OPTIMAL PLACEMENT OF ACTUATORS IN FIBER REINFORCED POLYMER COMPOSITE SHELL STRUCTURES USING GENETIC ALGORITHM

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CERTIFICATE

This is to certify that this report entitled, "**OPTIMAL PLACEMENT OF ACTUATORS IN FIBER REINFORCED POLYMER COMPOSITE SHELL STRUCTURES USING GENETIC ALGORITHM**" submitted by Anjana Satpathy in partial fulfillments for the requirements for the award of Bachelor of Technology Degree in Mechanical Engineering at National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by her under my supervision and guidance.

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ABSTRACT

Active vibration control of smart FRP composite structures finds use in high performance structures especially in light weight composite structures. Proper implementation of such smart structure systems demands complete understanding of their responses, optimal placement of sensors and actuators, and design of an appropriate control system. In the present work, an improved genetic algorithm (GA) based optimal collocated sensors and actuators of smart fiber reinforced polymer (FRP) composite shell structures has been presented. Layered shell finite elements have been formulated and the formulation has been validated for coupled electromechanical analysis of curved smart FRP composite structures having piezoelectric sensors and actuators patches. Modal analysis has been performed to transfer the coupled finite element equation to state space equation. An integer-coded GA-based open-loop procedure has been implemented for optimal placement of actuators for maximizing controllability index. This type of GA with uniform crossover and mutation technique has been developed to efficiently search for optimal locations of sensors/actuators.In this project, we have used integer coded GA to find optimal placement of actuators on spherical shell structures and semi-circular ring.

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CHAPTER 1

INTRODUCTION

Active vibration control in distributed structures is of practical interest because of the demanding requirement for guaranteed performance. This is particularly important in light-weight structures as they generally have low internal damping. An active vibration control system requires sensors, actuators, and a controller. The design process of such a system encompasses three main phases such as structural design, optimal placement of sensors and actuators and controller design. In vibration suppression of structures, locations of sensors and actuators have a major influence on the performance of the control system. It is well known that misplaced sensors and actuators lead to problems such as the lack of observability or controllability. Active vibration control is defined as a technique in which the vibration of a structure is reduced by applying counter force to the structure that is appropriately out of phase but equal in force and amplitude to the original vibration. As a result two opposing forces cancel each other, and structure essentially stops vibrating. Techniques like use of springs, pads, dampers, etc have been used previously in order to control vibrations. These techniques are known as 'Passive Vibration Control Techniques'. They have limitations of versatility and can control the frequencies only within a particular range of bandwidth. Hence there is a requirement for 'Active Vibration Control'. 'Active Vibration Control' makes use of 'Smart Structures'. This system requires sensors, actuators, a source of power and a compensator that performs well when vibration occurs. Smart Structures are used in bridges, trusses, buildings, mechanical systems, space vehicles, telescopes, and so on. The analysis of a basic structure can help improve the performance of the structures under poor working conditions involving vibrations. "A Smart Structure" means a structure that can sense an external disturbance and respond to that with active control in real time to maintain the mission requirements. A Smart Structure typically consists of a host structure incorporated with sensors and actuators coordinated by a controller. The integrated structured system is called Smart Structure because it has the ability to perform self diagnosis and adapt to environmental change. One promising application of such smart structure is the control and suppression of unwanted structural vibrations. Fig. 1 depicts the schematic representation of the basic elements of a smart structure

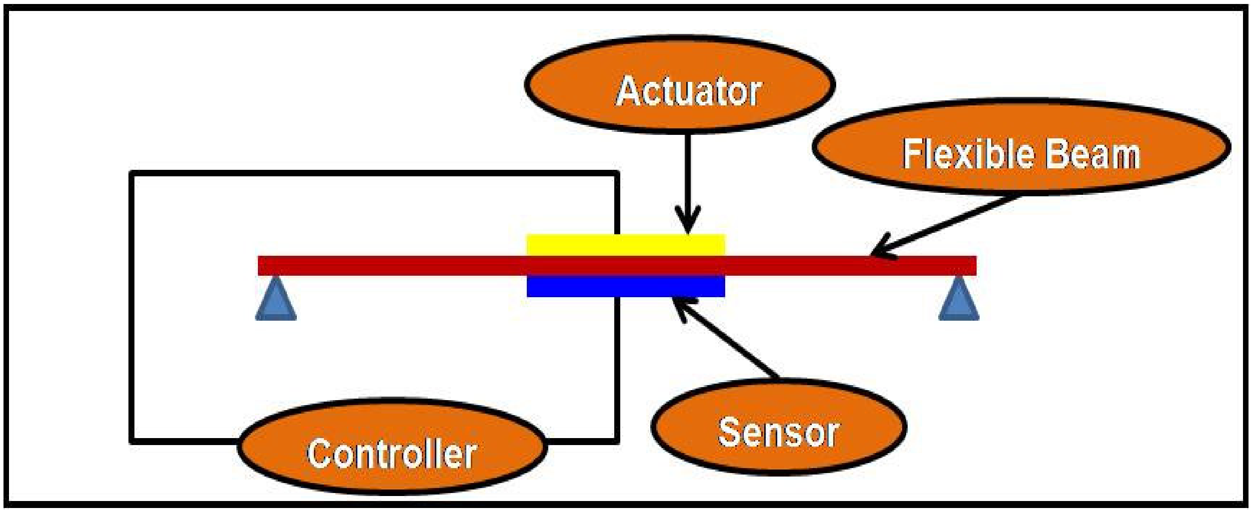


Fig. 1 Schematic representation of the basic elements of a Smart Structure

- **Sensor Patch**

It is bonded to the host structure (Beam). It is generally made up of piezoelectric crystals (one of the smartest materials). It senses the disturbance of the beam and generates a charge which is directly proportional to its strain. Direct piezoelectric effect is used here.

- **Controller**

The charge developed by the sensor is given to the controller. The controller lines the charge according to suitable control gain and then the charge is fed to the actuator. Controller also forms the feed back transfer function for this system.

- **Actuator Patch**

The lined up charge from the controller is fed to the actuator. An actuator is a piezoelectric patch bonded to the host. Due to the input voltage, actuator causes pinching action (or generates shear force along the surface of the host which acts as the damping force and helps in the attenuating vibration motion of the beam. Converse piezoelectric effect is used here.

A given structure can vibrate with many modes. The design of controller for all the modes is very difficult. However, all the modes do not contribute significantly to the overall disturbance. Hence, we filter out the modes which cause the maximum disturbance. Hence a controller can be designed to control only these modes.

In this project we have used an improved integer coded GA along with improved uniform crossover and mutation technique for determination of optimal placement of sensors and actuators. Optimal placement of PZT actuators on the curved smart FRP composite structures (i.e. semicircular ring, spherical and ellipsoidal shell panel) have been studied based on the controllability index, which is the singular value of the control input matrix. Higher the controllability index, the lower will be the electrical potential required for active control. Integer coded genetic algorithm has been applied to efficiently find the maximum controllability index.

1.1 FIBER REINFORCED POLYMER

A Fiber Reinforced Polymer (FRP) composite is defined as a polymer (plastic) matrix, either ***thermoset*** or ***thermoplastic***, that is reinforced (combined) with a fiber or other reinforcing material with a sufficient ***aspect ratio*** (length to thickness) to provide a discernable reinforcing function in one or more directions. FRP composites are different from traditional construction materials such as steel or aluminum. FRP composites are anisotropic (properties apparent in the direction of the applied load) whereas steel or aluminum is isotropic (uniform properties in all directions, independent of applied load). Therefore, FRP composite properties are directional, meaning that the best mechanical properties are in the direction of the fiber placement. Composites are similar to reinforced concrete where the rebar is embedded in an isotropic matrix called concrete.

