A Nouvelle Approach for Predicting The Shear Cracking Angle in RC and PC Beams Using Artificial Neural Networks

J. Hegger, Professor and Head, Institute of Structural Concrete, Technical University of Aachen, Germany

(heg@imb.rwth-aachen.de)

A. Sherif, Associate Professor, Civil Engineering Department, Helwan University-Mataria, Cairo, Egypt

(ecogim@idsc.net.eg)

M. M. Reda Taha, Assistant Professor, Department of Civil Engineering, University of New Mexico, NM, USA

(mrtaha@unm.edu)

S. Görtz, Zerna, Köpper & Partner Consulting Engineers, Bochum, Germany

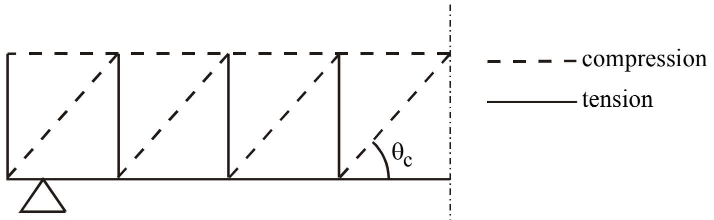
(sg@zkp.de)

Abstract

The truss model for predicting shear resistance of reinforced concrete beams has usually been criticized because of its underestimation of the concrete shear strength especially for beams with low shear reinforcement. Two challenges are commonly encountered in any truss model and are responsible for its inaccurate shear strength prediction. First: the cracking angle is usually assumed empirically and second the shear contribution of the arching action is usually neglected. This research introduces a nouvelle approach, by using Artificial Neural Network (ANN) for accurately evaluating the shear cracking angle of reinforced and prestressed concrete beams. The model inputs include the beam geometry, concrete strength, the shear reinforcement ratio and the prestressing stress if any. The model is trained using an extensive shear-testing database. The ANN model is then tested using parts of the database that were not used for training. The model prediction is compared to the empirical approach used by the DIN 1045-1 for predicting the shear cracking angle in addition to an own developed semi-empirical approach. It is concluded that a significant enhancement in the accuracy of predicting shear cracking angle of RC and PC beams can be achieved using the ANN model.

1 Introduction

The nonlinear behaviour of concrete is attributed to cracking. While the flexural behaviour of concrete elements have been well understood, the shear force transfer is far from being well explained. This is because of the fact that the presence of shear forces adds to the complexity of the problem in such a way, that until now no unified theory exists which is capable of fully describing the behavior of reinforced concrete elements subjected to shear. However, it is generally agreed that the shear capacity of a reinforced concrete beam may be divided into three components (Bhide et al. 1987, MacGregor 1997, Nawy 2002): a component resisted by the web V_{web}, a component resisted by the uncracked concrete compression zone $V_{concrete}$, and a component resisted by the dowel action of the tensile reinforcement V_{dowel}. For beams containing shear reinforcement (e.g. stirrups), V_{web} usually dominates and the typical crack pattern lead to the idealized truss model shown below. The idealized truss model consists of inclined concrete compressive struts and vertical tensile members representing the shear reinforcement. The shear capacity of this model is governed by the capacity of the tension ties (shear reinforcement) V_s and the capacity of the concrete compressive strut V_c:

