Numerical Solution of Interval and Fuzzy System of Linear Equations

A THESIS

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By

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DECLARATION

I hereby certify that the work which is being presented in the thesis entitled "**Numerical Solution of Interval and Fuzzy System of Linear Equations"** in partial fulfillment of the requirement for the award of the degree of Master of Science, submitted in the Department of Mathematics, National Institute of Technology, Rourkela is an authentic record of my own work carried out under the supervision of Dr. S. Chakraverty.

The matter embodied in this thesis has not been submitted by me or the award of any other degree.

Date: (SUPARNA DAS)

This is to certify that the above statement made by the candidate is correct to the best of my knowledge.

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SUPARNA DAS

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TABLE OF CONTENT

ABSTRACT

The system of linear equation has a great importance in many real life problem such as economics, Optimization and in various engineering field. We know that system of linear equations, in general is solved for crisp unknowns. For the sake of simplicity or for fuzzy computation it is taken as crisp value. In actual case the parameters of the system of linear equations are modeled by taking the experimental or observation data. So the parameters of the system actually contain uncertainty rather than the crisp one. The uncertainties may be considered in term of interval or fuzzy number.

Recently different authors have investigated these problems by various methods. These methods are described for the system having various type of fuzzy and non-fuzzy parameters. Although solutions are obtained by these methods are good but sometimes the method requires lengthy procedure and computationally not efficient.

Here, in this thesis detail study of linear simultaneous equations with interval and fuzzy parameter have been done. New methods have been proposed for the same.

The proposed methods have been tested for known problems viz. a circuit analysis, solved in the literature and the results are found to be in good agreement with the present. Next more example problems are solved using the proposed methods to have confidence on these new methods. There exists various type of fuzzy numbers. As such here the problems are also been solved by two types of fuzzy numbers. Corresponding plots of the solution for all the example problems are included in this thesis.

In view of the above analysis of the results, it is found that the proposed methods are simple and computationally efficient.

1. INTRODUCTION

In 1965, Lotfi Jadeh [8], a professor of electrical engineering at the University of California (Berkley), published the first of his papers on his new theory of Fuzzy sets and Systems. Since the 1980s, the mathematical theory of "unsharp amounts" has been applied with great succession in many different fields [9]. The literature on fuzzy arithmetic and its applications, often contains critical remarks as, "the standard fuzzy arithmetic does not take into account of all the information available, and the obtained results are more imprecise than necessary or in some cases, even incorrect" (remarks of Zhou) ([10],[11]).

The concept of a fuzzy number arises from phenomena which can be described quantitatively. These phenomena do not lend themselves to being characterized. For example, if we take a wire and measure its length from different view angles, we get different values. Thus, a fuzzy number is one which is described in terms of a number word such as approximately, nearly etc.

System of linear equations has various applications. Equations of this type are necessary to solve for getting the involved parameters. It is simple and straight forward when the variables involving the system of equations are crisp number. But in actual case the system variables cannot be obtained as crisp. Those are found by some experiment in general. So, these variables will either be an interval or a fuzzy number. i.e., the measurement of the length of the wire do not gives the crisp value in particular. As such there will be vagueness in the result of the experiment. So, to overcome the vagueness we may use the interval and fuzzy numbers in the place of crisp number[7].

Fuzzy linear systems have recently been studied by a good number of authors but only a few of them are mentioned here. A fuzzy linear system Ax=b where A is crisp and b is a fuzzy number vector have been studied by Friedman et al [1], Ma et al [2] and Allahviranloo [3-4].

There are many books having circuit analysis such as by Badrinarayanan and Nandini [5] which gives system of linear equations, where the resistive networks may include fuzzy or interval number. Thus we need to solve interval/fuzzy system of linear equations. Recently Taher et al [6] has investigated this type of system. Das et al [7] discussed the Fuzzy system of linear equation and its application to circuit analysis in this paper.

In this investigation we have discussed about interval and its arithmetic, fuzzy number, α-cut of a fuzzy number, various type of fuzzy numbers and its arithmetic. The concepts of these have been used for the numerical solution of system of linear equation. As such new methods are developed here to handle these fuzzy and interval problems in $\Re^+$. In special cases the solutions are also compared with the known results that are found in literature. As an application, investigation of circuit analysis have been done for interval/fuzzy input and interval/fuzzy source which governed by corresponding linear simultaneous equation.

2. AIM OF THE PROJECT

Solution of linear system of equations is well-known when the parameters involved are in crisp. As discussed, in actual practice we may not have the parameters in crisp form rather those are known in vague form that is with uncertainty.

Inclusion of uncertainty in the parameter makes the problem complex. Here uncertainty has been considered as either in interval or fuzzy form.

So, the aim of this investigation is to develop methodology to solve interval and fuzzy linear system of equations.

3. INTERVAL AND FUZZY ARITHMETIC

3.1. SYSTEM OF LINEAR EQUATIONS:

The n × n linear systems of equations (crisp) may be written as

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \ldots\ldots. + a_{1n}x_n &= y_1 \\
a_{21}x_1 + a_{22}x_2 + \ldots\ldots. + a_{2n}x_n &= y_2 \\
\vdots \qquad \vdots \qquad\qquad \vdots \quad &\quad \vdots \\
a_{n1}x_1 + a_{n2}x_2 + \ldots\ldots. + a_{nn}x_n &= y_n
\end{aligned} \qquad \cdots (1)$$

Where the coefficient matrix $A = (a_{ij}), 1 \le i, j \le n$, is a crisp $n \times n$ matrix.

Our aim is here to have the above system as either in interval or fuzzy variables and constants and their solution. As such in the following paragraphs we will first discuss preliminaries of interval arithmetic and then about fuzzy set and numbers.

3.2. Interval arithmetic:

An interval is a subset of $\Re$ such that $A = [a_1, a_2] = \{t \mid a_1 \le t \le a_2, a_1, a_2 \in \Re\}$.

If $A = [a_1, a_2]$ and $B = [b_1, b_2]$ are two intervals, then the arithmetic operations are:

ADDITION:

$$[a_1, a_2] + [b_1, b_2] = [a_1 + b_1, a_2 + b_2]$$

SUBTRACTION:

$$[a_1, a_2] - [b_1, b_2] = [a_1 - b_2, a_2 - b_1]$$

PRODUCT:

$$[a_1, a_2] \bullet [b_1, b_2] = [\min(a_1 b_1, a_1 b_2, a_2 b_1, a_2 b_2), \max(a_1 b_1, a_1 b_2, a_2 b_1, a_2 b_2)]$$

DIVISION:

$$[a_1, a_2]/[b_1, b_2] = [\min(a_1/b_1, a_1/b_2, a_2/b_1, a_2/b_2), \max(a_1/b_1, a_1/b_2, a_2/b_1, a_2/b_2)] \quad b_1, b_2 \neq 0$$

3.3. DEFINITION OF A FUZZY SET:

A fuzzy set can be defined as the set of ordered pairs such that $A = \{(x, \mu_A(x)) / x \in X, \mu_A(x) \in [0,1]\}$, where $\mu_A(x)$ is called the membership function or grade of membership of x [12].

3.4. DEFINITION OF A FUZZY NUMBER:

A fuzzy number is a convex normalized fuzzy set of the crisp set such that for only one $x \in X$, $\mu_A(x) = 1$ and $\mu_A(x)$ is piecewise continuous [12].

3.5. TYPES OF A FUZZY NUMBER:

Here we have discussed about 2 types of fuzzy number namely,

1) Triangular fuzzy number:

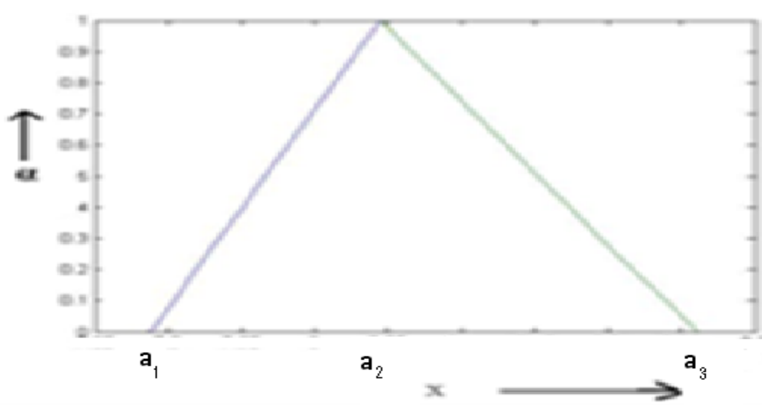

Fig .1 Triangular fuzzy number

A triangular fuzzy number (TFN) as shown in **Fig.1** is a special type of fuzzy number and its membership function is given by $\mu_{\bar{A}}(x)$ such that

$$\mu_{\bar{A}}(x) = \begin{cases} 0 \ , x \le a_1 \\ \dfrac{x-a_1}{a_2-a_1}, x \in [a_1, a_2] \\ \dfrac{a_3-x}{a_3-a_2}, x \in [a_2, a_3] \\ 0 \ , x \ge a_3 \end{cases}$$

2) Trapezoidal fuzzy number

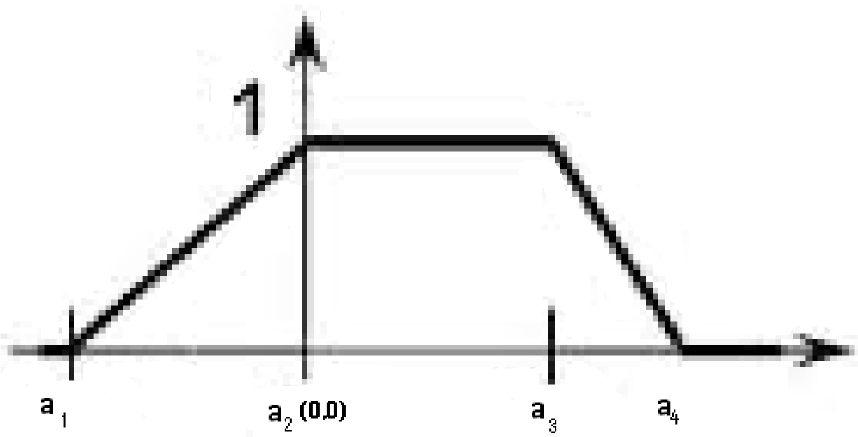

Fig. 2 Trapezoidal fuzzy number

A trapezoidal fuzzy number (Tr F N) as shown in **Fig.2** is a special type of fuzzy number and its membership function is given by $\mu_{\bar{A}}(x)$ such that

$$\mu_{\bar{A}}(x) = \begin{cases} 0 \ , x \le a_1 \\ \dfrac{x-a_1}{a_2-a_1}, x \in [a_1, a_2] \\ 1, x \in [a_2, a_3] \\ \dfrac{a_4-x}{a_4-a_3}, x \in [a_3, a_4] \\ 0 \ , x \ge a_4 \end{cases}$$

3.6. <u>DEFINITION OF ALPHA CUT:</u>

The crisp set of elements that belong to the fuzzy set $\bar{A}$ at least to the degree α is called the α-level set [12]:

$$A_\alpha = \{x \in X : \mu_{\bar{A}}(x) \geq \alpha\}$$

3.7. <u>CONVERSION FROM FUZZY NUMBER TO INTERVAL USING ALPHA CUT:</u>

1) Triangular fuzzy number to interval

Let, a triangular fuzzy number defined as $\overline{A} = (a_1, a_2, a_3)$

We can write the fuzzy interval in terms of α -cut interval as:

$$\overline{A_\alpha} = [\alpha(a_2 - a_1) + a_1, -\alpha(a_3 - a_2) + a_3]$$

2) Trapezoidal fuzzy number to interval

Let, a trapezoidal fuzzy number defined as $\overline{A} = (a_1, a_2, a_3, a_4)$

We can write the fuzzy interval in terms of α -cut interval as:

$$\overline{A_\alpha} = [\alpha(a_2 - a_1) + a_1, -\alpha(a_4 - a_3) + a_4]$$

3.8. <u>Fuzzy Arithmetic:</u>

As $\overline{A_\alpha}$ is now interval, so fuzzy addition, subtraction, multiplication and division are same as interval arithmetic.

4. FUZZY SOLUTION OF SYSTEM OF LINEAR EQUATION BY KNOWN METHOD

Now if the system (1) is fuzzy (Taher et al.[6]), then i th equation of the system may be written as

$$a_{i1}(\underline{x}_1,\overline{x}_1)+\ldots+a_{ii}(\underline{x}_i,\overline{x}_i)+\ldots+a_{in}(\underline{x}_n,\overline{x}_n)=(\underline{y}_i,\overline{y}_i)$$

We have

$$\underline{a_{i1}x_1}+\ldots\ldots+\underline{a_{ii}x_i}+\ldots\ldots+\underline{a_{in}x_n}=\underline{y}_i(\alpha)$$
$$\overline{a_{i1}x_1}+\ldots\ldots+\overline{a_{ii}x_i}+\ldots\ldots+\overline{a_{in}x_n}=\overline{y}_i(\alpha) \qquad 1\le \mathrm{i}\le\alpha \quad \cdots(2)$$

where notation α is considered for α -cut of fuzzy number.

From (2) we have two crisp n x n linear system for all i that can be extended to a 2n x 2n crisp linear system as follows (Taher et al.[6]),

$$SX=Y\rightarrow\begin{bmatrix} S_1\ge 0 & S_2\le 0 \\ S_2\le 0 & S_1\ge 0\end{bmatrix}\begin{bmatrix}\underline{X}\\ \overline{X}\end{bmatrix}=\begin{bmatrix}\underline{Y}\\ \overline{Y}\end{bmatrix} \quad \cdots(3)$$

$$\begin{Bmatrix} S_1\underline{X}+S_2\overline{X}=\underline{Y} \\ S_2\underline{X}+S_1\overline{X}=\overline{Y}\end{Bmatrix} \text{ or } \begin{Bmatrix} S_1\underline{X}-S_2\overline{X}=\underline{Y} \\ -S_2\underline{X}+S_1\overline{X}=\overline{Y}\end{Bmatrix} \quad \cdots(4)$$

Now let , $X=\{(\underline{x}_i(\alpha),\overline{x}_i(\alpha)),1\le i\le n\}$ denote the unique solution of SX = Y. The fuzzy number vector $U=\{(\underline{u}_i(\alpha),\overline{u}_i(\alpha)),1\le i\le n\}$ defined by

$$\underline{u}_i(\alpha)=\min\{\underline{x}_i(\alpha),\overline{x}_i(\alpha),\underline{x}_i(1)\}$$
$$\overline{u}_i(\alpha)=\max\{\underline{x}_i(\alpha),\overline{x}_i(\alpha),\underline{x}_i(1)\}$$

is the fuzzy solution of SX = Y. If $(\underline{x}_i(\alpha),\overline{x}_i(\alpha)),1\le i\le n$ are all triangular fuzzy numbers then $\underline{u}_i(\alpha)=\underline{x}_i(\alpha),\overline{u}_i(\alpha)=\overline{x}_i(\alpha),1\le i\le n$ and U is called a strong fuzzy solution. Otherwise, U is a weak fuzzy solution [4].

