Optimal Placement of Collocated Sensors and Actuators in FRP Composites Substrate

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by

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Date:13/05/2010 **Sudipto Mandal**

CERTIFICATE

This is to certify that the thesis entitled, "**Optimal Placement of Collocated Sensors and Actuators in FRP Composite Substrate"** submitted by Mr. **Sudipto Mandal** in partial fulfillment of the requirements for the degree of **Bachelor of Technology** in **Mechanical Engineering** at National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance. To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/ Institute for the award of any Degree or Diploma.

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ABSTRACT

In this thesis, Multi-Objective method is used for optimal placement of Collocated Sensors and Actuator, using integrated Genetic Algorithm. Optimal placement of piezoelectric sensors and actuators in a cantilever beam is found out by maximizing the controllability index and also observability index. First mode vibration is only considered for the present case. Finite element formulation for shell structure was used for the beam analysis by making the radius infinite, which results to the formulation for plate analysis. The cantilever beam was divided into twenty equal sections, where the piezoelectric material can be placed. In the present study four sensors and four actuators has been considered for collocated system. For non-collocated system four sensors was only considered. Results obtained from the work shows that the location for placement of piezoelectric material for non-collocated system is same as that obtained from multi-objective collocated system. Hence it can be deduced, it is not needed to find out the location for sensors and actuators separately rather controllability index for both can be found out together by using multi-objective collocated formulation.

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LIST OF SYMBOLS USED

$\varepsilon_{x'}$, $\varepsilon_{y'}$, $\gamma_{x'y'}$, $\gamma_{x'z'}$, $\gamma_{y'z'}$ = Five strain component in local co-ordinate.

$\{\sigma\}$ = Stress vector.

$\{\varepsilon\}$ = Strain vector.

$\{D\}$ = Dielectric displacement vector.

$\left[B_s^e\right]$ =Electric field gradient of sensors.

$\left[B_a^e\right]$ =Electric field gradient of actuator.

Φ_a^e =Degree of freedom at piezoelectric actuator.

Φ_s^e =degree of freedom at piezoelectric sensor.

$[M_{uu}]$ = Global mass matrix.

$[K_{uu}]$ = global elastic stiffness matrix.

$[K_{ua}]$ = global piezoelectric coupling matrix for actuator.

$[K_{us}]$ = global piezoelectric coupling matrix for sensors.

$[K_{aa}]$ = global dielectric stiffness matrix for actuatorr.

$[K_{ss}]$ = global dielectric stiffness matrix for sensors.

σ_{n_a} = number of actuator.

σ_{n_s} = Number of sensors.

f = Total fitness.

Ω_s =Controllability Index for sensors.

Ω_a =Controllability index for actuator.

$\gamma^{'}$ =Spill over (damping ratio).

1. INTRODUCTION

Optimal placement of sensors and actuators in FRP composites is of great interest to researchers as it results to the development of smart structures. Smart structures can automatically respond to external or internal disturbances. This is done by the use of sensors and actuators, which are integrated to the system. Piezoelectric materials are bonded to the structure, so when the external environment causes the structure to deform, the piezoelectric material also deforms. This causes the piezoelectric material to produce voltage, as in case of sensors. The voltage produced is then amplified by the controller, form the controller the output voltage is fed to the actuator. Depending on this voltage, the actuator actuates. Quartz is an example of piezoelectric material that is, it produces electrical voltage when it is strained in a particular direction. This phenomenon of quartz crystal was discovered by Pierre and Curie in 1880 [1]. Commonly used piezoelectric material are lead zirconate titanate (PZT) and poly-vinylidene fluoride (PVDF) [1].

Practically we face two kind of optimization problem discrete and continuous. Depending on this the optimization can be non-symmetric or symmetric [2]. Genetic algorithm is an example of symmetric optimization technique. Genetic Algorithm is an optimization method which uses the concept of natural biology. It is inspired by Darwin’s theory of natural selection. Gao et al [4]. (2000) used GA to solve discrete placement of sensors and actuators in a structure. With the help of GA’s large and complex optimization problems can be solved very easily [3], hence GA have become a common type of optimization method. Exhaustive process needs a long computational time, this can be avoided by the use of genetic algorithm. GA also increases the rate of convergence.

Optimization is done due to the following reasons:-

I) To reduce the cost.

ii) To find out the position where the sensor and actuator output will be maximum.

iii) The number of sensors and actuators are generally limited.

iv) To ensure that no extra sensor and actuator is used.

1.1 Composites

With the advancement of time and technology a stronger, high strength, with better mechanical properties are required to withstand extreme working conditions. Hence engineers are always in search of alternatives to steel and alloy to combat the high cost of repair and maintenance.

FRP stands for fiber reinforced polymer composites. It is a combination of two or more material in macroscopic scale. It consists of two phase that is fiber, matrix. Physical, chemical and mechanical properties of individual material are never lost in FRP. The combination is done so as to get properties that are better and consecutively more reliable. Examples of fibers are carbon or glass and commonly used matrix are polyester, epoxy or nylon. Composites comprises of resins, additives, fillers and reinforcement.

Table1: Difference between fiber reinforced polymer composites and traditional material:-

FRP COMPOSITES	**TRADITIONAL MATERIALS**
i) FRP are anisotropic which means that properties are apparent in the direction of applied load. ii) The properties shown by this kind of material are directional which means that best mechanical properties are in the direction of fiber placement.	i) Steel or Aluminum is isotropic. It exhibit uniform properties in all direction and it is independent of applied load. ii) Traditional material are non directional.