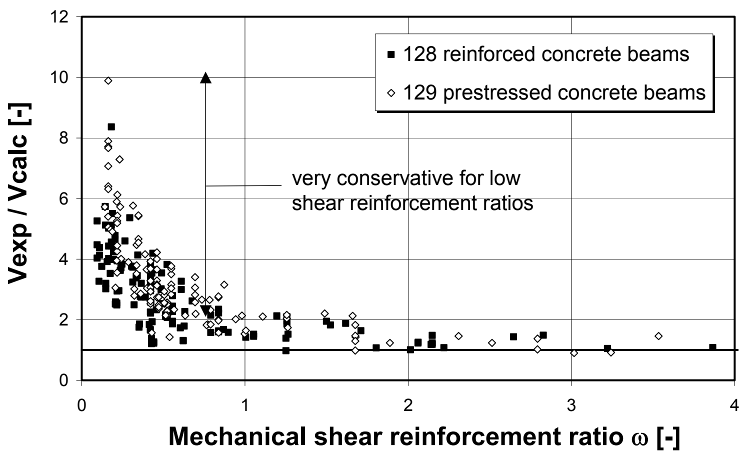


Idealized truss model for concrete

$$V_s = \rho_v f_y b_w z \cot\theta_c \tag{1}$$

$$V_c = \alpha_c f'_c b_w z \sin\theta_c \cos\theta_c \tag{2}$$

In this equation ρ_v is the geometrical shear reinforcement ratio, f_y is the yield strength of the reinforcement, b_w is the width of the section (web thickness in case of a profiled section), z is the internal lever arm, α_c is a reduction factor to account for the effect of transverse tensile trains on the compressive strength of the concrete which may be taken as 0.75 according to the German DIN 1045-1 Code (2001), f_c' is the concrete cylinder compressive strength, and θ_c is the angle of inclination of the concrete compressive strut and was originally assumed to be 45° (Mörsch 1908). Although a 45° truss model is easy to use in design, the model underestimates the shear capacity of reinforced concrete beams, especially for beams with a low shear reinforcement ratio, as shown below.



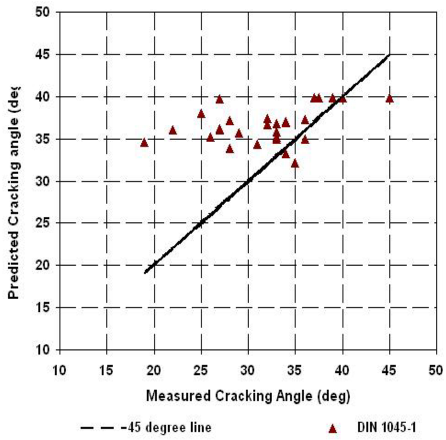
Experimental versus predicted shear capacity using a 45° truss model for concrete

The shear capacity of the truss model may be increased either by adding an empirical component called the concrete contribution to the capacity of the 45° truss model (Hegger et al. 2002), or by using a smaller angle θ_c for the compressive strut (e.g. Vecchio et al. (1986), Kuchma et al. (2000) and Reineck (2001)). The reduction of θ_c is explained in these models by shear friction and aggregate interlock across the cracks. However, the phenomenon of shear friction across the cracks was investigated and critizised by Hegger et al. (2004).

Despite the importance of the shear cracking angle in determining the shear capacity, only a few expressions have been developed to determine the cracking angle, and most of them are empirical and based purely on uncracked conditions. An example is the expression used by the DIN 1045-1:

$$\cot\beta_{cr} = 1.2 - 1.4\left(\frac{f_x}{f_c}\right) \tag{3}$$

Where β_{cr} is the cracking angle, f_x is the average normal stress along the cross section and f_c is the compressive strenght of concrete. The expression does not yield consistent prediction of the cracking angle as shown below and thereby overestimate the shear capacity of beams.



Experimental versus predicted shear cracking angle according to DIN 1045-1

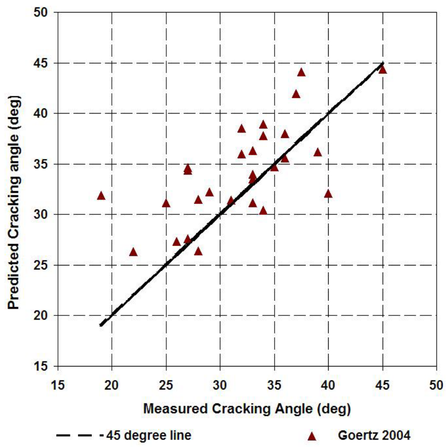
Investigations by Hegger et al. (2004) show that the cracking angle is influenced by both, the cracked as well as the uncracked parts. Görtz (2004) proposed the following half-empirical equation for the cracking angle β_{cr} as in equation (4) in which ω is the mechanical shear reinforcement ratio computed as by equation (5):