5. INTERVAL/FUZZY SOLUTION OF n×n SYSTEM OF LINEAR EQUATIONS

5.1. FIRST METHOD

Let, the system of equations be:

$$(\underline{a}_{11},\overline{a}_{11})(\underline{x}_1,\overline{x}_1)+(\underline{a}_{12},\overline{a}_{12})(\underline{x}_2,\overline{x}_2)+\ldots.+(\underline{a}_{1n},\overline{a}_{1n})(\underline{x}_n,\overline{x}_n)=(\underline{r}_1,\overline{r}_1)$$

$$(\underline{a}_{21},\overline{a}_{21})(\underline{x}_1,\overline{x}_1)+(\underline{a}_{22},\overline{a}_{22})(\underline{x}_2,\overline{x}_2)+\ldots.+(\underline{a}_{2n},\overline{a}_{2n})(\underline{x}_n,\overline{x}_n)=(\underline{r}_2,\overline{r}_2)$$

.

.

.

$$(\underline{a}_{n1},\overline{a}_{n1})(\underline{x}_1,\overline{x}_1)+(\underline{a}_{n2},\overline{a}_{n2})(\underline{x}_2,\overline{x}_2)+\ldots.+(\underline{a}_{nn},\overline{a}_{nn})(\underline{x}_n,\overline{x}_n)=(\underline{r}_n,\overline{r}_n)$$

$$\cdots(5)$$

Where all a_{ij} are in $\Re^+$

The above equations may be written equivalently as

$$\underline{a}_{11}\underline{x}_1+\underline{a}_{12}\underline{x}_2+\ldots+\underline{a}_{1n}\underline{x}_n=\underline{r}_1$$

$$\overline{a}_{11}\overline{x}_1+\overline{a}_{12}\overline{x}_2+\ldots+\overline{a}_{1n}\overline{x}_n=\overline{r}_1$$

$$\underline{a}_{21}\underline{x}_1+\underline{a}_{22}\underline{x}_2+\ldots+\underline{a}_{2n}\underline{x}_n=\underline{r}_2$$

$$\overline{a}_{21}\overline{x}_1+\overline{a}_{22}\overline{x}_2+\ldots+\overline{a}_{2n}\overline{x}_n=\overline{r}_2$$

.

.

$$\underline{a}_{n1}\underline{x}_1+\underline{a}_{n2}\underline{x}_2+\ldots+\underline{a}_{nn}\underline{x}_n=\underline{r}_n$$

$$\overline{a}_{n1}\overline{x}_1+\overline{a}_{n2}\overline{x}_2+\ldots+\overline{a}_{nn}\overline{x}_n=\overline{r}_n$$

$$\cdots(6)$$

Equations (6) can now be written in matrix form as:

$$
\begin{pmatrix}
\underline{a_{11}} & \underline{a_{12}} & . & . & . & \underline{a_{1n}} & 0 & 0 & . & . & . & 0 \\
0 & 0 & . & . & . & 0 & \overline{a_{11}} & \overline{a_{12}} & . & . & . & \overline{a_{1n}} \\
\underline{a_{21}} & \underline{a_{22}} & . & . & . & \underline{a_{2n}} & 0 & 0 & . & . & . & 0 \\
0 & 0 & . & . & . & 0 & \overline{a_{21}} & \overline{a_{22}} & . & . & . & \overline{a_{2n}} \\
. & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . \\
\underline{a_{n1}} & \underline{a_{n2}} & . & . & . & \underline{a_{nn}} & 0 & 0 & . & . & . & 0 \\
0 & 0 & . & . & . & 0 & \overline{a_{n1}} & \overline{a_{n2}} & . & . & . & \overline{a_{nn}}
\end{pmatrix}
\begin{pmatrix}
\underline{x_1} \\ \underline{x_2} \\ . \\ . \\ . \\ \underline{x_n} \\ \overline{x_1} \\ \overline{x_2} \\ . \\ . \\ . \\ \overline{x_n}
\end{pmatrix}
=
\begin{pmatrix}
\underline{r_1} \\ \overline{r_1} \\ \underline{r_2} \\ \overline{r_2} \\ . \\ . \\ . \\ \underline{r_n} \\ \overline{r_n}
\end{pmatrix}
\qquad \cdots(7)
$$

For clear understanding we now give the procedure with 2 equations and 2 unknowns.

So for2 equations and 2 unknowns we have:

$$
\begin{aligned}
&(\underline{a_{11}}, \overline{a_{11}})(\underline{x_1}, \overline{x_1}) + (\underline{a_{12}}, \overline{a_{12}})(\underline{x_2}, \overline{x_2}) = (\underline{r_1}, \overline{r_1}) \\
&(\underline{a_{21}}, \overline{a_{21}})(\underline{x_1}, \overline{x_1}) + (\underline{a_{22}}, \overline{a_{22}})(\underline{x_2}, \overline{x_2}) = (\underline{r_2}, \overline{r_2}) \qquad \ldots(8)\text{-}(9)
\end{aligned}
$$

Similar to equation (7) one may write (8) and (9) as

$$
\begin{bmatrix}
\underline{a_{11}} & \underline{a_{12}} & 0 & 0 \\
0 & 0 & \overline{a_{11}} & \overline{a_{12}} \\
\underline{a_{21}} & \underline{a_{22}} & 0 & 0 \\
0 & 0 & \overline{a_{21}} & \overline{a_{22}}
\end{bmatrix}
\begin{pmatrix}
\underline{x_1} \\ \underline{x_2} \\ \overline{x_1} \\ \overline{x_2}
\end{pmatrix}
=
\begin{pmatrix}
\underline{r_1} \\ \overline{r_1} \\ \underline{r_2} \\ \overline{r_2}
\end{pmatrix}
$$

This is a crisp system of equation and the above matrix equation may easily be solved now as below,

$$\underline{x}_1 = \frac{\underline{a}_{22}\underline{r}_1 - \underline{a}_{12}\underline{r}_2}{\underline{a}_{11}\underline{a}_{22} - \underline{a}_{12}\underline{a}_{21}}$$

$$\underline{x}_2 = \frac{\underline{a}_{11}\underline{r}_2 - \underline{a}_{21}\underline{r}_1}{\underline{a}_{11}\underline{a}_{22} - \underline{a}_{12}\underline{a}_{21}}$$

$$\overline{x}_1 = \frac{\overline{a}_{22}\overline{r}_1 - \overline{a}_{12}\overline{r}_2}{\overline{a}_{11}\overline{a}_{22} - \overline{a}_{12}\overline{a}_{21}}$$

$$\overline{x}_2 = \frac{\overline{a}_{11}\overline{r}_2 - \overline{a}_{21}\overline{r}_1}{\overline{a}_{11}\overline{a}_{22} - \overline{a}_{12}\overline{a}_{21}}$$

5.2. SECOND METHOD

In this method equation (8) and (9) are first written taking

LEFT:

$$\underline{a}_{11}\underline{x}_1 + \underline{a}_{12}\underline{x}_2 = \underline{r}_1$$

$$\overline{a}_{11}\overline{x}_1 + \overline{a}_{12}\overline{x}_2 = \overline{r}_1$$

$$\cdots (10 - (11)$$

RIGHT:

$$\underline{a}_{21}\underline{x}_1 + \underline{a}_{22}\underline{x}_2 = \underline{r}_2$$

$$\overline{a}_{21}\overline{x}_1 + \overline{a}_{22}\overline{x}_2 = \overline{r}_2$$

$$\cdots (12) - (13)$$

Solving (10) and (12); (11) and (13) we get,

$$\underline{x}_1 = \frac{\underline{a}_{22}\underline{r}_1 - \underline{a}_{12}\underline{r}_2}{\underline{a}_{11}\underline{a}_{22} - \underline{a}_{12}\underline{a}_{21}}$$

$$\underline{x}_2 = \frac{\underline{a}_{11}\underline{r}_2 - \underline{a}_{21}\underline{r}_1}{\underline{a}_{11}\underline{a}_{22} - \underline{a}_{12}\underline{a}_{21}}$$

$$\overline{x}_1 = \frac{\overline{a}_{22}\overline{r}_1 - \overline{a}_{12}\overline{r}_2}{\overline{a}_{11}\overline{a}_{22} - \overline{a}_{12}\overline{a}_{21}}$$

$$\overline{x}_2 = \frac{\overline{a}_{11}\overline{r}_2 - \overline{a}_{21}\overline{r}_1}{\overline{a}_{11}\overline{a}_{22} - \overline{a}_{12}\overline{a}_{21}}$$

Although methods 1 and 2 are same but this is shown in the above methods that we may solve for the left and right individually too.

5.3. THIRD METHOD

Here we first consider the equations (10) and (12),

$$\underline{a}_{11}\underline{x}_1 + \underline{a}_{12}\underline{x}_2 = \underline{r}_1$$

$$\underline{a}_{21}\underline{x}_1 + \underline{a}_{22}\underline{x}_2 = \underline{r}_2$$

Write the above as

$AX = b$ that is,

$$\begin{bmatrix} \underline{a}_{11} & \underline{a}_{12} \\ \underline{a}_{21} & \underline{a}_{22} \end{bmatrix} \begin{pmatrix} \underline{x}_1 \\ \underline{x}_2 \end{pmatrix} = \begin{pmatrix} \underline{r}_1 \\ \underline{r}_2 \end{pmatrix} \qquad \cdots (14)$$

From (14) we may have (if the inverse of the coefficient matrix exists) the given solution

$$\begin{pmatrix} \underline{x_1} \\ \underline{x_2} \end{pmatrix} = \begin{bmatrix} \dfrac{\underline{\underline{a_{22}}}\,\underline{\underline{r_1}} - \underline{\underline{a_{12}}}\,\underline{\underline{r_2}}}{\underline{a_{11}}\,\underline{a_{22}} - \underline{a_{12}}\,\underline{a_{21}}} \\ \dfrac{\underline{\underline{a_{11}}}\,\underline{\underline{r_2}} - \underline{\underline{a_{21}}}\,\underline{\underline{r_1}}}{\underline{a_{11}}\,\underline{a_{22}} - \underline{a_{12}}\,\underline{a_{21}}} \end{bmatrix}$$

$$\cdots(15)$$

Following the same way equations (11) and (13) may be used to get the solution

$$\begin{pmatrix} \overline{x_1} \\ \overline{x_2} \end{pmatrix} = \begin{pmatrix} \dfrac{\overline{\underline{\underline{a_{22}}}}\,\overline{\underline{\underline{r_1}}} - \overline{\underline{\underline{a_{12}}}}\,\overline{\underline{\underline{r_2}}}}{\overline{a_{11}}\,\overline{a_{22}} - \overline{a_{12}}\,\overline{a_{21}}} \\ \dfrac{\overline{\underline{\underline{a_{11}}}}\,\overline{\underline{\underline{r_2}}} - \overline{\underline{\underline{a_{21}}}}\,\overline{\underline{\underline{r_1}}}}{a_{11}\,a_{22} - a_{12}\,a_{21}} \end{pmatrix}$$

$$\cdots(16)$$

6. NUMERICAL EXAMPLE BY KNOWN AND PROPOSED METHOD

6.1. INTERVAL AND FUZZY EQUATIONS:

Let us first consider the following two interval equations in two unknowns

First of all we will start with an example given in Taher et al.[6] .

Here three different cases taking source and resistance as crisp or interval or fuzzy are considered in the circuit analysis. These cases are named as case I to case III in the following paragraph where current is taken as fuzzy in case I and case III and crisp in case II.

6.1.1. CASE-I:

Method of Taher et al.[6]:

First a circuit considering fuzzy system of linear equations applied to circuit analysis (Taher et al.[6]), having source as well as current as fuzzy and resistance as crisp has been considered. Related figure is shown in Fig. 3.

