FORCED CONVECTION IN CHANNELS WITH VISCOUS DISSIPATION

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

Bachelor of Technology
in
Mechanical Engineering

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CERTIFICATE

This is to certify that the thesis entitled "**FORCED CONVECTION IN CHANNELS WITH VISCOUS DISSIPSTION**" submitted by Mr. Anand Shukla in partial fulfilment of the requirements for the award of Bachelor of technology Degree in Mechanical Engineering at the National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

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ACKNOWLEDGEMENT

I deem it a privilege to have been the student of Mechanical Engineering stream in National Institute of Technology, ROURKELA.

My heartfelt thanks to Dr. Ramjee Repaka, my project guide who helped me to bring out this project in good manner with his precious suggestion and rich experience.

I take this opportunity to express our sincere thanks to our project guide for cooperation in accomplishing this project a satisfactory conclusion.

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ABSTRACT

Due to the reducing size of electronic equipments; the heat fluxes in them are increasing rapidly. As a result of the increased heat fluxes thermal management becomes indispensible for its longevity and hence it is one of the important topics of current research. The dissipation of heat is necessary for its proper function. The heat is generated by the resistance encountered by electric current. Unless proper cooling arrangement is designed, the operating temperature exceeds permissible limit. As a consequence, chances of failure get increased hazards.

Different phenomena have been observed in various works indicating that the mechanisms of flow and heat transfer in microchannels are still not understood clearly. There is little experimental data and theoretical analysis in the literature to elucidate the mechanisms. It is reasonable to assume that, as the dimensions of flow channels approach the micro-level, viscous dissipation could be too significant to be neglected due to a high velocity gradient in the channel. Thus, deviations from predictions using conventional theory that neglects viscous dissipation could be expected.

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Chapter 1

INTRODUCTION

INTRODUCTION

Since the discovery of electronic devices and computer, the technology has come a long way. Faster and smaller computers have led to the development of faster, denser and smaller circuit technologies which further has led to increased heat fluxes generating at the chip and the package level. Over the years, significant advances have been made in the application of air cooling techniques to manage increased heat fluxes. Air cooling continues to be the most widely used method of cooling electronic components because this method is easy to incorporate and is cheaply available. Although significant heat fluxes can be accommodated with the use of liquid cooling, its use is still limited in most extreme cases where there is no choice available.

In this study, the effects of viscous dissipation in microchannel flows are analyzed and examined theoretically. FORTRAN codes were used to solve the iterations. The values of temperature and velocity obtained; were used in calculating the nusselt's number and the graphs for it were drawn.

1.1 COOLING METHODS:

Various cooling methods are available for keeping electronic devices within their operating temperature specifications.

1.1.1 VENTING

Natural air currents flow within any enclosure. Taking advantage of this current saves on a long term component cost. Using a computer modeling package, a designer can experiment with component placement and the addition of enclosure venting to determine an optimum solution. When this solution fails to cool the device sufficiently, the addition of a fan is often the next step.

1.1.2 ENCLOSURE FANS

The increased cooling provided by adding a fan to a system makes it a popular part of many thermal solutions. Increased air flow also Increases the cooling efficiency of heat sinks, allowing a smaller or less efficient heat sink to perform adequately.

The decision to add a fan to a system depends on number considerations. Mechanical operation makes fans inherently less reliable than a passive system. In small enclosures, the pressure drop between the inside and the outside of the enclosure can limit the efficiency of the fan.

1.1.3 PASSIVE HEAT SINKS:

Passive heat sinks use a mass of thermally conductive material to move heat away from the device into the air stream, where it can be carried away. Heat sink designs include fins or other protrusions to increase the surface area, thus increasing its ability to remove heat from the device.

1.1.4 ACTIVE HEAT SINKS:

When a passive heat sink cannot remove heat fast enough, a small fan may be added directly to the heat sink itself, making the heat sink an active component. These active heat sinks, often used to cool microprocessors, provide a dedicated air stream for a critical device. Active heat sinks often are a good choice when an enclosure fan is impractical.

1.1.5 HEAT PIPES:

Heat pipes, a type of phase-change recalculating system, use the cooling power of vaporization to move heat from one place to another. Within a closed heat removal system, such as a sealed copper pipe, a fluid at the hot end (near a device) is changed into a vapor. Then the gas passes through a heat removal area, typically a heat removal area, typically a heat sink using either air cooling or liquid cooling techniques. The temperature reduction causes the fluid to recon dense into a liquid, giving off its heat to the environment. A heat pipe is a cost effective solution, and it spreads the heat uniformly throughout the heat sink condenser section, increasing its thermal effectiveness.

1.1.6 METAL BACKPLANES:

Metal-core primed circuit boards, stamped plates on the underside, of a laptop keyboard, and large copper pads on the surface of a printed circuit board all employ large metallic areas to dissipate heat.

1.1.7 THERMAL INTERFACES:

The interface between the device and the thermal product used to cool it is an important factor in the thermal solution. For example, a heat sink attached to a plastic package using double sided tape cannot dissipate the same amount of heat as the same heat sink directly in contain with thermal transfer plate on a similar package.

Microscopic air gaps between a semiconductor package and the heat sink, caused by surface non-uniformity, can degrade thermal performance. This degradation increases at higher operating temperature. Interface materials appropriate to the package type reduce the variability induced by varying surface roughness.

Since the interface thermal resistance is dependent upon allied force, the contact pressure becomes an integral design parameter of the thermal solution. If a package device can withstand a limited amount of contact pressure, it is important that thermal calculations use the appropriate thermal resistance for that pressure. The chemical compatibility of the interface materials with the package type is another important factor.

1.2 THERMAL OPTIONS FOR DIFFERENT PACKAGES:

Many applications have different constraints that favor one thermal solution over another. Power devices need to dissipate large amount of heat. The thermal solution for microprocessors must take space constraints into account. Surface mount and ball grid array technologies have assembly considerations. Notebook computers require efficiency in every area, including space, weight, and energy usage. While the optimum solution for anyone of these package types must be determined on a case-by-case basis, some solutions address specific issues, making them more suitable for a particular application.

