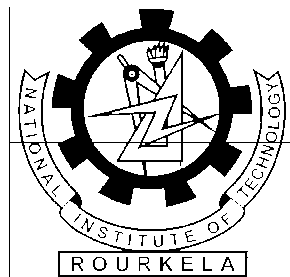


# AN NOVEL METHOD FOR ACOUSTIC NOISE CANCELLATION

A THESIS SUBMITTED IN  
PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF  
BACHELOR OF TECHNOLOGY  
IN  
ELECTRONICS & INSTRUMENTATION ENGINEERING.

By

SAMBIT KUMAR MISHRA  
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&  
NILIM CHANDRA SARMA  
Roll No. - 10507020



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**Department of Electronics & Communication Engineering  
National Institute of Technology, Rourkela**

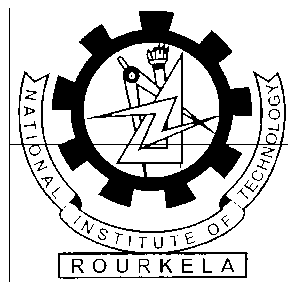
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Under the guidance of  
DR. GANAPATI PANDA



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**Department of Electronics & Communication Engineering  
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**National Institute of Technology  
Rourkela**

**CERTIFICATE**

This is to certify that the thesis titled “**A novel method for Acoustic Noise Cancellation**”, submitted by Sri Sambit Kumar Mishra (Roll No.- 10507037) and Sri Nilim Chandra Sarma (Roll No.- 10507020) in partial fulfillment of the requirements for the award of Bachelor of Technology Degree in Electronics and Instrumentation Engineering at National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/ Institute for the award of any Degree or Diploma.

Date:

Prof. Dr. G. Panda  
Department of E.C.E  
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Rourkela - 769008

# ACKNOWLEDGEMENT

I take this opportunity as a privilege to thank all individuals without whose support and guidance we could not have completed our project in this stipulated period of time.

First and foremost I would like to express my deepest gratitude to my Project Supervisor Prof. G. Panda, Head of the Department, Department of Electronics and Communication Engineering, for his invaluable support, guidance, motivation and encouragement throughout the period this work was carried out. His readiness for consultation at all times, his educative comments and inputs, his concern and assistance even with practical things have been extremely helpful.

I would also like to thank all professors and lecturers, and members of the department of Electronics and Communication Engineering for their generous help in various ways for the completion of this thesis. I also extend my thanks to my fellow students for their friendly co-operation.

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# TABLE OF CONTENTS

<b>List of Figures</b>	
<b>Abstract</b>	<b>1</b>
<b>1. Introduction</b>	<b>2</b>
<b>2. Adaptive Techniques applied to Acoustic Noise Cancellation</b>	<b>6</b>
2.1 The Least Mean Square Algorithm	7
2.2 Channel Equalization using LMS Algorithm	8
2.3 The Filtered-X LMS Algorithm	10
2.4 The Filtered-S LMS Algorithm	12
2.5 The Volterra Filtered-X LMS Algorithm	14
<b>3. The Hearing Aid Feedback Cancellation Model</b>	<b>16</b>
3.1 Identification of the External Path	19
3.2 Simulation Example	21
3.3 Simulation Output	22
<b>4. The Modified Hearing Aid Feedback Cancellation Model</b>	<b>23</b>
4.1 The Adaptive IIR LMS Algorithm	24
4.2 The Particle Swarm Optimization Algorithm	27
4.3 The Modified Hearing Aid Model	29
<b>5. Simulation Results and Discussion</b>	<b>30</b>
<b>6. Conclusion</b>	<b>40</b>
<b>7. References</b>	<b>42</b>

# LIST OF FIGURES

- Figure 1.1:** - Hearing aid with an internal feedback path used to cancel the acoustic feedback.
- Figure 2.1:** - Block diagram of adaptive transversal filter.
- Figure 2.2:** - Digital transmission system using channel equalization.
- Figure 2.3:** - Single Channel Filtered-X LMS adaptive controller.
- Figure 2.4:** - Block diagram of FSLMS algorithm.
- Figure 2.5:** - Block diagram of the second order VXLMS algorithm.
- Figure 3.1:** - Hearing aid with an internal feedback path used to cancel the acoustic feedback. The estimate of the external feedback path is achieved with FXLMS applied to filtered output and input signals.
- Figure 3.2:** - Impulse Response of the external feedback path and the adaptive model constituting the internal feedback path.
- Figure 4.1:** - Hearing aid with an adaptive IIR internal feedback path used to cancel the acoustic feedback.
- Figure 5.1:** - Simulation results showing the Learning curve and bit error rate for the LMS equalizer.
- Figure 5.2:** - Learning Curve for the Filtered-X LMS algorithm with  $\mu = 0.001$  and  $0.0025$ .
- Figure 5.3:** - Learning curve for Filtered-S LMS algorithm with  $\mu = 0.001$  &  $0.0025$ .
- Figure 5.4:** - Learning Curve for the second order Volterra Filtered-X LMS algorithm.
- Figure 5.5:** - Hearing Aid with an internal feedback path to cancel the external acoustic feedback.
- Figure 5.6:** - Minimization of Normalized Mean Square error for the FXLMS based feedback cancellation scheme.
- Figure 5.7:** - Minimization of Normalized Mean Square error for the adaptive IIR LMS based feedback cancellation scheme.
- Figure 5.8:** - Minimization of Normalized Mean Square error for the adaptive IIR PSO based feedback cancellation scheme.

## **ABSTRACT**

Over the last several years Acoustic Noise Cancellation (ANC) has been an active area of research and various adaptive techniques have been implemented to achieve a better online acoustic noise cancellation scheme. Here we introduce the various adaptive techniques applied to ANC viz. the LMS algorithm, the Filtered-X LMS algorithm, the Filtered-S LMS algorithm and the Volterra Filtered-X LMS algorithm and try to understand their performance through various simulations. We then take up the problem of cancellation of external acoustic feedback in hearing aid. We provide three different models to achieve the feedback cancellation. These are - the adaptive FIR Filtered-X LMS, the adaptive IIR LMS and the adaptive IIR PSO models for external feedback cancellation. Finally we come up with a comparative study of the performance of these models based on the normalized mean square error minimization provided by each of these feedback cancellation schemes.

# Chapter 1

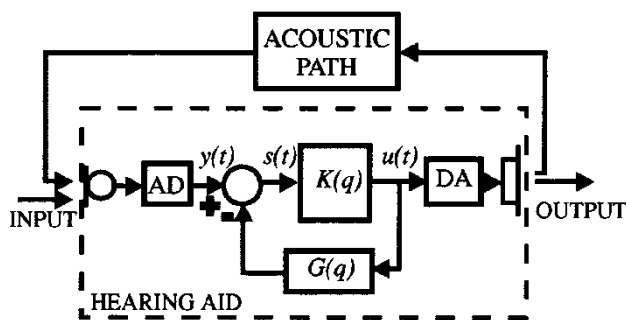
INTRODUCTION



A major complaint of hearing-aid users is acoustic feedback which is perceived as whistling or howling (at oscillation) or distortion (at sub-oscillatory intervals). This feedback occurs, typically at high gains, because of leakage from the receiver to the microphone. Acoustic feedback suppression in hearing-aids is important since it can increase the maximum insertion gain of the aid. The ability to achieve target insertion gain leads to better utilization of the speech bandwidth and, hence, improved speech intelligibility for the hearing-aid user. The acoustic path transfer function can vary significantly depending on the acoustic environment. Hence, effective acoustic feedback cancellers must be adaptive.

One way to reduce this problem is to cancel the acoustic feedback by an internal feedback path as in Fig. 1.1. The internal feedback path should have the same characteristics as the external feedback path from the input of the DA to the output of the AD. The output of the internal feedback path is subtracted from the microphone signal to remove the component of the microphone signal that constitutes feedback. The external feedback path will change when the hearing aid is used, when the user is chewing or placing the palm of the hand by the ear. Thus, it is desirable to continuously identify the external feedback path. The identification should be done without modifying the output signal of the hearing aid in such a way that the user could detect the modification. Another desirable characteristic of the identification algorithm is low computational complexity, as power consumption and size limit the processors used in digital hearing aids.

A number of different schemes that cancel the feedback as in Fig. 1.1, but uses alternative ways to identify the feedback path can be found in the literature. Some schemes identify



**Fig 1.1** Hearing aid with an internal feedback path used to cancel the acoustic feedback.

the feedback path in open loop by interrupting the throughput of the hearing aid and applying some probe signal, e.g., white noise, to the output. This may be disturbing for the user, especially if identification is required frequently. There is a second class of system where a probe signal (noise) is added to the output of the hearing aid and the identification is based on the information in the probe signal and the microphone signal. The main difference from the first class is that the identification can be performed without interrupting the throughput. The probe signal should have substantially lower level than the ordinary out of the hearing aid in order to be inaudible. A problem with this approach is that only a marginal part of the microphone signal will originate from the probe signal. This may reduce the accuracy of the achieved estimate. A third class of feedback cancellation schemes uses the output of the hearing aid and the microphone signal collected in closed loop as data for the identification. This corresponds to closed loop identification with the direct method. The output signal of the hearing aid can then be the sum of the ordinary output and a probe signal (as in the second class) or the ordinary output alone. An advantage over the second class is that a larger part of the microphone signal originates from the output signal, which can improve the accuracy of the identification. The disadvantage is that it is not only feedback that generates correlation between the output signal and the microphone signal. Input signals with substantial autocorrelation can also generate correlation between these signals. The feedback cancellation scheme will then predict and cancel more of the microphone signal than the signal via the external feedback. The performance of this scheme will thus depend on the input signal to the hearing aid.

Here, the characteristics of the recursive prediction error identification method Filtered-X LMS (FXLMS) applied to the feedback cancellation problem is analyzed. The data used for identification is the output and input signal of the hearing aid. The scheme would thus be classified to the third of the above classes of feedback cancellation schemes. The internal feedback path will then consist of a fixed filter in series with an adaptive filter with adjustable impulse response. The analyzed system also utilizes prefiltering of the data used for identification. FXLMS corresponds to system identification with an output error model. The feedback path is then identified under the assumption that the input signal to the system is white. The input signal to the hearing aid will in the identification be considered

as noise. Prefiltering can be seen as a way to modify the noise model. A good noise model (model of the input signal) is desirable in closed loop identification with the direct method as the error in the noise model may introduce bias in the estimate to which the adaptive filter converges.

