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## BACHELOR OF TECHNOLOGY IN ELECTRICAL ENGINEERING

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## CERTIFICATE

This is to certify that the thesis entitled "CONTROL AND DESIGNING OF THE DC MOTOR DRIVE USING P-I CONTROLLERS WITH THE HELP OF MATLAB SIMULATION" submitted by Praveen Kumar Singh and Kapil Arya in partial fulfilment of the requirements for the award of Bachelor of Technology Degree in Electrical Engineering at National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/ Institute for the award of any degree or diploma.

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#### Abstract

The paper describes the designing of a closed loop model of the dc motor drive for controlling speed. Accuracy and the dynamic responses are better in a closed loop system. In the closed loop system, acceleration and the deceleration of the motor can be controlled according to the requirement. In order to regulate drives automatically, the controlled variables are measured. A tachogenerator is used for this purpose. The output of the tachogenerators is compared with a preset reference voltage. The differences between these two signals are fed as an actuating signal to control the elements of the system. Control amplifiers are used as both comparators and amplifiers. As a small change in the voltage can cause a large change in the motor current and lead to a particular drive control feature. There are two closed paths in the dc control scheme. One is the outer speed control loop and the other is the inner current control loop. Both the loops are controlled with the help of P-I controllers having their own gains and the time constants according to the requirement of the system. The output of the speed P-I controller which is the function of the speed error is used to command for current P-I controller and compared with armature current. The current P-I controller output control the triggering circuit of the converter. The armature current tends to increase to a large value when the motor is accelerating. A current limit can be obtained in this system by limiting the maximum current command voltage. If the motor speed drops below the set point, the resulting error signal causes the gate pulse generator to advance the firing pulses, and increase the rectifier output voltage. Consequently the speed rises to the desired level.


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Chapter 1
Basics of DC motor.

## DIRECT CURRENT MOTOR

When the armature of a dc motor rotates under the influence of the driving torque ,the armature conductors move through the magnetic field and hence the emf is induced in them as in generator. The induced emf acts in opposite direction to applied voltage V (lenz's law) and is known as back emf or counter emf Eb .The back emf is always less than the applied voltage.

$$
E_{b}=P \phi Z N / 60 \mathrm{~A}
$$

Where $\mathrm{P}=$ number of poles.
$\phi=$ flux per pole in weber.
$Z=$ total numbers of armature conductors.
$N=$ speed of armature in rpm.
A = number of parallel paths $=2$--------------------(for wave winding)
=P---------------------(for lap winding)


Fig 1.1
Net voltage across the armature circuit $=\mathrm{V}-\mathrm{Eb}$
If $R a$ is the armature circuit resistance then, $l_{a}=(V-E b) / R a$
If the speed of the motor is high , then the back emf $\mathrm{Eb}_{\mathrm{b}}$ is large and hence the motor will draw less armature current and vice versa.

## ARMATURE TORQUE OF DC MOTOR

Let in DC motor,

$r=$ average radius of armature in motor
I = effective length of each conductor in meter.
$\mathrm{Z}=$ total number of armature conductors.
$A=$ number of parallel paths.
$i=c u r r e n t$ in each conductor $=I_{a} / A$.
I=effective length of each conductor in meter.
$B=$ average flux density in $w b / m^{\wedge} 2$.
$\phi=$ flux per pole in wb.
$\mathrm{P}=$ number of poles.

$$
\mathrm{T}_{\mathrm{a}}=(\mathrm{ZP} / 2 \pi \mathrm{~A}) \phi \mathrm{l}_{\mathrm{a}} \quad \mathrm{~N}-\mathrm{m}
$$

$$
\mathrm{T}_{\mathrm{a}}=\mathrm{K}_{\mathrm{a}} \phi \mathrm{l}_{\mathrm{a}} \quad \mathrm{~N}-\mathrm{m}
$$

Where

$$
K a=(Z P / 2 \Pi A)=\text { armature constant }
$$

so ,as $K a$ is constant the total armature torque is directly proportional to field flux and armature current.

## MOTORING MODE OF DC MOTOR



Fig 1.2
When the armature current la flows in opposition to the generated back emf Eb , dc motor operate in the motoring mode.

$$
V_{t}=E b+l_{a} R a
$$

GENERATING MODE OF DC MOTOR


Fig 1.3
When the armature current $l a$ is in direction of generated emf $E b$, the dc motor operates in the generating mode.

$$
V_{t}=E b-l_{a} R_{a}
$$

## Chapter 2

Four quadrant operation of rectifier bridge converter

## SINGLE-PHASE FULL WAVE BRIDGE CONVERTER



Fig 2.1
ASSUMPTIONS: 1. source voltage or the input voltage is sinusoidal.
2. Load current is constant and ripples free.

A single phase full converter bridge contains four SCR'S T1, T2, T3 and T4. Thyristor pair T1, T2 is simultaneously triggered and $\pi$ radians later, pair T3 and T4 is gated together. When $a$ is positive with respect to $b$, supply voltage waveform is shown as $V a b$. When $b$ is positive with respect to a, supply voltage waveform is shown dotted as Vba. Obviously, $\mathrm{Vab}=-\mathrm{V}$ ba. The current directions and voltage polarities are shown in fig.5. Load current or output current io is assumed continuous over the working range; this means that the load is always connected to the ac voltage source through the thyristors. Between $\omega \mathrm{t}=0$ and $\omega \mathrm{t}=\alpha ; \mathrm{T} 1, \mathrm{~T} 2$ are forward biased through already conducting SCR'S T3 and T4 and block the forward voltage. Until the SCR'S T1 and T2 are triggered, T3 and T4 continue to conduct even in the positive half. When the forward biased SCR'S T1 and T2 are triggered at $\omega \mathrm{t}=\alpha$, they get turned on. As a result, the supply voltage $\mathrm{Vm} \sin \alpha$ immediately appear across thyristors T3 and T4 as a reverse voltage and they get turned off by natural, or line commutation. So, when the SCR'S T1 and T2 are gated or triggered the potential at point a appear across the common anode point and the SCR T3 get turned off as a reverse voltage appear across it. The same thing happens in the lower half at the common cathode point. As T2 is triggered the potential at point b appear across the common cathode point and the thyristor T 4 gets turned off as a reverse voltage appear across it.


