“Mixed Convection Heat Transfer in the Entrance Region of an Inclined Channel”

A THESIS SUBMITTED IN PARTIAL FULFILLMENT

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BACHELOR OF TECHNOLOGY

IN

MECHANICAL ENGINEERING

By

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<u>CERTIFICATE</u>

This is to certify that the project entitled, "**Mixed Convection Heat Transfer in the Entrance Region of an Inclined Channel**" submitted by '**Mr.Gurpreet Singh** ' in partial fulfillments for the requirements for the award of Bachelor of Technology Degree in Mechanical Engineering at **National Institute of Technology, Rourkela (Deemed University)** is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the report has not been submitted to any other University / Institute for the award of any Degree or Diploma.

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13th may 2010 (Gurpreet Singh)

Abstract

In the present project we have tried to find the convection parameters related to the case of mixed convection in the entrance region of an inclined channel. We have tried to find the trend in the change in the values of various convection parameters related to mixed convection at different set values. We have done mathematical modeling using FORTRAN 77 and tried to follow a new scheme called SAR scheme whose detail we have shown at proper place. by this scheme we have calculated the convection parameters at different coordinates of the channel. This scheme is mainly an iteration process by which we have tried to find the error in each subsequent guesses and we have made an approach to reach the solution.

Nomenclature

Br Brinkman number, defined by, $\frac{\mu U_{ref}^2}{k\,\Delta T}$

D_h Hydraulic diameter, 2L

Ec Eckert number, $\frac{U_{ref}^2}{C_p\,\Delta T}$

g Acceleration due to gravity, m^2/s

Gr Grashof number, defined by, $\frac{g\,\beta\,\Delta T\,D_h^3}{\nu^2}$

k Thermal conductivity, W/m – K

K_1 Ratio of left wall temperature to inlet temperature ($= T_{w1}/T_i$)

K_2 Ratio of right wall temperature to inlet temperature ($= T_{w2}/T_i$)

L Spacing between the two plates, m

Nu_- Local Nusselt number at left wall, defined by, $\left.\frac{d\theta}{dY}\right|_{Y=-1/4}$

Nu_+ Local Nusselt number at right wall, defined by, $\left.\frac{d\theta}{dY}\right|_{Y=1/4}$

Nu_{b-} Nusselt number based on bulk mean temperature at left wall, defined by, $\frac{2Nu_-}{R_T + 2\theta^*}$

Nu_{b+} Nusselt number based on bulk mean temperature at right wall, defined by, $\frac{2Nu_+}{R_T - 2\theta^*}$

Pe Peclet number, defined by, $\frac{U_{ref}\,D_h}{\alpha}$

p_d Pressure that arises when the fluid is in motion, N/m^2

p_s Static pressure, when fluid velocity = 0 N/m^2

Re Reynolds number, defined by, $\frac{U_{ref}\,D_h}{\nu}$

R_T $\frac{T - T_{ref}}{\Delta T}$

T Dimensional temperature

T_b Dimensional bulk temperature

T_{ref} Reference temperature, defined by, $\frac{T_{w1} + T_{w2}}{2}$

T_i Inlet fluid temperature

T_{w1} Temperature of the wall at y = - L/2

T_{w2} Temperature of the wall at y = L/2

u Dimensional velocity in x direction, m/s

U_i Inlet velocity, m/s

U_{ref} Reference velocity, m/s

U Dimensionless velocity in X direction = u/U_{ref}

v Dimensional velocity in y direction, m/s

V Dimensionless velocity in Y direction, = v/U_{ref}

$\vec{V}$ Velocity vector

X Dimensionless axial distance

Y Dimensionless coordinate normal to the flow direction

x Dimensional axial distance

y Dimensional coordinate normal to the flow direction

x_f Dimensional entry length, m

X_f Dimensionless entry length

X^* X / Pe

Greek Symbols

μ Dynamic viscosity, kg/m-s

ν Kinematic viscosity, m^2/s

ρ Fluid density, kg/m^3

β Coefficient of thermal expansion

θ Dimensionless temperature

θ^* Non-dimensional bulk mean temperature of the fluid, defined by, $\frac{T_b - T_{ref}}{\Delta T}$

θ_b Non-dimensional temperature based on bulk mean temperature, defined by, $\frac{T - T_{ref}}{T_b - T_{ref}}$

CONTENTS

CHAPTER 1

1.1 Introduction

Heat transfer by forced convection in pipes has been the subject of investigation by many researchers for the past several decades, starting from Gratz [1] who pioneered the studies. Other studies include those of Callender [2], Nusselt [3]. Sparrow and Patankar [4] developed the relationships for Nusselt numbers for thermally developed duct flows for different boundary conditions.

The problem of forced convection in a channel between two parallel plate walls is a classical problem that has been revisited in recent years in connection with the cooling of electronic equipment using materials involving hyperporous media or microchannels. Recently published textbooks and handbooks, such as those by Bejan [37] and Kakac¸ et al. [38], devote substantial space to the case of symmetric heating but little to the more complicated case of asymmetric heating. However, this case is mentioned in Shah and London [39, pp. 155–157] and Kakac¸ et al. [38, pp. 3.31–3.32], where the key results are given, without details of derivation. (An outline derivation is given in Kays and Crawford [40].)

More recent studies deal with, flow through annuli, channels, with symmetric and asymmetric heating, particularly in the combined convection regime (Aung and Worku [5], Cheng, C.H., Kou, H.S., Huang, W.H. [6], Hamad and Wirtz [7], Barletta and Zanchini[8]). Also the channel / pipe / annuli are inclined at an arbitrary angle (Iqbal and Stachiewicz [9], Sabbagh, J.A., Aziz, A., El-Ariny, A.S., and Hamad, G. [10], Lavine, A. S., Kim, M.Y., and Shores, C.N. [11], Orfi, J., Galanis,N. and Nguyen, C.T. [12]). These studies are expected to provide insight needed to design cooling systems for electronic devices, solar energy devices, and chemical vapor deposition technique. Combined convection in channels of arbitrary inclination subjected to asymmetric heating, including dissipation finds practical applications. Asymmetric thermal boundary conditions may be thought of as due to a deliberate unequal temperature or fluxes imposed, or as due to unequal temperature jump owing to differing accommodation coefficients at the two walls in the rarefied ($Kn << 1$, where Kn is the Knudsen number) regime as relevant in micro-channel heat transfer.

