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DEVELOPMENT OF FUZZY RECEIVER FOR GSM APPLICATION

For degree of masters technology.
National Institute Of Technology, Rourkela,
India – 769008,
- May- 2005

A Thesis Report Submitted By

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Declaration of Originality

This Thesis was composed entirely by myself. The work reported herein was conducted exclusively by myself in the Department of Electronics and Instrumentation Engineering at National Institute of Technology, Rourkela.

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May 4, 2005

Abstract

A channel equalizer is one of the most important subsystems in any cellular communication receiver. It is also the subsystem that consumes maximum computation time in the receiver. Traditionally maximum-likelihood sequence estimation (MLSE) was the most popular form of equalizer. Owing to non-stationary characteristics of the communication channel MLSE receivers perform poorly. Under these circumstances maximum-aposteriori probability (MAP) receivers also called Bayesian receivers perform better. This thesis proposes a fuzzy receiver that implements MAP equalizer and provides a performance close to the optimal Bayesian receiver. Here Bit Error Rate (BER) has been used as the performance index.

This thesis presents an investigation on design of fuzzy based receivers for GSM application. Extensive simulation studies which shows that the performance of the proposed receiver is close to optimal receiver for variety of channel conditions in different receiver speeds where channel suffers from Rayleigh fading. The proposed receiver also provides near optimal performance when channel suffers from nonlinearities.

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N.I.T, Rourkela
(Deemed University)

Certificate

This is to certify that the work in this thesis entitled "**Development of Fuzzy Receiver for GSM application"** by Mr. Nihar Ranjan Panda (M.Tech. 4^{th} Sem, Roll No – 20307008) has been carried out under my supervision in partial fulfillment of the requirements for the degree of Masters of Technology in Electronics & Instrumentation Engineering during session 2003-2005 in the Department of in Electronics & Instrumentation Engineering, National Institute of technology, Rourkela, and this work has not been submitted elsewhere for a degree.

Dr. S. K. Patra
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Acknowledgement

On the submission of my Thesis report of **"Development of Fuzzy Receiver for GSM Application**", I would like to extend my gratitude & my sincere thanks to my supervisor Dr. Sarat Kumar Patra, Asst. Professor of Department of Electronics and Instrumentation Engineering for his constant motivation and support during the course of my work in the last six months. I would like to sincerely thank him for his patience and guidance in providing me a clear understanding about the intricate details of the topic and strengthening my foundation through out the work. I would like to offer my sincere thanks to

- Dr G. panda, Professor and Coarse Coordinator of M.Tech, Electronics & Instrumentation Engineering for his support and guidance.

- Dr. G. S. Rath for numerous valuable discussions on different topics and sharing his knowledge on pattern Recognition.

- Mrs. Sabita Tripathy, Sibsankar Nag, and Subrat kumar Biswal for keeping the lab open during the starting of my thesis work.

- All the faculty members of Department of Electronics and Instrumentation Engineering, National Institute of Technology, Rourkela for their generous help in various ways for overall completion of my thesis.

- My friends who made my two years stay happily in Rourkela an unforgettable rewarding experience.

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Chapter 1

Introduction

1.1 Introduction

Mobile cellular wireless systems operate under harsh and challenging channel conditions. The wireless channel is distinct and much more unpredictable than the wire line channel because of factors such as multipath and shadow fading, Doppler spread and time dispersion or delay spread. These factors are all related to variability that is introduced by the mobility of the user and the wide range of environments that may be encountered as a result.

Multipath [1] is a phenomenon that occurs as a transmitted signal is reflected by objects in the environment between the base station and a user. These objects can be buildings, trees, hills or even moving or stationary vehicles like buses, cars etc. The reflected signals arrive at the receiver with random phase offsets, because each reflection generally follows a different path to reach the user's receiver. The result is random signal fades as the reflections destructively (and constructively) superimpose on one another, which effectively cancels part of the signal energy for brief periods of time. The degree of cancellation or fading depends on the delay spread of the reflected signals as embodied by their relative phases and power.

Time dispersion represents distortion to the signal and is manifested by the spreading in time of the modulation symbols. This occurs when the channel is band-limited. This is considered when the coherence bandwidth of the channel is smaller than the modulation bandwidth. Time dispersion leads to intersymbol interference (ISI.) where the energy from one symbol spills over into another symbol which limits the data transmission rate and as a result, the BER is increased. ISI is also affected by fading.

Hence in mobile communication systems, the transmitted signal is subject to various corruptions, as it introduces ISI due to multipath radio propagation, co-channel interference (CCI) due to frequency reuse, Adjacent channel interference (ACI), fading due to mobile motion relative to base station, in presence of Additive Gaussian White Noise (AWGN).

The purpose of equalization is to compensate for these channel influences so that the original information can be found from the corrupted received signal. Adaptive equalization methods are needed in mobile communication field since the channel response is usually not known beforehand and it is often time varying. Additionally, equalization needs to be performed efficiently both in terms of computational complexity and probability of error. This thesis analyses different nonlinear equalizers like radial basis function (RBF) network based equalizer, fuzzy adaptive filters for GSM application.

1.2 Motivation Of Work

The field of digital data communications has experienced an explosive growth in recent years and its demand reaches at the peak as additional services are being added to existing infrastructure. The revolution in digital communication techniques can be attributed to the invention of the automatic linear adaptive equalizer in the late 1960's [2]. Adaptive equalizers have gone through many stages of development and refinement in the last 30 years. Early equalizers used linear adaptive filter algorithms with or without a decision feedback. Alternatively maximum likelihood sequence estimator (MLSE) [3] was implemented using Viterbi algorithm [4, 5].

Both forms of equalizers provided two extremities in-terms of performance achieved and the computational cost involved. The linear adaptive equalizers are simple in structure and easy to train but they suffer from poor performance in severe condition like varying channels as mobile radio channel. On the other hand the infinite memory MLSE provide good performance but at the cost of large computational complexity. Under lower memory constraints MLSE performance also degraded considerably.

As the state of the mobile radio channel always changes and multipath causes time dispersion of the digital information data causing inter-symbol-interference, makes too difficult to detect the actual information at the receiver. It requires adaptive equalizer to adjust its parameters during training to coup of with such fading environment but it needs large training data or sequences for the linear equalizers and also shows poor performance in case of this mobile radio channel.

The large computational complexity associated with the Viterbi algorithm and poor performance of linear equalizers led to the development of symbol-by-symbol equalizers using the maximum a-posteriori probability (MAP) principle these were also called Bayesian equalizers [6]. These Bayesian equalizers have been approximated using nonlinear signal processing techniques like Artificial Neural Networks (ANN) [7], radial basis function (RBF) [8], recurrent network [9], Kalman filters [10] etc. The study of new techniques provides adaptive equalizers which have the advantage of both good performance and comparatively low computational cost. The study of these nonlinear equalizers helps to achieve good performance for mobile channel. Hence different kinds of nonlinear equalizers have been discussed in this thesis.

Fuzzy systems have been used for equalization [11]. Subsequently they were used for equalization of non-linear channels. Fuzzy system based equalizers were used for mobile channels in [12]. This formed the motivation for development of new fuzzy equalizers suitable for GSM applications.

1.3 Background Literature Survey

The research in channel equalization started around 1960's. The earlier equalizers basic theory were of zero forcing equalizers. In 1960, LMS algorithm by Widrow and Hoff [13] shown the way to go for development of adaptive filters used for equalization purposes. In 1965, Lucky [2] used this LMS algorithm to design adaptive channel equalizers. As these equalizers were very simple to design got popularized but very soon their limitations were also revealed in the field of channel equalization. It was seen that these linear equalizers, in spite of best training, could not provide acceptable performance in case of highly dispersive channel and time varying channels. This is due to the fact that linear equalizers treat equalization as an inverse filtering problem whereas equalization can be treated as a pattern classification problem. This led to the investigation of other equalization techniques beginning with MLSE equalizer [3] and its Viterbi implementation [4] in 1970's. Another form of nonlinear equalizer was infinite impulse response (IIR) form of linear adaptive equalizer, where equalizer employs feedback termed as decision feedback equalizer (DFE) [14]. In between 1970's and 1980's , the research works carried out in this field were for the development of faster convergence and computationally efficient algorithms like recursive least square (RLS) algorithm, Kalman filters [10] etc. A review of the development of equalizers till 1985 is available in [15].

In the late 1980's, the beginning of development of field of adaptive neural network (ANN) [7] was seen. The large computational complexity associated with the Viterbi algorithm and poor performance of conventional equalizers with adaptive filters has led to the development of symbol-by-symbol equalizers [6]. These Bayesian equalizers have been approximated using nonlinear signal processing techniques like Artificial Neural Networks (ANN) [7], the multi layer perceptron (MLP) [16] which were computationally more efficient. Another form of nonlinear processors called radial basis function (RBF) [8] were first used for multidimensional functional interpolation. Subsequently these were used for equalization applications [17, 18]. In subsequent years, development of new training algorithms and equalizer structures using ANN [19, 20] and RBF [21] were also developed.

When mechanism of fading channels was first modeled in 1950's and 1960's, the ideas were primarily applied over the horizon communications covering a wide range of frequency bands. Although the fading effects in a mobile radio system are somewhat different than those in ionospheric and tropospheric channels, the early models are still quite useful to help characterize fading effects in mobile digital communication systems. These analysis are still valid for Ultra-high frequency (UHF) systems such as present day Personal communication systems (PCS).Under these conditions the Rayleigh fading manifestation and types of degradation and its mitigations are discussed [22, 23]. When multiple reflective paths are large in number in mobile radio communication and there is no line-of-sight component, the envelope of the received signal is statistically described by a Rayleigh probability density function (PDF). Few of the channel models that describe present day mobile cellular communication system are Clark and Gan's fading model [1], Okumara model [24], and Lee's model [25].

Global system for mobile communication (GSM) is a globally accepted standard for digital cellular communication and was established in 1982 to create a common European mobile telephone standard that would formulate specifications for a pan-European mobile cellular radio system operating at 900 MHz. This band was later extended to accommodate 1.8GHz carrier. India has adopted GSM as its mobile cellular communication standard. In this real world communication system the neural network equalizers [26] are used where two different neural network architectures have been used to realize a non-linear adaptive receiver for GSM signals. One is using the well-established back propagation technique, a recurrent network is build which has been trained considering different channels corrupted by ISI, fading and Doppler. This equalizer has shown better performances than the a classic coherent receiver and the other receivers based on a partially supervised Self Organizing Map in order to perform an effective real time learning .Also RBF equalizers used in GSM application [27] which proved the optimal one in terms of BER performance with acceptable computational complexity over linear equalizers, multi layer perceptron (MLP) network equalizer and viterbi equalizers.

Following the success of fuzzy logic system in different signal processing applications like system identification, equalization the Fuzzy Adaptive Filters (FAF) were designed using Fuzzy logic and they were trained with LMS and RLS algorithms [11]. Fuzzy filters are nonlinear filters that can incorporate linguistic information in the form of IF … THEN … fuzzy rules. These equalizers performed well but could not provide the Bayesian equalizer decision function. Additionally, the equalizer based on a fuzzy adaptive filter demanded high computational complexity. A new form of implementation of Bayesian equalizers using fuzzy filters proposed by Patra and Mulgrew [28] and used it to eliminate co-channel interference which reduced the computations but achieved a comparable performance with RBF equalizers. These equalizers provided an alternative implementation of RBF equalizers. Fuzzy adaptive filter is constructed from a changeable fuzzy IF … THEN rules which come either from human experts or by matching input-output pairs through an adaptation procedure. The adaptive algorithm adjusts the parameters of the membership functions which characterize the fuzzy concepts in the IF …THEN rules by minimizing some cost function. In Type-1 TSK (Takagi – Sugeno – Kang) fuzzy adaptive filter (FAF) the numerical data and linguistic information in a natural form, i.e. as fuzzy IF- THEN rules and input-output (I/O) data pairs, was proposed and applied to nonlinear channel equalization in [11]. The information to be processed by a FAF is oftenly uncertain due to uncertain linguistic knowledge and uncertain numerical values. In mobile communication, the mappings between input and output data pairs are uncertain due to channel dynamics. This numerical data uncertainty causes this FAF and other nonlinear filters to perform poorly. Linguistic and numerical uncertainties were more effectively handled by other types of fuzzy filters [29], where antecedent and consequent membership functions are fuzzy set which was introduced by Zadeh [30] as an extension of the concept of an ordinary fuzzy set, i.e. type -1 fuzzy set. A type -2 membership grade can be any set in [0, 1] — *primary membership* and corresponding to each primary membership, there is a *secondary membership,* which can also be [0, 1] that describes the possibilities for the primary membership which allows to handle linguistic uncertainties.

1.4 Thesis Contribution

This section outlines some of major contributions of the study presented in this thesis. In this thesis different linear equalizers like LMS and RLS as well as nonlinear equalizers like RBF, two types of fuzzy adaptive filter have been designed for GSM application.

The channel considered were here Rayleigh fading. The fuzzy equalizers described here are generally classified as nonlinear equalizers suitable for GSM environment. The digital communication problem is discussed first and then need for equalizers is established. A brief introduction of Global system for mobile channels and its frame structure is described which forms the basis of our design. Different forms of equalizers are reviewed and with this knowledge of equalizer techniques the two types of fuzzy logic based equalizers are proposed for GSM application and its advantage over the other are described. In the process of evaluation Bit Error rate (BER) has been used as performance measure.

This thesis presents a fuzzy implementation of maximum a-posteriori probability (MAP) equalizers based on Bayes's theory. All equalizers developed here are for Rayleigh fading channels corrupted with AWGN. One of these fuzzy receivers can be considered as fuzzy implementation of RBF receiver. The equalizer proposed here is in line with the fuzzy equalizer proposed in [28].

It is seen that the advantage provided by the fuzzy equalizers in terms of computational complexity and performance gain can provide efficient equalizer design for GSM application.

1.5 Thesis outline

Following this introduction the remaining part of the thesis is organized as under:
Chapter 2 provides the fundamental concepts of channel equalization and discusses different linear and nonlinear equalization techniques briefly. The channel characteristics that bring out the need for an equalizer in a communication system is also presented. Then the Rayleigh fading simulator is described and the simulation result is presented which shows how channel states vary with such fading. The chapter is also provides an introduction to GSM communication system.

Chapter 3 provides the description and the mathematical evaluation of fuzzy logic system from the RBF network equalizer with scalar channel states. This equalizer is termed as

Type-1 fuzzy equalizer. The performance of this equalizer has been compared with RBF equalizer, linear equalizers based on LMS and RLS training and fuzzy equalizer in [11] which is termed as Type-2 fuzzy equalizer for remaining part of the thesis.

Chapter 4 is devoted for the equalization of non-linear channels by using these non-linear equalizers and their performance were evaluated under GSM environment by BER calculation using Monte-Carlo simulations as performance measure.

Chapter 5 summarizes the work undertaken in this thesis and points to possible directions for future works.

Chapter 2

Background

2.1 Introduction

This thesis discusses the development of fuzzy system based channel equalizers for GSM environment for a variety of channel impairments. In order to establish the context and need for the work undertaken, it is necessary to discuss the fundamental concepts behind the work. This chapter brings out the need for an adaptive equalizer in a digital communication system (DCS) and describes the classification of adaptive equalizers. The chapter also discusses the characteristics of a mobile communication channel and introduces a Rayleigh fading simulator.

This chapter is organized as follows. Following this introduction, section 2.2 discusses the communication system in general, section 2.3 discusses the propagation channel model in a DCS, where general finite impulse response (FIR) filter model for intersymbol interference (ISI) channels is described and Section 2.4 describes a Rayleigh fading simulator with certain mobile carrier frequency, mobile speed and data rate. Section 2.5 presents the classification of equalizers with emphasis on symbol-by-symbol equalizers. Section 2.6 describes the GSM architecture and its frame structure. Section 2.7 describes fuzzy logic system briefly and finally section 2.8 provides the concluding remarks.

2.2 Digital Communication System

The Block diagram of a baseband model of a DCS is presented in Figure 2.1. As the analysis of the DCS with all the necessary blocks is very difficult due to complexity associated with all subsystems, communication system are studied in baseband frequency where the encoder, decoder, modulator and the demodulators have been removed.

The data source constitutes the signal generation system that generates the information to be transmitted. The work of the encoder in the transmitter is to encode the information bits before transmission so as to provide redundancy in the system. T his helps for the correction of errors at the receiver end. Some of typical coding schemes are convolutional codes, block codes and grey codes [31]. The digital data transmission requires very large bandwidth. The Efficient use of the available bandwidth is achieved through the transmitter filter, also called the modulating filter. The modulator on the other hand places the signal over a high frequency carrier for efficient transmission. Some of the typical modulation schemes used in digital communication are amplitude shift keying (ASK), frequency shift keying (FSK), pulse amplitude modulation (PAM) and phase shift keying (PSK) modulation. The channel is the medium through which information propagates from the transmitter to the receiver. At the receiver the signal is first demodulated to recover the baseband transmitted signal. The demodulated signal is processed by the receiver filter, also called receiver demodulating filter, which should be ideally matched to the transmitter filter and the channel. Hence physical channel can replace all filters in block diagram. The equalizer in the receiver removes the distortion introduced due to the channel impairments. The decision device provides the estimate of the encoded transmitted signal. The decoder reverses the work of the encoder and removes the encoding effect revealing the transmitted information symbols. This simplified communication system model, while maintaining the basic principle involved, is easy to analyze.

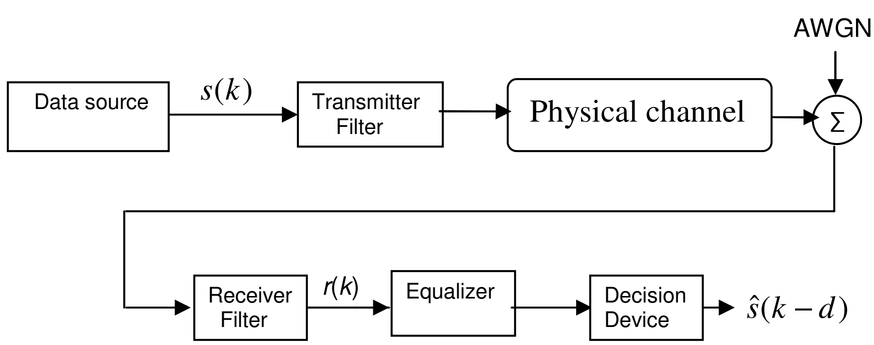


Figure 2.1: Baseband model of digital communication system

2.3 Propagation channel

This section discusses the channel impairments that limit the performance of a DCS. The DCS considered here is shown in Figure 2.1.The transmission of digital pulses over an analogue channel would require infinite bandwidth. An ideal physical propagation channel should behave like an ideal low pass filter represented by its frequency response,

$$H_c(f) = |H_c(f)| \exp(j\theta f) \qquad (2.1)$$

Where $H_c(f)$ represents the Fourier transform (FT) of the channel and θ is the phase response of the channel. The amplitude response of the channel $|H_c(f)|$ can be defined as,

$$H_c(f) = \begin{cases} k_1 & |f| \le w_c \\ 0 & |f| > w_c \end{cases} \qquad (2.2)$$

Where k_1 is a constant and w_c is the upper cutoff frequency. The channel group delay characteristic is given by

$$\tau(f) = -\frac{1}{2\pi}\frac{d\theta(f)}{df} = k_2 \qquad (2.3)$$

Where k_2 is an arbitrary constant. The conditions described in (2.2) and (2.3) constitute fixed amplitude and linear phase characteristics of a channel. This channel can provide distortion free transmission of analogue signal band limited to at least w_c. Transmission of the infinite bandwidth digital signal over a band limited channel of w_c will obviously cause distortion. This demands for the infinite bandwidth digital signal be band limited to at least w_c, to guarantee distortion free transmission. This work is done with the aid of transmitter and receiver filters shown in Figure (2.1). The combined frequency response of the physical channel, transmitter filter and the receiver filter can be represented as,

$$H(f) = H_T(f) H_C(f) H_R(f) \tag{2.4}$$

Where, $H_T(f)$, $H_C(f)$, $H_R(f)$ represent the FT of transmitter filter, propagation channel and the receiver filter respectively. When the receiver filter is matched to the combined response of the propagation channel and the transmitter filter, the system provides optimum signal to noise ratio (SNR) at the sampling instant. As channel impulse response is not known beforehand, hence the receiver filter impulse response $h_R(t)$ is generally matched to the transmitter filter impulse response $h_T(t)$. For this condition to be satisfied the frequency response of both the transmitter and receiver filters must be complex conjugate to each other. For the ideal channel case though it is very difficult but is possible by using a raised cosine filter.

2.3.1 FIR filter model

The cascade of the transmitter filter $h_T(t)$, the channel $h_C(t)$, the receiver matched filter $h_R(t)$ and the *T*- spaced sampler in the communication system shown in figure 2.1 can be modelled by a digital FIR filter. The noise at the equalizer input is correlated due to the presence of the matched filter. As it is easier to deal with white noise sequence in the equalizer, hence equalizer is generally preceded by a noise whitening filter. The combined channel due to the transmitter filter, propagation channel, the receiver filter, noise whitening filter and the *T*- spaced sampler can be modelled by the digital FIR filter shown in Figure 2.2.

Here the channel observed output $r(k)$ is the sum of the noise free channel output $\hat{r}(k)$ and the AWGN $\eta(k)$. The noise free output is the convolution sum of the transmitted sequence $s(k)$ with the channel taps a_i, $0 \leq i \leq n_c - 1$. The channel impulse response in z-domain can be represented by,

$$H(z) = \sum_{i=0}^{n_c-1} a_i z^{-i} \tag{2.5}$$

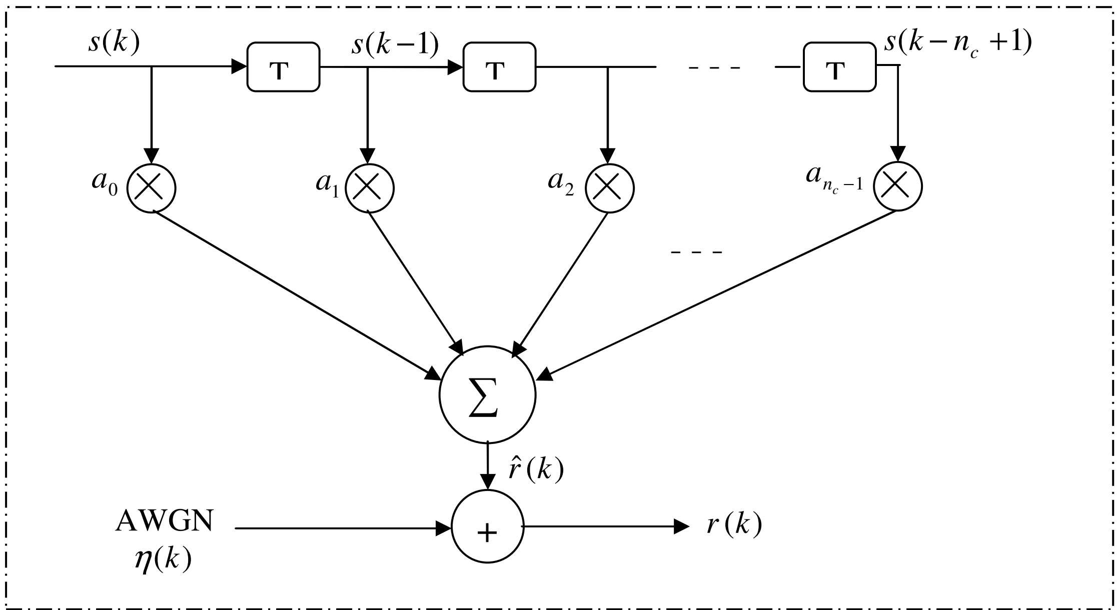


Figure 2.2: Finite impulse response filter channel model

2.4 Clarke's Model for Flat Fading (Rayleigh fading simulator)

Mobile radio channel varies with time due to fading, caused due to the interference between two or more versions of the transmitted signal which arrive at the receiver at slightly different times.

