ANALYSIS OF TRANSIENT HEAT CONDUCTION IN DIFFERENT GEOMETRIES

A THESIS SUBMITTED IN PARTIAL FULFILMENT

OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF TECHNOLOGY

IN

MECHANICAL ENGINEERING

By

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Rourkela

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CERTIFICATE

This is to certify that thesis entitled, "**ANALYSIS OF TRANSIENT HEAT CONDUCTION IN DIFFERENT GEOMETRIES**" submitted by **Miss Pritinika Behera** in partial fulfillment of the requirements for the award of **Master of Technology Degree** in Mechanical Engineering with specialization in "**Thermal Engineering**" at National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by her under my supervision and guidance.

To the best of my knowledge, the matter embodied in this thesis has not been submitted to any other university/ institute for award of any Degree or Diploma.

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ABSTRACT

Present work deals with the analytical solution of unsteady state one-dimensional heat conduction problems. An improved lumped parameter model has been adopted to predict the variation of temperature field in a long slab and cylinder. Polynomial approximation method is used to solve the transient conduction equations for both the slab and tube geometry. A variety of models including boundary heat flux for both slabs and tube and, heat generation in both slab and tube has been analyzed. Furthermore, for both slab and cylindrical geometry, a number of guess temperature profiles have been assumed to obtain a generalized solution. Based on the analysis, a modified Biot number has been proposed that predicts the temperature variation irrespective the geometry of the problem. In all the cases, a closed form solution is obtained between temperature, Biot number, heat source parameter and time. The result of the present analysis has been compared with earlier numerical and analytical results. A good agreement has been obtained between the present prediction and the available results.

Key words: *lumped model, polynomial approximation method, transient, conduction, modified Biot number*

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NOMENCLTURE

B	Biot Number
k	Thermal conductivity
h	Heat transfer coefficient
m	Order of the geometry
r	Coordinate
R	Maximum coordinate
S	Shape factor
t	Time
T	Temperature
V	Volume
g	Internal heat generation
G	Dimensionless internal heat generation
x	Dimensionless coordinate
PAM	Polynomial approximation
P	Modified Biot number

Greek symbol

α	Thermal diffusivity
τ	Dimensionless time
θ	Dimensionless temperature
$\bar{\theta}$	Dimensionless average temperature

Subscripts

$\circ$	Initial
∞	Infinite

CHAPTER 1

INTRODUCTION

CHAPTER 1

INTRODUCTION

1.1 GENERAL BACKGROUND

Heat transfer is the study of thermal energy transport within a medium or among neighboring media by molecular interaction, fluid motion, and electro-magnetic waves, resulting from a spatial variation in temperature. This variation in temperature is governed by the principle of energy conservation, which when applied to a control volume or a control mass, states that the sum of the flow of energy and heat across the system, the work done on the system, and the energy stored and converted within the system, is zero. Heat transfer finds application in many important areas, namely design of thermal and nuclear power plants including heat engines, steam generators, condensers and other heat exchange equipments, catalytic convertors, heat shields for space vehicles, furnaces, electronic equipments etc, internal combustion engines, refrigeration and air conditioning units, design of cooling systems for electric motors generators and transformers, heating and cooling of fluids etc. in chemical operations, construction of dams and structures, minimization of building heat losses using improved insulation techniques, thermal control of space vehicles, heat treatment of metals, dispersion of atmospheric pollutants. A thermal system contains matter or substance and this substance may change by transformation or by exchange of mass with the surroundings. To perform a thermal analysis of a system, we need to use thermodynamics, which allows for quantitative description of the substance. This is done by defining the boundaries of the system, applying the conservation principles, and examining how the system participates in thermal energy exchange and conversion.

1.2 MODES OF HEAT TRANSFER

Heat transfer generally takes place by three modes such as conduction, convection and radiation. Heat transmission, in majority of real situations, occurs as a result of combinations of these modes of heat transfer. Conduction is the transfer of thermal energy between neighboring molecules in a substance due to a temperature gradient. It always takes place from a region of higher temperature to a region of lower temperature, and acts to equalize temperature differences. Conduction needs matter and does not require any bulk motion of matter. Conduction takes place in all forms of matter such as solids, liquids, gases and plasmas. In solids, it is due to the combination of vibrations of the molecules in a lattice and the energy transport by free electrons. In gases and liquids, conduction is due to the collisions and diffusion of the molecules during their random motion. Convection occurs when a system becomes unstable and begins to mix by the movement of mass. A common observation of convection is of thermal convection in a pot of boiling water, in which the hot and less-dense water on the bottom layer moves upwards in plumes, and the cool and denser water near the top of the pot likewise sinks. Convection more likely occurs with a greater variation in density between the two fluids, a larger acceleration due to gravity that drives the convection through the convecting medium. Radiation describes any process in which energy emitted by one body travels through a medium or through space absorbed by another body. Radiation occurs in nuclear weapons, nuclear reactors, radioactive radio waves, infrared light, visible light, ultraviolet light, and X-rays substances.

1.3 HEAT CONDUCTION

Heat conduction is increasingly important in modern technology, in the earth sciences and many other evolving areas of thermal analysis. The specification of temperatures, heat sources, and heat flux in the regions of material in which conduction occur give rise to analysis of temperature distribution, heat flows, and condition of thermal stressing. The importance of such conditions has lead to an increasingly developed field of analysis in which sophisticated mathematical and increasingly powerful numerical techniques are used. For this we require a classification of minimum number of space coordinate to describe the temperature field. Generally three types of coordinate system such as one-dimensional, two-dimensional and three-dimensional are considered in heat conduction. In one dimensional geometry, the temperature variation in the region is described by one variable alone. A plane slab and cylinder are considered one-dimensional heat conduction when one of the surfaces of these geometries in each direction is very large compared to the region of thickness. When the temperature variation in the region is described by two and three variables, it is said to be two-dimensional and three-dimensional respectively. Generally the heat flow through the heat transfer medium dominates with only one direction. When no single and dominate direction for the heat transfer exist, the conduction problem needs to be solved by more than one dimensions.

A particular conduction circumstances also depends upon the detailed nature of conduction process. Steady state means the conditions parameters such as temperature, density at all points of the conduction region are independent of time. Unsteady or transient heat conduction state implies a change with time, usually only of the temperature. It is fundamentally due to sudden change of conditions. Transient heat conduction occurs in cooling of I.C engines, automobile engines, heating and cooling of metal billets, cooling and freezing of food, heat treatment of metals by quenching, starting and stopping of various heat exchange units in power insulation, brick burning, vulcanization of rubber etc. There are two distinct types of unsteady state namely periodic and non periodic. In periodic, the temperature variation with time at all points in the region is periodic. An example of periodic conduction may be the temperature variations in building during a period of twenty four hours, surface of earth during a period of twenty four hours, heat processing of regenerators, cylinder of an I.C engines etc. In a non-periodic transient state, the temperature at any point within the system varies non-linearly with time. Heating of an ingot in furnaces, cooling of bars, blanks and metal billets in steel works, etc. are examples of non-periodic conduction.

1.4 HEAT CONDUCTION PROBLEMS

The solution of the heat conduction problems involves the functional dependence of temperature on various parameters such as space and time. Obtaining a solution means determining a temperature distribution which is consistent with conditions on the boundaries.

1.4.1 One Dimensional analysis

In general, the flow of heat takes place in different spatial coordinates. In some cases the analysis is done by considering the variation of temperature in one-dimension. In a slab one dimension is considered when face dimensions in each direction along the surface are very large compared to the region thickness, with uniform boundary condition is applied to each surface. Cylindrical geometries of one-dimension have axial length very large compared to the maximum conduction region radius. At a spherical geometry to have one-dimensional analysis a uniform condition is applied to each concentric surface which bounds the region.

1.4.2 Steady and unsteady analysis

Steady state analysis

A steady-state thermal analysis predicts the effects of steady thermal loads on a system. A system is said to attain steady state when variation of various parameters namely, temperature, pressure and density vary with time. A steady-state analysis also can be considered the last step of a transient thermal analysis. We can use steady-state thermal analysis to determine temperatures, thermal gradients, heat flow rates, and heat fluxes in an object which do not vary over time. A steady-state thermal analysis may be either linear, by assuming constant material properties or can be nonlinear case, with material properties varying with temperature. The thermal properties of most material do vary with temperature, so the analysis becomes nonlinear. Furthermore, by considering radiation effects system also become nonlinear.

Unsteady state analysis

Before a steady state condition is reached, certain amount of time is elapsed after the heat transfer process is initiated to allow the transient conditions to disappear. For instance while determining the rate of heat flow through wall, we do not consider the period during which the furnace starts up and the temperature of the interior, as well as those of the walls, gradually increase. We usually assume that this period of transition has passed and that steady-state condition has been established.

In the temperature distribution in an electrically heated wire, we usually neglect warming up-period. Yet we know that when we turn on a toaster, it takes some time before the resistance wires attain maximum temperature, although heat generation starts instantaneously when the current begins to flow. Another type of unsteady-heat-flow problem involves with periodic variations of temperature and heat flow. Periodic heat flow occurs in internal-combustion engines, air-conditioning, instrumentation, and process control. For example the temperature inside stone buildings remains quite higher for several hours after sunset. In the morning, even though the atmosphere has already become warm, the air inside the buildings will remain comfortably cool for several hours. The reason for this phenomenon is the existence of a time lag before temperature equilibrium between the inside of the building and the outdoor temperature. Another typical example is the periodic heat flow through the walls of engines where temperature increases only during a portion of their cycle of operation. When the engine warms up and operates in the steady state, the temperature at any point in the wall undergoes cycle variation with time. While the engine is warming up, a transient heat-flow phenomenon is considered on the cyclic variations.

1.4.3 One Dimensional unsteady analysis

In case of unsteady analysis the temperature field depends upon time. Depending on conditions the analysis can be one-dimensional, two dimensional or three dimensional. One dimensional unsteady heat transfer is found at a solid fuel rocket nozzles, in reentry heat shields, in reactor components, and in combustion devices. The consideration may relate to temperature limitation of materials, to heat transfer characteristics, or to the thermal stressing of materials, which may accompany changing temperature distributions.

1.5 DESCRIPTION OF ANALYTICAL METHOD AND NUMERICAL METHOD

In general, we employ either an analytical method or numerical method to solve steady or transient conduction equation valid for various dimensions (1D/2D). Numerical technique generally used is finite difference, finite element, relaxation method etc. The most of the practical two dimensional heat problems involving irregular geometries is solved by numerical techniques. The main advantage of numerical methods is it can be applied to any two-dimensional shape irrespective of its complexity or boundary condition. The numerical analysis, due to widespread use of digital computers these days, is the primary method of solving complex heat transfer problems.

The heat conduction problems depending upon the various parameters can be obtained through analytical solution. An analytical method uses Laplace equation for solving the heat conduction problems. Heat balance integral method, hermite-type approximation method, polynomial approximation method, wiener–Hopf Technique are few examples of analytical method.

1.6 LOW BIOT NUMBER IN 1-D HEAT CONDUCTION PROBLEMS

The Biot number represents the ratio of the time scale for heat removed from the body by surface convection to the time scale for making the body temperature uniform by heat conduction. However, a simple lumped model is only valid for very low Biot numbers. In this preliminary model, solid resistance can be ignored in comparison with fluid resistance, and so the solid has a uniform temperature that is simply a function of time. The criterion for the Biot number is about 0.1, which is applicable just for either small solids or for solids with high thermal conductivity. In other words, the simple lumped model is valid for moderate to low temperature gradients. In many engineering applications, the Biot number is much higher than 0.1, and so the condition for a simple lumped model is not satisfied. Additionally, the moderate to low temperature gradient assumption is not reasonable in such applications, thus more accurate models should be adopted. Lots of investigations have been done to use or modify the lumped model. The purpose of modified lumped parameter models is to establish simple and more precise relations for higher values of Biot numbers and large temperature gradients. For example, if a model is able to predict average temperature for Biot numbers up to 10, such a model can be used for a much wider range of materials with lower thermal conductivity.

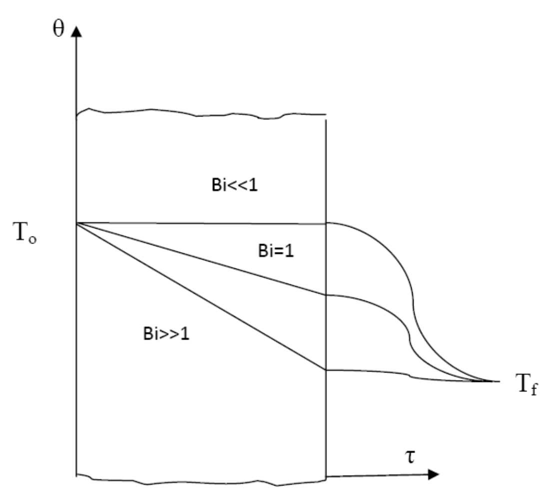

Fig 1.1: Schematic of variation of Biot number in a slab

Fig 1.1 shows the variation of temperature with time for various values of Biot number. The fig 1.1 predicts that for higher values Biot number temperature variation with respect to time is higher. When Biot number is more than one the heat transfer is higher which require more time to transfer the heat from body to outside. Thus the variation of temperature with time is negligible. Whereas as gradually the Biot number increase, the heat transfer rate decrease, and thus it results to rapid cooling. Fig 1.1 predicts, how at Biot number more than one the temperature variation with time is more as compared to Biot number with one and less than one.