1.2.1 POWER DEVICES:

Newer power devices incorporate surface mount compatibility into the power-hungry design. These devices incorporate a heat transfer plate on the bottom of the device, which can be wave soldered directly to the printed circuit board.

Metal-core substrates offer a potential solution to power device cooling, provided there are no other heat-sensitive devices in the assembly, and the cost of the board can be justified.

1.2.2 MICROPROCESSORS:

As microprocessor technology advances, the system designer struggles to keep ahead of the increase in the thermal output of both (the voltage regulator and the microprocessor. The use of active heat sinks allows concentrated, dedicated cooling of the microprocessor, without severely impacting space requirements. For some applications, specially designed passive heat sinks facilitate the use of higher-powered voltage regulators in the same footprint, eliminating the need for board redesign.

1.2.3 BGAs:

While BGA-packaged devices transfer more heat to the board then leaded devices, the type of package can affect the ability to dissipate sufficient heat to maintain high device reliability.

All plastic packages insulate the top of the device making heat dissipation through top mounted heat sinks difficult and more expensive. Metal heat spreaders incorporated into the top of the package enhance the ability to dissipate power from the chip.

For some lower power devices flexible copper spreaders attach to pre-applied double sided tape, offering a "*quick-fix*" for border line applications. As the need to dissipate more power increases, the optimum heat sink becomes heavier. To prevent premature failure caused by ball shear, well designed of the self heat sinks include spring loaded pins or clips that allow the weight of the heat sinks to be burn by the PC-board instead of the device.

Chapter 2

THEORY

THEORY

2.1 HEAT TRANSFER

Heat is defined as energy transferred by virtue of temperature difference or gradient. Being a vector quantity, it flows with a negative temperature gradient. In the subject of heat transfer, it is the rate of heat transfer that becomes the prime focus. The transfer process indicates the tendency of a system to proceed towards equilibrium. There are 3 distinct modes in which heat transfer takes place:

2.1.1 CONDUCTION:

Conduction is the transfer of heat between 2 bodies or 2 parts of the same body through molecules. This type of heat transfer is governed by Fourier`s Law which states that – "Rate of heat transfer is linearly proportional to the temperature gradient". For 1-D heat conduction-

$$q_k = -k \frac{dT}{dx} \tag{2.1}$$

2.1.2 RADIATION:

Thermal radiation refers to the radiant energy emitted by the bodies by virtue of their own temperature resulting from the thermal excitation of the molecules. It is assumed to propagate in the form of electromagnetic waves and doesn’t require any medium to travel. The radiant heat exchange between 2 gray bodies at temperature T1 and T2 is given by:

$$Q_{1-2} = \sigma A_1 F_{1-2} (T_1^4 - T_2^4) \tag{2.2}$$

2.1.3 CONVECTION:

When heat transfer takes place between a solid surface and a fluid system in motion, the process is known as Convection. When a temperature difference produces a density difference that results in mass movement, the process is called Free or Natural Convection.

When the mass motion of the fluid is carried by an external device like pump, blower or fan, the process is called Forced Convection. In convective heat transfer, Heat flux is given by:

$$q(x) = h_x (T_w - T_\infty) \tag{2.3}$$

2.2 CONVECTION

Convection in the most general terms refers to the internal movement of currents within fluids (i.e. liquids and gases). It cannot occur in solids due to the atoms not being able to flow freely.

Convection may cause a related phenomenon called advection, in which either mass or heat is transported by the currents or motion in the fluid. A common use of the term *convection* relates to the special case in which adverted (carried) substance is **heat**. In this case, the heat itself may be an indirect cause of the fluid motion even while being transported by it. In this case, the problem of heat transport (and related transport of other substances in the fluid due to it) may become especially complicated.

Convection is of two types:-

1. Forced convection

2. Free Convection

2.2.1 FORCED CONVECTION

When the density difference is created by some means like blower or compressor and due to which circulation takes place then it is known as forced convection

2.2.2 FREE CONVECTION

Density variation happens naturally then it is called free convection

Here we are concerned for Newtonian fluid only (fluid that follows Newton law of cooling)

$$\tau = \mu \frac{\partial u}{\partial y} \qquad (2.4)$$

2.3 Nondimensionalization

Nondimensionalization is the partial or full removal of units from a mathematical equation by a suitable substitution of variables. This technique can simplify and parameterize problems where measured units are involved. It is closely related to dimensional analysis. In some physical systems, the term scaling is used interchangeably with nondimensionalization, in order to suggest that certain quantities are better measured relative to some appropriate unit. These units refer to quantities intrinsic to the system, rather than units such as SI units. Nondimensionalization is not the same as converting extensive quantities in an equation to intensive quantities, since the latter procedure results in variables that still carry units.

To nondimensionalize a system of equations, one must do the following:

1. Identify all the independent and dependent variables.
2. Replace each of them with a quantity scaled relative to a characteristic unit of measure to be determined.
3. Divide through by the coefficient of the highest order polynomial or derivative term.
4. Choose judiciously the definition of the characteristic unit for each variable so that the coefficients of as many terms as possible become 1.
5. Rewrite the system of equations in terms of their new dimensionless quantities.

The last three steps are usually specific to the problem where nondimensionalization is applied. However, almost all systems require the first two steps to be performed.

As an illustrative example, consider a first order differential equation with constant coefficients:

$$a\frac{dx}{dt} + bx = Af(t).$$

1. In this equation the independent variable here is t, and the dependent variable is x.
2. Set $x = \chi x_c,\ t = \tau t_c$.