Several statistical models have been suggested to explain the observed statistical nature of a mobile channel. The first model proposed by Ossana [32] was based on interference of wave's incident and reflected from the flat sides of randomly located buildings. Though this model predicts flat fading power spectra with the agreement of the measurements done in suburban areas but it assumes the existence of a direct path between the transmitter and receiver, and is limited to a restricted range of reflection angles. Hence it is inflexible and inappropriate for urban areas where the direct path is almost blocked by buildings and other obstacles. Clarke developed a model based on scattering and widely used.

In mobile radio channel, the Rayleigh distribution is commonly used to describe the statistical time varying nature of the received envelope of a flat fading signal, or the envelope of an individual multipath component. If the mobile radio channel has a constant gain and linear phase response over a bandwidth which is greater than the bandwidth of the transmitted signal, then the received signal will undergo flat fading. In flat fading, the multipath structure of the channel is such that the spectral characteristics of the transmitted signal are preserved at the receiver. However the strength of the received signal changes with time, due to fluctuation in the gain of the channel caused by multipath.

It is well known that the envelope of the sum of the two quadrature Gaussian noise signals obeys rayleigh distribution. The Rayleigh distribution has a probability density function (PDF) given by,

$$p(r) = \begin{cases} \dfrac{r}{\sigma^2}\exp\left(-\dfrac{r^2}{2\sigma^2}\right) & (0 \le r \le \infty) \\ 0 & (r \le 0) \end{cases} \tag{2.6}$$

Where σ is the root mean square (rms) value of the received voltage signal before envelope detection, and σ^2 is the time-average power of the received signal before envelope detection.

2.4.1 Simulation of Clarke and Gan's fading Model:-

Clarke developed a model [33] where the statistical characteristics of the electromagnetic fields of the received signal at the mobile are reduced from scattering. The model assumes a fixed transmitter with vertically polarized antenna. The fields incident on the mobile antenna is assumed to be comprised of N azimuthal plane waves with arbitrary carrier phases, arbitrary azimuthal angles of arrival, and each wave having equal average magnitude as due to the absence of direct line-of-sight path, the scattered components arriving at receiver will experience similar attenuation over small-scale distances. It is often useful to simulate multipath fading channels in hardware. Every wave that is incident on the mobile undergoes a Doppler shift due to the motion of the receiver and arrives at the receiver at the same time. For the *n*th wave arriving at an angle α_n to the axis the mobile is moving, the Doppler shift in Hertz is given by

$$f_n = \frac{\nu}{\lambda} \cos \alpha_n \tag{2.7}$$

Where ν is the mobile velocity and λ is the wavelength of the incident wave.

A popular simulation method uses the concept of in-phase and quadrature modulation paths to produce a simulated signal representing equation below with spectral and temporal characteristics very closed to measured data

$$E_c(t) = T_c(t) \cos(2\pi f_c t) - T_s(t) \sin(2\pi f_c t) \tag{2.8}$$

Where $$T_c(t) = E_0 \sum_{n=1}^{N} C_n \cos(2\pi f_n t + \phi_n) \tag{2.9}$$

$$T_s(t) = E_0 \sum_{n=1}^{N} C_n \sin(2\pi f_n t + \phi_n) \tag{2.10}$$

Where $T_c(t)$ and $T_s(t)$ are Gaussian random processes at any time t. $T_c(t)$ and $T_s(t)$ are uncorrelated zero-mean Gaussian random variables with equal variance.

The envelope of the received E-field, $E_z(t)$ is given by

$$\left|E_z(t)\right| = \sqrt{T_c^{\,2}(t) + T_s^{\,2}(t)} = r(t) \tag{2.11}$$

Hence, according to Clarke and Gan's Rayleigh fading model, two independent Gaussian low pass noise sources are used to produce in-phase and quadrature fading branches. Each Gaussian source may be formed by summing two independent Gaussian random variables which are orthogonal (i.e. g = a + jb, where a & b are real Gaussian random variables and g is complex Gaussian).by using the spectral filter defined by equation below (2.12) to shape the random signals in frequency domain accurate time domain waveforms of Doppler fading can be produced by using an Inverse Fast Fourier transform (IFFT) at the last stage of the simulator.

Frequency response of the shaping filter is given by,

$$S_{E_Z}(f) = \frac{1.5}{\pi f_m \sqrt{1-\left(\frac{f-f_c}{f_m}\right)^2}} \tag{2.12}$$

Where f_c is the carrier frequency and f_m is the maximum Doppler frequency shift which basically depends on the vehicle speed and the direction of relative motion of mobile w.r.to the base station and on f_c.

The simulator can be implemented in following steps:

- The number of frequency domain points (N) used to represent$\sqrt{}$ $S_{E_z}(f)$ and the maximum Doppler frequency shift (f_m) are specified. The value of N to be used usually is chosen as power of two.
- The frequency spacing between adjacent spectral lines as $\Delta f = 2f_m /(N-1)$ is computed. This defines the time duration of a fading waveform, $T = 1/\Delta f$.

- Complex Gaussian random variables for each of N/2 positive frequency components of noise source are generated.
- Negative frequency components by conjugating positive frequency of the noise source by conjugating positive frequency values are constructed and assign at negative frequency values.
- In-phase & quadrature noise sources are multiplied with fading Spectrum $\sqrt{S_{E_Z}(f)}$.
- Inverse Fast Fourier Transform (IFFT) is performed on resulting frequency domain signal from the in-phase & quadrature arms to get two N length time series and the squares of each signal point in time are added to create N point time series like under the radical of equation (2). Note that each quadrature arm should be a real signal after the IFFT to model equation (1).
- Square root of the sum obtained in step-6 are taken to obtain N point time series of a simulated Rayleigh fading signal with the proper Doppler spread and time correlation.

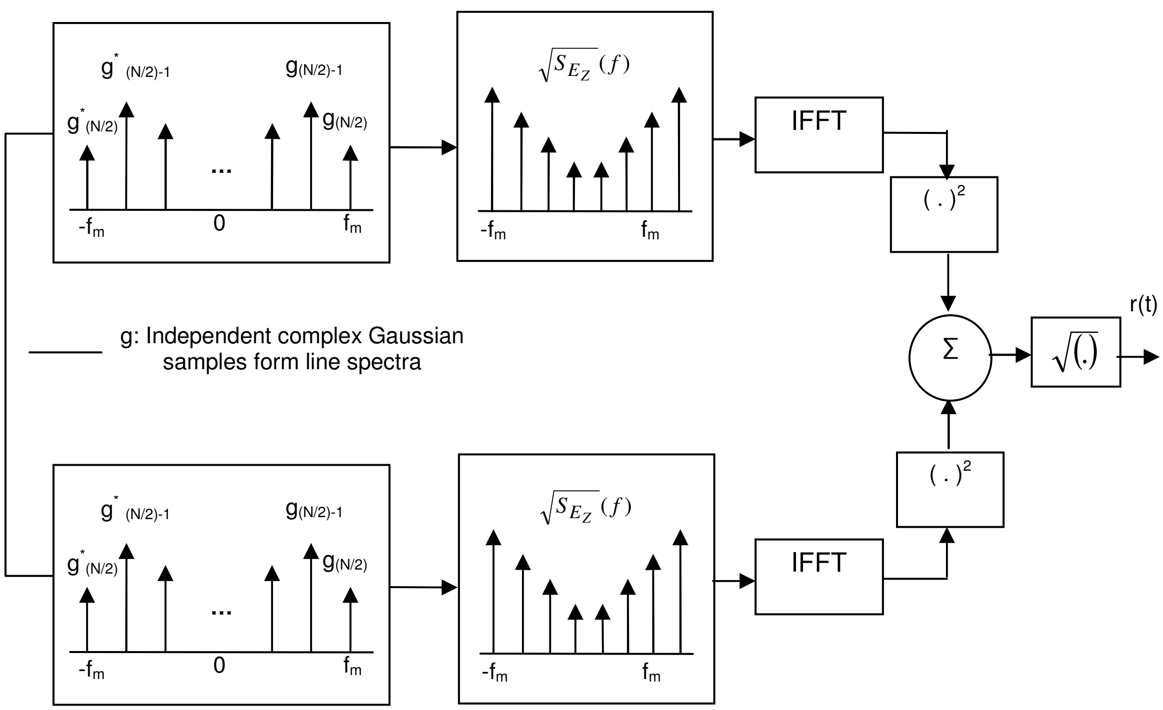


Figure 2.3: Frequency domain implementation of a Rayleigh fading simulator at baseband

2.4.2 Statistical model for multipath fading channel

For a multipath fading simulator with many resolvable components, Clarke and Gan's Rayleigh fading model can be used to alter the probability distribution of the individual multipath components in the simulator of Figure 2.4. Proper care must be taken to implement the IFFT such that each arm of Figure 2.3 produces a real time domain signal as given by $T_c(t)$ and $T_s(t)$ in equation (2.9) and (2.10).

The number of frequency domain points (N) used to represent $\sqrt{S_{E_Z}(f)}$ and s(t) was to be specified and the applied signal should be multiplied with r(t), the output of fading simulator. To determine the impact of more than one multipath component, a convolution can be performed as shown in Figure 2.4.

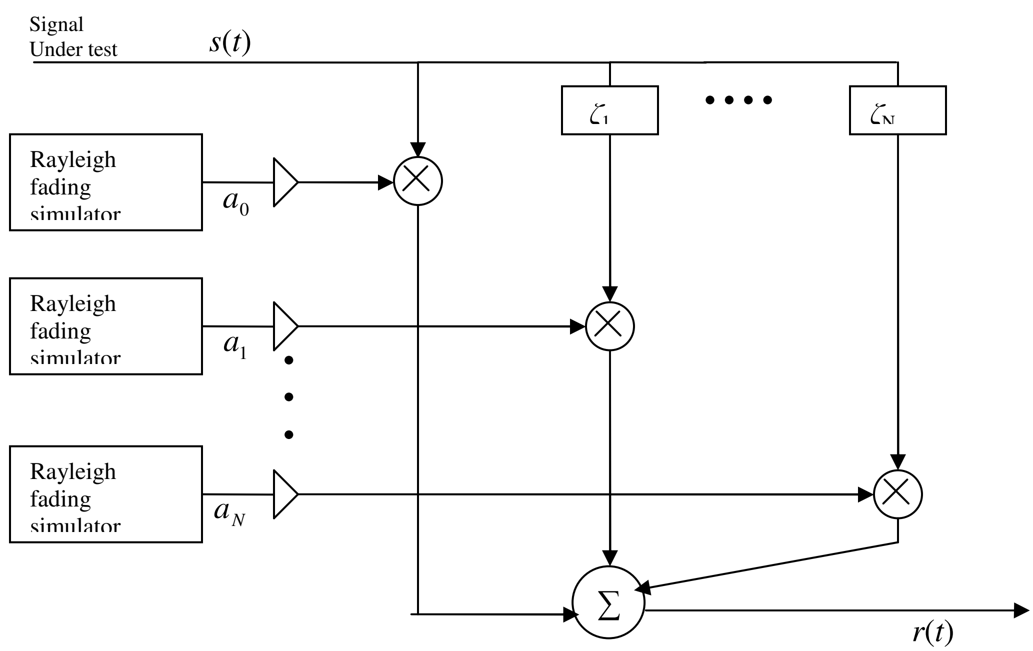


Figure 2.4: Application of Rayleigh fading simulator to determine performance in a Wide range of channel conditions.

Simulation Result:

Figure 2.5 presents a typical Rayleigh fading simulated envelope where the mobile carrier frequency of 900MHz is taken and vehicle speed of 120 km/hr is considered. This figure describes how the signal level changes with respect to the time elapsed. For this simulation 256 numbers of frequency spacing points were taken. This shows the changes in signal level about its root mean square (rms) values in Decibel scale for certain time ranges in milliseconds.

In Figure 2.6 below it has been shown how channel transfer function varies with the Rayleigh fading environment so as this variation is not known beforehand hence the equalizer needs to estimate the channel transfer function during the transmission of known training data.

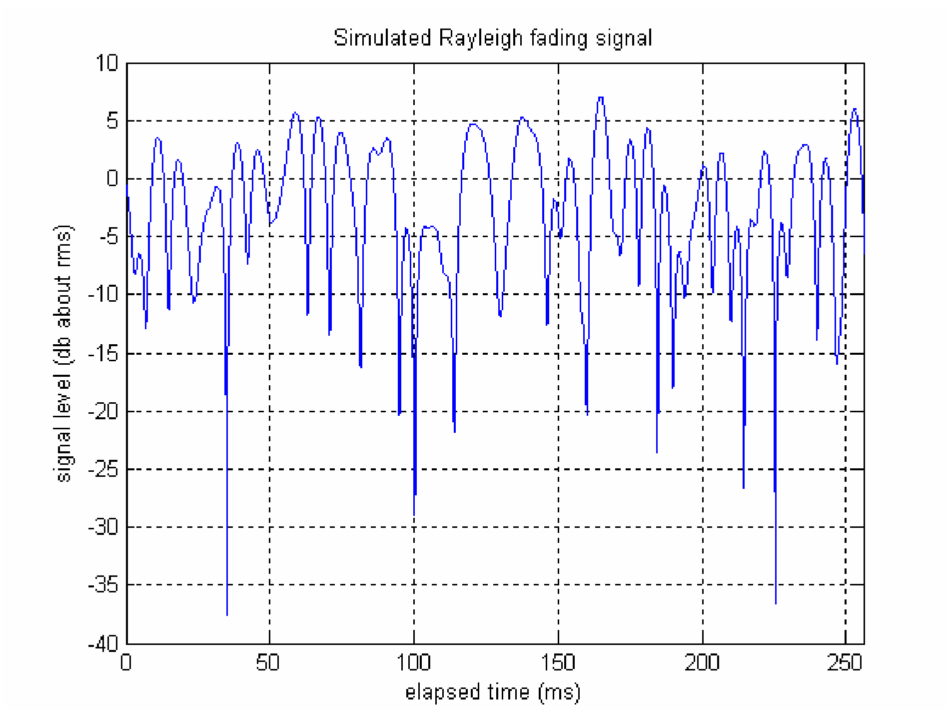


Figure 2.5: A typical Rayleigh fading envelope

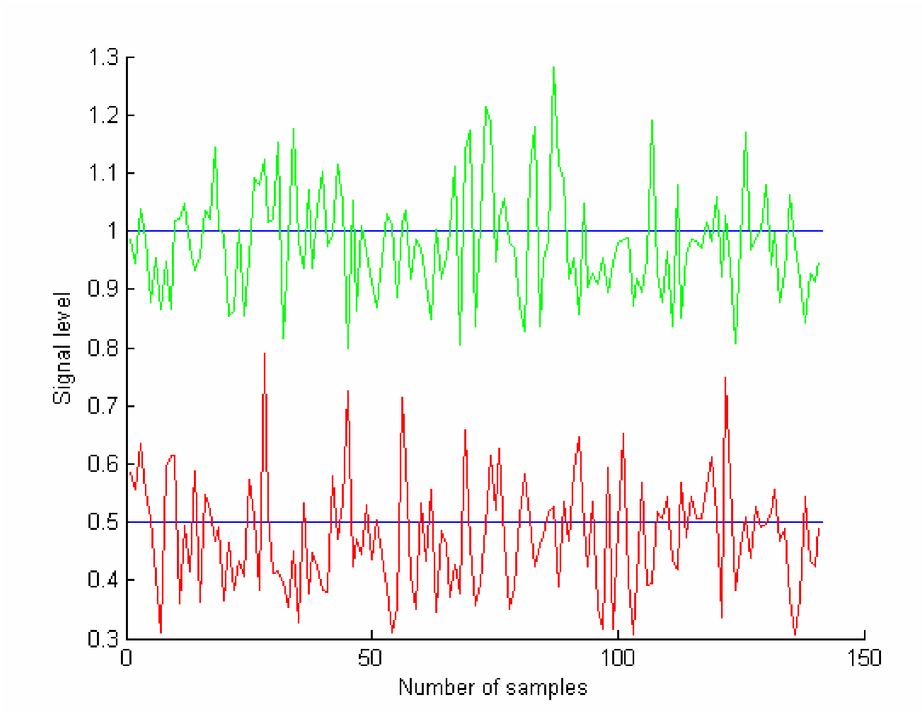


Figure 2.6: The realization of the time varying coefficients of channel having transfer function $H(z) = 1.0 + 0.5\,z^{-1}$.

2.5 Equalizer classification

This thesis discusses the performance of nonlinear receivers for mobile channel equalization problem. That's why a brief discussion on classification of equalizers is given here. In general the family of adaptive equalizers can be classified as supervised equalizers and unsupervised equalizers. The channel distortions introduced into the transmitted signal in the process of transmission can be conveniently removed by transmitting a training signal or pilot signal periodically during the transmission of information. A replica of this pilot signal is available at the receiver and the receiver uses this to update its parameter during the training period. These kinds are known as supervised equalizer.

However the constraints associated with communication systems like digital television and digital radio do not provide the scope for the use of a training signal. In this situation the equalizer needs some form of unsupervised or self recovery methods to update its parameters so as to provide near optimal performance. These equalizers are called blind equalizers. After training, the equalizer is switched to decision directed mode, where the equalizer can update its parameters based on the past detected samples.

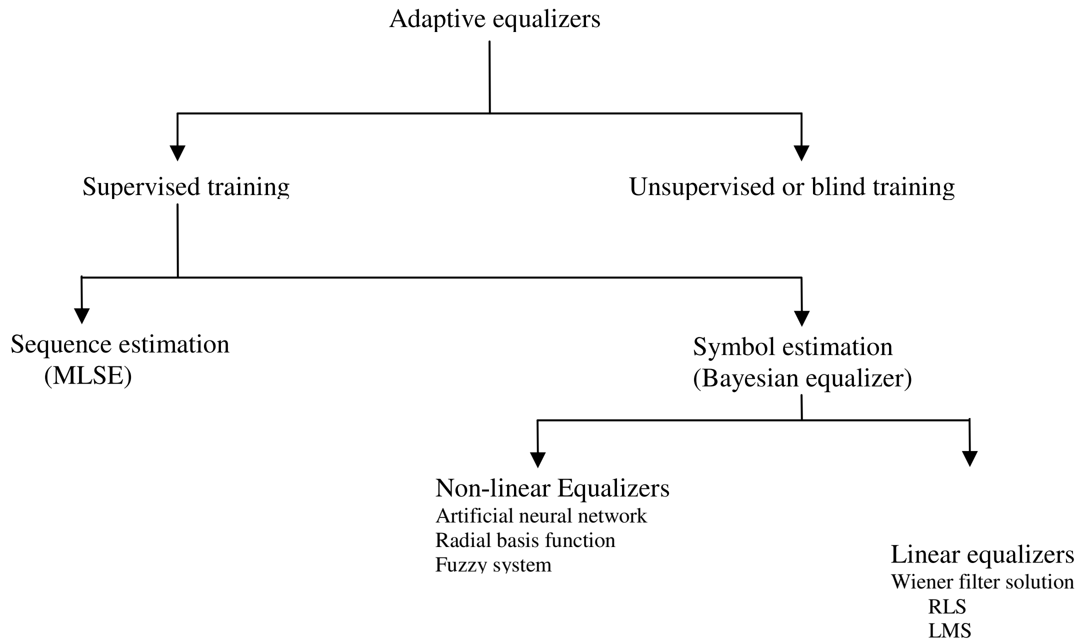


Fig 2.7: classification of equalizers

The process of supervised equalization can be achieved in two forms .These are *sequence estimation* and *symbol-by-symbol estimation*. The sequence estimator uses the sequence of past received samples to estimate the transmitted symbol. Hence this form of equalizer is called infinite memory equalizer and termed as MLSE [3] and can be implemented by viterbi algorithm [4]. The symbol-by-symbol equalizer on the other hand works as a finite memory equalizer and use fixed number of input samples to detect the transmitted symbols. A finite memory Bayesian equalizer can provide performance comparable to the MLSE but with a reduced computational complexity.

These linear equalizers treat equalization as inverse filtering and during process of training optimize a certain performance criteria like minimum mean square error (MMSE) or amplitude distortion. Linear equalizers trained with MMSE criterion provide the Wiener filter solution.

Recent advances in nonlinear signal processing techniques have provided a rich variety of nonlinear equalizers. Some of the equalizers are developed with these processing techniques are based on ANN [7], perceptrons, MLP [16], RBF networks [8], fuzzy filters [11] and fuzzy basis functions which treat equalization as a pattern classification problem.

2.6 Global System for Mobile (GSM)

Global system for mobile (GSM) [1] is a second generation cellular system standard that was developed to solve the fragmentation problems of the first cellular systems in Europe. GSM was the world's first cellular system to specify digital modulation and network level architectures and services. Around 1980s many countries developed their own cellular system, which was incompatible with other's equipment and operation which confines the use of mobile equipments within national boundaries with limited operation. The Europeans realized this early on, and in 1982 the Conference of European Posts and Telegraphs (CEPT) formed a study group called the Group Special Mobile (GSM) to study and develop a pan-European public land mobile system. The proposed system had to meet certain criteria:

- Good subjective speech quality
- Low terminal and service cost
- Support for international roaming
- Ability to support handheld terminals
- Support for range of new services and facilities
- Spectral efficiency
- ISDN compatibility

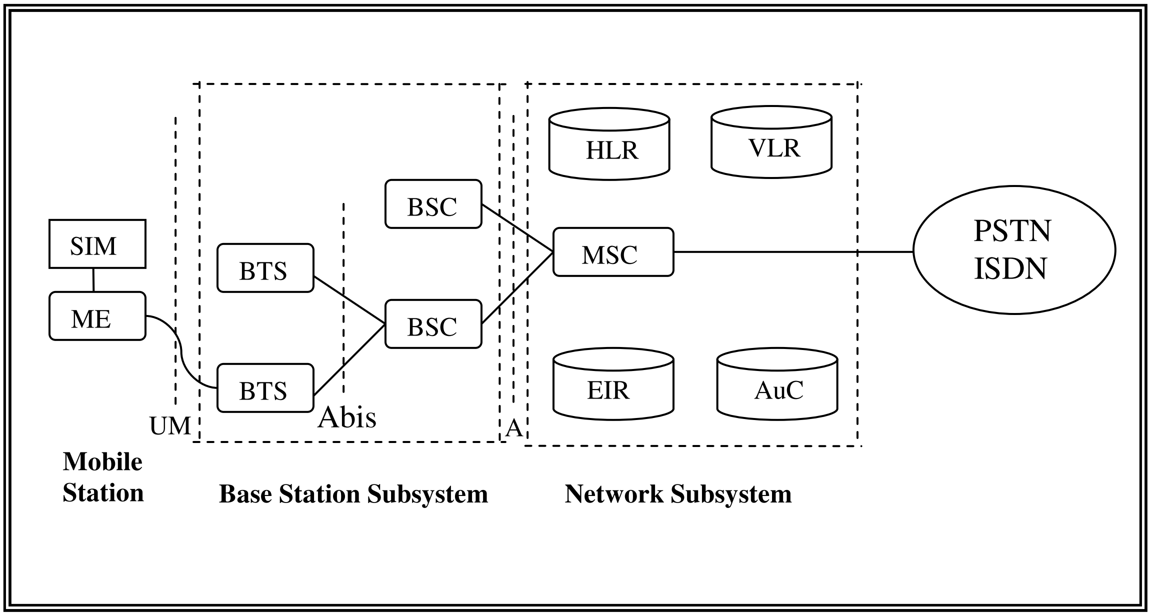
In 1989, GSM responsibility was transferred to the European Telecommunication Standards Institute (ETSI), and phase I of the GSM specifications were published in 1990. Commercial service was started in mid-1991, and by 1993 there were 36 GSM networks in 22 countries. The acronym GSM now aptly stands for Global System for Mobile communications since 1992 which was changed for marketing reasons.