1.7 SOLUTION OF HEAT CONDUCTION PROBLEMS

For a heat conduction problem we first define an appropriate system or control volume. This step includes the selection of a coordinate system, a lumped or distribution formulation, and a system or control volume. The general laws except in their lumped forms are written in terms of coordinate system. The differential forms of these laws depend on the direction but not the origin of the coordinates, whereas the integral forms depend on the origin as well as the direction of the coordinates. Although the differential forms apply locally, the lumped and integral forms are stated for the entire system or control volume. The particular law describing the diffusion of heat (or momentum, mass or electricity) is differential, applies locally, and depends on the direction but not the origin of coordinates. The equation of conduction may be an algebraic, differential or other equation involving the desired dependent variable, say the temperature as the only unknown. The governing equation (except for its flow terms) is independent of the origin and direction of coordinates. The initial and/or boundary condition pertinent at governing equation are mathematical descriptions of experimental observations. We refer to the conditions in time as the initial condition and the condition in space as the boundary conditions. For an unsteady problem the temperature of a continuum under consideration must be known at some instant of time. In many cases this instant is most conveniently taken to be the beginning of the problem. This we say as Initial (volume) conditions. Similarly for boundary condition prescribe parameters like temperature, heat flux, no heat flux (insulation), heat transfer to the ambient by convection, heat transfer to the ambient by radiation, prescribed heat flux acting at a distance, interface of two continuum of different conductivities, interface of two continua in relative motion, Moving interface of two continua(change of phase).

For the surface temperature of the boundaries it is specified to be a constant or a function of space and/or time. This is the easiest boundary condition from the view point of mathematics, yet a difficult one to materialize it physically. The heat flux across the boundaries is specified to be a constant or a function of space and/or time. The mathematical description of this condition may be given in the light of Kirchhoff’s current law; that is the algebraic sum of heat fluxes at a boundary must be equal to zero. Here after the sign is to be assuming positive for the heat flux to the boundary and negative for that from the boundary. Thus remembering the Fourier’s law as heat flux is independent of the actual temperature distribution, and selecting the direction of heat flux conveniently such that it becomes positive. A special case of no heat flux (insulation) from previous one is obtained by inserting heat flux as zero. When the heat transfer across the boundaries, of a continuum cannot be prescribed, it may be assumed to be proportional to the temperature difference between the boundaries and the ambient. Which we may call as Newton’s cooling Law. The importance of radiation relative to convection depends to a large extent, on the temperature level. Radiation increases rapidly with increasing temperature. Even at room temperature, however, for low rates of convection to air, radiation may contribute up to fifty percent of the total heat transfer. Prescribed heat flux involves in any body surrounded by the atmosphere, capable of receiving radiant heat, and near a radiant source (a light bulb or a sun lamp) or exposed to the sun exemplifies the forgoing boundary condition. Interface of two continuums of different conductivities which are called composite walls and insulated tubes have a common boundary, the heat flux across this boundary are elevated from both continua, regardless of the direction of normal. A second condition may be specified along this boundary relating the temperature of the two continua. When two solid continua in contact, one moving relative to other, we say Interface of two continua in relative motion. The friction brake is an important practical case of the forgoing boundary conditions. When part of a continuum has temperature below the temperature at which the continuum changes from one phase to another by virtue of the lubrication or absorption of heat, there exist a moving boundary between the two phase. For problems in this category, the way in which the boundary moves has to be determined together with the temperature variation in the continuum.

After formulating the governing equation and boundary conditions, we have converted the parameters to dimensionless values. Based on this application various approximation methods is employed for solutions.

1.8 OBJECTIVE OF PRESENT WORK

1. An effort will be made to predict the temperature field in solid by employing a polynomial approximation method.
2. Effort will be made analyze more practical case such as heat generation in solid and specified heat flux at the solid surface is investigated.
3. Effort will be made to obtain new functional parameters that affect the transient heat transfer process.
4. It is tried to consider various geometries for the analysis.

1.9 LAYOUT OF THE REPORT

Chapter 2 discuss with the literature review of different types of problems. Chapter 3 deals with the theoretical solution of different heat conduction problems (slab/tube) by employing polynomial approximation method. Chapter 4 reports the result and discussion obtained from the present theoretical analysis. Chapter 5 discuss with the conclusion and scope of future work.

CHAPTER 2

LITERATURE SURVEY

CHAPTER 2

LITERATURE SURVEY

2.1 INTRODUCTION

Heat conduction is increasingly important in various areas, namely in the earth sciences, and in many other evolving areas of thermal analysis. A common example of heat conduction is heating an object in an oven or furnace. The material remains stationary throughout, neglecting thermal expansion, as the heat diffuses inward to increase its temperature. The importance of such conditions leads to analyze the temperature field by employing sophisticated mathematical and advanced numerical tools.

The section considers the various solution methodologies used to obtain the temperature field. The objective of conduction analysis is to determine the temperature field in a body and how the temperature within the portion of the body. The temperature field usually depends on boundary conditions, initial condition, material properties and geometry of the body.

Why one need to know temperature field. To compute the heat flux at any location, compute thermal stress, expansion, deflection, design insulation thickness, heat treatment method, these all analysis leads to know the temperature field.

The solution of conduction problems involves the functional dependence of temperature on space and time coordinate. Obtaining a solution means determining a temperature distribution which is consistent with the conditions on the boundaries and also consistent with any specified constraints internal to the region. P. Keshavarz Æ M. Taheri[1] and Jian Su [2] have obtained this type of solution.

2.2 ANALYTICAL SOLUTIONS

Keshavarz and Taheri [1] have analyzed the transient one-dimensional heat conduction of slab/rod by employing polynomial approximation method. In their paper, an improved lumped model is being implemented for a typical long slab, long cylinder and sphere. It has been shown that in comparison to a finite difference solution, the improved model is able to calculate average temperature as a function of time for higher value of Biot numbers. The comparison also presents model in better accuracy when compared with others recently developed models. The simplified relations obtained in this study can be used for engineering calculations in many conditions. He had obtained the temperature distribution as:

$$\theta = \exp\left(-\frac{B(m+1)(m+3)}{m+B+3}\tau \right)$$

Jian Su [2] have analyzed unsteady cooling of a long slab by asymmetric heat convection within the framework of lumped parameter model. They have used improved lumped model where the heat conduction may be analyzed with larger values of Biot number. The proposed lumped models are obtained through two point Hermite approximations method. Closed form analytical solutions are obtained from the lumped models. Higher order lumped models, ($H_{1.1}/H_{0,0}$ approximation) is compared with a finite difference solution and predicts a significance improvement of average temperature prediction over the classical lumped model. The expression was written as

$$\theta = \exp\left(-\frac{3(B_1+B_2+2B_1B_2)}{2(3+2B_1+2B_2+B_1B_2)}\tau \right)$$

Su and Cotta [3] have modeled the transient heat transfer in nuclear fuel rod by an improved lumped parameter approach. Average fuel and cladding temperature is derived using hermite approximation method. Thermal hydraulic behavior of a pressurized water reactor (PWR) during partial loss of coolant flow is simulated by using a simplified mathematical model. Transient response of fuel, cladding and coolant is analyzed

Correa and Cotta [4] have directly related to the task of modeling diffusion problems. The author presented a formulation tool, aimed at reducing, as much as possible and within prescribed accuracy requirements, the number of dimensions in a certain diffusion formulation. It is shown how appropriate integration strategies can be employed to deduce mathematical formulations of improved accuracy In comparison, with the well-established classical lumping procedures. They have demonstrated heat conduction problems and examined against the classical lumped system analysis (CLSA) and the exact solutions of the fully differential systems.

A. G. Ostrogorsky [5] has used Laplace transforms, an analytical solution for transient heat conduction in spheres exposed to surroundings at a uniform temperature and finite Bi numbers. The solution is explicit and valid during early transients, for Fourier numbers Fo>0.3.

Alhama and Campo [6] depicted a lumped model for the unsteady cooling of a long slab by asymmetric heat convection. The authors took the plausible extension of the symmetric heat convection implicating an asymmetric heat convection controlled by two Bi number $Bi_1=Lh_1/k$ at the left surface and $Bi_2=Lh_2/k$ at the right surface.

Clarissa et al. [7] has depicted the transient heat conduction in a nuclear fuel rod by employing improved lumped parameter approach. The authors have assumed circumferential symmetry heat flux through the gap modeled. Hermite approximation for integration is used to obtain the average temperature and heat flux in the radial direction. The authors have claimed significant improvement over the classical lumped parameter formulation. The proposed fuel rod heat conduction model can be used for the stability analysis of BWR, and the real-time simulator of nuclear power plants.

H. Sadat [8] made an analysis on unsteady one-dimensional heat conduction problem using perturbation method. He has predicted the average temperature for simple first order models at the centre, and surface. He have used a slab, the infinite cylinder and the sphere for the analysis.

Gesu et al. [9] depicted an improved lumped-parameter models for transient heat conduction in a slab with temperature-dependent thermal conductivity. The improved lumped models are obtained through two point Hermite approximations for integrals. The author compared with the numerical solution of a higher order lumped model.

Ziabakhsh and Domairry [10] analyzed has the natural convection of a non-Newtonian fluid between two infinite parallel vertical flat plates and the effects of the non-newtonian nature of fluid on the heat transfer. The homotopy analysis method and numerical method are used for solution. The obtained results are valid for the whole solution domain.

Chakarborty et al. [11] presented the conditions for the validity of lumped models by comparing with the numerical solution obtained by employing finite element methods.

Ercan Ataer [12] presented the transient behavior of finned-tube, liquid/gas cross flow heat exchangers for the step change in the inlet temperature of the hot fluid by employing an analytical method. The temperature variation of both fluids between inlet and outlet is assumed to be linear. It is also assumed that flow rates and inlet conditions remain fixed for both fluids, except for the step change imposed on the inlet temperature of the hot fluid. The energy equation for the hot and cold fluids, fins and walls are solved analytically. The variation of the exit temperatures of both fluids with time are obtained for a step change in the inlet temperature of the hot fluid. The dynamic behavior of the heat exchanger is characterized by time constant. This approach is easier to implement and can easily be modified for other heat exchangers.

H. Sadat [13] presented a second order model for transient heat conduction in a slab by using a perturbation method. Iit is shown that the simple model is accurate even for high values of the Biot number in a region surrounding the center of the slab.

Monteiro et al. [14] analyzed the integral transformation of the thermal wave propagation problem in a finite slab through a generalized integral transform technique. The resulting transformed ODE system is then numerically solved. Numerical results are presented for the local and average temperatures for different Biot numbers and dimensionless thermal relaxation times. The author have compared with the previously reported results in the literature for special cases and with those produced through the application of the Laplace transform method.

Gesu at al. [15] studied the transient radiative cooling of a spherical body by employing lumped parameter models. As the classical lumped model is limited to values of the radiation-conduction parameters, N_{rc}, less than 0.7, the authors have tried to propose improved lumped models that can be applied in transient radiative cooling with larger values of the radiation-conduction parameter. The approximate method used here is Hermite approximation for integrals. The result is compared with numerical solution of the orginal distributed parameter model which yield significant improvement of average temperature prediction over the classical lumped model.

Shidfara et al. [16] identified the surface heat flux history of a heated conducting body. The nonlinear problem of a non-homogeneous heat equation with linear boundary conditions is considered. The objective of the proposed method is to evaluate the unknown function using linear polynomial pieces which are determined consecutively from the solution of the minimization problem on the basis of over specified data.

Liao et al. [17] have solved by employing homotopy method the nonlinear model of combined convective and radiative cooling of a spherical body. An explicit series solution is given, which agrees well with the exact and numerical solutions. The temperature on the surface of the body decays more quickly for larger values of the Biot number, and the radiation–conduction parameter N_{rc} . This is different from traditional analytic techniques based on eigen functions and eigen values for linear problems. They approached the independent concepts of eigen functions and eigen values. The author claims to provide a new way to obtain series solutions of unsteady nonlinear heat conduction problems, which are valid for all dimensionless times varying from $0 \leq \tau < +\infty$.