Advantages Of Fiber Reinforced Polymer Composites

Various advantages of Fiber reinforced composites are as:-

i) The have very less weight.

ii) FRP has non-magnetic properties.

iii) Strength to weight ratio is high.

iv) They are thermally non-conductive.

v) FRP are stronger than steel.

vi) Electrically non-conductive.

Application of fiber reinforced polymer composites

FRP have many properties which are better than metal, it is because of this desirable properties that use FRP is very vast. Common application is in construction of new structures, repairing of damaged structures, aerospace engineering, automotive.

1.2 Smart Structures

FRP when integrated with sensors and actuators become smart structures. Smart structures can automatically adapt to adverse external environment. Hence, it results in better performance at adverse working conditions like high temperature, e.t.c. This is achieved by implementing proper active control means. Smart material can sense and actuate in controlled manner. This is done with the help sensors and actuators which are bonded properly to the smart structure. Sensors and actuators are piezoelectric material that is it produces electric voltage when a strain is applied to it or vice-versa. Smart structures minimize vibration and can also detect fault if there is any. The advantages of smart structure are as:-

i) sensor/actuator collocation simplifies controller design.

ii) Distributed sensors and actuators tend to have better observability and controllability properties with respect to the flexible modes [5].

Adaptronics is another term used for smart structure. As Sensors and actuators are properly bonded to the structure, when the structure is displaced then the sensors also get displaced by same amount, generating voltage. This voltage is amplified by controller and an output voltage is calculated, which in turn is supplied to the actuator. Finally the actuator actuates and negative displacement is provided to the structure. Hence it can be seen that the smart structures can perform self diagnosis.

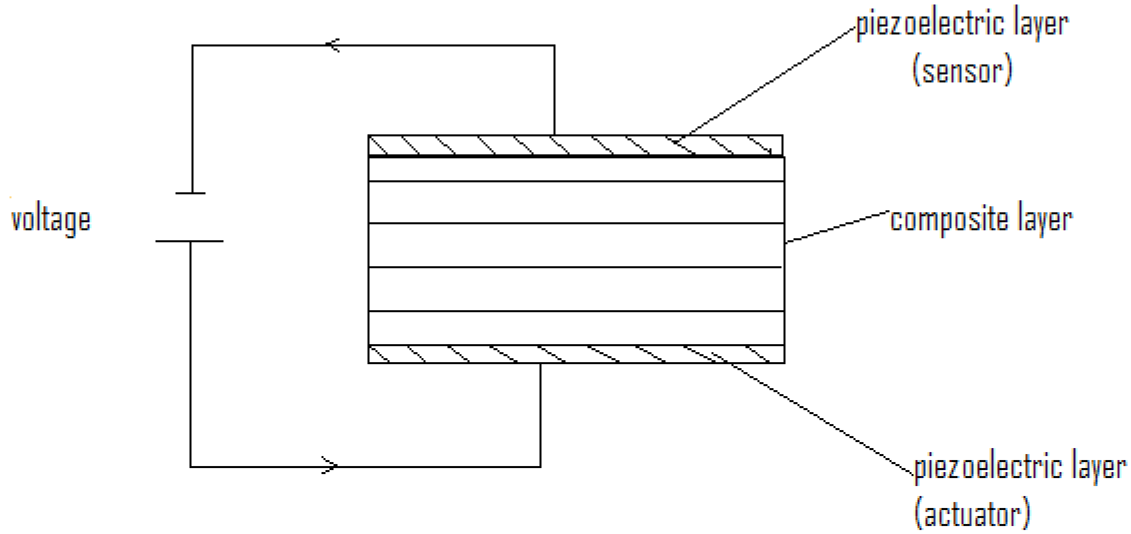

Fig 1:- A simple smart structure

A smart structure basically comprises of following 3 elements:-

i) Sensors:- The sensor which is bonded to the cantilever beam on deformation of the beam it generates voltage. They are generally made up of piezoelectric material e.g quartz. It works on the principle that when strain is generated voltage is produced.

ii) Controller:- The voltage generated by the sensors is received by the controller. Controller amplifies or decreases the value of voltage depending on the conditions with the help of proper Control gain. The output of controller is the input for actuator. Controller also forms the feedback transfer function for this system [6].

iii) Actuator: - The output of controller is fed to the actuator. Actuator works on the converse principle of sensors that is when voltage is given to it, actuators deform.

The deformation created by actuator is just opposite of the deformation produced by the system.

Smart structures are of two types:

i) Closed loop

ii) Open loop.

2. LITERATURE REVIEW

2.1 General

Three basic requirement, to make an active vibration control system are sensor, actuator and a controller [9,10]. The design process of such a system is divided into three phase optimal placement of sensors and actuators, sensors and actuator design, controller design [10]. Vibration control is done so as to have better system performance, and this can be achieved with the help of optimal placement of sensors and actuators. Optimization problem are broadly classified into two groups. They are discrete and continuous. Smart composites material has many properties which are better than metallic material e.g high strength to weight ratio, cheap, high strength, high stiffness, etc. Because of this desirable properties that smart material have increased its uses in aerospace engineering and different structures. Use of piezoelectric material results in more accurate control. Therefore it is required to optimize their location and many work on this field have been done and many are still going on.