2.6.1 Architecture of the GSM Network

A GSM network is composed of several functional entities, whose functions and interfaces are specified. Figure 2.8 shows the layout of a generic GSM network. The GSM network can be divided into three broad parts. The Mobile Station is carried by the subscriber. The Base Station Subsystem controls the radio link with the Mobile Station. The Network Subsystem, the main part of which is the Mobile services Switching Center (MSC), performs the switching of calls between the mobile users, and between mobile and fixed network users. The MSC also handles the mobility management operations. Not shown is the Operations and Maintenance Center, which oversees the proper operation and setup of the network. The Mobile Station and the Base Station Subsystem communicate across the Um interface, also known as the air interface or radio link. The Base Station Subsystem communicates with the Mobile services Switching Center across the A interface.

2.6.2 Frame structure of GSM

The method chosen by GSM to use the limited radio spectrum efficiently, is a combination of Time- and Frequency-Division Multiple Access (TDMA/FDMA). The FDMA part involves the division by frequency of the (maximum) 25 MHz bandwidth into 124 carrier frequencies spaced 200 kHz apart. One or more carrier frequencies are assigned to each base station. Each of these carrier frequencies is then divided in time, using a TDMA scheme. The fundamental unit of time in this TDMA scheme is called a *burst period* and it lasts 15/26 ms. Eight burst periods are grouped into a *TDMA frame* (120/26 ms), which forms the basic unit for the definition of logical channels. One physical channel is one burst period per TDMA frame.



SIM subscriber identity module
BSC Base Station Controller
MSC Mobile Switching Center
ME Mobile Equipment
HLR Home Location Register
AuC Authentication Center
EIR Equipment Identity Register
BTS Base Transceiver Station
VLR Visitor Location Register

Figure 2.8: GSM Architecture

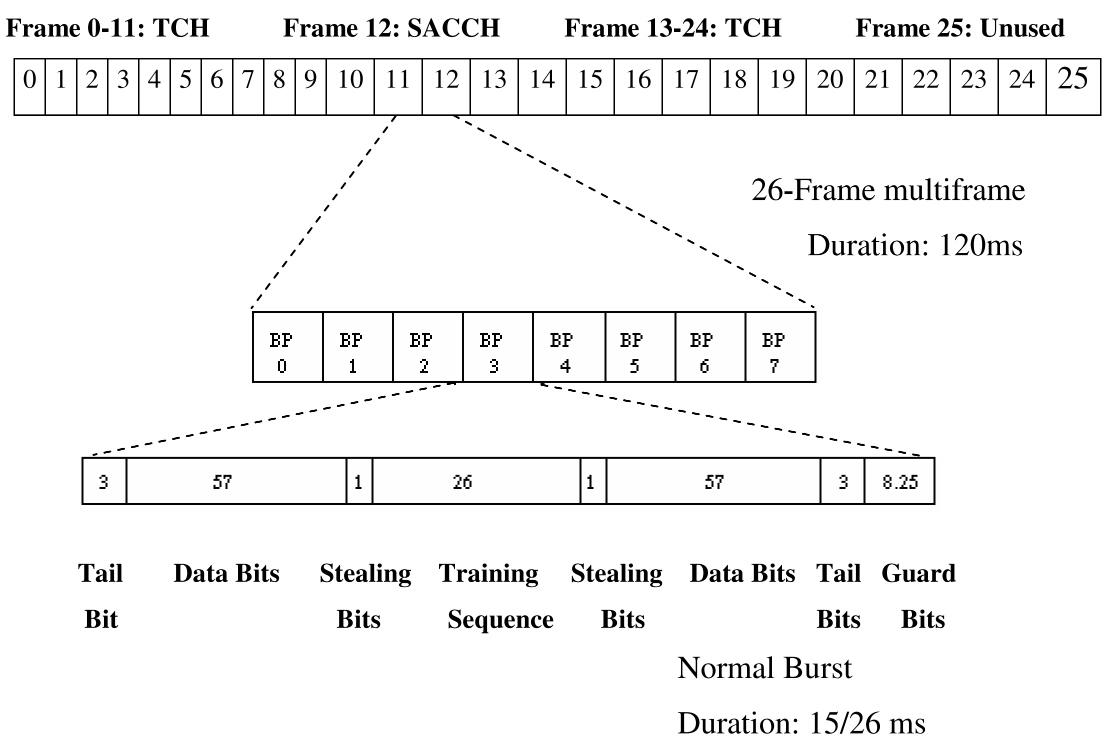


Figure 2.9: Organization of Bursts, TDMA frames and multiframes for speech and data

Each user transmits a burst of data during the time of slot assigned to it. It is a normal burst which contains 148 bits transmitted at a rate 270.83333 kbps. Out of total 148 bits per time slot (TS), 114 are information bearing bits which are transmitted as two 57 bit sequences close to the beginning and end of the burst. The midable consists of a 26 bit training sequence which allows the adaptive equalizer in the mobile or base station receiver to analyze the radio channel characteristics and train equalizer parameter before decoding the user data.

On either side of the midable, there are control bits called stealing flags. These two flags are used to distinguish whether the TS contains voice or control data. During a frame, a GSM subscriber unit uses one TS to transmit, one TS to receive, and may use the six spare time slots to measure signal strength on five adjacent base stations as well as its own base stations. In this thesis the performance of all the equalizers are compared for GSM frame structure.

2.7 Fuzzy logic system

This thesis discusses performance of fuzzy receivers for mobile communication channels. For this reason a brief introduction to fuzzy logic system is presented here.

The Basic building block of fuzzy logic system is presented Figure 2.10. Here the fuzzifier converts the real world crisp input sample $x_i(k)$ to a fuzzy output $F_i^{\,l}$ described by the membership function $\psi_i^{\,l}$. This provides the degree to which the input scalar $x_i(k)$ belongs to the fuzzy set $F_i^{\,l}$. The inference engine provides the relationship between the fuzzy input in terms of membership functions and the fuzzy output of the controller using a set of IF …. THEN rules derived from the rule base. The rule l in the fuzzy rule base can be defined as

$$R^{(l)} \;:\; \text{IF} \quad x_1 \text{ is } F_1^{\,l} \text{ and } \ldots \text{ and } x_n \text{ is } F_n^{\,l} \quad \text{THEN} \quad y \text{ is } G^l \qquad (3.12)$$

The Defuzzifier converts the inference G^l to provide the crisp output $y(k)$. In fuzzy system the rule base is generated in advance with expert knowledge of the system under consideration. The learning properties in fuzzy system are achieved with the adaptation and learning block that uses the available information in the system.

The available linguistic rules can also be applied in the adaptation algorithm. These types of system is called adaptive neuro fuzzy filter (ANFF) [34] and they possesses the ability to incorporate training like neural networks and can also use the rule bases from human experts as in fuzzy system. Wang et. al. [35] presented fuzzy basis function (FBF) and used a combination of these function for universal approximation and later on used them as fuzzy filter [11] for channel equalization. Based on these concepts different types of fuzzy adaptive filters have been proposed [36, 37] which uses complex RLS and LMS adaptive algorithms and different form of fuzzy equalizer is used in [38].

Most of the fuzzy equalizers developed in recent years have structures similar to the LMS and RLS fuzzy filters proposed in [11] by Mendel. But proposed fuzzy filter implementation using RLS is computationally complex as its rule base grows exponentially with the number of rules in each dimension and using LMS though simple, but suffers from performance degradation if initial parameters are not selected properly. In 1998, Patra and Mulgrew [28] proposed a similar form of fuzzy equalizer alleviating the problems associated with that of fuzzy filter proposed by Mendel and used a modified form of this filter to mitigate co-channel interference (CCI) [39].

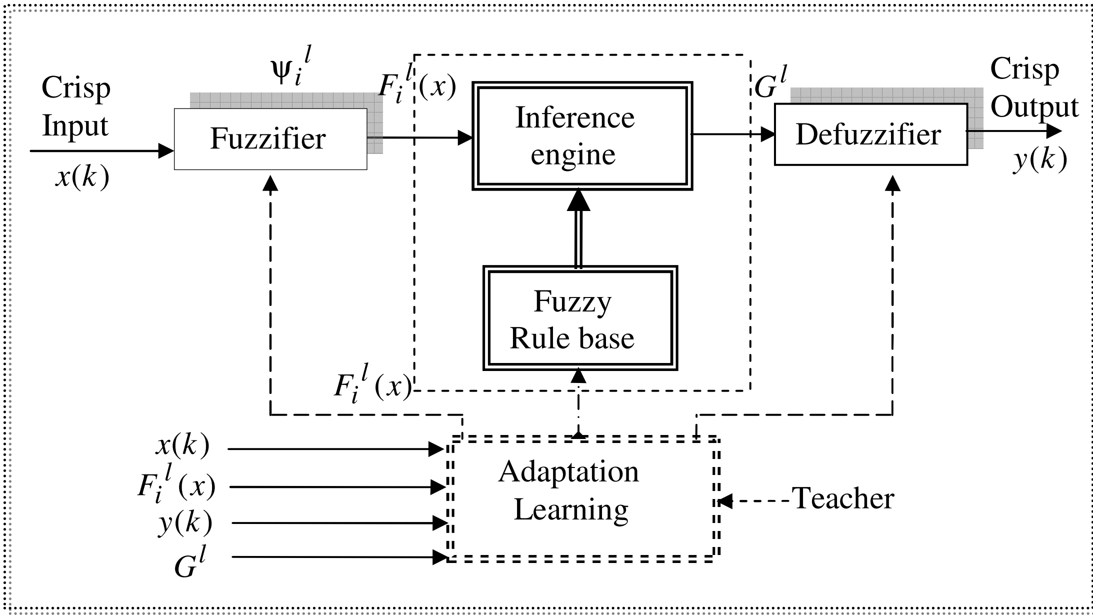


Figure 2.10: A typical fuzzy logic system

But the concept of type-2 fuzzy sets was introduced by Zadeh [30] as an extension of the concept of an ordinary fuzzy set i.e. type-1 fuzzy set. This concept is then used in the equalization of nonlinear time varying channels to design a type-2 fuzzy adaptive filter by Mendel in 2000 [29] which considers the linguistic and numerical uncertainties in mobile communication field where the mapping between input and output data pairs are uncertain due to the channel dynamics. This thesis analyzes the performance of RBF equalizer with two types of FAFs for GSM environment and linear equalizers using LMS and RLS algorithms.

2.8 Conclusion

This chapter provided a brief introduction to different aspects dealt in this thesis. The chapter discusses communication system in general and the propagation FIR channel model. In this chapter Clarke's Rayleigh fading model and the effects of the fading envelope on channel transfer function was described through simulation. The architecture and the frame structure of GSM system have been discussed. Fuzzy system was introduced here. These concepts will be used in subsequent chapters of this thesis.

Chapter 3

Fuzzy Based Equalizers for Linear Channel

3.1 Introduction

Channel equalization is a nonlinear classification problem. Even when channel is linear, the channel equalization problem is still a nonlinear one. Under many circumstances the nonlinear decision boundary can be approximated by a linear boundary. This is the best performance a linear equalizer can provide and therefore it suffers from performance degradation. Owing to this suboptimal performance of linear equalizers, it is always desirable to explore new nonlinear equalization algorithms that can provide a performance trade off with computational complexity against the optimal MAP Bayesian equalizer.

This chapter discusses the development of fuzzy nonlinear equalizer which can be considered as a fuzzy implementation of a Bayesian equalizer and also discusses the Type-2 fuzzy logic system and Type-2 fuzzy adaptive filter. The chapter organizes as follows.

- Section 3.2 discusses the decision function of Bayesian equalizer.
- Section 3.3 presents the design of RBF equalizer and the channel states which forms the centers of RBF equalizer.
- Section 3.4 describes the fuzzy implementation of the Bayesian equalizer and fuzzy equalizer structure.
- Section 3.5 discusses the advantages of fuzzy equalizer over the Bayesian equalizer implemented by RBF and a normalized Bayesian equalizer with with scalar states and some simulation results
- Section 3.6 presents the training of the fuzzy equalizer and some simulation results with discussion.

- Section 3.7 describes the Type-2 FAF, its structure and its decision function.
- Section 3.8 provides some extensive simulation results comparing among all equalizers at different channels, equalizer parameters, vehicle speeds.
- Section 3.9 gives the concluding remarks of this chapter.

3.2 Bayesian Equalizer decision function

A discrete time model of the baseband digital communication system using Transversal Equalizer (TE) structure [12] is presented below in Figure 3.1. Here channel is modeled as an FIR filter as in Figure 2.2. The equalizer uses an input vector $r(k) \in \Re^p$, the p dimensional space. The term p is the equalizer order (i.e. number of taps in equalizer) and the channel order is n (n+1 taps). The equalizer provides decision function $\Im\{r(k)\}$ based on the input vector which is then passed through a decision device to provide the estimate of transmitted signal $\hat{s}(k-d)$ where d is the delay associated with equalizer decision. The communication system is assumed to be a two level PAM system where the transmitted sequence $s(k)$ is drawn from an independent identically distributed (i.i.d.) sequence comprising of {±1} symbols.

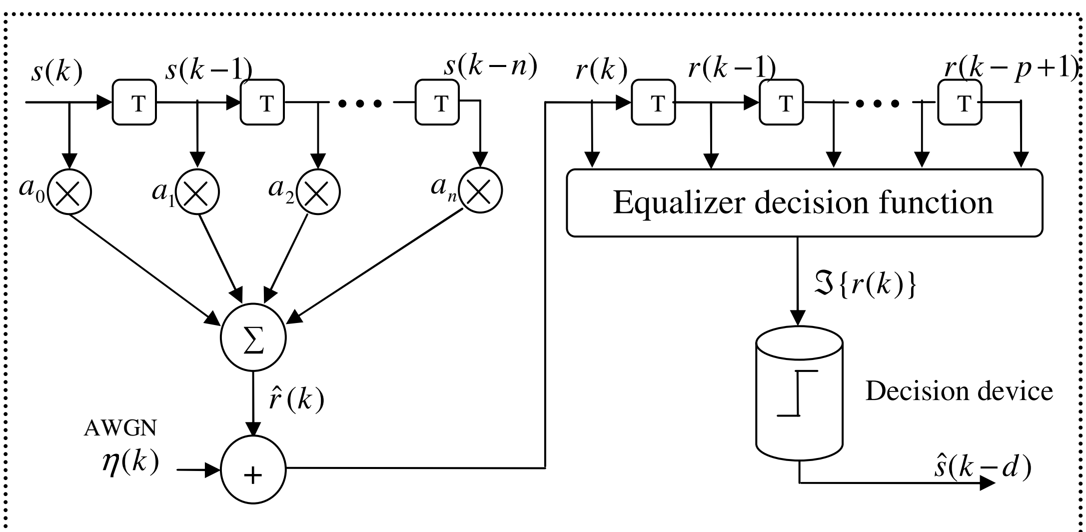


Figure3.1: Discrete time model of DCS using Transversal Equalizer structure

The noise source $\eta(k)$ is assumed to be zero mean white Gaussian with a variance of σ_η^2. The received signal $r(k)$ at the sampling instant k can be represented as,

$$r(k) = \hat{r}(k) + \eta(k) \qquad (3.1)$$

$$= \sum_{i=0}^{n} a_i(k)\, s(k-i) + \eta(k)$$

The equalizer uses the received signal vector $r(k) = [r(k),\ r(k\text{-}1),\ \ldots\ ,\ r(k\text{-}p\text{+}1)]^T \in \Re^p$ to estimate the delayed transmitted symbol $s(k-d)$. The equalizer with its decision function and a memoryless detector to quantize the real valued output from decision function $\Im\{r(k)\}$, provides an estimate of the transmitted signal. The memoryless detector is implemented using a sgn (x) function given by,

$$\operatorname{sgn}(x) = \begin{cases} +1 & if \quad x \geq 0 \\ -1 & if \quad x < 0 \end{cases} \qquad (3.2)$$

The process of equalization considered as classification problem in which the equalizer partitions into two regions corresponding to each of the transmitted sequences +1 / –1. The locus of the points which separate these two regions is called as decision boundary. The boundary which provides the minimum probability of misclassification is the Bayesian decision boundary derived with the MAP criterion.

The presence of AWGN makes the channel observation vector $r(k)$ a random process having a conditional Gaussian density function centered at each noise free received vector $\hat{r}(k)$. Given this to be the channel state $\hat{r}(k) = c_j$, $1 \leq j \leq N_s$, the conditional probability density distribution of the observed vector is,

$$P(r(k) \mid c_j) = \left(2\pi\sigma_\eta^{\,2}\right)^{-m/2} \exp\left(-\frac{\left\| r(k) - c_j \right\|^2}{2\sigma_\eta^{\,2}}\right) \qquad (3.3)$$

Where $\|.\|$ constitute the Euclidean distance. If the received signal vector is perturbed sufficiently to cross the decision boundary due to presence of AWGN, mis-classification results. To minimize the probability of mis-classification for a given received signal vector $r(k)$, the transmitted symbol should be estimated based on $s(k) \in \{\pm 1\}$ having maximum *a-posteriori* probability $P(\, s(k-d) = s \mid r(k))$ [40]. The decision device at the equalizer output provides a decision

$$\hat{s}(k-d) = \operatorname{sgn}\left(\Im\{r(k)\}\right) = \begin{cases} +1 & \text{if } \Im\{r(k)\} \geq 0 \\ -1 & \text{if } \Im\{r(k)\} < 0 \end{cases} \tag{3.4}$$

Where $\Im\{\, r(k)\,\}$ is the Bayesian equalizer decision function that compares the *a-posteriori* probabilities of the binary transmitted symbol, i.e.,

$$\Im\{r(k)\} = P\left(s(k-d) = +1 \mid r(k)\right) - P\left(s(k-d) = -1 \mid r(k)\right) \tag{3.5}$$

Where $P\left(s(k-d) = +1 \mid r(k)\right)$ and $P\left(s(k-d) = -1 \mid r(k)\right)$ are the *a-posteriori* probabilities that the transmitted signal is +1 / -1 respectively, having observed the signal vector $r(k)$. Using Bayes rule *a-priori* and the state conditional probabilities can be calculated in terms of the channel and noise statistics. If the transmitted symbols is i.i.d., the *a-priori* probabilities of the transmitted signals for +1 and -1 have equal value of $\frac{1}{2}$.

Calculating State conditional probability and applying Baye's rule [40] with some mathematical evaluation, we found the decision function of the Bayesian equalizer [12] as

$$\Im\{r(k)\} = \sum_{i=1}^{N_s} w_i \exp\left(-\frac{\|r(k)-c_i\|^2}{2\sigma_\eta{}^2}\right) \tag{3.6}$$

Where $w_i = +1$, if $c_i \in C_d{}^+$ and $w_i = -1$, if $c_i \in C_d{}^-$ and $C_d{}^+$, $C_d{}^-$ represents the positive channel states categories and negative channel state categories respectively.

3.3 Radial Basis Function Equalizer

The RBF network was originally developed for interpolation in multidimensional space. The schematic of this RBF network with m inputs and a scalar output is presented in Figure 3.2.This network can implement a mapping $f_{rbf} : \mathbb{R}^m \rightarrow \mathbb{R}$ by the function,

$$f_{rbf}\{x(k)\} = \sum_{i=1}^{N_r} w_i \phi(\|x(k) - \rho_i\|) \tag{3.7}$$

Where $x(k) \in \mathbb{R}^m$ is the input vector, $\phi(.)$ is the Radial Basis function from $\mathbb{R}^+$ to $\mathbb{R}$, w_i, $1 \leq i \leq N_r$ are weights and $\rho_i \in \mathbb{R}^m$ are known RBF centers. This RBF structure can be extended for multi dimensional output as well. Possible choices for the radial basis function $\phi(\gamma)$ include, a thin plate spline

$$\phi(\gamma) = \frac{\gamma}{\sigma_r^2} \log\left(\frac{\gamma}{\sigma_r}\right) \tag{3.8}$$

a multi quadric,

$$\phi(\gamma) = \sqrt{\gamma^2 + \sigma_r^2} \tag{3.9}$$

an inverse multi- quadric,

$$\phi(\gamma) = \frac{1}{\sqrt{\gamma^2 + \sigma_r^2}} \tag{3.10}$$

and Gaussian kernel,

$$\phi(\gamma) = \exp\left(-\frac{\gamma^2}{2\sigma_r^2}\right) \tag{3.11}$$

In this thesis radial basis function $\phi(\gamma)$ with Gaussian kernel is taken, where σ_r^2 controls the radius of influence of each basis function and determines how rapidly the function approaches 0 with γ . The Gaussian kernel provide bounded and localized properties such that $\phi(\gamma) \rightarrow 0$ as $\gamma \rightarrow \infty$.

The training of RBF networks involves setting the parameters for centers, ρ_i, spread parameter, σ_r and the linear weights ω_i. The RBF networks are easy to train since the training of centers, spread parameter and the weights can be done sequentially and the network offers a nonlinear mapping, maintaining its linearity in parameter structure at the output layer. One of the most popular scheme employed for training the RBF in a supervised manner is to estimate the centers using a clustering algorithm like κ-means clustering [41] and setting $\sigma_r^{\ 2}$ to an estimate of input noise variance calculated from the center estimation error. The output layer weights can be trained using popular stochastic gradient LMS algorithm. Other schemes for RBF training involve selecting a large number of centers initially and use the orthogonal least squares (OLS) [17] algorithm to pick a subset of the centers that provides near optimal performance. The MLP back propagation algorithm [42] can also be used to train the RBF centers.

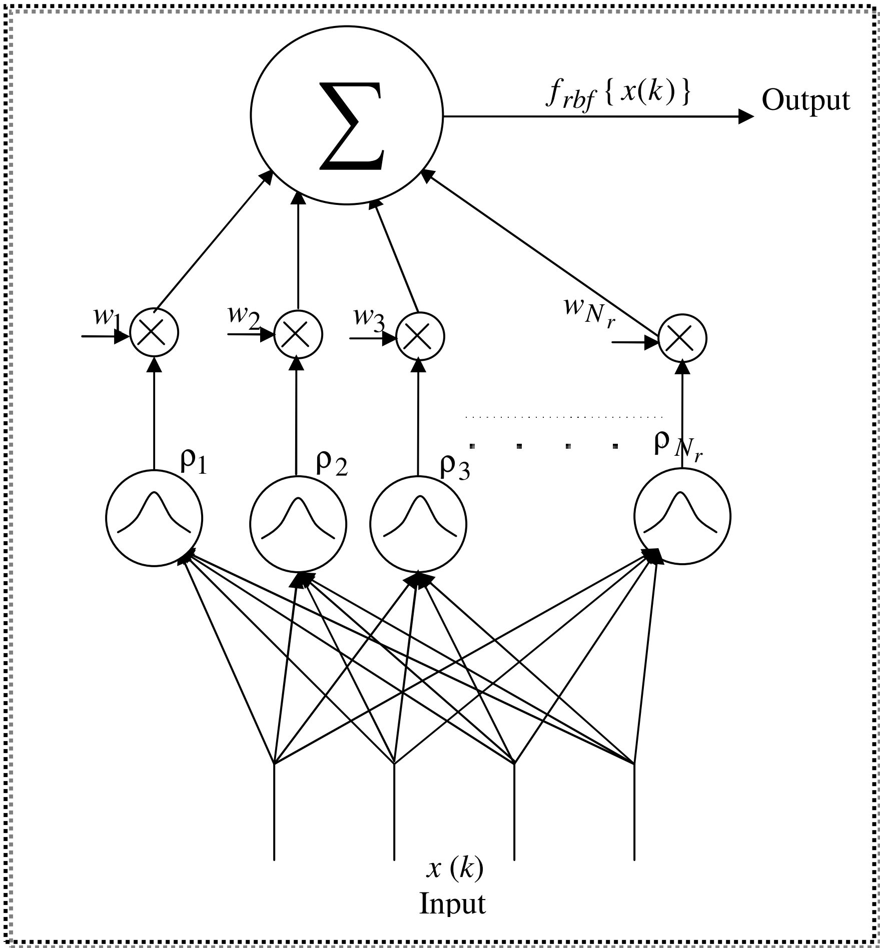


Figure 3.2: A radial basis function network

The close relationship between RBF network and Bayesian equalizer was found [43] which provide the idea to implement the Bayesian equalizer with the RBF. In these equalizers supervised k-means clustering [41] provides the estimate of the centers while linear weights are estimated by LMS algorithm.