F. de Monte [18] developed a new type of orthogonality relationship and used to obtain the final series solution of one-dimensional multilayered composite conducting slabs subjected to sudden variations of the temperature of the surrounding fluid. This gives the relationship between the eigen values for the different regions and then yields a transcendental equation for the determination of the eigen values in a simpler way. They have investigated the errors that involve due to neglecting of higher order terms

Pontedeiro et al. [19] presented an improved lumped-differential formulation for one-dimensional transient heat conduction in a heat generating cylinder with temperature-dependent thermo-physical properties typical of high burn-up nuclear fuel rods. Average temperature and heat flux in the radial direction are obtained by employing two-points Hermite approximations for integrals. Transient heat conduction in a nuclear fuel rod was computed with the thermo-physical properties represented by the MATPRO correlations. The problem was solved by using the symbolic/numerical computation software system MATHEMATICA. The solutions of the model is compared with the numerical solutions of the original distributed parameter formulation of the transient heat conduction problem, simulation results of SBLOCA obtained by RELAP5/MOD3

Mikhailov and Cotta [20] obtained a solution for the hyperbolic heat conduction equation where amplitude and phase lag of temperature oscillations in a slab subjected to a harmonic heat flux are presented. They have shown the example for evaluating the temperature oscillations in any desired location for specified parameters. Their result demonstrates the resonance phenomena that increases the relaxation time and decreases the damping of temperature amplitude.

Campo and Villase [21] have made a comparative study on the distributed and the lumped-based models. They have presented the controlling parameter is the radiation-conduction parameter, N_r by taking a sink temperature at zero absolute. The transient radiative cooling of small spherical bodies having large thermal conductivity has not been critically examined in with the transient convective cooling. Thus they have validated the solution by the family of curves for the relative errors associated with the surface-to center temperatures followed a normal distribution in semi log coordinates.

Lin et al. [22] determined the temperature distributions in the molten layer and solid with distinct properties around a bubble or particle entrapped in the solid during unidirectional solidification by employing of heat-balance integral method. The model is used to simulate growth, entrapment or departure of a bubble or particle inclusion in solids encountered in manufacturing and materials processing, MEMS, contact melting, processes and drilling, etc. They have derived the heat-balance equation by integrating unsteady elliptic heat diffusion equations and introducing the Stefan boundary condition. Due to the time-dependent irregular shapes of phases, they have assumed the quadratic temperature profiles as the functions of longitudinal coordinate and time. The temperature coefficients in distinct regions are determined by solving the equations governing temperature coefficients derived from heat-balance equations. The temperature field obtained is validated by using finite difference method. The authors provide an effective method to solve unsteady elliptic diffusion problems experiencing solid–liquid phase changes in irregular shapes.

Kingsley et al. [23] considered the thermochromic liquid to measure the surface temperature in transient heat transfer experiments. Knowing the time at which the TLC changes colour, hence knowing the surface temperature at that time, they have calculated the heat transfer coefficient. The analytical one-dimensional solution of Fourier conduction equation for a semi-infinite wall is presented. They have also shown the 1D analytical solution can be used for the correction of error. In this case the approximate two-dimensional analysis is used to calculate the error, and a 2D finite-difference solution of Fourier equation is used to validate the method.

Sheng et al. [24] investigated the transient heat transfer in two-dimensional annular fins of various shapes with its base subjected to a heat flux varying as a sinusoidal time function. The transient temperature distribution of the annular fins of various shapes are obtained as its base subjected to a heat flux varying as a sinusoidal time function by employing inverse Laplace transform by the Fourier series technique.

Sahu et al. [25] depicted a two region conduction-controlled rewetting model of hot vertical surfaces with internal heat generation and boundary heat flux subjected to a constant wet side heat transfer coefficient and negligible heat transfer from dry side by using the Heat Balance Integral Method. The HBIM yields the temperature field and quench front temperature as a function of various model parameters such as Peclet number, Biot number and internal heat source parameter of the hot surface. The authors have also obtained the critical internal heat source parameter by considering Peclet number equal to zero, which yields the minimum internal heat source parameter to prevent the hot surface from being rewetted. The approximate method used, derive a unified relationship for a two-dimensional slab and tube with both internal heat generation and boundary heat flux.

Faruk Yigit [26] considered taken a two-dimensional heat conduction problem where a liquid becomes solidified by heat transfer to a sinusoidal mold of finite thickness. He has solved this problem by using linear perturbation method. The liquid perfectly wets the sinusoidal mold surface for the beginning of solidification resulting in an undulation of the solidified shell thickness. The temperature of the outer surface of the mold is assumed to be constant. He has determined the results of solid/melt moving interface as a function of time and for the temperature distribution for the shell and mold. He has considered the problem with prescribed solid/melt boundary to determine surface temperature.

Vrentas and Vrentas [27] proposed a method for obtaining analytical solutions to laminar flow thermal entrance region problems with axial conduction with the mixed type wall boundary conditions. They have used Green's functions and the solution of a Fredholm integral equation to obtain the solution. The temperature field for laminar flow in a circular tube for the zero Peclet number is presented.

Cheroto et al. [28] modeled the simultaneous heat and mass transfer during drying of moist capillary porous media within by employing lumped-differential formulations They are obtained from spatial integration of the original set of Luikov's equations for temperature and moisture potential. The classical lumped system analysis is used and temperature and moisture gradients are evaluated. They compared the results with analytical solutions for the full partial differential system over a wide range of the governing parameters.

Kooodziej and Strezk [29] analyzed the heat flux in steady heat conduction through cylinders having cross-section in an inner or an outer contour in the form of a regular polygon or a circle. They have determined the temperature to calculate the shape factor. They have considered three cases namely hollow prismatic cylinders bounded by isothermal inner circles and outer regular polygons, hollow prismatic cylinders bounded by isothermal inner regular polygons and outer circles, hollow prismatic cylinders bounded by isothermal inner and outer regular polygons. The boundary collocation method in the least squares sense is used. Through non-linear approximation the simple analytical formulas have been determined for the three geometries.

Tan et al. [30] developed a improved lumped models for the transient heat conduction of a wall having combined convective and radiative cooling by employing a two point hermite type for integrals. The result is validated by with a numerical solution of the original distributed parameter model. Significant improvement of average temperature over the classical lumped model is obtained.

Teixeira et al. [31] studied the behavior of metallic materials. They have considered the nonlinear temperature-dependence neglecting the thermal–mechanical coupling of deformation. They have presented formulation of heat conduction problem. They estimated the error using the finite element method for the continuous-time case with temperature dependent material properties.

Frankel el at. [32] presented a general one-dimensional temperature and heat flux formulation for hyperbolic heat conduction in a composite medium and the standard three orthogonal coordinate systems based on the flux formulation. Basing on Fourier's law, non-separable field equations for both the temperature and heat flux is manipulated. A generalized finite integral transform technique is used to obtain the solution. They have applied the theory on a two-region slab with a pulsed volumetric source and insulated exterior surfaces. This displays the unusual and controversial nature associated with heat conduction based on the modified Fourier's law in composite regions.

CHAPTER 3

THEORTICAL ANALYSIS OF CONDUCTION PROBLEMS

CHAPTER 3

THEORTICAL ANALYSIS OF CONDUCTION PROBLEMS

3.1INTRODUCTION

In this chapter four different heat conduction problems are considered for the analysis. These include the analysis of a rectangular slab and tube with both heat generation and boundary heat flux. Added a hot solid with different temperature profiles is considered for the analysis

3.2 TRANSIENT ANALYSIS ON A SLAB WITH SPECIFIED HEAT FLUX

We consider the heat conduction in a slab of thickness 2R, initially at a uniform temperature T_0, having heat flux at one side and exchanging heat by convection at another side. A constant heat transfer coefficient (h) is assumed on the other side and the ambient temperature (T_∞) is assumed to be constant. Assuming constant physical properties, k and α, the generalized transient heat conduction valid for slab, cylinder and sphere can be expressed as:

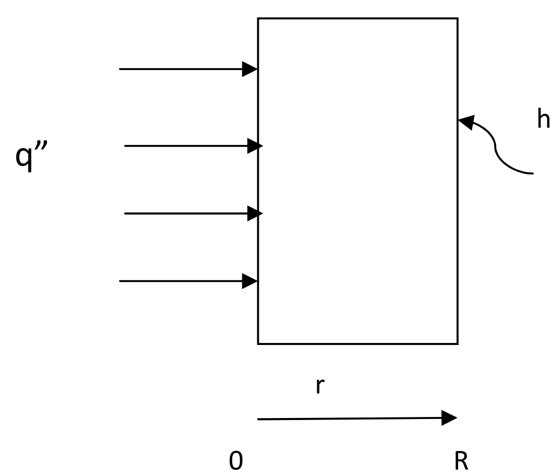

Fig 3.1: Schematic of slab with boundary heat flux

$$\frac{\partial T}{\partial t} = \alpha \frac{1}{r^m} \frac{\partial}{\partial r}\left(r^m \frac{\partial T}{\partial r} \right) \tag{3.1}$$

Where, m = 0 for slab, 1 and 2 for cylinder and sphere, respectively. Here we have considered slab geometry. Putting m=0, equation (3.1) reduces to

$$\frac{\partial T}{\partial r} = \alpha \frac{\partial^2 T}{\partial r^2} \tag{3.2}$$

Subjected to boundary conditions

$$\frac{\partial T}{\partial r} = -q \quad \text{at} \quad r = 0 \tag{3.3}$$

$$k\frac{\partial T}{\partial r} = -h\left(T - T_\infty\right) \quad \text{at} \quad r = R \tag{3.4}$$

Initial conditions: $T=T_0$ at $t=0$ (3.5)

Dimensionless parameters are defined as follows

$$\tau = \frac{\alpha t}{R^2}, \quad \theta = \frac{T - T_\infty}{T_0 - T_\infty}, \quad B = \frac{hR}{k}, \quad Q = -\frac{q''\delta}{k\left(T - T_\infty\right)}, \quad x = \frac{r}{R} \tag{3.6}$$

Using equations (3.6) the equation (3.2-3.5) can be written as

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial x^2} \tag{3.7}$$

Boundary conditions

$$\frac{\partial \theta}{\partial x} = -Q \tag{3.8}$$

Where $Q = \dfrac{q''R}{k\left(T_0 - T_\infty\right)}$ at $x = 0$

$$\frac{\partial \theta}{\partial x} = -B\theta \quad \text{at} \quad x = R \tag{3.9}$$

Solution procedure

Polynomial approximation method is one of the simplest, and in some cases, accurate methods used to solve transient conduction problems. The method involves two steps: first, selection of the proper guess temperature profile, and second, to convert a partial differential equation into an equation. This can then be converted into an ordinary differential equation, where the dependent variable is average temperature and independent variable is time. The steps are applied on dimensionless governing equation. Following guess profile is selected for the

$$\theta_p = a_0(\tau) + a_1(\tau)x + a_2(\tau)x^2 \tag{3.10}$$

Differentiating the above equation with respect to x we get

$$\frac{\partial \theta}{\partial x} = a_1 + 2a_2 x \tag{3.11}$$

Applying second boundary condition we have

$$a_1 + 2a_2 = -B\theta \tag{3.12}$$

Similarly applying first boundary condition at the differentiated equation we have

$$a_1 = -Q \tag{3.13}$$

Thus equation (3.11) may be written as

$$a_2 = \frac{Q - B\theta}{2} \tag{3.14}$$

Using equation (3.8) and (3.9) we get vale of a_0 as:

$$a_0 = -\left(\frac{a_1 + 2a_2}{B}\right) - a_1 - a_2 \tag{3.15}$$

Substituting the values of a_1 and a_2 we get

$$a_0 = \theta + Q - \left(\frac{Q - B\theta}{2}\right) \tag{3.16}$$

Average temperature for long slab, long cylinder and sphere can be written as:

$$\overline{\theta} = \frac{\int_v \theta dV}{\int dV} = \frac{\int_0^1 \theta x^m dx}{\int_0^1 x^m dx} = (m+1)\int_0^1 x^m \theta dx$$

m = 0 for slab, 1 and 2 for cylinder and sphere, respectively. Here we are using slab problem. Hence m=0. Average temperature equation used in this problem is

$$\overline{\theta} = \int_0^1 \theta dx \tag{3.17}$$

Substituting the value of θ and integrating we have

$$\overline{\theta} = \frac{Q}{6} + \left(\frac{1+B}{3}\right)\theta \tag{3.18}$$

Now, integrating non-dimensional governing equation we have

$$\frac{\partial \overline{\theta}}{\partial \tau} = -B\theta + Q \tag{3.19}$$

Substituting the value of $\bar{\theta}$ we have

$$\frac{\partial}{\partial \tau}\left(\frac{Q}{6} + \frac{1+B}{3}\theta\right) = -B\theta + Q \tag{3.20}$$

We may write the above equation as

$$\frac{\partial \theta}{\partial \tau} + U\theta - V = 0 \tag{3.21}$$

Integrating equation (3.21) we get an expression of dimensionless temperature as

$$\theta = \left(\frac{e^{-U\tau} + V}{U}\right) \tag{3.22}$$

Where: $V = \frac{Q}{1 + B/3}$, $U = \frac{B}{1 + B/3}$

Based on the analysis a closed form expression involving temperature, heat source parameter, Biot number and time is obtained for a slab.