In the following discussion the steady state characteristics of this system are analyzed. The analyzed system differs from many other systems where adaptive filters are used in that the system to be identified (the feedback path) is a part of a closed loop and that the data used for identification depends on previous estimates achieved with the adaptive filter. Thus, we cannot apply analysis of adaptive filters that assume that the system to be identified is operating in open loop or that the input signal to the system to be identified is independent of the adaptive filter.

# Chapter 2

ADAPTIVE TECHNIQUES APPLIED TO  
ACOUSTIC NOISE CANCELLATION

One of the major features of acoustic noise cancellation schemes is the identification of external feedback paths and cancellation of this feedback signal. Continuous adaptation to changing environment requires the use of adaptive systems for the identification purposes. Various techniques used for adaptive system identification have been implemented in acoustic noise cancellation problems. Some of these techniques have been discussed below.

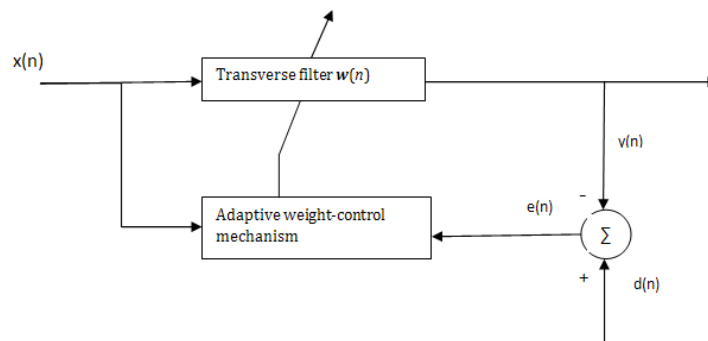
## 2.1 THE LEAST MEAN SQUARE (LMS) ALGORITHM

The Least Mean Square Algorithm is a linear adaptive filtering algorithm that consists of two basic processes:

1. A *filtering process* which involves
  - a) Computation of the output of a transverse filter produced by a set of tap inputs, and
  - b) Generating an estimation error by comparing this output to a desired response.
2. An *adaptive process*, which involves the automatic adjustment of the tap weights of the filter in accordance with the estimation error.

The LMS algorithm can be described in the form of three basic equations as follows

1. Filter output:  $y(n) = \mathbf{w}^H(n)\mathbf{x}(n)$
2. Estimation error:  $e(n) = d(n) - y(n)$
3. Tap weight adaptation:  $\mathbf{w}(n + 1) = \mathbf{w}(n) + 2\mu\mathbf{x}(n)e^*(n)$



**Fig 2.1** Block diagram of adaptive transversal filter

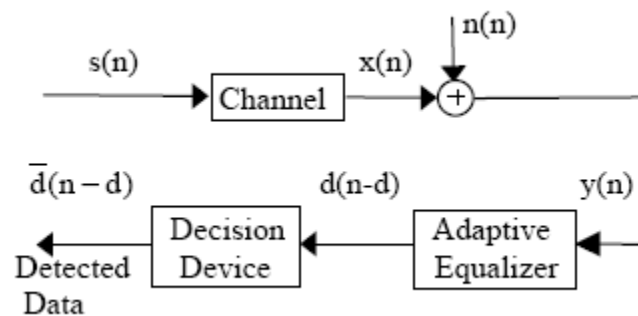
## 2.2 ADAPTIVE CHANNEL EQUALISATION USING LMS ALGORITHM

Traditionally, ISI problem is resolved by channel equalization in which the aim is to construct an equalizer such that the impulse response of the channel/equalizer combination is as close to  $z^{-d}$  as possible, where  $d$  is a delay. Frequently the channel parameters are not known in advance and moreover they may vary with time, in some applications significantly. Hence, it is necessary to use the adaptive equalizers, which provide the means of tracking the channel characteristics.

In Fig. 2.2, a digital transmission system using channel equalization is depicted. Here, as most often in practice, the analog signal is sampled at Nyquist rate and then the different samples are coded with binary sequences taking the two possible values  $-1$  and  $1$ . In Fig. 3.1  $s(n)$  denotes the transmitted signal where the sampling period  $T = 1$ . Actually  $s(n)$  is a random sequence of  $-1$ s and  $1$ s. Here the channel is dispersive and can be modeled by a FIR filter. Thus the channel output is written as

$$x(n) = \sum_{i=0}^N a_i s(n-i)$$

where  $N$  is filter order and  $a_i, 0 \leq i \leq N$ , are filter coefficients.



**Fig 2.2** Digital transmission system using channel equalization

In addition

$$y(n) = x(n) + n(n)$$

is the adaptive equalizer input, where  $n(n)$  is the inevitably present additive noise.

Obviously the problem to be considered is that of using the information represented in the observed equalizer inputs  $y(n)$  to produce an estimate of transmitted symbols in the sequence  $s(n)$ . There is a delay through the adaptive equalizer, which means that the estimate is delayed by  $d$  symbols. In Fig. 2.2 this is denoted as  $d(n - d)$  that is an estimate of  $s(n - d)$ . In the scheme shown in Fig. 2.2, a decision device applies a set of thresholds to recover the original data symbols selecting the symbol which is closest to the estimate  $d(n - d)$ . For example, if the transmitted signal takes values  $\pm 1$ , the decision device would simply be

$$\hat{d}(n - d) = \text{sign} [d(n - d)]$$

where  $\text{sign} [ \ ]$  is the sign function. Here the object is that  $\hat{d}(n - d) = s[d(n - d)]$ .

In such a digital transmission system a transversal filter can be used as an adaptive equalizer. In this arrangement the equalizer forms the linear combination of input samples as follows

$$d(n - d) = \sum_{i=0}^N w_i(n)y(n - d - i)$$

where  $w_j$ ,  $-d \leq j \leq d$ , are the filter coefficients. The object of the adaptive algorithm is to adjust the filter coefficients in such a manner that  $d(n - d) \approx s(n - d)$ . A criterion that can be applied to adapt the coefficients is the minimization of the output mean square error  $E[e^2(n - d)]$  where

$$e(n - d) = d(n - d) - s(n - d).$$

The estimation error is determined as the difference between the estimate and the original signal, which implies that the transmitted sequence is known in advance. In this case the equalizer operates in so-called training mode.

The second possible fashion of working is decision direction mode where

$$e(n - d) = d(n - d) - \hat{d}(n - d).$$

This manner of adaptation is acceptable for tracking a slowly varying channel, but cannot be used for initial channel identification. In this case the approach is to utilize a deterministic data sequence and a training period. The time it takes for training of the

equalizer is strongly dependent on the choice of adaptive algorithm. The main concern is to reduce the training time as much as possible and this suggests the usage of adaptive algorithms, which converge rapidly. Although the LMS algorithm has low adaptation rate, its strongest point is its low computation complexity.

### **2.3 THE FILTERED-X LEAST MEAN SQUARE ALGORITHM**

The filtered-X least-mean-square (LMS) algorithm is one of the most popular adaptive control algorithm used in DSP implementations of active noise and vibration control systems. There are several reasons for this algorithm's popularity. First, it is well-suited to both broadband and narrowband control tasks, with a structure that can be adjusted according to the problem at hand. Second, it is easily described and understood, especially given the vast background literature on adaptive filters upon which the algorithm is based. Third, its structure and operation are ideally suited to the architectures of standard DSP chips, due to the algorithm's extensive use of the multiply / accumulate (MAC) operation. Fourth, it behaves robustly in the presence of physical modeling errors and numerical effects caused by finite-precision calculations. Finally, it is relatively simple to set up and tune in a real-world environment.

The block diagram of a single channel filtered-X LMS adaptive feed forward controller is shown in Fig. 2.3, in which a sensor placed near a sound source collects samples of the input signal  $x(n)$  for processing by the system. This system computes an actuator output signal  $y(n)$  using a time-varying FIR filter of the form

$$y(n) = \sum_{l=0}^{L-1} w_l(n)x(n-l)$$

where ,  $w_l(n), 0 \leq l \leq L - 1$  are the controller coefficients at time  $n$  and  $L$  is the controller filter length. The acoustic output signal produced by the controller combines with the sound as it propagates to the quiet region, where an error sensor collects the combined signal. We model this error as



$$e(n) = d(n) + \sum_{m=-\infty}^{\infty} h_m y(n-m)$$

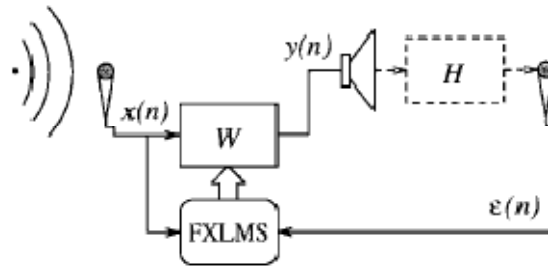
where  $d(n)$  is the undesired sound as measured at the error sensor and  $h_m$ ,  $-\infty < m < \infty$  is the plant impulse response. The filtered-X LMS coefficient updates are given by

$$w_l(n+1) = w_l(n) + \mu e(n) f(n-l)$$

where  $\mu$  is the algorithm step size, the filtered input sequence is computed as

$$f(n) = \sum_{m=1}^M h_m x(n-m)$$

and  $M$  is the FIR filter length of an appropriate estimate of the plant impulse response.



**Fig 2.3** Single Channel Filtered-X LMS adaptive controller

## **2.4 THE FILTERED-S LEAST MEAN SQUARE ALGORITHM**

Filtered-X least mean square (FXLMS) algorithm is most popular for Active Noise Control applications because of its simplicity. However, in the case of nonlinear noise processes, the FXLMS algorithm shows poor performance. It has been established that the performance of the controller can be considerably improved by using nonlinear control structures. Recently, two nonlinear adaptive algorithms, namely, filtered-S LMS (FSLMS) algorithm and Volterra filtered-X LMS (VFXLMS) algorithm have been proposed and the authors have shown that these algorithms can be used where FXLMS algorithm fails to perform. However, as each element of the nonlinear expansion has to be filtered through an estimated secondary path transfer function, these algorithms are more computationally intensive than the linear FXLMS algorithm.