It can be realized that the RBF decision function (3.7) by using Gaussian kernel of (3.11), and the Bayesian equalizer decision function of (3.6) are similar. The RBF network can provide a Bayesian decision function by setting RBF centers, ρ_i, to channel states c_i, RBF spread parameter, σ_r^2, to channel noise variance, σ_η^2 and the weights $w_i = +1$, if $c_i \in C_d^+$ and $w_i = -1$, if $c_i \in C_d^-$. The function changes for scalar states.

The RBF equalizers can provide optimal performance with small training sequence but they suffer from computational complexity. The number of RBF centers required in the equalizer increases exponentially with equalizer order and channel delay dispersion order. This increases all the computation exponentially. In a varied implementation [39] the RBF with scalar centers results in a reduction of computational complexity which is used in this thesis.

Bayesian decision function is given by

$$f(\mathrm{r(k)}) = \sum_{i=1}^{n_s} p_i \exp\left(\frac{-\left\| r(k) - c_i \right\|^2}{2\sigma_e^2} \right) \qquad (3.12)$$

Where $n_s = 2^{m+n_h-1}$ are number of channel states with $n_s^+ = n_s^- = \frac{n_s}{2}$, 'm' is the equalizer order, 'n_h' is the number of taps in the channel and p_i are weights associated with each centers. Also observed that each of the channel states has m components which can be represented as,

$c_i = \left[c_{i0}, c_{i1}, c_{i2}, \ldots c_{i(m-1)} \right]^T \in \mathbb{R}^m$

The Bayesian equalizer presented in (3.12) can be implemented with RBF networks [34]. The decision function given by (3.12) needs the channel states. The channel states can be estimated during the training period. This decision function reveals that the equalizer contains n_s channel states, each of m dimensions. The number of scalar channel states for any channel is $M = 2^{n_h}$. Each of the m components of the n_s channel states are taken M scalar channel states which form the estimate of the noise free received scalars.

3.3.1 Channel states calculation

To implement the Bayesian equalizer, the concept of channel states is introduced first. The equalizer input vector has been defined as, $r(k) = [\, r(k), r(k-1), \ \ldots, r(k-m+1)]^T \in \mathbb{R}^m$ and $r(k) = \hat{r}(k) + \eta(k)$. The vector $\hat{r}(k)$ is the noise free received signal vector and $\hat{r}(k) = [\, \hat{r}(k), \hat{r}(k-1), \ \ldots, \hat{r}(k-m+1)]^T \in \mathbb{R}^m$.

Each of these possible noise free received signal vectors constitutes a channel state. The channel states are determined by the transmitted symbol vector $s(k) = [\,s(k), s(k-1),\ \ldots\ , s(k-m-n_h+2)]^T \in \mathbb{R}^{\mathrm{m+nh-1}}$. Here $\hat{r}(k)$ can be represented as $\hat{r}(k) = \mathrm{H}[s(k)]$, Where matrix H is a $m \times (m+n_h-1)$ channel matrix.

$$H = \begin{bmatrix} a_0 & a_1 & \cdots & a_{n_h-1} & 0 & \cdots & 0 & \cdots & 0 \\ 0 & a_1 & \cdots & a_{n_h-2} & a_{n_h-1} & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \cdots & \cdots & \cdots & a_0 & \cdots & a_{n_h-1} \end{bmatrix} \qquad (3.13)$$

Since $s(k)$ has $n_s = 2^{m+n_h-1}$ combinations, $\hat{r}(k)$ has n_s states. These channel constructed with n_h sequences of $s(k)$, which can be denoted as

$$s_j(k) = \left[s_j(k), s_j(k-1), \cdots, s_j(k-m-n_h+2)\right]^T, \qquad 1 \le j \le n_s \qquad (3.14)$$

The corresponding channel states are denoted as,

$$c_j = \hat{r}(k) = H\left[s_j(k)\right], \qquad 1 \le j \le n_s \qquad (3.15)$$

For a channel $H(z) = 0.5 + 1.0z^{-1}$ with equalizer length is m = 2, d = 0 and n_s = 8 channel states. This channel is a non-minimum phase channel with its zero outside the circle, the scalar channel states are shown in Table-1. The scalar channel states are the noise free channel output which is decided by the possible combinations of the inputs.

No.	$s(k)$	$s(k-1)$	$\hat{r}(k)$	c_j
1	1	1	1.5	c_1
2	1	-1	-0.5	c_2
3	-1	1	0.5	c_3
4	-1	-1	1.5	c_4

Table-3.1: scalar channel state calculation for channel $H(z) = 0.5 + 1.0z^{-1}$

The channel states for this equalizer are presented from the scalar states in Table-2 and are located at $\hat{r}(k)$ with its components taken from scalars $[\hat{r}(k), \hat{r}(k-1)]^T$.

The channel state matrix $C_d = \{c_j\}, 1 \le j \le n_s$, can be partitioned into two subsets depending on the transmitted symbol $s(k-d)$, i.e.

$$C_d = C_d{}^+ \cup C_d{}^- \qquad (3.16)$$

No.	c_j	$s(k)$	$s(k-1)$	$s(k-2)$	$\hat{r}(k)$	
					$\hat{r}(k)$	$\hat{r}(k-1)$
1	c_1	1	1	1	1.5	1.5
2	c_2	1	1	-1	1.5	-0.5
3	c_3	1	-1	1	-0.5	0.5
4	c_4	1	-1	-1	-0.5	-1.5
5	c_5	-1	1	1	0.5	1.5
6	c_6	-1	1	-1	0.5	-0.5
7	c_7	-1	-1	1	-1.5	0.5
8	c_8	-1	-1	-1	-1.5	-1.5

Table-3.2: channel state calculation for $H(z) = 0.5 + 1.0z^{-1}$

Where

$$C_d{}^+ = \{\hat{r}(k) \mid s(k-d) = +1\} \qquad (3.17)$$

$$C_d{}^- = \{\hat{r}(k) \mid s(k-d) = -1\}$$

Each of the sets of the channel states matrix C_d^{+} and C_d^{-} contain $\frac{n_s}{2}$ channel states. Here the channel states $c_j \in C_d^{+}$ are termed as positive channel states and $c_j \in C_d^{-}$ are termed as negative channel states.

3.4 Fuzzy adaptive filter

The fuzzy adaptive filter (FAF) was originally proposed by Wang and Mendel [22]. Fuzzy filters are nonlinear filters that can incorporate fuzzy IF … THEN rules from a human expert system. Wang & Mendel proposed LMS and RLS fuzzy filters. Later Patra and Mulgrew [28] proposed a fuzzy filter similar to the RLS FAF of Mendel which could be trained by LMS algorithm.

The filter considered here maps the real input vector $\mathbb{R}^m \rightarrow \mathbb{R}$ with the function

$$f_{faf}\{x(k)\} : U \in \mathbb{R}^m \rightarrow \mathbb{R} \tag{3.18}$$

Where $x(k) = [x_1(k), x_2(k), \cdots, x_i(k), \cdots, x_m(k)]^T$, $x_i(k) \in U \equiv \left[g_i^{-}, g_i^{+}\right]$ is the input to the fuzzy filter and g_i^{-}, g_i^{+} are the minimum and maximum limits for the input scalars $x_i(k)$. Here $f_{faf}\{x(k)\}$ is the fuzzy adaptive filter output, corresponding to the filter input $x(i)$. The filter minimizes the sum squared error performance index such that

$$e(k) = \sum_{i=0}^{k}\left[y(i) - f_{faf}\{x(i)\}\right]^2 \tag{3.19}$$

Where $y(i)$ is the desired filter output corresponding to the filter input $x(i)$ and $e(k)$ is the sum of the squared of errors that needs to be minimized.

3.4.1 Fuzzy filter design

Let a fuzzy adaptive filter of input vector of length m having a scalar output and

Gaussian membership function is considered here. Each element of the filter input is first fuzzified with a Gaussian membership function which is represented as

$$\psi_i{}^j(k) = \exp\left\{-\frac{1}{2}\left(\frac{x_i(k)-\delta_i{}^j}{\sigma_i{}^j}\right)^2\right\} \tag{3.20}$$

Where $\delta_i{}^j$ and $\sigma_i{}^j$ are the *j*th center and spread parameters respectively corresponding to the input space x_i, $1 \le i \le m$ such that the input scalar $x_i \in U \equiv \left[g_i^-, g_i^+\right]$ is completely covered. These parameters once selected remain fixed and the input associated with each membership functions $\psi_i{}^1, \psi_i{}^2, \cdots, \psi_i{}^{M_i}$, so that the filter is characterized by a total of $\sum_{i=1}^{m} M_i$ membership functions.

The FAF consists of fuzzy IF … THEN rules of the form

$R^{(1,1\cdots,1)}$: IF x_1 is $F_1{}^1$ x_2 is $F_2{}^1$ $\cdots$ x_m is $F_m{}^1$ THEN y is $\psi_1{}^1, \psi_2{}^1 \cdots \psi_m{}^1 \cdots$

$R^{i1,i21\cdots,im}$: IF x_1 is $F_1{}^{i1}$ x_2 is $F_1{}^{i2}$ $\cdots$ x_m is $F_m{}^{im}$ THEN y is $\psi_1{}^{i1}, \psi_2{}^{i2} \cdots \psi_m{}^{im}$ $\cdots$

$R^{M_1,M_2,\cdots,M_m}$: IF x_1 is $F_1{}^{M_1}$ x_2 is $F_2{}^{M_2}$ $\cdots$ x_m is $F_m{}^{M_m}$ THEN y is $\psi_1{}^{i1}, \psi_2{}^{i2} \cdots \psi_m{}^{im}$

Where each of the terms i_1, i_2, $\cdots$, i_m are single indices each ranging from 1 to M_i respectively. The filter considered here finds the following nonlinear function of the membership functions $\psi_i{}^j$ taking normalized RBF proposed by Chen et.al. [37] into consider so that,

$$f_{faf}\{x(k)\} = \sum_{i1=1}^{M_1}\sum_{i2=1}^{M_2}\cdots\sum_{im=1}^{M_m} v_l(k)\left\{\psi_1{}^{i1}(k)\,\psi_2{}^{i2}(k)\cdots\psi_m{}^{im}(k)\right\} \qquad (3.21)$$

Where $\nu(k)^{(i1,i2,\cdots,im)}$ is the weights associated with the fuzzy IF … THEN … rule $R^{i1,i21\cdots,im}$.

The weight parameter $\nu(k)^{(i1,i2,\cdots,im)}$ is updated during the adaptation procedure so as to minimum the cost function in (3.19), where the input to the filter weight is,

$$\varphi\{x(k)\} = \psi_1{}^{i1}\,\psi_2{}^{i2}\cdots\psi_m{}^{im} \qquad (3.22)$$

The filter function in (3.21) finds a weighted sum of all possible combinations of the products of the membership functions, taking one from each input, and this sum is scaled with the sum of all possible product combinations of the membership functions taking one from each input.

For equalization the number of fuzzy sets M_i for each input are set equal so that $M_1 = M_1 = \cdots = M_m = M$. The membership function centers $\delta_i{}^j$, $0 \le j \le M$, of the fuzzy adaptive filter were selected uniformly in the signal space $\left[g_i{}^- , g_i{}^+\right]$ and spread parameter σ^2 can be set equal to noise variance. For providing good performance the same set of membership function centers should be used for each of the dimension of the signal positions the FBFs. The use of the large number of basis functions increased complexity and also RLS training scheme increased the complexity of the equalizer during training. Thinking on the idea of Lee [44] of positioning the centers of fuzzy basis functions at the scalar channel states, Patra and Mulgrew proposed a new form of fuzzy equalizer [28] where the calculation of basis function which includes the exponential function as well as division only depends upon M which is independent of the equalizer order, instead of n_s which in turn exponentially related to the equalizer order.

3.4.2 Bayesian equalizer and its fuzzy implementation [28]

The optimal decision function for Bayesian equalizer (3.6) is derived from Baye's probability theory which can be represented as

$$f(r(k)) = \sum_{i=1}^{n_s} p_i \exp\left(-\frac{\|r(k)-c_i\|^2}{2\sigma_e^2} \right) \tag{3.23}$$

Where n_s is the number of channel states, equal to 2^{n_h+m-1} with $n_s^+ = n_s^- = \frac{n_s}{2}$, and p_i are the weights associated with each of the centers, $p_i = 1$ if $c_i \in n_s^+$ and $p_i = -1$. if $c_i \in n_s^-$. Each of channel state vector has m components which can be represented as $c_i = [c_{i0}, c_{i1}, c_{i2}, \ldots c_{i(m-1)}]^T \in \mathbb{R}^m$. Rewriting the squared norm of (3.23) as summation and exploiting the properties of exp function making summation into multiplications and changing squared of Euclidean norm $\|.\|^2$ to the absolute distance $|.|^2$, we yield another realization of Bayesian decision function given by a radial basis function network as

$$f\{\, r(k) \,\} = \sum_{i=1}^{n_s} p_i \left\{ \prod_{l=0}^{m-1} \exp\left(-\frac{|r(k-l)-c_{il}|^2}{2\sigma_e^2} \right) \right\} \tag{3.24}$$

The products of exponential functions associated with particular channel states are linearly combined to provide the decision function. Both of these functions require the knowledge of channel states for estimating the decision function. Channel state vectors are formed from the scalar channel states by using the predefined rules made by experts in Table-2. Scalar channel states are the noise free channel output from the possible combinations of the inputs as Table-1. c_{il} is the (l+1)th component of channel state vector c_i, corresponding to the (l+1)th component of the input vector $r(k)$.

The estimation of the Bayesian equalizer decision function given by (3.24) needs channel state calculation. Equalizer contains n_s channel states each of m dimensions for equalizer of order *m.* The number of scalar channel states for any channel is $M = 2^{n_h}$. Each of the m components of n_s channel states are taken from the set of M scalar channel states. With this understanding, the equalizer decision function can be presented as

$$f\{\ \mathrm{r(k)}\ \} = \sum_{i=1}^{n_s} p_i \left\{ \prod_{l=0}^{m-1} \phi_l{}^{ij} \right\} \tag{3.25}$$

Where $\phi_l{}^{ij}$ is a basis function of the form,

$$\phi_l{}^{ij} = \exp\left[-\frac{1}{2} \left(\frac{r(k-l) - c_l{}^{ij}}{\sigma_\eta} \right)^2 \right] \tag{3.26}$$

$\phi_l{}^{ij}$ is the basis function output generated from scalar center $c_l{}^{ij}$,corresponding to the $j + 1$ scalar center of $l + 1$ element of $r(k)$, where $0 \le 1 \le (m - 1)$ and $0 \le j \le (M - 1)$ in (3.20) corresponds to the channel states number added to convenience, l and j are sufficient to specify the parameter of the equalizer. In (3.25) computation of $\prod_{l=0}^{m-1} \phi_l{}^{ij}$ is the same as the computation of $\exp\left(-\frac{\|r(k) - c_i\|^2}{2\sigma_e{}^2} \right)$ in (3.24).

According to the fuzzy filter proposed by Patra and Mulgrew in [28], setting the membership function centers with scalar channel states, the spread parameter with the channel noise variance and generating the membership functions with (3.26), an equalizer with fuzzy filter in unnormalized form can be represented as

$$f_k\,(r(k)) = \sum_{i=1}^{n_c} \theta_i \left\{ \prod_{l=0}^{m-1} \psi_l{}^{ij} \right\} \tag{3.27}$$

Where $\psi_l{}^{ij}$ is the membership function generated from the scalar center $c_l{}^{ij}$, corresponding to the (j+1) center of the (l + 1) th input scalar by

$$\psi_i{}^j(k) = \exp\left\{ -\frac{1}{2} \left(\frac{\left| r(k-i) - C_j \right|^2}{\sigma_\eta{}^2} \right) \right\} \tag{3.28}$$

Where $1 \leq j \leq \mathrm{M}$ and $0 \leq l \leq \mathrm{m\text{-}1}$.

Here (3.27), θ is the free design parameter which will be adjusted during training process. Here n_c corresponds to all possible combinations of the membership function taking one from each input scalar and $n_c = M^m$. The membership function generated in (3.25) where ϕ is replaced by ψ . The subscript i is used for convenience and terms l, j specify all parameters of the function. The equalizer function (3.25) finds a weighted sum of the fuzzy basis functions (FBF's) given by

$$f_k\,(r(k)) = \prod_{l=0}^{m-1} \psi_l{}^j \tag{3.29}$$

From (3.27), it is noted that this fuzzy basis function (FBF) uses a singleton fuzzifier, product inference, center of gravity defuzzifier, and Gaussian membership function, and the fuzzy filter forms the linear combination of these FBF's. On observing the decision function of Bayesian equalizer with scalar channel states (3.25) taken as RBF in whole discussion, and fuzzy equalizer (3.27) it can be seen that RBF using scalar states has $n_s = 2^{n_h + m - 1}$ basis functions and fuzzy equalizer has $N_c = M^m = \left(2^{n_c}\right)^m$ basis functions. The number of basis functions in (3.25) is a subset of the basis functions in the fuzzy equalizer (3.27) with positioning the centers of the basis functions in (3.26) and the membership function centers (3.28) at same points and the center spread parameters are uniformly set to $\sigma_\eta{}^2$. That means $(N_c - n_s)$ rules are neglected to provide optimal performance. These n_s rules can be extracted with the knowledge of the combination of the scalar channel states forming the channel states. With this, the weights corresponding to n_s terms of the fuzzy filter can be assigned +1/-1 depending on the values of p_i in (3.25). Hence fuzzy equalizer can also be represented by (3.25) where only n_s FBFs out of possible N_c functions are used.

3.4.3 Fuzzy equalizer structure

The structure of the fuzzy equalizer is presented in Figure 3.3. In this fuzzy equalizer structure each input to the equalizer is first passed through a Gaussian membership function generator which gives the output $\psi_l{}^j$ by positioning its centers $C_l{}^j$ at the scalar channel states. Here *j* represents the fuzzy center at the scalar channel states. *l* corresponds to the equalizer input number.

The membership functions from $r(k-i)$, $1 \le i \le m-1$ are generated by passing the membership functions from *r*(*k*) through tapped delay line (TDL). The membership functions are to be generated from each of the received scalar and the equalizer input vector is formed from the time delayed samples of the received scalar.

With this the membership function for input scalar $r(k-1)$ will be the delayed membership functions for input *r*(*k*). This can be represented as

$$\psi_l{}^j(k) = \psi_{l-1}{}^j(k-1) \tag{3.30}$$

Where $1 \le l \le (m-1)$ and $0 \le j \le M-1$.

The inference block of the equalizer has n_s units. Each of these units receives only one $\psi_l^{\,j}$ from each of the *m* inputs to the equalizer, and the combination of these is decided by the combination of the scalar channel states constituting the channel states. The output of the inference units are suitably weighted and added to provide *a* and *b* which provide the function of the defuzzifier. The output of the equalizer is computed by the equalizer function (3.25) which is $(a-b)/(\mathrm{a}+\mathrm{b})$.

Example 3.1

Let the channel considered here is having transfer function $H(z)=0.5+1.0z^{-1}$ and the length of equalizer $m = 2$ and $d = 0$ and SNR = 10 dB. This provides $n_s = 8$ channel states and $M = 4$ scalar channel states. The channel states for this equalizer have been presented in Table-2. The *m*- directional n_s channel states composed of the components of *M* scalar channel states. The weights w_i of the equalizer decision function are +1 for c_1, c_2, c_3, c_4 and -1 for c_5, c_6, c_7, c_8.

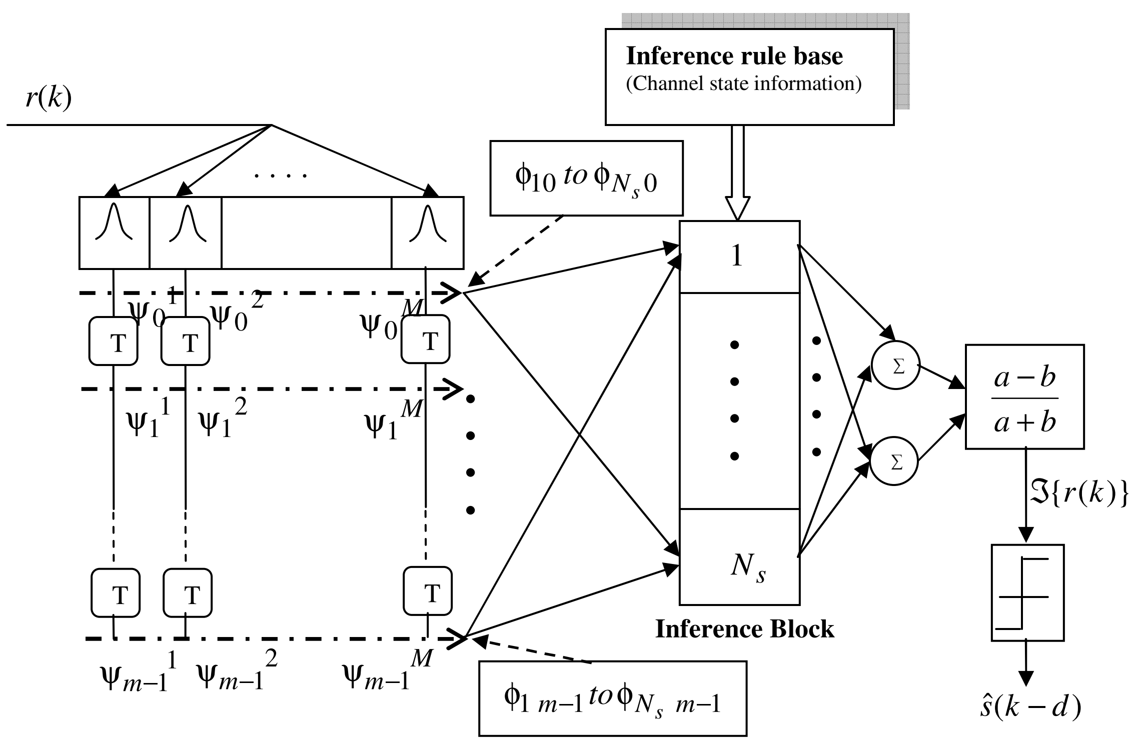


Figure 3.3: Structure of fuzzy implemented Bayesian equalizer

For fuzzy implementation the centers of the membership functions are positioned at scalar channel states +1.5, -0.5, 0.5 and -1.5. The membership functions $\psi_1{}^1, \psi_1{}^2, \psi_1{}^3$ and $\psi_1{}^4$ corresponding to $r(k-1)$, are delayed samples of $\psi_0{}^1, \psi_0{}^2, \psi_0{}^3$ and $\psi_0{}^4$ corresponding to $r(k)$. The inference block consists of $n_s = 8$ fuzzy IF … THEN rules. Here $\phi_{10} = \phi_{20} = \psi_0{}^1$, $\phi_{30} = \phi_{40} = \psi_0{}^2$, $\phi_{50} = \phi_{60} = \psi_0{}^3$, $\phi_{70} = \phi_{80} = \psi_0{}^4$ and $\phi_{11} = \phi_{51} = \psi_1{}^1$, $\phi_{21} = \phi_{61} = \psi_1{}^2$, $\phi_{31} = \phi_{71} = \psi_1{}^3, \phi_{41} = \phi_{81} = \psi_1{}^4$.These four product terms like, $\phi_{10}\phi_{11}, \phi_{20}\phi_{21}, \phi_{30}\phi_{31}, \phi_{40}\phi_{41}$ constitute the rules for $C_d{}^+$, are added to provide a, and $\phi_{50}\phi_{51}, \phi_{60}\phi_{61}, \phi_{70}\phi_{71}, \phi_{80}\phi_{81}$ constitute the rules for $C_d{}^-$, are added to provide b. The calculation of decision function is straight forward.

