of a cracked beam is obtained by taking the inverse of total flexibility matrix. This is the most convenient method to obtain the stiffness matrix of a cracked beam element. To find out the total flexibility matrix, the additional flexibility matrix due to the existence of the crack is added to the original flexibility matrix of the intact beam.

a) The original stiffness matrix for the intact beam

In the present analysis two noded beam elements with two degree of freedom (deflection and slope) per node is considered.

$$Ke = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \quad (4)$$

b) Stiffness matrix for cracked beam

The Fig.1 shows a typical cracked beam element with a rectangular cross section of breadth '*b*' and depth '*h*' with a crack of depth '*a*'. The left hand side end node '***i***' is assumed to be fixed, while the right hand side end node '***j***' is subjected to shearing force P_1 and bending moment P_2. The corresponding generalized displacements are denoted as q_1 and q_2.

L_c = Distance between the right hand side end node *j* and the crack location

Le = Length of the beam element

A = Cross-sectional area of the beam

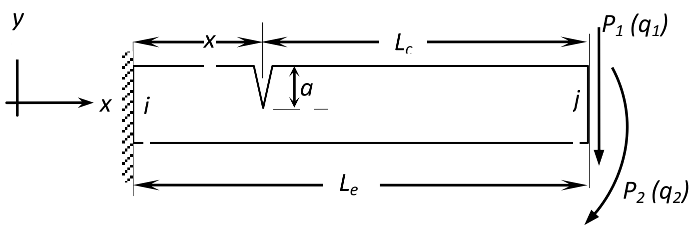

Figure 1.a typical cracked beam element subjected to shearing force and bending moment[v]

According to Zheng [i], the additional strain energy due to existence of crack can be expressed as

$$\pi = \int_A G\, dA_c \quad (5)$$

Where, G = the strain energy release rate and A_C = the effective cracked area

$$G = \frac{1}{E'}[(\sum_{n=1}^{2} K_{In})^2 + (\sum_{n=1}^{2} K_{IIn})^2 + k(\sum_{n=1}^{2} K_{IIIn})^2] \qquad (6)$$

Where, $E' = E$ for plane stress

$= E/1\text{-}\upsilon^2$ for plane strain

$k = 1+ \upsilon$

K_I, K_{II} *and* K_{III} = stress intensity factors for opening, sliding and tearing type cracks respectively.

Neglecting effect of axial force and for open cracks, Eq.7 can be written as

$$G = \frac{1}{E'}[(K_{I1} + K_{I2})^2 + K_{II1}{}^2] \qquad (7)$$

The expressions for stress intensity factors from earlier studies are given by,

$$K_{I1} = \frac{6P_1 L_c{}^2}{bh^2}\sqrt{\pi\xi}\ F_1\left(\frac{\xi}{h}\right)$$

$$K_{I2} = \frac{6P_2}{bh^2}\sqrt{\pi\xi}\ F_1\left(\frac{\xi}{h}\right)$$

$$K_{I2} = \frac{P_2}{bh}\sqrt{\pi\xi}\ F_{II}\left(\frac{\xi}{h}\right)$$

where,

s= ξ/h

$$F_I(s) = \sqrt{\frac{\tan\left(\pi s/_2\right)}{\left(\pi s/_2\right)}}\left[\frac{0.923 + 0.199\left(1 - sin\left(\pi s/_2\right)\right)^4}{cos\left(\pi s/_2\right)}\right]$$

$$F_{II}(s) = \frac{1.122 - 0.561s + 0.085s^2 + 0.180\, s^3}{\sqrt{1-s}}$$

$F_I(s)$ and $F_{II}(s)$ are correction factors for stress intensive factors.

From definition, the elements of the overall additional flexibility matrix C_{ij} can be expressed as

$$C_{ij} = \frac{\partial \delta_i}{\partial P_j} = \frac{\partial^2 \Pi_c}{\partial P_i \partial P_j}$$

Substituting the values,

$$C_{ij} = \frac{b}{E'}\frac{\partial^2}{\partial P_i \partial P_j}\int \left[\left\{\frac{6P_1 {L_c}^2}{bh^2}\sqrt{\pi\xi}\ F_1\left(\frac{\xi}{h}\right) + \frac{6P_2}{bh^2}\sqrt{\pi\xi}\ F_1\left(\frac{\xi}{h}\right)\right\}^2 + \left\{\frac{P_2}{bh}\sqrt{\pi\xi}\ F_{II}\left(\frac{\xi}{h}\right)\right\}^2\right] d\,\xi$$

Substituting $i, j\ (1, 2)$ values, we get

$$C_{11} = \frac{2\pi}{E'b}\left[\frac{36{L_c}^2}{h^2}\int_0^{\frac{a}{h}} x{F_1}^2(x)dx + \int_0^{\frac{a}{h}} x{F_{II}}^2(x)dx\right]$$

$$C_{12} = \frac{72\pi L_c}{E'bh^2}\int_0^{\frac{a}{h}} x{F_1}^2(x)dx = C_{21}$$

$$C_{22} = \frac{72\pi}{E'bh^2}\int_0^{\frac{a}{h}} x{F_1}^2(x)dx$$

Now, the overall flexibility matrix C_{ovl} is given by,

$$C_{ovl} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

Flexibility matrix C_{intact} of the intact beam element

$$C_{intact} = \begin{bmatrix} \frac{{L_e}^3}{3EI} & \frac{{L_e}^2}{2EI} \\ \frac{{L_e}^2}{2EI} & \frac{L_e}{EI} \end{bmatrix}$$

Total flexibility matrix C_{tot} of the cracked beam element

$$C_{total} = C_{intact} + C_{ovl}$$

$$C_{total} = \begin{bmatrix} \frac{{L_e}^3}{3EI} + C_{11} & \frac{{L_e}^2}{2EI} + C_{12} \\ \frac{{L_e}^2}{2EI} + C_{21} & \frac{L_e}{EI} + C_{22} \end{bmatrix}$$

c) Stiffness matrix K_c of a cracked beam element:
From the equilibrium conditions as shown in Fig.2,

$$(V_1 \quad \Theta_1 \quad V_2 \quad \Theta_2)^T = [L](V_2 \quad \Theta_2)^T$$

Where the transformation matrix is,

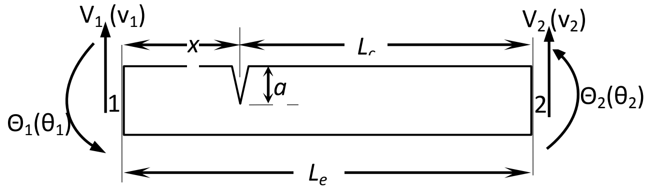

Figure 2.: typical cracked beam element subject to shearing force and bending moment (under the conventional fem coordinate system[v]

$$L = \begin{bmatrix} -1 & 0 \\ -L_e & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Hence the stiffness matrix K_c of a cracked beam element can be obtained as

$$K_c = L\, C_{tot}^{-1} L^T$$

Where L is the transformation matrix for equilibrium condition. Similarly standard procedure is carried out to find mass and geometric matrix.

