Multicriteria problem of regulation when planning building processes

Kochkarov A.M./Popova E.V./Zinchenko O.A.

The known problem of one machine tool or problem of the investor in onecriteria setting is formulated in [1,2]. Multicriteria setting of this problem consist in following

N invested projects reindexed i =1,2,...,n are considered, T_i -duration of ; construction, α_i -expected profit per unit of time from i project after putting it into operation, D_i -the time fixed, after the expiry of it a fine is paid each overdued unit of time α_i in unmber .Any admissible problem decision of investor represents one of $n!$ permutation of $x=(i_1,i_2,\dots,i_n)$ of numbers 1,2,...,n. $X=\{x\}$-is the set of all admissible solutions (SAS)of this problem.The vector objective function (VOF)

$$F(x)=((F_1(x),\dots,F_\nu(x),\dots,F_N(x)) \quad \textbf{(1)}$$

is determined on the set $X=\{x\}$ of all $n!$ permutation of $x=(i_1,i_2,\dots,i_n)$, consisting of minimization criterias , i. e. partial objective function (OF)

$$F_\nu(x)\in\{\varphi_\nu(x),\overline{\varphi}_\nu(x)\},1\le \nu,N,$$

where

$$\varphi_\nu(x)=\sum_{k=1}^{n}\alpha_{i_k}^{\nu}\max(t_{i_k}-D_{i_k}^{\nu},0)\to\min \quad \textbf{(2)}$$

where

$$t_{i_k}=\sum_{s=1}^{k}T_{i_s},$$

$$\overline{\varphi}_\nu(x)=\sum_{k=1}^{n}\alpha_{i_k}^{\nu}\max(D_{i_k}^{\nu}-t_{i_k},0)\to\min, \quad \textbf{(3)}$$

$$\psi_\nu(x)=\max_{1\le k\le n}\alpha_{i_k}^{\nu}\max(t_{i_k}-D_{i_k}^{\nu},0)\to\min, \quad \textbf{(4)}$$

$$\overline{\psi}_\nu(x)=\max_{1\le k\le n}\alpha_{i_k}^{\nu}\max(D_{i_k}^{\nu}-t_{i_k},0). \quad \textbf{(5)}$$

In literary soures the criterias (2) and (3) frequently have name as terms "criterion of kind MINMAX " accordingly .

The informal implication of OF(2) consists in the minimization of investor's total losses and that of OF(4) consist in the minimization of the worst outcome
(of the greatest fine) among the invested projects.
As a social-economic matter OF (2) optimuzes invector's effect and OF (4) optimizes those financial-economic safeguards invector can give the client . In other words ,the expression (2) - is the invector's OF,and the expression (4)-is the client's OF, for the optimization of this OF means nothing else than lowering of "risk plank"(the absolute level of risk [4]) for deptor - client .In real conditions the problem of rinding the investment policy naturally arises, which could of the same time and interdependently take into consideration the economic targets of both sides - investor's on the one hand , and deptlor - client's on the other.

The statement of the question like this means finding of some "compromise optimum". The last term implies some retreat of each of the side (in direction of deterioartion) from their optimums and adoption the policy of investment , compromisingly acceptable for both sides .The so-called Pareto law , well-known in economic theory,should be observed ,of course [5].

VOF (1) determines by self Pareto set (PS) [6,7] $\tilde{X} \subseteq X$ .For choice and acceptance of the best decision it is enough to have not all PS $\tilde{X}$, and only it's subset $X^0 \subseteq \tilde{X}$ which called a comlete set of alternatives (CSA).CSA is definded as a subset $X^0 \subseteq \tilde{X}$, with a minimum of capacity $|X^0|$, such , that $F(X^0) = F(\tilde{X})$, where $F(X^*) = \{F(x): x \in X^*\} \forall X^* \subseteq X$.

The problem consist that to find CSA and choose from it the most expedient decision with help those or other procedures of the theory of acceptance of the decisions [8],[9].

It is known , that at N=1 presence of the optimum decision on OF (2) is a NP-difficult problem [10].Such the statement is fair and in case of optimization on OF (3). Onecriteria proplem of the investor with OF (4) and (5) polinomially resolvable: the complexity of finding the optimum decision does not surpass O(n^2) [1].

Whether a formulated problem has by property of completenees [7] at $N \geq 2$? This property means , that for any natural numbers n and $N \geq 2$ there are such values of parameters, $\alpha_i^\nu, T_i, D_i^\nu, i = \overline{1,n}, \nu = \overline{1,N}$, at which quality are carried out $X^0 = \tilde{X} = X$ .Then in terminology [10] algorithm problem of finding CSA has exponential computing complexity.