The filter-bank implementation of the FSLMS algorithm is shown in Fig. 2.4. We can think of this adaptive filter as a bank of linear filters with the inputs modulated by sinusoidal nonlinear functions. In such a scheme, the acoustic path from the input microphone to the canceling loudspeaker is a primary path/plant which can be linear or nonlinear.  $d(n)$  is the primary noise at the canceling point and  $e(n)$  is the residual error sensed by the error microphone, which can be expressed by

$$e(n) = d(n) - h(n) * y(n)$$

Here,  $h(n)$  represents the impulse response of the secondary path transfer function and  $y(n)$  is the output of the adaptive filter, which is computed as

$$y(n) = \sum_{i=1}^{2P+1} y_i(n)$$

where

$$y_i(n) = \mathbf{s}_i^T(n) \mathbf{w}_i(n)$$

and  $\mathbf{w}_i(n) = [w_{i,0}(n) \ w_{i,1}(n) \ \dots \ w_{i,N-1}(n)]^T$ ,  $i = 1; \dots; 2P + 1$ , is an N-point coefficient vector of the controller at time n. Here, N represents the memory size, P is the order of the functional expansion and  $\mathbf{s}_i(n) = [s_i(n) \ s_i(n-1) \ \dots \ s_i(n-N + 1)]^T$  contains N recent samples of the functionally expanded reference input  $x(n)$ . Here,  $s_i(n)$  is defined as follows:

$$\begin{aligned} s_i(n) &= x(n), & \text{if } i &= 1; \\ \sin(l\pi x(n)), & & \text{if } i &\text{ is even and } l = 1, 2, \dots, P \\ \cos(l\pi x(n)), & & \text{if } i &\text{ is even and } l = 1, 2, \dots, P \end{aligned}$$

The weight update equations at time  $n - 1$  and  $n$ , respectively, can be written as

$$w_i(n) = w_i(n - 1) + \mu e(n - 1) f_i(n - 1)$$

$$w_i(n + 1) = w_i(n) + \mu e(n) f_i(n)$$

where  $f_i(n) = [f_i(n) f_i(n - 1) \dots f_i(n - N + 1)]^T$  is the vector of filtered, functionally expanded reference signal and  $f_i(n)$  is computed as

$$f_i(n) = \check{s}_i^T(n) \mathbf{h}$$

where  $\check{s}_i(n) = [s_i(n) s_i(n - 1) \dots s_i(n - L + 1)]^T$  and  $\mathbf{h} = [h_0 h_1 \dots h_{L-1}]^T$  is the coefficient vector of the transfer function of the secondary path and  $L$  is the length of the secondary path filter.

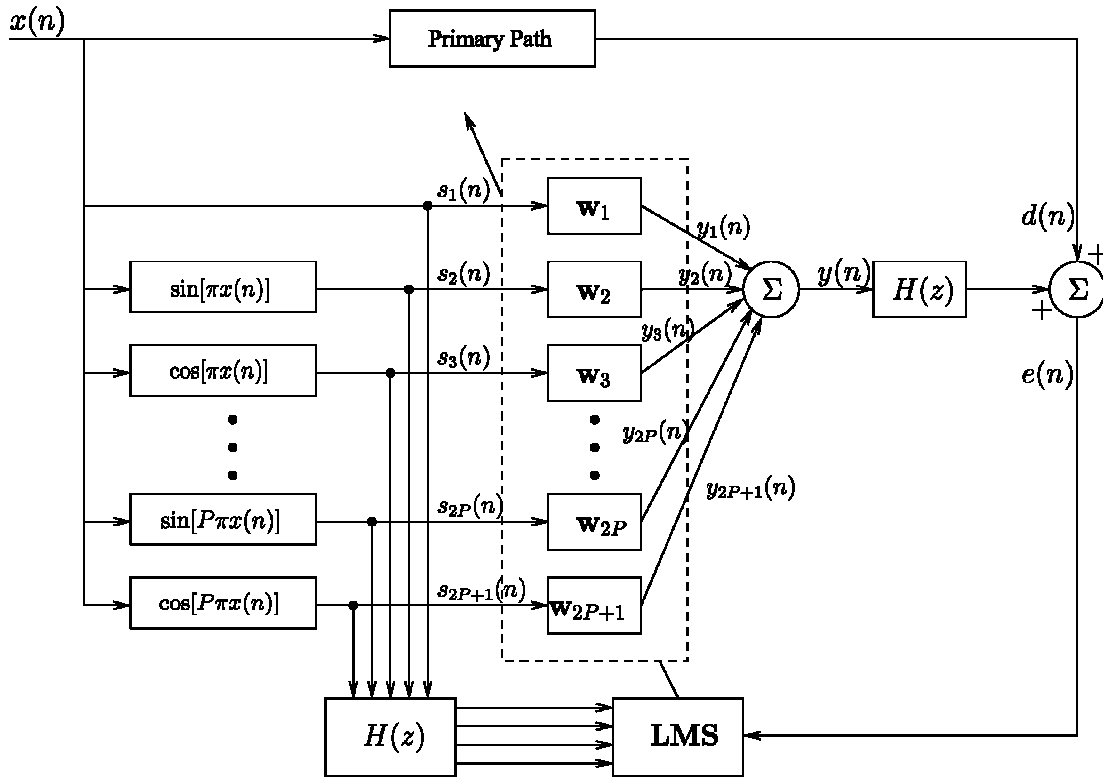


Fig 2.4 Block diagram of FSLMS algorithm

## 2.5 THE VOLTERRA FILTERED-X LEAST MEAN SQUARE ALGORITHM

The block diagram of the second-order Volterra filtered-X LMS (VFXLMS) algorithm for active noise control application is depicted in Fig 2.5. Here,  $e(n)$  is the residual noise, and can be expressed as

$$e(n) = d(n) - h(n)*y(n)$$

In the structure shown in Fig. 5.3,  $y(n)$  can be computed as

$$y(n) = \sum_{k=0}^N y_k(n)$$

where

$$y_k(n) = \mathbf{u}_k^T(n) \mathbf{w}_k(n)$$

where the weight vectors are defined as follows:

$$\begin{aligned} \mathbf{w}_k(n) &= [w_{k,0}(n); w_{k,1}(n); \dots; w_{k,(N-1)}(n)]^T; \text{ for } k = 0 \\ &= [w_{k,0}(n); w_{k,1}(n); \dots; w_{k,(N-k)}(n)]^T; \text{ for } k = 1; \dots; N \end{aligned}$$

By this definition,  $w_0(n)$  and  $w_1(n)$  are of length  $N$  and  $w_2(n); w_3(n); \dots; w_N(n)$  are of length  $N - 1; N - 2; \dots; 1$ , respectively. Hence, the total number of filter coefficients of the ANC structure =  $N(N + 3)/2$ . The input vector is defined as

$$\begin{aligned} \mathbf{u}_k(n) &= [u_k(n); u_k(n - 1); \dots; u_k(n - N + 1)]^T; \text{ for } k = 0 \\ &= [u_k(n); u_k(n - 1); \dots; u_k(n - N + k)]^T; \text{ for } k = 1; \dots; N \end{aligned}$$

where

$$\begin{aligned} u_k(n) &= x(n), & \text{for } k = 0; \\ &= x(n)x(n - l), & \text{for } k > 0 \text{ and } l = 0, 1, \dots, N-1; \end{aligned}$$

and  $x(n)$  is the reference noise signal.

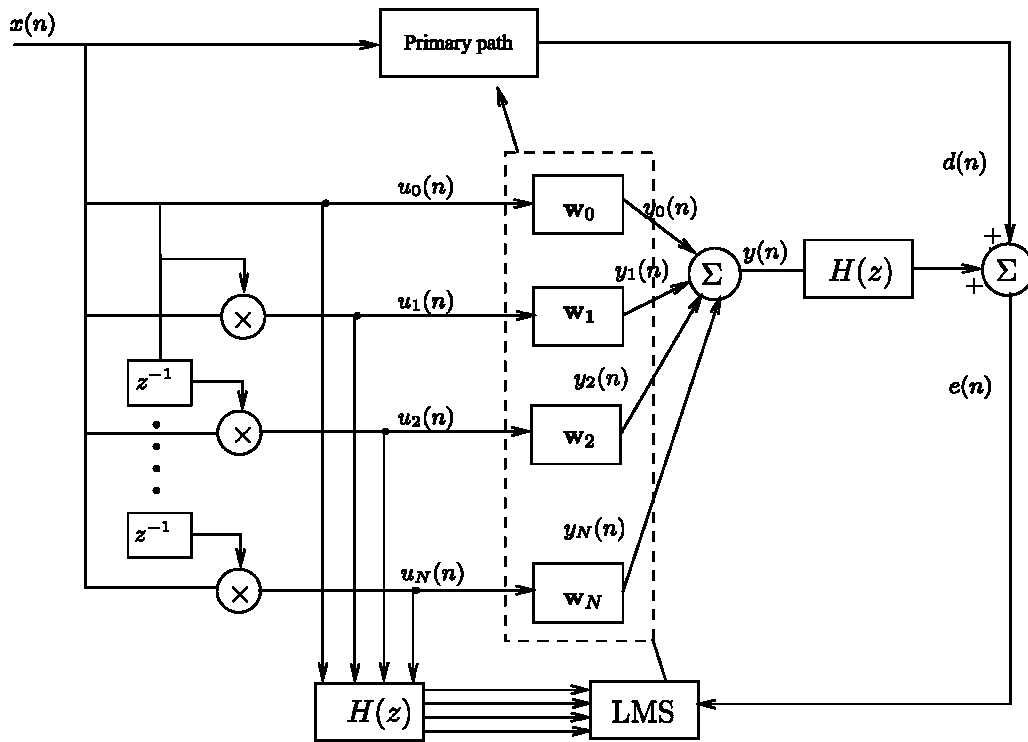


Figure 2.5 Block diagram of the second order VXLMS algorithm

# Chapter 3

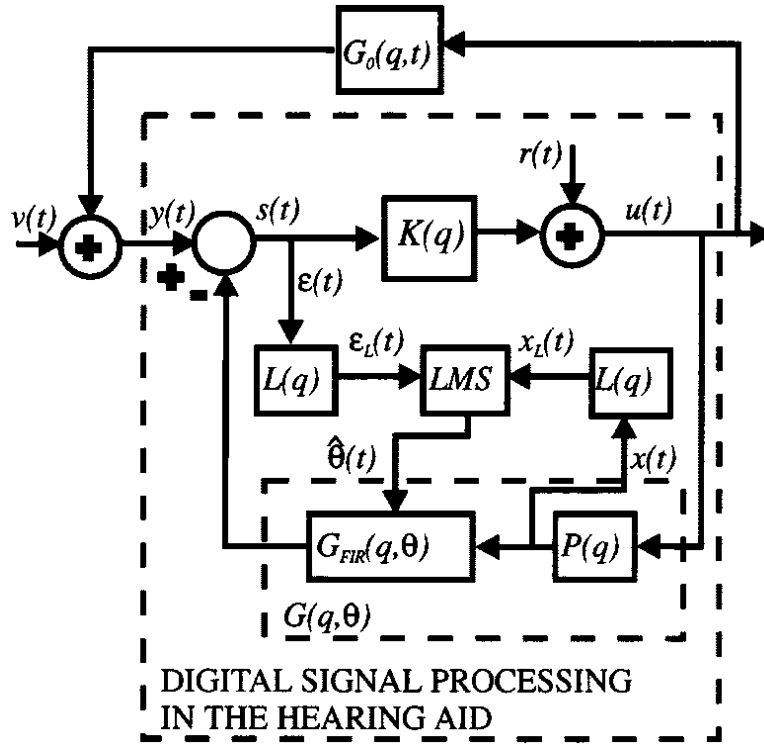
## THE HEARING AID FEEDBACK CANCELLATION MODEL

### **THE HEARING AID FEEDBACK CANCELLATION MODEL**

Acoustic feedback that causes a hearing aid to oscillate is one of the most common complaints of hearing aid users. In order to reduce this, the external feedback path can be cancelled by an internal feedback path having the same characteristics as the external path. Fig. 3.1 shows a model of the external feedback path cancellation scheme using an adaptive FIR filter.  $y(t)$  is the input signal to the hearing aid (output of the AD converter).  $t$  denotes discrete time.  $u(t)$  is the output of the hearing aid (input to the DA-converter).  $K(q)$  is the signal processing in the forward path of the hearing aid used to adjust the input signal of the hearing aid to the impaired ear.  $q$  is the time delay operator:  $q^{-1}(t) = u(t-1)$ .