In this study we have considered the change in mass matrix and geometric matrix to negligible for cracks.

Hence the mass matrix M_e of a cracked beam for a flexural beam is

$$Me = \frac{\rho AL}{420} \begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix}$$

Chapter 4

RESULT AND DISCUSSION

Numerical results and discussion:

Quantitative results on the effects of various parameters on the vibration of beams with crack are presented. The results are studied as:

- Convergence study
- Comparison with previous studies(with Shiffrin et al)
- Numerical Results

Convergence study:

For more accuracy, the convergence study for the beam has been done. Based on finite element method, more the number of elements more is the accuracy. Fig.4.1.1 shows that, for 16 elements the variation of frequency is very small. So, in this study we have considered 16 elements.

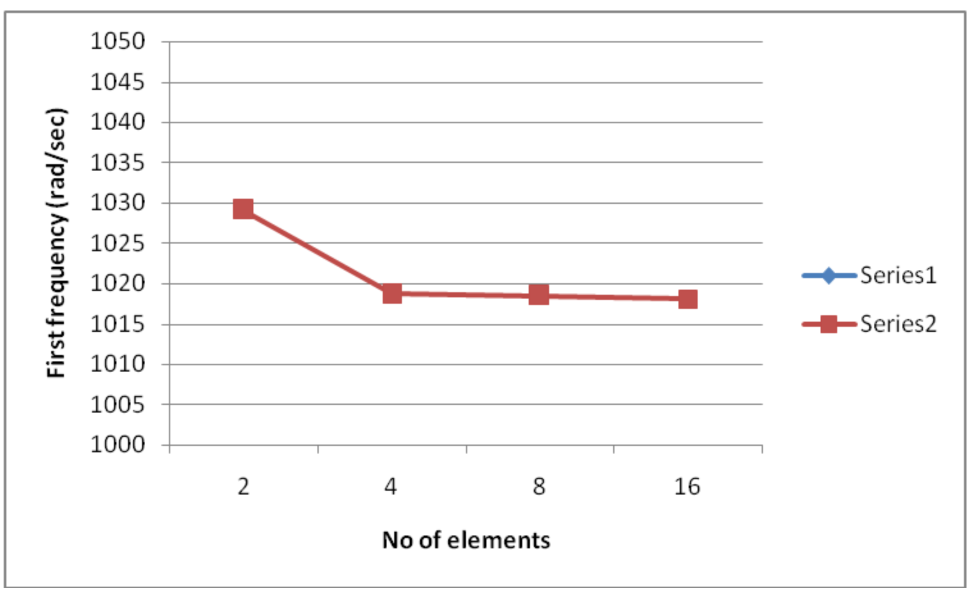

Figure 3.convergence graph

Comparison with previous studies:

The Finite element analysis is carried out for free vibration of cracked beams for various positions of crack and crack depths considered by Shifrin *et al.*[iii]. The beam under analysis has the following properties : length l=0.8m, rectangular cross-section with width b=0.02 m and height h=0.02m, a first crack with position x1=0.12m and depth a1=2mm, a second crack with variable position from the fixed end and a depth of 2, 4 and 6mm. The ratio between the first natural frequencies of cracked and uncracked beam is shown in Table 4.1.1 with the results of Shifrin *et al.* [iii]. The values are interpolated using graph analysis software (DigitizeIt_win) from reference [iii]. It is clear that there exists an fair agreement between the results obtained from present FEM with previous studies.

Table 1. comparison with previous study

Location of 2^{nd} crack	Rcd=0.1		Rcd=0.2		Rcd=0.3	
	Present analysis	Shiffrin et al.	Present analysis	Shiffrin et al.	Present analysis	Shiffrin et al.
0.05	0.9926	0.994	0.971	0.984	0.937	0.968
0.1	0.9931	0.994	0.974	0.987	0.943	0.974
0.2	0.9945	0.995	0.979	0.990	0.953	0.981
0.4	0.9961	0.997	0.985	0.996	0.967	0.994
0.5	0.9965	0.997	0.986	0.997	0.969	0.997
0.6	0.9967	0.997	0.987	0.997	0.971	0.997
0.7	0.9967	0.997	0.987	0.997	0.972	0.997

Numerical results

The study is then carried out for vibration of a cantilever beams with crack. For this, the geometry and material properties of the cantilever beam are:

Breath of the beam	= 0.2 m	Depth of the beam	= 0.2 m
Length of the beam	= 0.3 m	Unit Weight	= 7850 kg/m^3
Elastic modulus of the beam	= 206×10^9 N/m^2	Poisson's Ratio	= 0.28

For this analysis a first crack position is fixed at a distance of x1 = 0.2L from fixed end with relative crack depth 0.2 and a second crack is introduced with variable position from clamped end. The variation of first natural frequency of vibration with location of crack (y = 0.2L to 0.8L) for different relative crack depth (rcd = 0.2 to 0.8) is shown in fig.4. The variations of natural frequencies as expected are decreasing with increase in crack depth. However this change is more rapidly decreasing for the cracks nearing the fixed end and increasing relative crack depth.