This results in the equation

$$a\frac{x_c}{t_c}\frac{d\chi}{d\tau} + bx_c\chi = Af(\tau t_c) \stackrel{\text{def}}{=} AF(\tau).$$

3. The coefficient of the highest ordered term is in front of the first derivative term. Dividing by this gives

$$\frac{d\chi}{d\tau} + \frac{bt_c}{a}\chi = \frac{At_c}{ax_c}F(\tau).$$

4. The coefficient in front of χ only contains one characteristic variable tc, hence it is easiest to choose to set this to unity first:

$$\frac{bt_c}{a} = 1 \Rightarrow t_c = \frac{a}{b}.$$

Subsequently,

$$\frac{At_c}{ax_c} = \frac{A}{bx_c} = 1 \Rightarrow x_c = \frac{A}{b}.$$

5. The final dimensionless equation in this case becomes completely independent of any parameters with units:

$$\frac{d\chi}{d\tau} + \chi = F(\tau).$$

Some of the dimensionless numbers are:

2.3.1 REYNOLDS NUMBER

In fluid mechanics, the Reynolds number is the ratio of inertial forces ($v_s\rho$) to viscous forces (μ/L) and consequently it quantifies the relative importance of these two types of forces for given flow conditions. Thus, it is used to identify different flow regimes, such as laminar or turbulent flow.

It is one of the most important dimensionless numbers in fluid dynamics and is used, usually along with other dimensionless numbers, to provide a criterion for determining dynamic similitude. When two geometrically similar flow patterns, in perhaps different fluids with possibly different flow-rates, have the same values for the relevant dimensionless numbers, they are said to be dynamically similar.

It is named after Osborne Reynolds (1842–1912), who proposed it in 1883. Typically it is given as follows:

$$Re = \frac{\rho v_s L}{\mu} = \frac{v_s L}{\nu} = \frac{\text{Inertial forces}}{\text{Viscous forces}} \qquad (2.5)$$

Where:

v_s - Mean fluid velocity,

L - Characteristic length,

μ - (Absolute) dynamic fluid viscosity,

ν - Kinematic fluid viscosity: ν = μ / ρ ,

ρ - Fluid density.

For flow in pipes for instance, the characteristic length is the pipe diameter, if the cross section is circular, or the hydraulic diameter, for a non-circular cross section.

Laminar flow occurs at low Reynolds numbers, where viscous forces are dominant, and is characterized by smooth, constant fluid motion, while turbulent flow, on the other hand, occurs at high Reynolds numbers and is dominated by inertial forces, producing random eddies, vortices and other flow fluctuations.

The transition between laminar and turbulent flow is often indicated by a critical

Reynolds number (Re_{crit}), which depends on the exact flow configuration and must be determined experimentally. Within a certain range around this point there is a region of gradual transition where the flow is neither fully laminar nor fully turbulent, and predictions of fluid behavior can be difficult. For example, within circular pipes the critical Reynolds number is generally accepted to be 2300, where the Reynolds number is based on the pipe diameter and the mean velocity v_s within the pipe, but engineers will avoid any pipe configuration that falls within the range of Reynolds numbers from about 2000 to 3000 to ensure that the flow is either laminar or turbulent.

For flow over a flat plate, the characteristic length is the length of the plate and the characteristic velocity is the free stream velocity. In a boundary layer over a flat plate the local regime of the flow is determined by the Reynolds number based on the distance measured from the leading edge of the plate. In this case, the transition to turbulent flow occurs at a Reynolds number of the order of 10^5 or 10^6.

2.3.2 NUSSELT NUMBER

The Nusselt number is a dimensionless number that measures the enhancement of heat transfer from a surface that occurs in a 'real' situation, compared to the heat transferred if just conduction occurred. Typically it is used to measure the enhancement of heat transfer when convection takes place.

$$Nu_L = \frac{hL}{k_f} = \frac{\text{Convective heat transfer}}{\text{Conductive heat transfer}} \quad (2.6)$$

Where

L = characteristic length, which is simply Volume of the body divided by the Area of the body (useful for more complex shapes)

k_f = thermal conductivity of the "fluid"

h = convection heat transfer coefficient

Selection of the significant length scale should be in the direction of growth of the boundary layer. A salient example in introductory engineering study of heat transfer would be that of a horizontal cylinder versus a vertical cylinder in free convection.

Several empirical correlations are available that are expressed in terms of Nusselt number in the elementary analysis of flow over a flat plate etc. Sieder-Tate, Colburn and many others have provided such correlations.

For a local Nusselt number, one may evaluate the significant length scale at the point of interest. To obtain an average Nusselt number analytically one must integrate over the characteristic length. More commonly the average Nusselt number is obtained by the pertinent correlation equation, often of the form Nu = Nu (Ra, Pr).

The Nusselt number can also be viewed as being a dimensionless temperature gradient at the surface.

2.3.3 PRANDTL NUMBER

The Prandtl number is a dimensionless number approximating the ratio of momentum diffusivity (viscosity) and thermal diffusivity. It is named after Ludwig Prandtl.

It is defined as:

$$Pr = \frac{\nu}{\alpha} = \frac{\text{viscous diffusion rate}}{\text{thermal diffusion rate}} \quad (2.7)$$

Where:

ν is the kinematic viscosity, $\nu = \mu / \rho$.

α is the thermal diffusivity, $\alpha = k / (\mu c_p)$.

Typical values for *Pr* are:

- around 0.7 for air and many other gases,
- around 7 for water
- around 7×10^{21} for Earth's mantle
- between 100 and 40,000 for engine oil,
- between 4 and 5 for R-12 refrigerant
- around 0.015 for mercury

For mercury, heat conduction is very effective compared to convection: thermal diffusivity is dominant. For engine oil, convection is very effective in transferring energy from an area, compared to pure conduction: momentum diffusivity is dominant.

In heat transfer problems, the Prandtl number controls the relative thickness of the momentum and thermal boundary layers.

The mass transfer analog of the Prandtl number is the Schmidt number.