$v(t)$  is the input signal to the hearing aid without any feedback. Thus,  $v(t)$  is the desired input to  $K(q)$ .  $G_o(q,t)$  is the external feedback path from the input of the DA to the output of the AD that is to be estimated. The DA-converter, the hearing aid receiver (speaker), the acoustic feedback path, other possible feedback paths, the microphone and the AD-converter give the characteristics of  $G_o(q,t)$ . It is assumed that  $G_o(q,t)$  is linear. Therefore it can be useful to use some kind of nonlinearity in  $K(q)$  to limit the level of  $u(t)$ , which otherwise could cause saturation of the receiver and a nonlinear  $G_o(q,t)$ .  $\hat{G}_o(q,t)$  is the estimate of  $G_o(q,t)$  that is generated by the FXLMS algorithm and is used to predict the feedback signal.

$G_o(q,\theta)$  consists of a fixed filter  $P(q)$  in series with an adjustable FIR-filter  $G_{FIR}(q,\theta(t))$  are the parameters of the adjustable FIR-Filter and are the parameters used at time  $t$ . The two filters denoted  $L(q)$  are prefilters used to modify the signals used to update the parameters, and thereby modify the criteria of the adaptive filter. It is assumed that  $K(q)$  or both  $G_o(q,\theta(t))$  and  $G_o(q,t)$  has an initial delay. The system would otherwise not be computable.  $s(t)$  is the input signal with estimated feedback subtracted and is thus an estimate of the desired input  $v(t)$ .  $r(t)$  is some probe signal uncorrelated to  $v(t)$  added to the output of the hearing aid.  $r(t)$  is not required in the system, but can be used to reduce the bias in the limiting estimate that the adaptive filter converges.



**Fig. 3.1** Hearing aid with an internal feedback path used to cancel the acoustic feedback. The estimate of the external feedback path is achieved with FXLMS applied to filtered output and input signals.

The transfer function of the digital signal processing in the hearing aid from  $y(t)$  to  $u(t)$  will, with a linear  $K(q)$ , be

$$K(e^{j\omega}) / (1 + K(e^{j\omega})G(e^{j\omega}, \theta(t)))$$

where  $\omega$  is frequency expressed in rad/s. The transfer function of the entire system from  $v(t)$  to  $u(t)$  is then

$$K(e^{j\omega}) / (1 + K(e^{j\omega})G_{\sim}(e^{j\omega}, \theta(t)))$$

where  $G_{\sim}(e^{j\omega}, \theta(t)) = G_0(e^{j\omega}) - G(e^{j\omega}, \theta(t))$

The system given by can be compared with the desired function  $K(e^{j\omega})$ . The system will be stable if  $|K(e^{j\omega})G_{\sim}(e^{j\omega}, \theta(t))| < 1$  at all frequencies and we would like to minimize this quantity to minimize the deviation from  $K(q)$ .



### **3.1 IDENTIFICATION OF THE EXTERNAL FEEDBACK PATH**

The input of the hearing aid is a mix between the desired input signal  $v(t)$  and the feedback

$$y(t) = G_0(q,t)u(t) + v(t)$$

It is  $G_0(q,t)$  that we want to find an estimate of, which then is used to cancel the feedback.  $u(t)$  is a known signal as it is the output signal of the hearing aid.  $v(t)$  is unknown as is thus considered as a noise in the identification process.

The model of the system used in the identification uses a parameterized model of the feedback and a fixed model of the generator of the noise.

$$y(t) = G_0(q,t)u(t) + H(q)e(t)$$

$G(q,\theta)$  is the model of the feedback implemented by the fixed filter  $P(q)$  and a FIR filter  $G_{\text{FIR}}(q,\theta)$  with adjustable parameters  $\theta$ .  $P(q)$  defines fixed poles and zeros of  $G(q,\theta)$  that is not altered as  $\theta$  is modified. The length of the impulse response of  $G_{\text{FIR}}(q,\theta)$  and the number of coefficients in  $\theta$  is  $d$ . The noise in the model is generated by filtering a white noise  $e(t)$  through a monic and inversely stable filter  $H(q)$ . The used model is given by

$$\begin{aligned} G(q,\theta) &= P(q) G_{\text{FIR}}(q,\theta) \\ &= P(q)[q^{-1}, q^{-2}, q^{-3}, \dots, q^{-d}] \theta \\ \theta &= [b_1, b_2, b_3 \dots b_d]^T \\ H(q) &= 1 \end{aligned}$$

The optimal one step ahead predictor of  $y(t)$  of a system given by  $G(q,\theta)$  and  $H(q)$  is

$$y'(t) = H^{-1}(q) G(q,\theta)u(t) + [1 - H^{-1}(q)]y(t)$$

The one step ahead prediction with model structure used is thus

$$\begin{aligned} y'(t) &= \varphi(t)^T \theta \\ \varphi(t) &= P(q)[q^{-1}, q^{-2}, q^{-3}, \dots, q^{-d}]u(t) \end{aligned}$$

and the prediction error is

$$\epsilon(t) = y(t) - \varphi(t)^T \theta$$

The criteria that the adaptive filter tries to minimize is

$$V(\theta) = 0.5 * E[\epsilon_F(t)^2]$$

$$\epsilon_F(t) = L(q) \epsilon(t)$$

$L(q)$  is a linear predictor used to give a frequency specific weight of the error

The estimate is updated according to

$$\theta(t+1) = \theta(t) + \gamma(t) L(q) \varphi(t) L(q) \epsilon(t)$$

where the adaptation gain  $\gamma(t)$  is a positive scalar that controls the step size.

The identification method can be classified as FXLMS with prefiltering of data. The signal processing of the hearing aid can be summarized by

$$\varphi(t) = P(q)[q^{-1}, q^{-2}, q^{-3}, \dots, q^{-d}]u(t) \quad (a)$$

$$\varphi_L(t) = L(q) \varphi(t) \quad (b)$$

$$s(t) = \epsilon(t) = y(t) - \varphi(t)^T \theta(t) \quad (c)$$

$$\epsilon_L(t) = L(q) \epsilon(t) \quad (d)$$

$$\theta(t+1) = \theta(t) + \gamma(t) \varphi_L(t) \epsilon_L(t) \quad (e)$$

$$u(t) = K(q)s(t) + r(t) \quad (f)$$

The steps (a) – (e) are the steps used to estimate and cancel the feedback while (f) is the forward path of the hearing aid.

### 3.2 SIMULATION EXAMPLE

The signal processing in the forward path is

$$K(q) = -1$$

The feedback cancellation was defined by

$$L(q) = 1 - 2.01q^{-1} + q^{-2}$$

$$P(q) = q^{-61} - 1.8q^{-62} + 0.81q^{-63}$$

$$r(t) = 0$$

$$\gamma(t) = 2 \cdot 10^{-5}$$

$$d = 32$$

The probe signal added to the output  $r(t)$  is not used. The used  $P(q)$  is a high pass filter and is a very rough approximation of the external feedback path. the initial delay of  $P(q)$  was chosen to get the impulse response of the adaptive filter in approximately the same window as the impulse response of the external feedback path.  $L(q)$  was a high pass filter that can be suitable if the input signal has most energy at low frequencies.

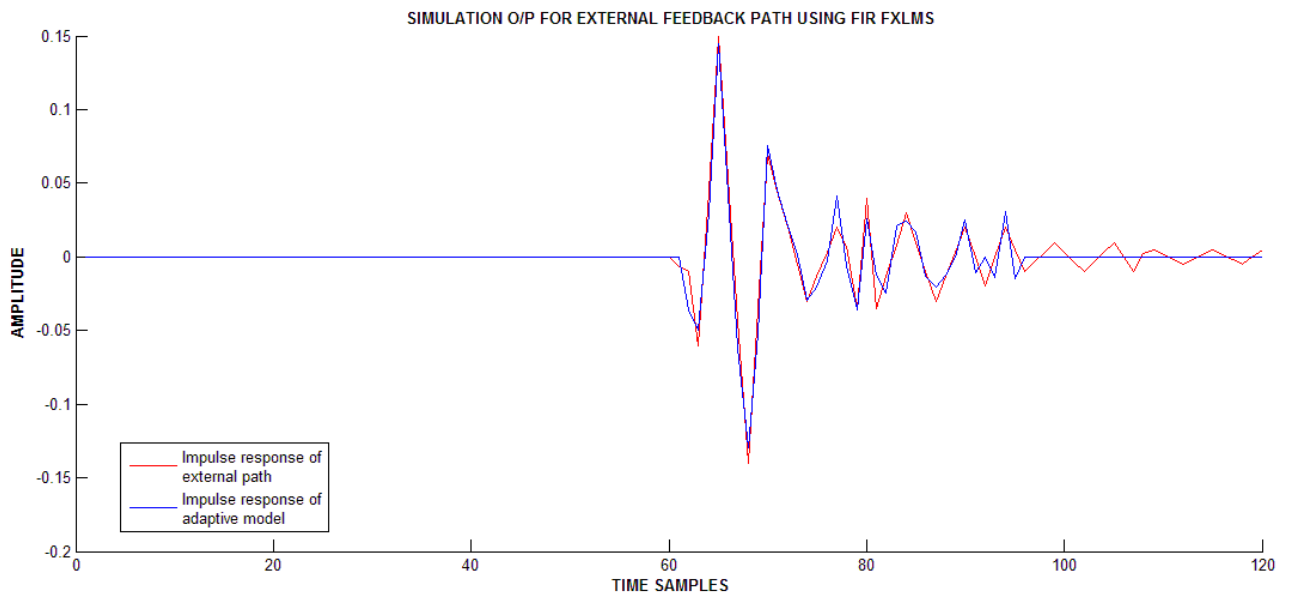
The feedback path of a Behind The Ear (BTE) hearing aid on a human subject identified in the normal situation  $G_{ON}(q)$  is used to generate  $G_O(q)$  used in simulations. The feedback path is identified as FIR filter with an initial delay of 50 samples and the 70 coefficients. A sampling frequency of 15.75 KHz was used. The impulse response of the external feedback path was close to zero for the 62 first lags.