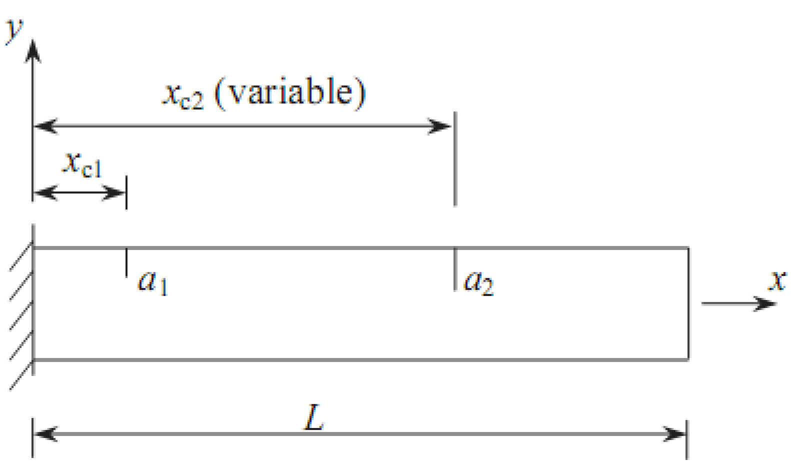

Figure 4.A cantilevered beam with two cracks while the location of the second crack is variable[v]

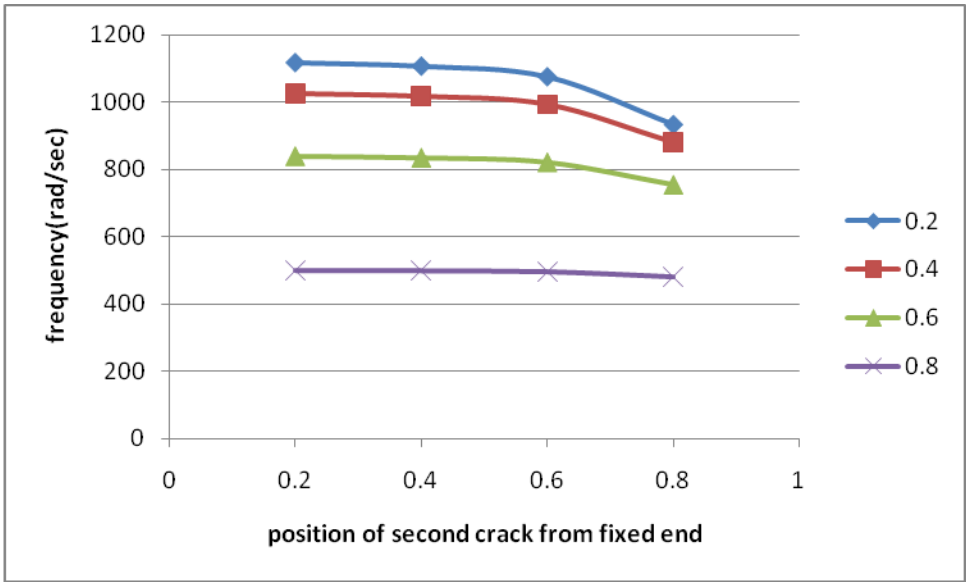

Figure 5. 1st crack at 0.2L and 2nd crack at 0.4L

Table 2 Variation of excitation frequency in (rad/sec) for x1 = 0.2L and x2=0.4L and (a/h) = 0.2,0.4

Position of 2nd crack	Rcd=0.2				Rcd=0.4			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	1109.420	7002.155	19189.509	36090.066	1070.025	6635.329	18728.847	35342.695
0.4	1019.229	6990.063	18778.448	34291.589	157.33	1052.283	2933.367	5289.872
0.6	835.511	6959.644	18009.794	31885.769	818.362	6561.114	17830.787	30390.389
0.8	498.174	6862.588	16887.142	29761.758	494.481	6445.204	16848.079	27912.302

Table 3 Variation of excitation frequency in (rad/sec) for x1 = 0.2L and x2=0.4L and (a/h) = 0.6,0.8

Position of 2nd crack	Rcd=0.6				Rcd=0.8			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	968.545	5938.337	17979.244	34148.400	682.170	4924.462	17079.105	32643.702
0.4	907.037	5874.453	17825.272	31622.827	659.962	4752.896	17028.707	29890.084
0.6	770.514	5747.644	17474.801	28149.518	602.083	4394.725	16897.604	25959.584
0.8	483.401	5533.906	16754.174	25092.371	432.835	3780.012	16520.989	22273.905

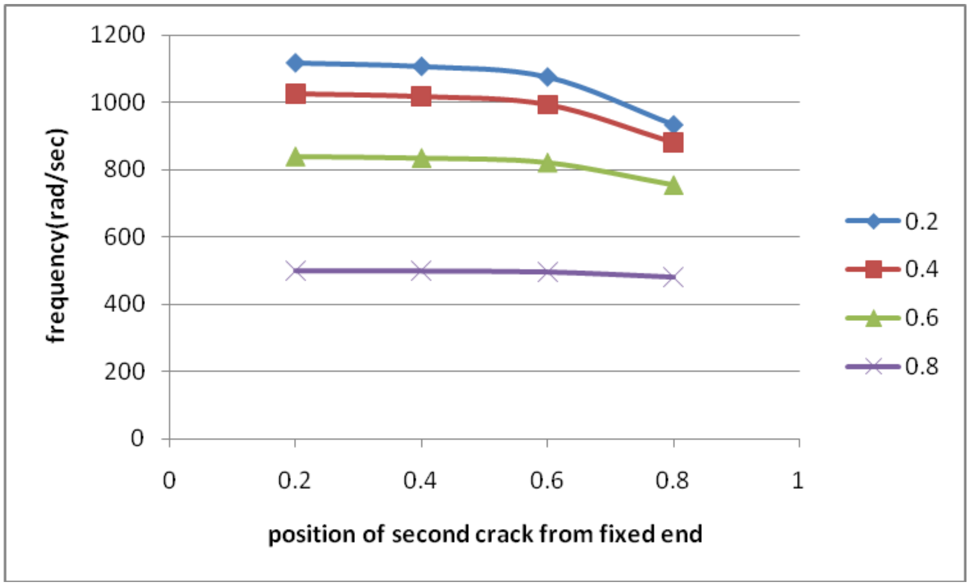

Figure 6 1st crack at 0.2L and 2nd crack at 0.6L

Table 4 Variation of excitation frequency in (rad/sec) for x1 = 0.2L and x2=0.6L and (a/h) = 0.2,0.4

Position of 2nd crack	Rcd=0.2				Rcd=0.4			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	118.333	6957.218	19091.298	36325.231	1026.198	6946.476	18632.847	34749.021
0.4	1107.852	6451.81	18396.611	36023.811	1018.118	6435.557	17949.232	34574.545
0.6	1076.339	5417.672	17349.957	35504.908	993.663	5383.025	16926.403	34251.048
0.8	933.5324	3672.165	16263.18	34676.481	879.247	3577.650	15877.464	33617.112

Table 5 Variation of excitation frequency in (rad/sec) for x1 = 0.2L and x2=0.6L and (a/h) = 0.6,0.8

