COMPUTER-AIDED ANALYSIS OF BALANCING OF MULTI-CYLINDER INLINE AND V ENGINES

Project report submitted in partial fulfillment

Of requirement for degree of

Bachelor of technology

In

Mechanical engineering

By

RAHUL SHARMA

ROLL NO: 10603043

Department of Mechanical Engineering

2009-2010

COMPUTER-AIDED ANALYSIS OF BALANCING OF MULTI-CYLINDER INLINE AND V ENGINES

Project report submitted in partial fulfillment

Of requirement for degree of

Bachelor of technology

In

Mechanical engineering

By

RAHUL SHARMA

ROLL NO: 10603043

Under the guidance of

Prof. N.KAVI

Department of Mechanical Engineering

2009-2010

National institute of Technology

CERTIFICATE

This is to certify that the project entitled **"COMPUTER-AIDED ANALYSIS OF BALANCING OF MULTI-CYLINDER INLINE AND V ENGINES"** by **Rahul Sharma** has been carried out under my supervision in partial fulfillment of the requirement of the Degree of Bachelor of Technology during session 2009-10 in department of Mechanical engineering, National Institute of Technology, Rourkela and this work has not been submitted elsewhere for a Degree.

Place: Rourkela
Date:

Prof. (Dr.) N. Kavi
Professor
Department of Mechanical Engineering
National Institute of Technology, Rourkela

ACKNOWLEDGEMENT

I wish to express my heartfelt thanks and deep gratitude to **Prof. N. Kavi** for his excellent guidance and whole hearted involvement during my project work. I am also indebted to him for his encouragement, affection and moral support throughout the project. I am also thankful to him for his time he has provided with practical guidance at every step of the project work.

I would also like to sincerely thank **Prof. K.P.Maity** who with their valuable comment and suggestion during the viva voce helped me immensely. I would also like to thank him because they was the one who constantly evaluated, corrected me and had been the guiding light for me.

Lastly, I would like to express my deepest thanks to my friends who were always there to help me whenever I was in any kind of problem during my project work.

Rahul Sharma

CONTENTS

ABSTRACT

The reciprocating engines are widely used as a source of power generation in various mechanical applications ranging from power generation to automobiles. These engines are subjected to noise, vibrations and harshness caused due to unbalanced inertia forces and moments which further cause complications in their operation. To minimise the unbalance, the reciprocating engines are analysed for the unbalanced forces and moments for different configuration of cylinders and firing order of multi-cylinder Inline and V engines. The C++ programs have been developed for this analysis to minimise the time and calculations. The Inline and V engines are compared on the basis of resultant unbalanced forces and moments.

CHAPTER # 1

INTRODUCTION

A reciprocating engine is a heat engine that uses one or more reciprocating pistons to convert pressure into a rotating motion. These are used extensively in motor vehicles, power generators and aircrafts. The engines using more than one cylinder are multi-cylinder engines which are used extensively nowadays. These engines can be inline, v-type or radial.

Inline engines are usually found in four- and six-cylinder configurations, with all cylinders aligned in one row, with no offset. They have been used in automobiles, locomotives and aircraft, although the term in-line has a broader meaning when applied to aircraft engines.An inline engine is considerably easier to build than an otherwise equivalent horizontally opposed or V engine, because both the cylinder bank and crankshaft can be milled from a single metal casting, and it requires fewer cylinder heads and camshafts. In-line engines are also much smaller in overall physical dimensions than designs like the radial, and can be mounted in any direction. Straight configurations are simpler than their V-shaped counterparts. They have a support bearing between each piston as compared to "flat and V" engines which have support bearings between every two pistons.

A V engine is a common configuration for an internal combustion engine. The cylinders and pistons are aligned, in two separate planes or 'banks', so that they appear to be in a **"V"** when viewed along the axis of the crankshaft. The V configuration generally reduces the overall engine length, height and weight compared to an equivalent inline configuration. Various cylinder bank angles of V are used in different engines; depending on the number of cylinders, there may be angles that work better than others for stability. The generally and widely used V angle is 90˚. V engines are generally used in automobiles and aircrafts.

V configurations are well-balanced and smooth, while some are less smoothly running than their equivalent straight counterparts. The V10 and crossplane V8 engine can be balanced with counterweights on the crankshaft. V12, being in effect two straight-6 engines married together, always have even firing and exceptional balance regardless of angle.

CHAPTER # 2

LITERATURE REVIEW

Vigen H. Arakelian and M. R. Smith [1] have worked for the solutions of the problem of the shaking force and shaking moment balancing of planar mechanisms by different methods based on the generation of the movements of counterweights. Some special cases are examined, such as balancing methods based on the copying properties of pantograph systems that carry the counterweights.

Esat and H. Bahai [2] have worked on complete force balancing of planar linkage using the criterion of Tepper and Lowen, then it can be fully force and moment balanced using geared counter-inertias. Tepper and Lowen have shown that complete force balancing of planar linkage is possible using simple counterweights provided that from every point on the linkage there exists a contour to the ground by way of revolute joints only.

W Harry Close, Wieslaw Szydlowski and Christopher Downton [3] have worked on balance of all forces and moments created by pistons, connecting rods and crankshaft of the Collins family of 12, 8, 6 and 4 cylinder engines. With the Scotch Yoke mechanism there are no secondary forces, or higher order forces, and thus the counter-balancing needed can be determined exactly.

H. D. Desai [4] has worked on computer aided kinematic and dynamic analysis of a horizontal slider crank mechanism used for single-cylinder four stroke internal combustion engine. This investigation furnishes the complete kinematic history of the driven links and the bearing loads for the complete working cycle of the engine mechanism.

V. Arakelian and M. Dahan [5] have worked on the solution of the shaking force and shaking moment balancing of planar and spatial linkages. The conditions for balancing are formulated by the minimization of the root-mean-square value of the shaking moment. There are two cases considered: mechanism with the input link by constant angular velocity and mechanism with the input link by variable angular velocity.

J. Singh and B. Singh [6] have worked on the design of crankshaft for complete balancing of primary unbalanced force in reciprocating engine.

CHAPTER # 3

BALANCING

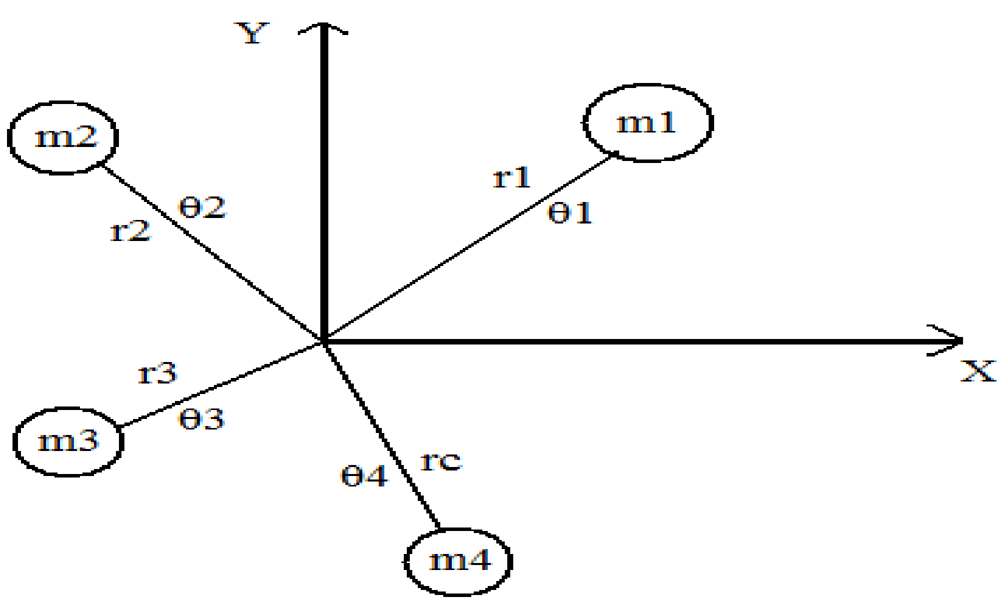
An unbalance of forces is produced in rotary or reciprocatory machinery due to the inertia forces associated with the moving masses. Balancing is the process of designing or modifying machinery so that the unbalance is reduced to an acceptable level and if possible is eliminated completely. The unbalance forces exerted on frame by the moving machine members are time varying, impart vibratory motion to frame and produce noise. Balancing of bodies is necessary to avoid noise, vibrations and harshness which could cause catastrophic failure of machinery.

The most common approach to balancing is by redistributing the masses which may be accomplished by addition or removal of masses from various machine members. There are two basic types of unbalance, rotating unbalance and reciprocating unbalance, which may occur separately or in combination.

BALANCING OF ROTATING MASSES

1. STATIC BALANCING

A system of rotating masses is said to be in static balance if the combined mass centre of the system lies on the axis of rotation.



The above figure shows a rigid rotor rotating with a constant angular velocity ω rad/s with four masses in same transverse plane but at different angular and radial positions.

For static balance,

$$\sum mr\cos\theta = m_1r_1\cos\theta_1 + m_2r_2\cos\theta_2 + m_3r_3\cos\theta_3 + m_4r_4\cos\theta_4 = 0$$

and $$\sum mr\sin\theta = m_1r_1\sin\theta_1 + m_2r_2\sin\theta_2 + m_3r_3\sin\theta_3 + m_4r_4\sin\theta_4 = 0$$

DYNAMIC BALANCING

When several masses rotate in different planes, the centrifugal forces, in addition to being out of balance , also forms couple .A system of rotating masses is in dynamic balance when there doesn't exist any resultant centrifugal force as well as resultant couple.

