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Deflection Calculation of RC Beams: Finite Element Software Versus Design Code Methods

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Summary

The paper investigates accuracy of deflection predictions made by the finite element package ATENA and design code methods ACI and EC2. Deflections have been calculated for a large number of experimental reinforced concrete beams reported by three investigators. Statistical parameters have been established for each of the technique at different load levels, separately for the beams with small and moderate reinforcement ratio.

1 Introduction

Civil engineers for analysis of reinforced concrete structures can choose between traditional design code methods and numerical techniques. Although design code methods ensure safe design, they do not reveal the actual stress-strain state of cracked structures and often lack physical interpretation. Numerical methods which were rapidly progressing within last decades are based on universal principles and can include all possible effects such as material nonlinearities, concrete cracking, creep and shrinkage, reinforcement slip, etc, being responsible for complexity of this material. In order to choose a particular calculation method, engineers should be aware of accuracy of different techniques.

This paper investigates accuracy of deflection predictions made by the finite element package ATENA and the design code methods ACI and EC2. Deflections have been calculated for a large number of experimental reinforced concrete beams reported by three investigators. Statistical parameters have been established for each of the technique at different load levels, separately for the beams with small and moderate reinforcement ratio.

2 Deflection calculation methods of design codes

In this section, two design code deflection calculation techniques, i.e. the ACI (ACI Committee 318 1989) and the EC2 (ENV 1992-1-1 1992) methods are briefly described.

2.1 Deflection analysis by ACI method

The curvature of a reinforced concrete member is determined by the classical expression $\kappa = M/E_cI$ where E_cI is the flexural stiffness. Constant modulus of elasticity of concrete, E_c, was offered for all loading stages, but moment of inertia, I, is varying (Branson 1977). Thus, for the elastic stage, I_g is written as for the gross concrete section ignoring reinforcement and for the load corresponding to the steel yielding I_{cr} is calculated as for the cracked section. For loading points between the concrete cracking and yielding of the steel, the following equation was derived to express the transition from I_g to I_{cr} that was observed in experimental data (Branson 1977):

$$I_e = \left(\frac{M_{cr}}{M}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M}\right)^3\right] I_{cr} \quad (1)$$

Here M is the external moment; $M_{cr}=f_r I_g/y_t$ is the cracking moment; $f_r=0.643(f_c')^{0.5}$ [Mpa] is the modulus of rupture; y_t is the distance from centroid to extreme tension fibre; f_c' is the compressive concrete cylinder strength. Deflection for simple beams can be assessed from

$$f = s\kappa l_o^2 \quad (2)$$

where s is the factor depending on a loading case and supporting conditions; κ is the curvature corresponding to the maximum moment, and l_0 is the beam span.

2.2 Deflection analysis by EC2 method

In the EC2 model, a reinforced concrete member is divided into two regions: region I, uncracked, and region II, fully cracked. In region I, both the concrete and steel behave elastically, while in region II the reinforcing steel carries all the tensile force on the member after cracking. Average curvature is expressed as

$$\kappa = (1-\xi)\kappa_1 + \xi\kappa_2 \quad (3)$$

where κ_1 and κ_2 correspond to the curvatures in regions II, and I respectively. A distribution coefficient ξ indicates how close the stress-strain state is to the condition causing cracking. It takes a value of zero at the cracking moment and approaches unity as the loading increases above the cracking moment. The relation gives it

$$\xi = \beta_1\beta_2(\sigma_{sr}/\sigma_s)^2 \quad (4)$$

where β_1 is a coefficient taking into account the bond properties of the reinforcement, it is taken 1 for deformed bars and 0.5 for plain (smooth) bars; β_2 is a coefficient assessing the duration and nature of the loading, it takes a value of 1 for short-term loads and 0.5 for sustained or cyclic loads; σ_{sr} and σ_s are the stresses in the tension steel calculated on the basis of a fully cracked section respectively under the cracking load and the load considered.