1.2. Literature Review

The following table gives the major studies pertaining to the flow and heat transfer through channels, annuli and pipes.

S.No.	Author	Geometry	Flow Model	Heat Transfer Model
1.	Kays [13]	Circular tube	Developing flow	Forced convection, Uniform tube temperature and uniform tube heat flux
2.	Sparrow [14]	Rectangular duct	Developing flow	Forced convection, Analytical study
3.	Reynolds [15]	Circular tube	Fully developed flow	Forced convection, Uniform heat flux, Analytical solution
4.	Mercer, Pearce, and Hitchcock, [16]	Parallel flat plates	Developing flow	Forced convection, Both plates are at uniform temperature, experimental study
5.	Morton [17]	Vertical tube	Fully developed flow	Mixed convection, Uniform wall heat flux. Analytical solution
6.	Barletta [18]	Vertical channel	Fully developed flow	Mixed convection with viscous dissipation. Walls are at same or at different temperatures, Approximate perturbation solution
7.	Barletta [19]	Vertical circular duct	Fully developed flow	Mixed convection with viscous dissipation and uniform wall temperature
8.	Barletta , Rossi di Schio [20]	Vertical circular tube	Fully developed flow	Mixed convection with viscous dissipation and uniform wall heat flux
9.	Choi,D.K., and Choi,D.H. [21]	Horizontal tube	Fully Developed flow	Mixed convection, Upper half of the duct wall is insulated and lower wall subjected to uniform heat flux
10.	Barletta and Zanchini [22]	Inclined channel	Fully developed flow	Mixed convection with viscous dissipation and uniform wall temperature

11.	Lavine [23]	Inclined parallel plates	Fully developed flow	Opposing mixed convection
12.	Lavine[24]	Inclined parallel plates	Fully developed flow	Aiding mixed convection
13.	Barletta et.al. [25]	Vertical circular duct	Fully Developed flow	Non-axisymmetric mixed convection
S.No.	Author	Geometry	Flow Model	Heat Transfer Model
14.	Bohne and Obermeier [26]	Vertical, inclined cylindrical annulus	Fully Developed flow	Combined Free and Forced Convection. Experimental study
15.	Lawrence and Chato [27]	Vertical tube	Developing laminar flow	Mixed convection, Uniform heat flux and uniform wall temperature
16.	Zeldin Schmidt [28]	Vertical tube	Developing laminar flow	Combined free and forced, convection isothermal wall Experimental study
17.	Moutsoglou and Kwon[29]	Vertical tube	Onset of flow reversal in developing flow	Mixed convection, Uniform heat flux and uniform wall temperature
18.	Morcos Abou-Allail [30]	Inclined multi rectangular channel	Buoyancy effects in the entrance region	Mixed convection, Numerical solution
19.	Cheng and Yuen [31]	Heated inclined pipes	Entrance region	Visualization studies on secondary flow pattern for mixed convection Isothermally heated pipe
20.	Choudury and Patankar [32]	Inclined isothermal tube	Convection in the entrance region	Combined forced and free laminar convection , Isothermal tube
21.	Rao and Morris [33]	Vertical parallel plates	Fully developed flow	Superimposed laminar mixed convection, One wall is heated at uniform heat flux and other is thermally insulated
22.	Yao [34]	Vertical channel	Hydrodynamically and thermally developing flow	Mixed convection Symmetric UWT or UHF
23.	Barletta and Zanchini [35]	Inclined channel	Steady periodic Flow	Mixed convection, The temperature of one wall is stationary and the temperature of other wall is a sinusoidal function of time

Chapter 2

2.1. Lacunae

1. Most studies assumed fully developed conditions or invoked boundary layer approximations.
2. Within the framework of Boussinesq approximation (for mixed convection studies).
3. Symmetrically heated wall boundary condition, constant temperature or uniform heat flux.
4. Particularly the studies [17-20, 22] including dissipation dealt with fully developed conditions only.

2.2. Objective

To study mixed convection in the entry region of an arbitrarily inclined channel subjected to asymmetric heating including dissipation.

2.3. Motivation

1. Examine the criterion to define fully developed thermal condition when asymmetrically heated.
2. Mixed convection when the channel is arbitrarily inclined is of practical importance in several electronic cooling configurations.
3. When the channel width is small, (micro channels), asymmetric thermal conditions can be expected irrespective of imposed boundary conditions, bring temperature jump or velocity jump boundary conditions, which may not be equal, even if walls are subjected to equal temperature or heat flux.

Chapter 3

3.1. Mathematical Formulation

The physical model considered is that of a channel of width L with both left and right walls being maintained at constant temperatures T_{w1} and T_{w2} are subjected to q_1 and q_2 respectively.

The channel is inclined at an angle φ with the direction of gravity. The flow enters at a uniform velocity U_i and uniform temperature T_i. The x-component of velocity is along the longitudinal direction of the channel and y-component of velocity is along the transverse direction of the channel. The flow is buoyancy aided when the flow is in upward direction and is buoyancy opposed when flow is in downward direction. The physical model and the coordinate system are shown in Fig. 1.