The input signal to the system  $v(t)$  was generated by filtering a white noise with a variance one through a filter  $H_O(q)$  given by

$$H_O(q) = (1 + 2q^{-1} + q^{-2}) / (1 - 1.45q^{-1} + 0.57q^{-2})$$

$H_O(q)$  is a second order filter (low pass) with a cutoff frequency of approximately 1 KHz and was used as a very rough approximation of normal speech. The  $L(q)$  used has a high pass characteristics and will to some extent whiten the spectrum of  $v(t)$ .

### 3.3 SIMULATION OUTPUT



**Fig 3.2** Impulse Response of the external feedback path and the adaptive model constituting the internal feedback path

# Chapter 4

## THE MODIFIED HEARING AID FEEDBACK CANCELLATION MODEL

The hearing aid external feedback cancellation model described in the previous section has the drawback that because of the finite impulse response of the adaptive FIR filter the characteristics of the internal feedback path does not match entirely with the external path. This is clearly evident from the simulation output shown in Fig 3.2. One way to overcome this problem is to use an adaptive IIR filter. An adaptive IIR filter consists of a number of poles and zeros which are tuned with the help of a learning or adaptive algorithm. We have used two algorithms to tune the parameters of the adaptive IIR filter –

- a) The adaptive IIR Least Mean Square Algorithm.
- b) The Particle Swarm Optimization Algorithm.

#### **4.1 THE ADAPTIVE IIR LMS ALGORITHM**

Adaptive IIR filters are attractive for the same reasons that IIR filters are attractive: many fewer coefficients may be needed to achieve the desired performance in some applications. However, it is more difficult to develop stable IIR algorithms, they can converge very slowly, and they are susceptible to local minima. Nonetheless, adaptive IIR algorithms are used in some applications (such as low frequency noise cancellation) in which the need for IIR-type responses is great. In some cases, the exact algorithm used by a company is a tightly guarded trade secret. Most adaptive IIR algorithms minimize the prediction error, to linearize the estimation problem, as in deterministic or block linear prediction.

$$y_k = \sum_{n=1}^L (v_n^k y_{k-n}) + \sum_{n=0}^L (w_n^k x_{k-n})$$

Thus the coefficient vector and the signal vectors are

$$W_k = \begin{pmatrix} v_1^k \\ v_2^k \\ \vdots \\ v_L^k \\ w_0^k \\ w_1^k \\ \vdots \\ w_L^k \end{pmatrix} \quad U_k = \begin{pmatrix} y_{k-1} \\ y_{k-2} \\ \vdots \\ y_{k-L} \\ x_k \\ x_{k-1} \\ \vdots \\ x_{k-L} \end{pmatrix}$$

The error is

$$\epsilon_k = d_k - y_k = d_k - W_k^T U_k$$

An LMS algorithm can be derived using the approximation

$$E[\epsilon_k^2] = \epsilon_k^2$$

$$\hat{\nabla}_k = \frac{\partial}{\partial W_k} (\epsilon_k^2) = 2\epsilon_k \frac{\partial}{\partial W_k} (\epsilon_k) = 2\epsilon_k \begin{pmatrix} \frac{\partial}{\partial v_1^k} (\epsilon_k) \\ \vdots \\ \frac{\partial}{\partial \epsilon_k} (w_1^k) \\ \vdots \end{pmatrix} = -2\epsilon_k \begin{pmatrix} \frac{\partial}{\partial v_1^k} (y_k) \\ \vdots \\ \frac{\partial}{\partial v_L^k} (y_k) \\ \frac{\partial}{\partial w_0^k} (y_k) \\ \vdots \\ \frac{\partial}{\partial w_L^k} (y_k) \end{pmatrix}$$

Now

$$\frac{\partial}{\partial v_i^k} (y_k) = \frac{\partial}{\partial v_i^k} \left( \sum_{n=1}^L (v_n^k y_{k-n}) + \sum_{n=0}^L (w_n^k x_{k-n}) \right) = y_{k-n} + \sum_{n=1}^L \left( v_n^k \frac{\partial}{\partial v_i^k} (y_{k-n}) \right) + 0$$

$$\frac{\partial}{\partial w_i^k} (y_k) = \frac{\partial}{\partial w_i^k} \left( \sum_{n=1}^L (v_n^k y_{k-n}) + \sum_{n=0}^L (w_n^k x_{k-n}) \right) = \sum_{n=1}^L \left( v_n^k \frac{\partial}{\partial w_i^k} (y_{k-n}) \right) + x_{k-n}$$

Let

$$\alpha_i^k = \frac{\partial}{\partial w_i^k} (y_k), \beta_i^k = \frac{\partial}{\partial v_i^k} (y_k)$$

Then

$$\hat{\nabla}_k = \left( \beta_1^k \quad \beta_2^k \quad \dots \quad \beta_L^k \quad \alpha_0^k \quad \dots \quad \alpha_L^k \right)^T$$

and the IIR LMS algorithm becomes

$$y_k = W_k^T U_k$$

$$\alpha_i^k = x_{k-i} + \sum_{j=1}^L \left( v_j^k \alpha_i^{k-j} \right)$$

$$\beta_i^k = y_{k-i} + \sum_{j=1}^L \left( v_j^k \beta_i^{k-j} \right)$$

$$\hat{\nabla}_k = -2\epsilon_k \left( \beta_1^k \quad \beta_2^k \quad \dots \quad \alpha_0^k \quad \alpha_1^k \quad \dots \quad \alpha_L^k \right)^T$$

$$W_{k+1} = W_k - U \hat{\nabla}_k$$

where the  $\mu$  may be different for the different IIR coefficients. Stability and convergence rate depends on these choices, of course.



## **4.2 THE PARTICLE SWARM OPTIMIZATION ALGORITHM**

Particle swarm optimization is a stochastic, population-based computer algorithm for problem solving. It is a kind of swarm intelligence that is based on social-psychological principles and provides insights into social behavior, as well as contributing to engineering applications. PSO emulates the swarm behavior of insects, animals herding, birds flocking and fish schooling where these swarms search for food in a collaborative manner. Each member in the swarm adapts its search patterns by learning from its own experience and other members' experience.

The swarm is typically modeled by particles in multidimensional space that have a position and a velocity. These particles fly through hyperspace (i.e.,  $\mathbb{R}^n$ ) and have two essential reasoning capabilities: their memory of their own best position and knowledge of the global or their neighborhood's best. In a minimization optimization problem, problems are formulated so that "best" simply means the position with the smallest objective value. Members of a swarm communicate good positions to each other and adjust their own position and velocity based on these good positions.

So a particle has the following information to make a suitable change in its position and velocity:

A *global best* that is known to all and immediately updated when a new best position is found by any particle in the swarm.

The *local best*, which is the best solution that the particle has seen.

The particle position and velocity update equations in the simplest form that govern the PSO are given by

*Velocity update:*

$$v_i(t+1) = w * v_i(t) + c1 * rand * (localbest(t) - x_i(t)) + c2 * rand * (globalbest(t) - x_i(t))$$

*Position update:*

$$x_i(t+1) = x_i(t) + v_i(t+1)$$

where  $c_1$  and  $c_2$  are constants;  $\text{rand}$  is a random number between 0 and 1 and  $w$  is the inertia weight vector. A large inertia weight ( $w$ ) facilitates a global search while a small inertia weight facilitates a local search. By linearly decreasing the inertia weight from a relatively large value to a small value through the course of the PSO run gives the best PSO performance compared with fixed inertia weight settings.

As the swarm iterates, the fitness of the global best solution improves (decreases for minimization problem). It could happen that all particles being influenced by the global best eventually approach the global best, and from there on the fitness never improves despite however many runs the PSO is iterated thereafter. The particles also move about in the search space in close proximity to the global best and not exploring the rest of search space. This phenomenon is called 'convergence'. If the inertial coefficient of the velocity is small, all particles could slow down until they approach zero velocity at the global best. The selection of coefficients in the velocity update equations affects the convergence and the ability of the swarm to find the optimum. One way to come out of the situation is to reinitialize the particles positions at intervals or when convergence is detected.

The linearly decreasing inertia weight is given by:

$$w = w_{\max} - (w_{\max} - w_{\min}) * k / I$$

where

$k$  = search number

$I$  = total number of iterations

$w_{\max}$  = maximum value of inertia weight

$w_{\min}$  = minimum value of inertia weight

### 4.3 THE MODIFIED HEARING AID MODEL

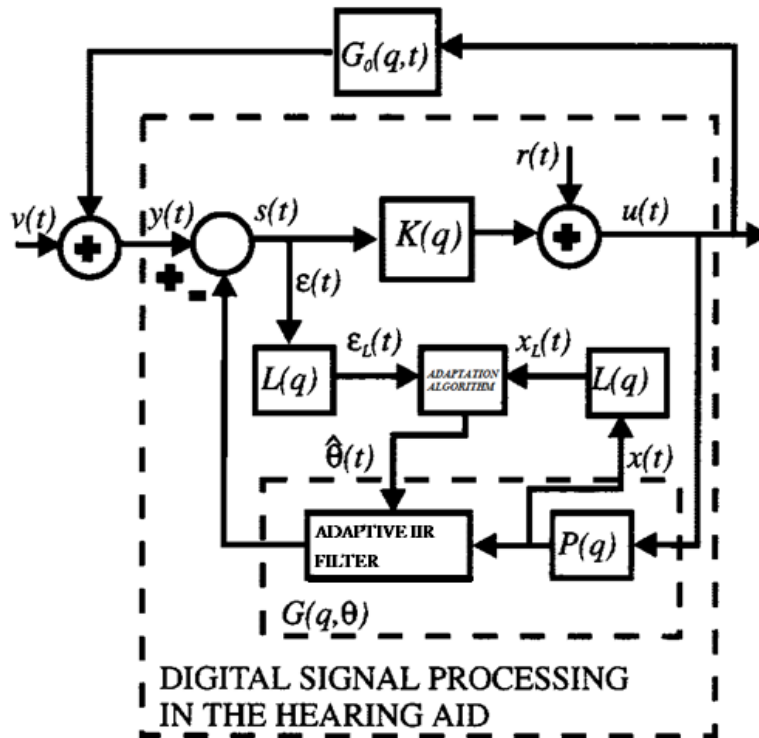


Fig 4.1 Hearing aid with an adaptive IIR internal feedback path used to cancel the acoustic feedback.