3.3 TRANSIENT ANALYSIS ON A TUBE WITH SPECIFIED HEAT FLUX

We consider the heat conduction in a tube of diameter 2R, initially at a uniform temperature T_0, having heat flux at one side and exchanging heat by convection at another side. A constant heat transfer coefficient (h) is assumed on the other side and the ambient temperature (T_∞) is assumed to be constant. Assuming constant physical properties, k and α, the generalized transient heat conduction valid for slab, cylinder and sphere can be expressed as:

$$\frac{\partial T}{\partial t} = \alpha \frac{1}{r^m} \frac{\partial}{\partial r}\left(r^m \frac{\partial T}{\partial r} \right)$$

Where, m = 0 for slab, 1 and 2 for cylinder and sphere, respectively. Here we have considered tube geometry. Putting m=1, equation (3.1) reduces to

$$\frac{\partial T}{\partial t} = \alpha \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r} \right) \tag{3.23}$$

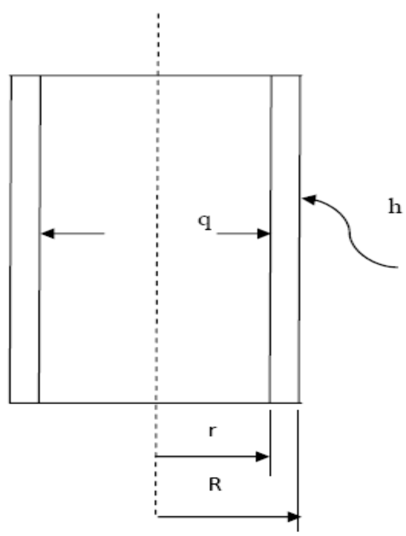

Fig 3.2: Schematic of a tube with heat flux

Subjected to boundary conditions

$$k \frac{\partial T}{\partial r} = -q'' \quad \text{at} \quad r = R_1 \tag{3.24}$$

$$k\frac{\partial T}{\partial r} = h\left(T - T_\infty\right) \quad \text{at} \quad r = R_2 \tag{3.25}$$

Initial conditions: $T=T_0$ at $t=0$ (3.26)

Dimensionless parameters are defined as follows

$$\varepsilon = \frac{R_1}{R_2}, \quad \theta = \frac{T - T_\infty}{T_0 - T_\infty}, \quad B = \frac{hR}{k}, \quad \tau = \frac{\alpha t}{R^2}, \quad x = \frac{r}{R} \tag{3.27}$$

Using equations (3.27) the equation (3.23-3.26) can be written as

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{x}\frac{\partial}{\partial x}\left(x\frac{\partial \theta}{\partial x}\right) \tag{3.28}$$

Boundary conditions as

$$\frac{\partial \theta}{\partial x} = -B\theta \quad \text{at} \quad x = 1 \tag{3.29}$$

$$\frac{\partial \theta}{\partial x} = -Q \quad \text{at} \quad x = \varepsilon \tag{3.30}$$

SOLUTION PROCEDURE

The guess temperature profile is assumed as

$$\theta_p = a_0\left(\tau\right) + a_1\left(\tau\right)x + a_2\left(\tau\right)x^2$$

Differentiating the above equation we get:

$$\frac{\partial \theta_p}{\partial x} = a_1 + 2a_2 x \tag{3.31}$$

Applying first boundary condition we have

$$a_1 + 2a_2 x = -Q \quad \text{at} \quad x = \varepsilon$$

$$a_1 + 2a_2 \varepsilon = -Q \tag{3.32}$$

Applying second boundary condition we have

$$a_1 + 2a_2 = -B\theta \tag{3.33}$$

Subtracting the equation (3.30) from (3.29) we get

$$a_2 = \frac{B\theta - Q}{2(\varepsilon - 1)} \tag{3.34}$$

Substituting the value at equation (3.30) we have

$$a_1 = \frac{-B\theta(\varepsilon - 1) - (B\theta - Q)}{\varepsilon - 1} \tag{3.35}$$

Using second boundary conditions we have

$$a_0 = -\left(\frac{a_1 + 2a_2}{B}\right) - a_1 - a_2 \tag{3.36}$$

Thus substituting the value of a_1 and a_2 at yhe expression of a_0 we get the following value

$$a_0 = \frac{2\theta(\varepsilon - 1) + 2B\theta(\varepsilon - 1) + (B\theta - Q)}{2(\varepsilon - 1)} \tag{3.37}$$

We may write the average temperature equation as

$$\bar{\theta} = \int_{m\varepsilon}^{1} x^m \theta dx$$

Where m=1 for cylinderical co-ordinate

Thus the above equation may be written as

$$\bar{\theta} = \int_{\varepsilon}^{1} \theta x dx \tag{3.38}$$

Substituting the value of θ and integrating equation (3.35) we get

$$\bar{\theta} = \left(\frac{a_0}{2} + \frac{a_1}{3} + \frac{a_2}{4}\right) - \left(\frac{a_0\varepsilon^2}{2} + \frac{a_1\varepsilon^2}{2} + \frac{a_1\varepsilon^3}{3} + \frac{a_2\varepsilon^4}{4}\right) \tag{3.39}$$

ε is the ratio of inside diameter and outside diameter of the cylinder

$$\left(\frac{a_0\varepsilon^2}{2} + \frac{a_1\varepsilon^2}{2} + \frac{a_1\varepsilon^3}{3} + \frac{a_2\varepsilon^4}{4}\right) = 0 \tag{3.40}$$

Thus considering $\varepsilon = 0$ and substituting the value of a_0, a_1, a_2 at equation (3.40) we get the value of $\bar{\theta}$ as

$$\overline{\theta} = \theta\left(\frac{1}{2}+\frac{B}{8}\right)+\frac{Q}{24} \tag{3.41}$$

Integrating the non-dimensional governing equation with respect to r we get

$$\frac{\partial}{\partial x}\int_{\varepsilon}^{1}\theta x dx = -B\theta + Q \tag{3.42}$$

Using equation (3.35) we may write the above equation as

$$\frac{\partial\overline{\theta}}{\partial\tau} = -B\theta + Q \tag{3.43}$$

Substituting the value of $\bar{\theta}$ at above equation we get

$$\frac{\partial\theta}{\partial\tau} = -\frac{B\theta}{(4+B)/8}+\frac{Q}{(4+B)/8} \tag{3.44}$$

We may write the above equation as

$$\frac{\partial\theta}{\partial\tau} = -U\theta + V \tag{3.45}$$

Integrating equation (3.45) we get an expression of dimensionless temperature as

$$\theta = \left(\frac{e^{-U\tau}+V}{U}\right) \tag{3.46}$$

Where: $U = \frac{B}{(4+B)/8}$, $V = \frac{Q}{(4+B)/8}$

Based on the analysis a closed form expression involving temperature, heat source parameter, Biot number and time is obtained for a tube.

3.4 TRANSIENT ANALYSIS ON A SLAB WITH SPECIFIED HEAT GENERATION

We consider the heat conduction in a slab of thickness 2R, initially at a uniform temperature T_0, having heat generation (G) inside it and exchanging heat by convection at another side. A constant heat transfer coefficient (h) is assumed on the other side and the ambient temperature (T_∞) is assumed to be constant. Assuming constant physical properties, k and α, the generalized transient heat conduction valid for slab, cylinder and sphere can be expressed as

$$\frac{\partial T}{\partial t} = \alpha \frac{1}{r^m} \frac{\partial}{\partial r}\left(r^m \frac{\partial T}{\partial r} \right) + G \tag{3.47}$$

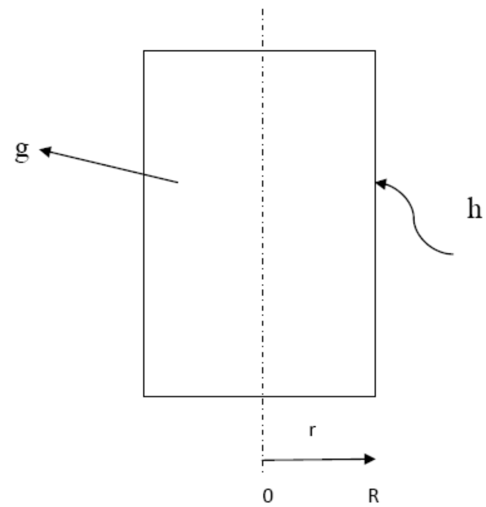

Fig 3.3: Schematic of slab with heat generation

Where, m = 0 for slab, 1 and 2 for cylinder and sphere, respectively. Here we have considered slab geometry. Putting m=0, equation (3.47) reduces to

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial r^2} + G \tag{3.48}$$

Boundary conditions $\quad k\frac{\partial T}{\partial r} = 0$ at $r = 0$ (3.49)

$$k\frac{\partial T}{\partial r} = -h\left(T - T_\infty\right) \quad \text{at } r = R \tag{3.50}$$

Initial condition $\quad T=T_0$ at t=0 (3.51)

Dimensionless parameters defined as

$$\theta = \frac{T - T_\infty}{T_0 - T_\infty}, \quad B = \frac{hR}{k}, \quad \tau = \frac{\alpha t}{R^2}, \quad x = \frac{r}{R}, \quad G = \frac{g_0 R^2}{k\left(T_0 - T_\infty\right)} \tag{3.52}$$

Using equations (3.52) the equation (3.48-3.51) can be written as

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial x^2} + G \tag{3.52}$$

Initial condition $\theta\left(x, 0\right) = 1$ (3.53)

Boundary condition $\frac{\partial \theta}{\partial x} = 0$ at $x = 0$ (3.54)

$$\frac{\partial \theta}{\partial x} = -B\theta \quad \text{at} \quad x = R \tag{3.55}$$

SOLUTION PROCEDURE

The guess temperature profile is assumed as

$$\theta_p = a_0\left(\tau\right) + a_1\left(\tau\right)x + a_2\left(\tau\right)x^2$$

Differentiating the above equation with respect to x we get

$$\frac{\partial \theta_p}{\partial x} = a_1 + 2a_2 x \tag{3.56}$$

Applying first boundary condition we have

$$a_1 + 2a_2 x = 0 \tag{3.57}$$

Thus, $a_1 = 0$ (3.58)

Applying second boundary condition we have

$$a_1 + 2a_2 = -B\theta \tag{3.59}$$

Thus,
$$a_2 = -\frac{B\theta}{2} \tag{3.60}$$

We can also write the second boundary condition as

$$\frac{\partial\theta}{\partial x} = -B\left(a_0 + a_1 + a_2\right) \tag{3.61}$$

Using equation (3.55) , (3.58)and (3.60-3.61) we have

$$a_0 = \theta\left(1 + \frac{B}{2}\right) \tag{3.62}$$

Average temperature equation used in this problem is

$$\overline{\theta} = \int_0^1 \theta dx \tag{3.63}$$

Substituting the value of θ we have

$$\overline{\theta} = \int_0^1 \left(a_0 + a_1 x + a_2 x^2\right) dx \tag{3.64}$$

Integrating equation (3.64) we have

$$\overline{\theta} = a_0 + \frac{a_1}{2} + \frac{a_2}{3} \tag{3.65}$$

Substituting the value of a_0, a_1, a_2 we have

$$\overline{\theta} = \theta\left(1 + \frac{B}{3}\right) \tag{3.66}$$

Integrating non-dimensional governing equation we have

$$\int_0^1 \frac{\partial\theta}{\partial\tau} dx = \int_0^1 \frac{\partial^2\theta}{\partial x^2} dx + \int_0^1 G dx$$

Thus we get
$$\Rightarrow \int_0^1 \frac{\partial \theta}{\partial \tau} dx = -B\theta + G \tag{3.67}$$

Taking the value of average temperature we have

$$\frac{\partial \overline{\theta}}{\partial \tau} = -B\theta + G \tag{3.68}$$

Substituting the value of average temperature at equation (3.62) we have

$$\frac{\partial \theta}{\partial \tau} = \frac{-B\theta}{\left(1+\frac{B}{3}\right)} + \frac{G}{\left(1+\frac{B}{3}\right)}$$

$$\Rightarrow \frac{\partial \theta}{\partial \tau} = -U\theta + V \tag{3.69}$$

Simplifying the equation (3.69) we have

$$\Rightarrow \frac{\partial \theta}{U\theta - V} = -\partial \tau \tag{3.70}$$

$$\Rightarrow \frac{1}{U} \ln\left(U\theta - V\right) = -\tau \tag{3.71}$$

Thus the temperature can be expressed as

$$\theta = \frac{e^{-\tau U} + V}{U} \tag{3.72}$$

Where $U = \frac{B}{\left(1+B/3\right)}$, $V = \frac{G}{\left(1+\frac{B}{3}\right)}$

Based on the analysis a closed form expression involving temperature, internal heat generation parameter, Biot number and time is obtained for a slab.