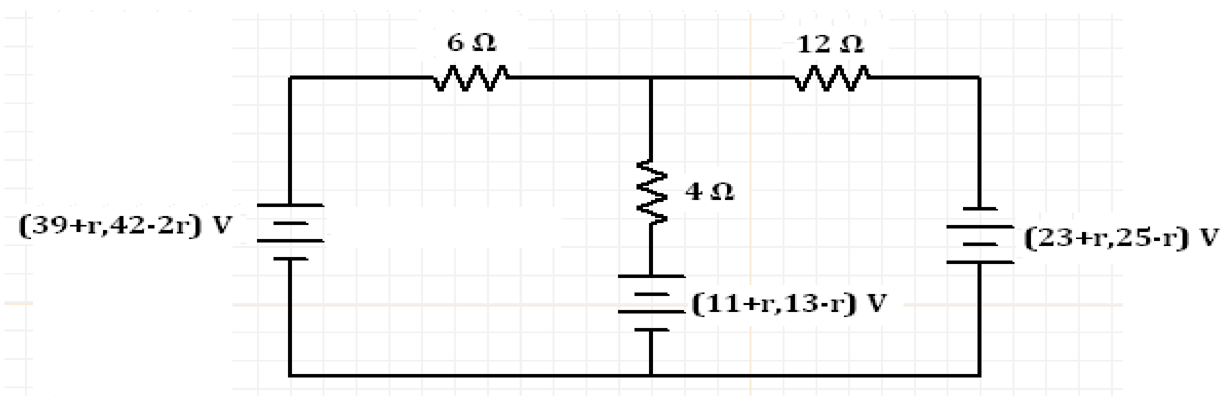

Fig.3. A circuit with fuzzy current, fuzzy source and crisp resistance

From the first and second loop, Kirchhoff's second law gives,

$$6I_1 + 4(I_1 - I_2) + (11+r, 13-r) - (39+r, 42-2r) = 0 \quad \ldots(17)$$

$$-4(I_1 - I_2) + 12I_2 - (23+r, 25-r) - (11+r, 13-r) = 0 \quad \ldots(18)$$

$$\Rightarrow 10I_1 - 4I_2 = (39+r, 42-2r) - (11+r, 13-r)$$

$$-4I_1 + 16I_2 = (23+r, 25-r) + (11+r, 13-r)$$

The above may be written as

$$S = \begin{bmatrix} S_1 & S_2 \\ S_2 & S_1 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 & -4 \\ 0 & 16 & -4 & 0 \\ 0 & -4 & 10 & 0 \\ -4 & 0 & 0 & 16 \end{bmatrix}, \quad \hat{Y} = \begin{bmatrix} 26+2r \\ 34+2r \\ 31-3r \\ 38-2r \end{bmatrix}, \quad I = \begin{bmatrix} \underline{I_1} \\ \underline{I_2} \\ \overline{I_1} \\ \overline{I_2} \end{bmatrix}$$

Now writing $SI = \hat{Y}$, as in (3), we have,

$$10\underline{I_1} - 4\overline{I_2} = 26 + 2r$$

$$16\underline{I_2} - 4\overline{I_1} = 34 + 2r$$

$$-4\underline{I_2} + 10\overline{I_1} = 31 - 3r$$

$$-4\underline{I_1} + 16\overline{I_2} = 38 - 2r \quad \ldots(19-22)$$

So, the solution in interval form may be written as

$$I_1 = \left(\frac{71}{18} + \frac{1}{6}r, \frac{79}{18} - \frac{5}{18}r\right) = (3.944 + 0.167r, 4.389 - 0.278r)$$

$$I_2 = \left(\frac{29}{9} + \frac{1}{18}r, \frac{121}{36} - \frac{1}{12}r\right) = (3.22 + 0.056r, 3.361 - 0.833r)$$

Corresponding plot of the solutions are given in Fig.4

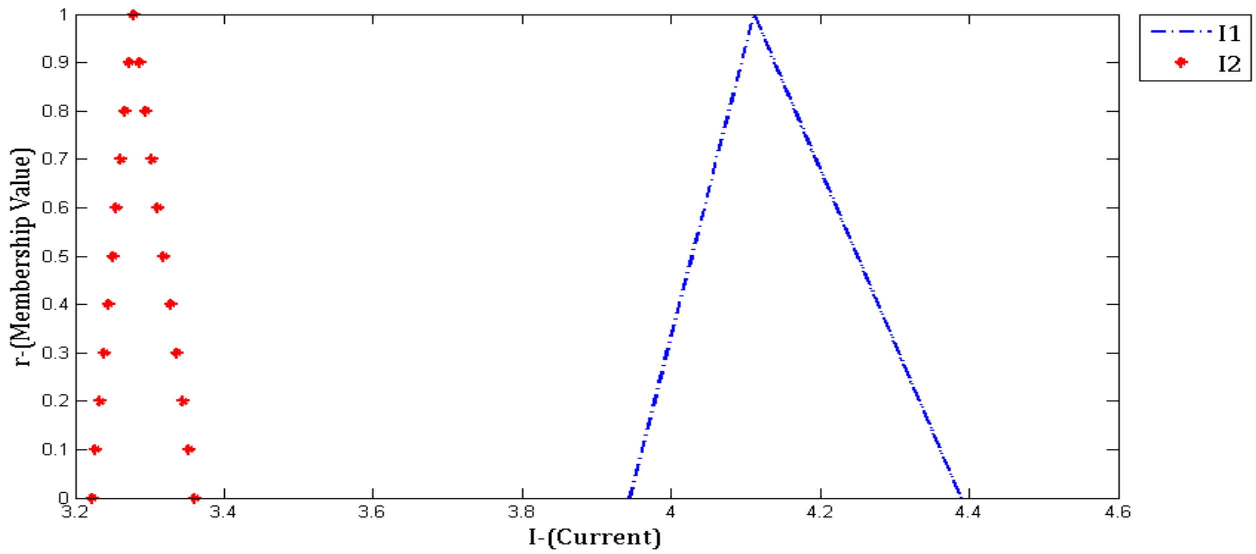

Fig.4.The solution of the system for case I

6.1.2. METHOD 2:

Now, the above problem is solved using the proposed methods viz. first to third. The problem is given as,

$$6I_1 + 4(I_1 - I_2) + (11 + r, 13 - r) - (39 + r, 42 - 2r) = 0$$
$$-4(I_1 - I_2) + 12I_2 - (23 + r, 25 - r) - (11 + r, 13 - r) = 0$$

$\cdots(23)-(24)$

This can be written as

$$6(\underline{I_1}, \overline{I_1}) + 4((\underline{I_1}, \overline{I_1}) - (\underline{I_2}, \overline{I_2})) = (26 + 2r, 31 - 3r)$$
$$-4((\underline{I_1}, \overline{I_1}) - (\underline{I_2}, \overline{I_2})) + 12(\underline{I_2}, \overline{I_2}) = (34 + 2r, 38 - 2r)$$

$\cdots(25)-(26)$

Simplifying the above equations we can get,

$$10\underline{I}_1 - 4\overline{I}_2 = 26 + 2r$$
$$10\overline{I}_1 - 4\underline{I}_2 = 31 - 2r$$

And

$$-4\underline{I}_1 + 16\overline{I}_2 = 38 - 2r$$
$$-4\overline{I}_1 + 16\underline{I}_2 = 34 + 2r$$

$$\cdots(27) - (30)$$

First method

We can get the matrix directly as

$$\begin{bmatrix} 10 & 0 & 0 & -4 \\ 0 & 10 & -4 & 0 \\ 0 & -4 & 16 & 0 \\ -4 & 0 & 0 & 16 \end{bmatrix} \begin{pmatrix} \underline{I}_1 \\ \overline{I}_1 \\ \underline{I}_2 \\ \overline{I}_2 \end{pmatrix} = \begin{pmatrix} 26+2r \\ 31-3r \\ 34+2r \\ 38-2r \end{pmatrix}$$

So, the solution in interval form may be written as

$$I_1 = (\underline{I}_1, \overline{I}_1) = \left(\frac{71}{18} + \frac{1}{6}r, \frac{79}{18} - \frac{5}{18}r\right) = (3.944 + 0.167r, 4.389 - 0.278r)$$
$$I_2 = (\underline{I}_2, \overline{I}_2) = \left(\frac{29}{9} + \frac{1}{18}r, \frac{121}{36} - \frac{1}{12}r\right) = (3.22 + 0.056r, 3.361 - 0.833r)$$

Second method

From (27) ; (29) and (28) ; (30) we get the result as:

$$(\underline{I}_1, \overline{I}_2) = \left(\frac{71}{18} + \frac{1}{6} r, \frac{121}{36} - \frac{1}{12} r \right) = (3.944 + 0.167r, 3.361 - 0.833r)$$

$$(\overline{I}_1, \underline{I}_2) = \left(\frac{79}{18} - \frac{5}{18} r, \frac{29}{9} + \frac{1}{18} r \right) = (4.389 - 0.278r, 3.22 + 0.056r)$$

$$\text{So}, I_1 = (\underline{I}_1, \overline{I}_1) = \left(\frac{71}{18} + \frac{1}{6} r, \frac{79}{18} - \frac{5}{18} r \right) = (3.944 + 0.167r, 4.389 - 0.278r)$$

$$I_2 = (\underline{I}_2, \overline{I}_2) = \left(\frac{29}{9} + \frac{1}{18} r, \frac{121}{36} - \frac{1}{12} r \right) = (3.22 + 0.056r, 3.361 - 0.833r)$$

As mentioned actually both method turns out to be same and so the results are also exactly same.

Third method

From equation (27) and (29) we can write

$$AX = b$$

$$\begin{bmatrix} 10 & -4 \\ -4 & 16 \end{bmatrix} \begin{pmatrix} \underline{I}_1 \\ \underline{I}_2 \end{pmatrix} = \begin{pmatrix} 26 + 2r \\ 38 - 2r \end{pmatrix}$$

$$X = \begin{pmatrix} \underline{I}_1 \\ \underline{I}_2 \end{pmatrix} = A^{-1} b = \begin{pmatrix} 3.944 + 0.167r \\ 3.361 - 0.833r \end{pmatrix}$$

Similarly, from equation (28) and (30) we get,

$$X = \begin{pmatrix} \overline{I}_1 \\ \underline{I}_2 \end{pmatrix} = A^{-1} b = \begin{pmatrix} 4.389 - 0.278r \\ 3.22 + 0.056r \end{pmatrix}$$

6.2. <u>CASE-II:</u> Next example gives the solution for a circuit with source, current and resistance all in interval. Corresponding figure is shown in Fig.5

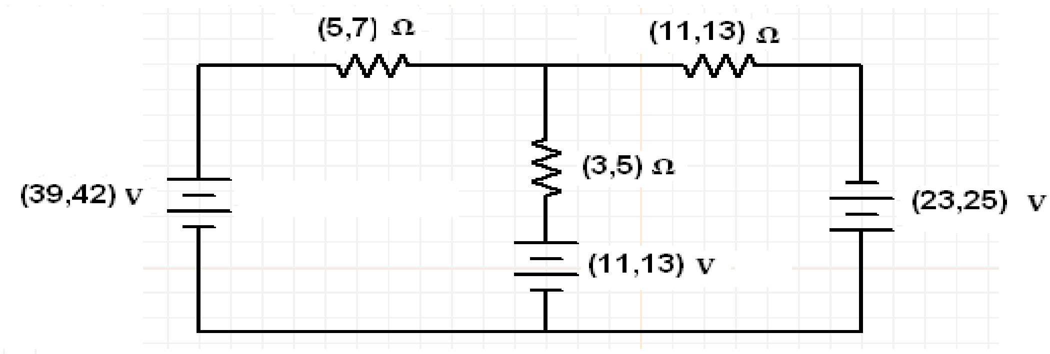

Fig.5.A circuit with current, source and resistance in interval

From the above circuit we can write

$$(5,7)(\underline{I}_1,\overline{I}_1)+(3,5)((\underline{I}_1,\overline{I}_1)-(\underline{I}_2,\overline{I}_2))=(26+2r,31-3r)$$
$$-(3,5)((\underline{I}_1,\overline{I}_1)-(\underline{I}_2,\overline{I}_2))+(11,13)(\underline{I}_2,\overline{I}_2)=(34+2r,38-2r)$$

From the first and second loops and simplifying the interval equations we have the following equations

$$\begin{aligned}
&8\underline{I}_1-3\overline{I}_2=26\\
&12\overline{I}_1-5\underline{I}_2=31\\
&5\overline{I}_1+16\underline{I}_2=34\\
&-3\underline{I}_1+16\overline{I}_2=38
\end{aligned} \qquad \cdots(31)-(34)$$

Solution of the equations (31) to (34) are:

$$\underline{I}_1 = \frac{530}{119} = 4.454$$
$$\overline{I}_1 = \frac{666}{167} = 3.99$$
$$\underline{I}_2 = \frac{563}{167} = 3.37$$
$$\overline{I}_2 = \frac{382}{119} = 3.21$$

Here lower nodes are larger than the upper nodes and it is called as weak solution. If the right hand side '0' is considered as interval which includes zero for example as [-3,3] then the equations after some calculation can be written as

$$8\underline{I_1} - 3\overline{I_2} = 23$$
$$12\overline{I_1} - 5\underline{I_2} = 34$$
$$5\overline{I_1} + 16\underline{I_2} = 31$$
$$-3\underline{I_1} + 16\overline{I_2} = 41$$

$$\cdots (35) - (38)$$

Solution of (35) to (38) gives,

$$\underline{I}_1 = \frac{491}{119} = 4.12$$
$$\overline{I}_1 = \frac{699}{167} = 4.18$$
$$\underline{I}_2 = \frac{542}{167} = 3.24$$
$$\overline{I}_2 = \frac{397}{119} = 3.33$$

In this case first we get a weak solution. Then a strong solution is obtained when the right hand side '0' arbitrarily changed to interval which includes '0'. This case is then simulated by reducing the interval width as shown in Fig.6. The corresponding solutions are given as

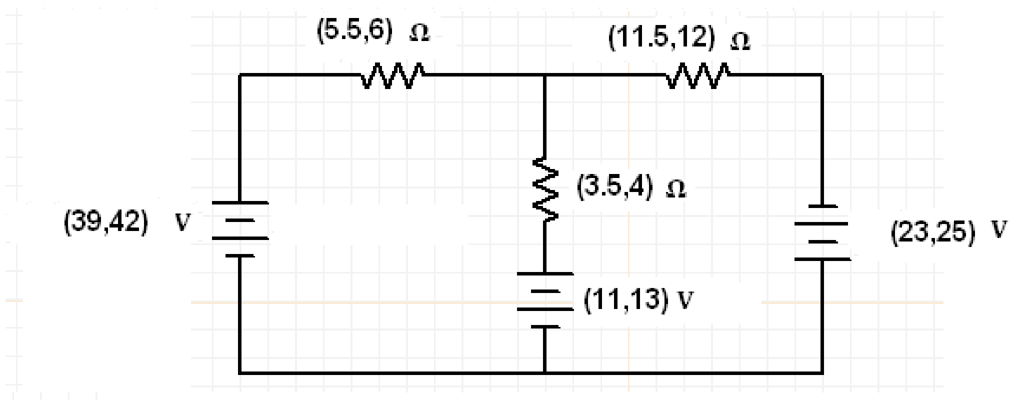

Fig.6.A circuit with interval current, source and resistance

$$\overline{I_1} = \frac{1233}{278} = 4.435$$

$$\underline{I_1} = \frac{2144}{509} = 4.21$$

$$\underline{I_2} = \frac{464}{139} = 3.34$$

$$\overline{I_2} = \frac{1732}{509} = 3.40$$

6.3. <u>CASE-III</u>: The above example taking resistance and voltage both in fuzzy has been discussed now and the corresponding circuit is shown in Fig. 7. The solution may easily be written by following lengthy calculations as in Case I and II which are given in Fig. 8.