3 Finite element package ATENA

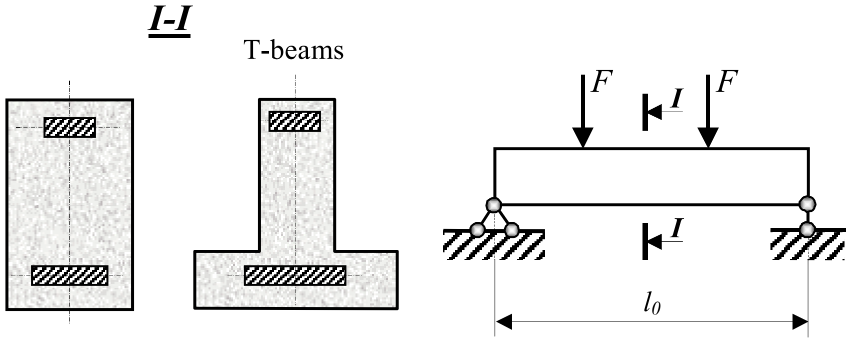
The formulation of constitutive relations in FE package ATENA is considered in the plane stress state. A smeared approach is used to model the material properties, such as cracks. Material properties defined for a point are valid within a certain material volume associated with the entire finite element (Červenka, Jendele and Červenka 2003).

The concrete model used in the analysis has included the following effects of concrete behaviour: non-linear behaviour in compression including hardening and softening; fracture of concrete in tension based on the nonlinear fracture mechanics; biaxial strength failure criterion, reduction of compressive strength after cracking; tension stiffening effect; reduction of the shear stiffness after cracking (variable shear retention); fixed direction crack model.

A fictitious crack model based both on the fracture energy and the exponential crack-opening law (Hordijk 1991) was used as the softening model of concrete.

4 Experimental data employed for comparative deflection analysis

Present analysis employs experimental data of 49 reinforced concrete beams reported by three investigators (Figarovskij 1962, Artemjev 1959, Gushcha 1968). The experimental specimens can be categorized as the beams with small and medium reinforcement ratio. Only five beams had reinforcement ratio above 1% (and below 1.3%) and 22 beams had it below 0.7% taken as a limit of lightly reinforced beams.

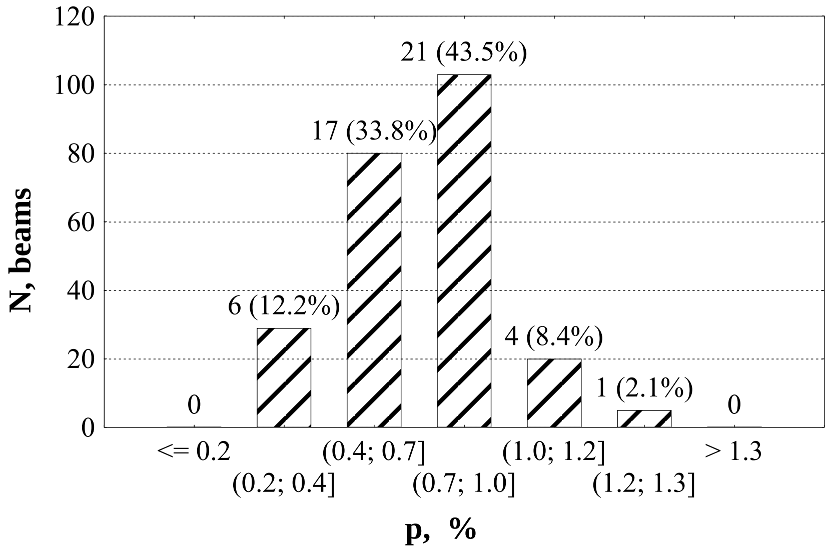


Cross-sections and loading of test beams

All the beams were tested under a four-point bending scheme. Most of the beams had a rectangular, but some an inverted T section. Six beams were reinforced with plain bars (Figarovskij 1962) while the remaining beams were reinforced with deformed bars.