3.2. Governing Equations: General Formulation

Governing equations for steady, laminar two dimensional flow of an incompressible, Newtonian fluid with constant fluid properties and invoking Boussinesq approximation to describe buoyancy forces are as follows

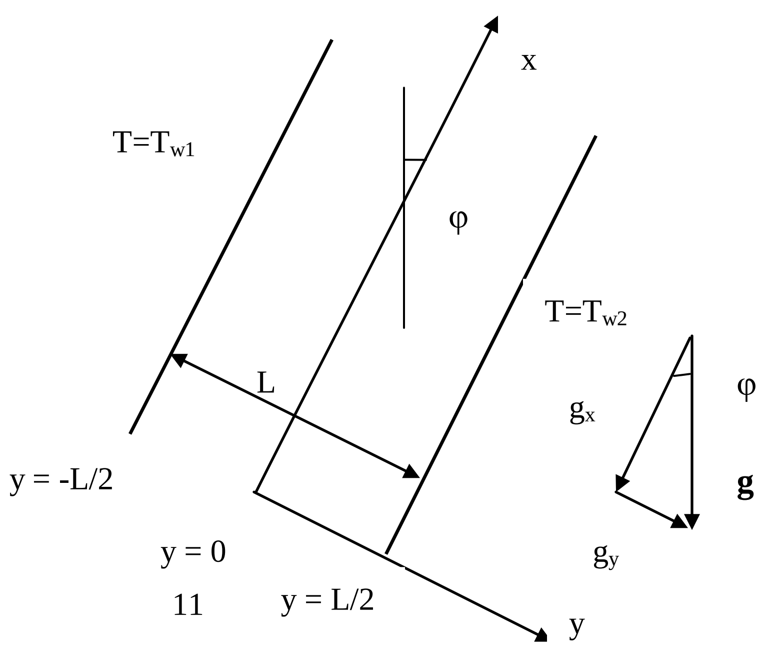


Figure 1. Physical model and Coordinate system

Continuity Equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

x-Momentum Equation

$$\rho_{ref}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p_d}{\partial x}+\mu\nabla^2 u-g(\rho-\rho_{ref})\cos\varphi \tag{2}$$

y-Momentum Equation

$$\rho_{ref}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p_d}{\partial y}+\mu\nabla^2 v+g(\rho-\rho_{ref})\sin\varphi \tag{3}$$

Conservation of Energy Equation

$$\rho_{ref}\,C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\mu\left[\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2+2\left(\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right)\right] \tag{4}$$

In Eqs. (2) and (3), it may be noted that

$$p = p_d + p_s \tag{5}$$

where p is the pressure in the fluid at a point, p_s is the static pressure(i.e., the pressure that exists, when $\vec{V}\equiv 0$) and p_d is the dynamic pressure that arises when the fluid is in motion.

has been used along with

$$\frac{\partial p_s}{\partial x}=-\rho_{ref}\,g\,\mathrm{Cos}\,\varphi \tag{6}$$

$$\frac{\partial p_s}{\partial y}=\rho_{ref}\,g\,\mathrm{Sin}\,\varphi \tag{7}$$

In addition, the auxiliary equation describing the variation of density with temperature within the frame work of Boussinesq approximation is given by,

$$\rho=\rho_{ref}\left[1-\beta(T-T_{ref})\right] \tag{8}$$

Where ρ_{ref} is the reference density

β is the thermal coefficient of expansion

Boundary Conditions

$u = U_i$ at $x = 0$ for $-L/2 \le y \le +L/2$

$u = 0$, $v = 0$ at $y = \pm L/2$ for all $x > 0$

$T = T_i$ at $x = 0$ for $-L/2 \le y \le +L/2$

$T = T_{w1}$ at $y = - L/2$ for all $x > 0$

$T = T_{w2}$ at $y = + L/2$ for all $x > 0$

$$\frac{\partial u}{\partial x} = 0,\ v = 0 \text{ for } x > x_{fd}, \text{ for } -L/2 \le y \le +L/2 \tag{9}$$

Note: $T_{w1}=T_{w2}$ correspond to symmetric heating.

x_{fd} is the entry length, i.e., the distance required for the flow/ thermal field to become fully developed. $x > x_{fd}$ represents fully developed condition.

In addition, the fully developed condition leads to a certain non-dimensional temperature remaining constant with x for $x > x_{fd}$.

Non-Dimensional Equations

The following dimensionless variables have been introduced to render the governing equations non-dimensional.

$$U = \frac{u}{U_{ref}},\ V = \frac{v}{U_{ref}},\ X = \frac{x}{D_h},\ Y = \frac{y}{D_h},\ \theta = \frac{T - T_{ref}}{\Delta T},\ \overline{p}_d = \frac{p_d}{\rho_{ref} U_{ref}^2},\ \overline{\rho} = \frac{\rho}{\rho_{ref}} \tag{10}$$

The governing equations given by Eqs. (1), (2), (3) and (4) non-dimensionalised using the non-dimensional variables defined by Eq. (10) take the following form

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{11}$$

$$\left(U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} \right) = -\frac{\partial \overline{p}_d}{\partial X} + \frac{1}{Re}\nabla^2 U + \frac{Gr}{Re^2}\theta\cos\varphi \tag{12}$$

$$\left(U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} \right) = -\frac{\partial \overline{p}_d}{\partial Y} + \frac{1}{Re}\nabla^2 V - \frac{Gr}{Re^2}\theta\sin\varphi \tag{13}$$

$$\left(U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} \right) = \frac{1}{Pe}\left(\frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2} \right) + \frac{Ec}{Re}\left[\left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial Y} \right)^2 + 2\left(\left(\frac{\partial U}{\partial X} \right)^2 + \left(\frac{\partial V}{\partial Y} \right)^2 \right) \right] \tag{14}$$

U_{ref} and ΔT in Eq. (10) are chosen depending on the wall boundary condition and the problem studied.

$$U_{ref} = U_i \tag{15}$$

$\Delta T = (T_{w2} - T_{w1})$ if $T_{w2} \neq T_{w1}$ for constant but unequal wall temperatures. (16)

$= \dfrac{\nu^2}{C_p D_h^2}$ if $T_{w2} = T_{w1}$ for constant and equal wall temperatures. (17)

$= (T_i - T_{ref})$ when the flow is thermally developing. (18)

In Eq. (18) T_{ref} is defined by,

$T_{ref} = (T_{w1}+T_{w2})/2$ (19)

$T_{ref} = T_i$, (20)