The fuzzy equalizer developed here uses an FBF with product inference and COG defuzzifier. Owing to the close relationship of this equalizer with the Bayesian equalizer, this equalizer can also be implemented with an RBF with scalar centers [39]. However, use of a fuzzy system to implement this equalizer provides the possibility of using other forms of inference rules and defuzzification processes. This can provide some of the alternate forms of fuzzy implementation of the Bayesian equalizer, which are discussed below..

1) Inference Rule: The fuzzy equalizer discussed above works with product inference which is described as RBF in this thesis. The output of each of the n_s inference rules are generated with the product rule. It is also seen from the membership function generator (3.28) that the membership for any input is $0 < \psi_l{}^j \leq 1$. Hence the output of any of the inference rules will be in the range of (0, 1] and will always be less than the smallest membership input to the rule. For this reason the product inference rule can be approximated by minimum inference rule. With this modification the equalizer decision function would be

$$\Im\{r(k)\} = \sum_{i=1}^{n_s} \left\{ \min_{l=0}^{m-1} \phi_l^{ij} \right\} \tag{3.31}$$

Where $\min_{l=0}^{m-1}$ selects the minimum of the inputs to each of the components of the inference block. With this, the computation of the products has been replaced by comparisons which are easy to implement in hardware. This minimum inference fuzzy equalizer is termed as fuzzy in this thesis.

2) Defuzzification Process*:* The output layer of the fuzzy equalizer [see (3.25) and (3.27)] finds a weighted sum of the inference rules and normalizes this with the inference output. The weights associated with the inference rules are +1/-1 . It is seen that the rule nearest to the input vector would provide the maximum output, and the contribution from the remaining rule will be minimal. These characteristics of the decision function can be utilized by replacing the COG defuzzifier with a maximum defuzzifier. This defuzzifier can be combined either with product inference or with the minimum inference. With this, the equalizer decision function with product inference can be represented as,

$$\Im\{r(k)\} = p_{\max} \max_{i=1}^{n_s} \left\{ \prod_{l=0}^{m-1} \phi_l^{ij} \right\} \tag{3.32}$$

and with minimum inference

$$\Im\{r(k)\} = p_{\max} \max_{i=1}^{n_s} \left\{ \min_{l=0}^{m-1} \phi_l^{ij} \right\} \tag{3.33}$$

Here $\max_{i=1}^{n_s}$ corresponds to the maximum of the available n_s inferences and $p_{\max}$ is the weight associated with the maximum inference. With this decision function, (3.27) and (3.33) use maximum defuzzification, where the output of the equalizer is based on the maximum of the n_s inference rules and the weight associated with it. The equalizer

(3.32) uses product inference where as (3.33) uses the minimum inference rule. In both of these defuzzification processes, computation of weighted sum of the inference is replaced by a comparison operation.

With the above analysis, a variety of fuzzy equalizers to approximate the Bayesian decision function can be designed. These equalizers can provide alternative equalizer architectures with a reduction in computational complexity. In this thesis, we use minimum inference and center of gravity (COG) defuzzifier.

3.5 Advantages of fuzzy equalizer

The fuzzy implementation of Bayesian equalizer provides the Bayesian equalizer decision function. Major advantages of such equalizer over the RBF implementation of Bayesian equalizer are lower computational complexity.

Computational Complexity:

After training is complete, the equalizer parameters are fixed and the actual detection of transmitted symbols starts. The computational requirements of a fuzzy equalizer of product inference and centroid defuzzification (Normalized Bayesian equalizer with scalar states (NBESS) noted in table 3.3 below) and RBF implemented Bayesian equalizer are the same. The computations required for estimating each of the samples with the Bayesian equalizer and its RBF implementation, fuzzy equalizer with product inference and the fuzzy equalizer of minimum inference and Centroid defuzzification are listed in Table-3.3. The second part of the table provides the typical computational requirements for an equalizer with $m = 4$, $n_c = 3$ and $M = 8$. From this table, the following points can be arrived at with regards to the computational advantages of fuzzy implementation of Bayesian equalizer.

• Fuzzy implementation of the Bayesian equalizer provides a significant reduction in addition, division and $\exp(x)$ evaluations.

• The time shift property of the membership function generation provides a considerable reduction in evaluation of functions and division.

• Evaluation of exp and division functions in a Bayesian equalizer are related to N_s which in turn is exponentially related to the sum of the equalizer and channel order but in the fuzzy equalizer it is related to M which is exponentially related to the equalizer order only. Hence, with the increase in the equalizer order the reduction in computational complexity for fuzzy equalizer over the Bayesian equalizer is exponentially related.

Equalizer Type	Add / Sub	Mul	Div	e^{-x}
Bayesian (RBF)	$2mN_s$	mN_s	N_s	N_s
NBESS	$M + N_s$	$M + mN_s$	$M + 1$	M
Fuzzy	$M + N_s$	$M + N_s$	$M + 1$	M
Bayesian (RBF)	512	256	64	64
NBESS	72	264	9	8
Fuzzy	72	72	9	8

Table – 3.3: Computational complexity comparisons for Bayesian equalizers, NBESS and the fuzzy equalizers

3.6 Fuzzy equalizer training

As described above the fuzzy equalizers are developed with the knowledge of channel states which describes the centers of fuzzy membership functions and the weights which are free parameters to be updated timely. These equalizer parameters can be estimated during the transmission of known training bits and after training the equalizer can use its previous decisions in decision directed mode to update its parameter. In mobile communication system the channel is no longer constant due to fading discussed in previous chapter, which needs channel estimation during training period. The process of estimating these parameters are described below.

3.6.1 Channel state estimation

The estimation of decision function needs the knowledge of channel states which depends upon the channel information but it is not known beforehand, so the channel states can be estimated during training period is described below.

❖ Considering small training sequence the channel model can be identified from the channel outputs available and the known training input information. First the channel is estimated and from the knowledge of channel the states are estimated. The channel estimator uses RLS adaptive algorithm to identify the channel transfer function iteratively during training. All training should be completed in 26 samples as GSM specification. With this knowledge of channel, it is straight-forward to find the scalar channel states taking all possible combinations of channel input as in Table-3.1 and the channel states are estimated by the combination of these scalar states as shown in Table-3.2. This technique is valid when the channel transfer function is linear.

❖ The scalar channel states can also be computed with scalar supervised clustering where scalar states in conjunction with the training signal can provide the scalar states combinations that form the channel states [43]. This process has been presented in section 3.6.1. The number of scalar channel states depends only on the channel order and hence requires a small length of training sequence compared to vector channel state estimation. The scalar channel states always occur in pairs so that $C_j = C_{M-j+1}$, $1 \le j \le M$. This further would require only estimation of $\frac{M}{2} = 2^{n_c - 1}$ scalar states. This has been seen in [23] that within around 30 iterations for a three tap channel, the estimated scalar channel states converge to the desired state. However in this chapter the first method has been adopted.

3.6.2 Equalizer weight update

Once scalar channel states have been estimated, fuzzy rules can be formed. The equalizer is constructed with the weights of inference rules assigned +1/ -1, depending on whether rule belongs to positive or negative channel states. Estimating the channel states and the noise statistics can involve some error. To compensate for these effects, the weights associated with the rules can be fine tuned with the LMS algorithm. This step would require only few samples as the initial weight assignment is very close to the final values. This process would not require additional training overhead, since the training signal used to estimate the channel states are reused for equalizer weight training. During the process of detection of samples the channel varies due to fading in a mobile communication system. To compensate for this effect the channel states are continuously modified in a decision directed mode using the estimated samples. The states are updated using LMS algorithm. This provides the equalizer the capability to track the channel variation due to movement of the mobile w.r.to. the base station.

3.6.3 Results and Discussion:

In the section 3.4 the development of fuzzy equalizer and its structure was described followed by its advantage and its training. The actual performance of equalizers was evaluated by computer simulation. During the simulation Bit Error Rate (BER) was used as the performance index. This section presents the BER performance of fuzzy equalizers for a variety of parameters. The BER performance of equalizers was computed using Monte-Carlo simulation. During the process 10^5 bits of data were transmitted and BER observed for a variety of AWGN. The BER Vs SNR at receiver input were plotted for performance analysis.

All simulations were conducted using MATLAB with Windows Xp operating system on Intel P-IV @ 3.8GHZ HTT processor and 256MB RAM. Uniform random sequences were generated and transmitted through the channel. The channels were affected the ISI, AWGN along with Rayleigh fading. Output of the channel was fed to the equalizer and the detected samples at the equalizer were compared with suitable transmitted sample for BER evaluation. In all cases the data frame as specified by the GSM was used. Only 26 training bits were transmitted and following this 114 information bits were transmitted. This set of 140 bits set considered at the receiver as a frame. During simulation 715 frames were transmitted to allow the transmission of 10^5 numbers of bits.

NOTE: In all the simulation plots, we denote RBF as fuzzy implementation of Bayesian equalizer with product inference, fuzzy with the minimum inference and Bayesian equalizer as Bayesian equalizer implemented using RBF.

First the BER performance of a fuzzy receiver was compared with equalizer based on RBF (fuzzy implementation of Bayesian equalizer with product inference), Bayesian equalizer (RBF implementation of Bayesian equalizer) and with the linear equalizers trained by RLS and LMS algorithms for SNR = 2.5dB to 20dB, using Monte Carlo simulations. Here the performance of all the equalizers were evaluated under same conditions. The channel of transfer function considered was $H(z) = 0.5 + 1.0z^{-1}$ and Rayleigh fading simulator was used at carrier frequency of 2GHz and mobile speed 13.5 km/hr. Transmitted data rate was conducted at 270.8Kb/sec. The channel is a non-minimum phase channel. This result was drawn for GSM environment where 26 training data were taken and 114 data involved in tracking period during which the error performance of the equalizers were evaluated. During the training period of all the equalizers the scalar channel states were estimated and the channel states were calculated. The equalizer of order 2 was taken and propagation detection delay of one symbol was considered. From the simulation result, it is observed that Bayesian equalizer shown the optimal performance and the fuzzy implementation of Bayesian equalizer with product inference which is also fuzzy implemented RBF has shown performance close to the optimal and followed by fuzzy equalizer with reduced computations.

It is observed that the linear equalizers using RLS and LMS algorithms fail to provide desirable results for GSM environment with such low training symbols in fading condition. During the training period the weights of these equalizers were adjusted adaptively by RLS and LMS algorithms by taking errors form the comparison of actual output of equalizer with the desired one and then allowed to freeze during the tracking period where errors were calculated and counted to prove its performance. Here it is seen that LMS equalizer provides worst performance. It is well known that LMS equalizer require long training sequence [45]. The RLS equalizer performs better than LMS since the RLS equalizer is able to adjust its parameter very quickly. The Bayesian and RBF equalizers provide the best performance since they provide optimum decision. The reduced complexity fuzzy equalizer suffers from slight performance degradation compared to optimum equalizer. However the performance of RBF, Bayesian and fuzzy equalizers are nearly similar. These nonlinear equalizers provide nearly 4dB performance gain over RLS equalizer and 7dB compared to LMS equalizer at BER of 10^{-3}.

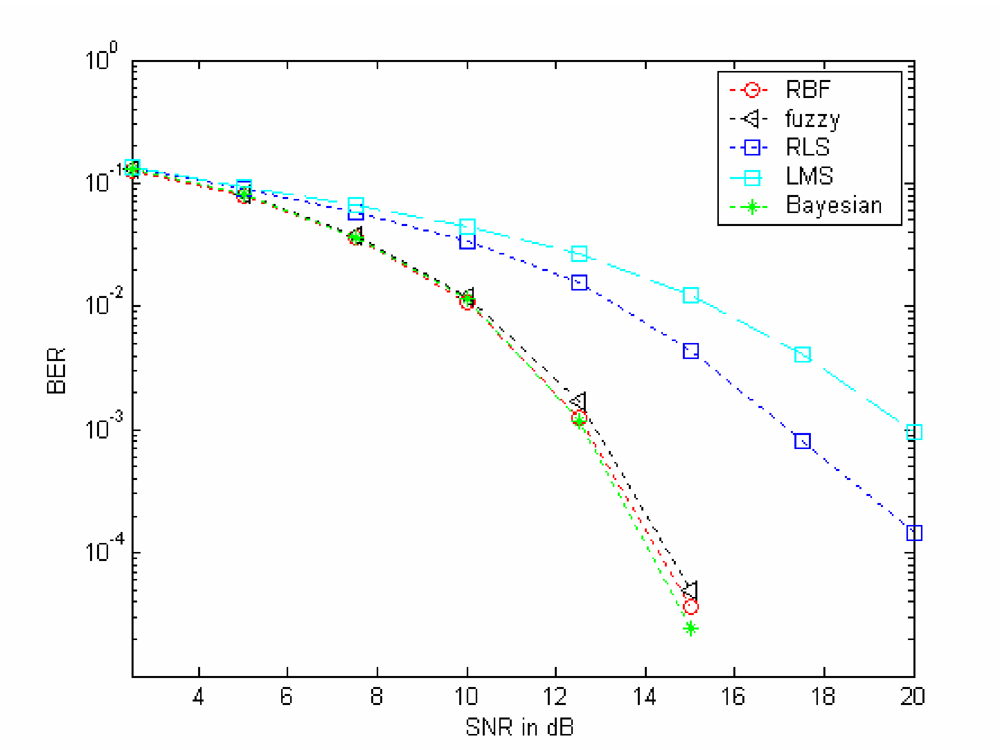


Figure 3.4: BER Vs SNR plot for channel $H(z) = 0.5 + 1.0z^{-1}$ ***and m = 2, d = 1***

In order to investigate further simulation were conducted for another channel having transfer function $H(z) = 0.3482 + 0.8704\,z^{-1} + 0.3482\,z^{-2}$ where 3 multipaths were taken into account and simulation was done under same condition as before. Three independent Rayleigh fading simulators were implemented due to the presence of three multipaths. The equalizer order here taken as m = 2 and propagation detection delay of one symbol was considered. Figure 3.5 presents the BER performance for the equalizer.

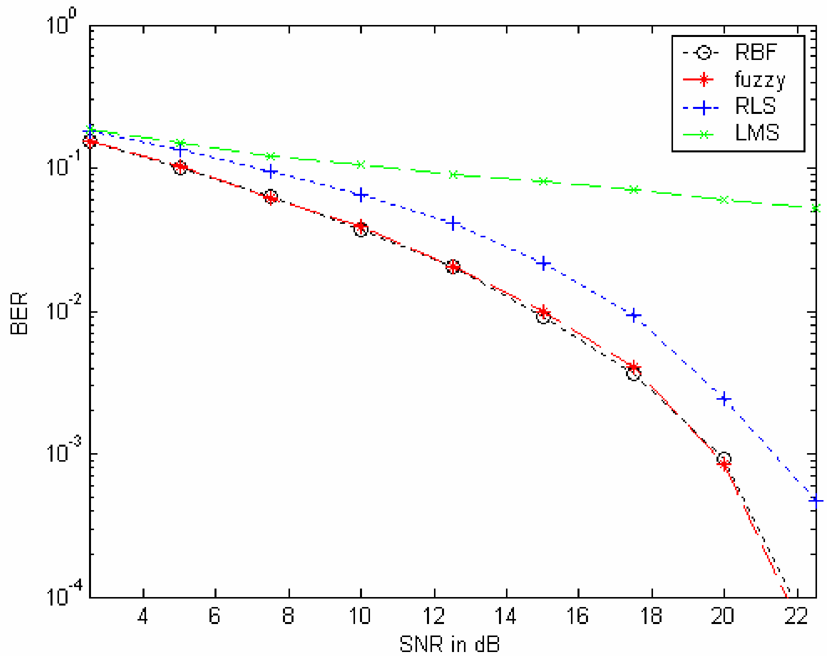


Figure 3.5: BER Vs SNR plot for channel $H(z) = 0.3482 + 0.8704\, z^{-1} + 0.3482\, z^{-2}$ ***and m = 2, d = 1***

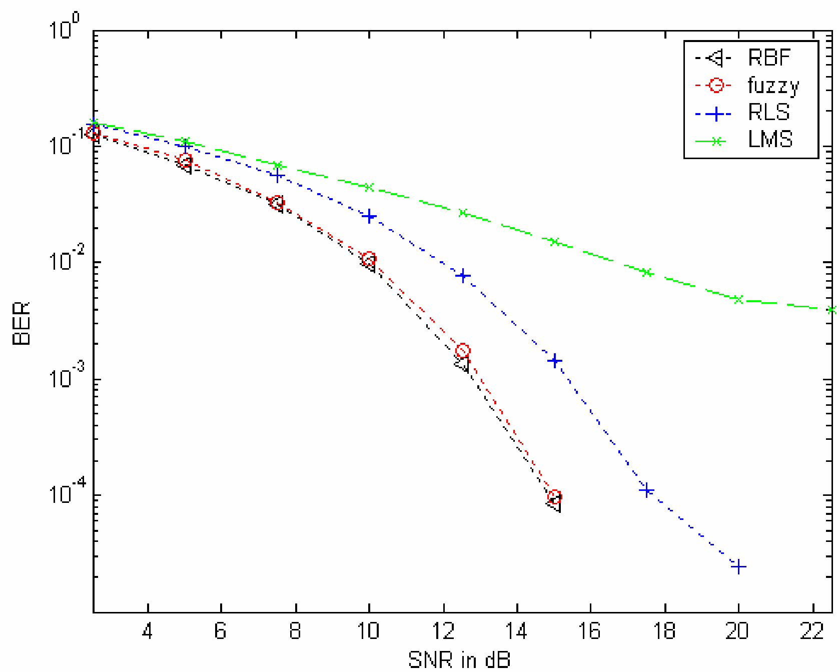


Figure 3.6: BER Vs SNR plot for channel $H(z) = 0.2682 + 0.9296\, z^{-1} + 0.2682\, z^{-2}$ ***and m = 3, d = 1***

The following points can be observed from the BER curves for different equalizer in Figure 3.5. For this channel the LMS equalizer performance never goes better than 10^{-2}. RLS performs better than LMS but has a 1.5dB performance degredation at 10^{-3} BER. It is again observed that the fuzzy equalizer has shown close performance to the Bayesian equalizer. Similarly Figure 3.6 presents BER performance for a channel having transfer function of $H(z) = 0.2682 + 0.9296\, z^{-1} + 0.2682\, z^{-2}$.

3.7 Type-2 fuzzy adaptive filter [29]

Quite often, the information to be processed by a FAF is uncertain due to uncertain linguistic knowledge and uncertain numerical values. In IF… THEN rules concerning fuzzy concepts such as *slowly time varying, moderately time varying,* or *rapidly time varying,* experts may not agree on how to represent these linguistic labels using fuzzy membership functions. So information coming from the experts contains linguistic uncertainty. As in mobile communication the mappings between input and output data pairs are uncertain due to the channel dynamics, this numerical data uncertainty causes type-1 fuzzy adaptive filter (FAF) which is discussed before, i.e. conventional fuzzy equalizer and other nonlinear filters to perform poorly. Hence Mendel proposed a new FAF [29] which is based on the type-2 fuzzy sets introduced by Zadeh [30] in 1975 which is an extension of the concept of an ordinary fuzzy set i.e. type-1 fuzzy set.

The type-2 fuzzy sets have grades of membership that are themselves fuzzy. A type-2 membership grade can be any subset in [0,1] .it has primary membership function and corresponding to each primary membership, there is secondary membership which can also be in [0, 1] that defines the possibilities for the primary membership.

Karnik and Mendel [46, 47] established a complete type-2 fuzzy logic system (FLS) theory to handle linguistic and numerical uncertainties. A type-2 FLS includes fuzzifier, rule base, fuzzy inference engine, and output processor. The output processor includes a type-reducer and a defuzzifier; it generates a type-1 fuzzy set output from the type reducer and a crisp number from the defuzzifier.A type-2 FLS (just as a type-1 FLS) is characterized by IF-THEN rules, but its antecedent or consequent sets are now type-2. General type-2 FLSs is computationally intensive because type-reduction is very intensive. Things simplify a lot when secondary membership functions (MFs) are interval sets (in this case, the secondary memberships are either zero or one). Mendel proposed type-2 FAF, which is an unnormalized output interval type-2 FLS and applied it to equalization of time-varying channels

3.7.1 Structure of a type-2 fuzzy logic system

The structure of a type-2 FLS is shown in Figure 3.7. It is very similar to the structure of a type-1 FLS. For a type-1 FLS, the *output processing* block only contains the defuzzifier. Here we focus only on the similarities and differences between the two FLS's. All blocks of this structure are described in [40].

The fuzzifier maps the crisp input into a fuzzy set. This fuzzy set can, in general, be a type-2 set. In the general type-1 case, we generally have "IF-THEN" rules, where the l th rule has the form

$$R^{(l)} : \text{IF} \quad x_1 \text{ is } F_1^{\,l} \text{ and } \dots \text{ and } x_n \text{ is } F_n^{\,l} \quad \text{THEN} \quad y \text{ is } G^l$$

Where x_i s are inputs; $F_i^{\,l}$ s are antecedent sets ($i = 1,2,\ldots,n$); y is the output and G^l s are consequent sets. The distinction between type-1 and type-2 is associated with the nature of the membership functions, which is not important while forming rules; hence the structure of the rules remains exactly the same in the type-2 case, the only difference being that now some or all of the sets involved are of type-2; so, the l th rule in a type-2 FLS has the form

$$R^{(l)} : \text{IF} \quad x_1 \text{ is } \tilde{F}_1^{\,l} \text{ and } \dots \text{ and } x_n \text{ is } \tilde{F}_n^{\,l} \quad \text{THEN} \quad y \text{ is } \tilde{G}^l .$$

It is not necessary that all the antecedents and the consequent be type-2 fuzzy sets. As long as one antecedent or the consequent set is type-2, we will have a type-2 FLS.