Figure 2.2 (for $\alpha<90$ )
Thyristors T1 and T2 conducts from $\omega \mathrm{t}=\alpha$ to $\omega \mathrm{t}=\pi+\alpha$. In other words, T1 and T2 conduct for $\pi$ radians. As in the given figure, prior to the triggering of the SCR'S T1, T2 SCR'S T3 and T4 were conducting. So till $\alpha$ a negative voltage appear across the load. At $\omega t=\alpha$ the SCR'S T1 and T2 are triggered and voltage across the load shoots to instantaneous value of positive half at $\omega \mathrm{t}=\alpha$. The SCR'S T3 and T4 are triggered at $\omega \mathrm{t}=\pi+\alpha$. So, after $\omega \mathrm{t}=\pi, \mathrm{T} 1$ and T2 continue conducting even in the negative half till the other two thyristors are triggered. Likewise, waveform of the current iт1 through T1 (or it2 through T2) is shown to flow $\pi$ radians in fig 6.NOTE: we have assumed the load current to be continuous and constant. So the load current it is shown by a straight line. Because of this assumption, the source current changes to a square wave. So just prior to $\alpha$ the output voltage was $\mathrm{V}_{\text {ba }}$. The moment we trigger T1, potential of a is same as the positive dc bias, that is the cathode potential of T3 and a negative voltage appears across T3, it turns off. Hence the name line commutation.

## EQUIVALENT CIRCUITS FOR TRIGGERING ANGLE LESS THAN 90



Fig $2.3(\alpha$ to $\pi+\alpha)$
(a) From $\omega t=\alpha$ to $\omega t=\pi+\alpha$.

T1 and T2 get triggered and conduct for $\pi$ radians.
$\mathrm{V}_{\mathrm{o}}=\mathrm{V}_{\mathrm{s}}=\mathrm{V}_{\mathrm{m}} \sin \omega \mathrm{t}$.
$I_{s}=I L$

At $\omega t=\pi+\alpha$ T3 and T4 are triggered. As T3 and T4 are triggered a negative voltage appears across T1 and T2 and they stop conducting or in other words they get turned off.


$$
(\pi+\alpha \text { to } 2 \pi+\alpha)
$$

(b)From $\omega t=\pi+\alpha$ to $\omega t=2 \pi+\alpha$.

At $\pi+\alpha$ T3 and T4 starts conducting and conducts for $\pi$ radians. At $2 \pi+\alpha$, T1 and T2 are again triggered and the whole cycle repeats. So, all the four thyristors conduct for $\pi$ radians.

As there are two pulses per cycle it is also known as two pulse converter.

## AVERAGE VALUE OF OUTPUT VOLTAGE

$$
\begin{gather*}
\pi+\alpha \\
V_{0}=1 / \pi \int V_{m} \sin \omega t d(\omega t)=V_{m} / \pi(-\cos \alpha) \mid=\left(2 V_{m} / \pi\right) \cos \alpha \\
\alpha \quad \alpha \\
V_{0}=2 V_{m} / \pi \cos (\alpha)
\end{gather*}
$$

The rms value of the output voltage is found to be

$$
\text { Vor }=V_{m} / 2=V_{s}
$$

## TWO QUARANT OPERATIONS.

$\Rightarrow V o=+v e \quad$ FOR $0<\alpha<\pi / 2$

$$
=-v e \quad F O R \pi / 2<\alpha<\pi
$$


figure 2.4
$\Rightarrow$ Note that the expression for average value of output voltage is valid only if the load current is continuous. If the load current is discontinuous then this expression cannot be used to find the average value of the output voltage. For $\alpha$ varying from 0 to $\pi / 2$ the average value of output voltage is positive and for $\alpha$ varying from $\pi / 2$ to $\pi$, the average value of output voltage is negative. The load current is unidirectional.


## CASE 2 : WHEN THE TRIGGERING ANGLE IS GREATER THEN 90( $\alpha>90)$



Fig 2.5

Till we trigger T1 and T2, T3 and T4 are conducting. Output voltage is Vba which is negative in the positive half so we get a negative voltage till $\alpha$. At $\alpha$ we have triggered T1 and T2, now output voltage jumps to Vab and it continues till $\pi+\alpha$ until you have triggered T3 and T4. At $\pi+\alpha$ when we trigger T3 and T4 the output voltage jumps to Vba
which is positive in the negative half and it continues till T 1 and T 2 are triggered at $2 \pi+\alpha$. Hence in this way the cycle continues. We can see from figure 11 that the average value of the output voltage across the load is negative that is the area under the curve from 0 to $\alpha$ is much higher than area under the curve from $\alpha$ to $\pi$. The source current waveform is same as that of for $\alpha<90$. The only difference is that it just shifts a bit more to the right as the triggering angle has been increased. But note that the average value of the output voltage is negative only when the load current is continuous what we have assumed in the beginning.

## EQUIVALENT CIRCUITS FOR TRIGGERING ANGLE GREATER THAN 90



In $0<\omega \mathrm{t}<\pi$,
Fig 2.6
From $0<\omega t<\alpha$ :
Potential of point $b$ is greater than that of point $a$, so voltage across load is negative.
$\mathrm{V}_{\mathrm{o}}=\mathrm{V}_{\mathrm{ba}}(-\mathrm{VE})$.
At $\omega \mathrm{t}=\alpha$ (Immediately after T1 and T2 are triggered)
$=>\mathrm{T} 1$ and T 2 are ON and T3 and T4 are OFF.