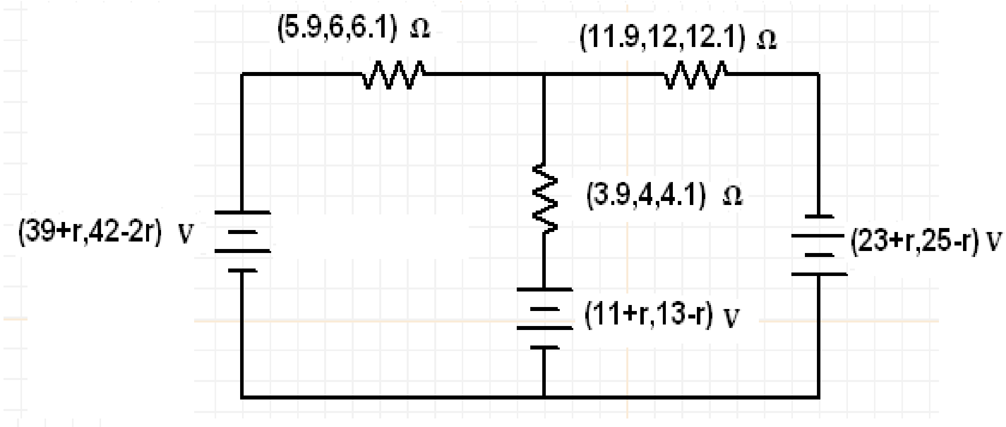

Fig.7. A circuit with fuzzy source, fuzzy current and fuzzy resistance

So we can write the equation as

$$(5.9,6,6.1)(\underline{I}_1,\overline{I}_1)+(3.9,4,4.1)((\underline{I}_1,\overline{I}_1)-(\underline{I}_2,\overline{I}_2))=(26+2r,31-3r)$$
$$-(3.9,4,4.1)((\underline{I}_1,\overline{I}_1)-(\underline{I}_2,\overline{I}_2))+(11.9,12,12.1)(\underline{I}_2,\overline{I}_2)=(34+2r,38-2r)$$

Transferring in interval form and simplifying we get the equation as

$$(0.1\alpha+5.9,-0.1\alpha+6.1)(\underline{I}_1,\overline{I}_1)+(0.1\alpha+3.9,-0.1\alpha+4.1)(\underline{I}_1-\overline{I}_2,\overline{I}_1-\underline{I}_2)=(26+2r,31-3r) \quad \ldots(39)$$
$$-(0.1\alpha+3.9,-0.1\alpha+4.1)(\underline{I}_1-\overline{I}_2,\overline{I}_1-\underline{I}_2)+(0.1\alpha+11.9,-0.1\alpha+12.1)(\underline{I}_2,\overline{I}_2)=(34+2r,38-2r) \quad \ldots(40)$$
$$\Rightarrow (0.2\alpha+9.8)\underline{I}_1-(0.1\alpha+3.9)\overline{I}_2=26+2r$$
$$(-0.2\alpha+10.2)\overline{I}_1-(-0.1\alpha+4.1)\underline{I}_2=31-3r$$
$$-(-0.1\alpha+4.1)\overline{I}_1+16\underline{I}_2=34+2r$$
$$-(0.1\alpha+3.9)\underline{I}_1+16\overline{I}_2=38-2r$$
$$\cdots(41)-(44)$$

First method

From the equation (39) and (40) one may write as:

$$\begin{bmatrix} 0.2\alpha+9.8 & -(0.1\alpha+3.9) & 0 & 0 \\ 0 & 0 & (-0.2\alpha+10.2) & -(-0.1\alpha+4.1) \\ 0 & 0 & -(-0.1\alpha+4.1) & 16 \\ -(0.1\alpha+3.9) & 16 & 0 & 0 \end{bmatrix} \begin{pmatrix} \underline{I}_1 \\ \overline{I}_2 \\ \overline{I}_1 \\ \underline{I}_2 \end{pmatrix} = \begin{pmatrix} 26+2r \\ 31-3r \\ 34+2r \\ 38-2r \end{pmatrix}$$

$$\underline{I}_1 = \frac{20(\alpha^2-140\alpha-2821)}{\alpha^2-242\alpha-14159}$$
$$\overline{I}_2 = \frac{20(\alpha^2+8\alpha-2369)}{\alpha^2-242\alpha-14159}$$
$$\overline{I}_1 = \frac{20(\alpha^2+216\alpha-3177)}{\alpha^2+238\alpha-14639}$$
$$\underline{I}_2 = \frac{10(\alpha^2+18\alpha-4739)}{\alpha^2+238\alpha-14639}$$

Second method

From (41)-(44) we get

$$(0.2\alpha+9.8)\underline{I}_1-(0.1\alpha+3.9)\overline{I}_2 = 26+2r$$
$$(-0.2\alpha+10.2)\overline{I}_1-(-0.1\alpha+4.1)\underline{I}_2 = 31-3r$$
$$-(-0.1\alpha+4.1)\overline{I}_1+16\underline{I}_2 = 34+2r$$
$$-(0.1\alpha+3.9)\underline{I}_1+16\overline{I}_2 = 38-2r$$

The solution can be obtained as

$$\underline{I}_1 = \frac{20(\alpha^2-140\alpha-2821)}{\alpha^2-242\alpha-14159}$$
$$\overline{I}_2 = \frac{20(\alpha^2+8\alpha-2369)}{\alpha^2-242\alpha-14159}$$
$$\overline{I}_1 = \frac{20(\alpha^2+216\alpha-3177)}{\alpha^2+238\alpha-14639}$$
$$\underline{I}_2 = \frac{10(\alpha^2+18\alpha-4739)}{\alpha^2+238\alpha-14639}$$

Third method

From equation (41) and (44) we can write

$$AX = b$$

$$X = \begin{pmatrix} \underline{I_1} \\ \overline{I_2} \end{pmatrix} = A^{-1}b = \begin{pmatrix} \dfrac{20(\alpha^2 - 140\alpha - 2821)}{\alpha^2 - 242\alpha - 14159} \\ \dfrac{20(\alpha^2 + 8\alpha - 2369)}{\alpha^2 - 242\alpha - 14159} \end{pmatrix}$$

Similarly, from equation (42) and (43) we get,

$$X = \begin{pmatrix} \overline{I_1} \\ \underline{I_2} \end{pmatrix} = A^{-1}b = \begin{pmatrix} \dfrac{20(\alpha^2 + 216\alpha - 3177)}{\alpha^2 + 238\alpha - 14639} \\ \dfrac{10(\alpha^2 + 18\alpha - 4739)}{\alpha^2 + 238\alpha - 14639} \end{pmatrix}$$

Corresponding graph is shown in figure 8

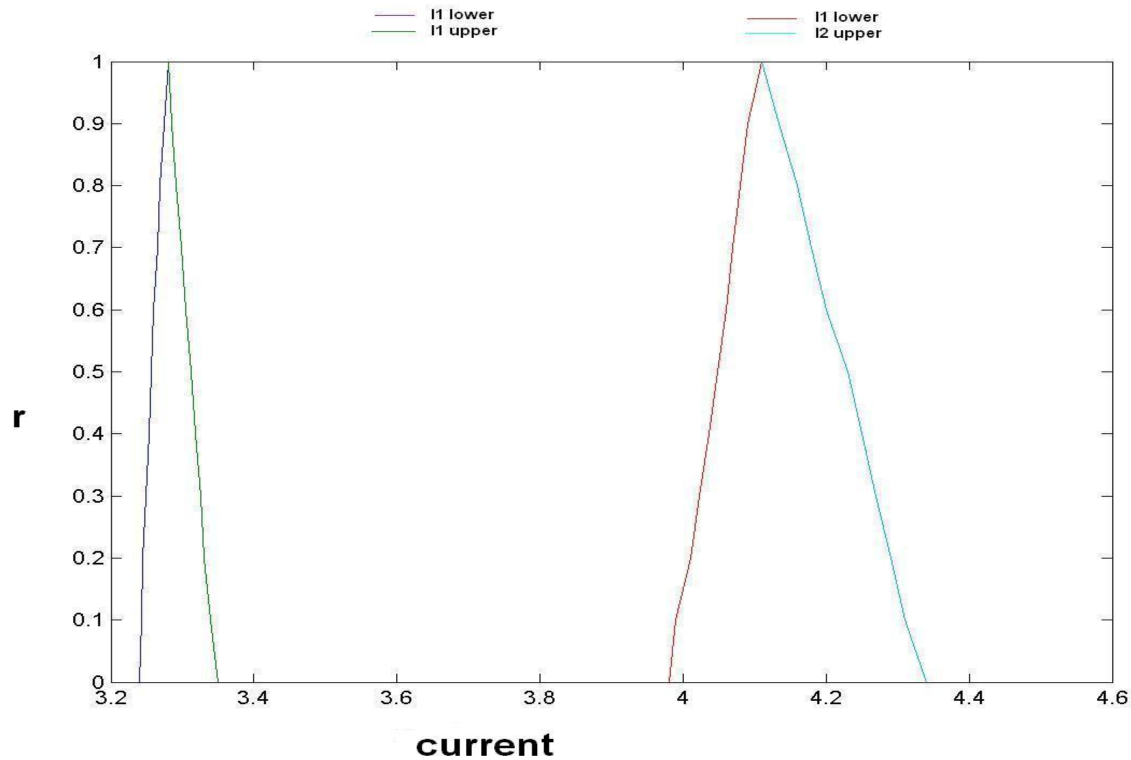

Fig.8. the solution of the system from the above example

7. NUMERICAL EXAMPLES OF CRISP, INTERVAL AND FUZZY EQUATION BY FIRST, SECOND AND THIRD PROPOSED METHOD

7.1 EXAMPLE 1:

In this problem taking three crisp equation with three unknowns:

$$0.4(\underline{x_1},\overline{x_1}) + 1.4(\underline{x_2},\overline{x_2}) + 0.3(\underline{x_3},\overline{x_3}) = 0.1$$
$$0.15(\underline{x_1},\overline{x_1}) + 0.14(\underline{x_2},\overline{x_2}) + 6.1(\underline{x_3},\overline{x_3}) = 0.14$$
$$5.1(\underline{x_1},\overline{x_1}) + 0.3(\underline{x_2},\overline{x_2}) + 0.2(\underline{x_3},\overline{x_3}) = 0.14$$

These equations we can write as

$$(0.4,0.4)(\underline{x_1},\overline{x_1}) + (1.4,1.4)(\underline{x_2},\overline{x_2}) + (0.3,0.3)(\underline{x_3},\overline{x_3}) = (0.1,0.1)$$
$$(0.15,0.15)(\underline{x_1},\overline{x_1}) + (0.14,0.14)(\underline{x_2},\overline{x_2}) + (6.1,6.1)(\underline{x_3},\overline{x_3}) = (0.14,0.14)$$
$$(5.1,5.1)(\underline{x_1},\overline{x_1}) + (0.3,0.3)(\underline{x_2},\overline{x_2}) + (0.2,0.2)(\underline{x_3},\overline{x_3}) = (0.14,0.14)$$

. . .(45)-(47)

1. First method

From (45)-(47) we get,

$$\begin{bmatrix} 0.4 & 1.4 & 0.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 1.4 & 0.3 \\ 0.15 & 0.14 & 6.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.15 & 0.14 & 6.1 \\ 5.1 & 0.3 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5.1 & 0.3 & 0.2 \end{bmatrix} \begin{pmatrix} \underline{x_1} \\ \underline{x_2} \\ \underline{x_3} \\ \overline{x_1} \\ \overline{x_2} \\ \overline{x_3} \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.1 \\ 0.14 \\ 0.14 \\ 0.14 \\ 01.4 \end{pmatrix}$$

we get the solution as

$$\underline{x_1} = 0.0231$$
$$\underline{x_2} = 0.0603$$
$$\underline{x_3} = 0.0210$$

$$\overline{x_1} = 0.0231$$
$$\overline{x_2} = 0.0603$$
$$\overline{x_3} = 0.0210$$

2. Second method

$$0.4\underline{x_1} + 1.4\underline{x_2} + 0.3\underline{x_3} = 0.1$$
$$0.4\overline{x_1} + 1.4\overline{x_2} + 0.3\overline{x_3} = 0.1$$
$$0.15\underline{x_1} + 0.14\underline{x_2} + 6.1\underline{x_3} = 0.14$$
$$0.15\overline{x_1} + 0.14\overline{x_2} + 6.1\overline{x_3} = 0.14$$
$$5.1\underline{x_1} + 0.3\underline{x_2} + 0.2\underline{x_3} = 0.14$$
$$5.1\overline{x_1} + 0.3\overline{x_2} + 0.2\overline{x_3} = 0.14$$

. . . (48)-(53)

Solving (48), (50), (52) and (49), (51), (53), the solutions comes as

$$\underline{x_1} = 0.0231$$
$$\underline{x_2} = 0.0603$$
$$\underline{x_3} = 0.0210$$

$$\overline{x_1} = 0.0231$$
$$\overline{x_2} = 0.0603$$
$$\overline{x_3} = 0.0210$$

3. Third method

$AX = b$

$X = A^{-1}b$

$$X = \begin{pmatrix} \underline{x_1} \\ \underline{x_2} \\ \underline{x_3} \end{pmatrix} = \begin{pmatrix} 0.0231 \\ 0.0603 \\ 0.0210 \end{pmatrix}$$

Similarly,

$$X = \begin{pmatrix} \overline{x_1} \\ \overline{x_2} \\ \overline{x_3} \end{pmatrix} = \begin{pmatrix} 0.0231 \\ 0.0603 \\ 0.0210 \end{pmatrix}$$