$$\cot\beta_{cr} = 1 + \left(\frac{0.015}{\omega}\right) - 0.18\left(\frac{f_x}{f_{ct}}\right) \qquad \& \quad \beta_{cr} \geq 25\,\mathrm{deg} \tag{4}$$

$$\omega = \frac{\rho\, f_y}{f_c^{'}} \tag{5}$$

f_{ct} is the tensile strength of the concrete as predicted using CEB-FIP model code 90 (1993):

$$\mathrm{f_{ct}} = 1.4\left(\frac{\mathrm{f_c^{'}}}{10}\right)^{2/3} \tag{6}$$



Experimental versus predicted shear cracking angle according to Görtz (2004)

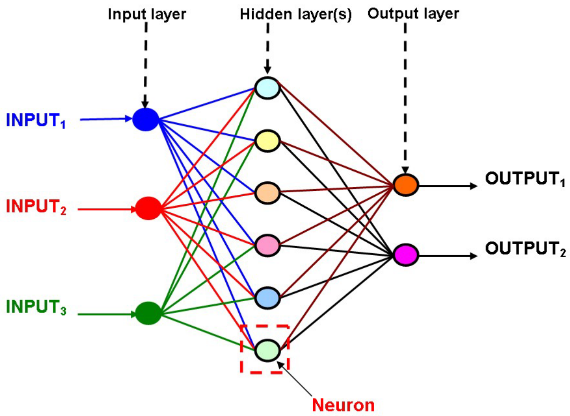
The predicted shear cracking angles by Görtz (2004) showed much better agreement with measured cracking angles compared with angles predicted by DIN 1045-1 as shown above.

2 Artificial Neural Networks

Artificial neural networks (ANN) are an artificial intelligence tool that were first proposed more than two decades ago for modeling systems that have complex nonlinear input/output relationships. Neuron computing, a technology of ANN, is a powerful tool for solving nonlinear problems that involve mapping input data to nonlinear output data without having any prior knowledge about the mathematical process involved. ANN have been succesfully used in many engineering applications including image processing, water quality predictions, satellite mapping problems (e.g. Kulakarni et al. 1994, Cheang et al. 2003) and most recently in predicting time-dependent deformations of structures (Reda Taha et al. 2003).

2.1 Structure of ANN

ANN are networks of many simple processors (neurons) operating in parallel, each possibly having a small amount of local memory. ANN consist of densly interconnected processing units that utilize parallel computation algorithms. The network consists of an input layer, an output layer and number of layers between the input and the output layers known as hidden layers. A representative sample of ANN architecture is shown below consisting of an input layer with three input parameters, an output layer with two output parameters and a single hidden layer with six neurons. The basic advantage of ANN is that they can learn from representative data examples (Haykin 1999). While ANN do not provide a closed form mathematical model, they do offer accurate models based on the learning procedure.



Sample representation of artificial neural network (Reda Taha et al. 2003)

The smallest network unit (the neuron) receives its input through a connection that multiplies the value of the input by a scalar weight "W" and adds a bias "b". The sum of the weighted inputs and their weights and biases is the argument for a transfer function "f" that produces the neuron output. The pattern of connectivity in the network is represented by a weight vector W. The initial values for the weights and biases of the network can be arbitrary chosen. By adjusting the weights (W) and the biases (b) the network can exhibit any desired output. The process of adjusting the weights and the biases of the network is known as training. In other words, an ANN learns from examples (of known input/output sequences) and exhibits some capability for generalization beyond the training data (Haykin 1999). Transfer functions for the neurons are needed to introduce non-linearity into the network. Transfer functions commonly used in feedforward neural networks include linear, log-sigmoid and tan-sigmoid transfer functions (Jang et al. 1997). These transfer functions have outputs ranging between 0 and 1 and are suitable for backpropagation networks because they are differentiable (Demuth et al. 2001).