Composition

Composites are composed of:

- **Resins** - The primary functions of the resin are to transfer stress between the reinforcing fibers, act as a glue to hold the fibers together, and protect the fibers from mechanical and environmental damage. The most common resins used in the production of FRP grating are ***polyesters*** (including orthophthalic-“ortho” and isophthalic-“iso”), ***vinyl esters*** and ***phenolics***.
- **Reinforcements** - The primary function of fibers or reinforcements is to carry load along the length of the fiber to provide strength and stiffness in one direction. Reinforcements can be oriented to provide

tailored properties in the direction of the loads imparted on the end product. The largest volume reinforcement is glass fiber.

- **Fillers** - Fillers are used to improve performance and reduce the cost of a composite by lowering compound cost of the significantly more expensive resin and imparting benefits as shrinkage control, surface smoothness, and crack resistance.
- **Additives** - Additives and modifier ingredients expand the usefulness of polymers, enhance their processability or extend product durability

Each of these constituent materials or ingredients play an important role in the processing and final performance of the end product.

There are a wide variety of processes available to the composites manufacturer to produce cost efficient products like pultrusion and various other molding processes.

CHAPTER 2

LITERATURE REVIEW

Vibration suppression performance in both active and passive damping decisively depends on the number, shape, size and location of the piezoelectric ceramic elements used as sensors and actuators [1,2]. The same holds for shape control, vibroacoustic control and structural health monitoring. Depending on the complexity of the structure, analytic or numerical models might prove more appropriate to describe its behavior. A number of different objective functions, design variables, constraints and solution methods can be applied for the optimization of a target application. The following subsections reviews a representative portion of the work performed in the last decade towards the optimal placement of sensors and actuators for vibration suppression. The articles reviewed here have been classified based on the optimization algorithm used.

2.1 Parameter Variation

Informal optimization consisting of parameter variation studies can deliver useful insight into the optimization task, in particular if the solution space can be explored with a reasonable number of configurations. This is the case for simple structures such as beams. While investigating the multiple mode passive vibration suppression with piezoelectric materials and resonant shunts, Hollkamp [3] estimated the generalized electromechanical coupling coefficient of a pair of piezoelectric ceramic tiles attached to a cantilever beam at different locations. Kang *et al* [4] optimized the placement of piezoelectric collocated sensor/actuator pairs for active vibration control of laminated beams by maximizing the structural damping index, a weighted sum of the achieved modal damping of each vibrational mode. Parametric studies were presented for the damping ratio as a function of the location of piezoelectric ceramic elements with given length and various outer-layer fiber orientations. Vibration suppression analysis of cantilever beam with piezoelectric sensors/actuators subjected to an exciting force has been performed by Zhang and Kirpitchenko [5]. They considered two sets of surface bonded piezoelectric patches with three locations of patches and experimentally showed that the damping of combined beam-piezoelectric patches system increased by 8-10 times in comparison to that of mechanical system. Formal optimization techniques, on the other hand, can be classified into deterministic methods and stochastic methods.

2.2 Deterministic Methods

Most mathematical programming methods work locally and are very efficient given that the assumptions on continuity, differentiability and convexity of the solution space are satisfied. Aside from the convexity assumption, this is mostly the case for basic structures such as beams and plates. Classical beam and plate structural models were used to derive cost functions for determining the optimum placement and thickness of embedded and surface mounted piezoactuators by Main et al. [6]. An optimization procedure was used to develop a design guide for simplified determination of piezoactuator size and placement. Li *et al* [7] presented an optimal design methodology for piezoelectric ceramic actuators/sensors and feedback gains towards the vibration suppression in flexible structures and studied the influence of the actuator/sensor pairs on the mass and stiffness properties of the composite structure. The proposed composite objective function included the control performance as well as the added mass. However, the gradient based optimization methods applied to the simple case of a beam structure, was prone to getting trapped in local optima. Kang *et al* [8] carried out an investigation on laminated plates where the optimization was carried out using the gradient method. Haramoto et al. [9] presented the optimal placement of two pairs of sensors and actuators in order to maximize the H_2 norm of the closed loop system for a simply supported beam using quasi-Newton method. Mukherjee and Joshi [10] obtained the actuator layout by minimizing the power consumption in order to achieve a specified displacement of plate structure using iterative procedure. Wang and Wang [11] proposed a controllability index for optimal locations and size of piezoelectric actuators for the beam model in order to maximize modal control forces and reported that higher the controllability index, the smaller would be the electrical potential required for active control. However, they did not consider control spillover of the higher order modes, which would give closed loop instability by maximizing modal control forces of the higher order modes. Seeger and Gabbert [12] proposed an optimization algorithm for the optimal positioning of collocated actuator/sensor patch pairs on a simply supported plate structure. Conjugate gradient method was applied to minimize the H_2-norm of the transfer function between an external excitation disturbance and the plate vibration amplitude. The constrained optimization algorithm used the augmented Lagrangian function in order to avoid patch overlapping. The quasi-modal sensor and quasi-modal actuator were developed for finding optimal placement and sizes of sensors and actuator on rectangular plate by Sun *et al* [13]. Sun and Tong [14] extended the investigation to simply supported closed- and open-form cylindrical shell structures. An energy based approach for optimal positioning of piezoelectric actuators and sensors on a flexible structure was presented by Leleu et al. [15]. First, a two-dimensional (2–D) model of a piezoelectric actuator bonded to a plate was obtained and then, a Ritz formulation was used to find a state model of the system in view of its control. Selection process for piezoelectric transducers (PZT) used as actuator elements for suppressing vibrations in a flexible beam system was discussed by Kermani et al. [16]. The effects of changing physical parameters such as the relative thickness of the piezoelectric ceramic with respect to the beam, the optimum location of the PZT actuator, and the length of the PZTs were studied based on the singular value decomposition of the controllability Grammian of the resulting system. Modal based correction methods were applied by Rose [17] for the placement of piezoelectric ceramic modules on a circular plate. These methods allow the negotiation of changes introduced by the piezoelectric element's mass and stiffness. The generalized electromechanical coupling coefficient was maximized by applying gradient-based methods in a two-step approach. Halim and Moheimani [18] suggested a criterion for the optimal placement of collocated piezoelectric ceramic actuator/sensor pairs on a thin plate using modal and spatial controllability. The spatial controllability was used to find the optimal placement of collocated actuator/sensor pairs for effective average vibration reduction over the entire structure, while maintaining modal controllability and observability of selected vibration modes. Sun and Tong [19] presented an investigation into design optimization of actuator patterns for static shape control of composite plates with piezoelectric actuator patches. An energy optimization based method for finding the optimal control voltages that can actuate a structure shape close to the desired one within a given error was described. Emilio et al. [20] proposed a simultaneous search for an optimal topology of a flexible structure as well as the optimal position of the piezoceramic in the design. The method was implemented based on the SIMP (‘Solid Isotropic Material with Penalization’) material model and the examples presented were limited to two-dimensional models.