2.3.4 BRINKMAN NUMBER

The **Brinkman Number** is a dimensionless group related to heat conduction from a wall to a flowing viscous fluid, commonly used in polymer processing. There are several definitions; one is

$$N_{Br} = \frac{\eta U^2}{\kappa(T_w - T_0)} \qquad (2.8)$$

Where

N_{Br} (or Br)= the Brinkman Number

η = fluid viscosity (dynamic)

U = fluid velocity

κ = thermal conductivity of fluid

T_0 = bulk fluid temperature

T_w = wall temperature

In, for example, a screw extruder, the energy supplied to the polymer melt comes primarily from two sources (i) viscous heat generated by shear between parts of the flow moving at different velocities (ii) direct heat conduction from the wall of the extruder. The former is supplied by the motor turning the screw, the latter by heaters. The Brinkman Number is a measure of the ratio of the two.

2.3.5 PECLET NUMBER

The Peclet number is a dimensionless number used in calculations involving convective heat transfer. It is the ratio of the thermal energy convected to the fluid to the thermal energy conducted within the fluid. If Pe is small, conduction is important and in such a case, the major source of conduction could be down the walls of a tube. The Peclet number is the product of the Reynolds number and the Prandtl number. It depends on the heat capacity, density, velocity, characteristic length and heat transfer coefficient.

$$\mathrm{Pe} \equiv \frac{[\text{advection of heat}]}{[\text{conduction of heat}]} = \frac{|\mathbf{u} \cdot \nabla T|}{|\kappa \nabla^2 T|} = \frac{UL}{\kappa}, \qquad (2.9)$$

Where,

u = flow velocity

T = temperature

U = velocity scale

L = length scale

κ = thermal diffusivity.

2.4 NAVIER STOKE`S EQUATIONS:

The Navier Stoke`s equations are derived from conservation principles of

- Mass.

- Energy.
- Momentum.
- Angular momentum.
- Equation of continuity.

Conservation of mass is written as:

$$\frac{\partial p}{\partial t} + \nabla.(\rho V) = \frac{D\rho}{Dt} + \rho\nabla.V = 0 \quad (2.10)$$

Where ρ is the mass density (mass per unit volume), and v is the velocity of the fluid. In the case of an incompressible fluid, ρ does not vary along a path-line and the equation reduces to:

$$\nabla \cdot \mathbf{v} = 0 \quad (2.11)$$

Conservation of Momentum equation for an incompressible fluid is:

$$\rho\frac{Du}{Dt} = \rho g_x - \frac{\partial P}{\partial x} + \mu\left(\Delta^2 .u\right) \quad (2.12)$$

The Navier-Stokes Continuity equation for cylindrical coordinates is:

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} \quad (2.13)$$

Note that the Navier-Stokes equations can only describe fluid flow approximately and that, at very small scales or under extreme conditions, real fluids made out of mixtures of discrete molecules and other material, such as suspended particles and dissolved gases, will produce different results from the continuous and homogeneous fluids modeled by the Navier-Stokes equations.

$$\rho\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}\right)= -\frac{\partial P}{\partial z}+\mu\left(\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right)+F_z \quad (2.14)$$

Note that the Navier-Stokes equations can only describe fluid flow approximately and that, at very small scales or under extreme conditions, real fluids made out of mixtures of discrete molecules and other material, such as suspended particles and dissolved gases, will produce different results from the continuous and homogeneous fluids modeled by the Navier-Stokes equations.

2.5 DISCRETIZATION METHODS:

The stability of the chosen discretization is generally established numerically rather than analytically as with simple linear problems. Special care must also be taken to ensure that the discretization handles discontinuous solutions gracefully. The Euler equations and Navier-Stokes equations both admit shocks, and contact surfaces.

Some of the discretization methods being used are:

2.5.1 FINITE VOLUME METHOD:

This is the "classical" or standard approach used most often in commercial software and research codes. The governing equations are solved on discrete control volumes. This integral approach yields a method that is inherently conservative (i.e., quantities such as density remain physically meaningful):

$$\frac{\partial}{\partial t}\iiint Q dV + \iint F d\mathbf{A} = 0,$$

(2.15)

Where Q is the vector of conserved variables, F is the vector of fluxes (see Euler equations or Navier-Stokes equations), V is the cell volume, and is the cell surface area.

2.5.2 FINITE ELEMENT METHOD:

This method is popular for structural analysis of solids, but is also applicable to fluids. The FEM formulation requires, however, special care to ensure a conservative solution. The FEM formulation has been adapted for use with the Navier-Stokes equations. In this method, a weighted residual equation is formed:

$$R_i = \iiint W_i Q dV^e$$

(2.16)

Where R_i is the equation residual at an element vertex i, Q is the conservation equation expressed on an element basis, W_i is the weight factor and V^{e-} is the volume of the element.

2.5.3 FINITE DIFFERENCE METHOD:

This method has historical importance and is simple to program. It is currently only used in few specialized codes. Modern finite difference codes make use of an embedded boundary for handling complex geometries making these codes highly efficient and accurate. Other ways to handle geometries are using overlapping-grids, where the solution is interpolated across each grid.

$$\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} + \frac{\partial H}{\partial z} = 0 \tag{2.17}$$

Where Q is the vector of conserved variables, and F, G, and H are the fluxes in the x, y, and z directions respectively.

Boundary element method: The boundary occupied by the fluid is divided into surface mesh.

High resolution schemes are used where shocks or discontinuities are present. To capture sharp changes in the solution requires the use of second or higher order numerical schemes that do not introduce spurious oscillations. This usually necessitates the application of flux limiters to ensure that the solution is total variation diminishing.

2.6 HYDRAULIC DIAMETER

The hydraulic diameter d_h is commonly used when dealing with non-circular pipes, holes or ducts.