The turn ON and turn OFF are assumed to be instantaneous.
$\mathrm{V}_{\mathrm{o}}=\mathrm{V}_{\mathrm{ab}}=+\mathrm{VE}$ till $\alpha<\omega \mathrm{t}<\pi$.
$V_{o}=\quad-V E$ from $\pi<\omega t<\pi+\alpha$.
As $\pi+\alpha>\pi-\alpha$, the average value of the output voltage is negative when the triggering angle of the SCR'S are greater than 90 degrees.

## Chapter 3

Expression for power factor for a fully controlled rectifier bridge

## EXPRESSION FOR POWER FACTOR FOR A FULLY CONTROLLED BRIDGE

Source current is a square wave. The magnitude of the source current is lofrom $\alpha$ to $\pi+\alpha$ and it is $-l_{0}$ from $\pi+\alpha$ to $2 \pi+\alpha$. So if we write the Fourier series,

$$
\begin{aligned}
& X(\omega t)=l_{s}
\end{aligned}=l_{0} \quad \alpha<\omega t<\pi+\alpha,
$$



Fig 3.1

$$
x(\omega t)=a_{0}+\sum_{n=1}^{\infty}\left(a_{n} \cos \frac{n \pi \omega t}{L}+b_{n} \sin \frac{n \pi \omega t}{L}\right)
$$

## $2 \pi+\alpha$

Average component $=>a_{o}=1 / 2 \pi \int x(\omega t) d(\omega t)=0$
$\alpha$
as the given square function is an odd function $a_{n}=0$.
So all the cosine terms are zero.

$$
\begin{gathered}
\pi / 2 \\
b_{n}=4 / T \int x(\omega t) \sin (n \omega t) d(\omega t)=4 / 0 / 2 \pi[1-\cos (n \pi / 2)]
\end{gathered}
$$

0

Putting these values in the Fourier series, we get
$x(\omega t)=41 \circ / \pi(\sin \omega t+\sin 3 \omega t+\sin 5 \omega t+--------------)$
so the peak value of the fundamental component sinct is $4 I_{0} / \pi$ where $l_{0}$ is the magnitude of the square wave.

Rms value of the fundamental component sin $\omega \mathrm{t}$ :
Irms of $\mathrm{Is}_{\mathrm{s} 1}=(1 / \mathrm{V} 2) 4 \mathrm{I} \circ / \pi=2 \mathrm{~V} 21 \mathrm{o} / \pi$.
Rms value of the source current I :
Irms of $I_{s}=I$.
Displacement factor:
$\cos \theta_{1}=\cos (-\alpha)$ (lagging)
$\Rightarrow$ Power factor $=V_{s 1} I_{s 1} \cos (-\alpha) / V_{r m s} I_{r m s}=$ mean input power $/$ input volt amperes.
Only fundamental component of voltage and current wave is responsible for current transfer. Also we have assumed that input voltage is a sinusoid. So the rms value of the fundamental component of the input voltage is equal to Vrms itself. The rms value of the fundamental component of the load current is $2 \sqrt{ } 21 \circ / \pi$ and the rms value of the input source current is $\mathrm{lo} . \operatorname{Cos} \alpha$ is the displacement factor. That is the fundamental component of the input source current lags the source input voltage by an angle $\alpha$.

SO,

$$
\begin{gathered}
\text { Power factor }=\frac{\mathrm{Vrms} *(2 \mathrm{~V} 2 \mathrm{I} / / \pi) * \cos (\alpha) .}{\mathrm{V}_{\mathrm{rms}} * \mathrm{I}_{0}} \\
\text { P.F }=(2 \mathrm{~V} 2 / \pi) \operatorname{Cos} \alpha
\end{gathered}
$$

Note that he the power factor obtained is a lagging one.

## Chapter 4

Firing angle control circuit

## FIRING ANGLE CONTROL CIRCUIT



Fig 4.1 COMPLETE LAYOUT OF A FIRING CONTROL CIRCUIT

The entire firing angles are with reference to our mains circuit. Mains is 230 V . So for control level we have to step down the mains voltage. So, we will use a step down center tap transformer for stepping down the voltage to 10 V . We have a control level of +10 or -10 . This we give to a zero crossing detector. We can design a positive or negative zero crossing detector with the help of the operational amplifiers. The output of a positive zero crossing detector is shown in figure. This output we give to a ramp generator. The output we will feed to a comparator circuit. Comparator voltage is controlled with the help of a control voltage Ec. The comparator output is given to a monoshot. In this way we can generate the desired gate pulse for the thyristor.


Fig 4.2 OUTPUTS OF THE FIRING ANGLE CONTROL COMPONENTS
Positive side of the sinusoidal reference voltage after stepping down is for triggering S1 and S3 whereas the opposite waveform is for triggering S2 and S4 as shown in the figure. Output of the positive zero crossing detector is shown. Same we can get for negative waveform. This output is fed to ramp circuit shown. The ramp output is compared with the control voltage Ec for firing angle variation. In the ramp generator the capacitor slowly charges. As per the figure at the zero crossing we must discharge the capacitor. So, for that purpose these zero crossing points we have to detect again with the help of the monoshot as shown in the figure. We connect a transistor across the capacitor and the monoshot pulses are given to transistor. Due to this at zero crossing points the capacitor gets discharged. In this a ramp we can generate.

$$
\mathrm{Eo}=\mathrm{Emax} \operatorname{COS} \alpha
$$

Here $\alpha$ is varying proportional to the tc (control voltage) but the output is varying proportional to cos $\alpha$. This is not a linear control. For linear control output as well as the firing angle must vary proportional to $\cos \alpha$.