2.2 Research Work's Done

Optimal collocation of sensors/actuators was done by Arbel [11, 12] in the year 1981 by using the concept of controllability/observability of a system. Crawley and Luis (1987) used mechanism of actuation strain concept for simple elastic strain beam [27, 6, 7]. Geromel (1989) [13,10] considered the effect of transducer placement and designed a controller to reduce some performance criteria for a closed loop system. The reciprocating and governing relation for laminated piezoelectric plates was defined by Lee (1990) [28]. Control of plate model by integrating sensors and actuators has been studied by Tzou and Tsang [29] (1990). In the year 1992 R.H. Zhang [14, 15] showed that it was not sufficient to justify the assumption that optimal location is independent to the excitation. Onada and Hawanda (1992) [16, 17] used SA method to solve optimization problem which started at a high temperature. Based on linear quadratic optimization Lim (1992) [18,12] gave several optimization rule for the optimal placement of sensors/actuators and their size. Generalized electromechanical coupling coefficient for a pair of piezoelectric material attached to a simple cantilever beam was estimated by Hollkamp [19]. Two dimensional, eight node quadratic quadrilateral isoparametric element simply supported intelligent beam was studied by Ray et al (1994). Considering the velocity feedback, and pulse constant, Chen (1994) [30] derived the expression energy dissipated for closed loop. Hwang (1994) [21, 18] used finite element method and suggested an intuitive method for optimization of laminated composite plate bind with piezoelectric sensors and actuators pair. It was not possible to solve BMI problems using standard convex optimization software and hence iterative method was used by Ghaoui (1997) [10, 22]. In the year 1998, Kim and Kou [23] used FEM method to reduce the sound radiated during resonance from the structure that was mounted with sensors and actuators.

Lopez [10] modeled formulation for structures including electromagnetic coupling matrix using finite element method in the year 2000 Later Chen et al(2000) [15], proposed three dimensional shell structure, on which piezoelectric sensors and actuators was mounted. Active vibration control for collocated laminated beam using sensors and actuators was optimized by Kang *et al* [31, 6, 7], he maximized the structural damping index. Gradient based optimization was carried by Kang *et* a l [32 ,6 ,7]for laminated plate. From the studies made earlier it became clear that gradient based optimization for a simple beam structure tends to get trapped in local optima. A nine nodded quadrilateral shell mounted with piezoelectric layer was created by Balagurugan [6, 7] and Narayanan [25] (2001) for the analysis of semicircular layer with distributed layer of PZT. Lee et al [28] (2003) modified the existing nine nodded strain shell formulation, so as to use it for the study of an arch of thin cylinder. Zheng et al (2004) developed a higher order piezoelectric formulation to study the vibration of laminated structure. Kusculuoglu and Royston (2005) used first order shear deformation theory to formulate for composite plate embedded with piezoelectric layer and used it for composite plate analysis. Balagurugan and Narayanan [6, 7] again in the year 2007 presented a smart composite plate finite element. It had 48 elastic degree of freedom and 9 piezoelectric impact response of adaptive piezoelectric laminated plates. Vibration control of smart FRP composites structure, which was bonded by piezoelectric sensors and actuators, was done by Roy and Chakraborty in the year 2008 by minimizing the maximum displacement, using the concept of genetic algorithm.

2.3 Motivation And Objective

Many works have been done in the field of smart structure and to control its vibration using piezoelectric material, whose position have been determined by using the concept of Genetic Algorithm. From the above literature review we can deduce the following:-

Control spillover of higher modes was neglected during optimal placement of sensors and actuators.

i) Very less work have been done for multi-objective collocated system (both sensor and actuator together)

ii) Not much work has been done for sensor placement.

iii) Maximum work done in the past had used binary coded Genetic Algorithm; it is therefore time consuming method, as it needs larger number of generation to find an optimal solution.

The objective of the present work, considering the above stated points can be summarized as follows:-

i) To find out the optimal location for sensor placement.

ii) To find out an optimal position for sensors and actuators for a collocated system.

iii) In this thesis Integer Genetic Algorithm method is used as tool for optimization problem.

3. GENETIC ALGORITHM

Genetic algorithm was invented by John Holland [34]. Idea of evolutionary computation was introduced in the year 1960, I**. Rechenberg** [34] in his work "***Evolution strategies***". GA'S are a search and optimization tool, which work differently from classical search and optimization method. Genetic algorithms are inspired by Darwin's theory of evolution, hence inspired and motivated by the principle of natural genetics and natural selection. Genetic algorithms in particular became popular through the work of John Holland [34] in the early 1970s, and particularly his book *Adaptation in Natural and Artificial Systems* (1975). He used evolutionary biology such as inheritance, mutation, selection, and crossover. A set of known solution is taken as the start point for algorithm, this is known as population pool. From here two population with best fitness is selected, then reproduction between this two population is dine. This is done with the hope of getting new and better solution. Then the fitness of both the offspring and that of the parent population is checked. The population with better fitness is selected and is stored for further mutation and crossover, in next check.

Traditional optimization technique is of two types:-

i) Direct type.

ii) Gradient based.

Table 2: Difference between direct and gradient based are as:-

Direct	Gradient based
i) Here only objective function f(x) and constraint values g(x), h(k) are used to guide the search strategy.	I) Here gradient based method use the first or second order derivatives of the objective or the constraint to guide the search.
ii) They are usually slow and needs much function evaluation for convergence.	ii) They converge quickly to optimal solution.
iii) They can be applied to many problems without major change of algorithm.	iii) Program should be changed for different problem.