Position of 2nd crack	Rcd=0.6				Rcd=0.8			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	839.297	6918.93	17803.504	32602.316	498.972	6827.927	16636.382	30640.065
0.4	834.896	6400.222	17143.674	32529.988	498.053	6312.921	15993.393	30597.556
0.6	821.422	5318.004	16166.407	32369.993	495.199	5212.307	15050.063	30488.817
0.8	754.727	3406.698	15183.893	31918.321	480.025	3176.007	14124.85	30114.544

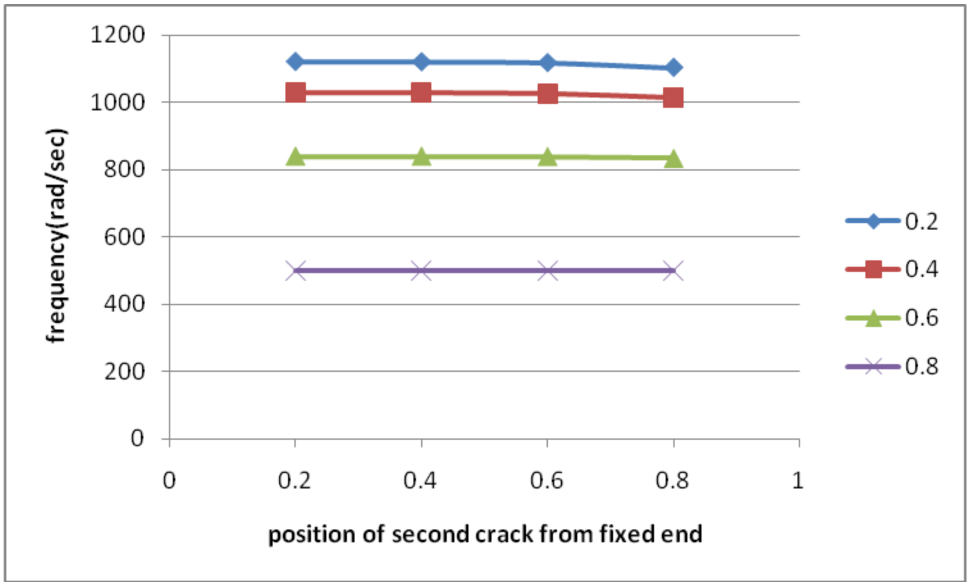

Figure 7 1st crack at 0.2L and 2nd crack at 0.8L

Table 6 Variation of excitation frequency in (rad/sec) for x1 = 0.2L and x2=0.8L and (a/h) = 0.2,0.4

Position of 2nd crack	Rcd=0.2				Rcd=0.4			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	1121.028	7096.268	18994.459	35302.425	1028.272	7086.686	18575.278	33739.885
0.4	1120.202	6976.376	17883.456	32969.180	1027.638	6966.291	17566.02	31525.896
0.6	1117.621	6605.562	15372.778	29914.573	1025.66	6593.648	15209.343	28562.637
0.8	1103.338	5040.085	11634.452	27654.645	1014.723	5018.758	11579.916	26364.886

Table 7 Variation of excitation frequency in (rad/sec) for x1 = 0.2L and x2=0.8L and (a/h) = 0.6,0.8

Position of 2nd crack	Rcd=0.6				Rcd=0.8			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	840.423	7060.597	17804.597	31616.063	499.206	6967.304	16697.154	29699.505
0.4	840.08	6939.952	16963.460	29501.711	499.136	6850.597	16037.116	27644.306
0.6	839.014	6565.910	14883.461	26574.713	498.916	6486.867	14299.703	24648.01
0.8	833.127	4979.606	11459.953	24395.117	497.706	4917.438	11160.509	22355.266

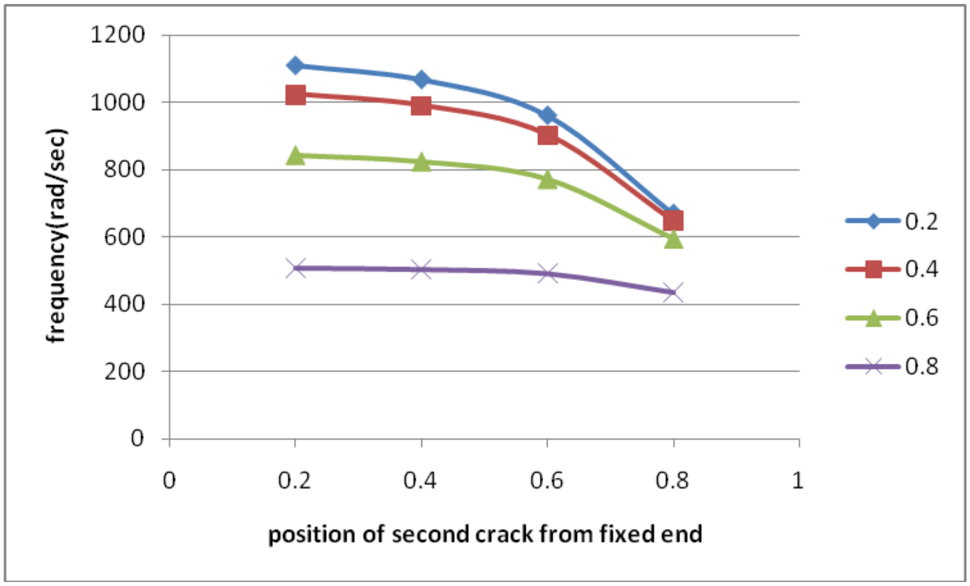

Figure 8 1st crack at 0.4L and 2nd crack is at 0.2L

Table 8 Variation of excitation frequency in (rad/sec) for x1 = 0.4L and x2=0.2L and (a/h) = 0.2,0.4

Position of 2nd crack	Rcd=0.2				Rcd=0.4			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	1109.948	7013.684	19114.003	36193.503	1023.407	7008.414	18615.168	34463.490
0.4	1067.908	6675.271	18554.864	35648.632	990.192	6663.687	18203.683	33581.760
0.6	961.029	6026.508	17652.612	34702.384	903.329	5983.609	17483.908	32142.834
0.8	668.503	5071.795	16553.582	33196.760	648.401	4931.835	16508.286	30409.979

Table 9 Variation of excitation frequency in (rad/sec) for x1 = 0.4L and x2=0.2L and (a/h) = 0.6,0.8

Position of 2nd crack	Rcd=0.6				Rcd=0.8			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	844.256	6993.546	17703.665	32192.644	507.719	6937.349	16466.903	30238.081
0.4	825.339	6637.503	17505.311	30770.579	503.524	6570.004	16430.053	28316.286
0.6	773.142	5895.539	17098.595	28498.811	491.002	5738.792	16333.044	25193.895
0.8	595.367	4624.632	16388.814	26145.086	435.057	4045.017	16043.069	21911.861