2.1.1 Backpropagation algorithm

The learning rule (or training algorithm) is a procedure for modifying the weights and biases of the network. Learning rules fall into two broad categories: supervised learning and unsupervised learning. In supervised learning, the learning rule is provided with a known input/output set of data and an algorithm is then used to adjust the weights and biases of the network in order to move the network outputs closer to the targets. In unsupervised learning the weights and biases of the network are modified according to the inputs only.

The basic learning rule in feedforward networks is the gradient descent method which is a classic technique for minimizing a given function defined on a multidimensional input space. The gradient descent method requires finding a gradient vector "g" in which each element is defined as the derivative of an error measure with respect to a network parameter. The procedure for finding this gradient vector "g" is known as "backpropagation" because the gradient vector is calculated in a direction opposite to the flow of data in the network (Jang et al. 1997). The simplest and most common approach for implementing the backpropagation criterion to update the network weights and biases is described as:

$$X_{k+1} = X_k - \mu_k g_k \tag{7}$$

where X is the vector of different weights and biases utilized inside the network, μ_k is a small positive constant that controls the step size of the iterative changes during the learning process and known as the learning coefficient and g_k is the gradient of the mean square estimation error. The task of the backpropagation algorithm is to minimize the overall error measure of the network so that the network prediction matches the desired output. The sum of the squared errors (E) is the most commonly used error measure.

$$E = \sum_{i=1}^{N} (d_i - y_i)^2 \tag{8}$$

where y_i is the i^{th} element in the network output vector and d_i is the i^{th} element in the desired output vector and N is the total number of outputs predicted by the network. The error measure E is minimized by altering the network weights (W) and biases (b) so that the desired output is achieved by the network. In depth discussions about the mathematical bases of the backpropagation algorithm is beyond the scope of this work but can be found in almost all neural network textbooks (Haykin et al. 1999, Ross 2004).

3 Artificial Neural Network for predicting the shear cracking angle

3.1 ANN models used

Four feedforward artificial neural networks (ANN) for modelling shear cracking angle of reinforced and prestressed concrete beams are developed. The four networks consist of N layers each includes R neurons. Five input parameters for the first two networks (ANN_I and ANN_II) include the shear span ratio (a/d), the mean axial stress (f_x), the shear reinforcement ratio (ρ_v), the yield stress of the steel (f_y) and the tensile strength of concrete (f_{ct}) as per equation (4).

The input parameters for the second two networks (ANN_III and ANN_IV) are reduced to four parameters only by excluding the (a/d) ratio which proved not to have any significant effect on the results. Thus, the input layer for the first two networks includs five neurons, while the input layer for the second two networks includs only four neurons. The efficiency of linear, tan-sigmoid and log-sigmoid transfer functions are also examined. The number of layers, the number of neurouns in each layer and the transfer function in each layer are listed in Table 1.

Table 1: Structure of ANN used to predict shear cracking angle

Network	N	R1	R2	R3	R4	F1	F2	F3	F4
ANN_I	3	5	10	1	--	L	L	L	--
ANN_II	4	5	10	2	1	L	T	L	L
ANN_III	4	4	6	6	1	L	L	L	L
ANN_IV	4	4	6	8	1	L	T	G	L

***** *N (Number of layers),* R_i *(Number of neurons in layer i)*
***** *L (Linear), T (Tan-sigmoid), G (Log-sigmoid) transfer functions***

The four networks utilize the backpropagation training algorithm as the learning rule for the network with the Levenberg-Marquardt weights update criterion (Haykin 1999). This criterion is based on the gradient descent method with a small modification that speeds up the training procedure minimizing the mean square estimation error (Werbos 1990, Ooyen et al. 1992). The training is implemented using the Neural Network Toolbox of MATLAB® (Demuth et al. 2003).