The evaluation function is provided by the programmer, this program calculates the fitness of all the Population. Based on this fitness, selection is done. Following the selection process, reproduction and mutation is done. This is continued until an optimal solution is achieved or the program is made to run for a particular number of generations. Genetic algorithm can be categorized as following [33]:-

Selection

The most basic type of selection is roulette-wheel selection [33]. According to roulette-wheel selection procedure, the population with best fitness has the highest probability of getting selected. Based on this selection procedure two individual are selected. Other type of fitness selection method rates the fitness of all the solutions and then selects the solution with best fitness. The latter type of selection is time consuming and hence is not much preferred.

Eg :- roulette-wheel selection procedure, tournament selection procedure.

Crossover

Crossover is done so as to produce offspring that may have better fitness. There are many type of crossover but the most common type is single point crossover. In single point crossover, a locus is chosen at which the remaining alleles are swapped from one parent to the other [33]. This can be understood by following diagram.

PARENT 1		**PARENT 2**	
1011010	010100110	0011010	110110101
CHILD 1		**CHILD 2**	
1011010	110110101	0011010	010100110

Fig 2:- Crossover shown

From above it can see that the child takes one part of the chromosome from each parent. This type of crossing is called single point crossing since there is only one crossing point. Crossing does not necessarily occur in every step. It depends on the probability of crossing that is given by the programmer.

Mutation

Mutation is done so as to maintain genetic diversity [33]. After the selection and reproduction, a pool of population left, which may be identical to other population in a pool or may differ. To reduce the probability of similarity of population in a pool, mutation operation is performed. Here a random solution is taken and is changed by small value or replace completely. The probability of mutation is generally between .001 or .002.

Before: 1101101001101110

After: 1101100001101110

Fig 3:-Mutation shown

Termination

The program continues until certain conditions are fulfilled, that is until we reach an optimal solution, where the solution start's to converge.

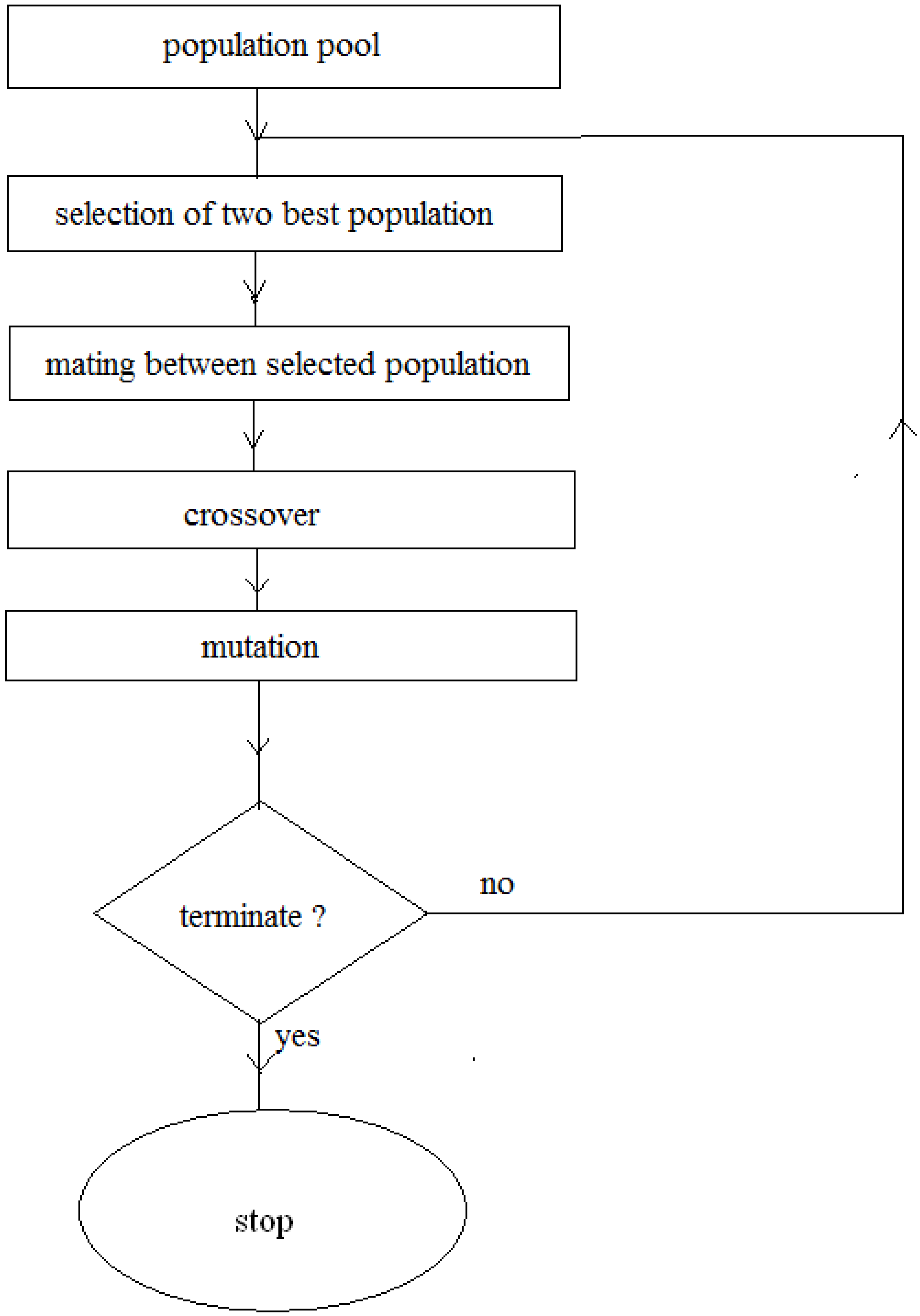

Fig 4: Schematic representation for genetic algorithm

3.1 Integer coded GA for optimal placement of sensors and actuators

Optimal location of piezoelectric sensors is found out by the value of fitness, which is the controllability index. Fitness of sensor can be written as:-