2.3 Stochastic Methods

Engineering design problems, are often of a discrete nature (e.g. the number of actuators), so that the above methods described in the previous subsection are not applicable or tend to get trapped in local optima. In order to overcome these limitations, the scientific community has put significant effort into the investigation of stochastic optimization methods. Stochastic optimization methods can handle search spaces involving both discrete and continuous domains, non-convex objective functions, and objective functions or constraints lacking differentiability. A drawback is that stochastic search methods are often computationally expensive. Genetic algorithm (GA) has been extensively used for optimization of engineering problems in recent times and some of the important works in this direction are described here. Rao *et al* [21] were the first to apply genetic algorithms to the problem of optimal actuators placement in an actively controlled two-bay truss. The dissipation energy of the active controller was maximized for a fixed number of three actuators. A strategy for determining the optimal number of actuators and their respective locations in the active vibration control of a 72-bar space truss was presented by Yan and Yam [22] where the eigenvalues of the energy correlative matrix of the input control force were used to determine an optimal number of actuators for vibration control. They reported that depending on the desired controllability level, these can be equal to or less than the number of degrees of freedom to be controlled. Using a binary-encoded genetic algorithm, Bishop and Striz [23] demonstrated the optimal placement of passive ideal viscous dampers on space trusses subjected to different loading. The kinetic and strain energy remaining in a system at the end of a full time-domain transient analysis, as well as the number of actuators, were combined to form a penalty function. Abdullah et al. [24] used genetic algorithm to simultaneously place collocated sensor/actuator pairs in multi-storey building while using output feedback as the control law in terms of minimizing the quadratic performance i.e. weighted energy of the system. They found optimal gain using Davidon-Fletcher-Powell gradient-based optimization algorithm by choosing weighting matrices [Q] and [R] using trial and error and concluded that the decision variables in this optimization problem were greatly dependent on the selection of weighting matrices. They also used binary coded GA with the length of the gene string as the number of floors in multi-storey building, which led to large number of function evaluations and large number of generations to reach near optimal solution. Richardson and Abdullah [25] used a real-encoded genetic algorithm for optimal placement of sensors and active tendon mechanisms on high-rise civil structures which were susceptible to vibrations due to earthquakes, hurricanes or other abnormal loads such as explosions. The proposed method allowed for the simultaneous determination of the optimal controller gains. However, real-encoded genetic algorithm is more suitable for continuous search space where structural responses are obtained analytically. Results by Gaudenzi *et al* [26] provided insight into the problem of optimal placement, sizing and loading of piezoelectric actuators for damping beam vibrations. A fundamental solution, formulated for a single piezoelectric actuator pair, was used in the framework of a genetic algorithm optimization. A float-encoded genetic algorithm for the integrated optimization of piezoelectric actuator and sensor locations and feedback gains for active vibration control was introduced by Zhang *et al* [27] and concluded that the float-encoded genetic algorithm was less likely to become trapped in local minima compared to the adaptive binary genetic algorithm and converged faster to the solution. A cantilever beam was presented as an optimization example, for which the performance function is based on maximizing the dissipation energy of the active controller. However, float-encoded genetic algorithm was also appropriate for continuous search space. A similar problem was tackled by Yang *et al* [28] and they presented a simultaneous optimization method considering several design variables such as placement of collocated piezoelectric sensors/actuators, size of sensor/actuator and feedback control gain for vibration suppression of simply supported beam by minimizing the equivalent total mechanical energy of the system. However, they did not consider input energy in the used objective function i.e. equivalent total mechanical energy. This type of chromosome representation used will not be feasible for multi input system with more sensors and actuators and it will also lead to more trial and error to impose bound for the entire feedback control gain matrix elements. The same authors later extended the method cited above to the investigation of plates and cylindrical shells [29] with dynamic constraints, included directly in the modified real-encoded genetic algorithm, and penalizes overlapping piezoelectric patches. Binary coded genetic algorithms based on the open loop performance were used by Han and Lee [30] to find efficient locations for six sensors and two actuators out of 99 possible sub-areas on a cantilever composite plate. Two criteria for the optimal placement of piezoelectric actuators for vibration control were suggested by Sadri *et al* [31] using modal controllability and the controllability Grammian. The number of actuators, their sizes and their optimal locations for maximum controllability of isotropic plates were determined using genetic algorithms. They used Gray coded genetic algorithm to find the eight coordinates of two piezoelectric actuators in a simply supported plate based on the open loop performance. However, this type of Gray coded GA leads to increased string length. The authors later applied the modal controllability as a criterion for optimal placement of piezoelectric actuators for panel flutter suppression [32]. Quek *et al* [33] used the classical direct pattern search method to maximize the active damping of a laminated composite plate. The starting point for the pattern search was selected based on the maxima of integrated normal strains consistent with the size of the collocated piezoelectric sensor/actuator pair used. Optimization performance indices were based on modal and system controllability. Guo et al. [34] presented a sensor placement optimization performance index based on the damage detection in the two dimensional truss structures using binary coded genetic algorithm. Li et al. [35] proposed two level genetic algorithms (TLGA) for optimal placement of active tendon actuators in multi storey building by minimizing the maximum top floor displacement. This proposed TLGA might be feasible for this type of optimization problem and for active vibration control of large-scale structures with complete electromechanical analysis considering PZT sensors/actuators but this will not be computationally feasible because there will more possible actuators locations. The positions of four piezoelectric patches for adaptive feed-forward control were chosen out of 64 candidate locations on a cantilever aluminum plate by [36] and concluded that the maximization of the controllability Grammian through a genetic algorithm guaranteed a minimum control force for minimizing the vibration response at three selected points of the plate. Wang *et al* [37] addressed the topology optimization of collocated sensors/ actuators pairs for torsional vibration control of a laminated composite cantilever plate using output feedback control. They used binary coded genetic algorithm for optimization, which was not computationally efficient for actuator/sensor location in terms of number of function evaluations, and generations for convergence. Liu et al. [38] used a spatial H_2 norm of the closed loop transfer matrix for finding the optimal nodal points for sensing displacement and applying actuation for the control of a fixed-fixed plate. This method did not address a complete coupled electromechanical analysis and used binary coded genetic algorithm leading to very large number of generations for convergence. Optimal placements and sizes of sensors and actuators attached to an inflated torus were found by Jha and Inman [39] using a binary encoded genetic algorithm. Performance indices were defined using modal controllability (minimum energy requirement) and observability (maximum output energy for a good signal to-noise ratio). Belloli and Ermanni [40] presented optimum placement of piezoelectric ceramic elements for vibration suppression of rear wing of a race car. The optimization procedure included a knowledge-based CAD model, an FE model and an evolutionary algorithm optimization loop controlled by the proprietary software tool DynOPS.

2.4 Motivation and Objectives

Even though many works have been reported in the broad area of active vibration control of smart structures, there are still scopes and need for improvement in better understanding of behavior of smart shell structures for achieving better actuation and superior control performance of such structures. From the exhaustive literature review, the following important observations have been made.

i) A large number of works are available in the form of beam and plate finite elements for analysis of piezo-laminated smart FRP structures, not many works are available in the form of shell finite element for such structures.

ii) Many existing literatures in optimal placement of sensors/actuators have used GA but they require large number of generations and function evaluations for reaching near optimal solution.

Keeping the above points in mind, the specific objectives of the present thesis have been laid down as

i) Development of a shell finite element capable of analysis coupled electro- mechanical responses of smart FRP shell structures

ii) Development of an improved GA based optimal placement scheme for achieving better controllability of such structures

CHAPTER 3

GA FOR OPTIMAL PLACEMENT OF ACTUATORS

3.1 GENETIC ALGORITHM

A **genetic algorithm (GA)** is a search technique used in computing to find exact or approximate solutions to optimization and search problems. Genetic algorithms are categorized as global search heuristics. Genetic algorithms are a particular class of evolutionary algorithms (also known as evolutionary computation) that use techniques inspired by evolutionary biology such as inheritance, mutation ,selection and crossover (also called recombination).

The current framework of GAs was first proposed by Holland[41] and his student Jong[42], and was finally popularized by another of his students, Goldberg[43].