The definition of the hydraulic diameter is:

$$d_h \equiv 4 \, \frac{\text{cross-sectional-area of duct}}{\text{wetted perimeter of duct}}$$

$$d_h = 4\ A \,/\, p \tag{2.18}$$

Where:

d_h = hydraulic diameter

A = area section of the duct

p = wetted perimeter of the duct

2.6.1 Use of hydraulic diameter:

Estimating the turbulent length-scale: For fully-developed flow in non-circular ducts the turbulent length scale can be estimated as $0.07\, d_h$. This is as useful estimation for setting turbulence boundary conditions for inlets that have fully developed flow.

Computing Reynolds number: The hydraulic diameter is often used when computing the dimensionless Reynolds number for non-circular ducts.

2.6.2 Hydraulic diameters for different duct-geometries:

Using the definition above the hydraulic diameter can easily be computed for any type of duct-geometry. Below follows a few examples.

Circular pipe:

For a circular pipe or hole the hydraulic diameter is:

$$d_h = 4\,\frac{\frac{\pi d^2}{4}}{\pi d} = d \tag{2.19}$$

Where d is the real diameter of the pipe. Hence, for circular pipes the hydraulic diameter is the same as the real diameter of the pipe.

Rectangular tube:

For a rectangular tube or hole with the width a and the height b the hydraulic diameter is:

$$d_h = 4\,\frac{ab}{2a+2b} = 2\,\frac{ab}{a+b} \qquad (2.20)$$

Coaxial circular tube:

For a coaxial circular tube with an inner diameter d_i and an outer diameter d_o the hydraulic diameter is:

$$d_h = 4\,\frac{\frac{\pi d_o^2}{4} - \frac{\pi d_i^2}{4}}{\pi d_o + \pi d_i} = d_o - d_i \qquad (2.21)$$

Chapter 3

MATHEMATICAL MODELLING

MATHEMATICAL MODELLING

It is known that the viscous dissipation terms in the governing energy conservation equation are commonly and conveniently neglected for describing conventional flow situations. However, in microchannel flows, the existence of a large velocity gradient may result in significant errors by ignoring the effects of viscous dissipation. Therefore, in this analysis, the conventional theory will be employed taking into consideration the viscous dissipation effects to analyze the characteristics of microchannel flows. The governing equations used to analyze the diffusion effect in the microchannel flow are given by,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial r}+\frac{v}{r}=0 \tag{3.1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}\right)=-\frac{\partial p}{\partial x}+\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right)+\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)\right) \tag{3.2}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial r}\right)=-\frac{\partial p}{\partial r}+\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial v}{\partial r}\right)-\frac{v}{r^2}+\frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right)\right) \tag{3.3}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}\right)=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(k\frac{\partial T}{\partial r}\right)+\mu\Phi \tag{3.4}$$

where Φ is the dissipation function due to the viscous diffusion, and is given by,

$$\Phi=2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial r}\right)^2+\left(\frac{v}{r}\right)^2\right]+\left(\frac{\partial u}{\partial r}+\frac{\partial v}{\partial x}\right)^2 \tag{3.5}$$

U and v are the velocity components in the axial (x) and radial (r) directions respectively. P is the pressure, ρ is the density,μ is the viscosity of the fluid, and c_p is the specific heat capacity at constant pressure.

Conservation of Thermal Energy Equation

When the flow is fully developed, assuming constant thermophysical properties and including axial conduction and viscous dissipation, the equation for conservation of thermal energy can be written as,

$$U\left(\frac{\partial \theta}{\partial X}\right)=\left(\frac{1}{Pe}\right)\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right)+\left(\frac{Br}{Pe}\right)\left(\frac{dU}{dY}\right)^2 \qquad (3.6)$$

X^* (= X/Pe) is the normalized axial coordinate. X, Y, U, θ and Pe in Eq. (3.6) have been defined as $X = x/D_h$, $Y = y/D_h$, $U = u/u_{avg}$, $\theta = (T - \overline{T}_w)/(T_i - \overline{T}_w)$, $Pe = (u_{avg} D_h)/\alpha = Re\, Pr$

$Re = (\rho\, u_{avg} D_h)/\mu$ and $Pr = (\mu\, C_p)/k$

Br is the Brinkman number.

U, is given by,

$$U = u/u_{avg} = (3/2)\{1 - (4Y)^2\} \qquad (3.7)$$

Eq. (3.6) has been rewritten in terms of X^* to facilitate evaluation of effect of axial conduction in the presence of viscous dissipation. Equation (3.6) takes the form,

$$U\left(\frac{\partial \theta}{\partial X^*}\right)=\left(\frac{1}{Pe^2}\right)\left(\frac{\partial^2 \theta}{\partial X^{*2}}\right)+\frac{\partial^2 \theta}{\partial Y^2}+Br\left(\frac{dU}{dY}\right)^2 \qquad (3.8)$$

Equation (3.8) is subjected to the boundary conditions including the downstream limiting conduction condition. The non-dimensional boundary conditions are as following.

$$\left.\begin{array}{llll} \theta = 1 & \text{at } X^* = 0 & \text{for} & -1/4 \le Y \le +1/4 \\ \theta = -(1-A)/(1+A) & \text{at } Y = -1/4 & \text{for} & \text{all } X^* > 0 \\ \theta = (1-A)/(1+A) & \text{at } Y = 1/4 & \text{for} & \text{all } X^* > 0 \end{array}\right\} \qquad (3.9)$$

where A, termed the asymmetry parameter is defined by,

$$A = \Delta T_2 / \Delta T_1 \tag{3.9a}$$

where ΔT_2 and ΔT_1 are the excess wall temperatures relative to the fluid inlet temperature, T_i. Thus,

$$\Delta T_2 = T_{w2} - T_i \tag{3.9b}$$

$$\Delta T_1 = T_{w1} - T_i \tag{3.9c}$$

$$\theta = 4Y\,\{(1 - A)/(1 + A)\} - 192\, Br\, Y^4 + (3/4)\, Br \qquad \text{for } X^* \geq X^*_{cl} \tag{3.10a}$$