$$\Omega_s = \left\{ \begin{array}{ll} \prod_{i=1}^{n_s} \sigma_i & \text{observed spillover considered} \\ \prod_{i=1}^{n_s} \sigma_i - \gamma' \sum_{i=1}^{n_s} \sigma_i^R & \text{if } \sum_{i=1}^{n_s} \sigma_i \rangle \gamma' \sum_{i=1}^{n_s} \sigma^R{}_i \\ \prod_{i=1}^{n_s} \sigma_i - \gamma' \sum_{i=1}^{n_s} \sigma_i^R \times 10^{-2} & \text{otherwise} \end{array} \right\} \quad (1)$$

The fitness of actuator is given by the following formulae [34, 6, 7] :-

$$\Omega_a = \left\{ \begin{array}{ll} \prod_{i=1}^{n_a} \sigma_i & \text{observed spillover considered} \\ \prod_{i=1}^{n_a} \sigma_i - \gamma' \sum_{i=1}^{n_a} \sigma_i^R & \text{if } \sum_{i=1}^{n_a} \sigma_i \rangle \gamma' \sum_{i=1}^{n_a} \sigma^R{}_i \\ \prod_{i=1}^{n_a} \sigma_i - \gamma' \sum_{i=1}^{n_a} \sigma_i^R \times 10^{-2} & \text{otherwise} \end{array} \right\} \quad (2)$$

Where,

n_s =number of sensors.

n_a =number of actuator.

Various steps that are followed during the optimization process using integer coded GA are discussed below [6, 7] :-

i) Initial chromosomes are selected randomly from a set of population pool. It depends on the number of sensors and actuators that are being used.

ii) The fitness of all the chromosomes that were selected is calculated, with the help of above given formulae.

iii) Different genetic operations, i.e selection, mating, crossover and mutation is done, so that the parent chromosome generates an offspring.

iv) The program runs until it fulfills the conditions, i.e until the fitness converges.

v) It is only after the convergence of fitness that the computation is terminated. Only after this we get the best controllability value and this is selected as the optimal location for sensors/actuators.

4. FINITE ELEMENT FORMULATION FOR OPTIMAL PLACEMENT OF SENSORS AND ACTUATORS

4.1 Strain displacement relationship

When the cantilever beam gets deflected then the piezoelectric layer that is bind to the structure also gets deflected. The deflection produces strain which is related to displacement by the relation given below. It is this strain that is used by the sensors to generate voltage. Neglecting the normal strain in thickness direction the five strain component in the local coordinate is given [6, 7] by following:-

$$[\varepsilon] = \begin{bmatrix} \varepsilon_{x'} \\ \varepsilon_{y'} \\ \gamma_{x'y'} \\ \gamma_{x'z'} \\ \gamma_{y'z'} \end{bmatrix} = \begin{bmatrix} \dfrac{du'}{dx'} \\ \dfrac{dv'}{dy'} \\ \dfrac{dv'}{dx'} + \dfrac{du'}{dy'} \\ \dfrac{du'}{dz'} + \dfrac{dw'}{dx'} \\ \dfrac{dv'}{dz'} + \dfrac{dw'}{dy'} \end{bmatrix} \tag{3}$$

With the help of transformation matrix we can get the local derivatives from the global derivatives of u, v, w. The strain-displacement matrix in global coordinate can be found out from the displacement derivative.

The strain-displacement in global coordinate is related to strain component to nodal variable is given by the following equation [6, 7]

$$\{\varepsilon\} = \sum_{k=1}^{8} \left[\left(B_u \right)_k^e \right] \left\{ d_k^e \right\} = \left[B_u^e \right] \left\{ d^e \right\} \tag{4}$$

4.2 Direct and converse piezoelectric patch relation:-

The relation between the direct and converse piezoelectric patch is given by the following equation[6, 7] :-

$$\{D\} = [e]\{\varepsilon\} + [\in][E] \tag{5}$$

$$\{\sigma\} = [C]\{\varepsilon\} - [e]^T \{E\} \tag{6}$$

Where,

$\{D\} =$ Dielectric displacement vector.

$\{\sigma\} =$ Stress vector.

$\{\varepsilon\} =$ Strain vector.

4.3 Potential developed in the piezoelectric patch(sensor) :-

The potential that is developed by the sensors is given by the following formulae[6, 7] :-

$$\{-E_a^e\} = [B_a^e]\{\phi_a^e\} = \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{h_a} \end{bmatrix} \{\phi_a^e\} \tag{7}$$

$$\{-E_s^e\} = [B_s^e]\{\phi_s^e\} = \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{h_a} \end{bmatrix} \{\phi_s^e\} \tag{8}$$

Where,

Subscript a & s refers to actuator and sensor respectively.

Subscript e refers to the parameter at element level.

$\left[B_s^e\right]$=Electric field gradient of sensor(matrix).

$\left[B_a^e\right]$=Electric field gradient of actuator(matrix)

Various assumption made here are as following:-

i) At the top of the piezoelectric actuator and sensors the electrical degree of freedom is considered to be one $\left(\phi_a^e, \phi_s^e\right)$.

ii) Through out the element the electrical potential is considered to be constant.

iii) The component of electric field is always dominant in thickness direction.