## COSINE COMPARISION METHOD:

We want our control voltage $t$ vary proportional to $\cos \alpha$.
So,

$$
\mathrm{Ec}=\mathrm{e} \cos \alpha
$$

$$
\cos \alpha=\mathrm{Ec} / \mathrm{e}
$$

Our output voltage Eo $=E \max \cos \alpha$.

$$
\begin{aligned}
& \text { Eo }=\operatorname{Emax}(\mathrm{Ec} / \mathrm{e})=\mathrm{kEc} . \\
& \qquad \mathrm{k}=(\mathrm{Emax} / \mathrm{e})=\mathrm{constant.}
\end{aligned}
$$

Where,
So, the output will be proportional to Ec.


Fig 4.3 VARIOUS OUTPUTS FOR COSINE COMPARISION TECHNIQUE
Here the reference waveform is integrated and a cosine waveform is generated. Positive as well as negative Ec is generated and is compared with the cosine waveform with the help of the operational amplifiers based comparators. Pulses are generated at each positive going pulses and pulse obtained due to comparison of +Ec and cosine waveform is send to the thyristors T1 and T3. Pulses obtained due to comparison of the -Ec and the cosine waveform are send to the thyristors T 2 and T 4 . So the firing angle is proportional to $\cos \alpha$. We can see from the following output waveforms that when the firing angle is zero, the output is maximum that is $\cos \alpha=\cos 0=1$. So the output obtained is maximum. When $\alpha=90$, the output is coming to be equal to zero.

## Chapter 5

Model of a DC motor

## SPEED CONTROL OF THE DC MOTOR



Fig 5.1
The output speed is compared with the reference speed. The error signal is fed into the speed controller. This controller output will vary whenever there is a mismatch in the reference speed and the speed feedback. Here we are controlling the dc motor. We are controlling the speed by controlling the armature voltage. The output of the controller is the control voltage Ec that controls the firing angle of the phase controlled converter. This converter output give the required Va required to bring motor back to the rated speed. Motor will have the tacho generator. Tacho voltage will not be perfectly dc and it will have ripple. So, we require to filter it out. So we need a filter. Also a gain is required to bring tacho output back to the controller level.

## MODEL OF A DC MOTOR



$$
V a=l a R a+L a(d l a / d t)+E b
$$

$\mathrm{Va}=$ armature voltage (volts)
$\mathrm{Eb}=$ motor back emf (volts)
$\mathrm{l} a=$ armature current (ampere)

Ra = armature resistance (ohms)
La = armature inductance (Henry)

$$
\begin{equation*}
\mathrm{Md}=\mathrm{Jd} \omega / \mathrm{dt}+\mathrm{ML} \tag{1}
\end{equation*}
$$

$M L=$ load torque $(N m) ; \quad J=$ moment of inertia $\left(\mathrm{kg} / \mathrm{m}^{2}\right) ; \omega=$ angular velocity ( $\mathrm{rad} / \mathrm{sec}$ )
now, $E b=K \Phi \omega$
also, $\quad \mathrm{Md}=\mathrm{K} \Phi \mathrm{la}^{2}$
also, $\quad \mathrm{la}(\mathrm{S})=(\mathrm{Va}-\mathrm{Eb}) /(\mathrm{Ra}+\mathrm{LaS})$
$\Rightarrow \quad l a(s)=(V a-K \Phi \omega)$
=> $\quad \mathrm{Ra}(1+\mathrm{LaS} / \mathrm{Ra})$

Also,

$$
\omega(\mathrm{s})=\underline{\mathrm{Md}-\mathrm{ML}=\underline{(\mathrm{K} \Phi \mathrm{la}-\mathrm{ML})}) .}
$$

$$
\mathrm{La} / \mathrm{Ra}=\mathrm{Ta} \text { (Armature time constant) }
$$



$$
\text { Now, } \quad \frac{\omega(\mathrm{s})}{\mathrm{Va}(\mathrm{~s})}=\frac{[\mathrm{k} \Phi / \mathrm{Ra}] / \mathrm{Js}(1+\mathrm{TaS})}{1+\left(\mathrm{k}^{2} \Phi^{2} / \mathrm{Ra}\right) / \mathrm{JS}(1+\mathrm{TaS})}
$$

On simplifying the above transfer function we get,

$$
\begin{equation*}
\underline{\omega(s)}= \tag{4}
\end{equation*}
$$

$\qquad$
$\qquad$
Now,

$$
\mathrm{Va}(\mathrm{~s}) \quad 1+\left(\mathrm{k}^{2} \Phi^{2} / \mathrm{Ra}\right) / \mathrm{JS}(1+\mathrm{TaS})
$$

$$
\begin{equation*}
\underline{\omega(s)}=\frac{(1 / k \Phi)--------------------------------1}{} \tag{5}
\end{equation*}
$$

## Now,

$$
\mathrm{Va}(\mathrm{~s}) \quad \mathrm{STm}(1+\mathrm{STa})+1
$$

$$
\mathrm{Tm}=\mathrm{JRa} /(\mathrm{k} \Phi)^{2}=\text { electromechanical time constant }
$$

Now we have to introduce Electromechanical Time Constant in transfer function Let us assume load torque ML = 0 and applying full voltage Va

```
Now
                    Va=KФ\omega(t)+laRa
(Assuming armature inductance = 0)
=>
Md= Jd\omega/dt = kФla
\[
V a=K \Phi \omega(t)+(J d \omega / d t) R a
\]
\[
\Rightarrow \quad \text { KФ }
\]
\[
\begin{equation*}
\Rightarrow \quad \frac{\mathrm{Va}}{\mathrm{~K} \Phi}=\omega(\mathrm{t})+\frac{\mathrm{JRa}(\mathrm{~d} \omega / \mathrm{dt})}{(\mathrm{K} \Phi)^{2}} \tag{7}
\end{equation*}
\]

At no load, when the machine reaches full speed
\(\omega\) (no load) \(=\mathrm{Va} / \mathrm{K} \Phi\)
putting this value in eq. (7)
```