BALANCING OF RECIPROCATING MASSES

Acceleration of the reciprocating mass of a slider-crank mechanism is given by

Therefore, the forces required to accelerate mass m is,

$$F= m\, r\omega^2 [\cos\theta + (\cos 2\theta)/n]$$

$$= m\, r\omega^2\cos\theta + m\, r\omega^2 (\cos 2\theta)/n$$

Primary force   secondary force

Maximum value of primary force = $mr\omega^2$

Maximum value of secondary force = $mr\omega^2 /n$

; n=L/R is much greater than unity; the secondary force is small compared with primary force and can be safely neglected for slow speed engines.

Consider a horizontal single-cylinder engine as shown in the figure drawn below. For approximately analyzing the unbalanced forces and moments, the connecting rod is replaced by two masses m_1 and m_2 at the ends of a massless rod. Thus, m_1 is an entirely reciprocating mass as it moves with the piston and m_2 is subject to pure rotation with the crank.

m_{rec} = Total mass (including m_1) of reciprocating parts

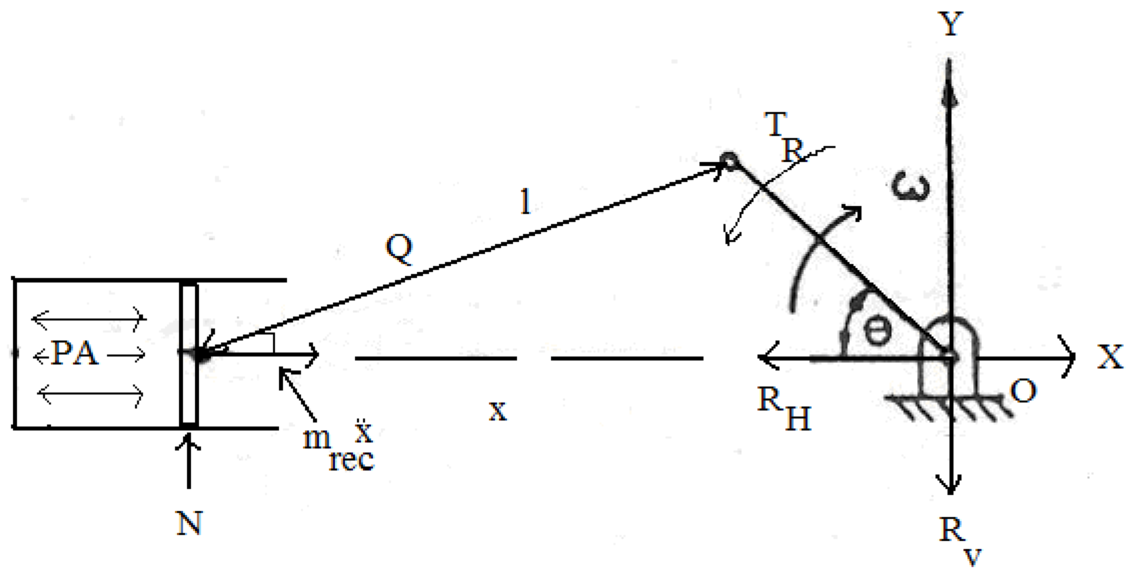
Q = thrust in connecting rod along the direction of connecting rod

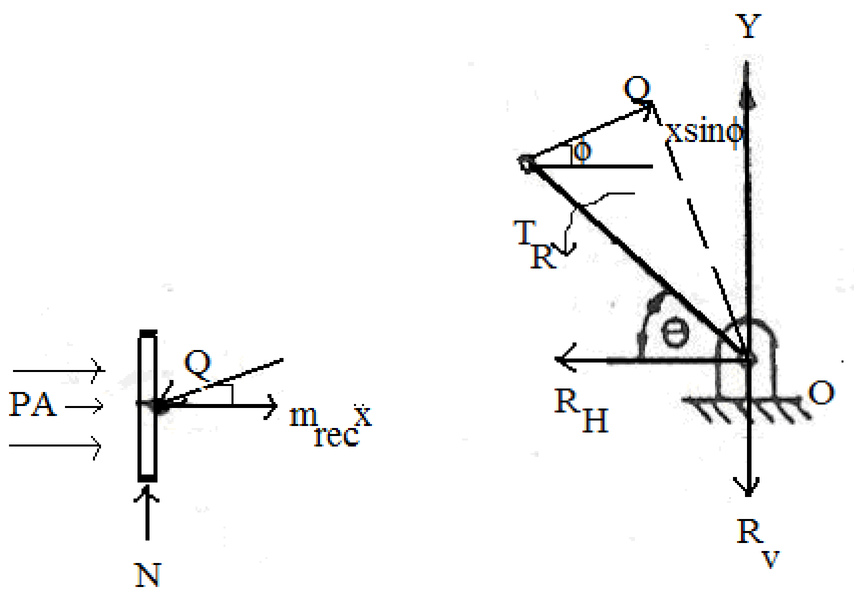
PA = gas force

T_R = load torque on the crank (supplied by surrounding structure and in turn is subjected to reaction)

N = normal thrust between the cylinder and the piston

R_H , R_V = horizontal and vertical reactions at crankshaft bearings





Now by force balance,

$Q\sin\phi = N$

$Q\cos\phi = PA + m_{rec}\,x''$

Taking moments about O,

$Q \times \sin\phi = T_R$

Balancing the forces on crank,

$R_H = Q\cos\phi$

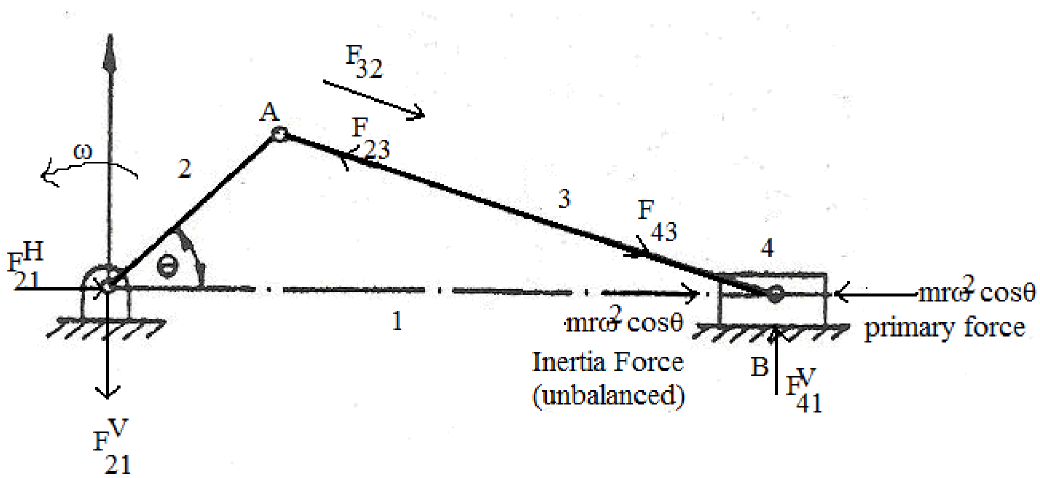
$R_V = Q\sin\phi$

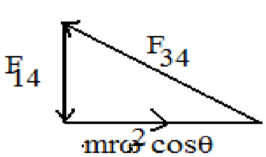
Also, $F_x = PA - R_H = m_{rec}\,x''$

And $F_y = R_v - N = 0$

Therefore, from this approximate analysis and assuming a balanced crank, we conclude that the only force transmitted to the foundation is in the line of stroke. The friction forces being internal always cancel each other.

Therefore, $F_x = m_{rec}\,\omega^2 r\,[\cos\theta + (\cos 2\theta)/n]$





The vertical forces F_{21}^{V} and F_{41}^{V} balance each other but form an unbalanced shaking couple.

PRIMARY AND SECONDARY BALANCE

Primary balance is the balance attained by compensating for the eccentricities of the rotating masses, including the connecting rods. Primary balance is controlled by adding or removing mass to or from the crankshaft, at each end, at the required radius and angle, which varies both due to design and manufacturing tolerances. Theoretically any conventional engine can be balanced perfectly for primary balance.

Secondary balance is attained by compensating partially or fully for:

- Kinetic energy of the pistons.
- Non-sinusoidal motion of the pistons.
- Motion of the connecting rods.
- Sideways motion of balance shaft weights.

The second of these is the main consideration for secondary balance. There are two main control mechanisms for secondary balance—matching the phasing of pistons along the crank, so that their second order contributions cancel, and the use of balance shafts which run at twice engine speed, and so can provide a balancing force.

No widely used engine configuration is perfectly balanced for secondary excitation. However by adopting particular definitions for secondary balance, particular configurations can be correctly claimed to be reasonably balanced in these restricted senses. In particular, the straight six, the flat six, and the V12 configurations offer exceptional inherent mechanical balance.

CHAPTER # 4

THEORETICAL ANALYSIS OF BALANCING OF MULTI-CYLINDER INLINE ENGINES

Multi-cylinder engines are used in applications where high power is required and it gives an advantage of better balance of forces and moments. It also provides better and even distribution of torque to crankshaft and facilitates the smooth running of engine.

The general configurations used are 2-cylinder, 3-cylinder, 4-cylinder, 5-cylinder 6-cylinder and 8-cylinder inline engines. The 6-cylinder and above configuration engines are completely balanced for forces and couples. The forces and couples for different cylinder numbers and firing order are analyzed below theoretically.

FIRING ORDER

In multi-cylinder engines, the crank arrangements are such that there is a smooth distribution of torque in the engine cycle as well as balance of inertia forces of the reciprocating masses. For example, in the 4-cylinder engine, a power stroke begins every 180˚ of crank angle in the following order of cylinder numbers: 1-3-4-2. This order is called firing order.