Genetic algorithms are implemented in a computer simulation in which a population of abstract representations (called chromosomes or the genotype of the genome) of candidate solutions (called individuals, creatures, or phenotypes) to an optimization problem evolves toward better solutions. Traditionally, solutions are represented in binary as strings of 0s and 1s, but other encodings are also possible. The evolution usually starts from a population of randomly generated individuals and happens in generations. In each generation, the fitness of every individual in the population is evaluated, multiple individuals are stochastically selected from the current population (based on their fitness), and modified (recombined and possibly randomly mutated) to form a new population. The new population is then used in the next iteration of the algorithm. Commonly, the algorithm terminates when either a maximum number of generations has been produced, or a satisfactory fitness level has been reached for the population. If the algorithm has terminated due to a maximum number of generations, a satisfactory solution may or may not have been reached.

Genetic algorithms find application in bioinformatics, phylogenetics, computational science, engineering, economics, chemistry, manufacturing, mathematics, physics and other fields.

A typical genetic algorithm requires:

1. a genetic representation of the solution domain,
2. a fitness function to evaluate the solution domain.

A standard representation of the solution is as an array of bits. Arrays of other types and structures can be used in essentially the same way. The main property that makes these genetic representations convenient is that their parts are easily aligned due to their fixed size, which facilitates simple crossover operations. Variable length representations may also be used, but crossover implementation is more complex in this case. Tree-like representations are explored in genetic programming and graph-form representations are explored in evolutionary programming.

The fitness function is defined over the genetic representation and measures the *quality* of the represented solution. The fitness function is always problem dependent. For instance, in the knapsack problem one wants to maximize the total value of objects that can be put in a knapsack of some fixed capacity. A representation of a solution might be an array of bits, where each bit represents a different object, and the value of the bit (0 or 1) represents whether or not the object is in the knapsack. Not every such representation is valid, as the size of objects may exceed the capacity of the knapsack. The *fitness* of the solution is the sum of values of all objects in the knapsack if the representation is valid, or 0 otherwise. In some problems, it is hard or even impossible to define the fitness expression; in these cases, interactive genetic algorithms are used.

Once we have the genetic representation and the fitness function defined, GA proceeds to initialize a population of solutions randomly, then improve it through repetitive application of mutation, crossover, inversion and selection operators.

3.1.1 Initialization

Initially many individual solutions are randomly generated to form an initial population. The population size depends on the nature of the problem, but typically contains several hundreds or thousands of possible solutions. Traditionally, the population is generated randomly, covering the entire range of possible solutions (the *search space*). Occasionally, the solutions may be "seeded" in areas where optimal solutions are likely to be found.

3.1.2 Selection

During each successive generation, a proportion of the existing population is selected to breed a new generation. Individual solutions are selected through a *fitness-based* process, where fitter solutions (as measured by a fitness function) are typically more likely to be selected. Certain selection methods rate the fitness of each solution and preferentially select the best solutions. Other methods rate only a random sample of the population, as this process may be very time-consuming.

Most functions are stochastic and designed so that a small proportion of less fit solutions are selected. This helps keep the diversity of the population large, preventing premature convergence on poor solutions. Popular and well-studied selection methods include roulette wheel selection and tournament selection.

3.1.3 Reproduction operator

The primary objective of the reproduction operator is to emphasize good solutions and eliminate bad solutions in a population, while keeping the population size constant. This is achieved by performing the following tasks as

i) Identify good (usually above-average) solutions in a population.

ii) Make multiple copies of good solutions.

iii) Eliminate bad solutions from the population so that multiple copies of good solutions can be placed in the population. There exist a number of ways to achieve the above tasks. Some common methods are tournament selection, proportionate selection, ranking selection and roulette wheel selection.

3.1.4 Crossover Operator

The reproduction operator cannot create any new solutions in the population. It only made more copies of good solutions at the expense of not-so-good solutions. Creation of new solutions is performed in crossover and mutation operators. Like reproduction operator, there exists a number of crossover operators in the GA literature, but in almost all crossover operators, two strings are picked from the mating pool at random and some portion of the strings are exchanged between the strings. In a single-point crossover operator, this is performed by randomly choosing a crossing site along the string and by exchanging all bits on the right side of the crossing site.

3.1.5 Mutation Operator

Crossover operator is mainly responsible for the search aspect of genetic algorithms, even though mutation operator is also used for this purpose sparingly. The need for mutation is to keep diversity in the population.

3.1.6 Termination

This generational process is repeated until a termination condition has been reached. Common terminating conditions are:

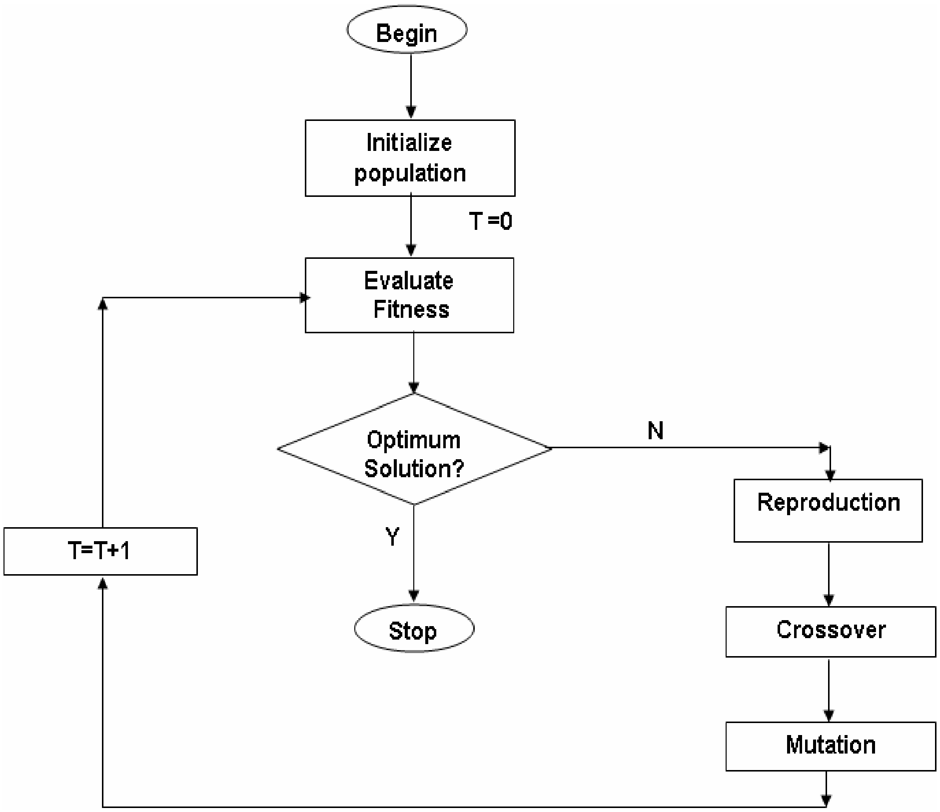
- A solution is found that satisfies minimum criteria
- Fixed number of generations reached
- Allocated budget (computation time/money) reached
- The highest ranking solution's fitness is reaching or has reached a plateau such that successive iterations no longer produce better results
- Manual inspection
- Combinations of the above

3.1.7 Simple generational genetic algorithm pseudo code

```
1. Choose initial population
2. Evaluate the fitness of each individual in the population
3. Repeat until termination: (time limit or sufficient fitness achieved)
    1. Select best-ranking individuals to reproduce
    2. Breed new generation through crossover and/or mutation (genetic operations) and give birth to offspring
    3. Evaluate the individual fitnesses of the offspring
    4. Replace worst ranked part of population with offspring
```

The Flowchart below represents the Basic Genetic Algorithm

Fig. 2 Flowchart representing Basic Genetic Algorithm



3.2 Shell Finite Element

Layered shell finite element has been formulated for analysis of smart laminated composite structures.In the present formulation, the kinematics has been described using a first-order shear deformation theory based on the Reissner–Mindlin assumptions. The basic assumptions made in the formulation are:

(a) straight line normal to the mid-surface may not remain straight during deformation,

(b) the strain energy corresponding to the stress component orthogonal to the mid-surface is disregarded.