3.5 TRANSIENT ANALYSIS ON A TUBE WITH SPECIFIED HEAT GENERATION

We consider the heat conduction in a tube of diameter 2R, initially at a uniform temperature T_0, having heat generation (G) inside it and exchanging heat by convection at another side. A constant heat transfer coefficient (h) is assumed on the other side and the ambient temperature (T_∞) is assumed to be constant. Assuming constant physical properties, k and α, the generalized transient heat conduction valid for slab, cylinder and sphere can be expressed as:

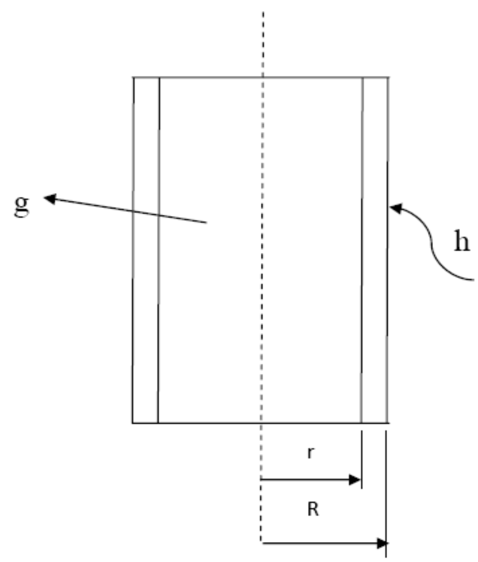

Fig 3.4: Schematic of cylinder with heat generation

$$\frac{\partial T}{\partial t} = \alpha \frac{1}{r^m} \frac{\partial}{\partial r}\left(r^m \frac{\partial T}{\partial r} \right) + G \tag{3.73}$$

Where, m = 0 for slab, 1 and 2 for cylinder and sphere, respectively. Here we have considered tube geometry. Putting m=1, the above equation reduces to

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial r^2} + G \tag{3.74}$$

Boundary conditions $k\frac{\partial T}{\partial r} = 0$ at $r = 0$ (3.75)

$$k\frac{\partial T}{\partial r} = -h\left(T - T_\infty\right) \quad \text{at } r = R \tag{3.76}$$

Initial condition $T=T_0$ at $t=0$ (3.77)

Dimensionless parameters defined as

$$\theta = \frac{T - T_\infty}{T_0 - T_\infty}, \quad B = \frac{hR}{k}, \quad \tau = \frac{\alpha t}{R^2}, \quad x = \frac{r}{R}, \quad G = \frac{g_0 R^2}{k\left(T_0 - T_\infty\right)} \tag{3.78}$$

Using equations (3.78) the equation (3.74-3.77) can be written as

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{x}\frac{\partial}{\partial x}\left(x\frac{\partial \theta}{\partial x}\right) + G \tag{3.79}$$

Initial condition $\theta\left(x,0\right) = 1$ (3.80)

Boundary condition $\frac{\partial \theta}{\partial x} = 0$ at $x = 0$ (3.81)

$$\frac{\partial \theta}{\partial x} = -B\theta \quad \text{at } x = 1 \tag{3.82}$$

SOLUTION PROCEDURE

The guess temperature profile is assumed as

$$\theta_p = a_0\left(\tau\right) + a_1\left(\tau\right)x + a_2\left(\tau\right)x^2 \tag{3.83}$$

Differentiating the above equation with respect to x we get

$$\frac{\partial \theta_p}{\partial x} = a_1 + 2a_2 x \tag{3.84}$$

Applying first boundary condition we have

$$a_1 + 2a_2 x = 0 \tag{3.85}$$

Thus, $$a_1 = 0 \tag{3.86}$$

Applying second boundary condition we have

$$a_1 + 2a_2 = -B\theta \tag{3.87}$$

Thus, $$a_2 = -\frac{B\theta}{2} \tag{3.88}$$

We can also write the second boundary condition as

$$\frac{\partial\theta}{\partial x} = -B\left(a_0 + a_1 + a_2\right) \tag{3.89}$$

Using equation (3.86), (3.83) and (3.88-3.89) we have

$$a_0 = \theta\left(1 + \frac{B}{2}\right) \tag{3.90}$$

Average temperature is expressed as

$$\overline{\theta} = \left(m+1\right)\int_0^1 x^m \theta dx$$

Substituting the value of θ and m we have

$$\overline{\theta} = 2\int_0^1 \left(a_0 x + a_1 x^2 + a_2 x^3\right) dx \tag{3.91}$$

Integrating the equation (3.91) we have

$$\overline{\theta} = 2\left(\frac{a_0}{2} + \frac{a_1}{3} + \frac{a_2}{4}\right) \tag{3.92}$$

Substituting the value of a_0 , a_1 and a_2 we have

$$\overline{\theta} = \theta\left(1 + \frac{B}{4}\right) \tag{3.93}$$

Integrating non-dimensional governing equation we have

$$\int_0^1 \frac{\partial \theta}{\partial \tau} x dx = \int_0^1 \left(\frac{1}{x}\frac{\partial}{\partial x}\left(x \frac{\partial \theta}{\partial x} \right) + G \right) x dx \tag{3.94}$$

Thus we get

$$\int_0^1 \frac{\partial \theta}{\partial \tau} x dx = -B\theta + \frac{G}{2} \tag{3.95}$$

Taking the value of average temperature we have

$$\frac{\partial \overline{\theta}}{\partial \tau} = -2B\theta + G \tag{3.96}$$

Substituting the value of $\overline{\theta}$ we have

$$\frac{\partial \theta}{\partial \tau} = -\frac{2B\theta}{\left(1 + B/4\right)} + \frac{G}{\left(1 + B/4\right)} \tag{3.97}$$

We may write the equation (3.97) as

$$\Rightarrow \frac{\partial \theta}{\partial \tau} = -U\theta + V \tag{3.98}$$

Where $U = \left(\dfrac{2B}{1 + B/4} \right)$, $V = \dfrac{G}{\left(1 + B/4\right)}$

Simplifying the above equation we have

$$\Rightarrow \frac{\partial \theta}{U\theta - V} = -\partial \tau \tag{3.99}$$

$$\Rightarrow \frac{1}{U} \ln\left(U\theta - V\right) = -\tau \tag{3.100}$$

Thus the temperature can be expressed as

$$\theta = \frac{e^{-\tau U} + V}{U} \tag{3.101}$$

Where $U = \left(\frac{2B}{1 + B/4} \right)$, $V = \frac{G}{\left(1 + B/4\right)}$

Based on the analysis a closed form expression involving temperature, internal heat generation parameter, Biot number and time is obtained for a tube.

3.6 TRAINSIENT HEAT CONDUCTION IN SLAB WITH DIFFERENT PROFILES

In this previous section we have used polynomial approximation method for the analysis. We have used both slab and cylindrical geometries. At both the geometries we have considered a heat flux and heat generation respectively. Considering different profiles, the analysis has been extended to both slab and cylindrical geometries. Unsteady state one dimensional temperature distribution of a long slab can be expressed by the following partial differential equation. Heat transfer coefficient is assumed to be constant, as illustrated in Fig 3.5. The generalized heat conduction equation can be expressed as

$$\frac{\partial T}{\partial t} = \alpha \frac{1}{r^m} \frac{\partial}{\partial r} \left(r^m \frac{\partial T}{\partial r} \right) \tag{3.102}$$

Where, m = 0 for slab, 1 and 2 for cylinder and sphere, respectively.

Boundary conditions are $\frac{\partial T}{\partial r} = 0$ at $r = 0$ (3.103)

$$k \frac{\partial T}{\partial r} = -h\left(T - T_\infty\right) \text{ at } r = R \tag{3.104}$$

And initial condition: $T=T_0$ at t=0 (3.105)

In the derivation of Equation (3.102), it is assumed that thermal conductivity is independent of

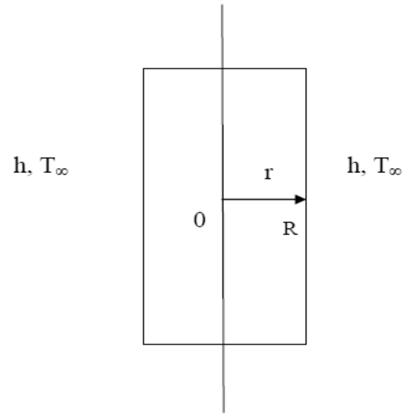

Fig 3.5: Schematic of slab

temperature. If not, temperature dependence must be applied, but the same procedure can be followed. Dimensionless parameters defined as

$$\theta = \frac{T - T_\infty}{T_0 - T_\infty}, \quad B = \frac{hR}{k}, \quad \tau = \frac{\alpha t}{R^2}, \quad x = \frac{r}{R} \tag{3.106}$$

For simplicity, Eq. (3.102) and boundary conditions can be rewritten in dimensionless form

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{x^m} \frac{\partial}{\partial x}\left(x^m \frac{\partial \theta}{\partial x} \right) \tag{3.107}$$

$$\frac{\partial \theta}{\partial x} = 0 \quad \text{at} \quad x = 0 \tag{3.108}$$

$$\frac{\partial \theta}{\partial x} = -B\theta \quad \text{at} \quad x = 1 \tag{3.109}$$

$$\theta = 1 \quad \text{at} \quad \tau = 0 \tag{3.110}$$

For a long slab with the same Biot number in both sides, temperature distribution is the same for each half, and so just one half can be considered

3.6.1 PROFILE1

The guess temperature profile is assumed as

$$\theta_p = a_0(\tau) + a_1(\tau)x + a_2(\tau)x^2 \tag{3.111}$$

Differentiating the above equation with respect to x we get

$$\frac{\partial \theta}{\partial x} = a_1 + 2a_2 x$$

Applying first boundary condition we have

$$a_1 + 2a_2 x = 0 \tag{3.112}$$

Thus

$$a_1 = 0 \tag{3.113}$$

Applying second boundary condition we have

$$a_1 + 2a_2 = -B\theta \tag{3.114}$$

Thus

$$a_2 = -\frac{B\theta}{2} \tag{3.115}$$

We can also write the second boundary condition as

$$\frac{\partial \theta}{\partial x} = -B\left(a_0 + a_1 + a_2\right) \tag{3.116}$$

Using the above expression we have

$$a_0 = \theta\left(1 + \frac{B}{2}\right) \tag{3.117}$$

Average temperature for long slab can be written as

$$\overline{\theta} = \int_0^1 \theta dx$$

Substituting the value of θ and integrating we have

$$\overline{\theta} = \theta + \frac{B\theta}{3} \tag{3.118}$$

Integrating non-dimensional governing equation we have

$$\int_0^1 x^m \frac{\partial \theta}{\partial \tau} dx = \int_0^1 \frac{\partial}{\partial x}\left(x^m \frac{\partial \theta}{\partial x} \right) dx$$

Simplifying the above equation we may write

$$\int_0^1 \frac{\partial \theta}{\partial \tau} dx = -B\theta \tag{3.119}$$

Considering the average temperature we may write

$$\frac{\partial \bar{\theta}}{\partial \tau} = -B\theta \tag{3.120}$$

Substituting the value of θ at equation (3.105) we have

$$\frac{\partial \bar{\theta}}{\partial \tau} = -\frac{3B\bar{\theta}}{B+3} \tag{3.121}$$

Integrating the equation (3.106) we may write as

$$\int_0^1 \frac{\partial \bar{\theta}}{\bar{\theta}} = -\int_0^1 \frac{3B}{B+3} \partial \tau \tag{3.122}$$

Thus by simplifying the above equation we may write

$$\bar{\theta} = \exp\left(-\frac{3B}{B+3} \tau \right) \tag{3.123}$$

Or we may write

$$\bar{\theta} = \exp(-P\tau) \tag{3.124}$$

Where

$$P = \frac{3B}{B+3} \tag{3.125}$$

Several profiles have been considered for the analysis. The corresponding modified Biot number, P, has been deduced for the analysis and is shown in Table 4.2.