ENGINE WITH N CYLINDERS INLINE

Angular Positions of cranks on the crankshaft

$\phi_1 = 0°$

$\phi_2 = \phi_2$

$\phi_3 = \phi_3$

: :

$\phi_{n-1} = \phi_{n-1}$

$\phi_n = \phi_n$

SHAKING FORCES

$\sum (\cos\phi) = (\cos\phi_1) + \cos\phi_2 + \cos\phi_3 + \dots\dots + \cos\phi_{n-1} + \cos\phi_n$

$\sum \sin\phi = \sin\phi_1 + \sin\phi_2 + \sin\phi_3 + \dots\dots + \sin\phi_{n-1} + \sin\phi_n$

$\sum \cos2\phi = \cos2\phi_1 + \cos2\phi_2 + \cos2\phi_3 + \dots\dots \cos2\phi_{n-1} + \cos2\phi_n$

$\sum \sin2\phi = \sin2\phi_1 + \sin2\phi_2 + \sin2\phi_3 + \dots\dots + \sin2\phi_{n-1} + \sin2\phi_n$

Primary force, $F_P = mr\omega^2 [\cos\theta \sum (\cos\phi) - \sin\theta \sum (\sin\phi)]$

And,

Secondary force, $F_S = mr\omega^2/n [\cos2\theta \sum (\cos2\phi) - \sin 2\theta \sum (\sin2\phi)]$

SHAKING COUPLES/MOMENTS

$\sum (a\cos\phi) = 0*(\cos\phi_1) + a*(\cos\phi_2) + 2a*(\cos\phi_3) + \dots\dots + (n-2)a*(\cos\phi_{n-1}) + (n-1)*(\cos\phi_n)$

$\sum (a\sin\phi) = 0*(\sin\phi_1) + a*(\sin\phi_2) + 2a*(\sin\phi_3) + \dots\dots + (n-2)\, a*(\sin\phi_{n-1}) + (n-1)*(\sin\phi_n)$

$\sum (a\cos2\phi) = 0*(\cos2\phi_1) + a*(\cos2\phi_2) + 2a*(\cos2\phi_3) + \dots\dots + (n-2)a*(\cos2\phi_{n-1}) + (n-1)*(\cos2\phi_n)$

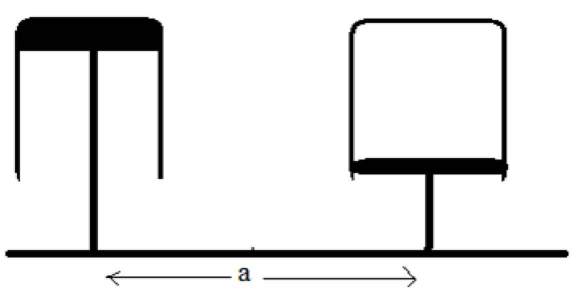
$\sum (a\sin2\phi) = 0*(\sin2\phi_1) + a*(\sin2\phi_2) + 2a*(\sin2\phi_3) + \dots\dots + (n-2)\, a*(\sin2\phi_{n-1}) + (n-1)*(\sin2\phi_n)$

Primary moment, $M_P = mr\omega^2 [\cos\theta \sum (a\cos\phi) - \sin\theta \sum (a\sin\phi)]$

And,

Secondary moment, $M_S = mr\omega^2/n [\cos2\theta \sum (a\cos2\phi) - \sin 2\theta \sum (a\sin2\phi)$

1) <u>2-cylinder engine (cranks at 180°, firing order: 1,2)</u>



$\sum (\cos\phi) = \cos0^\circ + \cos 180^\circ = 0$

$\sum (\sin\phi) = \sin0^\circ + \sin180^\circ = 0$

$\sum(\cos 2\phi) = \cos 0^\circ + \cos 360^\circ = 2$

$\sum(\sin 2\phi) = \sin 0^\circ + \sin 360^\circ = 0$

SHAKING FORCES

$F_P = mr\omega^2 [\cos\theta \sum(\cos\phi) - \sin\theta \sum(\sin\phi)] = 0$

$F_{SL} = mr\omega^2/n [\cos 2\theta \sum(\cos 2\phi) - \sin 2\theta \sum(\sin 2\phi)] = 2mr\omega^2/n (\cos 2\theta)$

$\sum(a\cos\phi) = -a$

$\sum(a\sin\phi) = 0$

$\sum(a\cos 2\phi) = a$

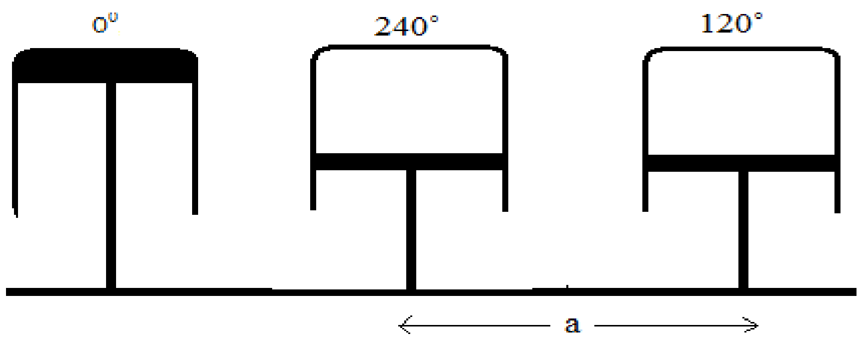
$\sum(a\sin 2\phi) = 0$

SHAKING MOMENTS

$M_P = mr\omega^2 [\cos\theta \sum(a\cos\phi) - \sin\theta \sum(a\sin\phi)] = -mr\omega^2 a\cos\theta$

$M_S = mr\omega^2/n [\cos 2\theta \sum(a\cos 2\phi) - \sin 2\theta \sum(a\sin 2\phi)] = mr\omega^2 a/n (\cos 2\theta)$

2) 3-cylinder engine (cranks at 120°)
Firing order: 1,3,2



$\sum(\cos\phi) = 0$

$\sum(\sin\phi) = 0$

$\sum(\cos 2\phi) = 0$

$\sum(\sin 2\phi) = 0$

SHAKING FORCES

$F_P = 0$

$F_S = 0$

$\sum (a\cos\phi) = -1.5a$

$\sum (a\sin\phi) = 0.866a$

$\sum (a\cos 2\phi) = -1.5a$

$\sum (a\sin 2\phi) = -0.866a$

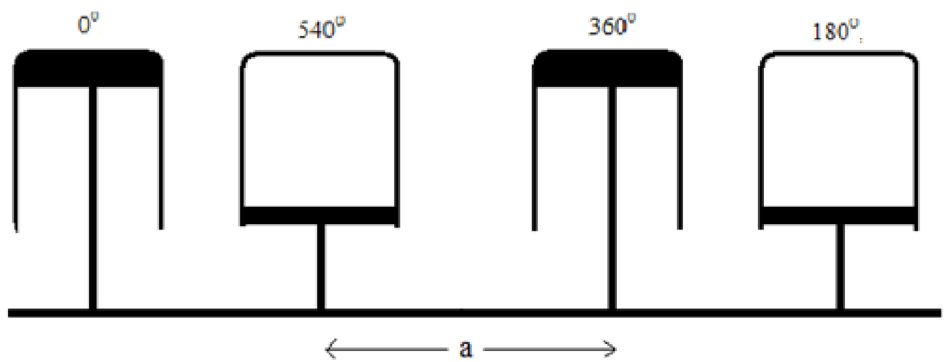
SHAKING MOMENTS

$M_P = -mr\omega^2 a[1.5\cos\theta + 0.866\sin\theta]$

$M_S = mr\omega^2 a/n\ [-1.5\cos 2\theta + 0.866\sin 2\theta]$

3) 4-cylinder engine(cranks at 180°)