Author	Beams	l_0	Height	Width	p	f_{cu}
		m	mm	mm	%	MPa
Figarovskij	30	3.0	248-254	179-181	0.20-1.26	10.5-36.0
Artemjev	15	3.0	250-264	176-187	0.80-0.91	18.8-53.4
Gushcha	4	3.6	306-312	133-162	0.28-0.97	30.0-40.8

Main characteristics of the experimental beams employed in the analysis



Number of beams of different reinforcement ratio intervals

Experimental mid-span deflections were compared with the ones predicted by all techniques. If available, deflections were taken at five load, F, levels, i.e. 40, 55, 60, 70 and 80 % of the theoretical ultimate load, F_{ult}, expressed in terms of relative load $\tilde{F}$.

$$\tilde{F} = F/F_{ult} \tag{5}$$

The experimental points below the cracking limit were excluded from the analysis. This was due to two reasons: 1) tests of some beams, particularly those later on subjected to long-term loading, were terminated prior to load $0.8F_{ult}$; 2) for some beams, particularly those with very small reinforcement ratios, the experimental cracking load has exceeded $0.4F_{ult}$. Therefore, for these beams the load intervals were widened and covered the range of 30 to 90 % of F_{ult}. Still no more than five experimental points were taken for each of the beam.

5 Statistical analysis of deflections predicted by different methods

This section statistically compares mid-span deflections of the test beams with predictions of the design code methods ACI and EC2 and FE package ATENA. Accuracy of a prediction at a point has been estimated by means of a relative error taken as

$$\Delta = f_{cal} / f_{obs} \tag{6}$$

where f_{obs} and f_{cal} are the experimental and the calculated deflections, respectively.

In the ideal case of infinite amount of experimental data, Δ as a random variable would follow the normal probability distribution law characterized by mean, μ_Δ, and variance, σ_Δ^2 (or standard deviation, σ_Δ). For limited amount of experimental data (as in case of present analysis), the corresponding characteristics, i.e. mean, m_Δ, and variance, s_Δ^2, as point estimates can be defined. Based on the obtained values for m_Δ and s_Δ, confidence intervals with a given confidence probability can be established for mean μ_Δ and standard deviation σ_Δ. In the ideal case when the model is fully adequate to experimental results, $\mu_\Delta=1$ and $\sigma_\Delta=0$. The postulate of minimum variance ($\sigma_\Delta^2 \to 0$) is decisive in terms of model accuracy (Zarubin and Krischenko 2001). Mean, μ_Δ, is a consistency parameter of a method. Although real fixed values of μ_Δ and σ_Δ cannot be determined, they can be statistically estimated and compared for different calculation methods.

The statistical analysis has been carried out in the following steps (Neter, Wasserman and Kutner 1990): the test for outliers; the test for constancy of variance, σ_Δ^2; the test of the relative error Δ for normality; statistical analysis of variance, σ_Δ^2, (ANOVA) and mean, μ_Δ, for different deflection prediction methods. The first three steps are to be carried out for validation of the statistical analysis whereas the last step covers the statistical analysis itself. Below each of the steps is described.

5.1 The test for outliers

Inaccurate experimental points *called the outliers* may violate consistency of the experimental data. If possible, such experimental points should be excluded from the statistical analysis; otherwise it might be inaccurate or even erroneous. In the present analysis, only one such point has been excluded.

5.2 The test for constancy of variance, σ_Δ^2

Statistical analysis results characterized by large standard deviation should be carefully dealt with. In such case care should be taken to make sure that the obtained results can be grouped around one center characterized by the same or similar variance, σ_Δ^2.

In the first statistical analysis carried out for the total experimental data (number of points N_p=237), large standard deviations were obtained for all three methods: 0.246 for the ACI, 0.294 for the EC2 and 0.244 for ATENA. This suggested splitting the experimental data into two groups based on reinforcement ratio ($p \leq 0.70\%$ and $p > 0.70\%$). Further analysis has shown that such grouping was not sufficient as standard deviation varied significantly for different load intervals. Therefore, the data within the two large groups was divided according to the level of the relative load $\tilde{F}$. The following six intervals of relative load have been distinguished:

$$\boldsymbol{I}:\quad 0.30 \le \tilde{F} < 0.40 \quad \boldsymbol{II}:\quad 0.40 \le \tilde{F} < 0.50 \quad \boldsymbol{III}:\quad 0.50 \le \tilde{F} < 0.60$$
$$\boldsymbol{IV}:\quad 0.60 \le \tilde{F} < 0.70 \quad \boldsymbol{V}:\quad 0.70 \le \tilde{F} < 0.80 \quad \boldsymbol{VI}:\quad 0.80 \le \tilde{F} \le 0.90 \tag{7}$$

Fourteen experimental points outside the relative load interval [0.30; 0.90] where excluded from the analysis.