The boundary conditions given in Eq. (9) become**,**

$U = 1$ at $X = 0$ for $-1/4 \leq Y \leq +1/4$

$U = 0$, $V = 0$ at $Y = \pm 1/4$ for all $X > 0$

$\theta = \dfrac{T_i - T_{ref}}{\Delta T}$ at $X = 0$ for $-1/4 \leq Y \leq +1/4$

$\theta = \dfrac{T_{w1} - T_{ref}}{\Delta T}$ at $Y = -1/4$ for all $X > 0$

$\theta = \dfrac{T_{w2} - T_{ref}}{\Delta T}$ at $Y = +1/4$ for all $X > 0$

$\dfrac{\partial U}{\partial X} = 0, V = 0, \dfrac{\partial \theta}{\partial X} = \dfrac{\theta}{\theta^*}\dfrac{\partial \theta^*}{\partial X}$ at $X > X_{fd}$ for $-1/4 \leq Y \leq +1/4$ (21)

Where X_{fd} , ($= x_{fd}/D_h$), is the non-dimensional distance for the flow to become fully developed, or entry length.

In Eq.(21), the condition on temperature gradient $\dfrac{\partial \theta}{\partial X}$ for $X > X_{fd}$ follows from the fully developed condition for temperature, $\dfrac{\partial \theta_b}{\partial X} = 0$ where θ_b is the non-dimensional temperature based on the mixed mean temperature T_b of the fluid. θ_b is defined by,

$\theta_b = \dfrac{T - T_{ref}}{T_b - T_{ref}}$ (22)

Where T_b, the mixed mean temperature is given by,

$T_b = \dfrac{\int_{-L/2}^{L/2} \rho C_p u T dy}{\int_{-L/2}^{L/2} \rho C_p u dy}$ (23)

Introducing θ^*, the non-dimensional bulk mean temperature, defined by,

$\theta^* = \dfrac{T_b - T_{ref}}{\Delta T}$ (24)

The fully developed condition $\frac{\partial \theta_b}{\partial X} = 0$ on non-dimensional temperature field leads to,

$$\frac{\partial \theta}{\partial X} = \frac{\theta}{\theta^*} \frac{\partial \theta^*}{\partial X} \tag{25}$$

Governing equations given by Eqs. (11), (12), (13) and (14) along with the boundary conditions given by Eq. (21), take specific form depending on the assumptions made and approximations invoked. In what follows work done so far employing the governing equations with specific simplifications is described.

In Eqs. (12), (13) and (14) the non-dimensional parameters, Gr, the Grashof number, Re, the Reynolds number, Ec, the Eckert number, Pe, the Peclet number are defined by,

$$Gr = \frac{g \beta \Delta T D_h^3}{\nu^2} \tag{26}$$

$$Re = \frac{U_{ref} D_h}{\nu} \tag{27}$$

$$Ec = \frac{U_{ref}^2}{C_p \Delta T} \tag{28}$$

$$Pe = \frac{U_{ref} D_h}{\alpha} \tag{29}$$

Also, when both the flow and temperature fields are fully developed and $u \frac{\partial T}{\partial x}$ term is neglected, Br, the Brinkman (see, § 8 (a)) number appears, which is defined by,

$$Br = \frac{\mu U_{ref}^2}{k \Delta T} = \frac{Ec.Pe}{Re} = Ec.Pr \tag{30}$$

3.3. Numerical Scheme (Successive Accelerated Replacement Scheme - SAR)

The basic philosophy of the Successive Accelerated Replacement (SAR) scheme as described in [42-45] is to guess an initial profile for each variable such that the boundary conditions are satisfied. Let the partial differential equation governing a variable, $\phi(X, Y)$, expressed in finite difference form be given by $\bar{\phi}_{M,N} = 0$ where M and N represent the nodal points when the non-dimensional height and length of the channel are divided in to a finite number of intervals MD, ND respectively. The guessed profile for the variable ϕ at any mesh point in general will not satisfy the equation. Let the error in the equation at (M,N) and kth iteration be $\bar{\phi}^{k}_{M,N}$

The (k+1)th approximation to the variable ϕ is obtained from,

$$\phi^{k+1}_{M,N} = \phi^{k}_{M,N} - \omega \frac{\bar{\phi}^{k}_{M,N}}{\partial \bar{\phi}^{k}_{M,N} / \partial \phi_{M,N}} \tag{31}$$

Where ω is an acceleration factor which varies between $0 < \omega < 2$. $\omega < 1$ represents under-relaxation and $\omega > 1$ represents over relaxation.

The procedure of correcting the variable ϕ at each mesh point in the entire region of interest is repeated until a set convergence criterion is satisfied. For example, the change in the variable at any mesh point between kth and (k+1)th approximation satisfies,

$$\left| 1 - \frac{\phi^{k}_{M,N}}{\phi^{k+1}_{M,N}} \right| < \varepsilon \tag{32}$$

Where ε is a prescribed small positive number.

To correct the guessed profile, each dependent variable has to be associated with one equation. It is natural to associate the variable with the equation, which contains the highest order derivative in that variable. For example, conservation of energy equation will be associated for correcting the temperature profile. The feature of using the corrected value of the variable immediately upon becoming available is inherent in this method.

Chapter 4

4.1 Laminar Forced Convection in a Channel, Thermally Developing Field.

Simplified Equations (Velocity Field Fully Developed and Boundary Layer Approximation in Entry Region)

The governing equations, when the flow is assumed to be hydrodynamically developed and thermally developing, in a parallel plate vertical channel with constant wall temperatures, neglecting axial conduction and buoyancy forces are obtained by setting $\varphi = 0$ and $v = 0$, $\frac{\partial u}{\partial x} = 0$ in the governing equations given by Eq.(1) to Eq.(4). They are,

x-Momentum Equation

$$-\frac{1}{\rho_{ref}}\frac{dp_d}{dx} + \nu\frac{d^2u}{dy^2} = 0 \tag{33}$$

Conservation of Energy

$$u\frac{\partial T}{\partial x} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{34}$$