a) Firing order: 1432



$\sum (\cos\phi) = 0$

$\sum (\sin\phi) = 0$

$\sum (\cos 2\phi) = 4$

$\sum (\sin 2\phi) = 0$

SHAKING FORCES

$F_P = 0$

$F_S = 4mr\omega^2/n\,[\cos 2\theta]$

$F_{S(max)} = 4mr\omega^2/n$

$\sum (a\cos\phi) = -2a$

$\sum (a\sin\phi) = 0$

$\sum (a\cos 2\phi) = 6a$

$\sum (a\sin 2\phi) = 0$

SHAKING MOMENTS

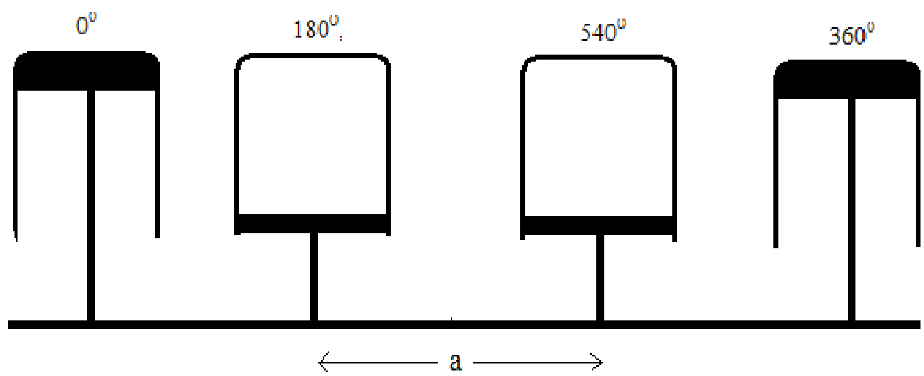
$M_P = -2mr\,\omega^2 a\cos\theta$

$M_{p\,(max)} = 2mr\omega^2\, a$

$M_s = 6mr\omega^2 a/n\,[\cos 2\theta]$

$M_{S\,(max)} = 6mr\omega^2\, a/n$

b) <u>Firing order: 1342</u>



$\sum (\cos\phi) = 0$

$\sum (\sin\phi) = 0$

$\sum (\cos 2\phi) = 4$

$\sum (\sin 2\phi) = 0$

SHAKING FORCES

$F_P = 0$

$F_S = 4mr\omega^2/n\ [\cos 2\theta]$

$F_{S(max)} = 4mr\omega^2/n$

$\sum (a\cos\phi) = 0$

$\sum (a\sin\phi) = 0$

$\sum (a\cos 2\phi) = 6a$

$\sum (a\sin 2\phi) = 0$

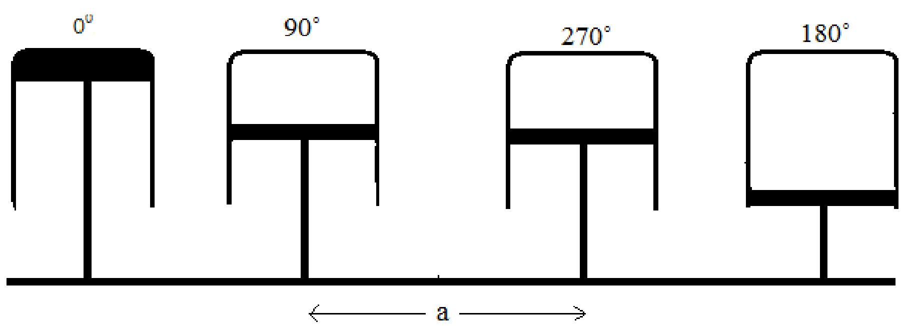
SHAKING MOMENTS

$M_P = 0$

$M_s = 6mr\omega^2 a/n\ [\cos 2\theta]$

$M_{S(max)} = 6mr\omega^2 a/n$

C) Firing Order: 1243 and cranks at 90˚



$\sum (\cos\phi) = 0$

$\sum (\sin\phi) = 0$

$\sum (\cos 2\phi) = 0$

$\sum (\sin 2\phi) = 0$

SHAKING FORCE

$F_P = 0$

$F_S = 0$

$\sum (a\cos\phi) = -3a$

$\sum (a\sin\phi) = -a$

$\sum (a\cos 2\phi) = 0$

$\sum (a\sin 2\phi) = 0$

SHAKING MOMENTS

$M_P = mr\omega^2 a\,[-3\cos\theta_1 + \sin\theta_1]$

$M_s = 0$

The above theoretical analysis can also be done for various engines with increasing number of cylinders like 5-cylinder, 6-cylinder and so on. But calculations will be much more, so we will now go for computer-aided analysis of balancing of multi-cylinder inline engines. A program has been developed in C++ for the required analysis. The program will evaluate all forces and couples acting on the engine for different number of cylinders and different firing orders.

C++ PROGRAM FOR DYNAMIC ANALYSIS OF MULTI-CYLINDER INLINE ENGINES

```
#include<iostream.h>

#include<stdio.h>

#include<conio.h>

#include<math.h>

#define N 6

void main()

{
```

```
int phi [N];
int seq [N];
float m,r,w,n,a,theta;
cout<<"Mention the firing order :\n";
for(int i=0;i<N;i++)
{
cin>>seq[i];
}

cout<<"enter mass :\n";
cin>>m;
cout<<"enter crank radius :\n";
cin>>r;
cout<<"enter angular velocity :\n";
cin>>w;
cout<<"enter n :\n";
cin>>n;
cout<<"enter cylinder pitch :\n";
cin>>a;
cout<<"enter theta :\n";
cin>>theta;
theta = (theta*M_PI)/180;
int delta;
cout<<"enter the crank angle diff. :\n";
cin>>delta;
```

```
phi[seq[0]-1]=0;//initial cylinder angle taken 0
for(i=1;i<N;i++)
{
phi[seq[i]-1]=i*delta;
}
for(i=0;i<N;i++)
{
cout<<"angle for cylinder "<<seq[i]<<" :"<<phi[seq[i]-1]<<"\n";
}
float sigmacos=0,sigmasin=0,sigma2cos=0,sigma2sin=0;
for(i=0;i<N;i++)
{
sigmacos = sigmacos + cos((phi[i]*M_PI)/180);
sigmasin = sigmasin + sin((phi[i]*M_PI)/180);
sigma2cos = sigma2cos + cos((2*phi[i]*M_PI)/180);
sigma2sin = sigma2sin + sin((2*phi[i]*M_PI)/180);
}
float fp=0;
fp = m*r*w*w*((cos(theta)*sigmacos)-(sin(theta)*sigmasin));
cout<<"\nThe primary force  is :"<<fp<<"\n";
float fs = (m*r*w*w*((cos(2*theta)*sigma2cos)-(sin(2*theta)*sigma2sin)))/n;
cout<<"The secondary force is :"<<fp<<"\n\n";
float asigmacos=0,asigmasin=0,asigma2cos=0,asigma2sin=0;
for(i=0;i<N;i++)
{
```

```
asigmacos = asigmacos + a*i*cos((phi[i]*M_PI)/180);

asigmasin = asigmasin + a*i*sin((phi[i]*M_PI)/180);

asigma2cos = asigma2cos + a*i*cos((2*phi[i]*M_PI)/180);

asigma2sin = asigma2sin + a*i*sin((2*phi[i]*M_PI)/180);

}

float mp=0,ms=0;

mp = m*r*w*w*((cos(theta1)*asigmacos)-(sin(theta1)*asigmasin));

ms = (m*r*w*w*((cos(2*theta)*asigma2cos)-(sin(2*theta)*asigma2sin)))/n;

cout<<"The primary moment is :"<<mp<<"\n";

cout<<"The secondary moment is :"<<ms<<"\n\n";

getch();

}
```

CHAPTER # 5

THEORETICAL ANALYSIS OF BALANCING OF MULTI-CYLINDER V-ENGINES

A V engine is a common configuration for an internal combustion engine. The axial planes in which the two sets of pistons reciprocate intersect at the crankshaft axis and form a V of angle β. In automotive installations, V-6 and V-8 engines are common in which β is either 60˚ or 90˚. The 90˚ V-angle is most preferred and generally used. Therefore, we have used $\beta = 90^\circ$ in this analysis. The V configuration generally reduces the overall engine length, height and weight compared to an equivalent inline configuration. The general configurations used are V2, V4, V6, V8, V10 and V12. The forces and couples for different cylinder numbers and firing order are analyzed below theoretically.

V Engine with n cylinders on each bank

Angular Positions of cranks on respective banks

LEFT BANK	RIGHT BANK
$\phi_1 = 0^\circ$	$\phi_7 = 0^\circ$
$\phi_2 = \phi_2$	$\phi_8 = \phi_2$
$\phi_3 = \phi_3$	$\phi_9 = \phi_3$
: :	: :
$\phi_{n-1} = \phi_{n-1}$	$\phi_{2n-1} = \phi_{n-1}$
$\phi_n = \phi_n$	$\phi_{2n} = \phi_n$

SHAKING FORCES

$$\sum (\cos\phi) = (\cos\phi_1) + \cos\phi_2 + \cos\phi_3 + \ldots\ldots + \cos\phi_{n-1} + \cos\phi_n$$

$$\sum \sin\phi = \sin\phi_1 + \sin\phi_2 + \sin\phi_3 + \ldots\ldots + \sin\phi_{n-1} + \sin\phi_n$$

$$\sum \cos2\phi = \cos2\phi_1 + \cos2\phi_2 + \cos2\phi_3 + \ldots\ldots \cos2\phi_{n-1} + \cos2\phi_n$$

$$\sum \sin2\phi = \sin2\phi_1 + \sin2\phi_2 + \sin2\phi_3 + \ldots\ldots + \sin2\phi_{n-1} + \sin2\phi_n$$

Since the two banks are identical, therefore the dynamic conditions of both are same except the crank angles θ_1 and θ_2 which are related to each other aS $\boldsymbol{\theta_2 = \theta_1 - 90°}$, where θ_1 is the angle which 1st crank of left bank makes with the axial plane of left bank and θ_2 is the angle which 1st crank of right bank makes with the axial plane of left bank. Hence we can analyse one bank and result can be similarly obtained for the other.

Primary force for left bank = F_{PL}

Primary force for right bank = F_{PR}

Secondary force for left bank = F_{SL}

Secondary force for right bank = F_{SR}

$F_{PL} = mr\omega^2 [\cos\theta_1 \sum (\cos\phi) - \sin \theta_1 \sum (\sin\phi)]$

$F_{PR} = mr\omega^2 [\cos\theta_2 \sum (\cos\phi) - \sin \theta_2 \sum (\sin\phi)]$

The resultant primary force F_P is given by:

$F_P = [F_{PL}^2 + F_{PR}^2]^{1/2}$

The angle μ which F_P makes with the axial plane of left bank is given by:

$\tan\mu = F_{PR}/ F_{PL}$

Similarly,

$F_{SL} = mr\omega^2/n [\cos2\theta_1 \sum (\cos2\phi) - \sin 2\theta_1 \sum (\sin2\phi)]$

$F_{SR} = mr\omega^2/n [\cos2\theta_2 \sum (\cos2\phi) - \sin 2\theta_2 \sum (\sin2\phi)]$

The resultant secondary force F_s is given by:

$F_S = [F_{SL}^2 + F_{SR}^2]^{1/2}$

The angle λ which F_S makes with the axial plane of left bank is given by:

$\tan \lambda = F_{SR}/ F_{SL}$

SHAKING COUPLES/MOMENTS

$\sum (a\cos\phi) = 0*(\cos\phi_1) + a*(\cos\phi_2) + 2a*(\cos\phi_3) + \ldots\ldots + (n-2)a*(\cos\phi_{n-1}) + (n-1)*(\cos\phi_n)$

$\sum (a\sin\phi) = 0*(\sin\phi_1) + a*(\sin\phi_2) + 2a*(\sin\phi_3) + \ldots\ldots + (n-2) a*(\sin\phi_{n-1}) + (n-1)*(\sin\phi_n)$