OR for $A = 1$,

$$\theta = -192\, Br\, Y^4 + (3/4)\, Br \qquad \text{for } X^* \geq X^*_{cl} \tag{3.10b}$$

3.1 NUMERICAL SCHEME: SAR

The basic philosophy of the SAR scheme is to guess a profile for each variable that satisfies the boundary conditions. Let the partial differential equation governing a variable, $\phi(X, Y)$, expressed in finite difference form be given by $\overline{\phi}_{M,N} = 0$ where (M, N) represent the nodal point, when the non-dimensional height and length of the channel are divided into a finite number of intervals MD, ND respectively. The guessed profile for the variable ϕ at any mesh point, in general, will not satisfy the equation. Let the error in the equation at (M, N) and kth iteration be $\overline{\phi}^{k}_{M,N}$

The (k+1)th approximation to the variable ϕ is obtained from,

$$\phi_{M,N}^{k+1} = \phi_{M,N}^{k} - \omega \left\{ \overline{\phi}_{M,N}^{k} \Big/ \left(\partial \overline{\phi}_{M,N}^{k} \Big/ \partial \phi_{M,N} \right) \right\} \tag{3.11}$$

where ω is an acceleration factor which varies between $0 < \omega < 2$. $\omega < 1$ represents under-relaxation and $\omega > 1$ represents over relaxation.

The procedure of correcting the variable ϕ at each mesh point in the entire region of interest is repeated until a convergence criterion is satisfied. The criterion is that, the normalized change in the variable at any mesh point between k^{th} and $(k+1)^{th}$ approximation satisfies,

$$\left| 1 - \left(\phi_{M,N}^{k} \Big/ \phi_{M,N}^{k+1} \right) \right| < \varepsilon \tag{3.12}$$

where ε, the error tolerance limit, is a prescribed small positive number.

To correct the guessed profiles, each dependent variable has to be associated with one equation. It is natural to associate the equation for a variable that contains the highest order derivative of that variable. For example, conservation of energy equation is associated for correcting the temperature profile.

3.2 Application of the SAR Scheme

Let *MD* and *ND* be the number of divisions in X^* and Y directions respectively. The intervals ΔX^* (in terms of the transformed coordinate) and ΔY are given by,

$$\Delta X^* = X_{fd}^* / MD \tag{3.13}$$

$$\Delta Y = 1/(2\,ND) \tag{3.14}$$

$X^*_{fd} = X_{fd}/Pe$ where X_{fd} is the non-dimensional axial distance needed for the temperature field to be fully developed.

According to the SAR scheme, the error $\bar{\theta}$ due to guessed values of θ at any mesh point (M, N) are obtained using the backward difference formula for convective term and central difference formula for the diffusion term in Eq. (3.8). The error $\bar{\theta}$ in finite difference form is as following.

$$\bar{\theta}_{M,N} = U_{M,N}\left(\frac{\theta_{M,N}-\theta_{M-1,N}}{\Delta X^*}\right) - \left(\frac{1}{Pe^2}\right)\left(\frac{\theta_{M+1,N}-2\theta_{M,N}+\theta_{M-1,N}}{(\Delta X^*)^2}\right) - \left(\frac{\theta_{M,N+1}-2\theta_{M,N}+\theta_{M,N-1}}{(\Delta Y)^2}\right)$$
$$- Br\left(\frac{U_{M,N+1}-U_{M,N-1}}{\Delta Y}\right)^2 \tag{3.15}$$

To correct the profile for θ according to Eq. (3.11) the following derivative becomes necessary,

$$\frac{\partial\bar{\theta}_{M,N}}{\partial\theta_{M,N}} = \left(\frac{U_{M,N}}{\Delta X^*}\right) + \frac{1}{Pe^2}\left(\frac{2}{\left(\Delta X^*\right)^2}\right) + \frac{2}{\left(\Delta Y\right)^2} \tag{3.16}$$

Boundary conditions given by Eq. (3.9) on θ in finite difference form become,

$$\left.\begin{array}{llll} \theta_{1,N} = 1 & \text{at } X^* = 0 & \text{for} & -1/4 \le Y \le +1/4 \\ \theta_{M,1} = -(1-A)/(1+A) & \text{at } Y = -1/4 & \text{for} & \text{all } X^* > 0 \\ \theta_{M,ND+1} = (1-A)/(1+A) & \text{at } Y = 1/4 & \text{for} & \text{all } X^* > 0 \end{array}\right\} \tag{3.17}$$

3.3 LOCAL NUSSELT NUMBER

The defining equation for the local heat transfer coefficient at x, h_{1x}, say, at the wall at $y = -L/2$ is given by,

$$-k\left(\partial T/\partial y\right)\Big|_{y=-L/2} = h_{1x}\,(T_{w1} - T_b) \quad (3.18)$$

In Eq. (3.18), T_b is the mixed mean temperature or the bulk mean temperature defined by,

$$T_b = \int_{-L/2}^{L/2} uTdy \Big/ \int_{-L/2}^{L/2} udy \quad (3.19)$$

From Eq. (3.18), it follows that Nu_{1x}, the local Nusselt number at $Y = -1/4$, based on the hydraulic diameter is given by,

$$Nu_{1x} = \left(h_{1x}\, D_h\right)/k = \{1/(\theta^* - \theta|_{Y=-1/4})\}\left(\partial\theta/\partial Y\right)\Big|_{Y=-1/4} \quad (3.20)$$

Similarly, the local Nusselt number at $Y = 1/4$, the second wall is given by,

$$Nu_{2x} = \left(h_{2x}\, D_h\right)/k = -\{1/(\theta^* - \theta|_{Y=1/4})\}\left(\partial\theta/\partial Y\right)\Big|_{Y=1/4} \quad (3.21)$$