    \omega(no load) = \omega(t)+\underline{JRa (d\omega/dt)}=\omega(\textrm{t})+\textrm{Tm}(\textrm{d}\omega/\textrm{dt})
    =>
(KФ) }\mp@subsup{}{}{2
KФ = Km
Tm}=\textrm{JRa}/(K\Phi) 2 = JRa/ (Km) 2',
=> J=Tm (Km) 2/ Ra

=> Js Js

```
putting the value of \(J\) in eq. (9)
\[
\omega(s)=\left[(\operatorname{Ra} / K m) \operatorname{la}(s)-M L R a /(K m)^{2}\right](1 / T m(s))
\]


Fig 5.2

Further simplifying eq. (5) we get
\[
\begin{equation*}
\Rightarrow \quad \frac{\omega(s)}{\mathrm{Va}(\mathrm{~s})}=\frac{(1 / \mathrm{Km})----------------------------(10)}{\left(1+S T m+S^{2} \mathrm{TaTm}\right)} \tag{10}
\end{equation*}
\]

But, the armature time constant Ta is much less than the electromechanical time constant Tm,

Ta \(\ll\) Tm

Now, \(1+S T m+S^{2} T a T m \approx 1+S(T a+T m)+S^{2} T a T m=(1+S T m)(1+S T a)\)
Whole this exercise is to represent the transfer function in the time constant form. If we get the transfer function in time constant form, we know the largest time constant that will delay the system. So if we use the PI controller, zero of the PI controller can be chosen so that this large delay can be cancelled. Delay means a lag and zero means a lead, so this will try to compensate the whole system.
\[
\begin{equation*}
=\quad \frac{\omega(s)}{\mathrm{Va}(\mathrm{~s})}=\frac{(1 / \mathrm{Km})---\cdots-----}{(1+\mathrm{STm})(1+\mathrm{STa})} \tag{11}
\end{equation*}
\]

This Tm and Ta determine the response of the system. These are the time constants of the system.


PROBLEM:

Let the machine is at standstill. It has not started. Suppose the operator give a high speed command. Let us say motor has to go to full speed. Because of electromechanical time constant it will take its own time to speed up. But the controller is very fast. Speed feedback is zero initially. So this will result in full controller output Ec. Converter gives the maximum voltage. But during starting back emf Eb is zero. So very large current flows which sometime exceed the motor rated current limit and it can damage the machine.

So, here the problem is current is not controlled. We are only looking at the speed and the voltage. For a machine there is not only the voltage rating but the current rating is also there. So any speed control must have a current control. So, before going to the machine one current control loop is there. That is, the applied voltage Va is now not dependent on the speed error but is on the current error. We must ensure that Va is applied in such a way that machine during positive and negative torque should not draw more than the rated current. So, an inner current loop is required.


Fig 5.3 COMPLETE LAYOUT OF THE DC DRIVE MECHANISM FOR CONTROLLING SPEED

\section*{Chapter 6}

Representation of converter

\section*{REPRESENTATION OF THE CONVERTER:}

Let the converter is a 3 phase fully controlled converter for a typical firing angle \(\alpha=60\).


Fig 6.1 RIPPLES WITH 60 DEGREE DURATION FOR 50 HZ WAVEFORM
Ripple will be six times the fundamental frequency. So the duration of each ripple will be 60 degree.

For 50 Hz supply
for 360 degrees, Time period \(T=(1 / 50)=0.02 \mathrm{~s}\).
For 60 degree, \(t=(0.02 * 60) / 360=3.3 \mathrm{~ms}\).
A change in converter firing angle occurs after every 60 degree. It's not instantaneous. That means a delay of 3.3 ms . It can have a maximum delay of 3.3 ms or a minimum of zero.
So, let us take an average \(\mathrm{Tr}=(3.3 \mathrm{~ms}+0) / 2=1.7 \mathrm{~ms}\).


So, converter can be represented as a first order delay with a gain. Suppose a +10v reference is coming from current controller. So if we are using a cosine firing with linear control, maximum output voltage should be obtained.
For a 3 phase converter, output voltage \(=2\) * 1.17 * Erms * \(\cos \alpha\)
Maximum voltage obtained when \(\alpha=0\).
Max. Voltage \(=2\) * 1.17 * Erms.
Control voltage is 10 V .
=> So, gain is:
\[
\mathrm{Kt}=(2 * 1.17 * \text { Erms }) / 10
\]

So, converter can be represented as first order lag with some gain Kt.
\[
\mathrm{Kt} /(1+\mathrm{STr})
\]


Fig 6.2 COMPLETE LAYOUT OF THE DC DRIVE SYSTEM

\section*{Chapter 7}

Designing of current controller

\section*{DESIGNING OF THE CURRENT CONTROLLER}

We will design the controller for the worst case. That is during starting when \(\mathrm{Eb}=0\) and a large current flows through the machine.


Fig 7.1
Transfer function:
\[
\begin{array}{ll} 
& \frac{\mathrm{la}(\mathrm{~S})(\mathrm{f})}{\mathrm{la}(\mathrm{~S})(\mathrm{ref})}  \tag{12}\\
\Rightarrow & \frac{[\mathrm{Kc}(1+\mathrm{TcS}) / \mathrm{TcS}](\mathrm{Kt} / 1+\mathrm{STt})[(1 / \mathrm{Ra}) /(1+\mathrm{STa})-\cdots-\cdots-----(12}{1+[\mathrm{Kc}(1+\mathrm{TcS}) / \mathrm{TcS}](\mathrm{Kt} / 1+\mathrm{STt})[(1 / \mathrm{Ra}) /(1+\mathrm{STa})(\mathrm{K} 2 / 1+\mathrm{ST} 2)}