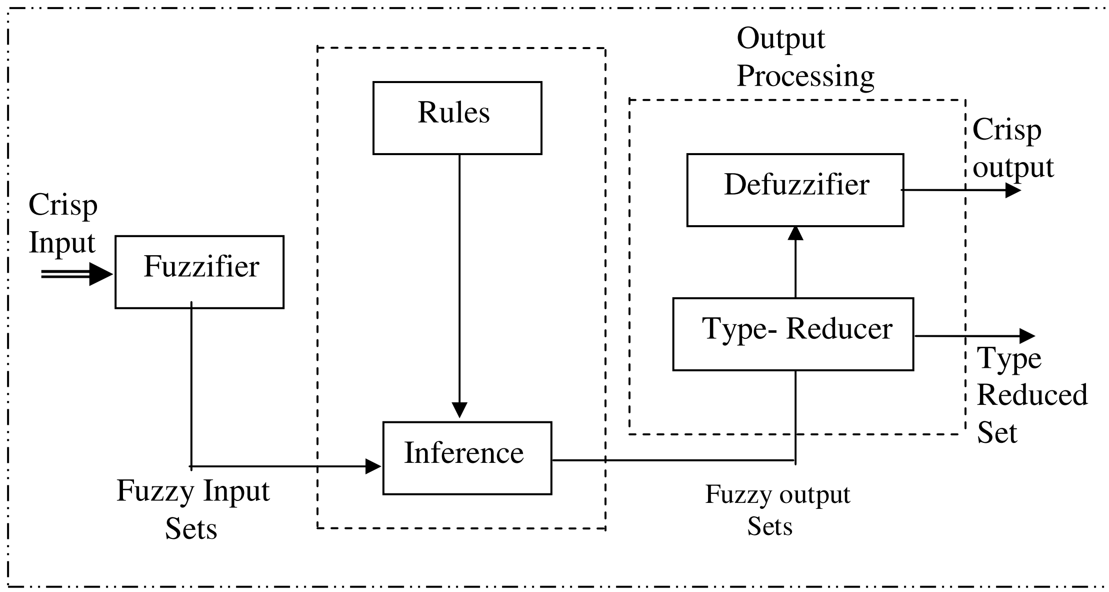


Figure 3.7 Structure of a type-2 FLS.

In a type-1 FLS, the inference engine combines rules and gives a mapping from input type-1 fuzzy sets to output type-1 fuzzy sets. Multiple rules may be combined using the union of sets operation or during defuzzification by weighted summation. In the type-2 case, the inference process is very similar. The inference engine combines rules and gives a mapping from input type-2 fuzzy sets to output type-2 fuzzy sets. To do this one needs to find unions and intersections of type-2 sets, as well as compositions of type-2 relations.

In a type-1 FLS, the defuzzifier produces a crisp output from the fuzzy set that is the output of the inference engine. In the type-2 case, the output of the inference engine is a type-2 set; so we use "extended versions" (using Zadeh's extension principle [30]) of type-1 defuzzification methods. This extended defuzzification gives a type-1 fuzzy set. Since this operation takes us from the type-2 output sets of the FLS to a type-1 set, we call this operation "type reduction" and the type-reduced set so obtained a "type-reduced set." Different types of type reduction processes are described in [47]. To obtain a crisp output from a type-2 FLS, we can defuzzify the type-reduced set. The most natural way of doing this seems to be by finding the centroid of the type-reduced set [47], however, there exist other possibilities like choosing the highest membership point in the type-reduced set. In the whole discussion of type-2 fuzzy the following notations will be used. A is a type-1 fuzzy set and the membership grade of

$x \in X$ in A is $\mu_A(x)$, which is a crisp number in [0, 1]. A type -2 fuzzy set in X is $\tilde{A}$ and the membership grade of $x \in X$ in $\tilde{A}$ is $\mu_{\tilde{A}}(x)$, which is a type-1 fuzzy set in [0, 1]. The elements of the domain of $\mu_{\tilde{A}}(x)$ are called primary memberships if x in $\tilde{A}$ and the memberships of the primary memberships in $\mu_{\tilde{A}}(x)$ are called secondary memberships of x in $\tilde{A}$.

3.7.2 Background and Decision function of Type-2 FAF

Type-2 FAF developed based on an unnormalized output interval type -1 FLS with applying the extension principle to the later.

A. Unnormalized output of Type-1 FLS

From previous discussion on Type-1 fuzzy system in section 3.4, we find that for a given input of $(x_1, x_2, \cdots, x_p)$, the final output of unnormalized first order type-1 FLS model is inferred as

$$y = \sum_{i=1}^{M} f^i y^i \tag{3.34}$$

Where f^i are rule firing strengths defined as

$$f^i = \bigcap_{k=1}^{p} \mu_{F_k{}^i}(x_k) \tag{3.35}$$

and "$\bigcap$" denotes a t-norm. When Gaussian membership functions and product t-norms are used, i.e.

$$\mu_{F_k^i}(x_k) = \exp\left[-\frac{1}{2}\left(\frac{x_k - m_k^i}{\sigma_k^i}\right)^2\right] \qquad (3.36)$$

Then (3.34) can be expressed as

$$y = \sum_{i=1}^{M} y_i \prod_{k=1}^{p} \exp\left[-\frac{1}{2}\left(\frac{x_k - m_k^i}{\sigma_k^i}\right)^2\right] \qquad (3.37)$$

Observe that (3.37) is identical to the output formula for a radial basis function network (3.24) when Gaussian membership function is used.

B. Extension principle

The extension principle allows the domain of a mapping or relation to be extended from points in the universe of discourse *U* to fuzzy subsets of *U*. when we need to extend the individual operation of the form $f(\theta_1, \cdots, \theta_n)$ to an operation $f(A_1, \cdots, A_n)$, we will not extend the individual operations, like multiplication, addition, etc., involved in *f*; rather we will use the following definition (3.38)

$$f(A_1, \cdots, A_n) = \int_{\theta_1} \cdots \int_{\theta_n} \mu_{A_1}(\theta_1) * \cdots * \mu_{A_n}(\theta_n) / f(\theta_1, \cdots, \theta_n) \qquad (3.38)$$

Where $\theta_i \in A_i$ for i= 1,2, … , n, and * denotes a *t*-norm .

It has been observed from the two theorems given in [29] that the meet and addition operations for interval sets are determined by two end points of each interval set. In a Type-2 FAF, the two end points are associated with type-1 MFs to which we refer as *upper* and *lower MFs*. Upper and lower MFs are two type-1 MFs, which are bounds for the footprint of uncertainty of an interval type-2 MF. The upper MF is a subset that has the maximum membership grade of the footprint of uncertainty and the lower MF is a subset which has the minimum membership grade of the footprint of uncertainty. We use the overbar (underbar) to denote the upper (lower) MF in this thesis.

For Gaussian primary MF with uncertain mean:

Consider the case of a Gaussian MF having a fixed standard deviation $\sigma_k{}^l$ and an uncertain mean that takes on values in $\left[m_{k1}{}^l, m_{k2}{}^l\right]$, i.e.

$$\mu_k{}^l(x_k) = \exp\left[-\frac{1}{2}\left(\frac{x_k - m_k{}^l}{\sigma_k{}^l}\right)^2\right] \tag{3.39}$$

Where $m_k{}^l \in \left[m_{k1}{}^l, m_{k2}{}^l\right]$ and $k =$ 1,2, … , p ; p is the number of antecedents l = 1, … , M and M is the number of rules. The upper MF,

$$\bar{\mu}_k{}^l(x_k) = \begin{cases} \mathrm{N}(m_{k1}{}^l, \sigma_k{}^l; x_k) & x_k < m_{k1}{}^l \\ 1 & m_{k1}{}^l \le x_k \le m_{k2}{}^l \\ \mathrm{N}(m_{k2}{}^l, \sigma_k{}^l; x_k) & x_k > m_{k2}{}^l \end{cases} \tag{3.40}$$

Where $\mathrm{N}(m_{k1}{}^l, \sigma_k{}^l; x_k) \cong \exp\left(-\frac{1}{2}\left(\frac{x_k - m_{k1}{}^l}{\sigma_k{}^l}\right)^2\right)$ and lower MF is

$$\underline{\mu}_k{}^l(x_k) = \begin{cases} \mathrm{N}(m_{k2}{}^l, \sigma_k{}^l; x_k) & x_k \le \left(m_{k1}{}^l + m_{k2}{}^l\right)/2 \\ \mathrm{N}(m_{k1}{}^l, \sigma_k{}^l; x_k) & x_k > \left(m_{k1}{}^l + m_{k2}{}^l\right)/2 \end{cases} \tag{3.41}$$

Output of a type-2 FAF:

In a type-2 FAF with a rule base of M rules, where each rule has p antecedents, the ith rule R^i is denoted as

$$R^i: \text{ IF } \; x_1 \text{ is } \tilde{F}_1{}^i \text{ and } \ldots \text{ and } x_p \text{ is } \tilde{F}_p{}^i \text{ THEN} \tag{3.42}$$

$$y^i = c_0{}^i + c_1{}^i x_1 + c_2{}^i x_2 + \cdots + c_p{}^i x_p$$

Where $i = 1, 2, \cdots, M$; $c_j{}^i \, (j = 0, 1, \cdots, p)$ are the consequent parameters that are crisp numbers; y^i is an output from the ith IF … THEN rule, which is a crisp number and the $\tilde{F}_k{}^i \; (k = 1, 2, \; \cdots, p)$ are type-2 fuzzy sets. Given an input,
$X = \left[x_1, x_2, \cdots, x_p\right]^T$, the firing strength of the *i*th rule is

$$F^i = \mu_{\tilde{F}_1{}^i}(x_1) \cap \mu_{\tilde{F}_2{}^i}(x_2) \cap \; \cdots \; \cap \mu_{\tilde{F}_p{}^i}\left(x_p\right) \qquad (3.43)$$

When interval type-2 sets are used in the antecedents, which means
$\mu_{\tilde{F}_K{}^i}(x_k)$ (k = 1, 2, … , p) is an interval set, and we denote

$$\mu_{\tilde{F}_K{}^i}(x_k) = \left[\underline{\mu}_{\tilde{F}_k{}^i}(x_k) \; , \; \overline{\mu}_{\tilde{F}_k{}^i}(x_k)\right] \cong \left[\underline{f}_k{}^i \; , \; \overline{f}_k{}^i\right] \qquad (3.44)$$

The type-2 FAF is then computed using the following results

1) In an interval type-2 FAF with meet under minimum or product t-norm, the firing strength in (3.43) for rule R^i is an interval set,

$$F^i = \left[\; \underline{f}^i \, , \overline{f^i} \;\right] \text{, where } (i = 1, 2 \; \ldots, M)$$

$$\underline{f^i} = \underline{\mu}_{\tilde{F}_1{}^i}(x_1) * \; \cdots \; * \; \underline{\mu}_{\tilde{F}_p{}^i}\left(x_p\right) = \bigcap_{k=1}^{p} \underline{f}_k{}^i \qquad (3.45)$$

and

$$\overline{f^{i}} = \bar{\mu}_{\tilde{F}_1^{i}}(x_1) * \cdots * \bar{\mu}_{\tilde{F}_p^{i}}(x_p) = \bigcap_{k=1}^{p} \bar{f}_k^{\,i} \qquad (3.46)$$

1) The extended weighted average $Y\left(F^1, \cdots, F^M\right)$ is also an interval set $\left[y_l, y_r\right]$ where

$$y_r = \sum_{i=1}^{M} \bar{f}^{i} y^{i} \qquad (3.47)$$

$$y_l = \sum_{i=1}^{M} \underline{f}^{i} y^{i} \qquad (3.48)$$

and $\quad y^{i} = c_0^{\,i} + c_1^{\,i} x_1 + c_2^{\,i} x_2 + \cdots + c_p^{\,i} x_p \qquad (3.49)$

2) The defuzzified output of type-2 FAF by using the theorems in [29] will be

$$y = \sum_{i=1}^{M} y^{i}\left(\underline{f}^{i}, \bar{f}^{i}\right) \Big/ 2 \qquad (3.50)$$

Designing the Type-2 FAF:

Let a channel transfer function considered here is $H(z) = 1.0 + 0.5z^{-1}$ where channel taps are 2 and for getting the inputs to the equalizer of order 2 i.e. $\left[\hat{r}(k), \hat{r}(k-1)\right]$, we have to take eight rules by making possible combinations of the input vectors taking $\left[s(k), s(k-1), s(k-2)\right]$ as discussed before for calculating channel states.

The output of the type-2 FAF will be computed using (3.50), where $y^{i} = w_l \ (l = 1, 2, \cdots, 8)$ Equals to +1 or -1, $\underline{f}^{\,l}$ is obtained from (3.43) and $\bar{f}^{\,l}$ is obtained from (3.46). As in (3.39)

$$\mu_{\tilde{F}_k^l}(x_k) = \exp\left[-\frac{1}{2}\left(\frac{x_k - m_k^l}{\sigma_e}\right)^2\right] \qquad (3.51)$$

and $\quad m_k^l \in \left[m_{k1}^l, m_{k2}^l\right], \quad k = 1,2.$

Below we let $m_1^l = \left[m_{11}^l, m_{21}^l\right]^T$ and $m_2^l = \left[m_{12}^l, m_{22}^l\right]^T$.

We use a clustering approach to estimate m_1^l and m_2^l. Suppose the number of training prototypes, $(s(k), r(k))$, is *N*. As $s(k)$ determines which cluster $r(k)$ belongs to; so the $N\, r(k)$ are classified into $n_s = 2^{n+p}$ clusters. Where in our example, $2^{p+n} = 2^{2+1} = 8$. Suppose N_l training prototypes belongs to the *l*th cluster l = 1,…,8 and the mean and standard deviation of these $r(k)$, $k = 1, \cdots, N_l$ are denoted $m_r^l\,(2\times 1)$ vector and $\sigma_r^l\,(2\times 1)$, respectively.

We let
$$m_1^l \cong m_r^l - \sigma_r^l \qquad (3.52)$$

$$m_2^l \cong m_r^l + \sigma_r^l \qquad (3.53)$$

Where l = 1, 2,…,8. Doing this assume that each cluster is centered at m_r^l. Consequently, $\left[m_{11}^l, m_{12}^l\right]$ is the range of the mean of the type-2 antecedent
Gaussian MF $\mu_{\tilde{F}_1^l}$ and $\left[m_{21}^l, m_{22}^l\right]$ is the range of the mean of $\mu_{\tilde{F}_2^l}$.

To complete the specification of the MFs in (3.51), we also need to estimate the standard deviation of noise (std) σ_e. By fixing signal to noise ratio (SNR) value, the std σ_r of $r(k)$ in the combined training and testing sequence. Then based on the fact that

$$\mathrm{SNR} = 10 \log_{10} \frac{\sigma_r^{\,2}}{\sigma_e^{\,2}} \tag{3.54}$$

We computed σ_e as

$$\sigma_e = \sigma_r \big/ 10^{SNR/20} \tag{3.55}$$

Here instead of going for the channel state calculation for fuzzy equalizer, we go for the mean and std of the clusters during training period of data and fixes the range of mean $\left[m_{k1}{}^{l} \,, \, m_{k2}{}^{l} \right]$ during the period of free transmission or tracking period.

3.8 Results and Discussions:

Mendel proposed Type-2 fuzzy adaptive filter is described in section 3.7 elaborately which can handles numerical and linguistic uncertaties. It is applied for linear and time varying channel and performance compared with the RBF noted above and fuzzy equalizers along with the linear equalizers. During the training period of Type-2 FAF, the mean and variances of channel state clusters which is due to the time varying nature of the channel were calculated in stead of the channel states in case of fuzzy and RBF implementation of Bayesian equalizer. Then during the tracking period these means are used as the centers of Gaussian *primary* and *secondary membership functions* which form the bounds of the footprint of uncertainties of Type-2 fuzzy adaptive filter.

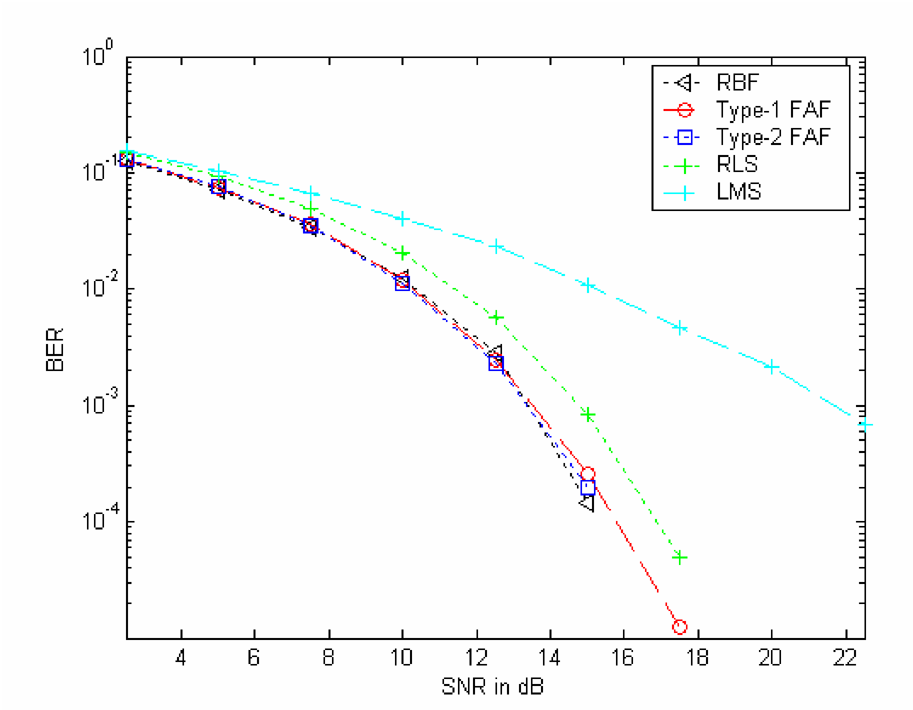


Figure 3.8: BER Vs SNR plots for a channel of transfer function

$$H(z) = 0.2682 + 0.9296\,z^{-1} + 0.2682\,z^{-2}, \; m = 2, \; d = 1$$

In Figure 3.8, the BER performance of RBF, Type-1 and Type-2 FAF are compared along with the linear equalizers using LMS and RLS algorithms for channel having transfer f unction of $H(z) = 0.2682 + 0.9296\,z^{-1} + 0.2682\,z^{-2}$.

For this simulation mobile channel having three multipaths were considered where a second order equalizer and the delay of one symbol was taken. In this simulation the Type-2 FAF is trained for a longer data than that of the other to give the result close to the optimal RBF.

The studies is again conducted for Channel $H(z) = 0.5 + 1.0\,z^{-1}$ with equalizer order m = 2 and one symbol delay.

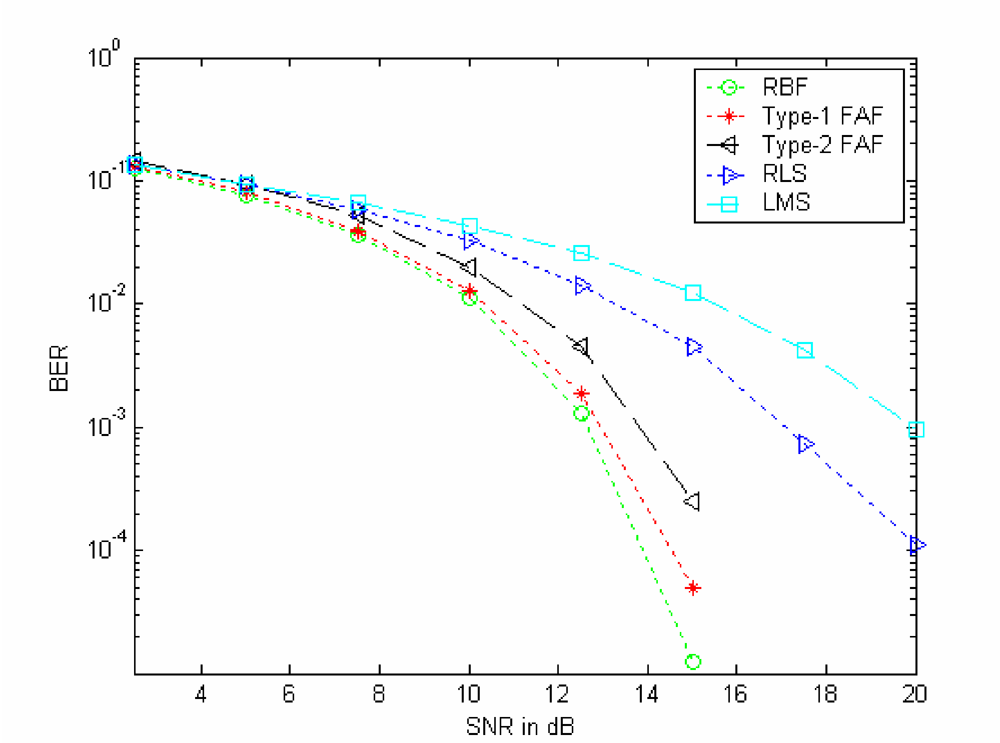


Figure 3.9: BER Vs SNR plots for a channel of transfer function

$H(z) = 0.5 + 1.0\, z^{-1}$, ***m = 2, d = 1***

For this simulation all the equalizers are trained for same number according to the GSM specification. In this Figure 3.9, we observe that the Type-1 FAF proposed in [28] provide nearly 1dB performance gain over Type-2 FAF proposed by Mendel [29] under GSM environment and nearly 5dB, 7dB compared to RLS and LMS equalizers respectively at BER of 10^{-3}. The Type-2 FAF provides nearly 4dB performance gain over RLS and 6dB over LMS equalizers. Also It is concluded that the RBF implementation of Bayesian equalizer using product inference gives the optimal result and Tyep-1 FAF using minimum inference gives the BER performance close to the RBF and Type-2 FAF though had shown desirable performance in large training data but could not afford suitable result for equalization of mobile channel in GSM environment.

Following this the performance of two types of fuzzy equalizers at different vehicle speeds are compared for GSM environment at different equalizer parameters and channels.

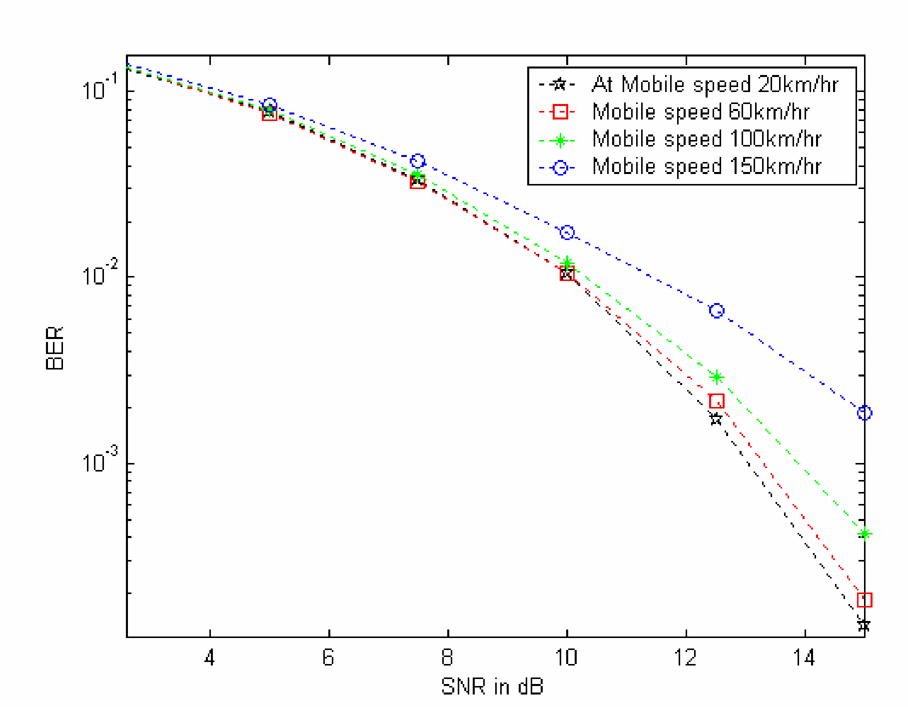


Figure3.10: BER performance of Type-1 FAF for channel $H(z) = 0.2682 + 0.9296\, z^{-1} + 0.2682\, z^{-2}$ ***at different Vehicle speeds, m = 3, d = 1.***

In this figure 3.10, we observed that the performance of Type-1 FAF degrades by increasing the vehicle speed which causes the increase in fading by increasing the Doppler frequency shift. It is observed that the performance of Type-1 FAF at vehicle speed of 20km/hr shown better, nearly 2.5dB compared to the vehicle speed at 150km/hr at BER of 10^{-3}. This study for the varying speed is again conducted for Type-2 FAF for a channel of $H(z) = 0.5 + 1.0\, z^{-1}$ in Figure 3.11. For this simulation a second order equalizer was taken and the propagation detection delay of one symbol has taken.