In Eqs. (3.20) and (3.21), θ^*, the non-dimensional bulk mean temperature is given by,

$$\theta^* = (T_b - \overline{T}_w)/(T_i - \overline{T}_w) = \int_{-1/4}^{1/4} U\theta dY \Big/ \int_{-1/4}^{1/4} UdY \quad (3.22)$$

When $A = 1$, noting $\theta|_{Y=-1/4} = \theta|_{Y=1/4} = 0$, Eqs. (3.20) and (3.21) yield,

$$Nu_{1x} = Nu_{2x} = Nu_x = (1/\theta^*)\left(\partial\theta/\partial Y\right)\Big|_{Y=-1/4} = -(1/\theta^*)\left(\partial\theta/\partial Y\right)\Big|_{Y=1/4} \quad (3.22)$$

Chapter 4

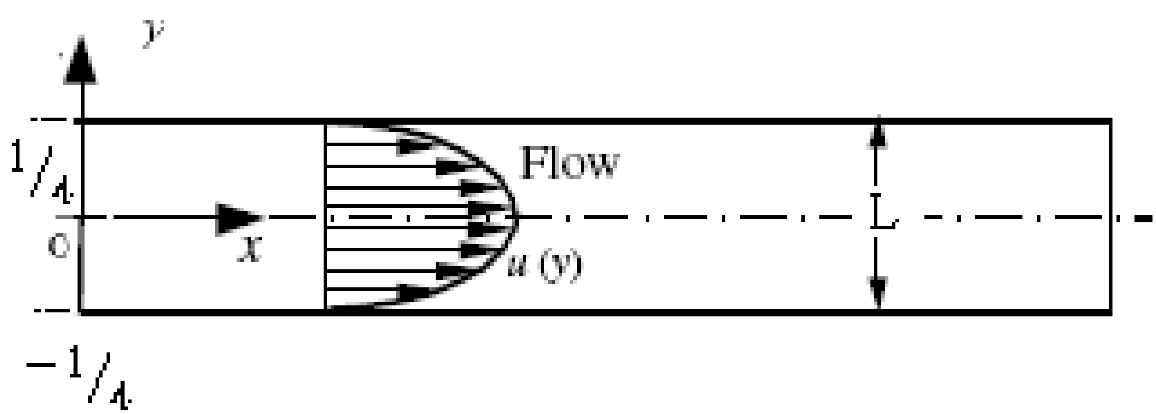
ABOUT THE PROJECT

ABOUT THE PROJECT

4.1 Objective

To analyze the thermal development of forced convection in a parallel plate channel filled by an incompressible fluid, with walls held at uniform temperature, and with the effects of axial conduction and viscous dissipation included. The analysis leads to expressions for the local Nusselt number, as a function of the dimensionless longitudinal coordinate and other parameters (Peclet number, Brinkman number).

4.2 Channel Geometry



X*=0 to 0.4 for M =1 to 1001,

Y=-0.25 to 0.25 for N=1 to 41,

Step size:

Δx =0.0004, Δy=0.0125

4.3 Procedure for evaluation of $Nu_{L,X}$

1. Based on SAR scheme velocity and temperature values for all the mesh points(X=1 to 1001, Y=1 to 41) were calculated.
2. Using trapezoidal rule; values for the bulk mean temperature were calculated.
3. Using 3-point forward discretization; values of $d\theta/dy$ were evaluated at the periphery of the channel(Y=0.25,-0.25).
4. Local Nusselt number values were calculated for different X* values.
5. Graph: Nu vs X was plotted for different peclet number values.

Chapter 5

RESULT AND DISCUSSION

5.1RESULT

$Nu_{1X}(=Nu_{2X})$ for Pe=1

Table: 1

X*	Nu_{1X}
0.002	119.5044
0.01	120.0414
0.05	120.0414
0.07	120.0414
0.1	120.0414
0.15	120.0414
0.2	120.0414
0.3	120.0414

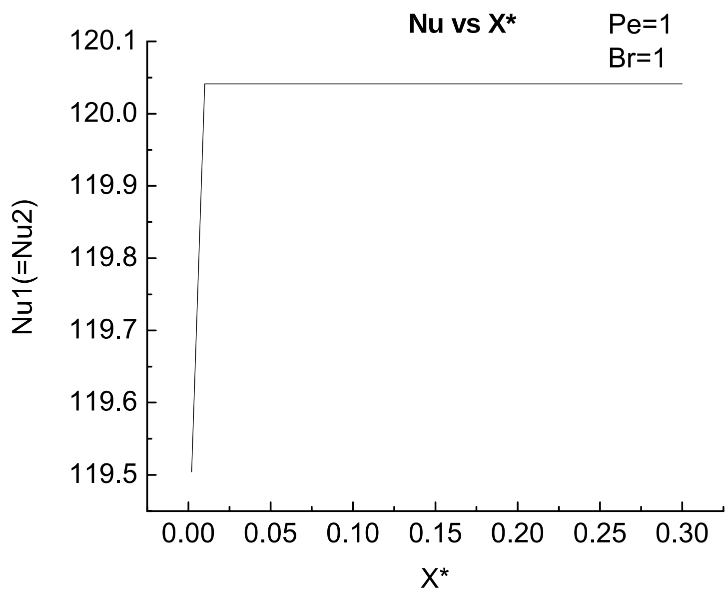


Figure: 1; A plot showing variation of local Nusselt number with X* for Pe=1, Br=1

Table: 2

$Nu_{1X}(=Nu_{2X})$ for Pe=10

X*	Nu_{1X}
0.002	11.05009
0.01	7.768944
0.05	7.518209
0.07	7.518401
0.1	7.519049
0.15	7.52275
0.2	7.539407
0.3	7.94603