\end{array}
\]

Tc is in our control. We choose Tc such that it can cancel one of the largest time constant in the transfer function. So, the order of the system will be reduced. Now, the response will be much faster. Converter delay Tr must be much less than armature time constant Ta.
So, let us make
\[
\mathrm{Tc}=\mathrm{Ta}
\]

Putting this value in equation (12)
\begin{tabular}{|c|c|}
\hline \(\underline{\mathrm{la}}(\mathrm{S})(\mathrm{f})\) & Kc (Kt/TaRa) (1+T2S) --------- \\
\hline \(\mathrm{la}(\mathrm{S})(\mathrm{ref})\) & \(\mathrm{S}(1+\mathrm{TtS})(1+\mathrm{T} 2 \mathrm{~S})+(\mathrm{KcKtK} 2) / \mathrm{TaRa}\) \\
\hline let \(\mathrm{Ko}=(\mathrm{K}\) & \\
\hline
\end{tabular}
\(\underline{l a(S)(f)}=\)
Ko (1+T2S) ----------------------
\(\mathrm{la}(\mathrm{S})\) (ref) \(\quad \mathrm{S}^{3} \mathrm{ttT2}+\mathrm{s}^{2}(\mathrm{Tt}+\mathrm{T} 2)+\mathrm{S}+\) KoK2

Tt and T2 are the converter lag and filter lag respectively. These are the smaller time constants. So, their product can be approximated to zero.
\(\mathrm{TtT} 2 \approx 0, \mathrm{Tt}+\mathrm{T} 2=\sigma\).
\(\underline{\mathrm{l}(\mathrm{S})(\mathrm{f})}=\)
\(\mathrm{Ko}(1+\mathrm{T} 2 \mathrm{~S})\)
\(\mathrm{la}(\mathrm{S})(\mathrm{ref})\)
\[
\begin{equation*}
S^{2} \sigma+S+\text { KoK } 2 \tag{15}
\end{equation*}
\]
\(\mathrm{la}(\mathrm{S})(\mathrm{f}) / \mathrm{la}(\mathrm{s})(\mathrm{ref})=(\mathrm{Ko} / \sigma)(1+\mathrm{T} 2 \mathrm{~S}) /\left[\mathrm{S}^{2}+(\mathrm{S} / \sigma)+(\mathrm{KoK} 2 / \sigma)\right]\)
\(S^{2}+(S / \sigma)+(K\) KK2/ \(\sigma) \approx S^{2}+2 \zeta \omega+\omega^{2}\)
\(\Rightarrow \omega=\vee(K o K 2) / \sigma ; \zeta=1 /(2 \sigma \omega)=1 /(2 \mathrm{~V}(\sigma K 2 K o)\)
this is a second order system. So, to get a proper response \(\zeta=0.707\)
\[
\begin{gathered}
1 / V(2)=1 / 2 \mathrm{~V}(\sigma K 2 \mathrm{Ko})=>\mathrm{Ko}=1 /(2 \mathrm{~K} 2 \sigma)=\mathrm{KcKt} /(\mathrm{RaTa}) \\
\mathrm{Kc}=(\mathrm{RaTa}) /(2 \mathrm{KtK} 2 \sigma)
\end{gathered}
\]
\(K o=K c K t /(R a T a)=1 / 2 K 2(T t+T 2)\)
\(\underline{\mathrm{la}(S)(f)}=\quad-\quad(1 / K 2)(1+T 2 S)----\cdots---------(16)\)
\(\mathrm{la}_{( }(\mathrm{S})\) (ref) \(\quad 2 \mathrm{~S}^{2} \sigma^{2}+2 \sigma \mathrm{~S}+1\)
thus on using a filter a zero is introduced in the transfer function. This zero may result in an overshoot. So we will use a filter with same time lag between the la (ref) and controller to cancel the effect of the current filter. Current loop time constant is much higher than the filter time constant. So a slight delay at la (ref) will not affect much. So we put a filter here.
\(\underline{\mathrm{la}(\mathrm{S})(\mathrm{f})}=\)
\[
\begin{equation*}
\frac{(1 / \mathrm{K} 2)(1+\mathrm{T} 2 \mathrm{~S})-}{2 S^{2} \sigma^{2}+2 \sigma S+1} \tag{17}
\end{equation*}
\]
\(\mathrm{la}(\mathrm{S})(\mathrm{ref})(1+\mathrm{ST} 2)\)
\[
\operatorname{la}(S)(f) / I_{a}(S)(r e f)=(1 / K 2) /\left(2 S^{2} \sigma^{2}+2 \sigma S+1\right)
\]

\section*{Chapter 8}

Designing of speed controller

\section*{DESIGINING OF THE SPEED CONTROLLER}

POLE ZERO CANCELLATION


Fig 8.1
\[
\begin{align*}
& \underline{\omega(\mathrm{s})}=(\mathrm{Kn} / \mathrm{K} 2)(\mathrm{Ra} / \mathrm{KmTmTn})\left(1+\mathrm{TnS} /(1+2 \sigma \mathrm{~S}) \mathrm{S}^{2}\right)  \tag{18}\\
& \Rightarrow \quad \omega s(r e f .) \quad 1+(\mathrm{KnRa} / \mathrm{K} 2 \mathrm{KmTmTn})\left(1+\mathrm{TnS} /(1+2 \sigma S) \mathrm{S}^{2}\right)(\mathrm{K} 1 / 1+\mathrm{T} 1 \mathrm{~S})
\end{align*}
\]

We can control Tn. We set Tn such that it cancels one of the largest time constant of the transfer function. So,
\[
T n=2 \sigma
\]

Let us take a case
\[
\begin{array}{ll} 
& \frac{\omega(\mathrm{s})}{\omega \mathrm{s}(\mathrm{ref})}=\frac{(\mathrm{KnRa} / \mathrm{K} 2 \mathrm{KmTmTn})(1+\mathrm{T} 1 \mathrm{~S})}{\mathrm{K} 2 \mathrm{KmTmTnS} \mathrm{~K}^{2}(1+\mathrm{T} 1 \mathrm{~S})+\mathrm{KnRaK} 1} \\
\Rightarrow & \omega(\mathrm{~s})=1 / \mathrm{S}\left(\mathrm{~S}^{2}+\alpha \mathrm{s}\right)
\end{array}
\]
we are not getting the \(S\) term coefficients in the denominator. Hence the damping constant is zero. So the system become little oscillatory and unstable.

\section*{OPTIMIZATION TECHNIQUE}

The dynamic performance of a control system is good if the controlled variable rapidly reaches the reference limit. For any frequency variation (between the bandwidth) of the input variable, the output should track the input variable instantaneously. For any frequency of input, output follows input as closely as possible.


Optimization technique aims at bringing the input/output gain as close to 1 over a wide frequency range (modulus hugging).
```