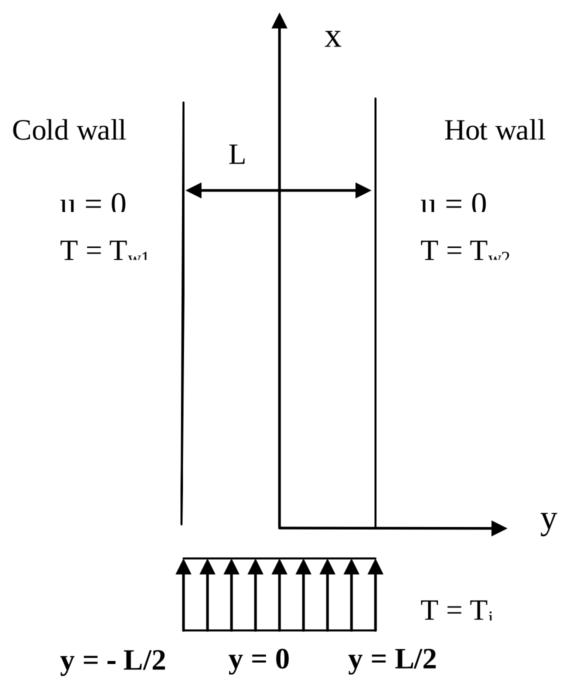


Figure 2. Physical model and Coordinate system

Where $p_d = p + \rho_{ref}\, g\, x$, is the difference between the pressure and the hydrostatic pressure.

Boundary Conditions

$u = 0$ at $y = \pm L/2$ for all $x > 0$

$T = T_i$ at $x = 0$ for $-1/4 < Y < 1/4$

$T = T_{w1}$ at $y = - L/2$ for all $x > 0$

$T = T_{w2}$ at $y = + L/2$ for all $x > 0$ (35)

Non-Dimensional Equations

The following dimensionless variables have been introduced to render the governing equations non-dimensional.

$$U = \frac{u}{U_{ref}},\ V = \frac{v}{U_{ref}},\ X = \frac{x}{D_h},\ Y = \frac{y}{D_h},\ \theta = \frac{T - \bar{T}_{ref}}{T_i - \bar{T}_{ref}} \tag{36}$$

U_{avg} is average velocity $= U_{ref} = -(dp_d/dx)D_h^2/(48\mu)$.

T_{w1}, T_{w2} are left and right wall temperatures.

T_i is the inlet temperature.

$T_{w1} = K_1\, T_i$

$T_{w2} = K_2\, T_i$

The governing equations given by Eqs. (53) and (54) non-dimensionalised using the non-dimensional variables defined by Eq. (56) take the following form.

$$\frac{d^2U}{dY^2} + 48 = 0 \tag{37}$$

$$Pe\left(U\frac{\partial\theta}{\partial X}\right) = \frac{\partial^2\theta}{\partial Y^2} \tag{38}$$

or

$$\left(U\frac{\partial\theta}{\partial X^*}\right) = \frac{\partial^2\theta}{\partial Y^2} \tag{39}$$

Where $X^* = X / Pe$

The boundary conditions given in Eq. (35) becomes,

$\theta = 1$ at $X = 0$ for $-1/4 < Y < 1/4$

$$U = 0,\ \theta = \frac{K_1 - K_2}{2 - K_1 - K_2} \text{ at } Y = -1/4 \text{ for all } X$$

$$U = 0,\ \theta = \frac{K_2 - K_1}{2 - K_1 - K_2} \text{ at } Y = 1/4 \text{ for all } X \tag{40}$$

It may be noted that $K_1 = K_2$ ($\neq 1$) represents symmetric heating, i.e., both the walls are at the same temperature but different from T_i and the boundary conditions, θ given by Eq. (40) become independent of K_1 and K_2.

Nusselt Number

The defining equation for calculating the heat transfer coefficient, say at the left wall is given by,

$$-k \left.\frac{\partial T}{\partial y}\right|_{y=-L/2} = h_1 (T_{w1} - T_b) \tag{41}$$

Local Nusselt number values based on D_h, are expressed in terms of non-dimensional temperature as,

$$Nu_{b-} = \frac{1}{\theta^* - \frac{K_1 - K_2}{2 - K_1 - K_2}} \left.\frac{\partial \theta}{\partial Y}\right|_{Y=-1/4} \tag{42}$$

$$Nu_{b+} = -\frac{1}{\theta^* - \frac{K_2 - K_1}{2 - K_1 - K_2}} \left.\frac{\partial \theta}{\partial Y}\right|_{Y=1/4} \tag{43}$$

Also when $K_1 = K_2$ ($\neq 1$), i.e., symmetric heating, $Nu_{b-} = Nu_{b+} = \frac{1}{\theta^*} \left.\frac{\partial \theta}{\partial Y}\right|_{Y=\pm 1/4}$ when $K_1 = K_2$ since $Nu_{b-} = Nu_{b+}$ shall be referred to as Nu_b.

4.2 Results and Discussion

Variation of Nusselt number Nu_b with X^* is shown in Figure 3. Nusselt number gradually decreases along the flow direction and reaches to a minimum and remains constant in the fully developed region. High Nusselt numbers in the entry region are due to high temperature gradients prevailing over there.

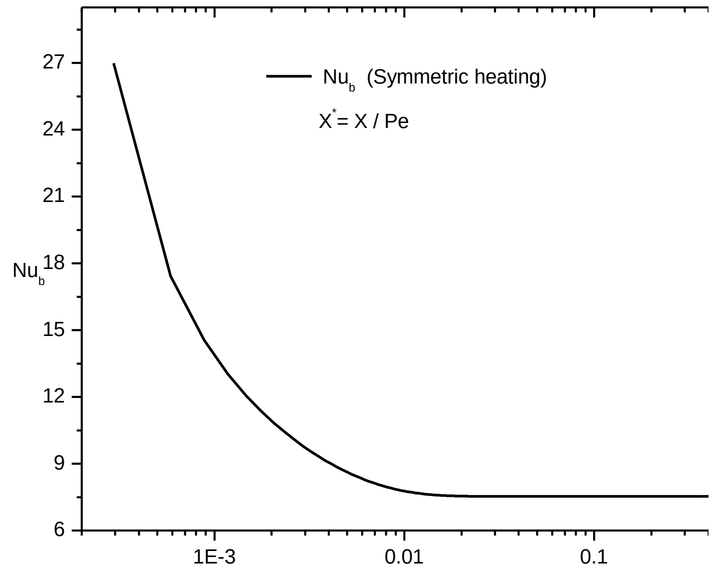


Figure 3. Nu_b at different positions of X^* for the case of symmetric heating

Average Nusselt number $\bar{N}u_{bx}$ up to a certain value of X^* for the case of symmetric heating is shown in Figure 3. $\bar{N}u_{bx}$ gradually decreases in the flow direction. The fully developed Nusselt number Nu_b obtained is 7.53 at $X^* = 0.04$ and up to this value of X^* the average Nusselt number obtained is 8.21.