Fig. 3 shows the general smart shell element with composite and piezoelectric layers. It has been assumed that the piezoelectric patches are perfectly bonded to the surface of the structure and the bonding layers are thin. The geometry and various coordinate systems of the degenerate shell element are shown in Fig. 4.

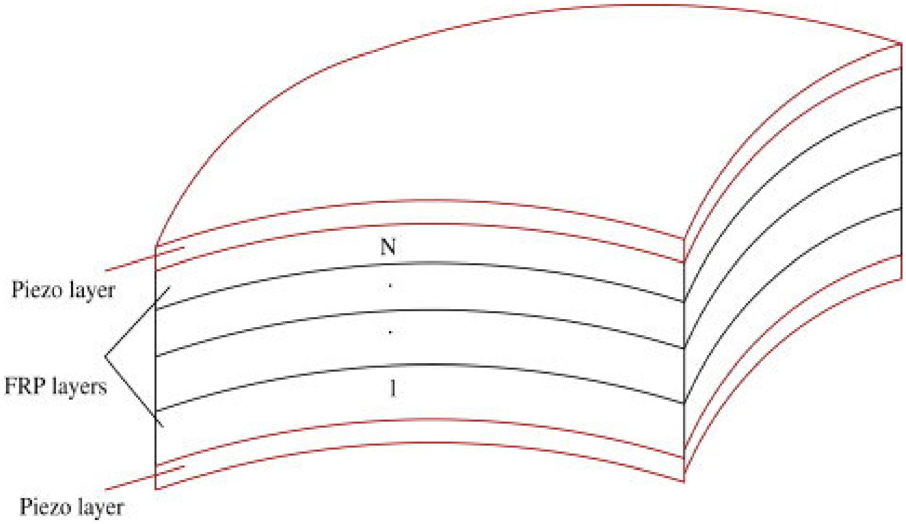


Fig.3 Smart layered shell element

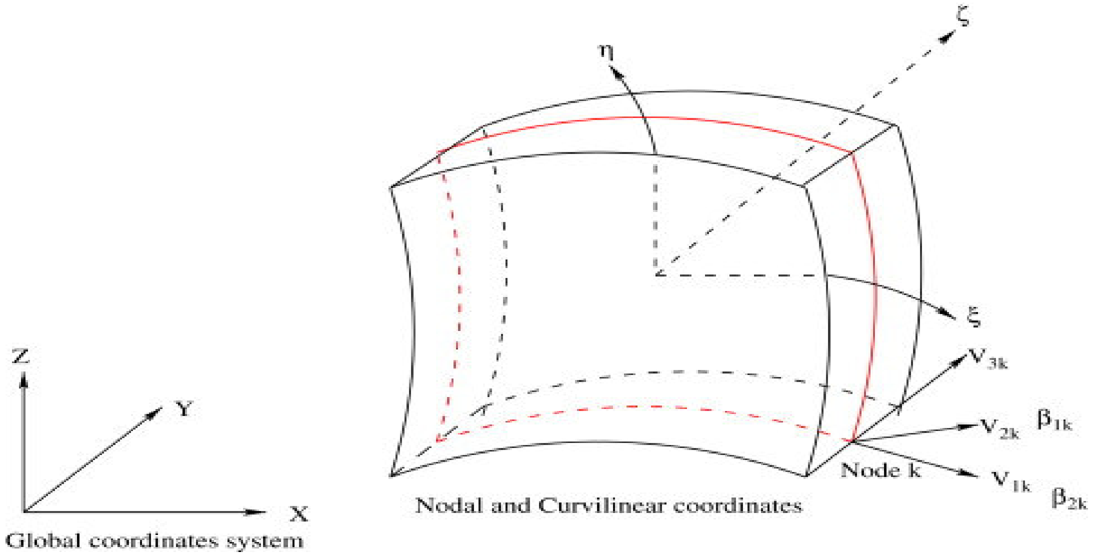


Fig. 4 Shell element with various coordinates system

3.3 State Space Representation

In control engineering, state space representation is a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations. To abstract from the number of inputs, outputs and states, the variables are expressed as vectors and the differential and algebraic equations are written in matrix form (the last one can be done when the dynamical system is linear and time invariant). The state space representation (also known as the "time-domain approach") provides a convenient and compact way to model and analyze systems with multiple inputs and outputs.

The global sets of dynamic equations for piezo- elastic analysis can be written as

$$[M_{uu}]\{\ddot{d}\}+[K_{uu}]\{d\}+\left[K_{u\phi}\right]\{\phi\}=\{F\} \tag{3.1}$$

$$\left[K_{\phi u}\right]\{d\}+\left[K_{\phi\phi}\right]\{\phi\}=\{G\} \tag{3.2}$$

The coupled piezoelectric static equations can be as follows

$$[K_{uu}]\{d\}+\left[K_{u\phi}\right]\{\phi\}=\{F\} \tag{3.3}$$

$$\left[K_{\phi u}\right]\{d\}+\left[K_{\phi\phi}\right]\{\phi\}=\{G\} \tag{3.4}$$

For open electrodes, charge can be expressed as

$$\{G\}=0 \tag{3.5}$$

Static displacement can be calculated from the Eq. (4.6)

$$\begin{bmatrix} [K_{uu}] & \left[K_{u\phi}\right] \\ \left[K_{\phi u}\right] & \left[K_{\phi\phi}\right] \end{bmatrix} \begin{Bmatrix} \{d_{th}\} \\ \{\phi\} \end{Bmatrix} = \begin{Bmatrix} F \\ G \end{Bmatrix} \tag{3.6}$$

Dynamic responses of piezolaminated structures can be calculated due to only dynamic loading from the Eq. (3.7)

$$[M]\left\{\ddot{d}_{dy}\right\}+[K]\{d_{dy}\}=\{F\}-\left[K_{u\phi a}\right]\{\phi_a\} \tag{3.7}$$

where $[M]$ is the overall global mass matrix, $[K]$ is the overall global elastic stiffness matrix and$[K_{ua}]$ is the global piezoelectric coupling matrices of actuator patches. The nodal dynamic displacement vector $d_{dy}(t)$ can be approximated by the modal superposition of the first 'r' modes as

$$\{d_{dy}(t)\}\approx[\psi]\{\eta(t)\} \tag{3.8}$$

where $[\psi]=[\psi_1\,\psi_2.........\psi_r]$ is the truncated modal matrix. The decoupled dynamic equations of Eq. (3.7) considering modal damping can be written as

$$\left\{\ddot{\eta}_i(t)\right\}+2\xi_{di}\omega_i\left\{\dot{\eta}_i(t)\right\}+\omega_i^2\{\eta_i(t)\}=[\psi]^T\{F\}-[\psi]^T[K_{ua}]\{\phi_a\} \tag{3.9}$$

where ξ_{di} is the damping ratio.