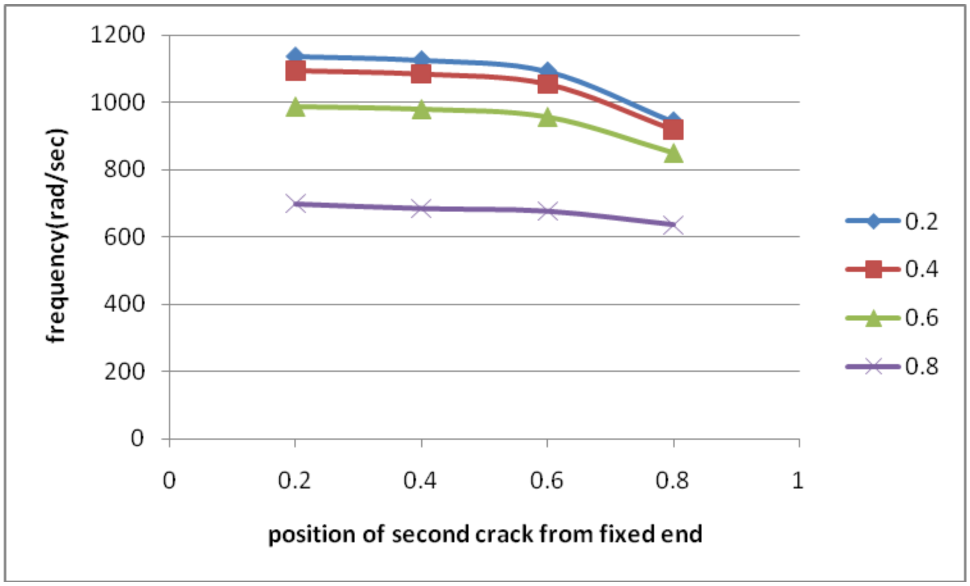

Figure 9 1st crack at 0.4L and 2nd crack is at 0.6L

Table 10 Variation of excitation frequency in (rad/sec) for x1 = 0.4L and x2=0.6 and (a/h) = 0.2,0.4

Position of 2nd crack	Rcd=0.2				Rcd=0.4			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	1136.546	6834.331	19046.067	36904.639	1094.251	6505.831	18493.421	36283.612
0.4	1125.492	6367.061	18283.455	36565.844	1084.262	6128.721	17580.721	36054.285
0.6	1092.322	5391.987	17121.531	35983.49	1054.238	5291.962	16154.415	35629.833
0.8	943.282	3703.117	15908.454	35092.934	918.056	3701.752	14645.967	34893.577

Table 11 Variation of excitation frequency in (rad/sec) for x1 = 0.4L and x2=0.6L and (a/h) = 0.6,0.8

Position of 2nd crack	Rcd=0.6				Rcd=0.8			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	986.281	5875.120	17607.825	35246.039	688.134	4946.037	16580.037	33793.785
0.4	978.761	5643.042	16432.017	35141.997	685.446	4862.281	15105.633	33764.76
0.6	956.051	5064.822	14502.785	34925.473	677.201	4614.782	12510.88	33693.547
0.8	850.348	3697.967	12367.385	34445.437	635.841	3685.199	9273.710	33463.669

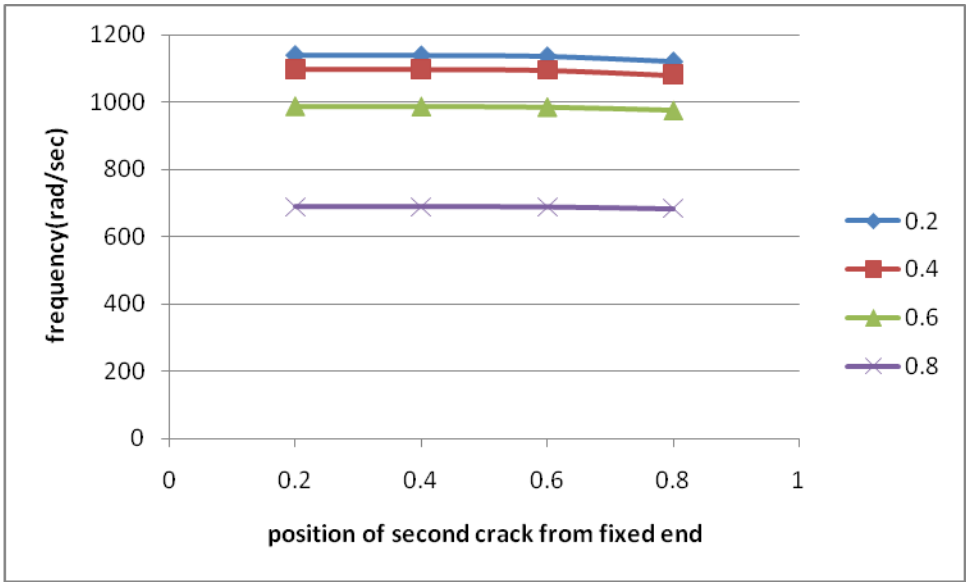

Figure 10 1st crack is at 0.4L and 2nd crack is at 0.8L

Table 12 Variation of excitation frequency in (rad/sec) for x1 = 0.4L and x2=0.8L and (a/h) = 0.2,0.4

Position of 2nd crack	Rcd=0.2				Rcd=0.4			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	1139.391	6961.855	18951.113	35932.668	1096.821	6606.359	18448.218	35423.026
0.4	1138.516	6852.517	17781.28	33640.328	1096.027	6521.036	17255.668	33376.304
0.6	1135.785	6511.913	15185.116	30601.791	1093.55	6250.75	14616.664	30519.744
0.8	1120.676	5025.793	11351.68	28306.163	1079.863	4964.833	10632.177	28255.921

Table 13 Variation of excitation frequency in (rad/sec) for x1 = 0.4L and x2=0.8L and (a/h) = 0.6,0.8

Position of 2nd crack	Rcd=0.6				Rcd=0.8			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	988.213	5934.143	17652.21	34557.814	688.822	4966.039	16733.711	33251.921
0.4	987.610	5883.853	16437.310	32879.089	688.603	4947.060	15530.707	31890.494
0.6	985.731	5720.945	13744.922	30313.395	687.921	4884.693	12828.680	29615.224
0.8	975.370	4814.383	9454.010	28104.155	684.173	4469.645	8120.348	27482.568