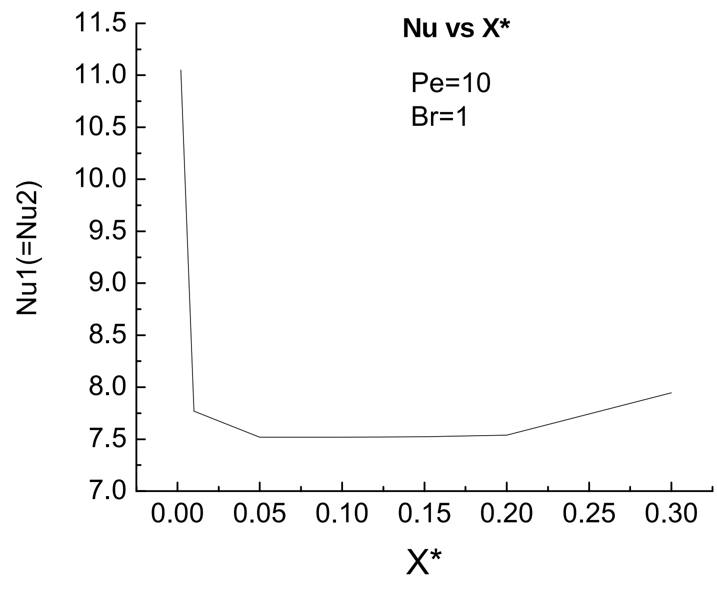


Figure: 2; A plot showing variation of local Nusselt number with X* for Pe=10, Br=1

Table: 3

$Nu_{1X}(=Nu_{2X})$ for Pe=20

X*	Nu_{1X}
0.002	11.05009
0.01	7.768944
0.05	7.518209
0.07	7.518401
0.1	7.519049
0.15	7.52275
0.2	7.539407
0.3	7.94603

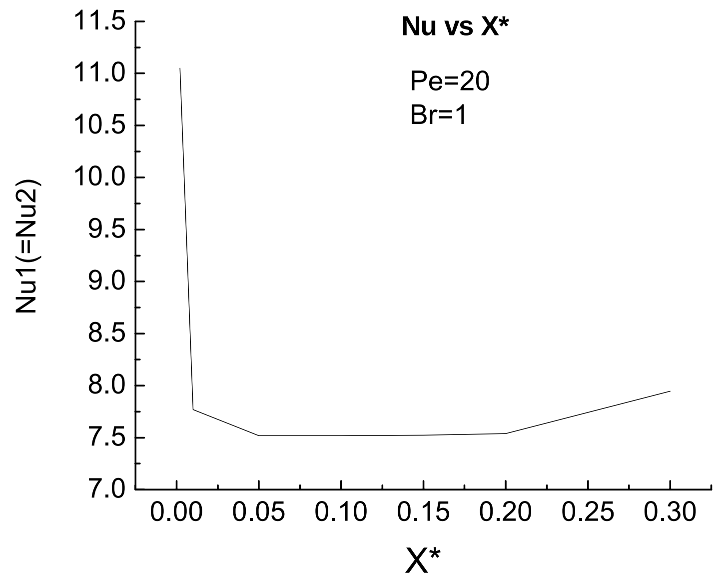


Figure: 3; A plot showing variation of local Nusselt number with X* for Pe=20, Br=1

Table: 4

$Nu_{1X}(=Nu_{2X})$ for Pe=50

X*	Nu_{1X}
0.002	11.05009
0.01	7.768944
0.05	7.518209
0.07	7.518401
0.1	7.519049
0.15	7.52275
0.2	7.539407
0.3	7.94603

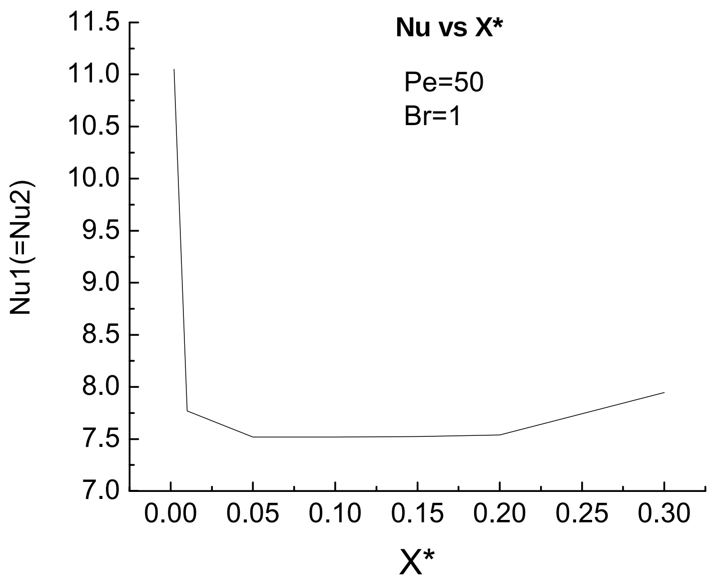


Figure: 4; A plot showing variation of local Nusselt number with X* for Pe=50, Br=1

5.2 DISCUSSION

Analysing the effects of adding an axial conduction term and a viscous dissipation term to the thermal energy equation for the problem of forced convection in a parallel-plate channel, with the temperature held constant at the walls. The Brinkman number and Peclet number have a significant effect on the developing Nusselt number.

Nusselt number for different X* and Peclet number (=1, 10, 20, 50) values was calculated. It was observed that Nusselt number attains a high value for small X* values and decreases almost asymptotically with rise in X* values.

A sharp change in the nature of graph is obtained for Pe=1 and Pe=10.Rise in Peclet number (above Pe=10) doesn't seem to have much impact on the Nusselt number values.

5.3 CONCLUSION

- Nusselt number decreases asymptotically with rise in the value of X*.
- A slight increase in the value of Nusselt number is obtained as approaching the end of channel.
- Small increments in Peclet number (above Pe=10), doesn't seem to have much impact on the values of local Nusselt number.

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