$\sum (a\cos2\phi) = 0*(\cos2\phi_1) + a*(\cos2\phi_2) + 2a*(\cos2\phi_3) + \ldots\ldots + (n-2)a*(\cos2\phi_{n-1}) + (n-1)*(\cos2\phi_n)$

$\sum (a\sin2\phi) = 0*(\sin2\phi_1) + a*(\sin2\phi_2) + 2a*(\sin2\phi_3) + \ldots\ldots + (n-2) a*(\sin2\phi_{n-1}) + (n-1)*(\sin2\phi_n)$

Primary moment for left bank = M_{PL}

Primary moment for right bank = M_{PR}

Secondary moment for left bank = M_{SL}

Secondary moment for right bank = M_{SR}

$M_{PL} = mr\omega^2 [\cos\theta_1 \sum (a\cos\phi) - \sin \theta_1 \sum (a\sin\phi)]$

$M_{PR} = mr\omega^2 [\cos\theta_2 \sum (a\cos\phi) - \sin \theta_2 \sum (a\sin\phi)]$

The resultant Primary Moment M_P is given by:

$M_P = [M_{PL}^2 + M_{PR}^2]^{1/2}$

The angle γ which M_P makes with the axial plane of left bank is given by:

$\tan\gamma = M_{PR}/ M_{PL}$

Similarly,

$M_{SL} = mr\omega^2/n [\cos2\theta_1 \sum (a\cos2\phi) - \sin 2\theta_1 \sum (a\sin2\phi)]$

$M_{SR} = mr\omega^2/n [\cos2\theta_2 \sum (a\cos2\phi) - \sin 2\theta_2 \sum (a\sin2\phi)]$

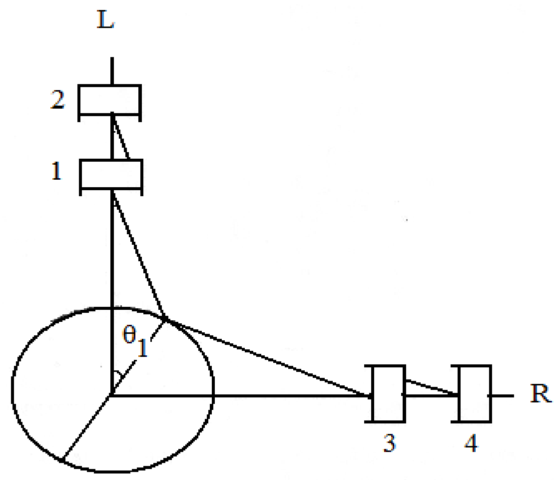
The resultant Secondary Moment M_S is given by:

$M_S = [M_{SL}^2 + M_{SR}^2]^{1/2}$

The angle ηwhich M_S makes with the axial plane of left bank is given by:

$\tan\eta = M_{SR}/ M_{SL}$

V-4 ENGINE ANALYSIS



LEFT BANK

$\phi_1 = 0°$

$\phi_2 = 180°$

RIGHT BANK

$\phi_3 = 0°$

$\phi_4 = 180°$

SHAKING FORCES

$\sum \cos\phi = \cos 0° + \cos 180° = 0$

$\sum \sin\phi = \sin 0° + \sin 180° = 0$

$\sum \cos 2\phi = \cos 0° + \cos 360° = 2$

$\sum \sin 2\phi = \sin 0° + \sin 360° = 0$

Therefore,

$F_{PL} = mr\omega^2 [\cos\theta_1 \sum (\cos\phi) - \sin \theta_1 \sum (\sin\phi)] = 0$

$F_{PR} = mr\omega^2 [\cos\theta_2 \sum (\cos\phi) - \sin \theta_2 \sum (\sin\phi)] = 0$

$F_{SL} = mr\omega^2/n\, [\cos 2\theta_1 \sum (\cos 2\phi) - \sin 2\theta_1 \sum (\sin 2\phi)] = 2mr\omega^2/n\, (\cos 2\theta_1)$

$F_{SR} = mr\omega^2/n\, [\cos 2\theta_2 \sum (\cos 2\phi) - \sin 2\theta_2 \sum (\sin 2\phi)] = 2mr\omega^2/n\, (\cos 2\theta_2)$

$= -2mr\omega^2/n\, (\cos 2\theta_1)$

Resultant secondary force, $F_S = [F_{SL}^2 + F_{SR}^2]^{1/2}$

$= 2\sqrt{2}\ mr\omega^2/n$

$\tan\lambda = F_{SR}/ F_{SL} = \tan 2\theta$

$\Rightarrow \lambda = 2\theta_1$

Hence the resultant secondary force has a constant magnitude but the direction varies twice as that of crank.

SHAKING COUPLES/MOMENTS

$\sum (a\cos\phi) = -a$

$\sum (a\sin\phi) = 0$

$\sum (a\cos 2\phi) = a$

$\sum (a\sin 2\phi) = 0$

$M_{PL} = mr\omega^2 [\cos\theta_1 \sum (a\cos\phi) - \sin \theta_1 \sum (a\sin\phi)] = - mr\omega^2 a\cos \theta_1$

$M_{PR} = mr\omega^2 [\cos\theta_2 \sum (a\cos\phi) - \sin \theta_2 \sum (a\sin\phi)] = -mr\omega^2 a\sin \theta_1$

$M_P = [M_{PL}^2 + M_{PR}^2]^{1/2} = mr\omega^2 a$

$\tan\gamma = M_{PR}/ M_{PL} = \tan\theta_1$

$\Rightarrow \gamma = \theta_1$

Similarly,

$M_{SL} = mr\omega^2/n [\cos2\theta_1 \sum (a\cos2\phi) - \sin 2\theta_1 \sum (a\sin2\phi)] = mr\omega^2 a/n (\cos2\theta_1)$

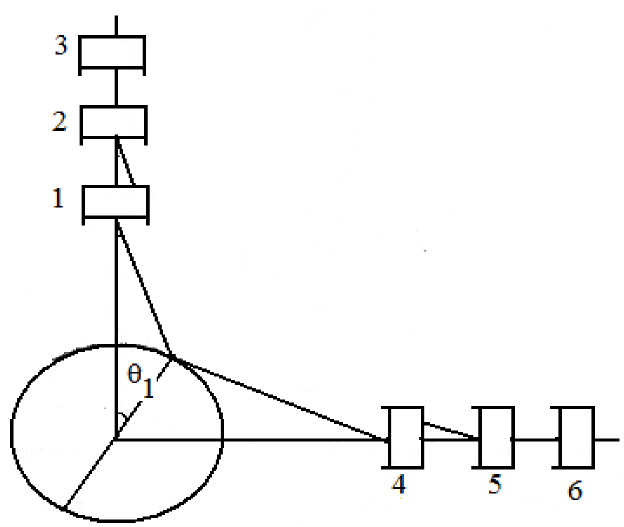
$M_{SR} = mr\omega^2/n [\cos2\theta_2 \sum (a\cos2\phi) - \sin 2\theta_2 \sum (a\sin2\phi)] = mr\omega^2 a/n (\cos2\theta_2) = -mr\omega^2 a/n (\cos2\theta_1)$

$M_S = [M_{SL}^2 + M_{SR}^2]^{1/2} = \sqrt{2}\, mr\omega^2 a/n (\cos2\theta_1)$

$\tan\eta = M_{SR}/ M_{SL} = -1$

$\Rightarrow \eta = 315°$

V-6 ENGINE ANALYSIS



LEFT BANK

$\phi_1 = 0°$

$\phi_2 = 240°$

$\phi_3 = 120°$

RIGHT BANK

$\phi_4 = 0°$

$\phi_5 = 240°$

$\phi_6 = 120°$

SHAKING FORCES

$\sum \cos\phi = \cos0° + \cos240° + \cos120° = 0$

$\sum \sin\phi = \sin0° + \sin240° + \sin120° = 0$

$\sum \cos2\phi = \cos0° + \cos480° + \cos240° = 0$

$\sum \sin2\phi = \sin0° + \sin480° + \sin240° = 0$

Therefore,

$F_{PL} = mr\omega^2 [\cos\theta_1 \sum (\cos\phi) - \sin \theta_1 \sum (\sin\phi)] = 0$

$F_{PR} = mr\omega^2 [\cos\theta_2 \sum (\cos\phi) - \sin \theta_2 \sum (\sin\phi)] = 0$

$F_{SL} = mr\omega^2/n [\cos2\theta_1 \sum (\cos2\phi) - \sin 2\theta_1 \sum (\sin2\phi)] = 0$

$F_{SR} = mr\omega^2/n [\cos2\theta_2 \sum (\cos2\phi) - \sin 2\theta_2 \sum (\sin2\phi)] = 0$

SHAKING COUPLES/MOMENTS

$\sum (a\cos\phi) = -1.5a$

$\sum (a\sin\phi) = 0.866a$

$\sum (a\cos2\phi) = -1.5a$

$\sum (a\sin2\phi) = -0.866a$

$M_{PL} = mr\omega^2 [\cos\theta_1 \sum (a\cos\phi) - \sin \theta_1 \sum (a\sin\phi)] = - mr\omega^2 a [1.5\cos \theta_1 + 0.866\sin \theta_1]$

$M_{PR} = mr\omega^2 [\cos\theta_2 \sum (a\cos\phi) - \sin \theta_2 \sum (a\sin\phi)] = mr\omega^2 a [1.5\sin \theta_1 - 0.866\cos \theta_1]$