<u>EXAMPLE 2:</u>

In this example 3 interval equations in 3 unknowns are considered,

$$(0.1,0.9)(\underline{x_1},\overline{x_1}) + (1,1.9)(\underline{x_2},\overline{x_2}) + (0.11,0.9)(\underline{x_3},\overline{x_3}) = (0.01,0.2)$$
$$(0.1,0.2)(\underline{x_1},\overline{x_1}) + (0.11,0.2)(\underline{x_2},\overline{x_2}) + (6,6.2)(\underline{x_3},\overline{x_3}) = (0.11,0.2)$$
$$(5,5.4)(\underline{x_1},\overline{x_1}) + (0.1,0.4)(\underline{x_2},\overline{x_2}) + (0.11,0.4)(\underline{x_3},\overline{x_3}) = (0.1,0.2)$$

. . .(54)-(56)

1. First method

Equation (54)-(56) we can write in matrix form as:

$$\begin{bmatrix} 0.1 & 1 & 0.11 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9 & 1.9 & 0.9 \\ 0.1 & 0.11 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0.2 & 6.2 \\ 5 & 0.1 & 0.11 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5.4 & 0.4 & 0.4 \end{bmatrix} \begin{pmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \underline{x}_3 \\ \overline{x}_1 \\ \overline{x}_2 \\ \overline{x}_4 \end{pmatrix} = \begin{pmatrix} 0.01 \\ 0.2 \\ 0.11 \\ 0.2 \\ 0.1 \\ 0.2 \end{pmatrix}$$

Solution is obtained as,

$$\underline{x}_1 = 0.0195$$
$$\underline{x}_2 = 0.0061$$
$$\underline{x}_3 = 0.0179$$

$$\overline{x}_1 = 0.0291$$
$$\overline{x}_2 = 0.0778$$
$$\overline{x}_3 = 0.0288$$

2. Second method

From (54)-(56) we get by dispersing left and right,

$$\begin{aligned} &0.1\underline{x}_1 + \underline{x}_2 + 0.11\underline{x}_3 = 0.01 \\ &0.9\overline{x}_1 + 1.9\overline{x}_2 + 0.9\overline{x}_3 = 0.2 \\ &0.1\underline{x}_1 + 0.11\underline{x}_2 + 6\underline{x}_3 = 0.11 \\ &0.2\overline{x}_1 + 0.2\overline{x}_2 + 6.2\overline{x}_3 = 0.2 \\ &5\underline{x}_1 + 0.1\underline{x}_2 + 0.11\underline{x}_3 = 0.1 \\ &5.4\overline{x}_1 + 0.4\overline{x}_2 + 0.4\overline{x}_3 = 0.2 \end{aligned}$$

. . . (57)-(62)

Solving the above as in equation (57)-(62) we have,

$$\underline{x_1} = 0.0195$$
$$\underline{x_2} = 0.0061$$
$$\underline{x_3} = 0.0179$$

$$\overline{x_1} = 0.0291$$
$$\overline{x_2} = 0.0778$$
$$\overline{x_3} = 0.0288$$

3. Third method

From (57), (59), (61) we can write,

$$AX = b$$
$$X = A^{-1}b$$
$$X = \begin{bmatrix} 0.1 & 1 & 0.11 \\ 0.1 & 0.11 & 6 \\ 5 & 0.1 & 0.11 \end{bmatrix}^{-1} \begin{pmatrix} 0.01 \\ 0.11 \\ 0.1 \end{pmatrix}$$
$$X = \begin{pmatrix} \underline{x_1} \\ \underline{x_2} \\ \underline{x_3} \end{pmatrix} = \begin{pmatrix} 0.0195 \\ 0.0061 \\ 0.0179 \end{pmatrix}$$

Similarly (58), (60) and (62) gives,

$$X = \begin{pmatrix} \overline{x_1} \\ \overline{x_2} \\ \overline{x_3} \end{pmatrix} = \begin{pmatrix} 0.0291 \\ 0.0778 \\ 0.0288 \end{pmatrix}$$

EXAMPLE 3:

Consider the equations as,

$$(0.1,0.4,0.9)(\underline{x_1},\overline{x_1})+(1,1.4,1.9)(\underline{x_2},\overline{x_2})+(0.11,0.3,0.9)(\underline{x_3},\overline{x_3})=(0.01,0.1,0.2)$$

$$(0.1,0.15,0.2)(\underline{x_1},\overline{x_1})+(0.11,0.14,0.2)(\underline{x_2},\overline{x_2})+(6,6.1,6.2)(\underline{x_3},\overline{x_3})=(0.11,0.14,0.2)$$

$$(5,5.1,5.4)(\underline{x_1},\overline{x_1})+(0.1,0.3,0.4)(\underline{x_2},\overline{x_2})+(0.11,0.2,0.4)(\underline{x_3},\overline{x_3})=(0.1,0.14,0.2)$$

Transferring the fuzzy equations into interval form we get

$$(0.3\alpha+0.1,-0.5\alpha+0.9)(\underline{x_1},\overline{x_1})+(0.4\alpha+1,-0.5\alpha+1.9)(\underline{x_2},\overline{x_2})+(0.19\alpha+0.11,-0.6\alpha+0.9)(\underline{x_3},\overline{x_3})=(0.09\alpha+0.01,-0.1\alpha+0.2)$$

$$(0.05\alpha+0.1,-0.05\alpha+0.2)(\underline{x_1},\overline{x_1})+(0.03\alpha+0.11,-0.06\alpha+0.2)(\underline{x_2},\overline{x_2})+(0.1\alpha+6,-0.1\alpha+6.2)(\underline{x_3},\overline{x_3})=(0.03\alpha+0.11,-0.06\alpha+0.2)$$

$$(0.1\alpha+5,-0.3\alpha+5.4)(\underline{x_1},\overline{x_1})+(0.2\alpha+0.1,-0.1\alpha+0.4)(\underline{x_2},\overline{x_2})+(0.09\alpha+0.11,-0.2\alpha+0.4)(\underline{x_3},\overline{x_3})=(0.04\alpha+0.1,-0.06\alpha+0.2)$$

. . . (63)-(65)

1. First Method

Equation (63)-(65) we can write in matrix form as,

$$\begin{bmatrix} 0.3\alpha+0.1 & 0.4\alpha+1 & 0.19\alpha+0.11 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.5\alpha+0.9 & -0.5\alpha+1.9 & -0.6\alpha+0.9 \\ 0.05\alpha+0.1 & 0.03\alpha+0.11 & 0.1\alpha+6 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.05\alpha+0.2 & -0.06\alpha+0.2 & -0.1\alpha+6.2 \\ 0.1\alpha+5 & 0.2\alpha+0.1 & 0.09\alpha+0.11 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.3\alpha+5.4 & -0.1\alpha+0.4 & -0.2\alpha+0.4 \end{bmatrix} \begin{pmatrix} \underline{x_1} \\ \underline{x_2} \\ \underline{x_3} \\ \overline{x_1} \\ \overline{x_2} \\ \overline{x_3} \end{pmatrix} = \begin{pmatrix} 0.09\alpha+0.01 \\ -0.1\alpha+0.2 \\ 0.03\alpha+0.11 \\ -0.06\alpha+0.2 \\ 0.04\alpha+0.1 \\ -0.06\alpha+0.2 \end{pmatrix}$$

$$\underline{x_1}=\frac{1/10(125\alpha^3+7993\alpha^2-409017\alpha-582021)}{(166\alpha^3-5386\alpha^2-1266749\alpha-2987081)}$$

$$\underline{x_2}=\frac{1/2(17\alpha^3+482\alpha^2-478099\alpha-36340)}{(166\alpha^3-5386\alpha^2-1266749\alpha-2987081)}$$

$$\underline{x_3}=\frac{(41\alpha^3-4387\alpha^2-31628\alpha-53460)}{(166\alpha^3-5386\alpha^2-1266749\alpha-2987081)}$$

$$\overline{x_1}=\frac{-2/5(34\alpha^3+2689\alpha^2-22150\alpha+44000)}{(32\alpha^3-6937\alpha^2+185000\alpha-604000)}$$

$$\overline{x_2}=\frac{-(40\alpha^3-1513\alpha^2-19830\alpha+47000)}{(32\alpha^3-6937\alpha^2+185000\alpha-604000)}$$

$$\overline{x_3}=\frac{1/5(250\alpha^3-8017\alpha^2+50050\alpha-87000)}{(32\alpha^3-6937\alpha^2+185000\alpha-604000)}$$

2. Second method

$$(0.3\alpha+0.1)\underline{x_1}+(0.4\alpha+1)\underline{x_2}+(0.19\alpha+0.11)\underline{x_3}=(0.09\alpha+0.01)$$
$$(-0.5\alpha+0.9)\overline{x_1}+(-0.5\alpha+1.9)\overline{x_2}+(-0.6\alpha+0.9)\overline{x_3}=(-0.1\alpha+0.2)$$
$$(0.05\alpha+0.1)\underline{x_1}+(0.04\alpha+0.11)\underline{x_2}+(0.1\alpha+6)\underline{x_3}=(0.03\alpha+0.11)$$
$$(-0.05\alpha+0.2)\overline{x_1}+(-0.06\alpha+0.2)\overline{x_2}+(-0.1\alpha+6.2)\overline{x_3}=(-0.06\alpha+0.2)$$
$$(0.1\alpha+5)\underline{x_1}+(0.2\alpha+0.1)\underline{x_2}+(0.09\alpha+0.11)\underline{x_3}=(0.04\alpha+0.1)$$
$$(-0.3\alpha+5.4)\overline{x_1}+(-0.1\alpha+0.4)\overline{x_2}+(-0.2\alpha+0.4)\overline{x_3}=(-0.6\alpha+0.2)$$

. . . (66)-(71)

We get from (66), (68), (70) and (67), (69), (71) respectively,

$$\underline{x_1}=\frac{1/10(125\alpha^3+7993\alpha^2-409017\alpha-582021)}{(166\alpha^3-5386\alpha^2-1266749\alpha-2987081)}$$
$$\underline{x_2}=\frac{1/2(17\alpha^3+482\alpha^2-478099\alpha-36340)}{(166\alpha^3-5386\alpha^2-1266749\alpha-2987081)}$$
$$\underline{x_3}=\frac{(41\alpha^3-4387\alpha^2-31628\alpha-53460)}{(166\alpha^3-5386\alpha^2-1266749\alpha-2987081)}$$

$$\overline{x_1}=\frac{-2/5(34\alpha^3+2689\alpha^2-22150\alpha+44000)}{(32\alpha^3-6937\alpha^2+185000\alpha-604000)}$$
$$\overline{x_2}=\frac{-(40\alpha^3-1513\alpha^2-19830\alpha+47000)}{(32\alpha^3-6937\alpha^2+185000\alpha-604000)}$$
$$\overline{x_3}=\frac{1/5(250\alpha^3-8017\alpha^2+50050\alpha-87000)}{(32\alpha^3-6937\alpha^2+185000\alpha-604000)}$$

3. Third method

We can write the equations as

$$AX=B$$

$$X=A^{-1}B$$

For (66), (68), (70) we have,

$$X=\begin{pmatrix}\underline{x_1}\\ \underline{x_2}\\ \underline{x_3}\end{pmatrix}=\begin{pmatrix}\dfrac{1/10(125\alpha^3+7993\alpha^2-409017\alpha-582021)}{(166\alpha^3-5386\alpha^2-1266749\alpha-2987081)}\\ \dfrac{1/2(17\alpha^3+482\alpha^2-478099\alpha-36340)}{(166\alpha^3-5386\alpha^2-1266749\alpha-2987081)}\\ \dfrac{(41\alpha^3-4387\alpha^2-31628\alpha-53460)}{(166\alpha^3-5386\alpha^2-1266749\alpha-2987081)}\end{pmatrix}$$

Also for (67), (69), (71) we have,

$$X = \begin{pmatrix} \overline{x_1} \\ \overline{x_2} \\ \overline{x_3} \end{pmatrix} = \begin{pmatrix} \dfrac{-2/5(34\alpha^3 + 2689\alpha^2 - 22150\alpha + 44000)}{(32\alpha^3 - 6937\alpha^2 + 185000\alpha - 604000)} \\ \dfrac{-(40\alpha^3 - 1513\alpha^2 - 19830\alpha + 47000)}{(32\alpha^3 - 6937\alpha^2 + 185000\alpha - 604000)} \\ \dfrac{1/5(250\alpha^3 - 8017\alpha^2 + 50050\alpha - 87000)}{(32\alpha^3 - 6937\alpha^2 + 185000\alpha - 604000)} \end{pmatrix}$$

Corresponding individual graph is shown in figure 9, 10 and 11 respectively.