Eq. (3.9) can be represented in state-space form as

$$\left\{\dot{X}\right\}=[A]\{X\}+[B]\{\phi_a\}+\left[\hat{B}\right]\{u_d\} \tag{3.10}$$

$[A]=\begin{bmatrix}[0] & [I]\\ \left[-\omega_i^2\right] & [-2\xi_{di}\omega_i]\end{bmatrix}$ is the system matrix, $[B]=\begin{bmatrix}[0]\\ -[\psi]^T[K_{ua}]\end{bmatrix}$ is the control matrix, $\left[\hat{B}\right]=\begin{bmatrix}[0]\\ [\psi]^T\{F\}\end{bmatrix}$ is the disturbance matrix, $\{u_d\}$ is the disturbance input vector, $\{\phi_a\}$ is the control input, and

$$\left\{\dot{X}\right\}=\begin{Bmatrix}\dot{\eta}\\ \ddot{\eta}\end{Bmatrix} \text{ and } \{X\}=\begin{Bmatrix}\eta\\ \dot{\eta}\end{Bmatrix} \tag{3.11}$$

Two types of sensor output equations have been considered for mechanical and thermo-mechanical loading. The sensor output equation [150] for mechanical loading can be written as

$$\{y\} = [C_0]\{X\} \tag{3.12}$$

where output matrix $[C_0]$ depends on the modal matrix $[\psi]$ and the sensor coupling matrix $\left[K_{\phi us}\right]$. And the sensor output equation for thermo-mechanical loading has been proposed as

$$\{y\} = -\left[K_{\phi\phi s}\right]^{-1}\left[\left[K_{\phi us}\right] \quad [0]\right]\{d\} \tag{3.13}$$

3.3.1 Controllability index for actuator location

The system controllability is a basis in the modern control theory. Wang and Wang [95] proposed a controllability index for actuator locations, which was obtained by maximizing the global control force, and this has been considered in the present study. The modal control force f_c applied to the system can be written as

$$\{f_c\} = [B]\{\phi_a\} \tag{3.14}$$

It follows from Eq. (3.14) that

$$\{f_c\}^T\{f_c\} = \{\phi_a\}^T[B]^T[B]\{\phi_a\} \tag{3.15}$$

Using the singular value analysis, $[B]$ can be written as $[B] = [M][S][N]^T$

where $[M]^T[M] = [I]$,$[N]^T[N] = [I]$ and $[S] = \begin{bmatrix} \sigma_1 & \cdots & 0 \\ 0 & \ddots & \vdots \\ \vdots & \cdots & \sigma_{n_a} \\ 0 & \cdots & 0 \end{bmatrix}$

where n_a is the number of actuators. Equation (4.15) can be rewritten as

$$\{f_c\}^T\{f_c\} = \{\phi_a\}^T[N][S]^T[S][N]^T\{\phi_a\} \quad \text{or} \qquad \left\|\{f_c\}\right\|^2 = \left\|\{\phi_a\}\right\|^2\left\|S\right\|^2 \tag{3.16}$$

Thus, maximizing this norm independently on the input voltage $\{\phi_a\}$ induces maximizing $\left\|S\right\|^2$. The magnitude of σ_i is a function of location and the size of piezoelectric actuators. Wang and Wang [1] proposed that the controllability index is defined by

$$Maximize\ \Omega = \prod_{i=1}^{n_a} \sigma_i \tag{3.17}$$

The higher the controllability index, the smaller will be the electrical potential required for control. The control spillover effects are a significant problem of active vibration control implementation on real structures. Therefore, a similar controllability index has been proposed in the present work incorporating residual modes of system/structures as follows

$$Maximize\ \Omega = \prod_{i=1}^{n_a} \sigma_i - \gamma' \prod_{i=1}^{n_a} \sigma_i^R \tag{3.18}$$

where σ_i^R are the components of $[S^R]$ corresponding to residual modes and γ' is a weight

3.4 GA for Optimal Placement

Most natural representation in the form of a string of integers specifying the locations of actuators has been used in this study. An integer coded genetic (IGA) algorithm with uniform crossover and mutation have been developed for optimal placement of actuators. In the present problem the design variables are the positions of the actuators, and are represented in a string of integers specifying the locations of actuators. The gene code is taken as $ac_1, ac_2, ..., ac_j, ..., ac_{n_a}$, where $ac_j \in (1, m)$ and is a positive integer number and *m* is the total number of locations for actuators in the structures/system. Uniform crossover and new mutation techniques for integer coded genetic algorithm have been discussed in the following subsections.

3.4.1 Uniform crossover

The steps involve in this crossover are

a) A random mask is generated

b) The mask determines which bits are copied from one parent and which from the other parent

c) Bit density in mask determines how much material is taken from the other parent

For example, if the randomly generated mask is 0110011000 and parents are1010001110 and 0011010010 then their offspring will be 0011001010 and 1010010110.

3.4.2 Mutation

A one-digit positive integer value $ac_j \in [1, m]$ is generated at random, which replaces the old one when mutating. If ac_j is equal to old one, then a new positive integer is selected again until they are different in the chromosome. The efficiency of the mutation could be improved greatly using the method.

3.4.3 Optimal Placement using IGA

The fitness value i.e. measure of controllability for the optimal actuators location has been proposed as follows

$$\Omega = \left\{ \begin{array}{l} \prod_{i=1}^{n_a} \sigma_i, \ \textit{without control spillover} \\ \\ \left\{ \begin{array}{l} \left(\prod_{i=1}^{n_a} \sigma_i - \gamma' \prod_{i=1}^{n_a} \sigma_i^R \right) \ \textit{if} \ \left(\prod_{i=1}^{n_a} \sigma_i \right) > \left(\gamma' \prod_{i=1}^{n_a} \sigma_i^R \right) \\ \quad \left(\prod_{i=1}^{n_a} \sigma_i - \gamma' \prod_{i=1}^{n_a} \sigma_i^R \right) \times 10^{-12}, \ \textit{otherwise} \end{array} \right\}, \textit{considering control spillover} \end{array} \right\}$$

(3.19)

The outline of optimization problem using IGA is as follows:

i) Initial chromosomes depending on the number of actuators and populations are chosen randomly.

ii) The fitness value (measure of controllability) is calculated for each chromosome.

iii) Genetic operators are applied to produce a new set of chromosomes.

iv) Steps (ii) to (iii) are repeated until the fitness converge

v) The computation is terminated after convergence of fitness and the chromosome based on the best controllability value is selected as the optimal locations of actuators.

CHAPTER 4

RESULTS AND DISCUSSIONS

Based on the formulations discussed above, a computer code has been developed for finite element analysis of smart shell structures followed by optimal actuator placement.

4.1 Structural validation

In order to verify the finite element code developed, a spherical shell made of graphite/epoxy with the four edges simply supported, having the following dimensions have been considered: $a/b=1$, $R_1=R_2=R$, $R/a=3$, $a/h=10$. Graphite/epoxy properties considered are as follows: $E_1=25E_2$, $G_{12}=G_{13}=0.5E_2$, $\upsilon_{12}=0.25$, $G_{23}=0.2E_2$. A 10×10 finite element mesh has been used to model this entire shell.

4.2 Validation for optimal actuators placement

A smart fiber reinforced polymer (FRP) cantilever beam made of GR/E has been considered to validate the code for optimal placement of actuators as well as to compare the performances of integer and binary-coded GA in terms of generation required to reach the optimal solution. In this analysis, four actuators and first mode of vibration have been considered. The length and width of the beam are taken as 0.2 and 0.01 m, respectively. The stacking sequence of the laminated beam structure considered is $[p/[0/0]_s/p]]$. Here '*p*' stands for piezo-patches one for sensing and the other for actuation. Thickness of each ply has been considered as 0.15 mm and that of piezo-patch is 0.5 mm. The mechanical, electrical and coupled material properties used in the present study have been listed in Table 1. Several important parameters used for integer- and binary-coded GA have been listed in Table 2. Optimal actuators placement based on the maximum controllability index is shown in Fig. 5. It could be clearly observed from Fig. 5 that the optimal locations of PZT actuators are at the root of the beam. This result is expected since the curvature of the first mode of vibration reaches its maximum value at the fixed end of the cantilever beam and a similar observation was also reported by Wang and Wang .Fig. 6 shows the convergence plot with number of generations for integer-coded and binary-coded GA and it could be observed that while the integer-coded GA converges at 31 generations, binary-coded GA converges only after 246 generations.