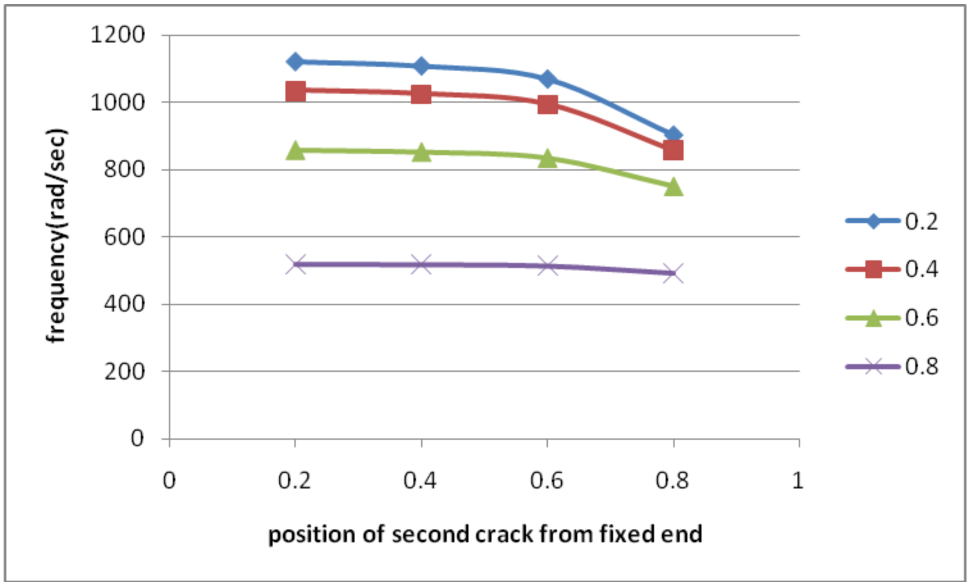

Figure 11 1st crack is at 0.6L and 2nd crack is at 0.2L

Table 14 Variation of excitation frequency in (rad/sec) for x1 = 0.6L and x2=0.2L and (a/h) = 0.2,0.4

Position of 2nd crack	Rcd=0.2				Rcd=0.4			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	1120.352	6947.579	19094.132	36176.031	1035.283	6943.632	18403.930	34771.562
0.4	1107.332	6421.413	18604.449	35353.261	1024.998	6418.012	17888.699	34140.394
0.6	1068.656	5380.207	17823.856	33979.123	994.207	5363.590	17084.964	33034.212
0.8	902.588	3709.700	16887.103	31792.062	857.335	3630.209	16169.214	31044.195

Table 15 Variation of excitation frequency in (rad/sec) for x1 = 0.6L and x2=0.2L and (a/h) = 0.6,0.8

Position of 2nd crack	Rcd=0.6				Rcd=0.8			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	857.493	6933.816	17218.892	32976.421	518.506	6901.823	15769.105	31466.228
0.4	851.638	6409.880	16668.379	32553.345	517.209	6385.020	15180.018	31187.507
0.6	833.858	5331.148	15836.590	31750.227	513.192	5277.346	14319.155	30601.309
0.8	749.443	3475.416	14956.429	30002.380	492.221	3241.657	13475.333	29045.479

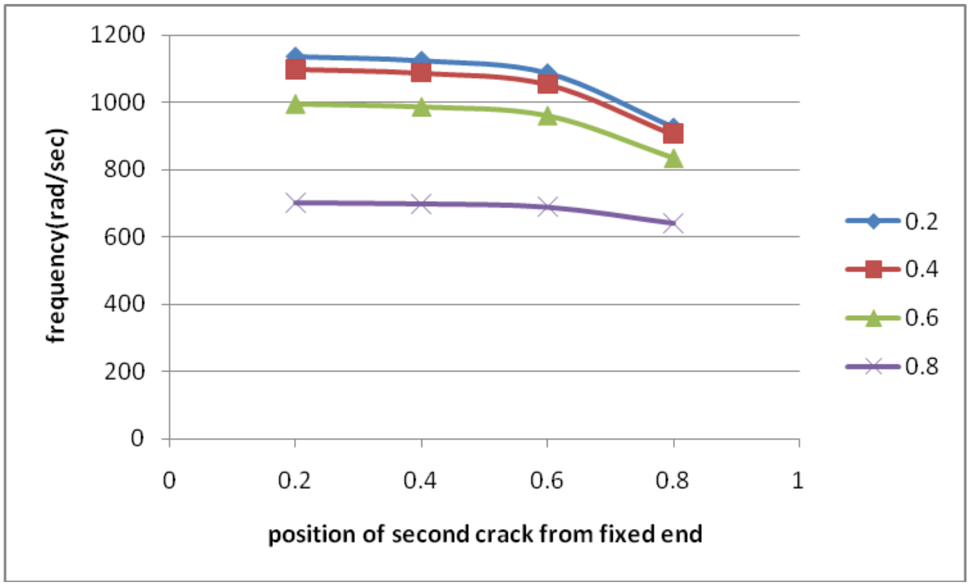

Figure 12 1st crack at 0.6L and 2nd crack is at 0.4L

Table 16 Variation of excitation frequency in (rad/sec) for x1 = 0.6L and x2=0.4L and (a/h) = 0.2,0.4

Position of 2nd crack	Rcd=0.2				Rcd=0.4			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	1137.030	6819.830	19135.111	36727.744	1097.360	6466.389	18688.387	35883.849
0.4	1124.701	6343.506	18493.129	36180.358	1086.137	6085.723	17904.635	35528.968
0.6	1087.929	5367.160	17485.780	35238.064	1052.608	5256.357	16635.167	34850.113
0.8	927.107	3719.617	16362.817	33752.491	904.538	3717.102	15204.141	33594.058

Table 17 Variation of excitation frequency in (rad/sec) for x1 = 0.6L and x2=0.4L and (a/h) = 0.6,0.8

Position of 2nd crack	Rcd=0.6				Rcd=0.8			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	994.762	5792.346	17958.393	34546.176	702.678	4807.474	17106.294	32999.911
0.4	986.163	5564.488	16913.555	34400.376	699.480	4731.695	15736.444	32964.313
0.6	960.338	5005.458	15117.524	34070.409	689.704	4510.949	13189.642	32864.020
0.8	843.172	3710.467	12987.947	33228.498	641.540	3690.298	9753.141	32464.323