For x_1

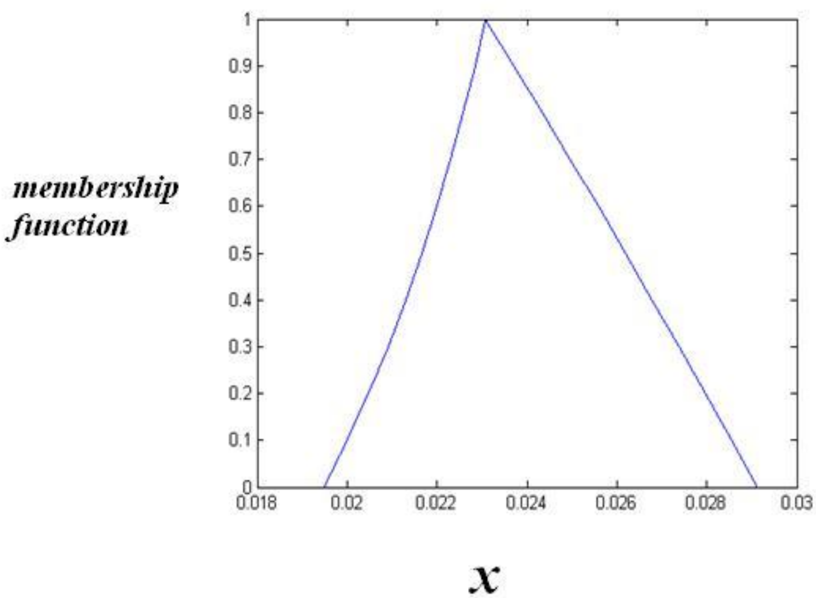

Fig. 9

For x_2

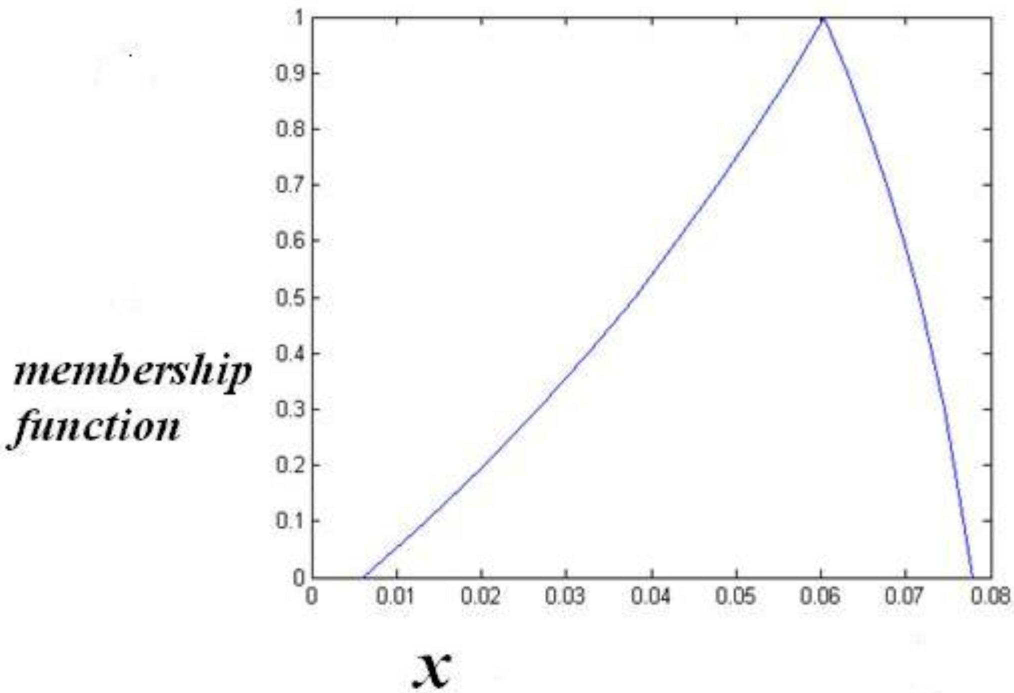

Fig. 10

For x_3

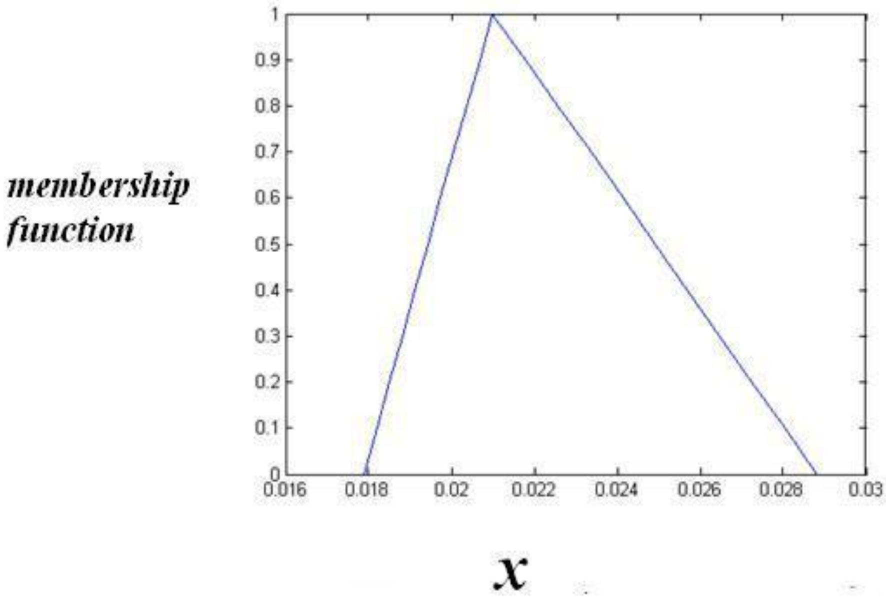

Fig.11

Plots for x_1, x_2 and x_3 are given all together in figure 12

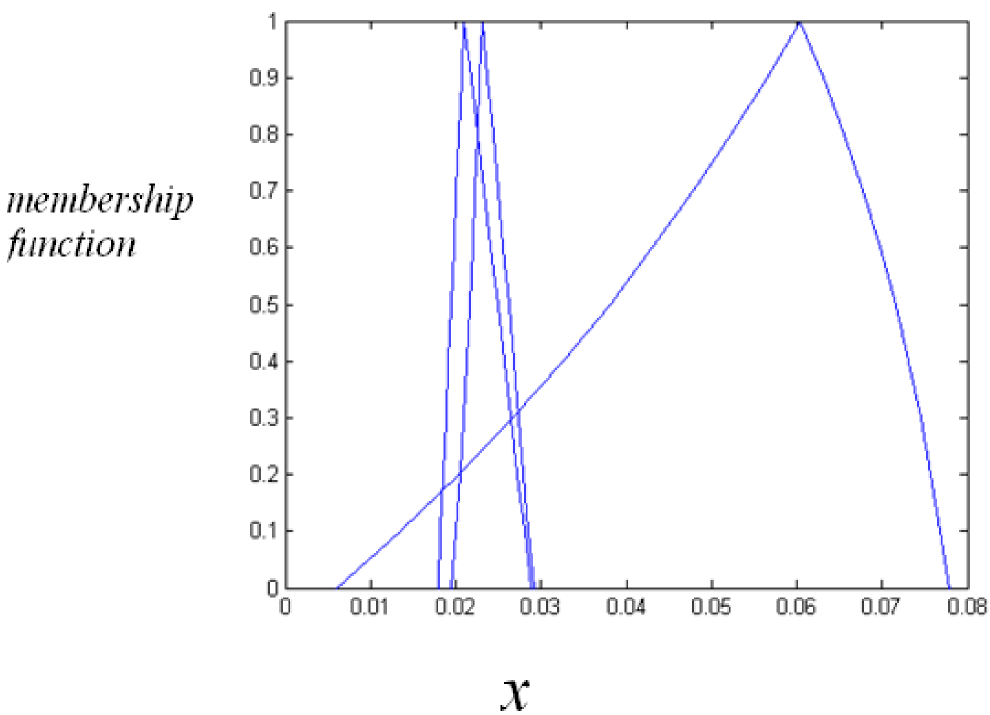

Fig. 12

Special case:

One may note that result for crisp case can be obtained simply by putting $\alpha = 1$ in fuzzy case. Moreover by substituting $\alpha = 0$ in fuzzy case we can obtain the results for the interval case.

EXAMPLE 4:

Let the equations be,

$$(0.1,0.4,0.6,0.9)(\underline{x_1},\overline{x_1}) + (1,1.4,1.6,1.9)(\underline{x_2},\overline{x_2}) + (0.11,0.3,0.5,0.9)(\underline{x_3},\overline{x_3}) = (0.01,0.1,0.15,0.2)$$
$$(0.1,0.15,0.18,0.2)(\underline{x_1},\overline{x_1}) + (0.11,0.14,0.18,0.2)(\underline{x_2},\overline{x_2}) + (6,6.1,6.15,6.2)(\underline{x_3},\overline{x_3}) = (0.11,0.14,0.16,0.2)$$
$$(5,5.1,5.2,5.4)(\underline{x_1},\overline{x_1}) + (0.1,0.3,0.35,0.4)(\underline{x_2},\overline{x_2}) + (0.11,0.2,0.3,0.4)(\underline{x_3},\overline{x_3}) = (0.1,0.14,0.16,0.2)$$

. . . (72)-(74)

Transforming the trapezoidal fuzzy numbers into interval,

$$(0.3\alpha + 0.1, -0.3\alpha + 0.9)(\underline{x_1},\overline{x_1}) + (0.4\alpha + 1, -0.3\alpha + 1.9)(\underline{x_2},\overline{x_2}) + (0.19\alpha + 0.11, -0.4\alpha + 0.9)(\underline{x_3},\overline{x_3}) = (0.09\alpha + 0.01, -0.05\alpha + 0.2)$$
$$(0.05\alpha + 0.1, -0.02\alpha + 0.2)(\underline{x_1},\overline{x_1}) + (0.03\alpha + 0.11, -0.02\alpha + 0.2)(\underline{x_2},\overline{x_2}) + (0.1\alpha + 6, -0.05\alpha + 6.2)(\underline{x_3},\overline{x_3}) = (0.03\alpha + 0.11, -0.04\alpha + 0.2)$$
$$(0.1\alpha + 5, -0.2\alpha + 5.4)(\underline{x_1},\overline{x_1}) + (0.2\alpha + 0.1, -0.05\alpha + 0.4)(\underline{x_2},\overline{x_2}) + (0.09\alpha + 0.11, -0.1\alpha + 0.4)(\underline{x_3},\overline{x_3}) = (0.04\alpha + 0.1, -0.04\alpha + 0.2)$$

. . . (75)-(77)

1. First method

We can write the above equations in matrix form as:

$$\begin{bmatrix} 0.3\alpha+0.1 & 0.4\alpha+1 & 0.19\alpha+0.11 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.3\alpha+0.9 & -0.3\alpha+1.9 & -0.4\alpha+0.9 \\ 0.05\alpha+0.1 & 0.03\alpha+0.11 & 0.1\alpha+6 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.02\alpha+0.2 & -0.02\alpha+0.2 & -0.05\alpha+6.2 \\ 0.1\alpha+5 & 0.2\alpha+0.1 & 0.09\alpha+0.11 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.2\alpha+5.4 & -0.05\alpha+0.4 & -0.1\alpha+0.4 \end{bmatrix} \begin{pmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \underline{x}_3 \\ \overline{x}_1 \\ \overline{x}_2 \\ \overline{x}_3 \end{pmatrix} = \begin{pmatrix} 0.09\alpha+0.01 \\ -0.05\alpha+0.2 \\ 0.03\alpha+0.11 \\ -0.04\alpha+0.2 \\ 0.04\alpha+0.1 \\ -0.04\alpha+0.2 \end{pmatrix}$$

$$\underline{x}_1 = \frac{1/10(125\alpha^3 + 7993\alpha^2 - 409017\alpha - 582021)}{(166\alpha^3 - 5386\alpha^2 - 1266749\alpha - 2987081)}$$

$$\underline{x}_2 = \frac{1/2(17\alpha^3 + 482\alpha^2 - 478099\alpha - 36340)}{(166\alpha^3 - 5386\alpha^2 - 1266749\alpha - 2987081)}$$

$$\underline{x}_3 = \frac{41\alpha^3 - 4387\alpha^2 - 31628\alpha - 53460}{(166\alpha^3 - 5386\alpha^2 - 1266749\alpha - 2987081)}$$

$$\overline{x}_1 = \frac{-1/10(29\alpha^3 + 10856\alpha^2 - 123600\alpha + 352000)}{21\alpha^3 - 6281\alpha^2 + 226540\alpha - 1208000}$$

$$\overline{x}_2 = \frac{-2/5(94\alpha^3 - 4659\alpha^2 - 39150\alpha + 235000}{21\alpha^3 - 6281\alpha^2 + 226540\alpha - 1208000}$$

$$\overline{x}_3 = \frac{(33\alpha^3 - 1490\alpha^2 + 13500\alpha - 34800)}{21\alpha^3 - 6281\alpha^2 + 226540\alpha - 1208000}$$

2. Second method

From (75), (76) and (77) we can write,

$$(0.3\alpha+0.1)\underline{x_1}+(0.4\alpha+1)\underline{x_2}+(0.19\alpha+0.11)\underline{x_3}=(0.09\alpha+0.01)$$
$$(-0.3\alpha+0.9)\overline{x_1}+(-0.3\alpha+1.9)\overline{x_2}+(-0.4\alpha+0.9)\overline{x_3}=(-0.05\alpha+0.2)$$
$$(0.05\alpha+0.1)\underline{x_1}+(0.03\alpha+0.11)\underline{x_2}+(0.1\alpha+6)\underline{x_3}=(0.03\alpha+0.11)$$
$$(-0.02\alpha+0.2)\overline{x_1}+(-0.02\alpha+0.2)\overline{x_2}+(-0.05\alpha+6.2)\overline{x_3}=(-0.04\alpha+0.2)$$
$$(0.1\alpha+5)\underline{x_1}+(0.2\alpha+0.1)\underline{x_2}+(0.09\alpha+0.11)\underline{x_3}=(0.04\alpha+0.1)$$
$$(-0.2\alpha+5.4)\overline{x_1}+(-0.05\alpha+0.4)\overline{x_2}+(-0.1\alpha+0.4)\overline{x_3}=(-0.04\alpha+0.2) \qquad \ldots(78)-(83)$$

$$\underline{x_1}=\frac{1/10(125\alpha^3+7993\alpha^2-409017\alpha-582021)}{(166\alpha^3-5386\alpha^2-1266749\alpha-2987081)}$$
$$\underline{x_2}=\frac{1/2(17\alpha^3+482\alpha^2-478099\alpha-36340)}{(166\alpha^3-5386\alpha^2-1266749\alpha-2987081)}$$
$$\underline{x_3}=\frac{41\alpha^3-4387\alpha^2-31628\alpha-53460}{(166\alpha^3-5386\alpha^2-1266749\alpha-2987081)}$$
$$\overline{x_1}=\frac{-1/10(29\alpha^3+10856\alpha^2-123600\alpha+352000)}{21\alpha^3-6281\alpha^2+226540\alpha-1208000}$$
$$\overline{x_2}=\frac{-2/5(94\alpha^3-4659\alpha^2-39150\alpha+235000}{21\alpha^3-6281\alpha^2+226540\alpha-1208000}$$
$$\overline{x_3}=\frac{(33\alpha^3-1490\alpha^2+13500\alpha-34800)}{21\alpha^3-6281\alpha^2+226540\alpha-1208000}$$

3. Third method

Equation (78)-(83) we can write as:

$$AX = B$$

$$X = A^{-1}B$$

$$X = \begin{pmatrix} \underline{x_1} \\ \underline{x_2} \\ \underline{x_3} \end{pmatrix} = \begin{pmatrix} \dfrac{1/10(125\alpha^3 + 7993\alpha^2 - 409017\alpha - 582021)}{(166\alpha^3 - 5386\alpha^2 - 1266749\alpha - 2987081)} \\ \dfrac{1/2(17\alpha^3 + 482\alpha^2 - 478099\alpha - 36340)}{(166\alpha^3 - 5386\alpha^2 - 1266749\alpha - 2987081)} \\ \dfrac{41\alpha^3 - 4387\alpha^2 - 31628\alpha - 53460}{(166\alpha^3 - 5386\alpha^2 - 1266749\alpha - 2987081)} \end{pmatrix}$$

$$X = \begin{pmatrix} \overline{x_1} \\ \overline{x_2} \\ \overline{x_3} \end{pmatrix} = \begin{pmatrix} \dfrac{-1/10(29\alpha^3 + 10856\alpha^2 - 123600\alpha + 352000)}{21\alpha^3 - 6281\alpha^2 + 226540\alpha - 1208000} \\ \dfrac{-2/5(94\alpha^3 - 4659\alpha^2 - 39150\alpha + 235000}{21\alpha^3 - 6281\alpha^2 + 226540\alpha - 1208000} \\ \dfrac{(33\alpha^3 - 1490\alpha^2 + 13500\alpha - 34800)}{21\alpha^3 - 6281\alpha^2 + 226540\alpha - 1208000} \end{pmatrix}$$

Corresponding individual graph is shown in figure 13, 14 and 15 respectively.