4.4 Finite element formulation:-

Finite element formulation(FEM) comprises of for parts structural mass matrix, stiffness mass matrix, dielectric conductivity matrix and piezoelectric coupling matrix [6, 7].

Structural mass :-

$$\left[M_{uu}^e\right] = \int_V \rho \left[N^T\right]\left[N\right] dV \tag{9}$$

Structural stiffness:-

$$\left[K_{uu}^e\right] = \frac{2}{T}\int_{-1}^{1}\int_{-1}^{1}\sum_{k=1}^{N}\frac{t_k - t_{k-1}}{2}\int_{-1}^{1}\left[B_u\right]^T\left[C\right]\left[B_u\right] |J|\, d\xi d\eta d\zeta \tag{10}$$

Dielectric conductivity:-

$$\left[K_{ss}^{e}\right] = -\frac{2}{T}\int_{-1}^{1}\int_{-1}^{1}\sum_{k=1}^{N}\frac{t_k - t_{k-1}}{2}\int_{-1}^{1}\left[B_{\phi}\right]^{T}\left[\in\right]\left[B_{\phi}\right]|J|\,d\xi d\eta d\zeta \qquad (11)$$

Piezoelectric coupling matrix:-

$$\left[K_{us}^{e}\right] = \frac{2}{T}\int_{-1}^{1}\int_{-1}^{1}\sum_{k=1}^{N}\frac{t_k - t_{k-1}}{2}\int_{-1}^{1}\left[B_{u}\right]^{T}\left[e\right]^{T}\left[B_{\phi}\right]|J|\,d\xi d\eta d\zeta \qquad (12)$$

Stifness mass matrix was evaluated by numerical integration using gauss quadrature $(3\times3\times2)$ scheme or selective integration scheme depending on the thickness of cantilever beam [6, 8].

The over all dynamic finite element equation thus become:-

$$\left[M_{uu}\right]\left\{d^{"}\right\} + \left[\left[K_{uu}\right] - \left[K_{ua}\right]\left[K_{aa}\right]^{-1}\left[K_{au}\right] - \left[K_{us}\right]\left[K_{ss}\right]^{-1}\left[K_{su}\right]\right]\left\{d\right\} = \left\{F\right\} - \left[K_{ua}\right]\left\{\phi_a\right\} \quad (13)$$

Where,

$\left[M_{uu}\right]$ = Global mass matrix.

$\left[K_{uu}\right]$ = Global elastic stiffness matrix.

$\left[K_{ua}\right]$ = Global piezoelectric coupling matrix for actuator.

$\left[K_{us}\right]$ = Global piezoelectric coupling matrix for sensors.

$\left[K_{aa}\right]$ = Global dielectric stiffness matrix for actuatorr.

$\left[K_{ss}\right]$ = Global dielectric stiffness matrix for sensors.

4.5 State-Space representation:-

The most easily excitable mobes are the lower modes as they have very low energy. The transformation matrix that can be uesed is Ψ_s , which nothing but truncated modal matrix between the generalized co-ordinate dt and the modal co-ordinate $d\eta$.

The displacement vector can be written as:

$$\{d(t)\} \approx [\psi]\{\eta(t)\} \tag{14}$$

Where,

$[\Psi]$ = [Ψ_1 Ψ_2 Ψ_3……..Ψ_r] is the truncated modal matrix.

The decoupled dynamic equation when modal damping is considered is given by:-

$$\{\eta_i(\ddot{t})\} + 2\xi_{di}\omega_i\{\eta_i(t)\} + \omega_i^2\{\eta_i(t)\} = [\psi]^T\{F\} - [\psi]^T[K_{ua}]\{\phi_a\} \tag{15}$$

Where,

ξ_{di} is the damping ratio.

The state-space form is given by the following:-

$$\{\dot{x}\} = [A]\{X\} + [B]\{\phi_s\} + [\hat{B}]\{u_d\} \tag{16}$$

Where,

$$[A] = \begin{bmatrix} [0] & [1] \\ -\omega_i^2 & 2\xi_{di}\omega_i \end{bmatrix}$$ known as symmetric matrix.

$$[B] = \begin{bmatrix} [0] \\ -[\psi]^T[K_{ua}] \end{bmatrix}$$ known as coupling matrix.

$$[\hat{B}] = \begin{bmatrix} [0] \\ -[\psi]^T[F] \end{bmatrix}$$ known as disturbance matrix.

$[u_d]$ is the disturbance input vector,

$[\phi_a]$ is the control input

$$\left[\dot{X}\right] = \begin{Bmatrix} \dot{\eta} \\ \ddot{\eta} \end{Bmatrix} \text{ and } \left[X\right] = \begin{Bmatrix} \eta \\ \eta \end{Bmatrix}$$

The sensor output equation is

$$\{y\} = [C_0][X]$$

Where $[c_0]$ depends on the modal matrix $[\psi]$ and the sensor coupling matrix $[k_{\phi w}]$

4.6 Controllability index for actuator location:-

With the help of this index we can find the fitness and hence ultimately we find the best actuator or sensor location. By maximizing the global controle force Wang and Wang [34] developed a controllability index actuator. Which is given by the following equation.

$$\{f_c\} = [B]\{\phi_a\} \tag{17}$$

Similarly we can write the controllability index for sensors as:-

$$\{f_c\} = [B]\{\phi_s\} \tag{18}$$

With the concept of singular value decomposition we can write the following equation:-

$$[B]=[M][S][S^T] \tag{19}$$

For actuator [S] given as [33,6,7]:-

$$[S] = \begin{pmatrix} \sigma_1 & \cdots & 0 \\ 0 & \ddots & \vdots \\ \vdots & \cdots & \sigma_{n_a} \\ 0 & \cdots & 0 \end{pmatrix} \tag{20}$$

Where,

σ_{n_a} = number of actuator.