3.7 TRAINSIENT HEAT CONDUCTION IN CYLINDER WITH DIFFERENT PROFILES

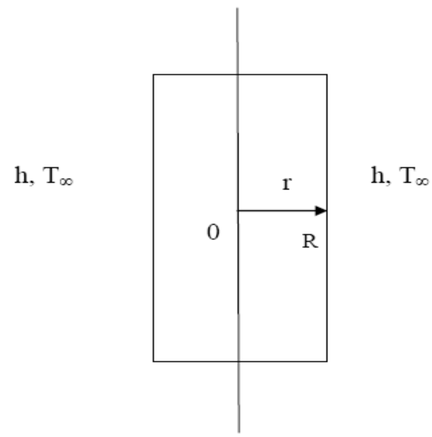

Fig 3.6: Schematic of cylinder

At the previous section we have assumed different profiles for getting the solution for average temperature in terms of time and Biot number for a slab geometry. A cylindrical geometry is also considered for analysis. Heat conduction equation expressed for cylindrical geometry is

$$\frac{\partial T}{\partial t} = \alpha \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r} \right) \qquad (3.126)$$

Boundary conditions are $\frac{\partial T}{\partial r} = 0$ at $r = 0$ (3.127)

$$k\frac{\partial T}{\partial r} = -h\left(T - T_{\infty} \right) \text{ at } r = R \qquad (3.128)$$

And initial condition $T=T_0$ at $t=0$ (3.129)

In the derivation of Equation (3.126), it is assumed that thermal conductivity is independent of temperature. If not, temperature dependence must be applied, but the same procedure can be followed. Dimensionless parameters defined as

$$\theta = \frac{T - T_\infty}{T_0 - T_\infty}, \quad B = \frac{hR}{k}, \quad \tau = \frac{\alpha t}{R^2}, \quad x = \frac{r}{R} \tag{3.130}$$

For simplicity, Eq. (3.126) and boundary conditions can be rewritten in dimensionless form

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{x}\frac{\partial}{\partial x}\left(x\frac{\partial \theta}{\partial x}\right) \tag{3.131}$$

$$\frac{\partial \theta}{\partial x} = 0 \quad \text{at} \quad x = 0 \tag{3.132}$$

$$\frac{\partial \theta}{\partial x} = -B\theta \quad \text{at} \quad x = 1 \tag{3.133}$$

$$\theta = 1 \quad \text{at} \quad \tau = 0 \tag{3.134}$$

For a long cylinder with the same Biot number in both sides, temperature distribution is the same for each half, and so just one half can be considered

3.7.2 PROFILE 1

The guess temperature profile is assumed as

$$\theta_p = a_0(\tau) + a_1(\tau)x + a_2(\tau)x^2 \tag{3.135}$$

Differentiating the above equation with respect to x we get

$$\frac{\partial \theta}{\partial x} = a_1 + 2a_2 x \tag{3.136}$$

Applying first boundary condition we have

$$a_1 + 2a_2 x = 0 \tag{3.137}$$

Thus

$$a_1 = 0 \tag{3.138}$$

Applying second boundary condition we have

$$a_1 + 2a_2 = -B\theta \tag{3.139}$$

Thus

$$a_2 = -\frac{B\theta}{2} \tag{3.140}$$

We can also write the second boundary condition as

$$\frac{\partial \theta}{\partial x} = -B\left(a_0 + a_1 + a_2\right) \tag{3.141}$$

From equation (3.117) we get

$$a_0 = \theta\left(1 + \frac{B}{2}\right) \tag{3.142}$$

Average temperature for long cylinder can be written as

$$\overline{\theta} = \left(m+1\right)\int_0^1 x^m \theta dx$$

Substituting the value of θ , m and integrating we get

$$\overline{\theta} = \theta\left(1 + \frac{B}{4}\right) \tag{3.143}$$

Integrating non-dimensional governing equation we have

$$\int_0^1 x^m \frac{\partial \theta}{\partial \tau} dx = \int_0^1 \frac{\partial}{\partial x}\left(x^m \frac{\partial \theta}{\partial x}\right) dx \tag{3.144}$$

Substituting the value of θ at equation (3.120) and from equation (3.114), (3.116), (118) we get

$$\int_0^1 \frac{\partial \theta}{\partial \tau} dx = -B\theta \tag{3.145}$$

Considering the average temperature we may write

$$\frac{\partial \overline{\theta}}{\partial \tau} = -2B\theta \tag{3.146}$$

Substituting the value of θ at above equation we have

$$\frac{\partial\overline{\theta}}{\partial\tau} = -\frac{8B\overline{\theta}}{4+B} \tag{3.147}$$

Integrating equation (3.123) we get

$$\int_0^1 \frac{\partial\overline{\theta}}{\overline{\theta}} = -\int_0^1 \frac{8B}{B+4}\partial\tau \tag{3.148}$$

Thus by simplifying the above equation we may write

$$\overline{\theta} = \exp\left(-\frac{8B}{B+4}\tau\right) \tag{3.149}$$

Or we may write

$$\overline{\theta} = \exp(-P\tau) \tag{3.150}$$

Where

$$P = \frac{8B}{B+4} \tag{3.151}$$

Several profiles have been considered for the analysis. The corresponding modified Biot number, P, has been deduced for the analysis and is shown in Table 4.3.

3.8 CLOSURE

At this section we have covered different heat conduction problems for the analysis. The analytical method used here is polynomial approximation method. Two problems are taken for heat flux, and two for heat generation. At the last a simple slab and cylinder is considered with different profiles. The result and discussion from the above analysis has been presented in the next chapter. Furthermore, the present prediction is compared with the analysis of P. Keshavarz and M. Taheri[1] , Jian Su [2] and E.J. Correa, R.M. Cotta [4].

CHAPTER 4

RESULT AND DISCUSSION

CHAPTER 4

RESULTS AND DISCUSSION

4.1 HEAT FLUX FOR BOTH SLAB AND CYLINDER

We have tried to analyze the heat conduction behavior for both Cartesian and cylindrical geometry. Based on the previous analysis closed form solution for temperature, Biot number, heat source parameter, and time for both slab and tube has been obtained. Fig 4.1 shows the variation of temperature with time for various heat source parameters for a slab. This fig contains Biot number as constant. With higher value of heat source parameter, the temperature inside the slab does not vary with time. However for lower value of heat source parameter, the temperature decreases with the increase of time.

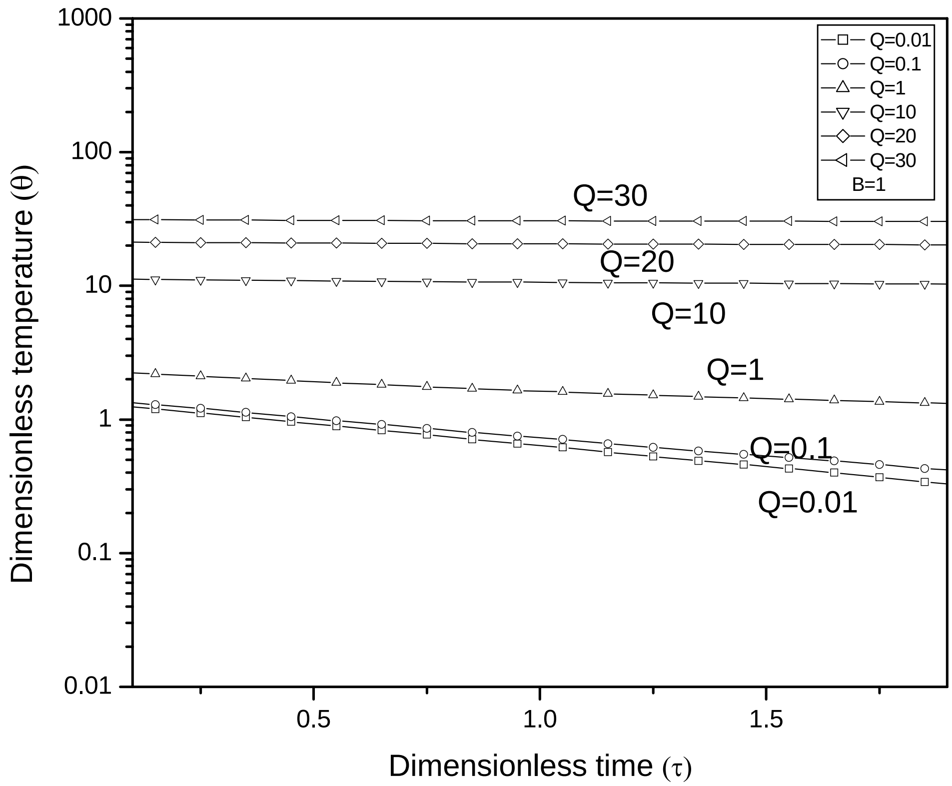

Fig 4.1 Average dimensionless temperature versus dimensionless time for slab, B=1

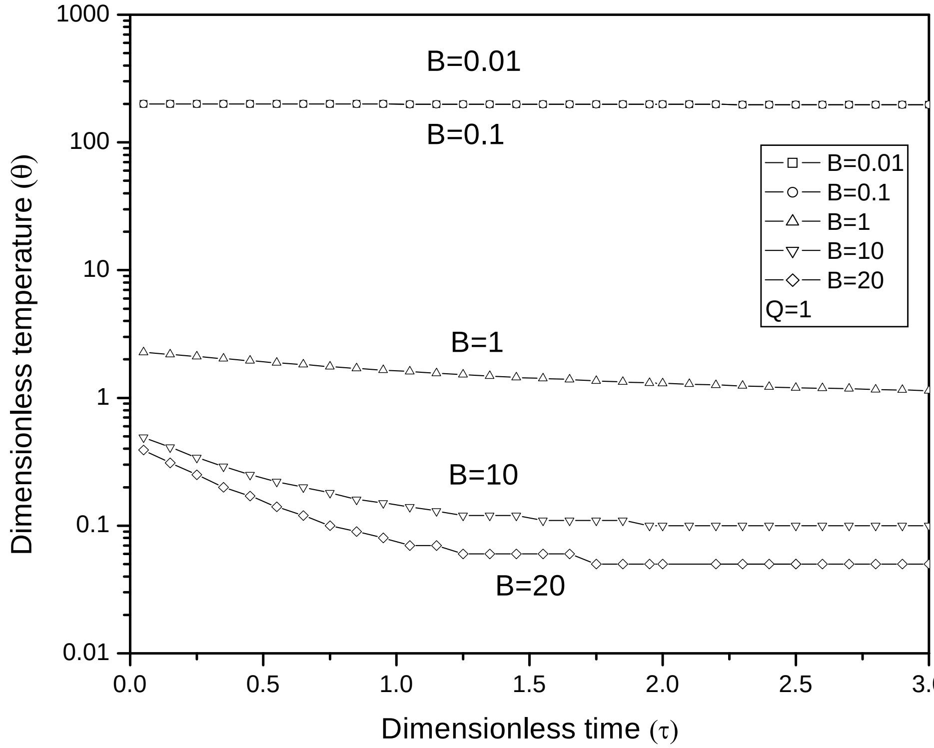

Fig 4.2 Average dimensionless temperature versus dimensionless time for slab, Q=1

Fig 4.2 shows the variation of temperature with time for various Biot numbers, having heat source parameter as constant for a slab. With lower value of Biot numbers, the temperature inside the slab does not vary with time. However for higher value of Biot numbers, the temperature decreases with the increase of time.

Similarly Fig (4.3) shows the variation of temperature with time for various heat source parameters for a tube. This fig contains Biot number as constant. With higher value of heat source parameter, the temperature inside the tube does not vary with time. However at lower values of heat source parameters, the temperature decreases with increase of time. Fig 4.4 shows the variation of temperature with time for various Biot numbers, having heat source parameter as constant for a tube. With lower value of Biot numbers, the temperature inside the tube does not vary with time. For higher value of Biot numbers, the temperature decreases with the increase of time.

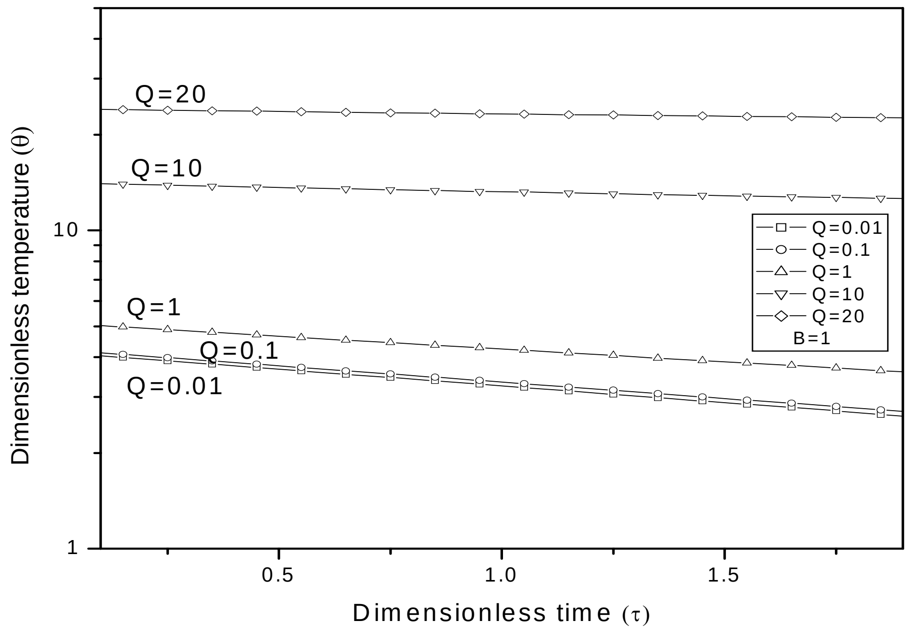

Fig 4.3 Average dimensionless temperature versus dimensionless time for cylinder, B=1