Graph for x_1

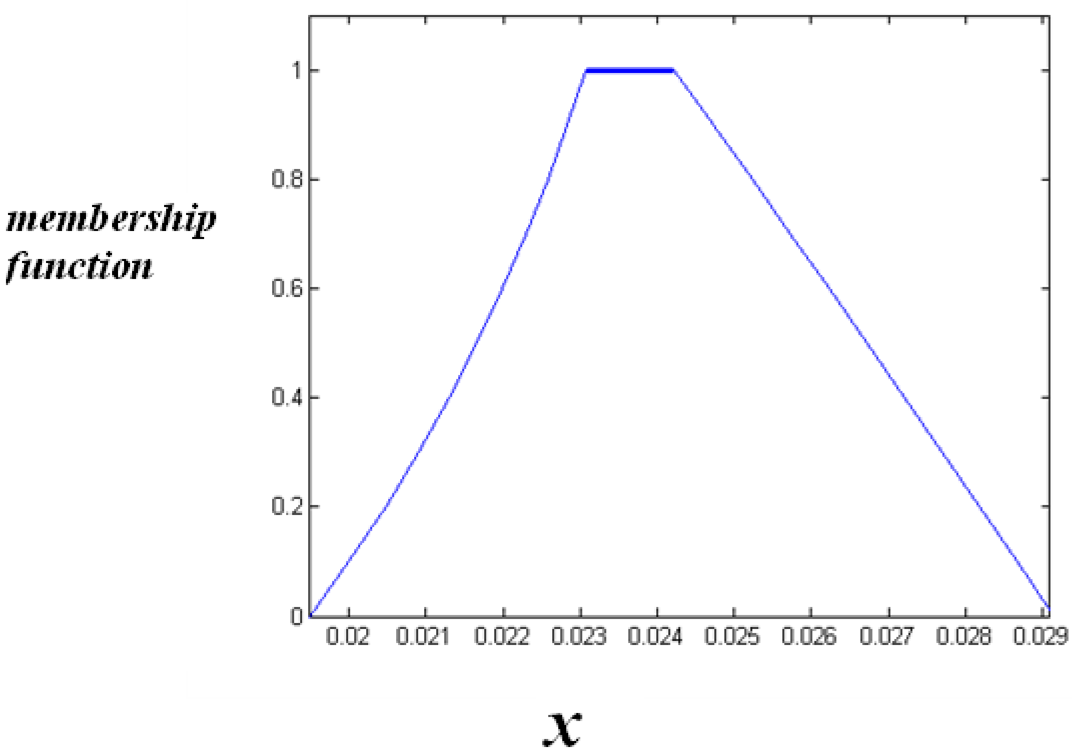

Fig. 13

Graph for x_2

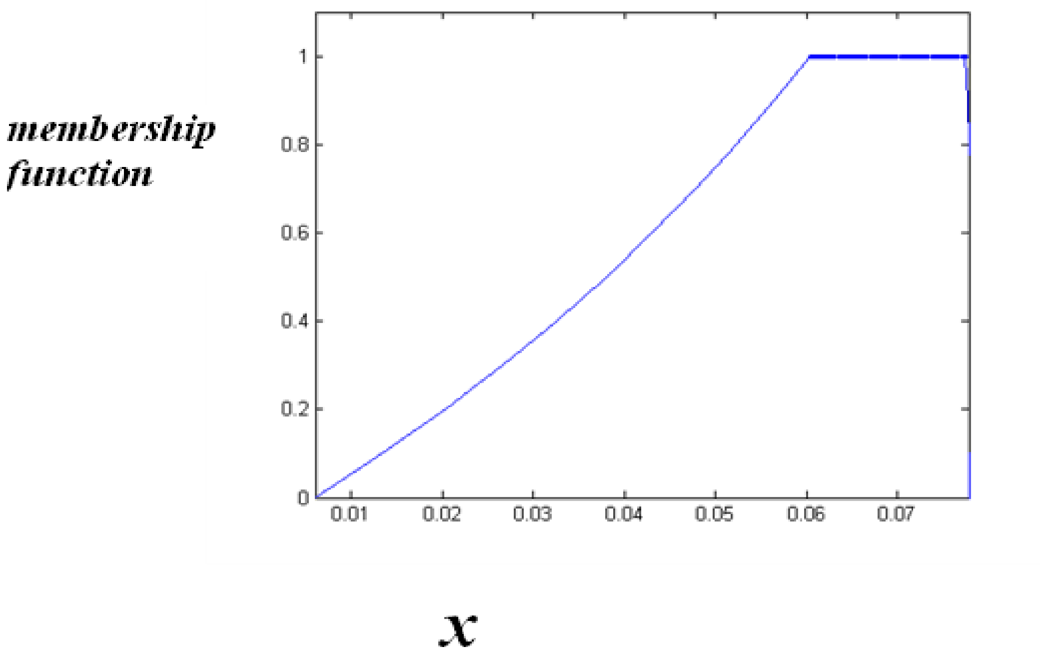

Fig. 14

Graph for x_3

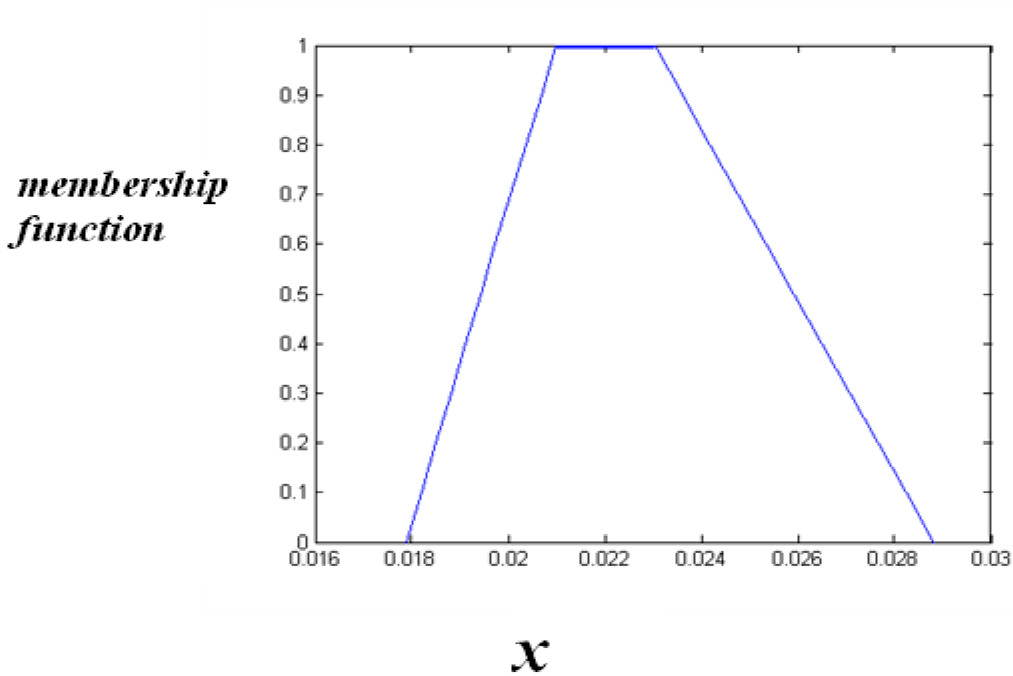

Fig. 15

Plots for x_1, x_2, x_3 given all together in figure 16.

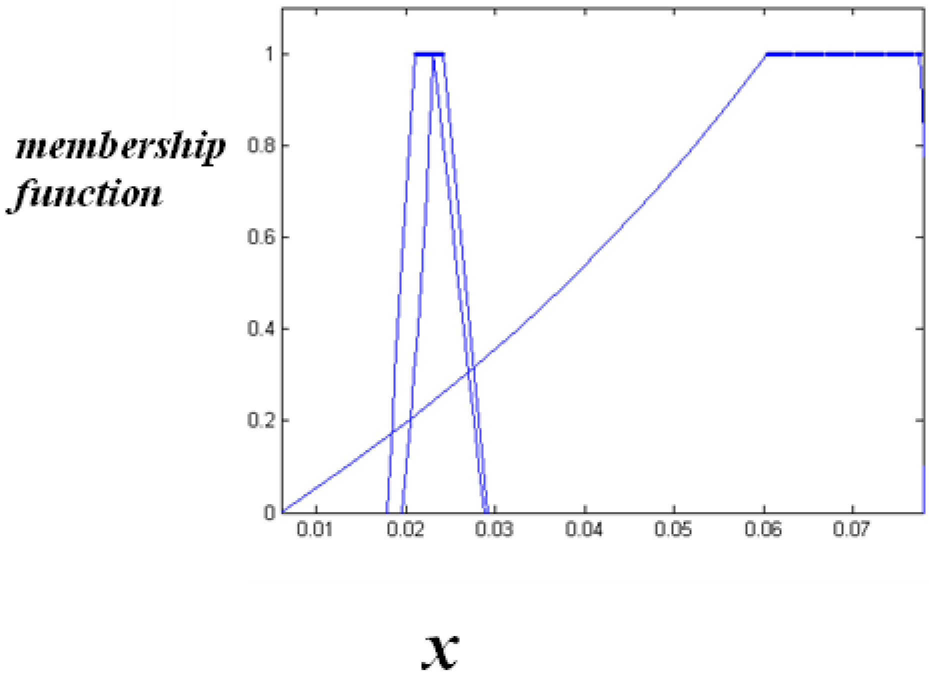

Fig. 16

Special Cases:

One may note that for $\alpha = 1$ we get different value with membership value 1 in fuzzy case. Moreover by substituting $\alpha = 0$ in fuzzy case we can obtain the results for the interval case. But at the point $a_2 = a_3$ we get the same for triangular fuzzy number.

7.2. EXAMPLE 1:

$$(2,3)(\underline{x}_1,\overline{x}_1)+(4,5)(\underline{x}_2,\overline{x}_2)=(1,2)$$
$$(3,4)(\underline{x}_1,\overline{x}_1)+(2,3)(\underline{x}_2,\overline{x}_2)=(18,19)$$

. . . (84)-(85)

1. First method

We have used equation (7) for this example as

$$\begin{bmatrix} 2 & 4 & 0 & 0 \\ 0 & 0 & 3 & 5 \\ 3 & 2 & 0 & 0 \\ 0 & 0 & 4 & 3 \end{bmatrix}\begin{pmatrix} \underline{x}_1 \\ \underline{x}_2 \\ \overline{x}_1 \\ \overline{x}_2 \end{pmatrix}=\begin{pmatrix} 1 \\ 2 \\ 18 \\ 19 \end{pmatrix}$$

whose solution may easily be obtained as

$$\underline{x}_1=8.75$$
$$\underline{x}_2=-4.12$$
$$\overline{x}_1=8.09$$
$$\overline{x}_2=-4.45$$

2. Second method

From (84) and (85) we get,

$$2\underline{x}_1+4\underline{x}_2=1$$
$$3\overline{x}_1+5\overline{x}_2=2$$
$$3\underline{x}_1+2\underline{x}_2=18$$
$$4\overline{x}_1+3\overline{x}_2=19$$

. . . (86)-(89)

Solving (86) and (88); (87) and (89), we have,

$$\underline{x_1} = 8.75$$
$$\underline{x_2} = -4.12$$
$$\overline{x_1} = 8.09$$
$$\overline{x_2} = -4.45$$

3. Third method

Equation (86) and (88) are obtained for the present example as:

$$AX = b$$
$$\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \begin{pmatrix} \underline{x_1} \\ \underline{x_2} \end{pmatrix} = \begin{pmatrix} 1 \\ 18 \end{pmatrix}$$
$$X = \begin{pmatrix} \underline{x_1} \\ \underline{x_2} \end{pmatrix} = A^{-1}b = \begin{pmatrix} 8.75 \\ -4.12 \end{pmatrix}$$

Similarly equations (87) and (89) give the direct solution as,

$$X = \begin{pmatrix} \overline{x_1} \\ \overline{x_2} \end{pmatrix} = \begin{pmatrix} 8.09 \\ -4.45 \end{pmatrix}$$

7.3. <u>EXAMPLE 1:</u>

Let, the equations be,

$$(4,4.2,4.4)(\underline{x_1},\overline{x_1})+(2,2.3,2.6)(\underline{x_2},\overline{x_2})=(1,1.5,2)$$
$$(3,3.1,3.3)(\underline{x_1},\overline{x_1})+(1,1.1,1.3)(\underline{x_2},\overline{x_2})=(5,5.2,5.5)$$

... (90)-(91)