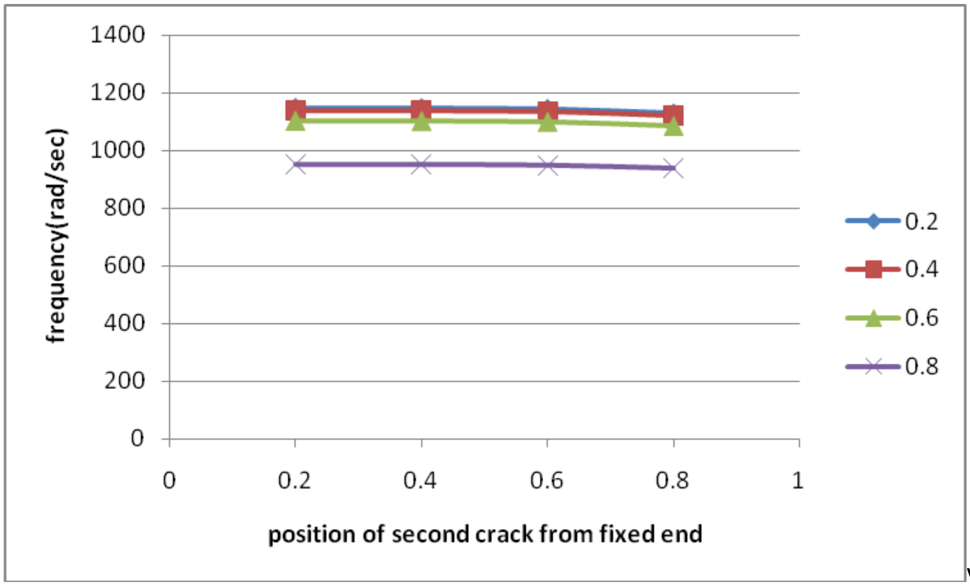

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Figure 13 1st crack is at 0.6L and 2nd crack is at 0.8L

Table 18 Variation of excitation frequency in (rad/sec) for x1 = 0.6L and x2=0.8 and (a/h) = 0.2,0.4

Position of 2nd crack	Rcd=0.2				Rcd=0.4			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	1149.051	6916.746	18908.733	35925.787	1137.673	6421.853	18284.867	35413.425
0.4	1148.155	6805.522	17859.786	33276.542	1136.789	6332.129	17478.482	32306.314
0.6	1145.357	6462.179	15411.362	29828.642	1134.031	6056.093	15362.105	28145.435
0.8	1129.872	4993.301	11580.974	27330.341	1118.779	4832.355	11408.67	25207.934

Table 19 Variation of excitation frequency in (rad/sec) for x1 = 0.6L and x2=0.8L and (a/h) = 0.6,0.8

Position of 2nd crack	Rcd=0.6				Rcd=0.8			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	1103.544	5408.619	17318.118	34585.281	950.606	3698.825	16278.717	33521.520
0.4	1102.701	5356.601	16831.224	30812.661	949.977	3686.771	16042.659	29245.631
0.6	1100.070	5196.374	15259.230	25543.753	948.018	3649.207	15064.315	22901.247
0.8	1085.560	4429.314	11014.194	21969.240	937.292	3447.395	10272.353	18766.913

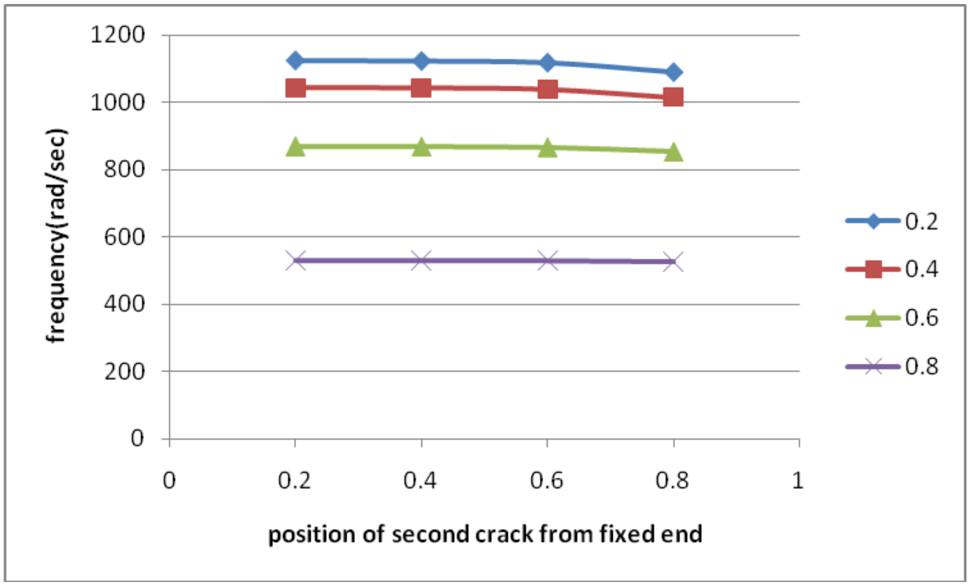

Figure 14 1st crack is at 0.8L and 2nd crack is at 0.2L

Table 20 Variation of excitation frequency in (rad/sec) for x1 = 0.8L and x2=0.2L and (a/h) = 0.2,0.4

Position of 2nd crack	Rcd=0.2				Rcd=0.4			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	1124.907	7072.907	18760.626	35394.475	1042.588	7061.067	18027.293	33944.205
0.4	1123.306	6878.252	17421.790	33408.644	1041.313	6870.418	16836.125	31892.163
0.6	1118.282	6314.429	14912.753	31095.443	1037.314	6311.982	14524.271	29473.198
0.8	1090.650	4472.858	11962.381	29311.955	1015.310	4461.474	11721.499	27638.060

Table 21 Variation of excitation frequency in (rad/sec) for x1 = 0.8L and x2=0.2L and (a/h) = 0.6,0.8

Position of 2nd crack	Rcd=0.6				Rcd=0.8			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	868.281	7038.706	16773.578	32144.117	529.031	6997.276	15273.218	30687.593
0.4	867.546	6854.522	15800.366	29967.849	528.865	6823.595	14500.944	28380.584
0.6	865.238	6306.811	13810.568	27330.120	528.344	6294.374	12850.982	25474.221
0.8	852.530	4439.864	11273.742	25338.788	525.474	4407.551	10644.652	23232.748