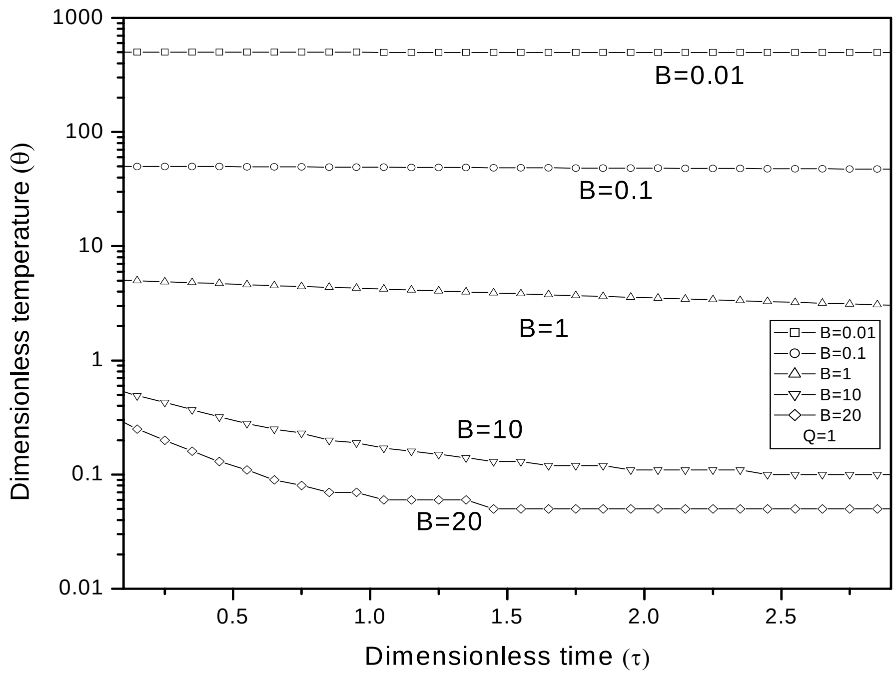

Fig 4.4 Average dimensionless temperature versus dimensionless time for cylinder, Q=1

4.2 HEAT GENERATION FOR BOTH SLAB AND TUBE

Fig (4.5) depicts the variation of temperature with time for various heat generation parameters for a slab. This fig contains Biot number as constant. With higher value of heat generation parameter, the variation of temperature inside the tube with time is less as compared to lower values of heat generation parameters. Fig (4.6) shows the variation of temperature with time for various Biot numbers, having constant heat generation parameter for a slab. With lower value of Biot numbers, the temperature inside the tube does not vary with time. As the Biot number increases, the temperature varies more with increase of time.

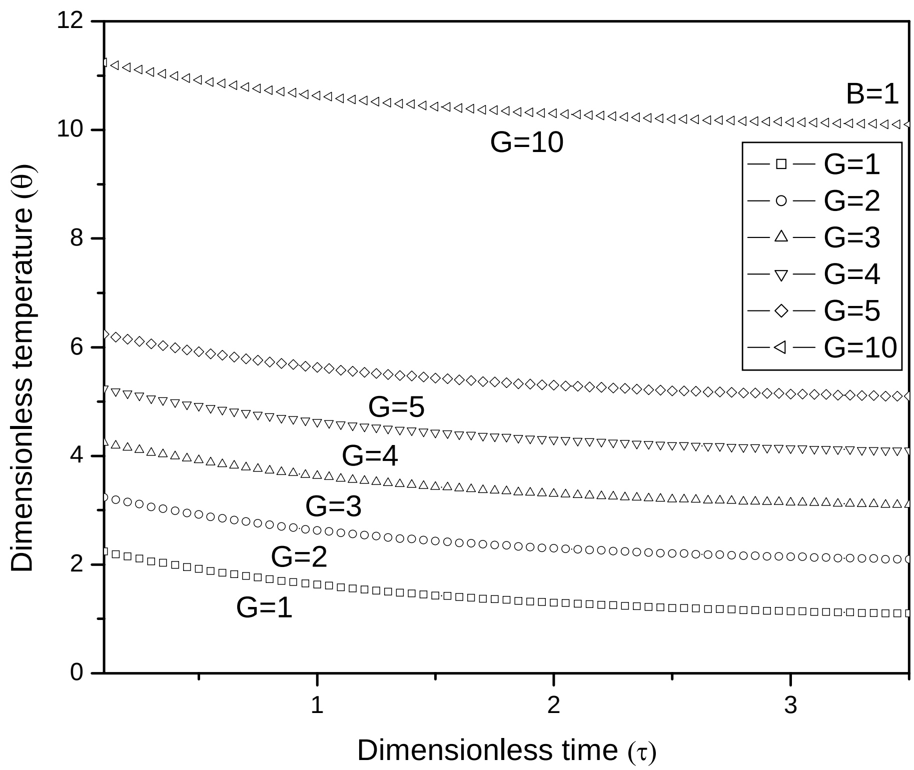

Fig4.5 Average dimensionless temperature versus dimensionless time in a slab with constant Biot number for different heat generation

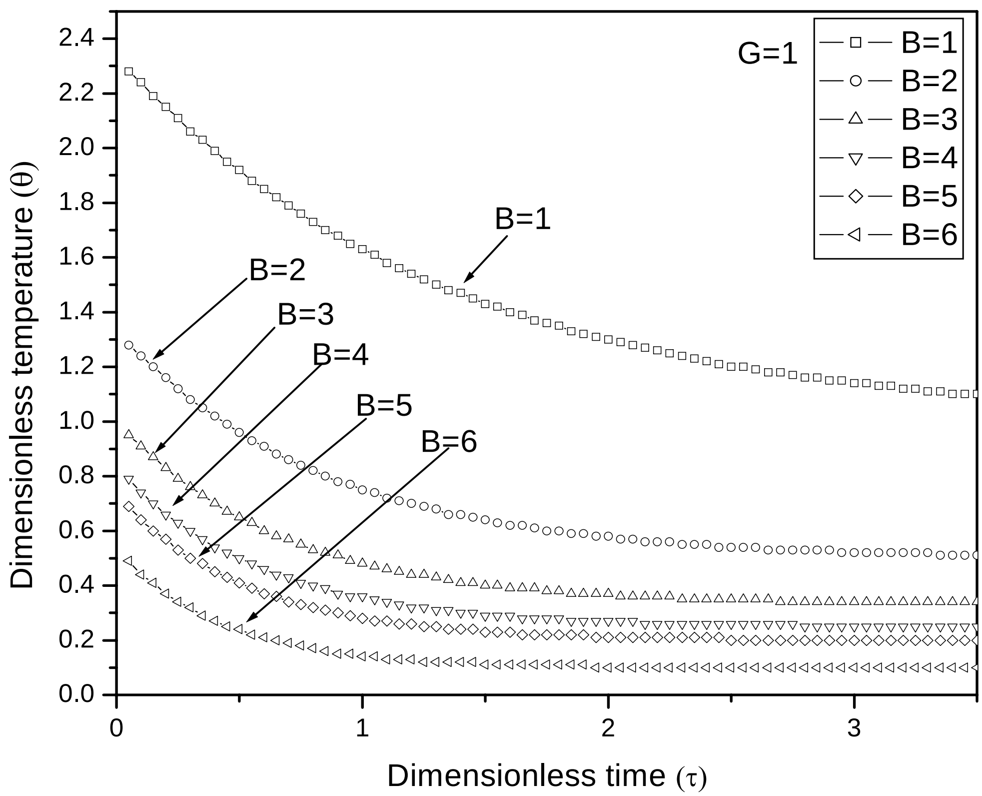

Fig4.6 Average dimensionless temperature versus dimensionless time in a slab with constant heat generation for different Biot number

Fig (4.7) depicts the variation of temperature with time for various heat generation parameters for a tube. This fig contains Biot number as constant. With higher value of heat generation parameter, the variation of temperature inside the tube with time is less as compared to lower values of heat generation parameters. Fig (4.8) shows the variation of temperature with time for various Biot numbers, having constant heat generation parameter for a tube. With lower value of Biot numbers, the temperature inside the tube does not vary with time. As the Biot number increases, the temperature varies more with increase of time.

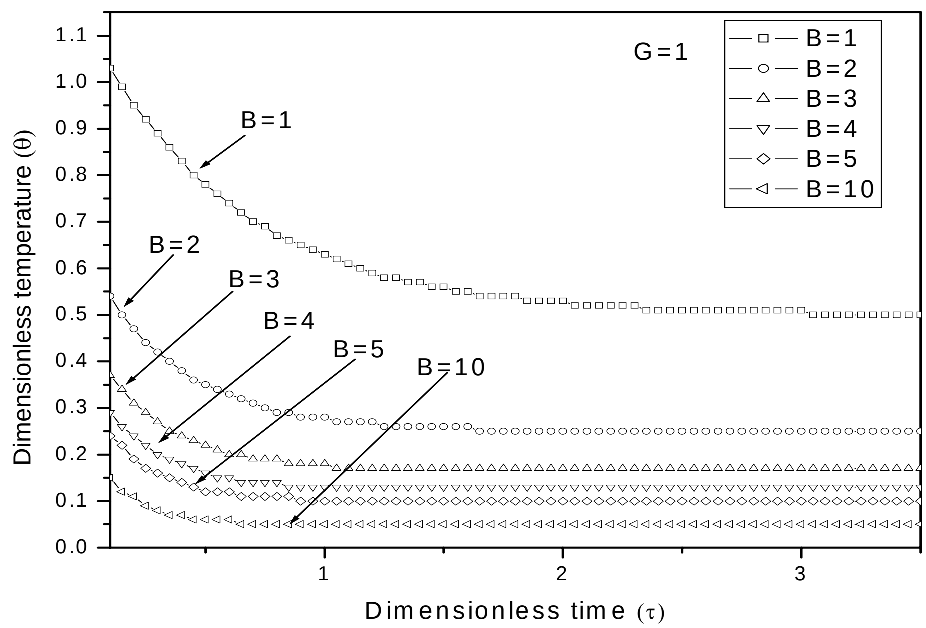

Fig 4.7 Average dimensionless temperature versus dimensionless time in a tube with constant heat generation for different Biot number

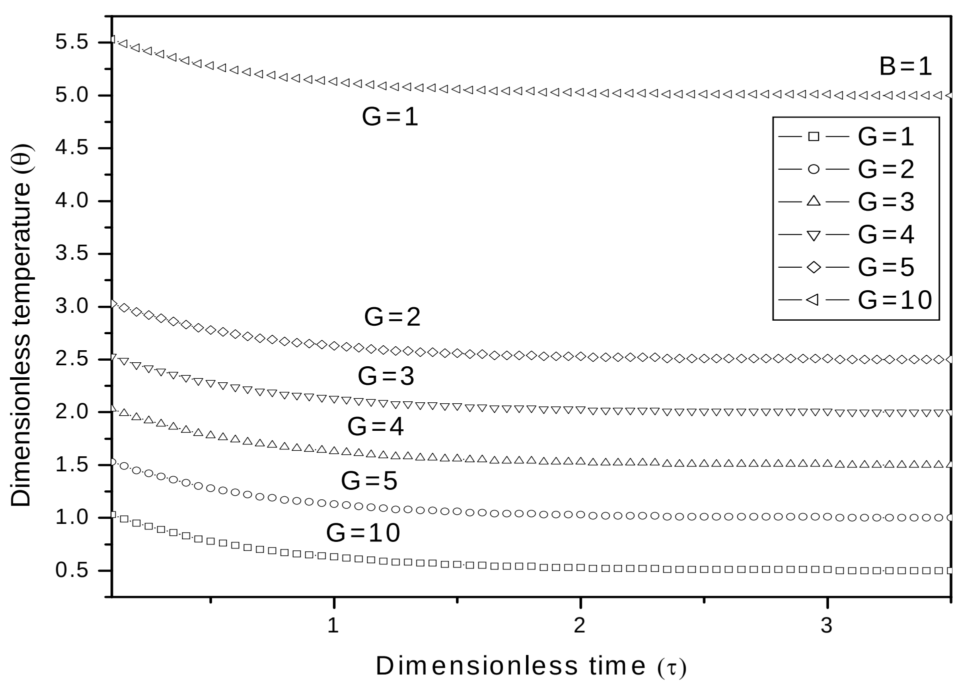

Fig 4.8 Average dimensionless temperature versus dimensionless time in a cylinder with constant Biot number for different heat generation

4.3 TRAINSIENT HEAT CONDUCTION IN SLAB WITH DIFFERENT PROFILES

We have considered a variety of temperature profiles to see their effect on the solution. Based on the analysis a modified Biot number has been proposed, which is independent of geometry of the problem. Fig (4.9-4.10) shows the variety of temperature with time for different values of modified Biot number, P. It is seen that, for higher values of P represent higher values of Biot number. Therefore the heat removed from the solid to surrounding is higher at higher Biot number. This leads to sudden change in temperature for higher value of P. This trend is observed in the present prediction and is shown in fig (4.9).