Under the assumption of normal probability distribution of the relative error, Δ, basic statistical parameters such as mean, m_Δ, and standard deviation, s_Δ, as point estimates of mean, μ_Δ, and standard deviation, σ_Δ, have been calculated for each load, $\tilde{F}$, interval. If standard deviation, σ_Δ, and mean, μ_Δ, under confidence level γ=0.10 were assumed to be equal for the adjacent load intervals, those intervals were combined into one.

	Reinforcement ratio *p*<=0.70%				**Reinforcement ratio *p*>0.70%**					
Intervals	***I-II***		***III***		***I***		***II***		***III-IV***	
Points	49		21		21		24		41	
Estimates	m_Δ	s_Δ	m_Δ	s_Δ	m_Δ	s_Δ	m_Δ	s_Δ	m_Δ	s_Δ
ATENA	1.440	0.259	1.184	0.179	1.076	0.212	1.027	0.122	0.990	0.078
ACI	1.188	0.301	0.995	0.239	1.224	0.271	1.073	0.134	0.966	0.055
EC2	1.454	0.304	1.165	0.202	1.116	0.239	0.984	0.128	0.900	0.060
Intervals	***IV***		***V-VI***		***V***		***VI***			
Points	16		17		17		16			
Estimates	m_Δ	s_Δ	m_Δ	s_Δ	m_Δ	s_Δ	m_Δ	s_Δ		
ATENA	1.073	0.113	1.028	0.090	0.992	0.061	1.003	0.097		
ACI	0.949	0.159	0.868	0.105	0.903	0.040	0.867	0.055		
EC2	1.076	0.163	0.989	0.148	0.851	0.041	0.818	0.057		

Mean and standard deviation for different load and reinforcement ratio intervals

5.3 The test of the relative error Δ for normality

Validation of the normal distribution law has been carried out for each of the methods using statistical procedures (the chi-square, the Kolmogorov-Smirnov, and Shapiro-Wilk tests), but is not presented herein. It can be concluded that the probability distribution of the relative error Δ practically follows the normal law with mean, μ_Δ, and standard deviation, σ_Δ. Therefore, further analysis employs the normal law of the probability distribution with the previously estimated characteristics: mean, m_Δ, and standard deviation, s_Δ.

5.4 Statistical analysis of variance, σ_Δ^2 (ANOVA), and mean, μ_Δ

The minimum of variance, σ_Δ^2, was taken as the accuracy criteria of the calculation techniques. Although real fixed values of σ_Δ^2 cannot be defined, σ_Δ^2 for different methods can be compared with each other using statistical procedures. Comparison of each technique with the remaining two in terms of σ_Δ^2 has been carried out for different *p* and relative load intervals.

σ^2-test	**Reinforcement ratio *p*<=0.70%**				**Reinforcement ratio *p*>0.70%**				
Intervals	***I-II***	***III***	***IV***	***V-VI***	***I***	***II***	***III-IV***	***V***	***VI***
ATENA-ACI	=	=	=	=	=	=	>	>	>
ATENA-EC2	=	=	<	<	=	=	>	>	>
ACI-EC2	=	=	=	<	=	=	=	=	=