$M_P = [M_{PL}^2 + M_{PR}^2]^{1/2}$

$\tan\gamma = M_{PR}/ M_{PL}$

Similarly,

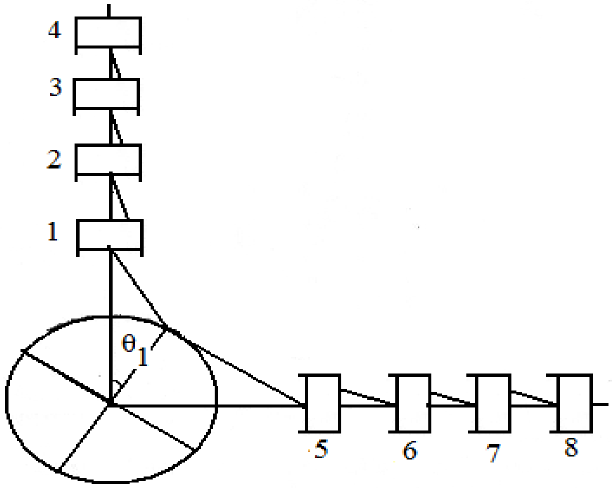
$M_{SL} = mr\omega^2/n [\cos2\theta_1 \sum (a\cos2\phi) - \sin 2\theta_1 \sum (a\sin2\phi)] = mr\omega^2 a/n [-1.5\cos \theta_1 + 0.866\sin \theta_1]$

$M_{SR} = mr\omega^2/n [\cos2\theta_2 \sum (a\cos2\phi) - \sin 2\theta_2 \sum (a\sin2\phi)] = mr\omega^2 a/n [1.5\cos \theta_1 - 0.866\sin \theta_1]$

$M_S = [M_{SL}^2 + M_{SR}^2]^{1/2}$

$\tan\eta = M_{SR}/ M_{SL}$

V-8 ENGINE ANALYSIS (cranks at 90°)



LEFT BANK

$\phi_1 = 0°$

$\phi_2 = 90°$

$\phi_3 = 270°$

$\phi_4 = 180°$

RIGHT BANK

$\phi_5 = 0°$

$\phi_6 = 90°$

$\phi_7 = 270°$

$\phi_8 = 180°$

SHAKING FORCES

$\sum \cos\phi = \cos 0° + \cos 90° + \cos 270° + \cos 180° = 0$

$\sum \sin\phi = \sin 0° + \sin 90° + \sin 270° + \sin 180° = 0$

$\sum \cos 2\phi = \cos 0° + \cos 180° + \cos 540° + \cos 360° = 0$

$\sum \sin 2\phi = \sin 0° + \sin 180° + \sin 540° + \cos 360° = 0$

Therefore,

$F_{PL} = mr\omega^2 [\cos\theta_1 \sum (\cos\phi) - \sin \theta_1 \sum (\sin\phi)] = 0$

$F_{PR} = mr\omega^2 [\cos\theta_2 \sum (\cos\phi) - \sin \theta_2 \sum (\sin\phi)] = 0$

$F_{SL} = mr\omega^2/n [\cos 2\theta_1 \sum (\cos 2\phi) - \sin 2\theta_1 \sum (\sin 2\phi)] = 0$

$F_{SR} = mr\omega^2/n [\cos 2\theta_2 \sum (\cos 2\phi) - \sin 2\theta_2 \sum (\sin 2\phi)] = 0$

SHAKING COUPLES/MOMENTS

$\sum (a\cos\phi) = -3a$

$\sum (a\sin\phi) = -a$

$\sum (a\cos 2\phi) = 0$

$\sum (a\sin 2\phi) = 0$

$M_{PL} = mr\omega^2 [\cos\theta_1 \sum (a\cos\phi) - \sin \theta_1 \sum (a\sin\phi)] = mr\omega^2 a[-3\cos \theta_1 + \sin \theta_1]$

$M_{PR} = mr\omega^2 [\cos\theta_2 \sum (a\cos\phi) - \sin \theta_2 \sum (a\sin\phi)] = mr\omega^2 a [-3\sin \theta_1 - \cos \theta_1]$

$M_P = [M_{PL}^2 + M_{PR}^2]^{1/2}$

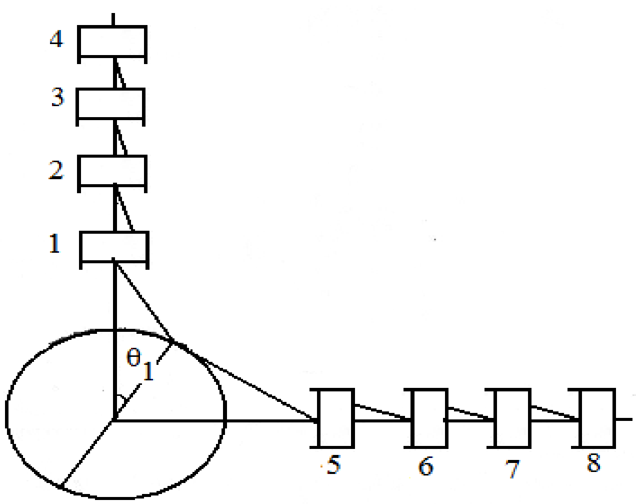
$\tan\gamma = M_{PR}/ M_{PL}$

Similarly,

$M_{SL} = mr\omega^2/n [\cos 2\theta_1 \sum (a\cos 2\phi) - \sin 2\theta_1 \sum (a\sin 2\phi)] = 0$

$M_{SR} = mr\omega^2/n [\cos 2\theta_2 \sum (a\cos 2\phi) - \sin 2\theta_2 \sum (a\sin 2\phi)] = 0$

V-8 ENGINE ANALYSIS (cranks at 180°)



LEFT BANK

$\phi_1 = 0°$

$\phi_2 = 180°$

RIGHT BANK

$\phi_5 = 0°$

$\phi_6 = 180°$

$\phi_3 = 180°$ $\phi_7 = 180°$

$\phi_4 = 0°$ $\phi_8 = 0°$

SHAKING FORCES

$\sum \cos\phi = \cos 0° + \cos 180° + \cos 180° + \cos 0° = 0$

$\sum \sin\phi = \sin 0° + \sin 180° + \sin 180° + \sin 0° = 0$

$\sum \cos 2\phi = \cos 0° + \cos 360° + \cos 360° + \cos 0° = 4$

$\sum \sin 2\phi = \sin 0° + \sin 360° + \sin 360° + \sin 0° = 0$

Therefore,

$F_{PL} = mr\omega^2 [\cos\theta_1 \sum (\cos\phi) - \sin \theta_1 \sum (\sin\phi)] = 0$

$F_{PR} = mr\omega^2 [\cos\theta_2 \sum (\cos\phi) - \sin \theta_2 \sum (\sin\phi)] = 0$

$F_{SL} = mr\omega^2/n\, [\cos 2\theta_1 \sum (\cos 2\phi) - \sin 2\theta_1 \sum (\sin 2\phi)] = 4mr\omega^2/n\, [\cos 2\theta_1]$

$F_{SR} = mr\omega^2/n\, [\cos 2\theta_2 \sum (\cos 2\phi) - \sin 2\theta_2 \sum (\sin 2\phi)] = -4mr\omega^2/n\, [\cos 2\theta_1]$

$F_S = 4\sqrt{2} mr\omega^2/n\, (\cos 2\theta_1)$

SHAKING COUPLES/MOMENTS

$\sum (a\cos\phi) = 0$

$\sum (a\sin\phi) = 0$

$\sum (a\cos 2\phi) = 6a$

$\sum (a\sin 2\phi) = 0$

$M_{PL} = mr\omega^2 [\cos\theta_1 \sum (a\cos\phi) - \sin \theta_1 \sum (a\sin\phi)] = 0$

$M_{PR} = mr\omega^2 [\cos\theta_2 \sum (a\cos\phi) - \sin \theta_2 \sum (a\sin\phi)] = 0$

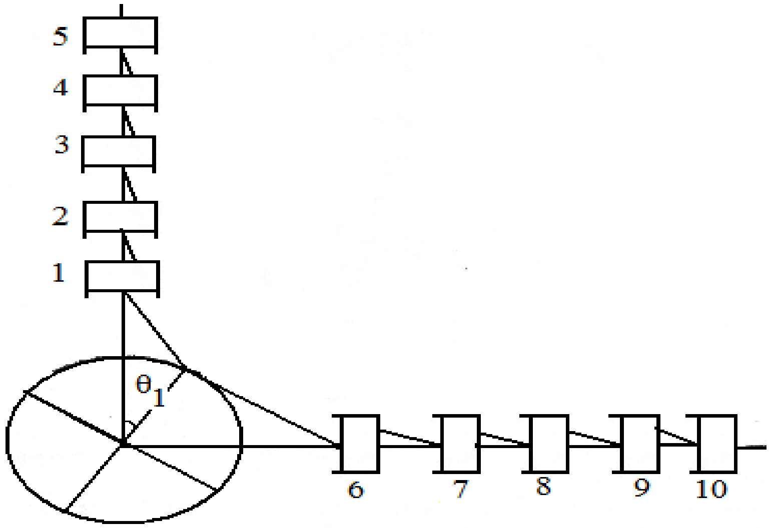
And,

$M_{SL} = mr\omega^2/n\, [\cos 2\theta_1 \sum (a\cos 2\phi) - \sin 2\theta_1 \sum (a\sin 2\phi)] = 6mr\omega^2 a/n\, (\cos 2\theta_1)$

$M_{SR} = mr\omega^2/n\, [\cos 2\theta_2 \sum (a\cos 2\phi) - \sin 2\theta_2 \sum (a\sin 2\phi)] = -\, 6mr\omega^2 a/n\, (\cos 2\theta_1)$