Table 1: Material properties of structural laminate and PZT

Material properties	Structural laminate	PZT
E_1	172.5 GPa	63.0 GPa
$E_2=E_3$	6.9 GPa	63.0 GPa
$G_{12}=G_{13}$	3.45 GPa	24.6 GPa
G_{23}	1.38 GPa	24.6 GPa
$\upsilon_{12}=\upsilon_{13}=\upsilon_{23}$	0.25	0.28
ρ	1600 kg m^{-3}	7600 kg m^{-3}
$e_{31}=e_{32}$	0.0	10.62 C m^{-2}
$\in_{11}=\in_{22}=\in_{33}$	0.0	0.1555×10^{-7} F m^{-1}

Table 2.Several important parameters for integer-and binary-coded GA

Number of genes to represent	Integer-coded GA	Binary-coded GA
One actuator	1	8
Length of the chromosome	4	32
Population size	10	10
Crossover probability	0.9	0.9
Mutation probability	0.1	0.1

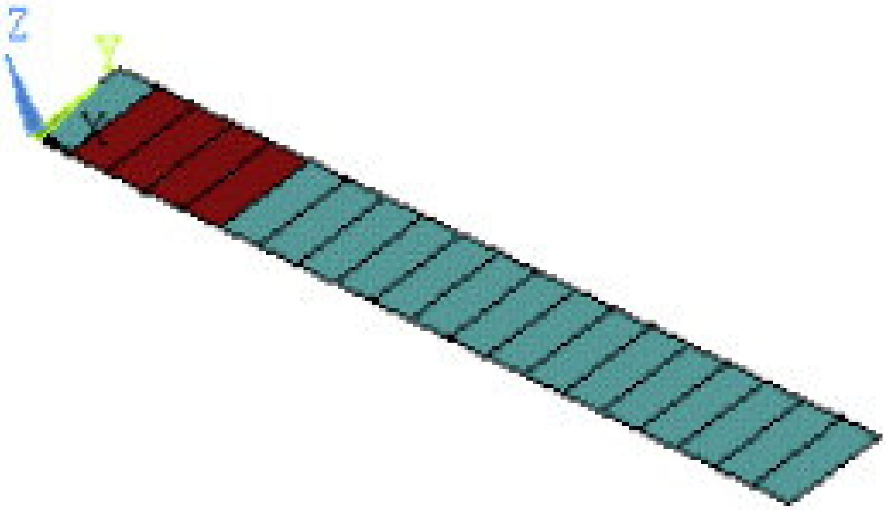


Fig. 5. Optimal location of four actuators on the beam substrate based on maximum controllability

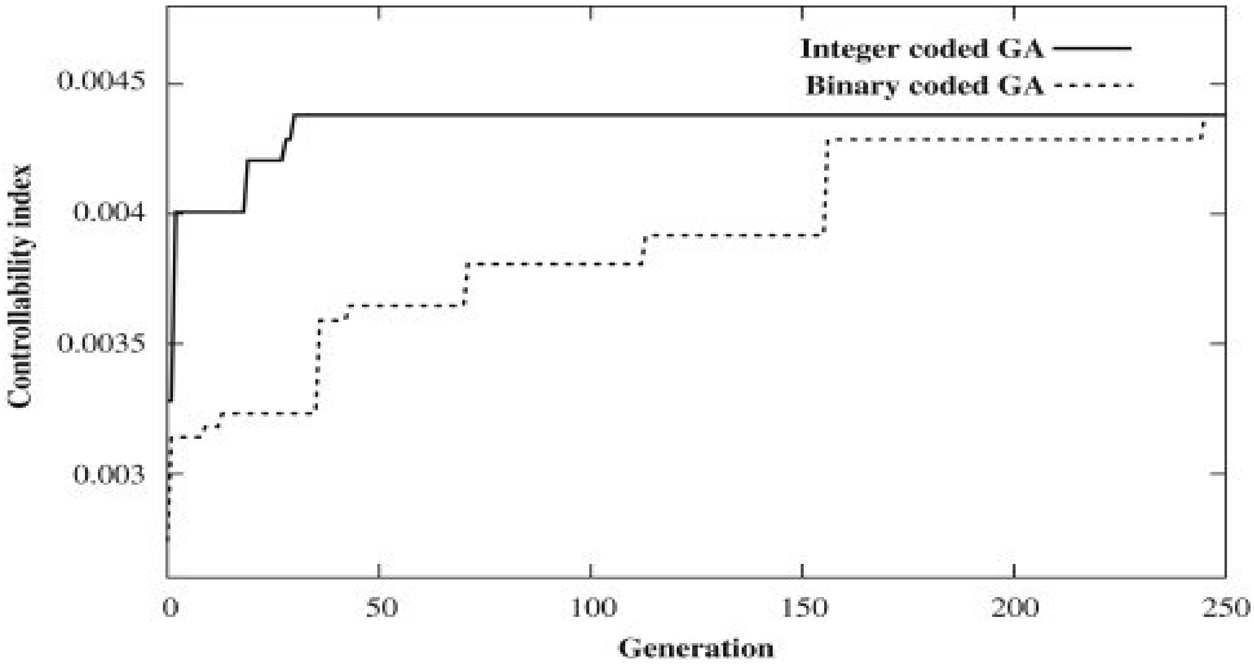


Fig. 6. Comparison of variation of controllability index with generation for the cantilever beam using integer- and binary-coded GA.

4.3 Optimal vibration control of a semi-circular ring

A simply supported smart FRP composite semi-circular arch under the action of impulse load at the center has been considered. The radius R_1 and R_2 of this panel have been considered to be 0.06 m and infinity respectively. The dimensions of the base are a=2R_1, a, b are the width of the base. The stacking sequence of the laminated spherical structure considered is [*p*/[0/90]$_s$/*p*]. Here '*p*' stands for piezo-patches one for sensing and the other for actuation. Fig. 7 shows optimal actuators placement based on the maximum controllability index considering six actuators. Fig. 8 presents the evolution of the best fitness value i.e. controllability index using GA after 50 generations. In this case, the maximum value of controllability index is 0.037 as shown in Fig. 8.

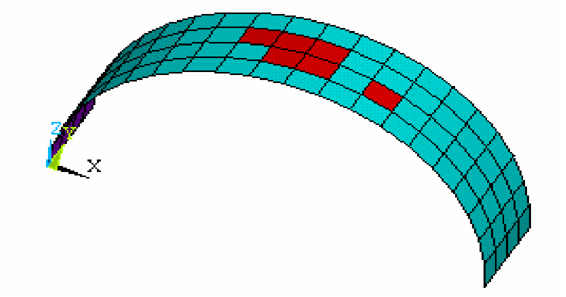


Fig. 7. Actuators location on the semicircular ring based on maximum controllability index.