Lemma 1 For OF a kind MINMAX (2) there is such set of values of parameters $\alpha_i, T_i, D_i, i = \overline{1,n}$, that for any pair $x', x'' \in X$ an inequality is carried out $\varphi(x') \neq \varphi(x'')$. The idea of the proof consists of ,that for objects of $i \in \{1,2,..,n\}$ values of the "specific penalties" α_i are detrmined agrees the formulas,

$$\alpha_i = 2^{m_i}, i = 1,2,3,..,n$$

where $m = \log_2 n^2$.

We shall accept also , that duration of investment period for all objects it is identical , for example $T_i = 1, i = \overline{1,n}$, and the time fixed $D_i = 0, i = \overline{1,n}$ .Then we'll get a concrete problem for which all the conditions of lemma1 are to be done .It is also valid:

Lemma 2 For OF of a kind MINSUM (3) there is such of parameters, $\alpha_i, T_i, D_i, i = \overline{1,n}$ that for any pair $x', x'' \in X$,anineqnality is carried out $\overline{\varphi}(x') \neq \overline{\varphi}(x'')$.

Let T_i for all objectes $i=\overline{1,n}$,and the time fixed $D_i^1=0, i=\overline{1,n}$ for criterion $F_1(x)$ of a kind (2) and $D_i=T=\sum_{i=1}^{n}T_i=n, i=\overline{1,n}$ for criterion $F_2(x)$ of a kind (3),then VOF

$$F(x)=(F_1(x),F_2(x)) \qquad \textbf{(6)}$$

consists of criterions

$$F_1(x)=\varphi_1=\sum_{k=1}^{n}k2^{m_{i_k}} \to \min \qquad \textbf{(7)}$$

$$F_2(x)=\varphi_2(x)=\sum_{k=1}^{n}(n-k)2^{m_{i_k}} \to \min \qquad \textbf{(8)}$$

Lemma 3 If VOF (6) is determined agrees (7),(8) , for any allowable decision $x=(i_1,i_2,..,i_n)\in X$ a sum

$$F_1(x)+F_2(x)=C \qquad \textbf{(9)}$$

Where *C*- constant, which does not depend on chosen seqnence *x*.

From Lemma 1,2 and 3 follows , that any pair of the decisions $x',x''\in X$ at certain above parameters is vector incomparable on VOF (6)-(8),i.e. is fair:

Theorem 1. The twocriteria problem of the investor with criteria of kind MINSUM (2),(3) has property of completeness, i. e. for any *n* there are such values of parameter $\alpha_i^v, T_i, D_i^v, i=\overline{1,n}, v=\overline{1,2}$, at which CSA X^0 , PS $\tilde{X}$ and GRS*X* coincide.

Taking into account, that copacity GRS$|X^0|=n!$ from Theorem 1 directly we recive, that the fair following basic statemente is:

Theorem 2 The algorithm problem of a finding CSA for twocriteria problem of the investor with criteria of a kind MINSUM has computing complexity,which grows exponential with growth of dimension *n*.

The leterature:

1. Perepeliza V.A., Mamedov A.A. Reseach of complexity and resolvability of vector problems on Graphs.-Cherkessk;K-ChTI,1995.-45 p.

2.Podinovsky V.V., Nogin V.D. The Pareto-optimum decisions multicriteria problem.-Moscow;Science,1982-256 p.

3.BerezovskyB.A., BorzenkoV.I.,Kempner L.M. Binary relation in multicriteria optimization.-Moscow;Science,1981.-150 p.

4.Vilkas A.I.,Maiminas E.Z.The decision:the theory,information,modeling.-Moscow:Radio and communication,1981.-328 p.

5.Dubov J.A.,Travkin S.I., Iakimes V.N. multicriteria models of formatiion and choice of variants of systems.-Moscow :Science,1986.-296 p.

6.Taha H.A. Operations research.An introducion. MacMillan Publ.Co., Inc. New York,1982.

7.Emelichev V.A., Perepeliza V.A. Complexity discretic multicriteria problem ‖ Discretic mathematic.-1994.-T.6, ¹1.-p.3-33.

8.Emelichev V.A. Perepeliza V.A. Complexity of vector optimization problem on Graphs.‖ Optimization 22.-1991.-¹6-p. 903-918.

9.Emelichev V.A., Perepeliza V.A. On cardinality of the set of alternatives in discrete manycriterion problems.║ Discrete Math.Appl.-1992-Vol 2, ^{1}5,-p.461-471.

10.Micheal R. Garey, David S. Johnson.Computers and Intractabillity.-Moscow: Mir,1982.-416p.