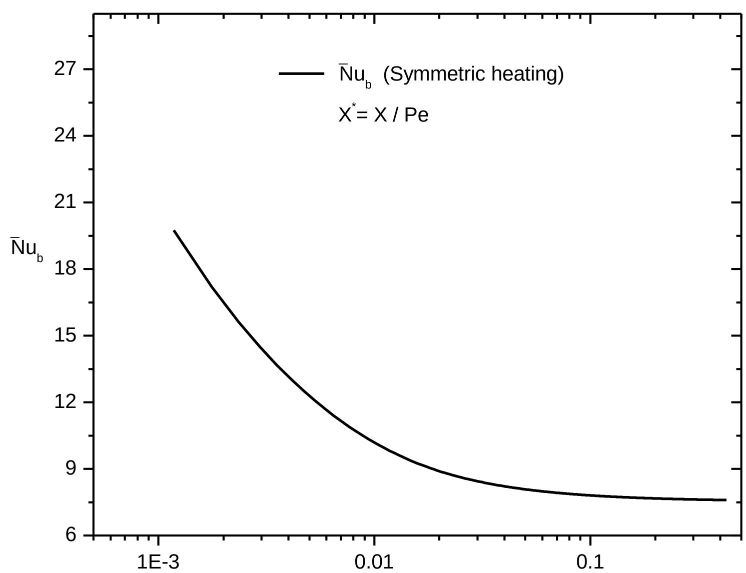


Figure 4(a). $\bar{N}u_b$ up to a certain value of X^* for the case of symmetric heating

Variation of average Nusselt number up to a certain value of X^* for different asymmetries is shown in Figure 4(a), (b) and in (c). From Figure 4(a) it is clear that for small asymmetry ($K_1 = 3$, $K_2 = 2.9$), the average Nusselt number is same for both symmetric and asymmetric cases up to a distance where bulk mean temperature equals to lower wall temperature. However, when the asymmetry is more, the average Nusselt number for the symmetric heating with that of asymmetric heating cases of $K_1 = 3$, $K_2 = 2$ and $K_1 = 3$, $K_2 = 1.5$, is different and this difference increases with asymmetry and also the average Nusselt number at the lower temperature wall $\bar{N}u_{b2}$ gradually diverges with increase in asymmetry in the flow direction. But the average of $\bar{N}u_{b1}$ and $\bar{N}u_{b2}$ is equal to $\bar{N}u_{b}$ of symmetric heating up to where bulk mean temperature becoming equal to lower wall temperature. Moreover, the bulk mean

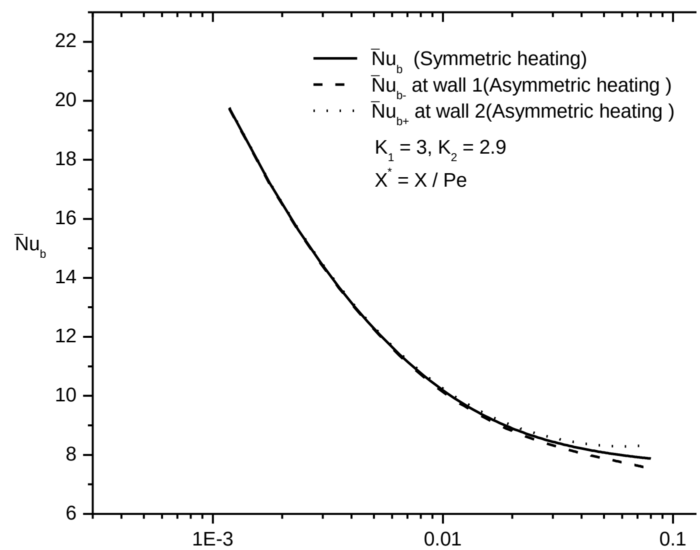


Fig 4 (b)

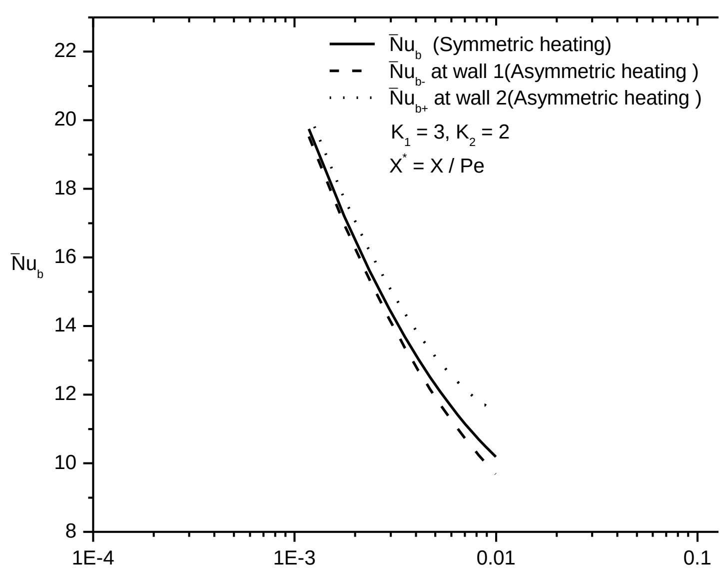


Fig 4 (c)