The model of the hearing aid is similar to that used previously. The only difference is that the adaptive filter is now an IIR filter instead of the FIR filter. The adaptive IIR filter consists of 20 taps in the feed forward path and 12 taps in the feedback path. The following additional parameters were used during simulation

Adaptive IIR LMS algorithm:

$\mu_1$  = step size in the feed forward path =  $5 \cdot 10^{-6}$

$\mu_2$  = step size in the feedback path =  $1.5 \cdot 10^{-6}$

Particle Swarm Optimization Algorithm:

Population size = 50

wmax = 0.9

wmin = 0.4

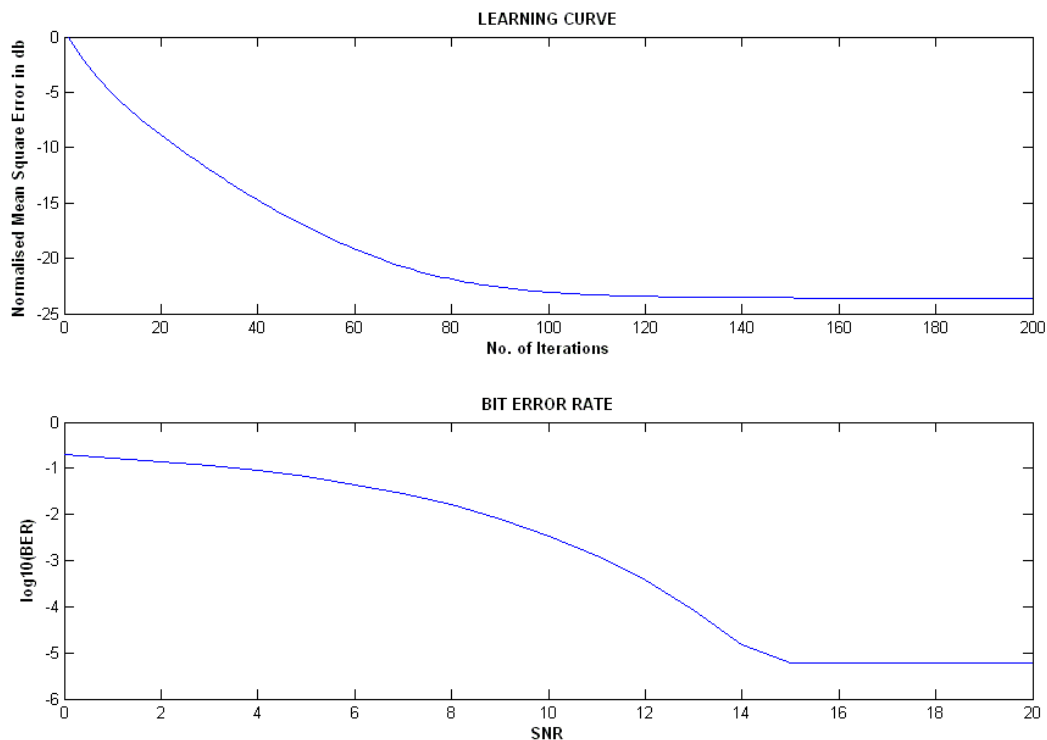
# Chapter 5

SIMULATION RESULTS  
AND DISCUSSION

### SIMULATION 1: Channel Equalization using LMS algorithm

Here we have taken the channel as a FIR filter whose impulse response is given by  $h(n) = [0.23, 0.96, 0.23]$ . During the training period, the input signal applied is a random signal which takes on values +1 or -1. White Gaussian noise of SNR 25 db is added to it. The values of the delay and step parameter are optimized. The equalizer tries to reduce the normalized mean square error of the output signal compared with the delayed sample of the original signal.

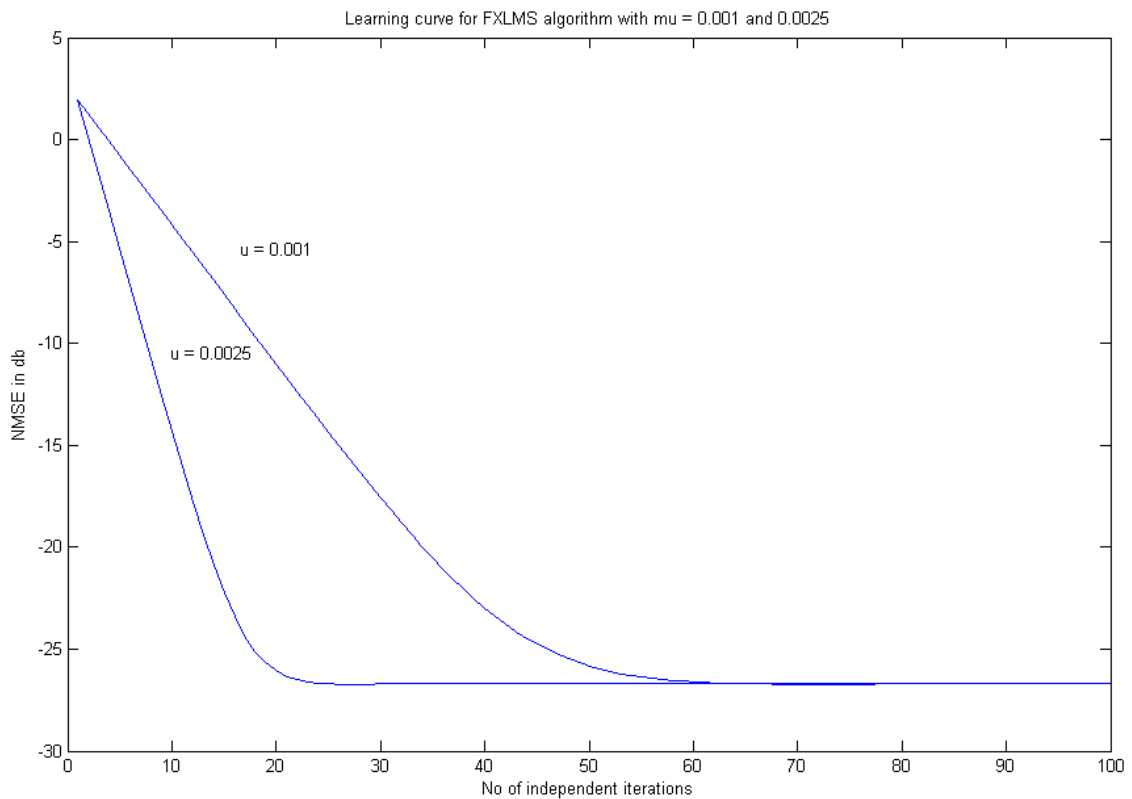
After the normalized mean square error has been reduced, the equalizer freezes the values of the weights and the detection period begins. In the detection process, we have taken a range of SNR values from 0 db to 20 db. The input signal is a random signal of  $10^6$  bits which take on values -1 or +1; on which is added White Gaussian noise of desired SNR.



**Figure 5.1** Simulation results showing the Learning curve and bit error rate for the LMS equalizer

## SIMULATION 2: Convergence Graph for Filtered-X LMS Algorithm

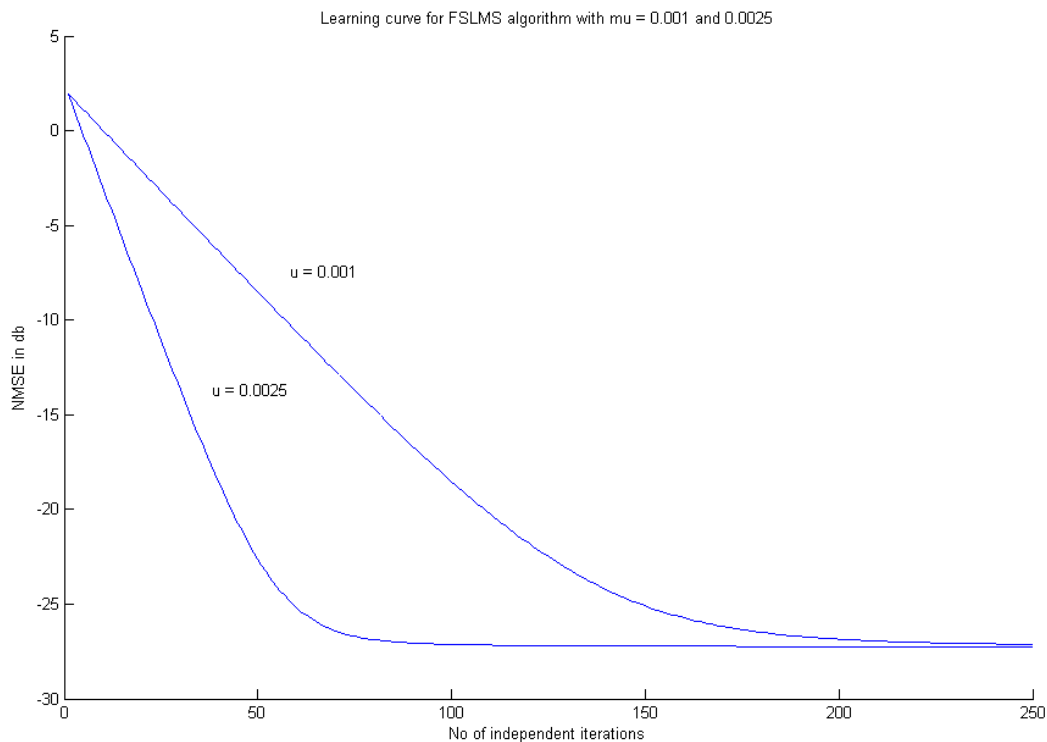
Here we have considered the primary path to be  $z^{-5} + 0.3z^{-6} + 0.2z^{-7}$ . The secondary path is an FIR filter with non minimum phase transfer function  $H(z) = z^{-2} + 1.5z^{-3} + z^{-4}$ . The reference noise is logistic chaotic with the recursive relation  $x(n + 1) = \lambda x(n)[1 - x(n)]$ , normalized to have unit signal power ( $\sigma^2_x = 1$ ), where  $x(0) = 0.9$  and  $\lambda = 4$ . The length of the adaptive filter  $N = 10$ . Fig. 5.2 shows the average learning curves for the FXLMS algorithm for  $\mu = 0.001$  and  $\mu = 0.0025$  respectively.