$M_S = 6\sqrt{2} mr\omega^2 a/n\, (\cos 2\theta_1)$

V-10 ENGINE ANALYSIS



LEFT BANK

$\phi_1 = 0°$

$\phi_2 = 216°$

$\phi_3 = 144°$

$\phi_4 = 72°$

$\phi_5 = 288°$

RIGHT BANK

$\phi_6 = 0°$

$\phi_7 = 216°$

$\phi_8 = 144°$

$\phi_9 = 72°$

$\phi_{10} = 288°$

SHAKING FORCES

$\sum \cos\phi = \cos0° + \cos216° + \cos144° + \cos72° + \cos288° = 0$

$\sum \sin\phi = \sin0° + \sin216° + \sin144° + \sin72° + \sin288° = 0$

$\sum \cos2\phi = \cos0° + \cos432° + \cos288° + \cos144° + \cos576° = 0$

$\sum \sin2\phi = \sin0° + \sin432° + \sin288° + \sin144° + \sin576° = 0$

Therefore,

$F_{PL} = mr\omega^2 [\cos\theta_1 \sum (\cos\phi) - \sin \theta_1 \sum (\sin\phi)] = 0$

$F_{PR} = mr\omega^2 [\cos\theta_2 \sum (\cos\phi) - \sin \theta_2 \sum (\sin\phi)] = 0$

$F_{SL} = mr\omega^2/n [\cos2\theta_1 \sum (\cos2\phi) - \sin 2\theta_1 \sum (\sin2\phi)] = 0$

$F_{SR} = mr\omega^2/n [\cos2\theta_2 \sum (\cos2\phi) - \sin 2\theta_2 \sum (\sin2\phi)] = 0$

SHAKING COUPLES/MOMENTS

$\sum (a\cos\phi) = -0.264a$

$\sum (a\sin\phi) = -0.3633a$

$\sum (a\cos 2\phi) = -4.736a$

$\sum (a\sin 2\phi) = -1.5389a$

$M_{PL} = mr\omega^2 [\cos\theta_1 \sum (a\cos\phi) - \sin \theta_1 \sum (a\sin\phi)] = mr\omega^2 a [-0.264\cos \theta_1 + 0.3633\sin \theta_1]$

$M_{PR} = mr\omega^2 [\cos\theta_2 \sum (a\cos\phi) - \sin \theta_2 \sum (a\sin\phi)] = mr\omega^2 a [-0.264\sin \theta_1 – 0.3633\cos \theta_1]$

$M_P = [M_{PL}^2 + M_{PR}^2]^{1/2}$

$\tan\gamma = M_{PR}/ M_{PL}$

Similarly,

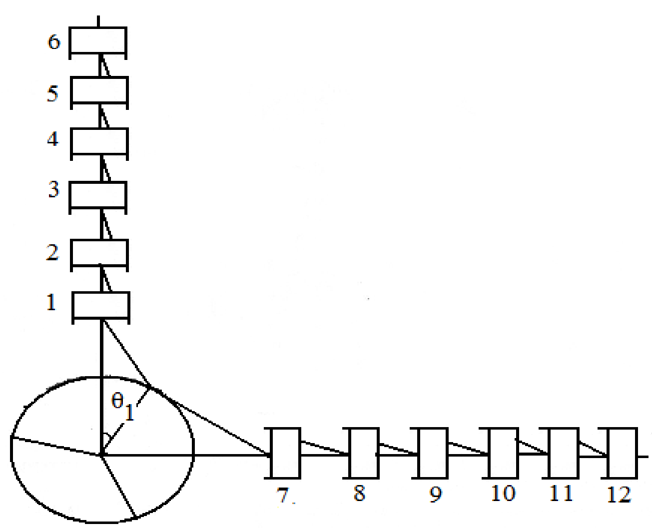
$M_{SL} = mr\omega^2/n [\cos 2\theta_1 \sum (a\cos 2\phi) - \sin 2\theta_1 \sum (a\sin 2\phi)] = mr\omega^2 a/n [-4.736\cos 2\theta_1 + 1.5389\sin 2 \theta_1]$

$M_{SR} = mr\omega^2/n [\cos 2\theta_2 \sum (a\cos 2\phi) - \sin 2\theta_2 \sum (a\sin 2\phi)] = mr\omega^2 a/n [4.736\cos 2\theta_1 - 1.5389\sin 2 \theta_1]$

$M_S = [M_{SL}^2 + M_{SR}^2]^{1/2}$

$\tan\eta = M_{SR}/ M_{SL}$

V-12 ENGINE ANALYSIS (cranks at 120°)



LEFT BANK RIGHT BANK

$\phi_1 = 0°$ $\phi_7 = 0°$

$\phi_2 = 240°$ $\phi_8 = 240°$

$\phi_3 = 120°$ $\phi_9 = 120°$

$\phi_4 = 120°$ $\phi_{10} = 120°$

$\phi_5 = 240°$ $\phi_{11} = 240°$

$\phi_6 = 0°$ $\phi_{12} = 0°$

SHAKING FORCES

$\sum \cos\phi = \cos0° + \cos240° + \cos120° + \cos120° + \cos240° + \cos0° = 0$

$\sum \sin\phi = \sin0° + \sin240° + \sin120° + \sin120° + \sin240° + \sin0° = 0$

$\sum \cos2\phi = \cos0° + \cos480° + \cos240° + \cos240° + \cos480° + \cos0° = 0$

$\sum \sin2\phi = \sin0° + \sin480° + \sin240° + \sin240° + \sin480° + \sin0° = 0$

Therefore,

$F_{PL} = mr\omega^2 [\cos\theta_1 \sum (\cos\phi) - \sin \theta_1 \sum (\sin\phi)] = 0$

$F_{PR} = mr\omega^2 [\cos\theta_2 \sum (\cos\phi) - \sin \theta_2 \sum (\sin\phi)] = 0$

$F_{SL} = mr\omega^2/n [\cos2\theta_1 \sum (\cos2\phi) - \sin 2\theta_1 \sum (\sin2\phi)] = 0$

$F_{SR} = mr\omega^2/n [\cos2\theta_2 \sum (\cos2\phi) - \sin 2\theta_2 \sum (\sin2\phi)] = 0$

SHAKING COUPLES/MOMENTS

$\sum (a\cos\phi) = 0$

$\sum (a\sin\phi) = 0$

$\sum (a\cos2\phi) = 0$

$\sum (a\sin2\phi) = 0$

$M_{PL} = mr\omega^2 [\cos\theta_1 \sum (a\cos\phi) - \sin \theta_1 \sum (a\sin\phi)] = 0$

$M_{PR} = mr\omega^2 [\cos\theta_2 \sum (a\cos\phi) - \sin \theta_2 \sum (a\sin\phi)] = 0$

$M_P = [M_{PL}^2 + M_{PR}^2]^{1/2} = 0$

Similarly,

$M_{SL} = mr\omega^2/n\,[\cos 2\theta_1 \sum (a\cos 2\phi) - \sin 2\theta_1 \sum (a\sin 2\phi)] = 0$

$M_{SR} = mr\omega^2/n\,[\cos 2\theta_2 \sum (a\cos 2\phi) - \sin 2\theta_2 \sum (a\sin 2\phi)] = 0$

$M_S = [M_{SL}^2 + M_{SR}^2]^{1/2} = 0$

Therefore, it is a completely balanced engine with no shaking forces and couples.

The above theoretical analysis can also be done for various engines with increasing number of cylinders like V-14, V-16 and so on. But calculations will be much more, so we will go for computer-aided analysis of balancing of multi-cylinder V engines. A program has been developed in C++ for the required analysis. The program will evaluate all forces and couples acting on the engine for different number of cylinders and different firing orders.

C++ PROGRAM FOR DYNAMIC ANALYSIS OF MULTI-CYLINDER V-ENGINES

```
#include<iostream.h>

#include<stdio.h>

#include<conio.h>

#include<math.h>

#define N 6

void main()

{

int phi [N];

int seq [N];

float m,r,w,n,a,theta1,theta2;

cout<<"Mention the firing order :\n";

for(int i=0;i<N;i++)

{
```

```
cin>>seq[i];
}

cout<<"enter mass :\n";
cin>>m;
cout<<"enter crank radius :\n";
cin>>r;
cout<<"enter angular velocity :\n";
cin>>w;
cout<<"enter n :\n";
cin>>n;
cout<<"enter cylinder pitch :\n";
cin>>a;
cout<<"enter theta1 :\n";
cin>>theta1;
theta1 = (theta1*M_PI)/180;
theta2 = ((theta1 - 90)*M_PI/180) ;
int delta;
cout<<"enter the crank angle diff. :\n";
cin>>delta;
phi[seq[0]-1]=0;//initial cylinder angle taken 0
for(i=1;i<N;i++)
{
phi[seq[i]-1]=i*delta;
}
```

```
for(i=0;i<N;i++)
{
cout<<"angle for cylinder "<<seq[i]<<" :"<<phi[seq[i]-1]<<"\n";
}
float sigmacos=0,sigmasin=0,sigma2cos=0,sigma2sin=0;
for(i=0;i<N;i++)
{
sigmacos = sigmacos + cos((phi[i]*M_PI)/180);
sigmasin = sigmasin + sin((phi[i]*M_PI)/180);
sigma2cos = sigma2cos + cos((2*phi[i]*M_PI)/180);
sigma2sin = sigma2sin + sin((2*phi[i]*M_PI)/180);
}
float forcepl=0,forcepr=0;
forcepl = m*r*w*w*((cos(theta1)*sigmacos)-(sin(theta1)*sigmasin));
forcepr = m*r*w*w*((cos(theta2)*sigmacos)-(sin(theta2)*sigmasin));
cout<<"\nprimary forces :"<<forcepl<<"  "<<forcepr<<"\n";
float forcesl = (m*r*w*w*((cos(2*theta1)*sigma2cos)-(sin(2*theta1)*sigma2sin)))/n;
float forcesr = (m*r*w*w*((cos(2*theta2)*sigma2cos)-(sin(2*theta2)*sigma2sin)))/n;
cout<<"secondary forces :"<<forcesl<<"   "<<forcesr<<"\n\n";
float asigmacos=0,asigmasin=0,asigma2cos=0,asigma2sin=0;
for(i=0;i<N;i++)
{
asigmacos = asigmacos + a*i*cos((phi[i]*M_PI)/180);
asigmasin = asigmasin + a*i*sin((phi[i]*M_PI)/180);
asigma2cos = asigma2cos + a*i*cos((2*phi[i]*M_PI)/180);
```

```
asigma2sin = asigma2sin + a*i*sin((2*phi[i]*M_PI)/180);
}
float mpl=0,mpr=0,msl=0,msr=0;
mpl = m*r*w*w*((cos(theta1)*asigmacos)-(sin(theta1)*asigmasin));
mpr = m*r*w*w*((cos(theta2)*asigmacos)-(sin(theta2)*asigmasin));
msl = (m*r*w*w*((cos(2*theta1)*asigma2cos)-(sin(2*theta1)*asigma2sin)))/n;
msr = (m*r*w*w*((cos(2*theta2)*sigma2cos)-(sin(2*theta2)*sigma2sin)))/n;
cout<<"primary moment values :"<<mpl<<"   "<<mpr<<"\n";
cout<<"secondary moment values :"<<msl<<"   "<<msr<<"\n\n";
float fp,fs,mp,ms,thetafp,thetafs,thetamp,thetams;
fp = sqrt(forcepl*forcepl + forcepr*forcepr);
 fs = sqrt(forcesl*forcesl + forcesr*forcesr);
mp = sqrt(mpl*mpl + mpr*mpr);
ms = sqrt(msl*msl + msr*msr);
thetafp = atan(forcepr/forcepl);
thetafs = atan(forcesr/forcesl);
thetamp = atan(mpr/mpl);
thetams = atan(msr/msl);
cout<<"the Resultant forces : "<<fp<<"   "<<fs<<"\n";
cout<<"the Resultant moments : "<<mp<<"   "<<ms<<"\n\n";


cout<<"the angles of Resultant moments : "<<thetamp<<"   "<<thetams<<"\n";
getch();
}
```