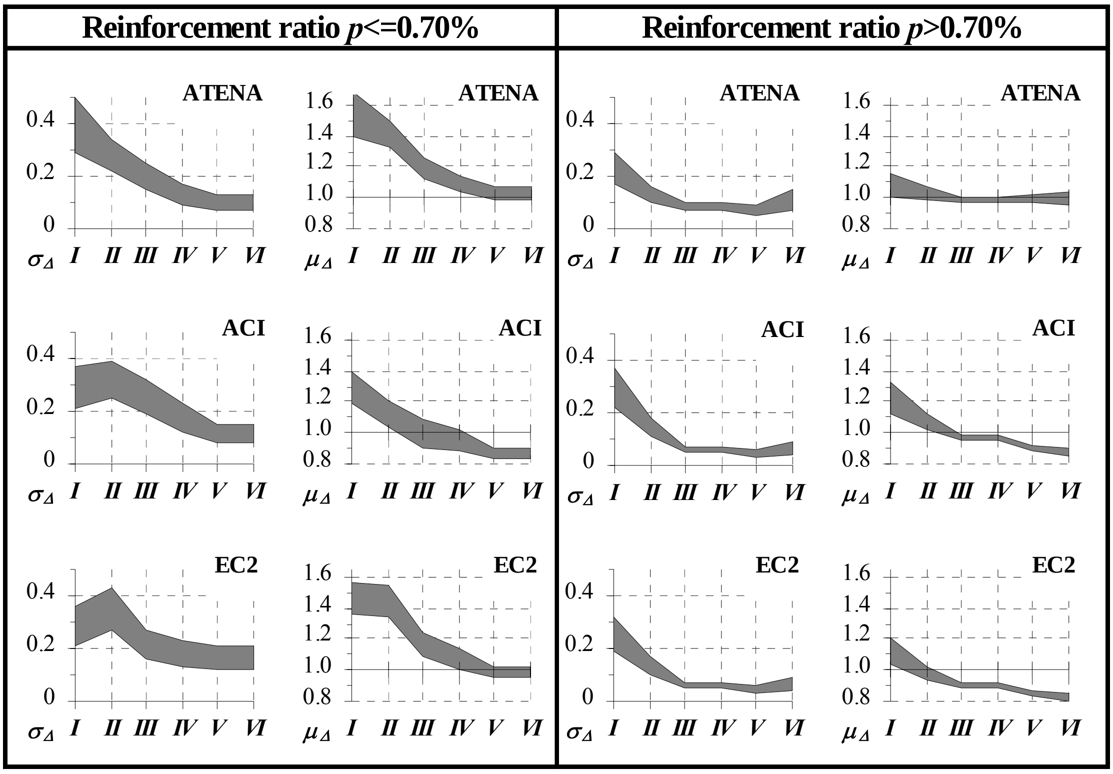
Test for variance, σ_Δ^2, under assumed probability $P=1-\alpha$=0.90

As noted previously, mean, μ_Δ, as the parameter characterizing consistency of a calculation method, ideally should be equal to unity. This as a two-sided test (alternatives: H_0: μ_Δ=1, and

H_a: $\mu_\Delta \neq 1$) has been statistically verified for all the calculation techniques with the assumed probability $P=1-\alpha=0.90$ where α (taken as 0.10) is a risk of making a Type I error (discarding true hypothesis) (Zarubin and Krischenko 2001).

μ-test	Reinforcement ratio p<=0.70%				Reinforcement ratio p>0.70%				
Intervals	*I-II*	*III*	*IV*	*V-VI*	*I*	*II*	*III-IV*	*V*	*VI*
ATENA	H_a	H_a	H_a	H_0	H_0	H_0	H_0	H_0	H_0
ACI	H_a	H_0	H_0	H_a	H_a	H_a	H_a	H_a	H_a
EC2	H_a	H_a	H_a	H_0	H_a	H_0	H_a	H_a	H_a