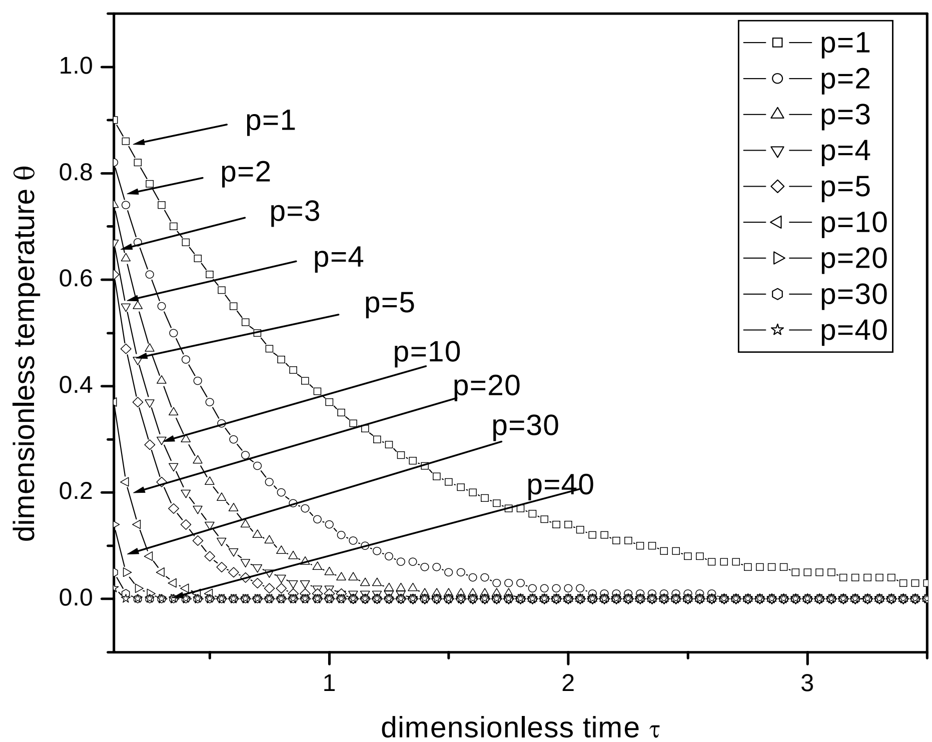

Fig 4.9 Variation of average temperature with dimensionless time, for P=1 to 40 for a slab.

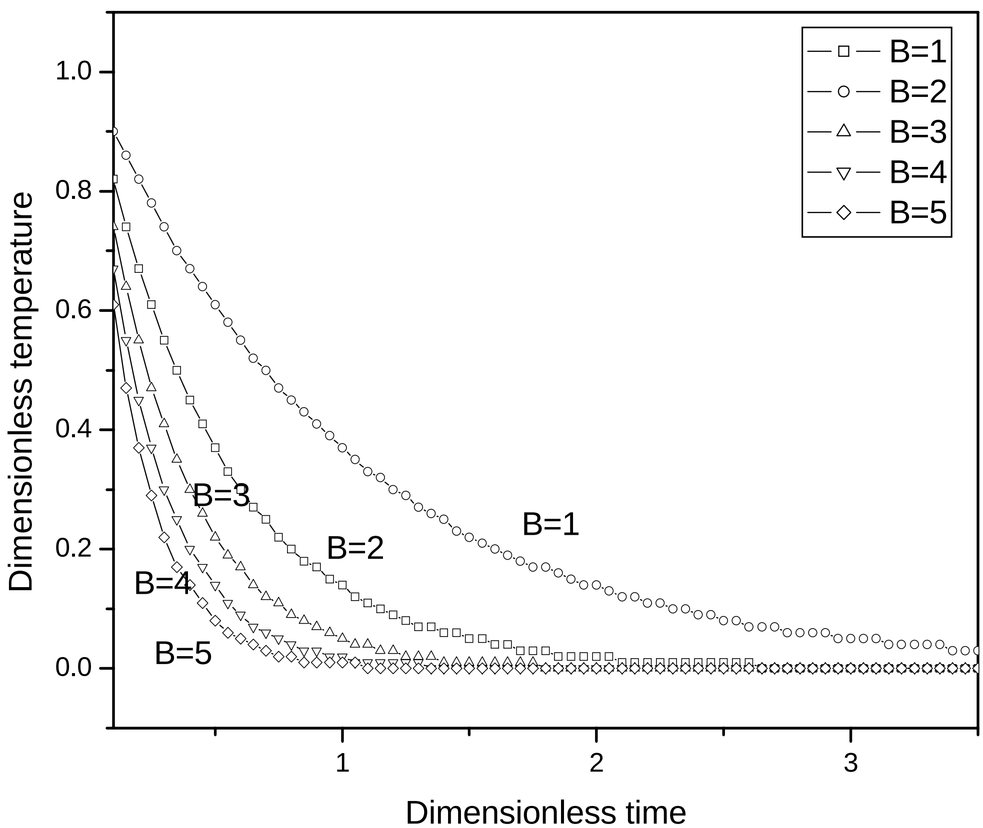

Fig 4.10 Variation of average temperature with dimensionless time, for B=1 to 5 for a slab

Fig (4.11) shows the comparison of present analysis with the other available results. These include classified lumped system analysis and exact solution by E.J. Correa and R.M. Cotta [4] of a slab. It is observed that the present prediction shows a better result compared to CLSA. The present prediction agrees well with the exact solution of E.J. Correa and R.M. Cotta [4] at higher time. However at shorter time, the present analysis under predicts the temperature in solid compared to the exact solution. This may be due to the consideration of lumped model for the analysis.

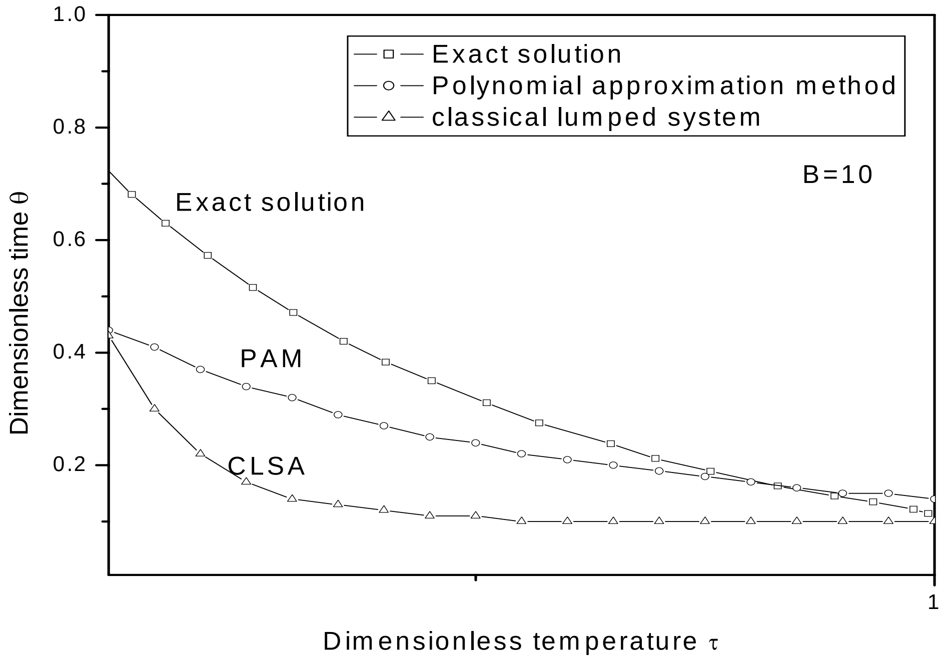

Fig 4.11 Comparison of solutions of PAM, CLSA and Exact solution for a slab having internal heat generation

4.4 TABULATION

Table 4.1 Comparison of solutions of average temperature obtained from different heat conduction problems

Average temperature	Slab with heat flux	Slab with heat generation	Tube with heat flux	Tube with heat generation
θ	$\theta = \left(\frac{e^{-U\tau}+V}{U}\right)$ Where $U = \frac{B}{1+B/3}$, $V = \frac{Q}{1+B/3}$	$\theta = \frac{e^{-\tau U}+V}{U}$ Where $U = \frac{B}{(1+B/3)}$, $V = \frac{G}{(1+\frac{B}{3})}$	$\theta = \left(\frac{e^{-U\tau}+V}{U}\right)$ Where $U = \frac{B}{(4+B)/8}$, $V = \frac{Q}{(4+B)/8}$	$\theta = \frac{e^{-\tau U}+V}{U}$ Where $U = \left(\frac{2B}{1+B/4}\right)$, $V = \frac{G}{(1+B/4)}$

Table 4.2 Comparison of modified Biot number against various temperature profiles for a slab

Si No	Profile	Value of P
1	$\theta_p = a_0(\tau) + a_1(\tau)x + a_2(\tau)x^2$	$P = \frac{3B}{B+3}$
2	$\theta = a_0 + a_1(x^2 - x) + a_2(x^3 - x^2)$	$P = \frac{13B}{13+12B}$
3	$\theta = a_0 + a_1(x^4 - x^2) + a_2(x^3 - x)$	$P = \frac{30B}{30+17B}$
4	$\theta = a_0 + a_1(x^4 - x^2) + a_2(x^2 - x)$	$P = \frac{30B}{30+13B}$
5	$\theta = a_0 + a_1(x^4 - x) + a_2(x^5 - x^3)$	$P = \frac{24B}{24+13B}$
6	$\theta = a_0 + a_1(x^4 - x^3) + a_2(x^4 - x)$	$P = \frac{20B}{20+21B}$

Table 4.3 Comparison of modified Biot number against various temperature profiles for a cylinder

Si No.	Profile	Value of P
1	$\theta_p = a_0(\tau) + a_1(\tau)x + a_2(\tau)x^2$	$P = \frac{8B}{B+4}$
2	$\theta = a_0 + a_1(x^2 - x) + a_2x^2$	$P = \frac{4B}{B+2}$
3	$\theta = a_0 + a_1(x^3 - x) + a_2x^3$	$P = \frac{30B}{3B+15}$
4	$\theta = a_0 + a_1(x^4 - x) + a_2(x^3 - x^2)$	$P = \frac{10B}{4B+5}$

CHAPTER 5

CONCLUSION & SCOPE FOR FUTURE WORK

CHAPTER 5
CONCLUSIONS & SCOPE FOR FUTURE WORK

5.1 CONCLUSIONS

An improved lumped parameter model is applied to the transient heat conduction in a long slab and long cylinder. Polynomial approximation method is used to predict the transient distribution temperature of the slab and tube geometry. Four different cases namely, boundary heat flux for both slab and tube and, heat generation in both slab and tube has been analyzed. Additionally different temperature profiles have been used to obtain solutions for a slab. A unique number, known as modified Biot number is, obtained from the analysis. It is seen that the modified Biot number, which is a function of Biot number, plays important role in the transfer of heat in the solid. Based on the analysis the following conclusions have been obtained.

1. Initially a slab subjected to heat flux on one side and convective heat transfer on the other side is considered for the analysis. Based on the analysis, a closed form solution has been obtained.

$$\theta = \left(\frac{e^{-U\tau} + V}{U} \right)$$

Where $U = \frac{B}{1 + B/3}$, $V = \frac{Q}{1 + B/3}$

2. A long cylinder subjected to heat flux on one side and convective heat transfer on the other side is considered for the analysis. Based on the analysis, a solution has been obtained.

$$\theta = \left(\frac{e^{-U\tau} + V}{U} \right)$$

Where $U = \frac{B}{(4 + B)/8}$, $V = \frac{Q}{(4 + B)/8}$

3. A slab subjected to heat generation at one side and convective heat transfer on the other side is considered for the analysis. Based on the analysis, a closed form solution has been obtained.

$$\theta = \frac{e^{-\tau U} + V}{U}$$

Where $U = \frac{B}{(4+B)/8}$, $V = \frac{G}{(1+\frac{B}{3})}$

4. A long cylinder subjected to heat generation at one side and convective heat transfer on the other side is considered for the analysis. Based on the analysis a closed form solution has been obtained.

$$\theta = \frac{e^{-\tau U} + V}{U}$$

Where $U = \left(\frac{2B}{1+B/4}\right)$, $V = \frac{G}{(1+B/4)}$

5. Based on the analysis a unique parameter known as modified Boit number obtained from the analysis and is shown in Table 2 and 3. With higher value of heat source parameter, the temperature inside the tube does not vary with time. However at lower values of heat source parameters, the temperature decreases with increase of time. With lower value of Biot numbers, the temperature inside the tube does not vary with time. For higher value of Biot numbers, the temperature decreases with the increase of time.

5.2 SCOPE FOR FURTHER WORK

1. Polynomial approximation method can be used to obtain solution of more complex problem involving variable properties and variable heat transfer coefficients, radiation at the surface of the slab.

2. Other approximation method, such as Heat Balance Integral method, Biots variation method can be used to obtain the solution for various complex heat transfer problems.

3. Efforts can be made to analyze two dimensional unsteady problems by employing various approximate methods.

CHAPTER 6

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