RESULTS AND DISCUSSION

The results computed from the C++ program developed for this analysis are tabulated below which provides the unbalanced forces and moments on different types of engines at various crank positions. The comparison has been done between Inline and V engines with 8, 10 and 12 cylinders on the basis of unbalanced forces and moments which are shown below along with discussion.

1. 8-Cylinder Engine

Inline-8

Crank angle θ (degree)	Primary Force (N)	Secondary Force (N)	Primary Moment(Nm)	Secondary Moment(Nm)
0	0	0	-17640	-3528
60	0	0	6455.9439	4819.25785
120	0	0	24096.2893	-1290.7422
180	0	0	17641.6343	-3528.6537
240	0	0	-6453.7112	4819.01842
300	0	0	-24095.691	-1289.8491
360	0	0	-17643.269	-3529.3073
420	0	0	6451.47847	4818.77882
480	0	0	24095.0921	-1288.956
540	0	0	17644.9025	-3529.9607
600	0	0	-6449.2457	4818.53906
660	0	0	-24094.493	-1288.0628
720	0	0	-17646.536	-3530.6141

V-8

Crank angle θ (degree)	Primary Force (N)	Secondary Force (N)	Primary Moment(Nm)	Secondary Moment(Nm)
0	0	12473.36	0	7484.018
60	0	-6236.01274	0	-3741.60865
120	0	-6238.01444	0	-3742.80967
180	0	12473.35979	0	7484.017872
240	0	-6234.01083	0	-3740.4075
300	0	-6240.01593	0	-3744.01056
360	0	12473.35914	0	7484.017486
420	0	-6232.0087	0	-3739.20622
480	0	-6242.0172	0	-3745.21132
540	0	12473.35807	0	7484.016844

600	0	-6230.00636	0	-3738.00482
660	0	-6244.01825	0	-3746.41195
720	0	12473.35657	0	7484.015944

The primary force for both configurations is zero but the primary moment is zero only in V-8 engine. Hence V-8 engine is balanced for both primary forces and moments, but there are higher secondary forces and moments as compared to inline-8. Since primary forces and moments are more dominating over the secondary ones, hence the V-8 engine is better balanced than inline-8.

2. 10-Cylinder Engine

Inline-10

Crank angle θ (degree)	Primary Force (N)	Secondary Force (N)	Primary Moment(Nm)	Secondary Moment(Nm)
0	0	0	4286.52	-7472.304
60	0	0	-47646.283	3146.12747
120	0	0	-51935.352	4326.51313
180	0	0	-4291.8469	-7472.1777
240	0	0	47643.2755	3144.86514
300	0	0	51937.671	4327.64904
360	0	0	4297.17382	-7472.0512
420	0	0	-47640.267	3143.6027
480	0	0	-51939.99	4328.7848
540	0	0	-4302.5007	-7471.9244
600	0	0	47637.2587	3142.34015
660	0	0	51942.3077	4329.92041
720	0	0	4307.8275	-7471.7973

V-10

Crank angle θ (degree)	Primary Force (N)	Secondary Force (N)	Primary Moment(Nm)	Secondary Moment(Nm)
0	0	0	1980.09	-5907.38501
60	0	0	1980.09	4615.793989
120	0	0	1980.09	1292.084861
180	0	0	1980.09	-5907.74061
240	0	0	1980.09	4615.023687
300	0	0	1980.09	1293.210681
360	0	0	1980.09	-5908.09601

420	0	0	1980.09	4614.253226
480	0	0	1980.09	1294.336457
540	0	0	1980.09	-5908.4512
600	0	0	1980.09	4613.482606
660	0	0	1980.09	1295.462189
720	0	0	1980.09	-5908.8062

The primary and secondary forces for both configurations are zero but there are moments acting on both.

Inline-10: $M_{P/max}$ = 51942 Nm, $M_{S/max}$ = 7472 Nm

V-10: $M_{P/max}$ = 1980 Nm, $M_{S/max}$ = 5908 Nm

$\Delta M_{P/max}$ = 49962 Nm, $\Delta M_{P/max}$ = 1564 Nm

The primary and secondary moments on V-10 are much lower than those in inline-8, hence V-10 is better balanced.

3. 12-Cylinder Engine

Inline-12

Crank angle θ (degree)	Primary Force (N)	Secondary Force (N)	Primary Moment(Nm)	Secondary Moment(Nm)
0	0	0	-13230	0
60	0	0	-33074.106	0
120	0	0	-19845.875	0
180	0	0	13227.1691	0
240	0	0	33073.7518	0
300	0	0	19848.3519	0
360	0	0	-13224.338	0
420	0	0	-33073.397	0
480	0	0	-19850.828	0
540	0	0	13221.5071	0
600	0	0	33073.0429	0
660	0	0	19853.3049	0
720	0	0	-13218.676	0

V-12

Crank angle θ (degree)	Primary Force (N)	Secondary Force (N)	Primary Moment(Nm)	Secondary Moment(Nm)
0	0	0	15276.68	-3742.00909
60	0	0	15276.68	3741.820744
120	0	0	15276.68	0.588675538
180	0	0	15276.68	-3742.40936
240	0	0	15276.68	3741.420285
300	0	0	15276.68	1.38936342
360	0	0	15276.68	-3742.8095
420	0	0	15276.68	3741.019696
480	0	0	15276.68	2.190051253
540	0	0	15276.68	-3743.20951
600	0	0	15276.68	3740.618979
660	0	0	15276.68	2.990739011
720	0	0	15276.68	-3743.6094

The primary and secondary forces for both configurations are zero but there is only primary moment on inline-12 while both primary and secondary moments act on V-12. Since primary moments dominate on their secondary counterparts, so we have compared the primary moments.

Inline-12: $M_{P/max}$ = 33074 Nm

V-8: $M_{P/max}$ = 17802 Nm

Since the primary moment on V-12 is much lower than that on inline-12, hence V-12 is much better engine than Inline-12 engine.

CONCLUSION

On comparing the inline and V-engines on the basis of forces and moments, it has been found out that V-configuration engines are better balanced and hence they will provide smooth operation free from noise, vibration and harshness to a greater extent as compared to inline engines. They are less complex and more compact with less weight, thus producing more power while taking less space. These are used in heavy automobiles and aircrafts where more power with compact size and minimum vibrations is required. The present analysis can further be applied by varying the firing orders for different configurations and also to multi-cylinder radial engines.

REFERENCES

[1] Vigen H. Arakelian and M. R. Smith ,Shaking Force and Shaking Moment Balancing of Mechanisms: A Historical Review With New Examples Journal of Mechanical Design, MARCH 2005, Vol. 127, pp 334-339

[2] I. Esat , H. Bahai, A theory of complete force and moment balancing of planer linkage mechanisms, Mechanism and Machine Theory 34 (1999) 903-922

[3] W Harry Close, Wieslaw Szydlowski and Christopher Downton, Perfect Engine Balance Via Collins Scotch Yoke Technology, SAE International, 941039 ,March 1994

[4] H. D. Desai, Computer Aided Kinematic and Dynamic Analysis of a Horizontal Slider Crank Mechanism Used For Single-Cylinder Four Stroke Internal Combustion Engine, Proceedings of the World Congress on Engineering 2009 Vol II WCE 2009, July 1 - 3, 2009, London, U.K.

[5] V. Arakelian and M. Dahan, Partial shaking moment balancing of fully force balanced linkages, Mechanism and Machine Theory, Volume 36, Issues 11-12, November 2001, Pages 1241-1252

[6] J. Singh, B. Singh, Design of crankshaft for complete balancing of primary unbalanced force in reciprocating engine, Journal of the Institution of Engineers (India), Part MC, Mechanical Engineering Division. Vol. 82, no. 3, Oct. 2001, pp. 138-142.

[7] Mechanisms & Dynamics of machinery:Mable & Reinholtz

[8] Therory of machines :S S Rathan

[9] Theory of machines:Abdulla Shariff

[10] Theory of machines & mechanism :Ghosh & Mallick