**Figure 5.2** Learning Curve for the Filtered-X LMS algorithm with  $\mu = 0.001$  and  $0.0025$

### SIMULATION 3: Convergence Graph for Filtered-S LMS Algorithm

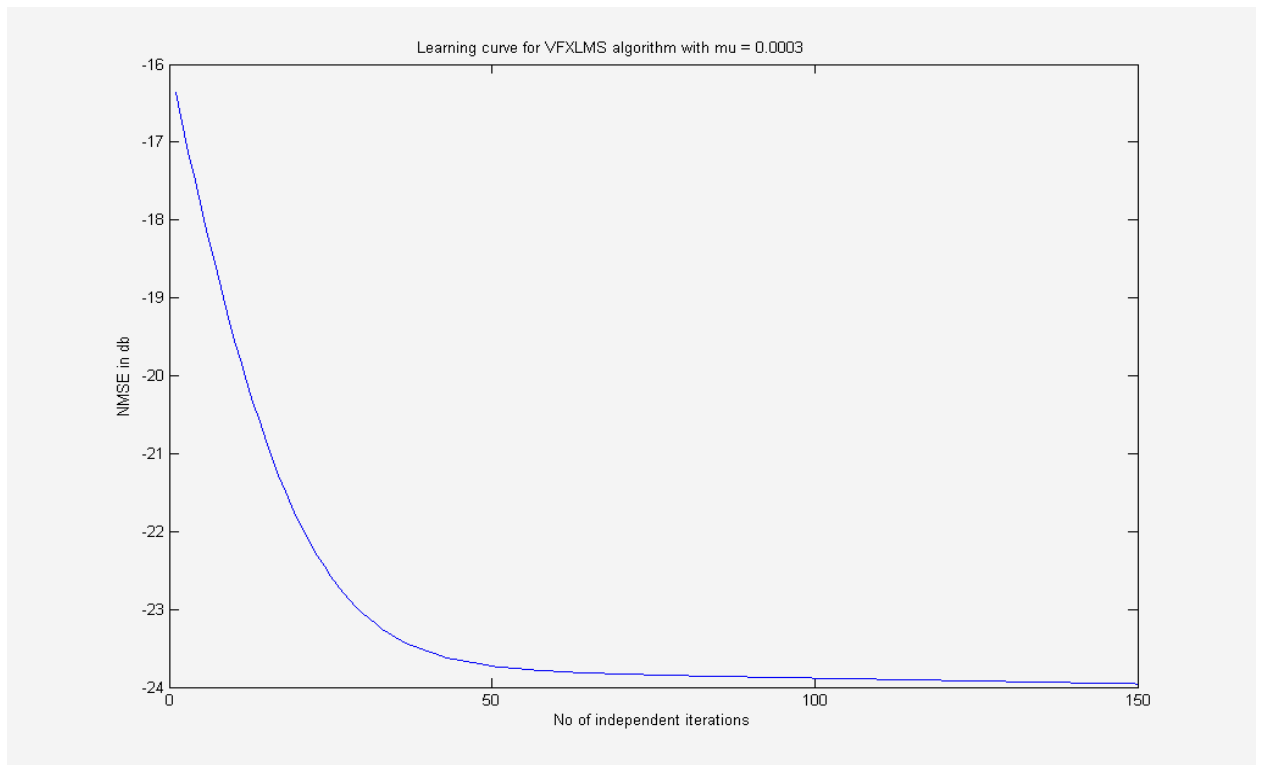
Here we provide the simulation results to prove the convergence characteristics of the FSLMS algorithm. For performance analysis, we use the normalized mean-square error (NMSE), defined as  $NMSE = 10\log_{10}\{E(e^2(n)/\sigma_d^2)\}$  where,  $\sigma_d^2$  is the power of the primary noise at the canceling point. The secondary path is considered to be an FIR filter with non minimum phase transfer function  $H(z) = z^{-2} + 1.5z^{-3} + z^{-4}$ . The reference noise is logistic chaotic with the recursive relation  $x(n+1) = \lambda x(n)[1 - x(n)]$ , normalized to have unit signal power ( $\sigma_x^2 = 1$ ), where  $x(0) = 0.9$  and  $\lambda = 4$ . The primary path is considered to be  $z^{-5} + 0.3z^{-6} + 0.2z^{-7}$ . Fig. 5.2 shows the average learning curves for the FSLMS algorithm for  $\mu = 0.001$  and  $\mu = 0.0025$  respectively. We consider the first-order functional expansion;  $P = 1$  and memory size  $N = 10$ .



**Figure 5.3** Learning curve for Filtered-S LMS algorithm with  $\mu = 0.001$  &  $0.0025$

### SIMULATION 4: Convergence Graph for Volterra Filtered-X LMS Algorithm

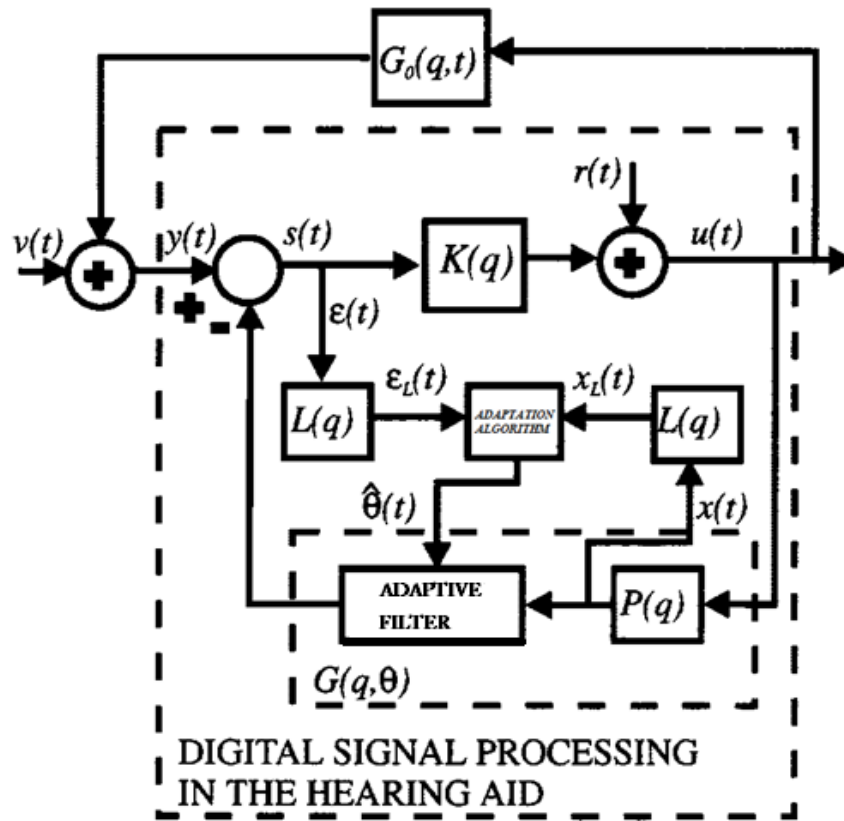
In this experiment, we assume that the primary noise at the canceling point is generated based on the third-order polynomial model,  $d(n) = t(n - 2) + 0.08t^2(n - 2) + 0.04t^3(n - 2)$ , where,  $t(n) = x(n) * f(n)$ ,  $f(n)$  denotes the impulse response of the transfer function  $F(z) = z^{-3} + 0.3z^{-4} + 0.2z^{-5}$ . The reference signal  $x(n)$  is the sum of a sinusoidal wave of 500 Hz, with a sampling rate of 8000 samples/s and a Gaussian noise of 40-dB signal-to-noise ratio (SNR). We consider the second-order Volterra filter and memory size  $N = 10$ . The secondary path is considered to be an FIR filter with non minimum phase transfer function  $H(z) = z^{-2} + 1.5z^{-3} + z^{-4}$ . Step size parameter  $\mu$  is taken as 0.0003.



**Figure 5.4** Learning Curve for the second order Volterra Filtered-X LMS algorithm



### SIMULATION 5: Error Minimization of Feedback Cancellation in Hearing Aid with Filtered-X LMS



**Fig 5.5** Hearing Aid with an internal feedback path to cancel the external acoustic feedback

In this experiment, we show the cancellation of the external feedback by an adaptive filter in series with a fixed filter. The adaptive filter used is a 32 tap FIR filter. The adaptation algorithm is the Least Mean Square Algorithm. The hearing aid model shown in Fig is characterized by the following parameters

$$L(q) = 1 - 2.01q^{-1} + q^{-2}$$

$$P(q) = q^{-61} - 1.8q^{-62} + 0.81q^{-63}$$

$$r(t) = 0$$

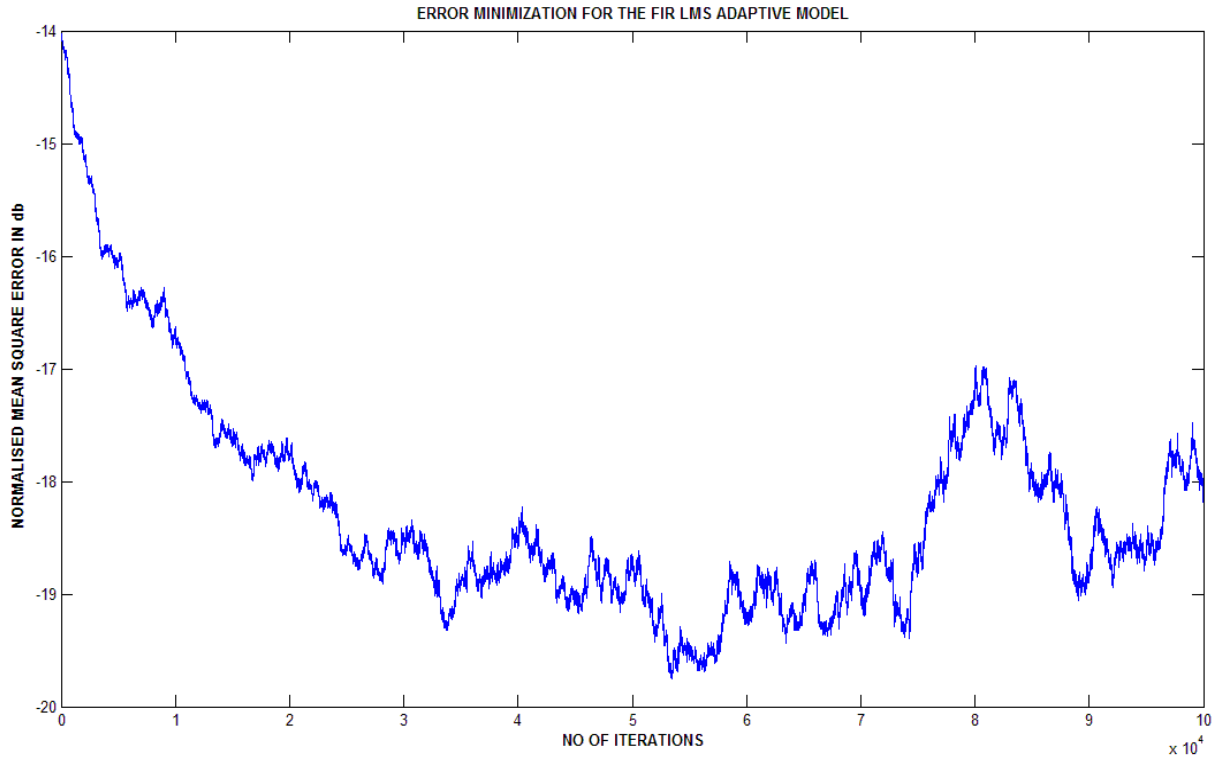
$$\gamma(t) = 2 \cdot 10^{-5}$$

$$d = 32$$

The input signal to the system  $v(t)$  was generated by filtering a white noise with a variance one through a filter  $H_0(q)$  given by

$$H_0(q) = (1 + 2q^{-1} + q^{-2}) / (1 - 1.45q^{-1} + 0.57q^{-2})$$

To determine the static characteristics of the feedback cancellation scheme, the cost function used is the normalized mean square error. Fig 5.6 shows the reduction in normalized mean square error with increasing number of iterations.