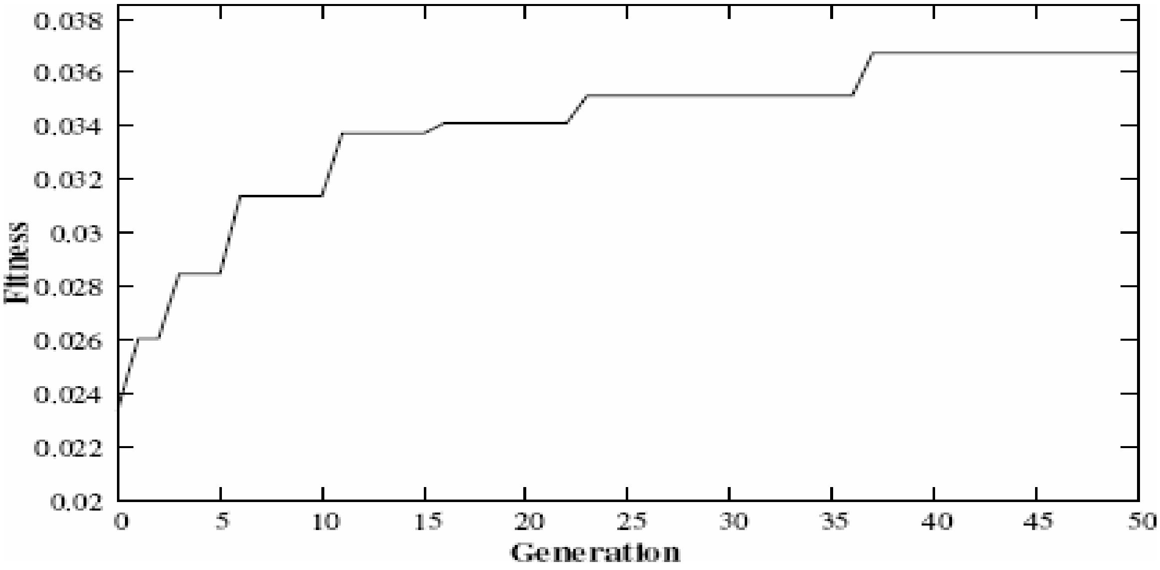


Fig. 8. Variation of controllability index with generation for semicircular ring.

4.4 Optimal vibration control of laminated spherical shell panel

A simply supported smart FRP composite shell panel on a square base (a=b=0.02 m) under the action of impulse load at the center has been considered. The radius (i.e. R_1=R_2=R) of this panel has been considered to be 0.06 m. The stacking sequence of the laminated spherical structure considered is $[p/[0/90]_s/p]$. Here '*p*' stands for piezo-patches one for sensing and the other for actuation. Thickness of each ply has been considered to be 0.25 mm and that of piezo-patch has been taken as 0.5 mm. A 10×10 finite element mesh has been considered to model this entire panel. Two types of piezo-patch locations viz. Placement1 has been considered to study influence of optimal placement on the input voltage of actuator and the closed-loop damping ratio. Placement1 stands for optimal actuators placement based on the maximum controllability index considering six actuators as shown in Fig. 10. Fig. 11 presents the evolution of the best fitness value i.e. controllability index using GA after 50 generations. In case of Placement1, the maximum value of controllability index is 0.680956 as shown in Fig. 11.

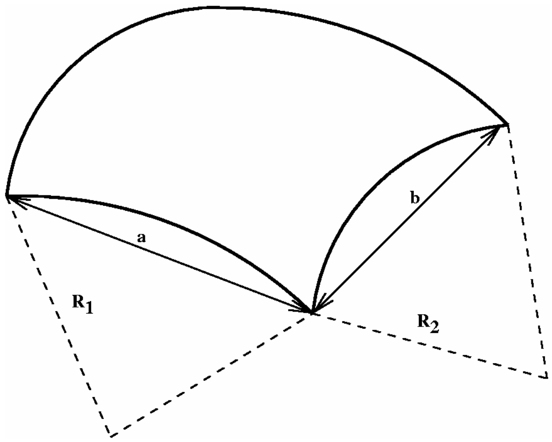


Fig. 9 Schematic Representation of a shell panel

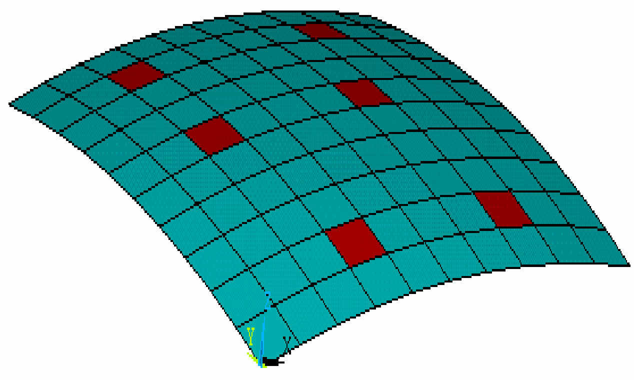


Fig.10. Actuators location on the spherical panel based on maximum controllability index.

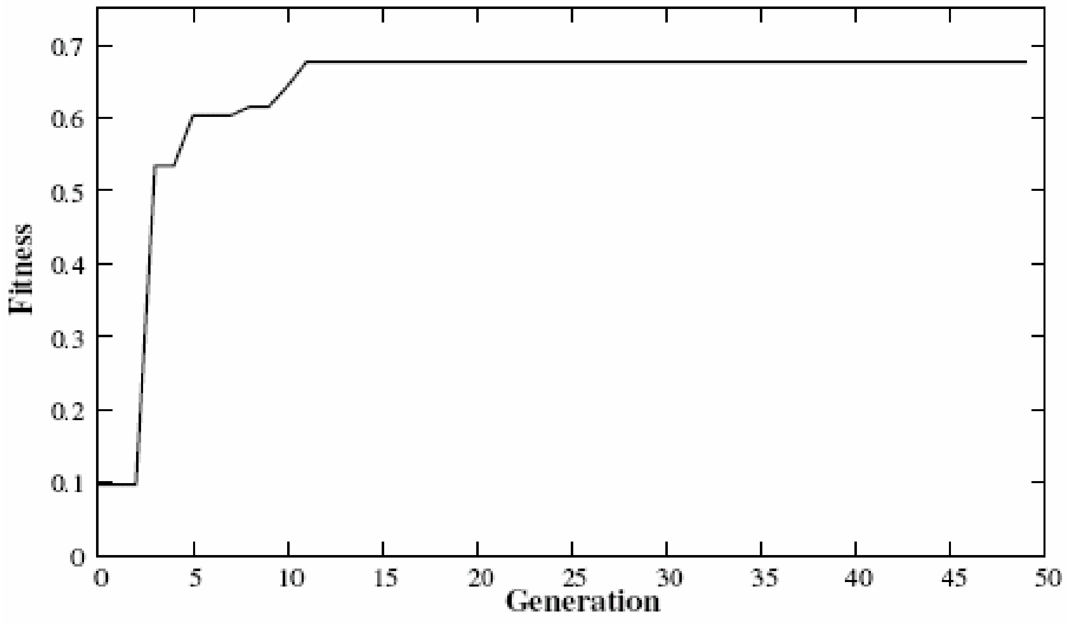


Fig.11. Variation of controllability index with generation for spherical panel.

CHAPTER 5

CONCLUSION AND SCOPE FOR FUTURE WORK

5.1 Conclusion

In this project we have developed an improved integer-coded GA-based program for optimal placement of actuators for active vibration control of smart FRP composite shell structures. This has been used in conjunction with the developed layered shell finite element procedure for coupled electromechanical analysis of smart shell structures. The present integer-coded GA-based optimal actuator location is especially advantageous for large structures where number of actuators is large. It has been observed that the proposed improved GA module leads to optimal locations of actuators. In this project we have used Integer-Coded GA for finding optimal placement of actuators for a semi-circular ring and a spherical shell and we have found better results, in the form of higher controllability as compared to previous work done in this field.

5.2 Scope for Future Works

- Multi-objective optimization where both structural design as well as control performance will be optimized
- Non-collocated sensors and actuators optimal locations
- Optimal sensors and actuators placement of large structures requiring large number of sensors and actuators
- Parallelizing the optimal placement evaluation

CHAPTER 6

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