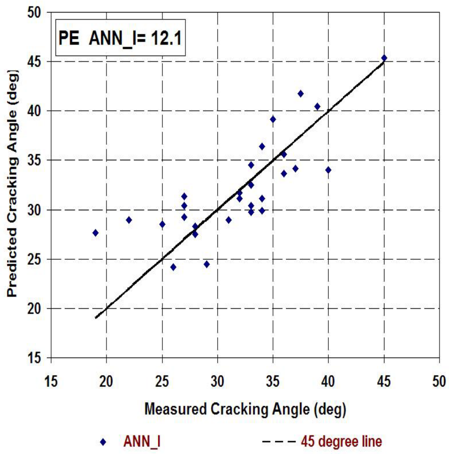
3.2 Training ANN model for predicting cracking angles

A learning matrix including 92 training samples drawn from the shear database is used in training each network. A target mean square error (MSE) of $1 * 10^{-4}$ was set to the network. The input parameters are normalized to allow fast convergence to the target MSE. The structure of the network and the choice of the transfer functions clearly affect the number of iterations needed during the training procedure for the network to converge. Convergence is assumed if the network achieves the target MSE or reaches the maximum number of iterations chosen equal to 1000.

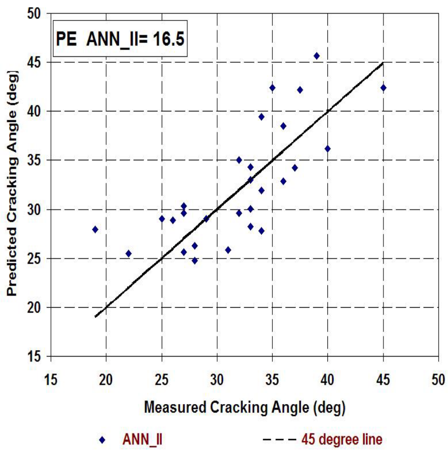
4 Results and Discussions

4.1 Testing ANN models for predicting shear cracking angles

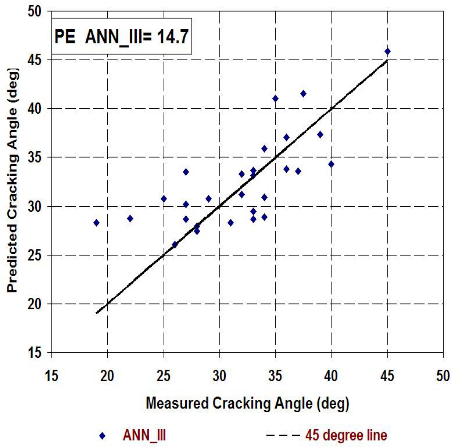
Each of the four networks is tested using a matrix of 28 data points never obsered during training of the networks. The shear cracking angle predicted by the networks is compared to the experimentally measured cracking angle and the semi-empirical equation that was early proposed by Görtz (2004). The predicted versus the measured cracking angles for the four ANN and the empirical formula are shown below.



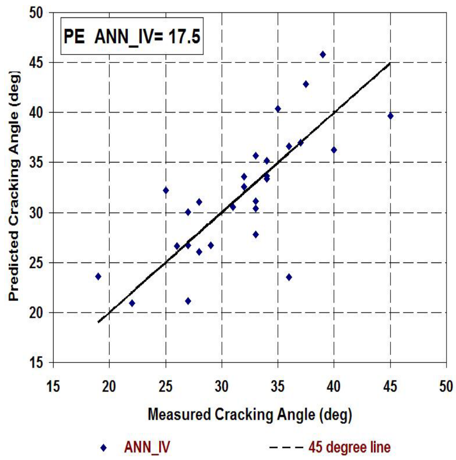
Experimental vs. predicted shear cracking angle ANN_I



Experimental vs. predicted shear cracking angle ANN_II



Experimental vs. predicted shear cracking angle ANN_III



Experimental vs. predicted shear cracking angle ANN_IV

The large number of parameters affecting ANN predictions and the large number of data points used for comparison between predicted and experimentally measured cracking angle, necessitate statistical analysis to provide means of cross validation. Statistical comparisons between predicted and measured cracking angle have been performed by estimating the prediction error (PE) which measures the average squared error between the predicted cracking angle obtained from the models and the experimentally measured cracking angle. The PE is described according to Martinez et al. (2002) as:

$$PE = \frac{1}{m}\sum_{i=1}^{m}(y_{ti} - y_{pi})^2 \qquad (9)$$

where y_{pi} is the predicted value and y_{ti} is the experimentally measured value and m represents the number of samples in each testing group. Prediction errors for the four ANN models and the formulae by DIN 1045-1 and Görtz (2004) are presented in Table 2.