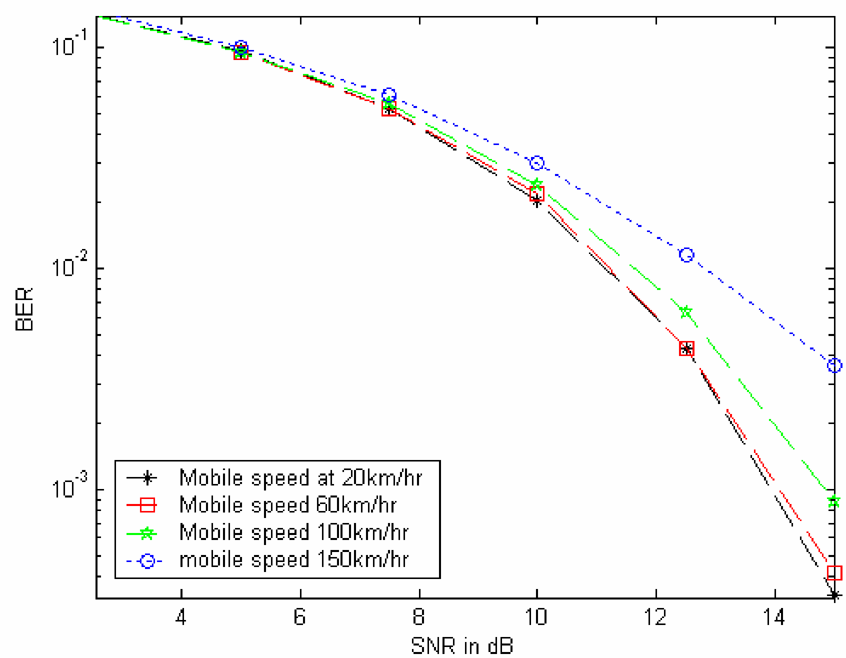


Figure3.11: BER performance of Type-2 FAF at different Vehicle speeds, m =2, d = 1.

Following this the performances of different equalizers at a speed of 20km/hr are evaluated in Figure 3.12. For this simulation a channel having transfer function $H(z)=0.5+1.0\,z^{-1}$ was considered.

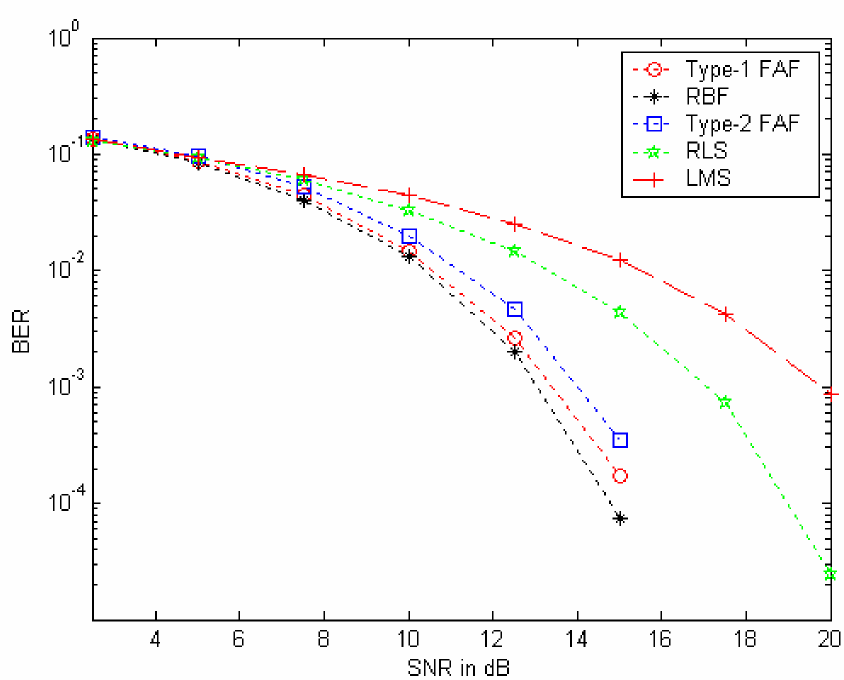


Figure 3.12: Performances of equalizers at vehicle speed of 20km/hr

From the Figure 3.12, we observe that the Type-1 FAF proposed in [28] provide was showing nearly same performance with fuzzy implementation of Bayesian equalizer with product inference (RBF) and nearly 1dB performance gain over Type-2 FAF proposed by Mendel [29] under GSM environment at a vehicle speed of 20km/hr. It was showing better performance gain of nearly 4dB, 6dB compared to RLS and LMS equalizers respectively at BER of 10^{-3}. The type-2 FAF provides nearly 2dB performance gain over RLS and 5dB over LMS equalizers.

3.9 Conclusion

In this chapter the Bayesian equalizer was implemented with fuzzy systems and the performance of fuzzy equalizer was evaluated. The following points can be drawn from the study presented in this chapter.

- The fuzzy equalizer (Type-1 FAF) provides an efficient implementation of the Bayesian equalizer.
- RBF equalizer (fuzzy implementation of Bayesian equalizer with product inference) and the computationally efficient fuzzy (Type-1) equalizer provide very little performance degradation in terms of BER.
- The computational complexity of RBF implementation of the Bayesian equalizer is related to $N_s = 2^{m+n_c-1}$ (dependent on m and n_c), whereas the complexity of the fuzzy equalizer is related to N_s for multiplications but is related to $M = 2^{n_c}$ i.e. dependent only on n_c for summation, exponentiation and divison.
- The training of fuzzy equalizer is very easy as it needs the estimation of M scalar parameters which causes fast training and ease of tracking in decision directed mode. This feature of fuzzy equalizer could make them suitable for use in mobile communication application.
- The Type-2 FAF proposed by Mendel did not provide suitable performance for the equalization of mobile channel in GSM environment.

Chapter 4

Fuzzy Equalizers for Nonlinear Channels

4.1 Introduction

For effective high-speed digital data transmission over a communication channel, the adverse effects of the dispersive channel causing intersymbol interference (ISI), the nonlinearities introduced by the modulation/demodulation process, transmitter and receiver amplifiers and the noise generated in the system are to be suitably compensated. The performance of the linear channel equalizers employing a linear filter with FIR using a least mean square (LMS) or recursive least-squares (RLS) algorithm is limited specially when the nonlinear distortion is severe. In such cases, nonlinear equalizer structures may be conveniently employed with added advantage in terms of lower bit error rate (BER), and higher convergence rate than those of a linear equalizer.

Nonlinear equalizers based on RBF network, Type-1 fuzzy logic system and Type-2 fuzzy logic system can perform mapping between its input and output space and are capable of forming decision regions with nonlinear decision boundaries. Some of the features of these equalizers were analyzed in earlier chapters. Further, because of nonlinear characteristics of the above equalizers and channel equalization being a nonlinear classification problem; these equalizers are best suited for channel equalization problem.

This chapter discusses the performance of the nonlinear equalizers based on RBF network, Type-1 and Type-2 fuzzy logic system discussed in this thesis for channels affected by nonlinearity. Following the introduction this chapter is organized as under.

- Section 4.2 describes the digital transmission system affected by nonlinearities and the channel states of time invariant and time-varying channel.
- Section 4.3 provides some extensive simulation results and the discussion to validate the performance of Type-1 and Type-2 FAF in case of a nonlinear channel.
- Section 4.4 provides concluding remarks.

4.2 Digital transmission system with nonlinearity in channel

The block diagram of the digital transmission system [48] with equalizer where the channel is affected by some nonlinearity is presented in figure 4.1.

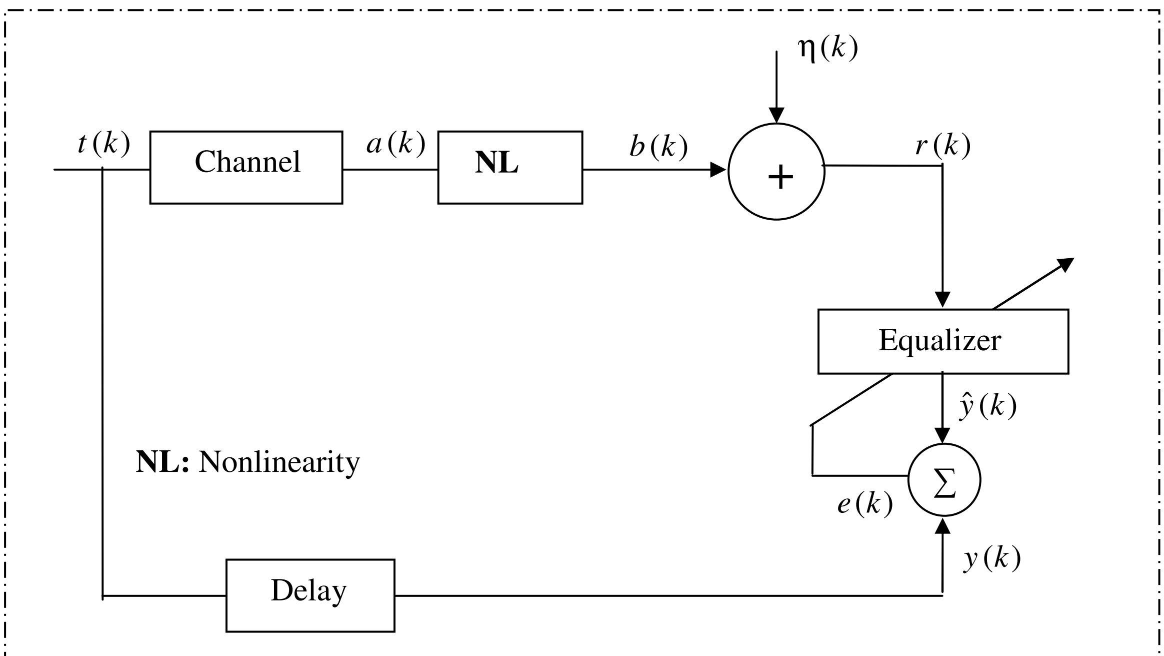


Figure 4.1: Digital transmission system with equalizer and nonlinearity

In this baseband structure of the digital communication system with the nonlinearity introduced at the output of the channel, the combined effect of the transmitter filter, the transmission medium, and other components are included in the "channel". A widely used model for a linear dispersive channel is a FIR model whose output at time instant k may be written as

$$a(k) = \sum_{n=0}^{N_c - 1} h(n)\, t(k-n) \qquad (4.1)$$

Where $h(n)$ are the values of the channel taps and N_c is the length of the FIR channel model.

$a(k)$ is the noise free channel output. If the nonlinear distortion caused by the channel is to be considered, the channel model is treated as nonlinear and its output may be expressed as

$$b(k) = \psi(a(k)) \qquad (4.2)$$

Or

$$a'(k) = \psi\big(t(k), t(k-1), \cdots, t(k-N_c+1)\ ;\ h(0), h(1), \cdots, h(N_c-1)\big) \qquad (4.3)$$

Where $\psi(\cdot)$ is some nonlinear function represented by "**NL**" block shown in Figure 4.1. The channel output is corrupted with AWGN $\eta(k)$ of variance σ^2 to produce $r(k)$, the signal received at the receiver. The purpose of the equalizer is to recover the transmitted symbol $t(k)$ or $t(k-\tau)$ from the knowledge of the received signal symbols without any error, where τ is the transmission delay associated with the physical channel. As the channel is affected by some nonlinearity due to either modulation/demodulation or amplifiers used in transmitter and receives, it is difficult to estimate the channel during training period. The scalar channel states can be directly estimated by k-mean clustering method [40] during the transmission of the known data and these scalar states formulate the channel states which forms the centers of the basis function of the equalizers. In the Figure 4.1, we have

$$r(k) = \sum_{i=1}^{N} c_i\, [a(k)]^i + \eta(k) \qquad (4.4)$$

Where N denotes the number of nonlinear terms. Considering $a(k) = 1.0 + 0.5z^{-1}$ and the $c_1 = 1$, $c_2 = 0$, $c_3 = -0.9$, the channel states of this channel $\hat{r}(k)$ can be calculated as below.

No.	$s(k)$	$s(k-1)$	$a(k)$	$\hat{r}(k)$
1	-1	-1	-1.5	1.5375
2	-1	1	-0.5	-0.3875
3	1	-1	0.5	0.3875
4	1	1	1.5	-1.5375

Table- 4.1: Calculation of channel states with nonlinearity

The channel states are calculated as Table-3.2. We focus on the case when the channel is time-varying i.e. when the channel coefficients a_1 and a_2 in (4.4) are time-varying which was simulated by Rayleigh fading simulator discussed above in section 2.4.

For a time varying channel, the channel coefficients a_i $(i = 0, 1, \cdots, n)$ are uncertain. The channel states are plotted in Figure 4.2 for a time invariant channel model of (4.3). This shows that it is a nonlinear channel whose channel states of category +1 and of -1 can not be separated by a single linear boundary which can be separated by a nonlinear decision boundary. Thus, a linear equalizer could not be able to classify properly which need a nonlinear model of classifier.

Realization of the time-varying coefficients and channel states are plotted in Figure 4.4 & Figure 4.5, where it shows how the channel transfer function varies with time and also it is observed that the channel states are now becomes eight clusters instead of eight individual points. These clusters illustrate that $\hat{r}_i$ is uncertain for all $i = 1, 2, \cdots, 8$ for a two tap channel taken here. We see that there are eight channel states and that $s(k)$ determines which cluster $\hat{r}(k)$ belongs to. The clusters $[\hat{r}(k), \hat{r}(k-1)]$ in the first four rows among eight rows like Table-3.2 are belongs to category +1 and the clusters in the last four rows belongs to the category of -1, which establishes the value of the weight parameters of the equalizer. In figure 4.5 it is shown that the channel states are moving in different clusters which cause the difficulty to estimate the channel.

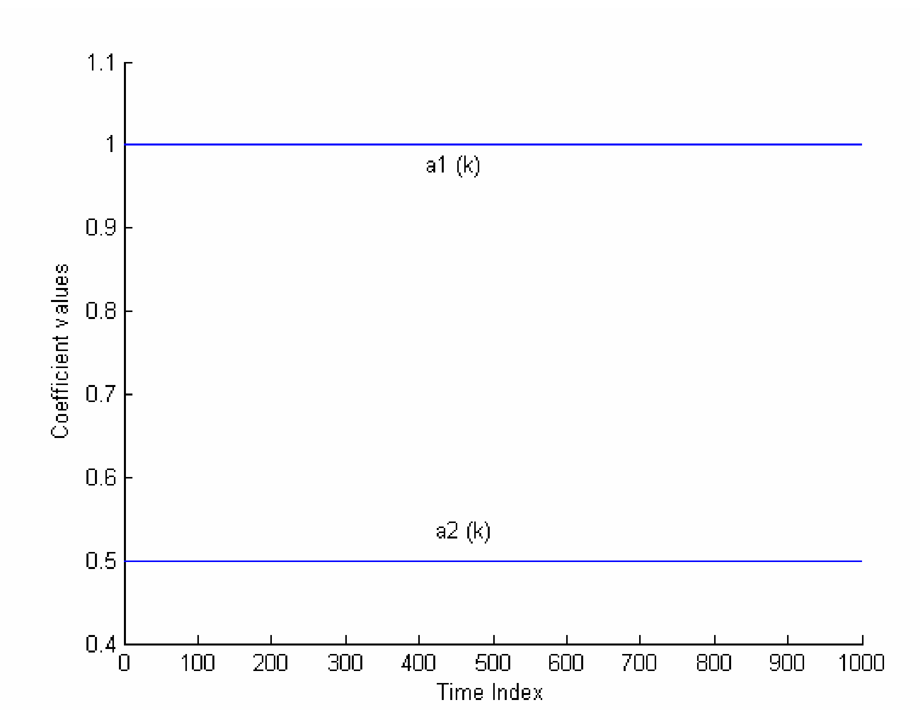


Figure 4.2: A time invariant channel with $a_1 = 1$ and $a_2 = 0.5$

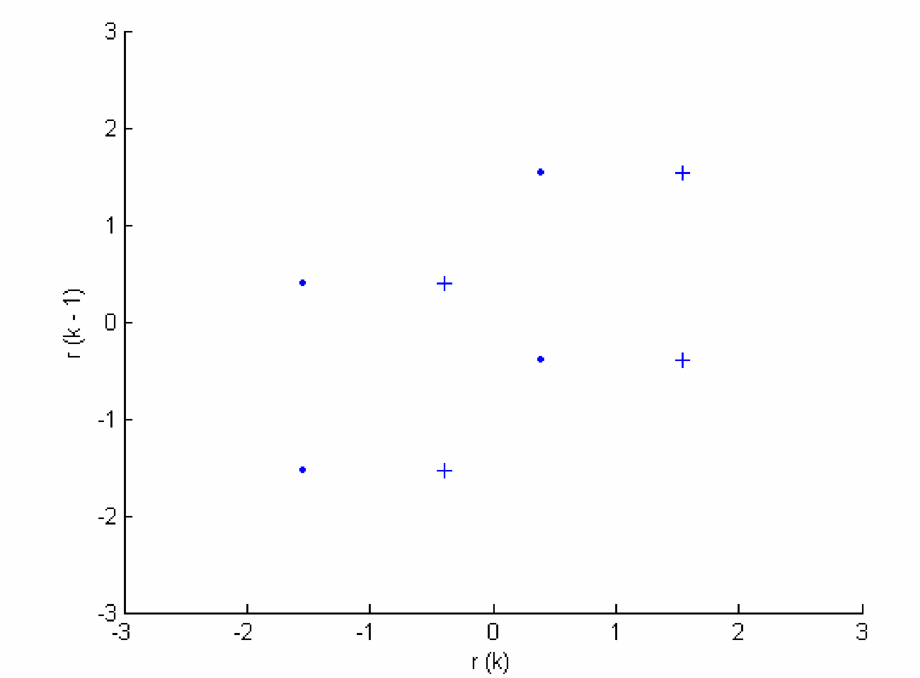


Figure 4.3: Noise free channel states of time invariant channel

where "." denotes the category $\hat{r}(k) = +1$ *and "+" denotes* $\hat{r}(k) = -1$

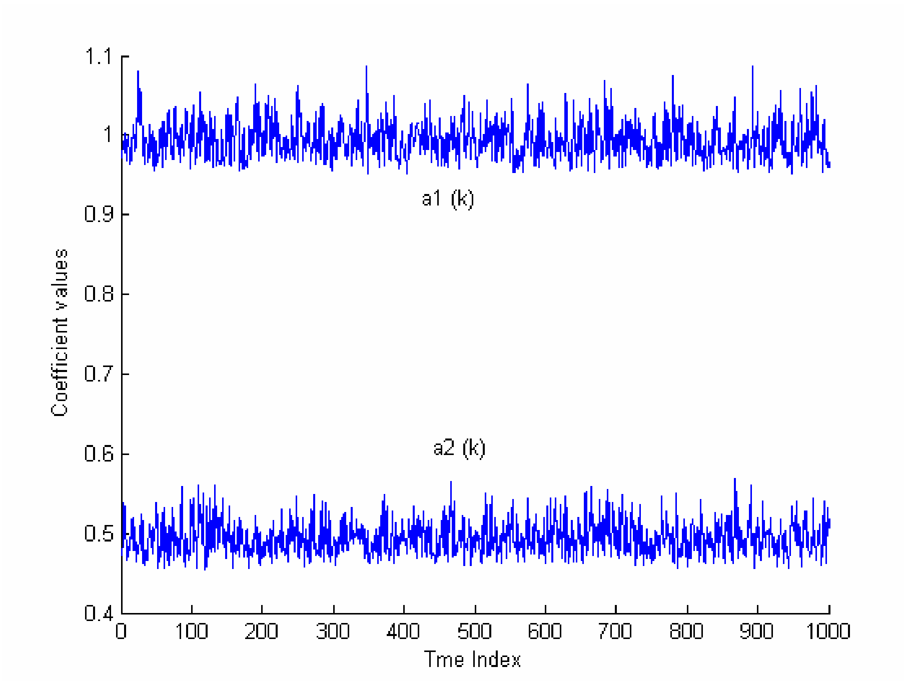


Figure 4.4: A time varying channel with Rayleigh faded

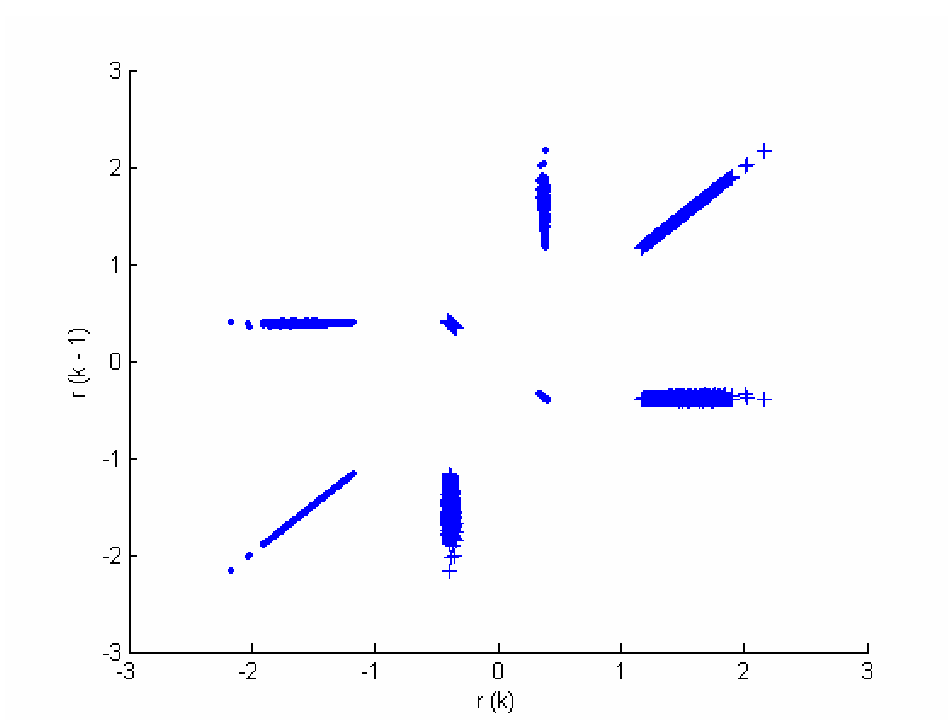


Figure 4.5: Channel states (noise free) of the above time varying channel

4.3 Simulation results and discussion

In Figures ((4.4), (4.5)) a time varying nonlinear channel was described where channel coefficients were uncertain. In the simulations below the nonlinear time-variant channel models were used.