=> G (j\omega) \approx1

```

\section*{GENERAL CONSIDERATION FOR OPTIMIZATION}

Simplifying eqt. (18) further we get,

Both T1 and \(\sigma\) are smaller time constants. So their product can be approximated to zero.
\[
1+S(2 \sigma+T 1)=1+\delta S . \quad \text { Where } \delta=(2 \sigma+T 1)
\]
\[
\Rightarrow \quad \frac{\omega(s)}{\omega s(r e f .)}=\frac{(\mathrm{KnRa} / \mathrm{K} 2 \mathrm{Km})(1+\mathrm{TnS})(1+\mathrm{T} 1 \mathrm{~S})-\cdots-\cdots-\cdots-\cdots-\cdots-\cdots-\cdots-\cdots-\cdots(20)}{S^{2} \operatorname{TmTn}(1+\delta S)+(\mathrm{KnRaK} 1 / \mathrm{K} 2 \mathrm{Km})(1+\mathrm{TnS})}
\]
\(\Rightarrow \quad \omega s\) (ref. \() \quad S^{3} T m T n \delta+S^{2} T m T n+(K o K 1 T n) S+K o K 1\)
The terms \((1+\operatorname{TnS})\) and ( \(1+\mathrm{T} 1 \mathrm{~S}\) ) in the denominator will be cancelled by using filters.
For a third order system:
\(F(j \omega)=b o+j \omega b 1 /\left[a o+j \omega a 1+(j \omega)^{2} a 2+(j \omega)^{3} a 3\right]\)
for low frequency \(\mathrm{bo}=\mathrm{ao}\) and \(\mathrm{b} 1=\mathrm{a} 1\)
\(|F(j \omega)|=\left(a o^{2}+\omega a 1^{2} / a o^{2}+\omega^{2}\left(a 1^{2}-2 a o a 2\right)+\omega^{4}\left(a 2^{2}-2 a 1 a 3\right)+\omega^{6}\left(a 3^{2}\right)\right)^{1 / 2}\)
For making this mod equivalent to 1 ; coefficients of \(\omega^{2}\) and \(\omega^{4}\) are made equal to zero
\[
\begin{aligned}
& \mathrm{ar}^{2}=2 \mathrm{a} o \mathrm{a} 2 \\
& \mathrm{a} 2^{2}=2 \mathrm{a} 1 \mathrm{a} 3
\end{aligned}
\]

Using filters at the \(\omega\) (ref) side:
\(\Rightarrow \quad \omega \mathrm{s}(\) ref. \() \quad \mathrm{S}^{2} \operatorname{TmTn}(1+2 \sigma S)(1+\mathrm{T} 1 \mathrm{~S})+(\mathrm{KnRaK} 1 / K 2 K m)(1+\operatorname{TnS})\)
But, \((1+2 \sigma S)(1+T 1 S)=1+\mathrm{T} 1 S+2 \sigma S+2 \sigma T 1 S^{2} \approx 1+S(2 \sigma+\mathrm{T} 1)+2 \sigma T 1 S^{2} \approx 1+\mathrm{S}(2 \sigma+\mathrm{T} 1)\)
\(\omega(\mathrm{s})\) \(\qquad\) \(=\)
\((\mathrm{KnRa} / \mathrm{K} 2 \mathrm{Km})(1+\mathrm{TnS})(1+\mathrm{T} 1 \mathrm{~S})\)
\(\omega(s)(r e f))(1 / 1+T n S)(1 / 1+T 1 S)\)
\(S^{3} \operatorname{TmTn} \delta+S^{2} T m T n+(K o K 1 T n) S+K o K 1\)
```

K0-----------------------
$\Rightarrow \quad \omega(s)(r e f)) \quad S^{3} T m T n \delta+S^{2} T m T n+(K o K 1 T n) S+$ KoK1
where $\mathrm{Ko}=(\mathrm{KnRa} / \mathrm{K} 2 \mathrm{Km})$
$(\text { KoK1Tn })^{2}=2 *$ KoK1 $*$ TmTn

$$
\mathrm{Tn}=4 \delta=(2 \sigma+\mathrm{T} 1)=4(2 \mathrm{Tt}+2 \mathrm{~T} 2+\mathrm{T} 1)
$$