**Fig 5.6** Minimization of Normalized Mean Square error for the FXLMS based feedback cancellation scheme

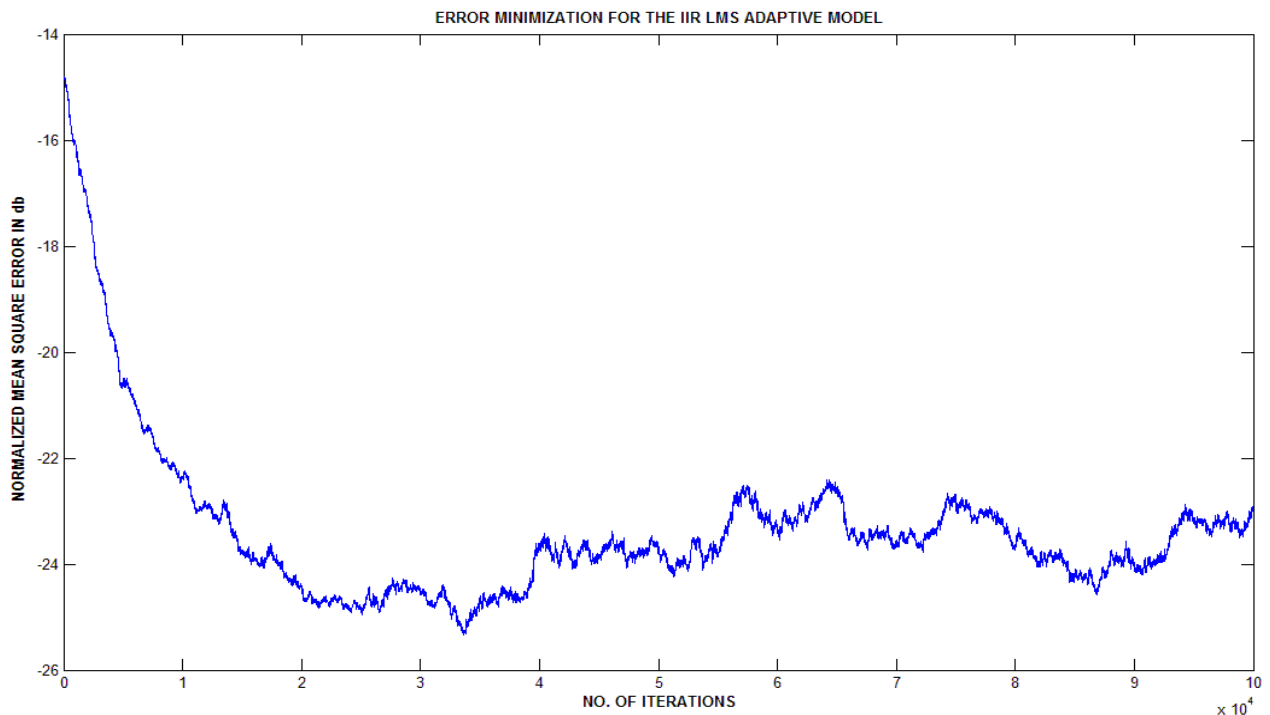
### SIMULATION 6: Error Minimization of Feedback Cancellation in Hearing Aid with IIR LMS

In this experiment, we use the hearing aid model shown in Fig 5.5. The adaptive filter used in this case is an IIR filter and the adaptation algorithm applied is the Least Mean Square algorithm for IIR filters. The IIR filter consists of 20 taps in the feed forward path and 12 taps in the feedback path, thus a total of 32 taps. The step size for each iteration is taken as

$\mu_1$  = step size in the feed forward path =  $5 \cdot 10^{-6}$

$\mu_2$  = step size in the feedback path =  $1.5 \cdot 10^{-6}$

To determine the static characteristics of the feedback cancellation scheme, the cost function used is the normalized mean square error. Fig 5.6 shows the reduction in normalized mean square error with increasing number of iterations.

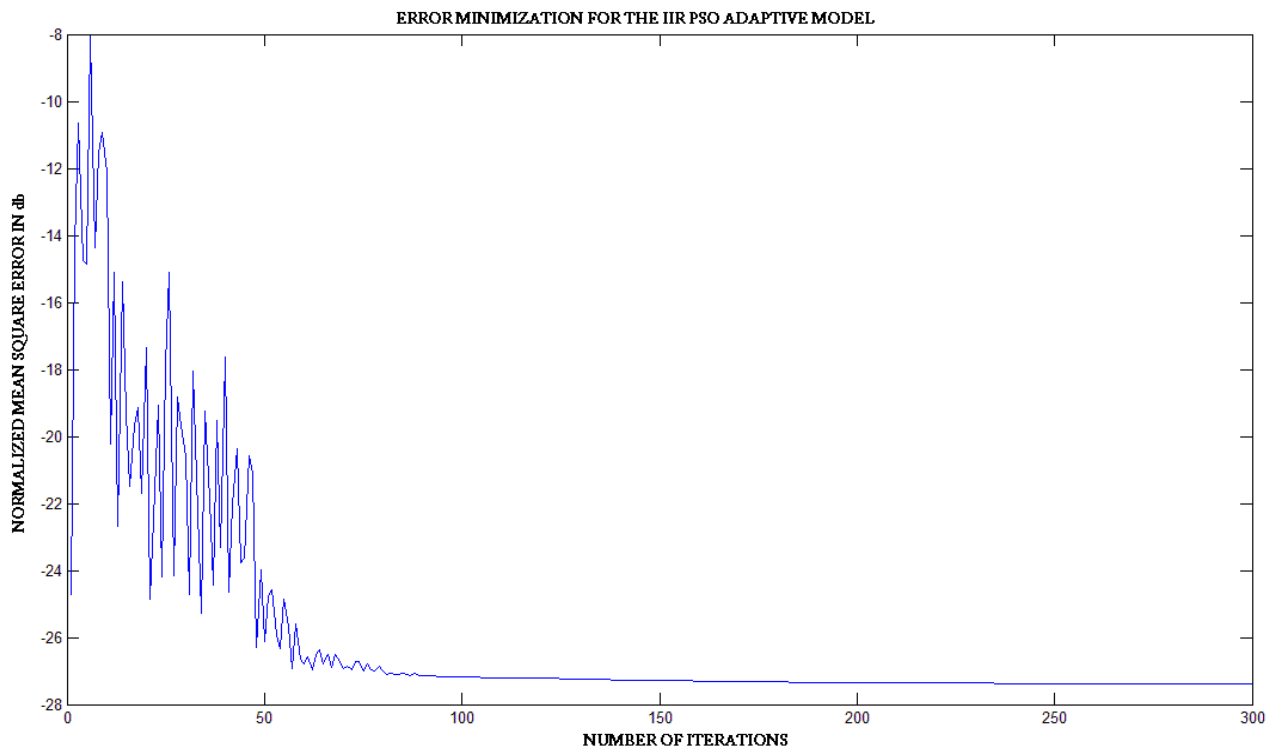


**Fig 5.7** Minimization of Normalized Mean Square error for the adaptive IIR LMS based feedback cancellation scheme

### **SIMULATION 7: Error Minimization of Feedback Cancellation in Hearing Aid with IIR PSO**

In this experiment, we use the hearing aid model shown in Fig 5.5. The adaptive filter used in this case is an IIR filter and the adaptation algorithm applied is the Particle Swarm Optimization Algorithm. The IIR filter consists of 20 taps in the feed forward path and 12 taps in the feedback path, thus a total of 32 taps. The population size is take as 50. Here we have applied a linearly decreasing inertia weight vector with its maximum and minimum values as 0.9 and 0.4 respectively.

To determine the static characteristics of the feedback cancellation scheme , the cost function used is the normalized mean square error. Fig 5.7 shows the reduction in normalized mean square error with increasing number of iterations.



**Fig 5.7** Minimization of Normalized Mean Square error for the adaptive IIR PSO based feedback cancellation scheme

## **DISCUSSION**

The above simulations and their output show that the IIR based feedback cancellation scheme in hearing aid gives better performance in terms of error minimization as compared to the FIR based scheme. The minimum normalized mean square error achieved in the FIR Filtered-X LMS based scheme is around -20 db whereas that for the IIR LMS based scheme is around -25db and for the IIR PSO based scheme it is around -27db. These figures clearly suggest that the IIR filters provide a better matching of the characteristics of the external feedback path and thus ensures a better feedback cancellation.

# Chapter 6

CONCLUSION

The various techniques that can be applied for acoustic noise cancellation have been introduced. Moreover, three models for the cancellation of external acoustic feedback in hearing aid have been discussed. These are - the adaptive FIR Filtered-X LMS, the adaptive IIR LMS and the adaptive IIR PSO models for external feedback cancellation. The results show that adaptive IIR based schemes provides a better feedback cancellation as compared to the adaptive FIR based schemes. However certain issues have to be taken into consideration while selecting the appropriate model for the purpose.

Although the IIR PSO and IIR LMS algorithms give better error minimization as compared to FIR LMS, the algorithms involve increased computational complexity. Also parameter selection for IIR LMS and PSO algorithms are less well defined than for FIR LMS.

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