The different nonlinearities considered for all the simulations in this chapter are represented as

NL # 1	$b(k) = \tanh(a(k))$
NL # 2	$b(k) = a(k) - 0.9\, a^3(k)$
NL # 3	$b(k) = a(k) + 0.2\, a^2(k) - 0.1a^3(k)$
Linear	$b(k) = a(k)$

Table 4.2 Different Nonlinearities considered.

In Figure 4.6 three different types of nonlinearity were considered for a channel of transfer function in z-domain $H(z)=1.0+0.5z^{-1}$. The performance of RBF equalizer (fuzzy implementation of Bayesian equalizer with product inference) was evaluated for GSM application with nonlinearities and linearity of the channel where the equalizer order of 2 and no propagation decision delay was considered.

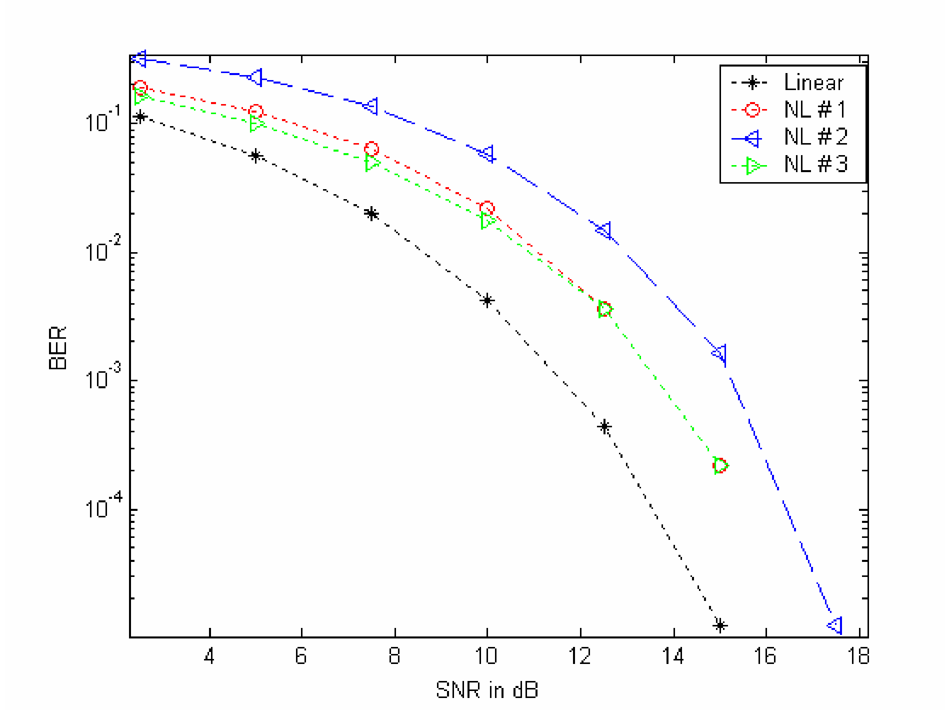


Figure 4.6: performance of RBF equalizer for different nonlinearities

We observed that the performance of RBF equalizer was different for different nonlinearities introduced in the channel. The performance of RBF equalizer for linear channel is showing nearly 2dB gain compared to the channel affected by the nonlinearities of NL # 1 and NL # 3 but showing nearly 4dB gain over the channel introduced by NL # 2 type of nonlinearity at BER of 10^{-3}.

For such nonlinear channels the channel states are evaluated by k- mean clustering algorithm during training. This simulation is conducted for high fading condition. Once the scalar states are estimated , the vector channel states are formed directly.

During training according to the different combinations of transmitted data $[s(k), s(k-1)]$ the received channel output $r(k)$ will be clustered and then their mean provides the channel state values which form the RBF centers.

The figure 4.7, shows the performance comparison of Type-1 FAF and Type-2 FAF with RBF for a non-minimum channel having transfer function of $H(z)=0.5+1.0z^{-1}$, where equalizer order of 2 and one symbol delay is taken. Nonlinearity type of NL # 2 has been used. Also in this figure the performance of linear equalizers using RLS and LMS were compared for this nonlinear channel condition. All the equalizers are trained for the same training data (26 symbols) as per the GSM specification.

From the BER plot it is seen that, the linear equalizers employing LMS and RLS algorithms for training of the equalizer perform poorly. The Type-2 fuzzy adaptive filter performs better than the linear equalizers but perform poorer than RBF equalizer and fuzzy equalizer proposed here under GSM environment for nonlinear channels. The performance degradation of nearly 1.5 dB of Type-2 fuzzy filter over Type-1 FAF proposed here at 10^{-3} BER evaluates the fuzzy equalizer. The fuzzy equalizer also shows close performance with the RBF equalizer with introduction of nonlinearity type NL # 2 in GSM specification and a performance gain of nearly 5dB over linear equalizer employing RLS algorithm.

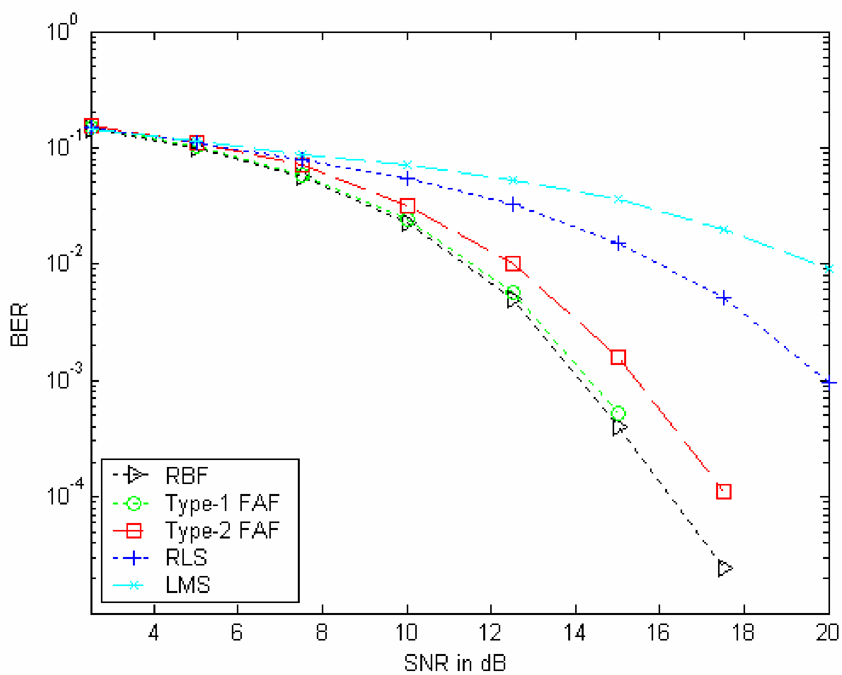


Figure 4.7: BER Vs SNR performance comparison of Type-2 and Type-1 FAF

with RBF for channel of $H(z) = 0.5 + 1.0z^{-1}$ ***and NL # 2***

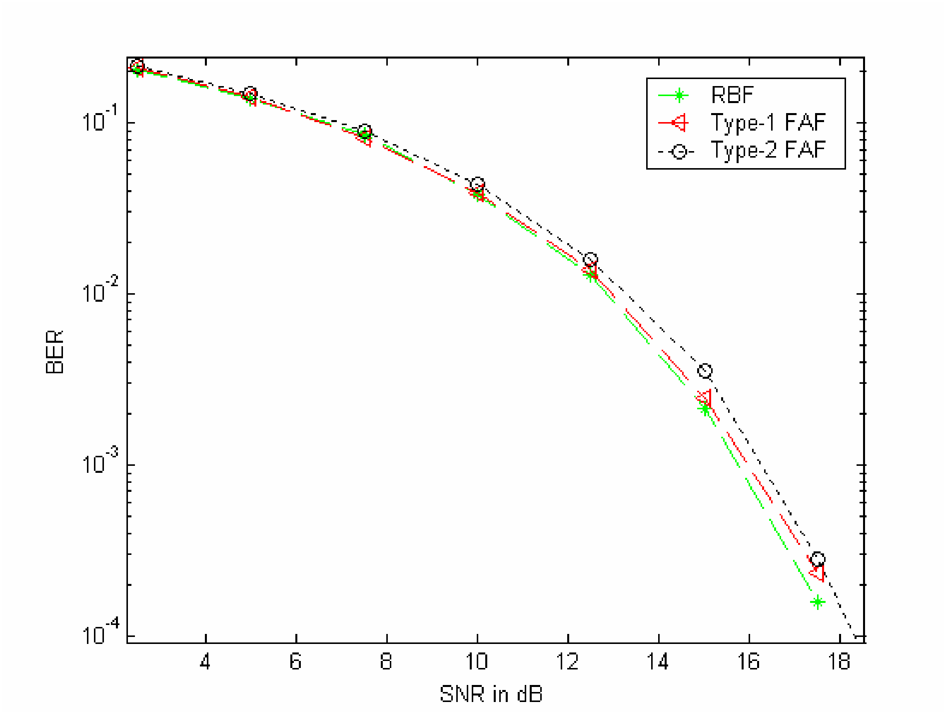


Figure 4.8: BER Vs SNR performance comparison of Type-2 and Type-1 FAF with RBF for channel of $H(z) = 0.5 + 1.0z^{-1}$ ***and NL # 1***

In order to investigate further, simulation were conducted for the performance comparison of Type-1 FAF, Type-2 FAF and RBF for another nonlinearity type NL # 1 of a channel having transfer function $H(z) = 0.5 + 1.0z^{-1}$. From the results it is seen that fuzzy equalizer proposed here provides better performance than Type-2 fuzzy filter in nonlinear channels. Figure 4.9 shows another example for channel of $H(z) = 1.0 + 0.5z^{-1}$ with introduction of nonlinearity type NL # 3.

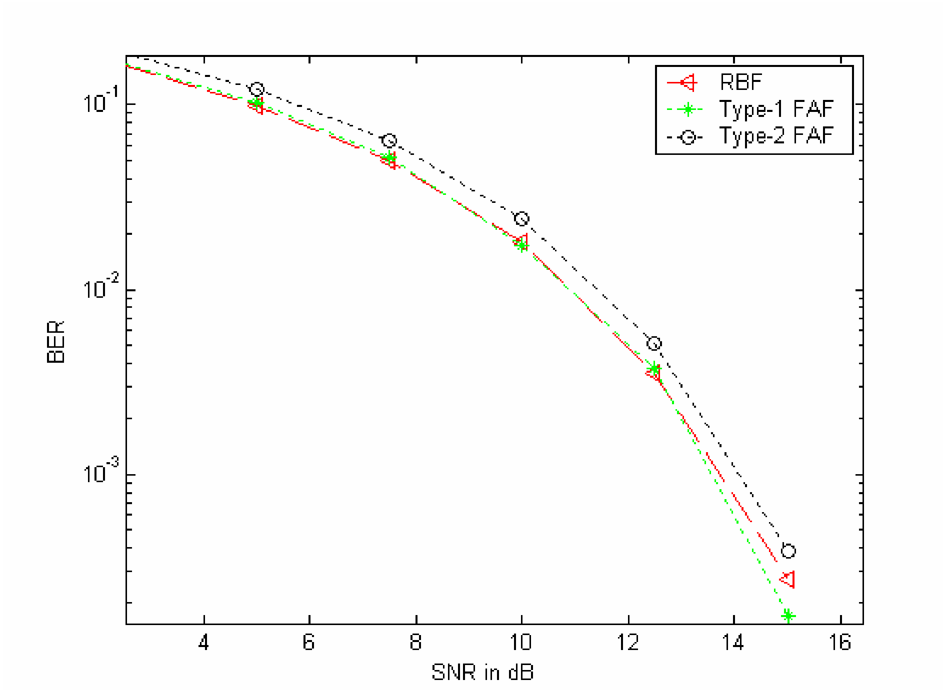


Figure 4.9:BER performance for a channel of $H(z)=1.0+0.5z^{-1}$ ***with NL # 3.***

In above figures in this chapter a constant velocity of 13.5km/hr was considered at carrier frequency of 2GHz and transmission rate of 270.8Kb/Sec.

Taking the consideration of different vehicle speeds, simulation were conducted in Figure 4.10 for a channel having transfer function of $H(z) = 0.447 + 0.894\,z^{-1}$ with affected by nonlinearity type NL # 3. This channel is a nonminimum phase channel and the vehicle speed of 20km/hr was considered at carrier frequency of 2GHz and data rate of 270.8Kb/Sec. The equalizer order of 2 and one propagation detection delay was considered. It is observed that Type-1 FAF performs close to the RBF equalizer. Type-2 FAF shows a performance degradation of nearly 0.5 dB compared

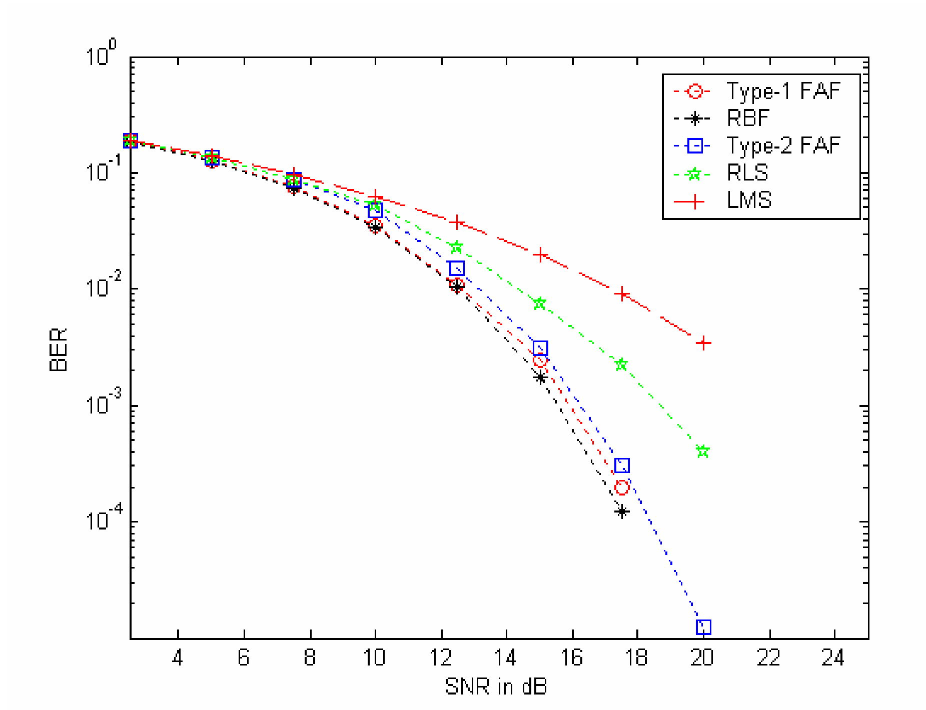


Figure 4.10: BER Vs SNR plot for a channel of $H(z) = 0.447 + 0.894\,z^{-1}$ ***for a vehicle speed of 20km/hr with nonlinearity NL # 3.***

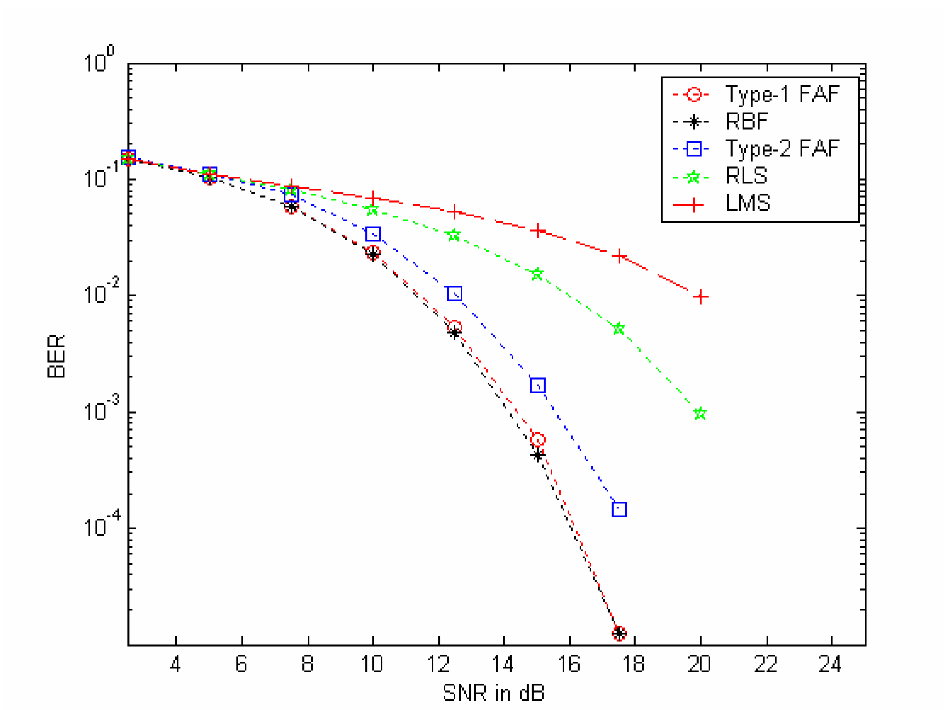


Figure 4.11: BER Vs SNR plot for a channel of $H(z)=0.5+1.0z^{-1}$ ***for a vehicle speed of 60km/hr with nonlinearity NL # 2.***

to fuzzy Type-1 filter but performs better than linear equalizers employing RLS and LMS algorithms. Figure 4.11 shows for a different vehicle speed of 60km/hr for a channel of transfer function $H(z)=0.5+1.0z^{-1}$ affected by the nonlinearity type NL # 2. For this simulation carrier frequency of 2GHZ has been considered.

Transmission rate of 270.8Kb/Sec was taken.

4.4 Conclusion

In this chapter performance of different nonlinear equalizers like Type-1 FAF, Type-2 FAF and RBF implementation of Bayesian equalizer are compared through simulation for different nonlinear channels at different channels and equalizer orders. The estimation of channel states are made with k-mean clustering method since channel estimation is difficult for nonlinear channel conditions. This chapter presented some BER performance plots for different nonlinear conditions for different channel. It has been shown that the linear equalizers do not perform for nonlinear channel with time-varying coefficients in GSM environment. The fuzzy Type-1 filter performs better than fuzzy Type-2 filter for all conditions.

Chapter 5

Conclusion

5.1 Introduction

The work undertaken in this thesis primarily discusses the two types of fuzzy system based channel equalizers in mobile communication system in GSM environment. The fuzzy implementation and RBF implementation of Bayesian equalizer based on MAP criteria has been presented. The capability of fuzzy equalizers in a GSM environment for a Rayleigh faded linear channels and nonlinear channels have been analyzed. A Type-2 fuzzy logic based equalizer proposed by Mendel [29] has been compared with proposed equalizer for GSM applications. This chapter summarizes the work reported in this thesis, specifying the limitations of the study and provides some pointers to future development.

Following this introduction section 5.2 lists the achievements from the work undertaken. Section 5.3 provides the limitations and section 5.4 presents few pointers towards the future work.

5.2 Achievements of the thesis

The work presented in this thesis can be classified for two parts. The first part presents the measure of the performance of two types of fuzzy system based equalizers for GSM application in linear fading channels and the other part is dedicated for nonlinear channels. Major points of the thesis, highlighting the contributions at each stage, are presented below.

Chapter 3 of this thesis presents fuzzy implementation of Bayesian equalizer [28]. It has seen that the Bayesian equalizer uses the estimates of noise free received vectors called channel states to formulate the decision function. It can be efficiently implemented using the estimates of noise free received scalars called scalar channel states which reduce computations considerably over conventional Bayesian equalizer. It can be implemented using RBF with scalar channel states. Subsequently, fuzzy implementation of Bayesian equalizer has been derived and this fuzzy equalizer gives suboptimal result with further reducing the computational complexity. The fuzzy implemented Bayesian equalizer uses Gaussian membership functions, product inference in the form of IF … THN rules and COG defuzzifier. This equalizer has been termed as fuzzy implemented RBF or simply RBF in the thesis. This RBF equalizer shown optimal performance in form of BER in GSM environment. The use of fuzzy system in implementing the Bayesian equalizer provides flexibility in the design of Bayesian equalizers with applying different inference rules and defuzzification process [12]. But in this thesis the fuzzy equalizer has been discussed and proposed with having minimum inference rule and COG defuzzifier. The parametric implementation of Bayesian equalizers using fuzzy systems make the equalizer traceable in GSM application providing the result close to the optimal with reduced computational complexity. The Type-2 FAF has been described and evaluated under GSM environment for channel equalization purposes. This has been used to compare performance of proposed equalizer. Type-2 FAF utilizes the mean of the channel state clusters formed by the fading of channel to formulate two membership functions (Upper and Lower) to decide its decision function. Type-2 FAF though provides result near optimal using large training data but could not perform well for GSM application.

Major contribution from this chapter is summarized here.

The performance evaluation of two types of fuzzy equalizers has been done under GSM environment with the comparison to the RBF implementation of Bayesian equalizer and other linear equalizers trained with RLS and LMS algorithm with different channels and equalizer orders. These equalizers are evaluated for linear channels with Rayleigh fading.

Chapter 4 of this thesis discusses the performance of fuzzy equalizers along with the linear equalizers under GSM environment for the nonlinear channels with Rayeligh fading. A block diagram of digital communication system with nonlinearities has been discussed where different types of nonlinearities were considered. In this chapter how the channel coefficients were changing with fading has been shown. The channel states with introduction of some nonlinearities have been shown and the movement of channel states along with certain dimensions with fading was described with simulation. The k-mean clustering method of training for nonlinear channels has been described as it is difficult to estimate the channel with presence of nonlinearities.

Major contribution form this chapter is summarized in a brief.

The Type-1 fuzzy adaptive equalizer performs close to the RBF equalizer which is optimal equalizers [43] and better than the Type-2 FAF proposed by Mendel for GSM application. Type-1 FAF can be trained in 26 training data which Type-2 could not. The linear equalizers trained with RLS and LMS algorithms does not show acceptable performance with such small training data and fading environment with the channels affected by nonlinearities.

5.3 Limitations of the work

This section presents some of the limitations of the work reported in this thesis.

In this thesis the fuzzy implementation of Bayesian equalizer has been validated for GSM application. This equalizers are related to (N^{n_c}) where N is the size of the symbol alphabet or constellation. This large complexity limits this form of equalizers to communication systems where channel dispersion is relatively small, of order of $n_c \approx 5$. The work undertaken in this thesis only considered 2-level PAM modulation where to increase the transmission speed the efficient modulation schemes like 4-level PAM, QPSK are needed. It can be extended to other efficient modulation schemes in line with RBF implementation of Bayesian equalizers [49].

Other issues like adjacent channel interference (ACI), co-channel interference (CCI), Rayleigh fading with different delays, timing recovery in the receiver were not considered.

5.4 Scope for the further research

By concluding this thesis, the following are some pointers for further works can be undertaken.

The suggested area in which research can be undertaken follows from the limitation of the work presented in this chapter. As decision feedback equalizers are simple and needs less training data, a decision feedback equalizer could provide alternative equalization strategy for GSM environment. Different efficient coding and modulation schemes could be considered in the simulation.

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Appendix – A

No	Impulse response	Zero location	Channel type
$H_1(z)$	$0.5+1.0z^{-1}$	-2.0	nonminimum phase
$H_2(z)$	$1.0+0.5z^{-1}$	-0.2	Minimum phase
$H_3(z)$	$0.447 + 0.894\,z^{-1}$	-2.0	nonminimum phase
$H_4(z)$	$0.2682+0.9296z^{-1}+0.2682z^{-2}$	-3.1484,-0.3176	mixed phase
$H_5(z)$	$0.3482+0.8704z^{-1}+0.3482z^{-2}$	-2.0,-0.5	mixed phase