$(\operatorname{TmTn})^{2}=2 *$ KoK1Tn $* \operatorname{TmTn} \delta$
$\Rightarrow \mathrm{Tm}=2{ }^{*} \mathrm{KoK} 1 * \delta=2$ * $\left.\mathrm{KnRa} / \mathrm{K} 2 \mathrm{Km}\right)^{*} \mathrm{~K} 1^{*} \delta$

$$
\mathrm{Kn}=\mathrm{TmKmK2} /(2 \mathrm{~K} 1 \mathrm{Ra} \delta)
$$

```

Putting the values of kn and km in the main transfer function, we get
\[
\omega(s)(f) / \omega(s)(r e f)=1 /\left(1+4 s \sigma+8 s^{2} \sigma^{2}+8 s^{3} \sigma^{3}\right.
\]

\section*{Chapter 9}

Designing of speed and current controller for a DC drive with the given specifications

\section*{DESIGNING OF SPEED AND CURRENT CONTROLLER FOR A DC DRIVE WITH THE FOLLOWING SPECIFICATION}
1. RATED POWER = 300 KW
2. MAXIMUM RATED VOLTAGE \(=460 \mathrm{~V}\) D.C.
3. MAXIMUM RATED CURRENT = 690 Amps.
4. BASE SPEED \(=500 \mathrm{RPM}=52.3 \mathrm{rad} / \mathrm{s}\)
5. MOMENT OF INERTIA (J) \(=84 \mathrm{Kg}-\mathrm{m} 2\)
6. BACK EMF CONSTANT \((\mathrm{Km})=8.5\) Volt-sec. \(/\) Rad.
7. ARMATURE RESISTANCE \((\mathrm{Ra})=0.02342\) Ohms.
8. ARMATURE INDUCTANCE (La) \(=0.7026 \mathrm{mH}\).
9. CONVERTER = 3-Ph Fully Controlled bridge converter
10. SPEED FEEDDBACK FILTER TIME CONSTANT (T1) \(=25 \mathrm{msec}\).
11. CURRENT FILTER TIME CONSTANT (T2) \(=3.5 \mathrm{msec}\).
12. MAXIMUM CURRENT LIMIT \(=1200\) Amps.

\section*{VARIOUS PARAMETERS FOR DESIGNING THE CURRENT CONTROLLER}
\(\mathrm{Kc}=(\mathrm{TaRa}) / 2 \mathrm{KtK}_{2}\left(\mathrm{Tt}+\mathrm{T}_{2}\right) ;\)

But, Tc = Ta;
\(\mathrm{Ta}=\mathrm{La} / \mathrm{Ra}=30 \mathrm{~ms}\).
\(K t=460 / 10=46\)
\(\mathrm{K}_{2}=10 / 1200=1 / 120\)
\(\mathrm{Tt}=1.7 \mathrm{msec}\).
\(\mathrm{T}_{2}=3.5 \mathrm{msec}\).

So Now,
\(K c=30 \times 0.02342 /(2 \times 46 \times 42(1.7+3.5))=0.1762\)
\(\mathrm{Tc}=\mathrm{Ta}=30 \mathrm{msec}\).

\section*{VARIOUS PARAMETERS FOR DESIGINING THE SPEED CONTROLLER}
\(\mathrm{Tn}=4 \delta=4\left(2 \sigma+T_{1}\right)=4\left(2\left(\mathrm{~T}_{1}+\mathrm{T}_{2}\right)+\mathrm{T}_{1}\right)=4(3.4+7+25) \mathrm{ms}=141.6 \mathrm{~ms}\).
\(\mathrm{Kn}=\mathrm{TmKmK}_{2} /\left(2 \mathrm{~K}_{1} \mathrm{Ra}\left(2 \mathrm{Tt}+2 \mathrm{~T}_{2}+\mathrm{T}_{1}\right)\right.\)
\(\mathrm{Tt}=1.7 \mathrm{msec}\).
\(\mathrm{T}_{1}=25 \mathrm{msec}\).
\(\mathrm{T}_{2}=3.5 \mathrm{msec}\).
\(\mathrm{Km}=8.5\) Volt-sec. / rad.
\(R a=0.02342\)
\(\mathrm{K}_{2}=1 / 120=0.0083\)
SPEED \(=500 \mathrm{rpm}=(52.3 \mathrm{rad} / \mathrm{sec}\).
FILTER GAIN K1 \(=10 / 52.3=0.19\)
\(\mathrm{Tm}=\mathrm{J} \times \mathrm{Ra} /(\mathrm{Km})=84 \times 0.02342 /(8.5)=27.2 \mathrm{msec}\).
\(K n=27.2 \times 8.5 \times 0.0083 / 2 \times 0.019 \times 0.0234 \times 35.4=6.12\)

\section*{UNIT STEP RESPONSE FOR CURRENT LOOP}

\section*{MATLAB PROGRAM}
>> s=tf('s');
>> \(\mathbf{g}=2218934 /\left(\mathbf{s}^{\wedge} 2+192.30 * \mathrm{~s}+18491\right)\);
\(\gg \mathrm{d}=0.1762+5.87 / \mathrm{s}\);
>> m=feedback( \(\mathbf{d}^{*}\) g,1);
>> step(m);


\section*{UNIT STEP RESPONSE FOR SPEED LOOP}

\section*{MATLAB PROGRAM}

\section*{>> s=tf('s');}
>> \(\mathrm{g}=99.74 /\left(\mathrm{s}^{\wedge} 2+14.12^{*} \mathrm{~s}+99.74\right)\);
>> d=6.2+43.71/s;
>> m=feedback(d*g,1);
>> step(m);

Step Response


\section*{Chapter 10}

Simulation of DC motor drive model


It consist of two control loops. The inner control loop is the current control loop and the outer control loop is the speed control loop. We are using the PI controllers for boh the speed as well as current control.

\section*{Chapter 11}
results

GRAPH FOR THE DC MOTOR SPEED RESPONSE


GRAPH FOR THE ARMATURE CURRENT RESPONSE


Time onsen \({ }^{0}\)

(ii) AT LOAD LESS THAN FULL LOAD

GRAPH FOR THE DC MOTOR SPEED RESPONSE


GRAPH FOR THE ARMATURE CURRENT RESPONSE


Time ofrsen [0]

(iii) AT LOAD MORE THAN FULL LOAD

GRAPH FOR THE DC MOTOR SPEED RESPONSE


GRAPH FOR THE ARMATURE CURRENT RESPONSE


GRAPH FOR THE SPEED ERRROR


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