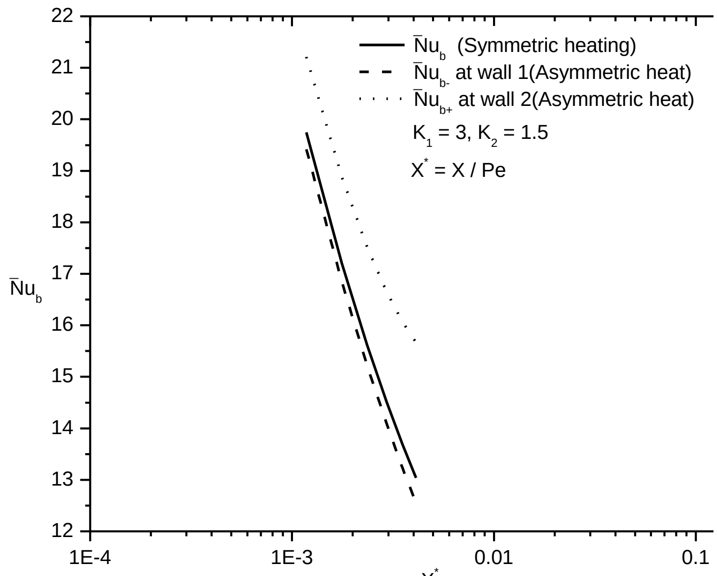
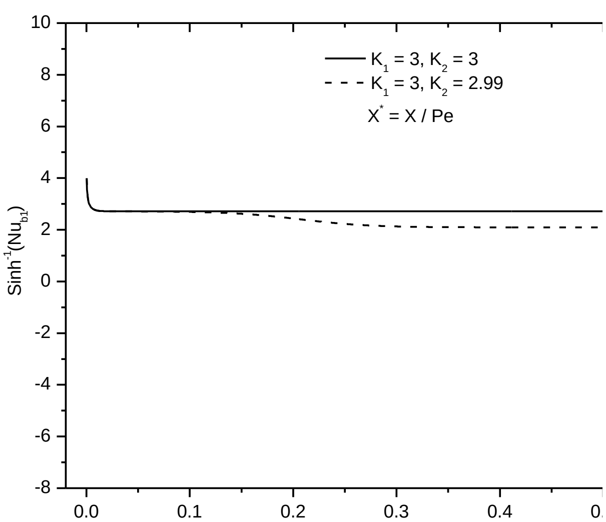


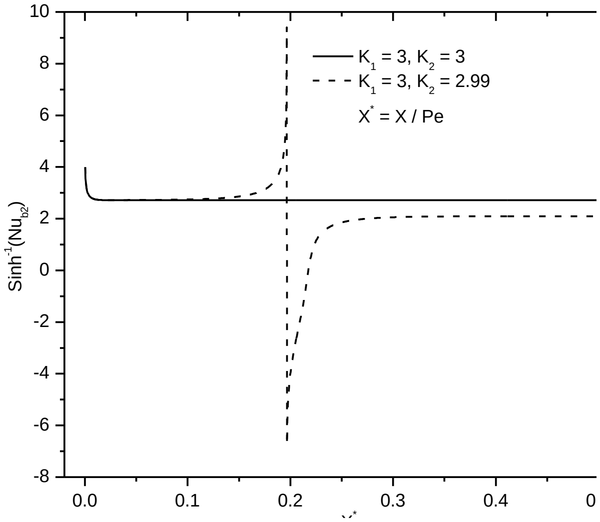
Fig 5

$\bar{N}u_b$ (symmetric heating, i.e. $K_1 = K_2 = 3$), $\bar{N}u_{b1}$ and $\bar{N}u_{b2}$ (asymmetric heating) up to a certain value of X^* for different asymmetries: (a) $K_1 = 3$, $K_2 = 2.9$, (b) $K_1 = 3$, $K_2 = 2$, (c) $K_1 = 3$, $K_2 = 1.5$

temperature becoming equal to lower wall temperature will shifts towards the inlet with increase in asymmetry. Considering local Nussel number, actually the difference in $Nu_{b\ fd}$ (Fully developed Nusselt number) for $T_{w1} = T_{w2}$ and $T_{w1} \approx T_{w2}$ is due to comparing Nu_b at $X^* = X_f^*$ for $T_{w1} = T_{w2}$ with Nu_b at $X^* = X_{fa}^*$, where X_{fa}^* is the fully developed length for asymmetric heating and $X_{fa}^* > X_f^*$. For example, as shown in Figure (b), X_f^* for symmetric heating ($K_1 = 3$, $K_2 = 3$) is 0.04 whereas the fully developed length for asymmetric heating ($K_1 = 3$, $K_2 = 2.99$), i.e. X_{fa}^* is 0.5097 which is much higher than X_f^*.



(a)



(b)

Figure 6:. $\mathrm{Sinh}^{-1}(Nu_b)$ at different positions of X^* for different asymmetries: (a) higher temperature wall (b) lower temperature wall

Chapter 5

5.1 Laminar Mixed Convection in a Channel, Thermally Developing Field.

The governing equations, when the flow is assumed to be hydrodynamically developed and thermally developing, in a parallel plate vertical channel with constant wall temperatures, neglecting axial conduction are obtained by setting $\varphi = 0$ and $v = 0$, $\frac{\partial u}{\partial x} = 0$ in the governing equations given by Eq.(1) to Eq.(4). The non-dimensionalised governing equations obtained by using non-dimensional variables given by Eq. (10) take the following form