4.7 Controllability index for sensors location:-

The controllability index for sensors is considered similar to that of actuator for the work done in this thesis. Hence the controllability index for sensor can be given as:-

$$[S] = \begin{pmatrix} \sigma_1 & \cdots & 0 \\ 0 & \ddots & \vdots \\ \vdots & \cdots & \sigma_{n_s} \\ 0 & \cdots & 0 \end{pmatrix} \tag{21}$$

Where,

σ_{n_s} = number of sensors.

Wang and Wang [19] concluded that the controllability index for actuator is given by :-

$$\Omega = \prod_{i=1}^{n_a} \sigma_i \tag{22}$$

From the above formulation controllability index for sensors can also be written as :-

$$\Omega = \prod_{i=1}^{n_s} \sigma_i \tag{23}$$

Hence the electric potential needed to control the system will be small when the controllability index is high. Therefore by maximizing the above controllability index we can get the fitness.

4.8 Multi-objective formulation for both sensors and actuator

By using multi-objective formulae we can find the optimal location for a collocated system, i.e for sensors and actuator working together. Multi objective equation for sensors and actuators together is given by the following equation:

$$f = w_1\Omega_a + w_2\Omega_s \tag{24}$$

$$w_1 + w_2 = 1 \tag{25}$$

Where , subscript a and s denotes actuator and sensors respectively. w_1 and w_2 are the weighing functions. Total fitness (controllability index) for multi-objective system is given by f .

5. RESULTS AND DISCUSSION

Based on the above formulation, that is for controllability index for sensors and the controllability index for a multi-objective system, following results were obtained.

5.1 Optimal position of sensors

In order to find out the optimal placement of sensors, a code was written by using GA optimization method. To find out the validation of the code written, a smart cantilever beam was considered. The beam is fixed at one end and it is divided into 20 equal parts, where the sensor can be placed. It is made up of 6 layers, out of which 2 layer, that is the upper and the lower are piezoelectric material (sensor) and the remaining layers are composite layer. Thickness of piezoelectric layer is considered to be 0.5 mm. Thickness of each composite layer is 2.5mm, hence the total thickness of composite layer is 2.5×4mm which is equal to 10 mm. the length of cantilever beam is considered to be 0.2m and the width of the beam is 0.01m. Number of sensor to be placed is four and only first mode of vibration is taken into consideration. The material property of all the layer is shown in Table 1 and Table 2.

Table 3 Material properties

Layer	Thickness of layer	μ_{12}	μ_{21}	u_1	u_2
Piezoelectric	5×10^{-4}	.28	.28	63×10^{9}	24.6×10^{9}
FRP	2.5×10^{-4}	.25	.01	172.5×10^{9}	6.9×10^{9}

Table 4 Material properties

Layer	G_{12}	G_{23}	permittivity	Density	Pyroelectric constant
Piezoelectric	24×10^{9}	24×10^{9}	1.55×10^{-9}	7600	-2.5×10^{-4}
FRP	3.45×10^{9}	1.38×10^{9}	3.45×10^{9}	1600	o

Table 5 Orientation Of Fiber

Layer	1	2	3	4	5	6
Angle	0	0	90	90	0	0

5.2 Results for sensor location:-

After providing various inputs material properties, eigen vector, piezoelectric material properties the program is run. The results of the program shows that the after 120 generation fitness converges. The optimal position of sensors is found out to be position 1, 2, 3, 4 that is the portion coloured with pink (as shown in Fig 5). This shows that the location of sensors is near the supported end, that is at the base of the cantilever beam. Fitness versus generation(convergence) plot is shown in figure 6.

The result that is obtained is justifyable as strain developed is maximum at the base for first order vibration and also curvature is maximum at the base. Since strain is directly proportional to curvature, the sensors will have sufficient strain at this points to generate the voltage that is needed to actuate the actuators.

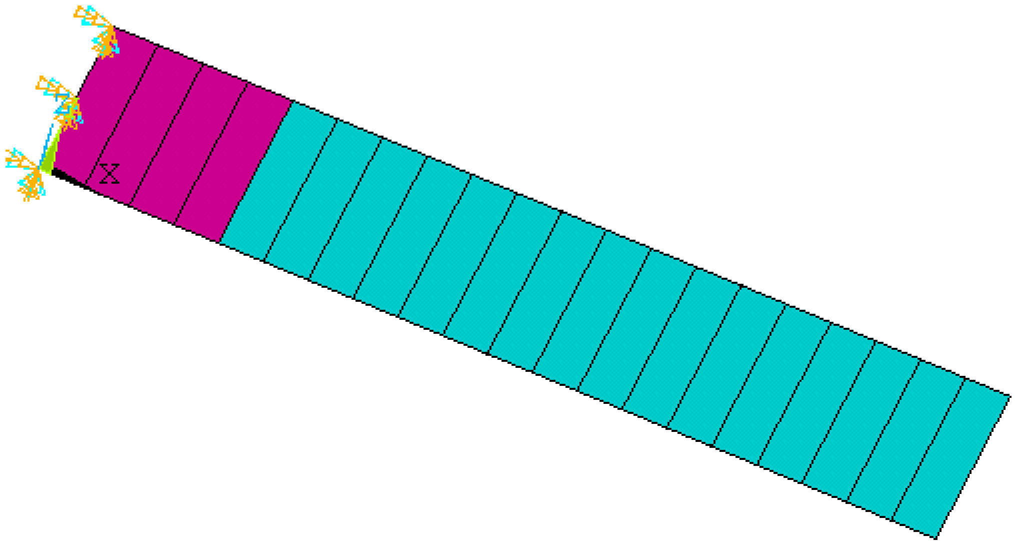

Fig 5: Location of sensors shown in cantilever beam.