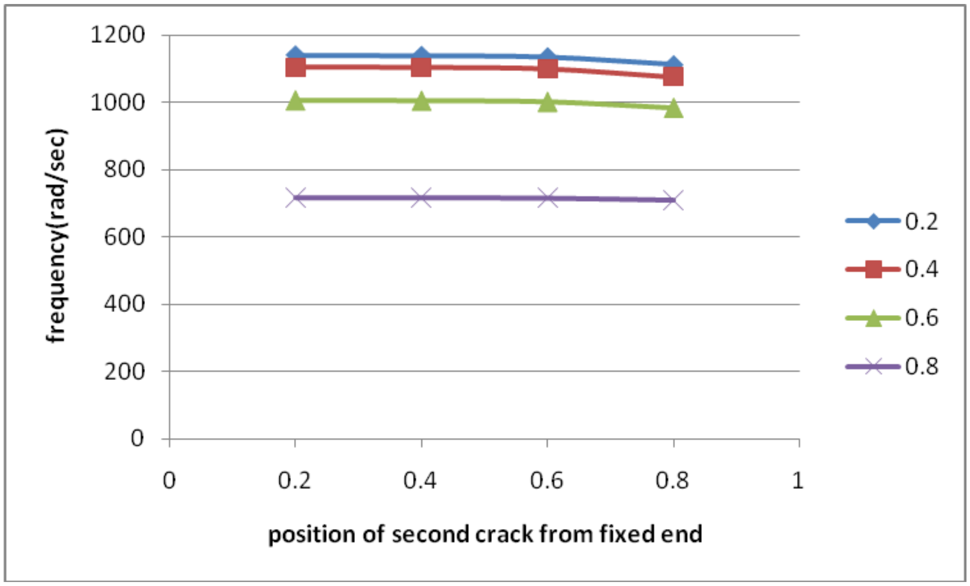

Figure 15 1st crack is at 0.8L and 2nd crack is at 0.4L

Table 22 Variation of excitation frequency in (rad/sec) for x1 = 0.8L and x2=0.4L and (a/h) = 0.2,0.4

Position of 2nd crack	Rcd=0.2				Rcd=0.8			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	1140.933	6928.700	18941.994	35774.906	1103.529	6521.105	18613.276	34731.822
0.4	1139.549	6776.031	17589.449	33821.108	1102.255	6404.291	17188.966	33075.845
0.6	1135.216	6318.492	14944.803	31443.120	1098.267	6045.407	14400.034	31007.344
0.8	111.332	4639.933	11614.127	29592.166	1076.311	4588.002	10807.672	29355.516

Table 23 Variation of excitation frequency in (rad/sec) for x1 = 0.8L and x2=0.4L and (a/h) = 0.6,0.8

Position of 2nd crack	Rcd=0.6				Rcd=0.8			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	1005.589	5759.712	18076.419	33167.664	718.502	4678.040	17457.290	31562.207
0.4	1004.585	5693.955	16550.173	31924.955	718.106	4656.154	15844.318	30670.487
0.6	1001.446	5484.844	13550.602	30309.917	716.866	4584.721	12655.304	29470.793
0.8	984.205	4456.893	9464.282	28959.691	710.070	4137.517	7896.581	28417.912

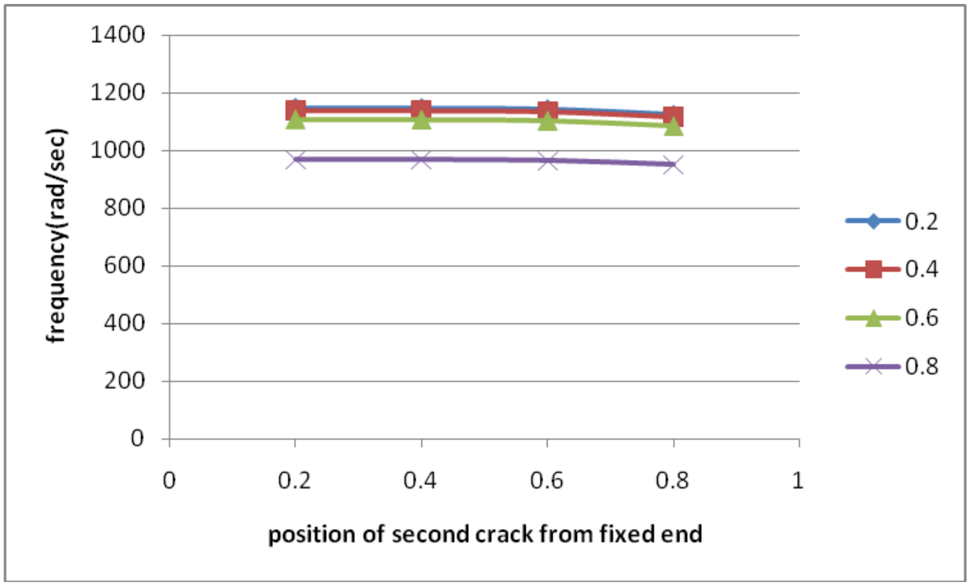

Figure 16 1st crack at 0.8L and 2nd crack is at 0.6L

Table 24 Variation of excitation frequency in (rad/sec) for x1 = 0.8L and x2=0.6L and (a/h) = 0.2, 0.4

Position of 2nd crack	Rcd=0.2				Rcd=0.4			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	1149.338	6915.574	18826.858	36030.228	1139.178	6432.722	18091.497	35715.396
0.4	1148.199	6782.074	17713.723	33564.813	1138.052	6323.637	17259.644	32799.471
0.6	1144.636	6378.350	15280.591	30435.191	1134.532	5994.672	15209.208	28872.410
0.8	1124.949	4801.208	11760.833	28094.325	1115.103	4660.127	11632.111	25937.208

Table 25 Variation of excitation frequency in (rad/sec) for x1 = 0.8L and x2=0.6L and (a/h) = 0.6, 0.8

Position of 2nd crack	Rcd=0.6				Rcd=0.8			
	W1	W2	W3	W4	W1	W2	W3	W4
0.2	1108.522	5425.273	16974.576	35196.129	967.434	3678.510	15829.892	34639.075
0.4	1107.440	5360.876	16502.249	31562.286	966.598	3663.723	15625.681	30221.902
0.6	1104.061	5165.780	15064.449	26297.192	963.992	3617.986	14824.06	23546.172
0.8	1085.456	4301.390	11319.374	22384.519	949.759	3382.109	10639.044	18617.629

Conclusions:

1. The frequencies of vibration of cracked beams decrease with increase of crack depth for crack at any particular location due to reduction of stiffness.
2. The effect of crack is more pronounced near the fixed end than at far free end.
3. The first natural frequency of free vibration decreases with increase in number of cracks.
4. The natural frequency decreases with increase in relative crack depth.

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