Table 2: Prediction error for the four ANN models as well as for the emperical formulae

Prediction Model	PE
DIN 1045-1 (2001)	47.9
Görtz (2004)	22.6
ANN_I	12.1
ANN_II	16.5
ANN_III	14.7
ANN_IV	17.5

It is obvious from Table 2 that the DIN 1045-1 has the highest prediction error. The expression developed by Görtz (2004) reduces the prediction error significantly. However, the cracking angle predictions using ANN have a smaller prediction error and consequently higher accuracy (e.g. ANN_I and ANN_III) than the emperical formulae. An interesting observation is that the accuracy of the ANN model does not necessarily increase with the complexity of the network. On the contrary, simple networks with one or two hidden layers that utilize linear transfer functions were capable of producing less prediction errors than networks with larger number of hidden layers and utilizing non-linear transfer functions.

Another interesting observation is that neglecting the shear span ratio (a/d) in the model did not detract from the ANN model ability to predict shear cracking angle. An average prediction error of ANN models ANN_III and ANN_IV of (16.1) is very close to the average prediction error of ANN models ANN_I and ANN_II (14.3) in which the (a/d) ratio was considered.

5 Summary and Conclusions

Four Artificial Neural Networks (ANN) models have been developed to investigate the potential use of feedforward neural networks in predicting shear cracking angle in reinforced and prestressed concrete beams. The ANN models require five input parameters which are the shear span ratio (a/d), the axial stress (f_x), the shear reinforcement ratio (ρ_v), the yield stress of the steel (f_y) and the tensile strength of concrete (f_{ct}). The four models were trained and tested using an experimental shear test database. The ANN models developed show a good ability to predict the shear cracking angle with a reasonable level of accuracy. The following are the main conclusions:

1. The accuracy of the ANN model does not necessarily increase with the increase in complexity of the network. On the contrary, simple networks with one or two hidden layers that utilize linear transfer functions were capable of producing less prediction errors than networks with larger number of hidden layers and utilizing non-linear transfer functions.

2. The prediction error of the ANN models developed was not greatly affected by the shear span ratio (a/d). This indicates that the shear cracking angle is not significantly influenced by the shear span ratio of the beam.

3. By comparing the results of the ANN models with empirical and semi-empirical expressions for determining the shear cracking angle it is concluded that ANN models yield lower PE values than those attained by emperical formulae and thus can result in high accuracy in predicting shear capacity of reinforeced and prestressed concrete beams.

6 Acknowledgment

The authors gratefully acknowledge the funding provided by the Arbeitgemeinschaft industrieller Forschungsvereinigungen, Germany, the Deutscher Beton- und Bautechnik Verein e.V, Germany and The University of New Mexico, NM, USA. Part of this work was performed during a research visit of A. Sherif to the RWTH Aachen financed by the Alexander von Humboldt Stiftung, Germany. This support is deeply appreciated.

7 References

Bhide, S.B., Collins, M.P. 1987. Reinforced Concrete Elements in Shear and Tension.

CEP-FIP Model Code 90, 1993. Model code for concrete structures. Comité Euro-International du Beton (CEB) – Fédération Internationale de la Précontrainte (FIP), Thomas Telford Ltd., London, UK

Chiang K-W, Noureldin A., and El-Sheimy N. 2003. Multi-sensors Integration using Neuron Computing for Land Vehicle Navigation. GPS Solutions, Springer-Verlag Heidelberg, V6 (4), pp. 209 - 218.