After transferring the fuzzy interval into α-cut interval, we have,

$$(0.2\alpha+4,-0.2\alpha+4.4)(\underline{x_1},\overline{x_1})+(0.3\alpha+2,-0.3\alpha+2.6)(\underline{x_2},\overline{x_2})=(0.5\alpha+1,-0.5\alpha+2)$$
$$(0.1\alpha+3,-0.2\alpha+3.3)(\underline{x_1},\overline{x_1})+(0.1\alpha+1,-0.2\alpha+1.3)(\underline{x_2},\overline{x_2})=(0.2\alpha+5,-0.3\alpha+5.5)$$

... (92)-(93)

1. First method

Equation (92) and (93) can be written in matrix form as,

$$\begin{bmatrix} 0.2\alpha+4 & 0.3\alpha+2 & 0 & 0 \\ 0 & 0 & -0.2\alpha+4.4 & -0.3\alpha+2.6 \\ 0.1\alpha+3 & 0.1\alpha+1 & 0 & 0 \\ 0 & 0 & -0.2\alpha+3.3 & -0.2\alpha+1.3 \end{bmatrix}\begin{pmatrix} \underline{x_1} \\ \underline{x_2} \\ \overline{x_1} \\ \overline{x_2} \end{pmatrix}=\begin{pmatrix} 0.5\alpha+1 \\ -0.5\alpha+2 \\ 0.2\alpha+5 \\ -0.3\alpha+5.5 \end{pmatrix}$$

$$\underline{x_1}=\frac{\alpha^2+130\alpha+900}{\alpha^2+50\alpha+200}$$
$$\underline{x_2}=\frac{\alpha^2-20\alpha-1700}{\alpha^2+50\alpha+200}$$
$$\overline{x_1}=\frac{-\alpha^2-138\alpha+1170}{2\alpha^2-37\alpha+286}$$
$$\overline{x_2}=\frac{4\alpha^2+37\alpha-1760}{2\alpha^2-37\alpha+286}$$

2. Second method

From (92) and (93) we can write,

$$(0.2\alpha+4)\underline{x_1}+(0.3\alpha+2)\underline{x_2}=0.5\alpha+1$$
$$(-0.2\alpha+4.4)\overline{x_1}+(-0.3\alpha+2.6)\overline{x_2}=-0.5\alpha+2$$
$$(0.1\alpha+3)\underline{x_1}+(0.1\alpha+1)\underline{x_2}=0.2\alpha+5$$
$$(-0.2\alpha+3.3)\overline{x_1}+(-0.2\alpha+1.3)\overline{x_2}=-0.3\alpha+5.5$$

. . . (94)-(97)

Solving the above, we have,

$$\underline{x_1}=\frac{\alpha^2+130\alpha+900}{\alpha^2+50\alpha+200}$$
$$\underline{x_2}=\frac{\alpha^2-20\alpha-1700}{\alpha^2+50\alpha+200}$$
$$\overline{x_1}=\frac{-\alpha^2-138\alpha+1170}{2\alpha^2-37\alpha+286}$$
$$\overline{x_2}=\frac{4\alpha^2+37\alpha-1760}{2\alpha^2-37\alpha+286}$$

3. Third method

From (94) and (96) we can write,

$$AX=b$$
$$X=A^{-1}b$$
$$=\begin{pmatrix}0.2\alpha+4 & 0.3\alpha+2\\ 0.1\alpha+3 & 0.1\alpha+1\end{pmatrix}^{-1}\begin{pmatrix}0.5\alpha+1\\ 0.2\alpha+5\end{pmatrix}$$
$$X=\begin{pmatrix}\underline{x_1}\\ \underline{x_2}\end{pmatrix}=\begin{pmatrix}\dfrac{\alpha^2+130\alpha+900}{\alpha^2+50\alpha+200}\\ \dfrac{\alpha^2-20\alpha-1700}{\alpha^2+50\alpha+200}\end{pmatrix}$$

Similarly, from (95) and (97), we have,

$$X = \begin{pmatrix} \overline{x_1} \\ x_2 \end{pmatrix} = \begin{pmatrix} \dfrac{-\alpha^2 - 138\alpha + 1170}{2\alpha^2 - 37\alpha + 286} \\ \dfrac{4\alpha^2 + 37\alpha - 1760}{2\alpha^2 - 37\alpha + 286} \end{pmatrix}$$

Corresponding individual graph is shown in figure 17

For x_1

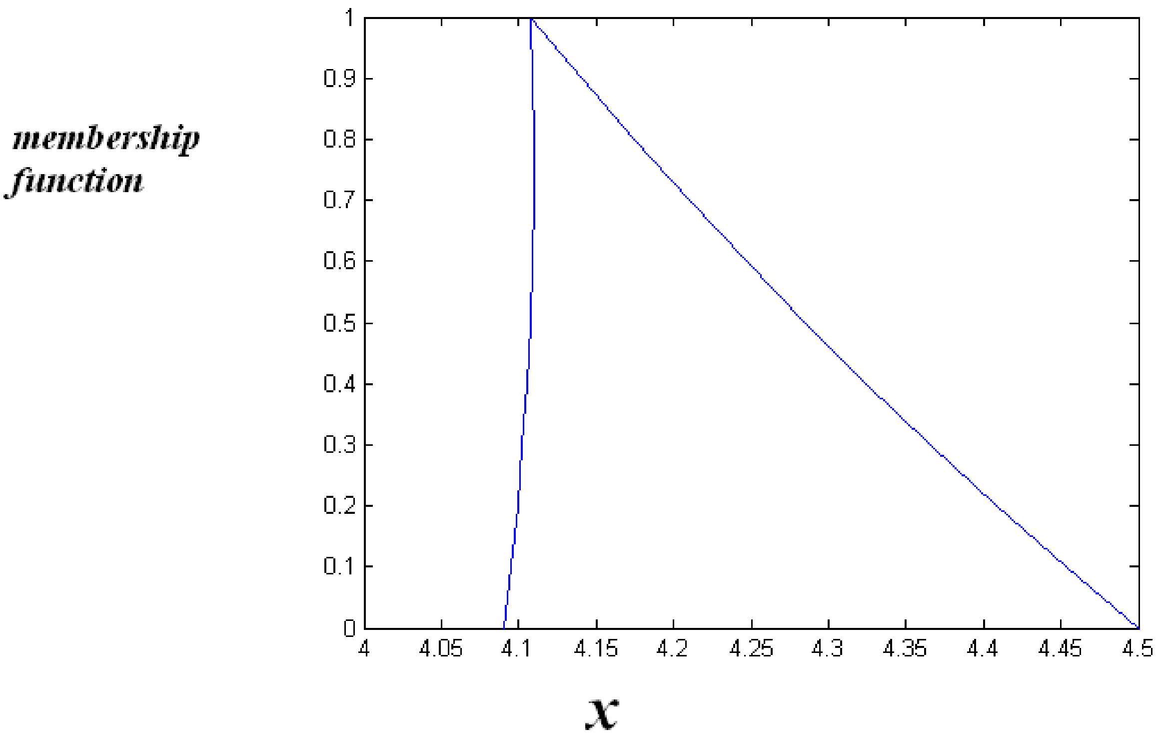

Fig .17

For x_2

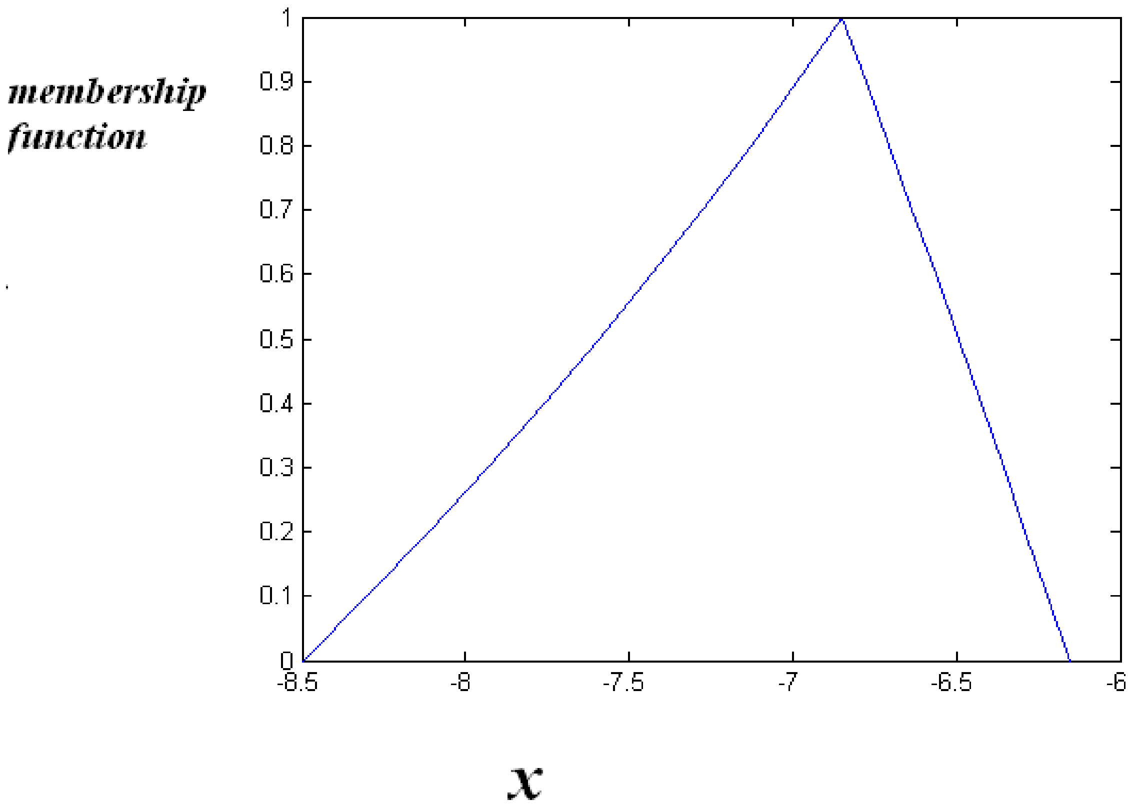

Fig .18

Plots of x_1 and x_2 are shown in figure 19 all together

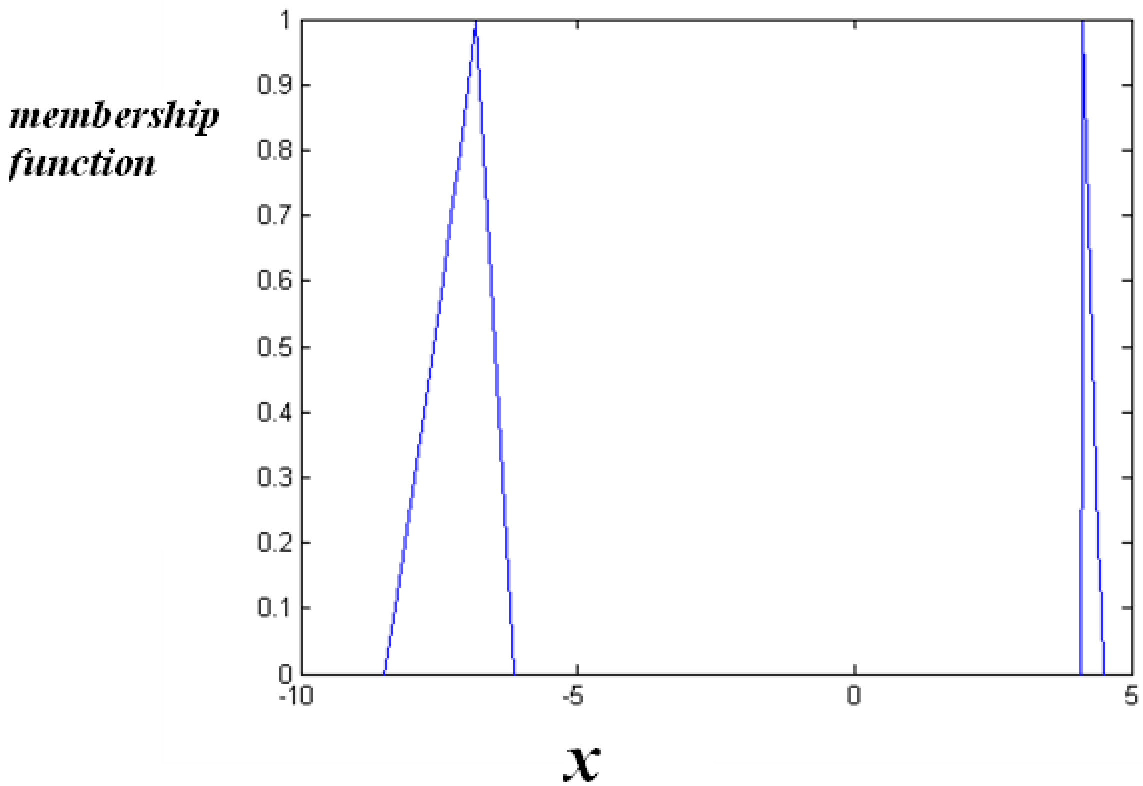

Fig .19

8. CONCLUSION

Present work demonstrates a new method for interval and fuzzy solution of fuzzy system of linear equations. This is applied first in a known problem of circuit analysis. As discussed earlier, the concepts of fuzzy number (that is, triangular fuzzy number, trapezoidal fuzzy number), $\alpha - \text{cut}$, have been used here to solve the numerical problems of system of linear equations. Few other example problems are also solved to have the efficiency of the proposed method.

Three different cases are considered in the above circuit analysis problem taking source and resistance as:

Case I: Crisp

Case II: Interval

Case III: Fuzzy.

where, current is taken as fuzzy in case I and case III and crisp in case II.

As mentioned above the other example problems are solved with interval, triangular fuzzy number, and trapezoidal fuzzy number to have the reliability and powerfulness of the proposed methods.

This investigation gives a new idea of solving the interval/fuzzy system of linear equations with simple computations.

9. FUTURE DIRECTION

In this project, methodologies have been developed to solve fuzzy system of linear equations. Simulation with different example problems shows that the proposed methods are easier and simple to handle in comparison with the existing methods.

The proposed method may very well be applied to other problems where we get linear simultaneous equations in interval or fuzzy form. Some subject area may be mentioned as in bio-mathematics, electrical engineering, computer science engineering, mechanical engineering etc where we may get the interval or fuzzy linear system of equations and those may be solved by the proposed methods.

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