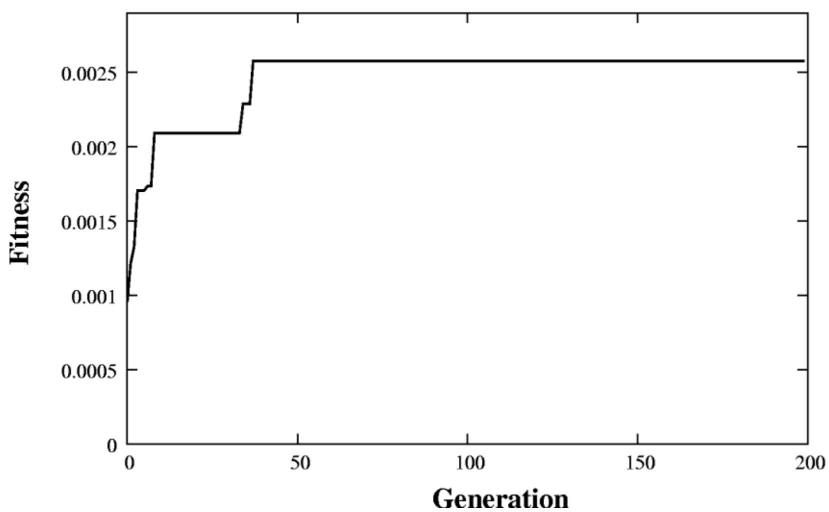

Fig 6:Fitness versus generation plot for non-collocated sensor location

5.3 Optimal position using multi-objective formulation for collocated system

Multi-objective formulation discussed above is used to find out the optimal location for both sensors and actuators working togather. The smart cantilever beam that was used for the non-collocated problem, that is for only the sensor placement, is used here. All the dimentions, material properties, number of layer of piezoelectric materia and number of layers of FRP, thickness remains same. The number of sensors to be placed is four and number of actuator to be placed is also four. The weighing function is taken to be equal. The value of w_1 and w_2 is taken to be 0.5 each.

5.4 Results for the collocated system:

After providing all the values like the material properties, weighing function and eigine vector the program written for multi-objective collocated system was made to run. It was seen that the collocated position for actuator and sensor is position 1, 2, 3, 4 as shown by the pink shade in the Fig 6. This means that the sensors are to be placed at the top surface of smart cantilever beam at position 1,2,3,4, and the actuators are placed at the bottom surface of the smart cantilever beam, that is just below the sensors. Fitness versus generation plot is shown in Fig 7.

The results that is obtained is justifiable as it is known that for a beam the maximum strain developed is at the base and also the maximum curvature is at the base. Therefore the voltage developed by the sensors will be more, if it is placed at base, which will result to the maximum actuation for the actuator. From the above observation, it is seen that the optimization of the collocated system by considering the multi-objective formulae is correct.

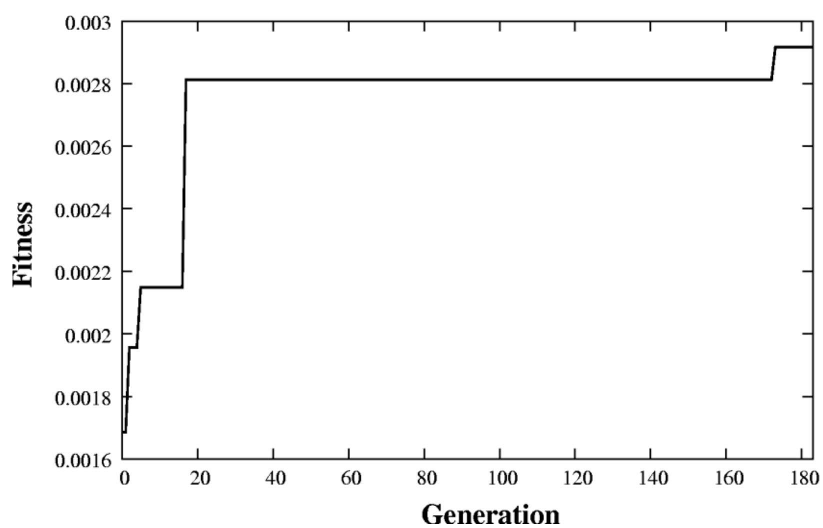

Fig 7: Fitness versus Generation plot for collocated system

6. CONCLUSION AND FUTURE SCOPE

The controllability index and observability index obtained from multi-objective formulae for collocated system shows that, it is same as the controllability index and ibservability index that is obtained for non-collocated system. Therefore the optimal placement of sensors and actuators for non-collocated system is same as that of multi-objective collocated system.

Hence, it is not necessary to find the sensor and actuator location seperately by running two separate code for sensor and actuator placement, instead the same result can be obtained by a running the multi-objective collocated system code only once. This results in the less time consumption and also reduces complicacy.

Non-collocated multi-objective optimization can be done in future.

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