Test for mean, μ_Δ, under assumed probability $P=1-\alpha=0.90$

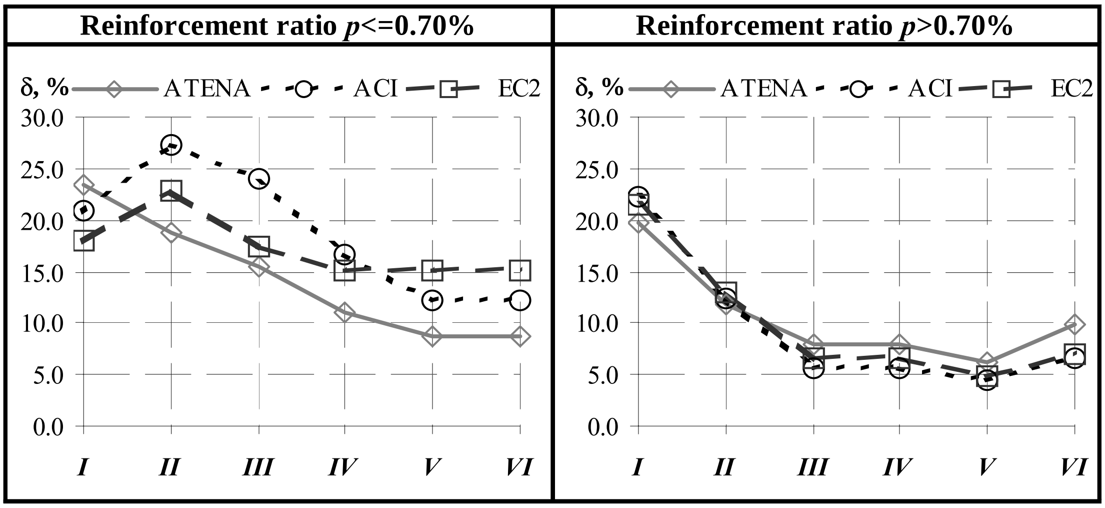


Confidence intervals of mean, μ_Δ, and standard deviation, σ_Δ, under confidence probability $\gamma=0.90$

Confidence intervals under given probability $\gamma=0.90$ have been established for mean, μ_Δ, and standard deviation, σ_Δ. Graphical presentation of the confidence intervals serves as an illustrative tool of the prediction errors for different reinforcement ratio and load levels. If these diagrams concerning variation of σ_Δ are intended for more qualitative observation of prediction differences of the calculation techniques, the test for consistency can be visually checked: alternative H_0 can be accepted, if unity is within the confidence intervals.

A combined estimation of standard deviation and mean, can be expressed in terms of coefficient of variation:

$$\delta_\Delta = s_\Delta / m_\Delta \tag{8}$$



Cofficient of variation δ_Δ for the methods uder investigation at different load and reinforcement ratio intervals

6 Results and conclusions

Comparative statistical analysis of deflections calculated by the ACI and EC2 design codes and finite element software package ATENA has been carried out for 49 experimental reinforced concrete beams reported by three investigators. Deflections were calculated at four or five loading levels for each of the beam within load interval ranging from 30 to 90% of the theoretical ultimate load. The data points corresponding to the loads smaller than the cracking load were excluded from the analysis. Accuracy of the predictions has been assessed by statistical parameters such as mean and standard deviation calculated for relative deflections, $\Delta=f_{cal}/f_{obs}$. The following conclusions can be drawn:

Lightly reinforced members can be considered as a particular case of deflection analysis as the stress-strain state and curvatures are significantly influenced by the effects of cracked tensile concrete. Since tensile strength of concrete is a highly dispersed value, far less accurate deflection predictions (in terms of standard deviations) have been made by all the methods for the lightly reinforced beams in comparison to the beams with moderate reinforcement ratios.

Standard deviations varied with change of load. The largest errors were obtained for the smallest loads, i.e. the loads close to the cracking ones. The errors have decreased with increase of load.

For the lightly reinforced beams, the ACI method and ATENA have provided similar accuracy in terms of variance under given probability, P=0.90. Less reliable predictions where made by the EC2 method for the load exceeding 60% of the ultimate load.

In most cases deflections were overestimated at the initial load intervals close to the cracking load. Justification for that were large standard deviations. Excepting the predictions by ATENA for the beams with moderate reinforcement ratio, there was a tendency of reduction of mean value with increasing load. At higher loads mean is well below unity for the ACI and EC2 methods. It can be explained by the effect of plastic strains in the compressive concrete at the advanced stress-strain states which is not assessed by the design code methods. Although the effect of plastic strains is less significant for underreinforced members, deflections are underestimated by the ACI method at large loads. This is not the case for the EC2 which for lightly reinforced beams has given very similar mean relative deflections to ATENA predictions.

For the beams with moderate reinforcement ratio (p>0.70%), all the methods have predicted deflections with similar accuracy excepting the load interval $0.50 \leq \tilde{F} < 0.70$ where predictions by the ACI and EC2 methods were more accurate in respect to ATENA. In fact, the above load interval covers the service load. Accurate prediction of deflections at service loads has been of prime importance to the design code methods.

In a combined estimation of standard deviation and mean, for lightly reinforced members ATENA has produced the most accurate estimates in terms of coefficient of variation.

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