Demuth H. and Beale, M. 2001. Neural network toolbox for use with MATLAB®. The Mathworks Inc., MA, USA

DIN 1045-1. 2001. Tragwerke aus Beton, Stahlbeton und Spannbeton, Teil 1: Bemessung und Konstruktion. Normenausschuß Bauwesen (NABau) im DIN Deutsches Institut für Normung e.V., Berlin, July

Görtz, S. 2004. Shear Cracking Behavior of Reinforced and Prestressed Beams made of Normal- and High Performance Concrete. Ph.D. Thesis submitted to the Faculty of Engineering, The Technical University of Aachen, Germany

Haykin, S. 1999. Neural networks: A comprehensive foundation. Prentice Hall Inc., Englewood Cliffs, N.J., USA.

Hegger, J., Görtz, S. 2000. Shear Cracking Behaviour of Prestressed Beams Made of High-Strength Concrete. PCI/FHWA/FIB International Symposium on High Performance Concrete, Orlando, Florida, 25-27 September

Hegger, J., Sherif, A, and Görtz, S. 2004. Analysis of Pre- and Post-cracking Shear Behavior of Prestressed Concrete Beams Using Innovative Measuring Techniques. ACI Structural Journal, Vol. 101, No. 2, pp. 183-192

Hegger, J., Görtz, S., Kommer, B., and Sherif, A. 2002. Load-Bearing Capacity of Prestressed Elements made out of High Performance Concrete (HPC). 6th International Symposium on Utilization of high Strength / High Performance Concrete, Leipzig, Germany, pp. 277-287

Jang, J., -S., R., Sun, C., -T. and Mizutani, E. 1997. Neuro-Fuzzy and soft computing, A computational approach to learning and machine intelligence Prentice Hall Inc., Englewood Cliffs, N.J., USA.

Kulkarni, A. D. 1994. Artificial neural networks for image understanding, Van Nostrand Reinhold, NY, USA.

Kuchma, D.A.; Bentz, E.C. 2000. Shear Capacity and Design Strength of Reinforced and Prestressed Members Cast with High Performance Concrete. PCI/FHWA/FIB International Symposium on High Performance Concrete, 25-27 September, pp. 626-634

MacGregor J. G. 1997. Reinforced Concrete: Mechanics and Design, 3rd edition, Prentice Hall, NY, USA.

Martinez, W. L. and Martinez, A. R. Computational statistics handbook with MATLAB®. Chapman & Hall/CRC Press, NY, USA, 2002.

Mörsch, E. 1908. Der Eisenbetonbau – Seine Theorie und Anwendung, Berlin

Nawy, E. G., 2002, Reinforced Concrete: A Fundamental Approach, 5th Edition, Prenitce Hall, NY, USA.

Ooyen, A. V. and Nienhuis, B. Improving the convergence of the back-propagation algorithm, Neural Networks, 1992, 5, 465-471.

Reda Taha, M.M., Noureldin, A., El-Sheimy, N. and Shrive, N. G. 2003. Artificial Neural Networks to Predict Creep With an Example Application to Structural Masonry. Canadian Journal of Civil Engineering , Vol. 30, N0. 3, pp. 523-532.

Reineck, K.-H. 2001. Hintergründe zur Querkraftbemessung in DIN 1045-1 für Bauteile aus Konstruktionsbeton mit Querkraftbewehrung. Bauingenieur, 76, April, pp. 168-179

Ross, T., J., 2004. Fuzzy logic with Engineering Applications, 2nd Edition, J. Wiley, UK.

Vecchio, J.V., Collins, M.P. 1986. The Modified Compression-Field Theory for Reinforced Concrete Elements Subjected to Shear. ACI Journal, 83(2), March-April, pp. 219-231

Werbos, P. J. 1990. Back propagation through time: what does and how to do it. Proceedings of the IEEE, Vol 78, pp. 1550-1560.

Zink, M. 1999. Zum Biegeschubversagen schlanker Bauteile aus Hochleistungsbeton mit und ohne Vorspannung. Dissertation, University of Leipzig.