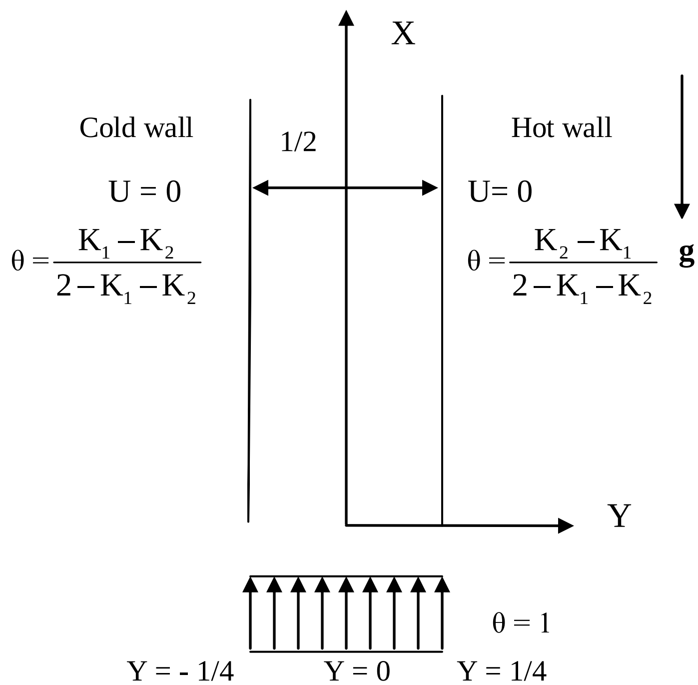


Figure 7. Physical model and Coordinate system

X-Momentum Equation

$$\frac{d^2U}{dY^2} + 48 + \frac{Gr}{Re}\theta = 0 \tag{44}$$

Conservation of Energy

$$Pe\left(U\frac{\partial\theta}{\partial X}\right) = \frac{\partial^2\theta}{\partial Y^2} + Br\left(\frac{dU}{dY}\right)^2 \tag{45}$$

or

$$\left(U\frac{\partial\theta}{\partial X^*}\right)= \frac{\partial^2\theta}{\partial Y^2}+Br\left(\frac{dU}{dY}\right)^2 \tag{46}$$

Where $X^* = X / Pe$

The boundary conditions are,

$\theta = 1$ at $X = 0$ for $-1/4 < Y < 1/4$

$U = 0,\ \theta = \dfrac{K_1 - K_2}{2 - K_1 - K_2}$ at $Y= -1/4$ for all X

$$U = 0,\ \theta = \frac{K_2 - K_1}{2 - K_1 - K_2} \text{ at } Y= 1/4 \text{ for all X} \tag{47}$$

Nusselt Number

The defining equation for calculating the heat transfer coefficient, say at the left wall is given by,

$$-k\left.\frac{\partial T}{\partial y}\right|_{y=-L/2} = h_1(T_{w1} - T_b) \tag{48}$$

Local Nusselt number values based on D_h, are expressed in terms of non-dimensional temperature as,

$$Nu_{b-} = \frac{1}{\theta^* - \dfrac{K_1 - K_2}{2 - K_1 - K_2}}\left.\frac{\partial\theta}{\partial Y}\right|_{Y=-1/4} \tag{49}$$

$$Nu_{b+} = -\frac{1}{\theta^* - \dfrac{K_2 - K_1}{2 - K_1 - K_2}}\left.\frac{\partial\theta}{\partial Y}\right|_{Y=1/4} \tag{50}$$

5.2 Results and Discussion

Variation of Nu_b (Nusselt number based on bulk mean temperature) with X^* for Br = 0 is shown In fig 8 for different values of Gr/Re in the case of symmetric heating. Nu_b is gradually decreasing along the flow direction and reaches to a minimum in the fully developed region. High Nusselt numbers in the entry region are due to high temperature gradients prevailing over there. Nu_b increases with Gr/Re in the developing region, whereas fully developed Nu_b is independent of Gr/Re. This needs further investigation along with asymmetric heating.

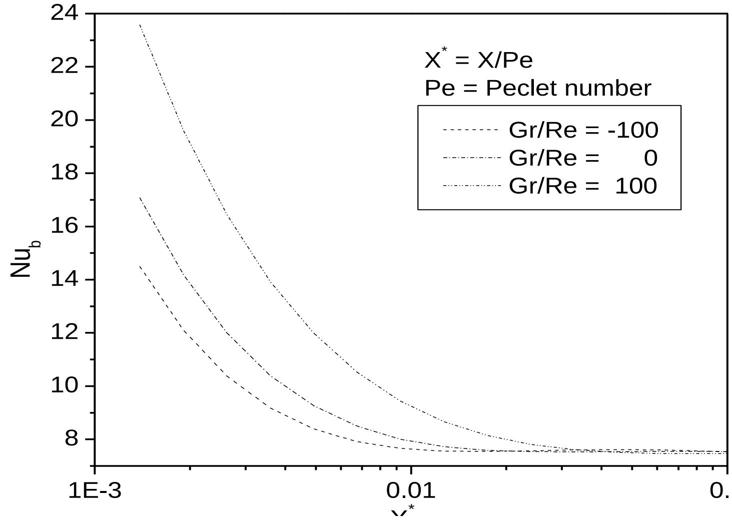


Figure 8 Nu_b at different positions of X* for Br = 0; symmetric heating, for different values of Gr/Re

Chapter 6

6.1 Conclusion

1. Nu_{fd} (fully developed Nusselt number) when $T_{w1} \neq T_{w2}$ is not the same as Nu_{fd} when $T_{w1} = T_{w2}$ even if (T_{w1} - T_{w2}) is small. Since X_{fd}^* ($T_{w1} \neq T_{w2}$) >> X_{fd}^* ($T_{w1} = T_{w2}$)
2. $\overline{Nu}_b$ ($T_{w1} \neq T_{w2}$) $\rightarrow$ $\overline{Nu}_b$ ($T_{w1} = T_{w2}$), if $T_{w1} \rightarrow T_{w2}$ for $X^* << X_{fd}^*$ or for all $X^* < X^*$ for which $T_b < T_{w1}$ where $T_{w1} > T_{w2}$. Thus there is no discontinuity in average Nusselt number $\overline{Nu}_b$ or in heat transfer.

6.2. Further Work

1. Study developing flow and thermal fields by employing full Navier-Stokes equations under asymmetric heating.
2. Include viscous dissipation when the channel is arbitrarily inclined under asymmetric heating.

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